THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

January 2014

Written Portion of the Comprehensive Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer two questions from each section. You will not receive additional credit, and may receive less credit, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

1. Consider a two-agent, two-good exchange economy $\mathcal{E} = (u_i, \mathbf{w}_i)_{i=1}^2$ specified by the utility functions

$$
\begin{aligned}
u_1(\mathbf{x}_1) &= \min\{x_{11}, x_{12}\} \\
u_2(\mathbf{x}_2) &= \max\{x_{21}, x_{22}\}
\end{aligned}
$$

(x_{ik} denotes i's consumption of the good k) and endowments

$$
\begin{aligned}
\mathbf{w}_1 &= (1, 0) \\
\mathbf{w}_2 &= (0, 1)
\end{aligned}
$$

(a) Find the set of (weak) Pareto efficient allocations for $\mathcal{E}$.

(b) Argue that $\mathcal{E}$ does not have a Walrasian equilibrium.

(c) Let $\mathcal{E}^{(2)}$ denote the two-fold replica of the economy $\mathcal{E}$, that is, $\mathcal{E}^{(2)}$ is an economy in which there are two consumers with utility function u_1 and endowment $\mathbf{w}_1$ and two consumers with utility function u_2 and endowment $\mathbf{w}_2$. Show that $\mathcal{E}^{(2)}$ does have a Walrasian equilibrium.

(d) Let $\mathcal{E}^{(n)}$ denote the n-fold replica of $\mathcal{E}$, that is, $\mathcal{E}^{(n)}$ is an economy in which there are n consumers with utility function u_1 and endowment $\mathbf{w}_1$ and n consumers with utility function u_2 and endowment $\mathbf{w}_2$. Under what conditions does $\mathcal{E}^{(n)}$ have a Walrasian equilibrium?

(e) Define an ε-Walrasian equilibrium $(\mathbf{p}^*, \mathbf{x}^*)$ in the same way as a Walrasian equilibrium with one amendment: for all i, if $\mathbf{x}_i$ is an affordable consumption bundle then $u_i(\mathbf{x}_i) - u_i(\mathbf{x}_i^*) \leq \varepsilon$. Show that for all $\varepsilon > 0$, the economy $\mathcal{E}^{(n)}$ has an ε-Walrasian equilibrium if n is large enough.

Section I (cont.)

I.2 Suppose that a consumer has a regular (complete and transitive) preference relation $\succeq$ on $\mathbb{R}^L_+$ and and wealth $w > 0$ (w is a number). Let $p^a, p^b \in \mathbb{R}^L_+$, let x^a be a commodity bundle in the consumer's demand set at (p^a, w), and let x^b be in the consumer's demand set at (p^b, w).

a) Decompose the demand change $x^b - x^a$ into a *substitution effect* and an *income effect*. You may use either the Hicks definition or the Slutsky definition, but be sure to state which you are using.

b) Using additional assumptions if necessary, prove that the substitution effect has a nonpositive inner product with the price change $p^b - p^a$. State your assumptions clearly, and be as general as you can.

I.3 Consider a production economy with a single firm, $\mathcal{E} = \left((\succeq_i, \mathbf{w}_i, \theta_{i1})_{i=1}^I, Y_1 \right)$. For each consumer i, $\succeq_i$ is a regular (complete and transitive) preference relation on $\mathbb{R}^L_+$, the endowment bundle $\mathbf{w}_i \in \mathbb{R}^L_+$ and θ_{i1} is consumer i's share of the firm. The production set of the firm is the nonpositive orthant, $Y_1 = -\mathbb{R}^L_+$.

a) Define a *competitive quasi-equilibrium* for this economy.

b) Let $\left((\mathbf{x}_i^*)_{i=1}^I, \mathbf{y}_1^*\right)$ be a Pareto efficient allocation. Using additional assumptions if necessary, prove that there exists a nonzero and nonnegative price vector $\mathbf{p}^* \in \mathbb{R}^L_+ \backslash \{0\}$, such that $\left(\mathbf{p}^*, \left((\mathbf{x}_i^*)_{i=1}^I, \mathbf{y}_1^*\right)\right)$ is a competitive quasi-equilibrium. State your assumptions clearly, and be as general as you can.

c) Did you assume (or make other assumptions that imply) that at least one consumer has nondecreasing preferences? If your answer is "Yes," give an example with two commodities and one consumer (and $Y_1 = -\mathbb{R}^2_+$) showing that this property is essential.

d) Did you assume (or make other assumptions that imply) that at least one consumer is locally non satiated? If your answer is "Yes," give an example with two commodities and one consumer (and $Y_1 = -\mathbb{R}^2_+$) showing that this property is essential.

Section II

II.1(a) Consider the following game that is played T times. First, players move simultaneously and independently. Then each player is informed about the actions taken by the other player in the first play and, given this, they play it again and so on. The payoff for the whole game is the sum of the payoffs a player obtains in the T plays of the game.

	a	b	c
A	3,2	5,0	0,1
B	2,6	4,4	1,2
C	1,2	0,3	2,3

(a.i) How many subgames are there in the repeated game?

(a.ii) Find the smallest value of T such that it is possible for B and b to be played in the first play of the game, in a subgame perfect equilibrium. Describe the strategies that constitute such an equilibrium. Consider only pure strategies.

(b) Consider a seller S of identical durable goods with three potential buyers, H, M, L with values for the good v_H, v_M, v_L respectively. Suppose the buyers and seller have a discount factor δ. (Remember a durable good is a good that gives a utility payoff in each period once purchased; the value is the discounted sum of such utilities and we are in a quasi-linear world, so you can assume the v_i are in the same units as prices.)

The market opens at two dates, 1 and 2. At date 1, S announces a price p_1, following which H, M, L simultaneously and independently decide to buy or not. Everyone observes who remains in the market for date 2. Given this, S announces a price p_2 at date 2 and the buyers remaining then decide whether to buy or not. If buyer i buys in period 1 at a price of p, his payoff is $v_i - p$. If he buys in period 2, his payoff in the game is $\delta(v_i - p)$ since he misses one period's utility. The seller's payoff is given by $p_1 x_1 + \delta p_2 x_2$, where x_t is the number of items sold at date t.

(b.i) Assume that $v_H > v_M > v_L > 0$ and $3v_L > 2v_M > v_H$. What is the subgame perfect pricing policy for S? Remember you have to specify a price p_2 for every combination of buyers remaining in the market at date 2. (If you need to you can assume $v_H > 2v_L$.)

(b.ii) Now suppose S has only two items to sell. What is the new subgame perfect pricing policy? Is S better off with two items or three items?

Section II (cont.)

II.2 Consider the following game-a coordination game with private information:

	A_2	B_2
A_1	$1-c_1, 1-c_2$	$-c_1, 0$
B_1	$0, -c_2$	$0, 0.$

Suppose the costs $c_i \epsilon\ [0, 1+\varepsilon]$, are independent draws from a uniform distribution on $[0, 1+\varepsilon]$. Player i knows c_i but not the other player's cost. The probability distributions are commonly known.

Find a symmetric Bayes Nash equilibrium (remember a Bayes Nash equilibrium is an equilibrium in strategies mapping information to actions). Consider equilibria of the following kind: Play B if $c_i \geq x$, play A otherwise.

II.3(a) Consider two probability distributions F and G on [0,1] with the same expected value. Suppose the densities of these distributions $f(.)$ and $g(.)$ respectively are related as follows:

There exist values a, b such that $g(x) \geq f(x)$ for x$\leq$ a and x $\geq$ b, with $f(x) \geq g(x)$ on [a,b], with each inequality being strict on some positive sub-interval.

Can one identify which of these distributions any risk-averse decision-maker would prefer? Prove your answer if yes, explain if no.

(b) In the moral hazard problem we went through in class, show that the property MLRP (in this case $\frac{d}{dy}(\frac{f_a(y,a)}{f(y,a)})$ is increasing in y) implies first-order stochastic dominance, in this case, $F_a(y,a) \leq 0$ and strictly negative for some range of y. Assume the support of the density of y is $[0,1]$.