Macro Candidacy Exam: January, 2014

Directions. There are five (5) questions worth a total of 180 possible points. The points assigned to each question and each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

(40) 1. An overlapping generations model with capital and money

Consider a discrete-time infinite horizon overlapping generations model with one good per date and one person per generation. Agents live for two periods, young and old. The lifetime utility of a person is $u(c_t^y) + \beta u(c_t^o)$, where c_t^y denotes consumption when young of generation t and c_t^o is consumption of generation t when old. Assume that $u(x) = \ln x$ and that $\beta > 0$. The economy starts at date 1 at which time the old member of generation 0 is endowed with $k_1 > 0$ amount of capital and $M_1 > 0$ amount of money. Each member of generation t for $t \geq 1$ is endowed with one unit of labor when young and nothing when old. Output at date-t is $l_t^{1-\alpha} k_t^{\alpha}$, where l_t is labor employed and k_t is capital employed and $\alpha \in (0, 1)$. Capital fully depreciates after use so that capital at date $t+1$ is output at date t minus total consumption at date t. Agents behave competitively in *all* markets.

The stock of money grows at the rate σ and additions to the stock are distributed to individuals proportionally to their holdings.

(5) (a) Find the constant amount of capital that maximizes total consumption. Call it k^*.

(15) (b) Show that an equilibrium in which money has no value gives rise to a linear first-order difference equation in $\ln k_t$ and that the solution converges to a steady state, denoted $\bar{k}$. Give a sufficient condition in terms of the parameters for this equilibrium to be inefficient.

(20) (c) Give a sufficient condition for existence of a steady state with valued money. Describe this valued-money steady state including the way it depends on σ.

(40) 2. A 'two-sector' stochastic growth model

Time is discrete and there is one good per date. Total output at date t, denoted y_t, is given by

$$y_t = z_t^1 f(k_t^1, l_t^1) + z_t^2 f(k_t^2, l_t^2)$$

where $z_t = (z_t^1, z_t^2)$ follows a first-order Markov process; f is homogeneous of degree one, strictly quasi-concave, and satisfies the usual Inada conditions; and

$$l_t^1 + l_t^2 \leq 1 \text{ and } k_t^1 + k_t^2 \leq k_t,$$

where k_t is the stock of capital at the beginning of date t, l_t^i is labor used in sector i and k_t^i is capital used in sector i. The representative agent maximizes expected discounted utility of consumption with period utility function $u : R_+ \to R$ that is strictly increasing and strictly concave and with discount factor $\beta \in (0, 1)$. Also,

$$k_{t+1} \leq y_t - c_t + (1 - \delta)k_t,$$

where c_t is consumption at t and where $\delta \in (0, 1)$. There are two versions of this economy concerning when decisions have to be made.

Version A: the allocation of capital and labor between the sectors at date t must be made before observing z_t, while the decision about current consumption is made after observing z_t.

Version B: identical to version A except that the allocation of labor is made after observing z_t.

(15) (a) Formulate each version as a dynamic programming problem.

(10) (b) In which version is welfare higher? Explain.

(5) (c) Provide a sufficient condition that implies that both sectors have strictly positive inputs at every date.

(5) (d) Suppose that an observer sees output and inputs for each sector and knows the function f. Would the observer see marginal products equated across the sectors? Explain.

(5) (e) Recently, a lot of attention has been given to the possibility that the *degree of uncertainty* is not constant. Does the above formulation allow for that possibility? Explain.

(40) 3. Equilibrium unemployment and training

Time is discrete and there is a nonatomic unit measure of infinitely-lived people, each of whom maximizes expected discounted income with a discount factor given by $\beta \in (0,1)$. The state of a person has two parts: one part is productivity, denoted ω, where either $\omega = \omega_L$ or $\omega = \omega_H$ and where $\omega_H > \omega_L > 0$; the other part is employment status, either employed or unemployed.

If employed, a person earns a wage equal to his/her productivity ω. With probability δ the job is destroyed and the person starts the next period in the unemployment state with productivity ω. In the unemployment state, a person with productivity ω receives goods from home production of $b\omega$, where $b \in [0,1]$. An unemployed person chooses whether to obtain training: if training is chosen, the cost $p\omega > 0$ is borne and the person starts the next period with productivity ω_H; otherwise, no training cost is incurred and productivity remains ω. Whether or not training is chosen and with probability γ, an unemployed person who begins with $\omega = \omega_H$ starts the next date with $\omega = \omega_L$. The probability of an unemployed person becoming employed is α.

(The parameters are β, ω_H, ω_L, δ, b, γ, and α.)

(10) (a) Set out the functional equation(s) for this model.

(10) (b) Find the steady-state fraction who are employed. Find the steady-state distribution of ω under the assumption that unemployed workers with $\omega = \omega_L$ train. Find the steady state distribution of ω under the assumption that unemployed workers do not train.

(10) (c) Suppose $\gamma = 0$. Provide a necessary and sufficient condition for unemployed workers to choose not to train. (Hint: For this case, conjecture that $v(\omega, i) = \omega h(i)$ for $i \in \{employed, unemployed\}$, where $v(\omega, i)$ is the value of starting a date in state (ω, i).)

(10) (d) Assume that the parameters are such that unemployed workers with low human capital choose not to train. How would you amend the model to study the effects of a government subsidy on training, while taking into account that any subsidy has to be financed? Would such a policy be desirable? (Hint: Do not attempt to work out all the details.)

(30) 4. Optimal monetary policy with commitment

We consider optimal monetary policy in a two-sector closed-economy model (sectors 1 and 2). The central bank's objective is to minimize the loss function

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{\phi_1}{2} \pi_{1,t+j}^2 + \frac{\phi_2}{2} \pi_{2,t+j}^2 + x_{t+j}^2 + \phi_P \left(z_{t+j} - z_{t+j}^E \right)^2 \right]$$

subject to

$$\pi_{1,t} = \beta E_t \left[\pi_{1,t+1} \right] + \kappa_1 x_t - \frac{\kappa_1}{2} \left(z_t - z_t^E \right),$$

$$\pi_{2,t} = \beta E_t \left[\pi_{2,t+1} \right] + \kappa_2 x_t + \frac{\kappa_2}{2} \left(z_t - z_t^E \right),$$

and

$$z_t = z_{t-1} + \pi_{1,t} - \pi_{2,t},$$

where E_t is the conditional expectation operator, x_t is the central bank's instrument,and β, ϕ_1, ϕ_2, ϕ_P, κ_1, and κ_2 are parameters. The random variable z_t^E is *i.i.d.* over time and has unit variance. The central bank chooses x_t after the shock z_t^E is realized and observed.

Suppose that the central bank can commit at date 0 to a contingent path for x_t.

(10) (a) Characterize, as completely as you can, the solution to the above problem.

(10) (b) Is the solution in (a) time-inconsistent? Explain.

(10) (c) Consider a special case of the model with $\kappa_1 = \kappa_2 = \kappa$ and $\phi_1 = \phi_2 = \phi$. In this case, is the solution in (a) time-inconsistent?

(30) 5. Money in a model with idiosyncratic preference shocks

There is a nonatomic unit measure of people and one good per discrete date. Each person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$ and date t period utility $z_t u(c_t)$. Here, $u : \mathbb{R}_+ \to \mathbb{R}$ and is bounded, strictly increasing, strictly concave, and satisfies $u'(0) = \infty$, while $z_t \in \{z^1, z^2\}$, $0 < z^1 < z^2$, and $z_t = z^i$ with probability $\pi^i \in (0, 1)$ and $\pi^1 + \pi^2 = 1$. Realizations of z_t are i.i.d. over time for a person and i.i.d. across people. Each person's endowment of date t good is $y > 0$, a constant. There is no aggregate risk. There is a fixed and unchanging stock of money, the percapita quantity of which is normalized to be unity.

At date t and after seeing the realization of z_t, each person chooses (m_{t+1}, c_t) subject to

$$c_t + m_{t+1} p_t \leq m_t p_t + y \text{ and } m_{t+1} \geq 0,$$

where m_t is the quantity of money held by the person at the start of date t and where the person treats p_t as unaffected by the person's choice. That is, the only market is a spot price-taking market at each date in which money is traded for the good.

(25) (a) Define a recursive equilibrium and a steady state.

(5) (b) Without attempting to prove anything, consider and discuss the following conjecture: there is a steady state with valued money.