THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Fall 2014

Candidacy Examination for

The Degree of Doctor of Philosophy

MACROECONOMIC THEORY

Please read the instructions carefully. You have 3-1/2 hours to complete the exam.

GOOD LUCK!

Macro Candidacy Exam: August, 2014

Directions. There are five (5) questions worth a total of 180 possible points. The points assigned to each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. A growth model with an external habit

Consider the one-sector non-stochastic growth model with a unit measure of identical agents. Each agent is endowed with one unit of labor per period and the same initial wealth in the form of capital. The period utility of an agent is $u(C, c)$, where C is the agent's consumption and c is *the average level of consumption in the previous period.* Assume that $u(\cdot, c)$ is strictly increasing, strictly concave, and twice differentiable. Also assume that the cross partial, $u_{Cc}(C, c)$, exists and is different from zero. Each agent maximizes discounted utility with discount factor $\beta \in (0, 1)$.

Each agent also has access to a constant returns-to-scale production function with capital and labor as inputs. Output per unit of labor net of depreciation is given by $f(k)$, where k is capital per unit of labor. Assume that $f(0) = 0$ and that $f(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable.

(10) (a) Suppose that each agent makes decisions taking the path of c as given. Define a recursive equilibrium. In doing so, be clear about what is in the state vector.

(15) (b) Suppose the representative agent believes that the average capital stock, k, evolves according to $k' = h(k, c)$ and that average consumption evolves according to $c' = z(k, c)$. Show that in a recursive competitive equilibrium these two functions **cannot be** independent of c. Explain why c is in the state space even though it has no affect on the technology.

(15) (c) Assume that there is a recursive equilibrium that satisfies your definition in part (a). Is the implied allocation the same as that implied by a solution to the planner's problem for that economy? Defend your answer.

2. An OLG model with money and storage

Consider an infinite-horizon, discrete-time overlapping generations model with one consumption good per date and one representative agent in each generation. Agents live two periods, working in youth and consuming only when old. Let $u(c_{t+1}) - g(n_t)$ be the preferences over consumption (c_{t+1}) and work (n_t) for the representative generation t agent. The function $u(\cdot)$ is strictly increasing and concave and $g(\cdot)$ is strictly increasing and strictly convex. The representative agent has a unit of time in youth to allocate between work and leisure and the production function is given by $y = n$, where y is output.

There are two ways to save: through money and physical storage of output, both of which are non-negative. Storage is stochastic. If k amount of the good is put into storage in period t, the result is $R_t k$ amount of the good at $t+1$, where $R_t \in \{r_1, r_2\}$, $R_t = r_i$ with probability $\pi_i > 0$, and $0 < r_1 < 1 < r_2$. The return on storage is *i.i.d.* over time. Agents in generation t know the return on storage, R_t, before making their decisions in period t. There is a fixed stock of money owned initially by the old. Markets for the exchange of the good for money operate each period and agents behave competitively.

The following questions are about a stationary rational expectations price-taking equilibrium (SREE) in which the state of the economy at date t is the realization R_t.

(10) (a) Define a SREE.

(20) (b) Suppose that $u(c) = c$ and that $g(n) = \frac{\theta}{2}n^2$, where $\theta > 0$. Show by construction that there are parameters for which there exists a SREE in which money has value.

3. Risk sharing without commitment

There are two infinitely-lived ex ante identical agents, $i = 1, 2$, and a single nonstorable consumption good each discrete date. Agents receive endowments of the consumption good stochastically: if one agent has endowment $1+\varepsilon$, the other agent has $1-\varepsilon$, where $\varepsilon \in [0, 1)$. Let $s_t \in S = \{1, 2\}$ denotes the agent who has endowment $1+\varepsilon$ at date t. Assume that $\{s_t\}_{t=0}^{\infty}$ is a sequence of *i.i.d.* random variables with $Prob\{s_t = 1\} = Prob\{s_t = 2\} = 1/2$. Each agent maximizes lifetime expected discounted utility with discount factor $\beta \in (0, 1)$ and period utility function u. Assume that u is twice continuously differentiable and satisfies $u'' < 0 < u'$ on $(0, \infty)$ and $u'(c) \to \infty$ as $c \to 0$.

Any risk-sharing arrangement is subject to reneging by either agent at any time and the consequence of any reneging is permanent autarky.

(10) (a) Formulate recursively the implied problem of optimal risk-sharing.

For parts (b) and (c), assume, as can be shown, that the solution to the part (a) problem has the following form: at every date, the agent with high endowment consumes $1+x$ and the agent with low endowment consumes $1-x$, with $x \in [0, \varepsilon]$ and where x depends on u, β, and ε.

(15) (b) For given u and β, find ε_1 such that the optimal solution is $x = \varepsilon$ (autarky) for all $\varepsilon \in [0, \varepsilon_1]$.

(10) (c) For given u and β, find ε_2 such that the optimal solution is $x = 0$ (first-best) for all $\varepsilon \in [\varepsilon_2, 1)$.

4. Optimal monetary policy with and without commitment when the price level and the wage are sticky

The central bank's problem is to minimize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\phi_1 (\pi_{t+j})^2 + \phi_2 \left(\pi_{t+j}^w\right)^2 + (x_{t+j})^2\right]$$

subject to

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa_1 x_t + \lambda_1 w_t,$$

$$\pi_t^w = \beta E_t \left[\pi_{t+1}^w\right] + \kappa_2 x_t - \lambda_2 w_t,$$

and

$$w_t = w_{t-1} + \pi_t - \pi_t^w - \varepsilon_t,$$

where ε_t is iid over time and has a unit variance, E_t is the conditional expectation operator, x_t is the central bank's instrument, and β, ϕ_1, ϕ_2, κ_1, κ_2, λ_1, and λ_2 are positive parameters. The central bank chooses x_t after the shock ε_t is realized and observed.

(20) (a) Suppose that the central bank can commit at date 0 to a contingent path for x_t. Is the solution under such commitment time-consistent? Defend your answer.

(20) (b) How would you go about finding a time-consistent solution to the above problem?

5. Capital accumulation with idiosyncratic and uninsurable endowments of labor

Consider a discrete-time economy with a nonatomic unit measure of infinitely-lived agents. Each agent maximizes discounted expected utility with period utility function $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma > 0$ and discount factor $\beta \in (0,1)$. Agents face idiosyncratic labor endowment shocks: agent i receives $l_{it} \in \{l^1, l^2, \ldots, l^N\}$ amount of labor at each date t, with $0 < l^1 < \ldots < l^N$. The shocks follow a Markov process with transition probabilities $\pi(l', l) = Pr\{l_{i(t+1)} = l' | l_{it} = l\}$ and $\pi(l', l) > 0$ for all (l', l). Shocks are *i.i.d.* across agents. Assume that the aggregate endowment of labor implied by the stationary distribution of π is 1 per capita. There is no insurance market, no borrowing, and agents can save only by accumulating capital.

The agents behave as price takers and rent all their labor and capital to price-taking firms who make use of a constant-returns-to-scale Cobb-Douglas production function, F. Also, capital depreciates at rate $\delta \in (0,1)$.

(10) (a) Define a steady state.

(15) (b) Outline the main ingredients you would use to prove that a steady state exists. (By *outline*, we mean that you should state the main steps you would use in a proof. You do not have time to provide detailed arguments that show that the required hypotheses hold.)

(10) (c) In a version without heterogeneous labor endowments, the per capita steady-state capital stock, k, satisfies $\partial F(k,1)/\partial k + (1-\delta) = 1/\beta$. Does that hold in the above model with heterogeneous labor endowments? Explain.