Economies of density and congestion in the sharing economy.*

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Abstract

The advancement of information technology in the developing world is side-passing moral hazard problems that prevent the creation of rental markets for durable goods. In an intend to reap the benefits from sharing capital services across small-scale producers, governments have increasingly subsidized these rental markets. While interventions focused mostly on equal access concerns, little is known about their efficiency as well as unintended consequences. This paper is the first one to study them. We focus on subsidies to the creation of rental markets for agricultural equipment. Using our own census of 40000 farmers in India, we document that small-scale producers are rationed out by market providers and that a government subsidized first-come-first-served dispatch system grants small-scale producersâ timely access to equipment. It also induces competition among providers, lowers rental costs, and access to equipment services for low-productivity producers. Some of these inframarginal producers locate further away from providers on average, inducing equipment transportation costs that may well overturn the productivity gains from equipment access. Through a structural model of search in rental services and optimal service dispatch, we find that, despite higher available service capacity, a market deregulation could induce higher productivity costs, i.e. longer delays due to congestion. Our findings imply that the government subsidies may have delayed scale consolidation and its efficiency gains.

JEL codes: O13, O18, O47, R41.

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1 Introduction

From courier services to machinery to car rides, the sharing economy is expanding to the developing world. Governments in the developing world have recently engaged in stimulating rental markets by either subsidizing new capital purchases, or by subsidizing set up costs to the creation of these markets through public/private partnerships. These interventions have distributional effects as well as efficiency effects that are not well understood. This paper is the first one to assess the impact of such interventions.

The development of rental markets for equipment is an important mechanism grant small scale producers access to capital and its technology. Indeed, the history of the path of mechanization for currently rich economies suggests that equipment rental markets were a stepping stone to that process (Binswanger, 1986). But where contract enforcement is weak and overall wealth is low, rental markets may not emerge, and if they do, they may take a form that rations out productive and/or smaller producers.

We focus on subsidies to the creation of rental markets for agricultural equipment, which have the potential to unravel productivity gains and employment reallocation away from agriculture, typical of the development process. Fairness concerns may lead to servicing farmers in relatively less dense areas, incurring additional transportation costs from moving equipment in space; or whose productivity is low. If equipment purchases are subsidized, rental markets are more likely to arise for operations that do not need to be performed in all farms at the same time (Binswanger, 1986). But those operations may not be those with the highest returns. Due to the time-sensitivity of agricultural activities demand is synchronous and equipment rental markets are likely to display congestion because service capacity is (relatively) fixed. By subsidizing increases in equipment supply the government could lower congestion, but it could also induce marginal farmers to demand services, slowing down service provision for productive farmers. At the same time, it increases the incentives of larger land-owners to demand services from market providers, increasing providers' market power. The balance of these effects dictates the efficiency and distributional impact of the development of these rental markets.

Our paper combines the first available transaction level dataset of equipment rental markets in the developing world, a census of 130 villages characterizing the full demand for services, and a novel structural model of frictional rental markets to assess market efficiency and the distributional impact of government subsidies. We use data on the two prevalent types of provision of mechanization rentals. The first one is the provision of services by single owners of equipment, often farmers who own agricultural equipment and rent them out ("market" providers). Given service capacity constraints, these providers prioritize the service of large orders or orders in densely populated locations. Consistent with the data, this prioritization induces delays for farmers with relatively small orders or in lower density locations. The second one is a private-public partnership that set up custom hiring centers (CHCs) with the objective of providing machinery for small and marginal farmers. These providers allocate equipment on a first-come-first-served basis ("fcfs" providers). In this case, the order size and location is uncorrelated with the timing of service. Inframarginal farmers are driven into the government subsidize platform, but the additional transportation costs associated to moving equipment in space may well overturn the gains in productivity to those farmers. Through counterfactual exercises, we quantify the strength of these channels and argue that general equilibrium forces in prices and the equilibrium sorting of farmers to providers are quantitatively important to assess the allocative efficiency of rental markets.

We start by characterizing rental markets in Karnataka, India. We use detailed transaction data from the private-public partnership since 2017, including hours requested, acreage serviced, implement and location. This data is linked to our newly collected survey data on demographics, farmer assets, productivity and engagement in the rental markets.First, we document that requests for capital rental services are spread across space and that there is higher engagement in rental markets by large-scale farmers (more than 4 acres of farmland) than by low-scale farmers (cropping less than 4 acres). However, the distribution of these large and small farmers in space is such that once we control for equipment ownership levels in neighbouring villages, there is no differential access between large and small scale farmers. Second, we document queuing for service fulfillment which varies throughout the season. This congestion effect is of first-order importance for the organic development of rental markets, and a key margin to assess the efficiency and distributional effects of the markets induced by the government subsidies. Third, because agricultural activities are highly time sensitive, the delay in the arrival of inputs associated to these queues is potentially costly. We quantify the costs of delays for productivity and profitability by estimating the optimal date for land preparation during a season. We choose land preparation because those are the processes for which the rental markets in Karnataka are most active. We compare profits for farmers that prepared the land away from that optimal date. We find that the average productivity cost for delay within a 10-day window of the optimal planting date is 8.5% per day.

To assess the merits of different dispatch systems for efficiency and productivity, we pose a novel structural model of frictional rental markets for equipment. There are two types of providers that differ in their order dispatch technology and face capacity constraints for service. There are two types of farmers that differ in their demand of equipment-hours and their locations in space. Their location induces costs in service provision because moving equipment in space is costly. Providers set rental rates and farmers choose in which provider to queue at, as in a standard directed search model. Consistently with the data, providers can serve up to three orders within a day, posing an interesting combinatorics problem given the provider's capacity. The main predictions of the model are that if small and large farmers are equally distributed in space, the efficient allocation is that small scale farmers only approach the fcfs provider, where their expected queueing time is lower. Large farmers should search for services both in the market and in the fcfs provider. However, their incentive to reach market providers increases with the share of small scale farmers in the market. The reason is that the queue length of large farmers at the market providers is lower. The cost of service is higher than from the fcfs, so that large farmers are indifferent in equilibrium. Disparities in the distribution of farmers in space have direct implications for the cost of provision.

To bring the model to the data we allow for additional heterogeneity in farmers equipment-hours requests and locations. We solve and estimate our structural model of optimal dispatch, using administrative data from the fcfs platform and the Census data for market providers and our survey data on farmers' productivity.The link between equipment-hours demanded, productivity and plot location is inferred directly from the data. Through a simulation exercise, we find that the government subsidized provision, which relies on a first-come-first-served dispatch, benefited small-holder farmers through declines in queueing time. However, this dispatch system entails equipment transportation costs that could overturn the gains to those farmers. Large farmers face lower delays when renting from market providers, and their location is such that when market providers minimize transportation costs, larger farmers disproportionally benefit from queuing with them.

A unique dimension of our problem is that location-specific demand congestion generates losses in productivity due to delays in service provision. Congestion is endogenous to the queueing and pricing decisions of market participants. We propose a tractable framework to study congestion through a queueing service model. We expand the directed search environment in Shi (2002) to allow for provision of multiple orders and service capacity constraints.[1] Delays are often overlooked as a barrier to technology adoption, yet is a potentially important mechanism in sectors like agriculture, where returns are extremely sensitive to the timing of farming activities. Delays are typically the result of synchronous queuing.

The notion that there might be scale economies associated to concentrating production in certain locations is explored by Holmes and Lee (2012) in the context of crop choices of adjacent plots. Bassi et al. (2020) documents the workings of rental markets for small carpentry producers in urban Uganda and argue that frictions in their setup are relatively limited. Distinctively, agricultural equipment needs to be moved in space to reap its benefits and the time sensitivity of agricultural production makes demand synchronous. We show that these two margins are key determinants of the efficiency of usage of capital services through the rental market, and its distributional and productivity impact.

The role of geography for the efficient allocation of factors of production in agriculture has been studied by Adamopoulos and Restuccia (2018). In their paper they focus on the characteristics of the soil and the potential yields of different plots to assess the degree of misallocation of factors of production. We instead consider the effects of congestion in capital service provision due to spatial misallocation of service providers and demand for those services. Spatial misallocation has been studied in the context of the rural-urban gap in labor income by Gollin et al. (2013); Lagakos et al. (2015).[2] In this paper we focus on the distributional effects across land-sizes and locations of equipment rental markets.

This paper also relates to the literature that emphasized barriers to adoption of technology in agriculture (Suri, 2011) and of mechanization in particular. Pingali et al. (1988) highlighted the role of contractual enforcement problems while Foster and Rosenzweig (2017) emphasizes economies of scale. Small poor farmers do not find it optimal to adopt capital in-

[1]As highlighted by Lagos (2000) and Sattinger (2002), queueing models are powerful to micro-found a matching process between, in this case, farmers' orders and service provision.

[2]Spatial misallocation has been studied in the context of taxation across US states by Fajgelbaum et al. (2018), while Hsieh and Moretti (2019) study misallocation of workers across MSAs.

tensive practices when they entail the purchase of equipment, a relative large expense whose services are used for a limited period in the season. High cost of capital relative to labor have been emphasized as a deterrent to technology adoption, (Pingali, 2007; Yang et al., 2013; Yamauchi, 2016; Manuelli and Seshadri, 2014). In this paper we focus on the role of geography and density in assuring access to capital services through rental markets.

2 Equipment rental markets.

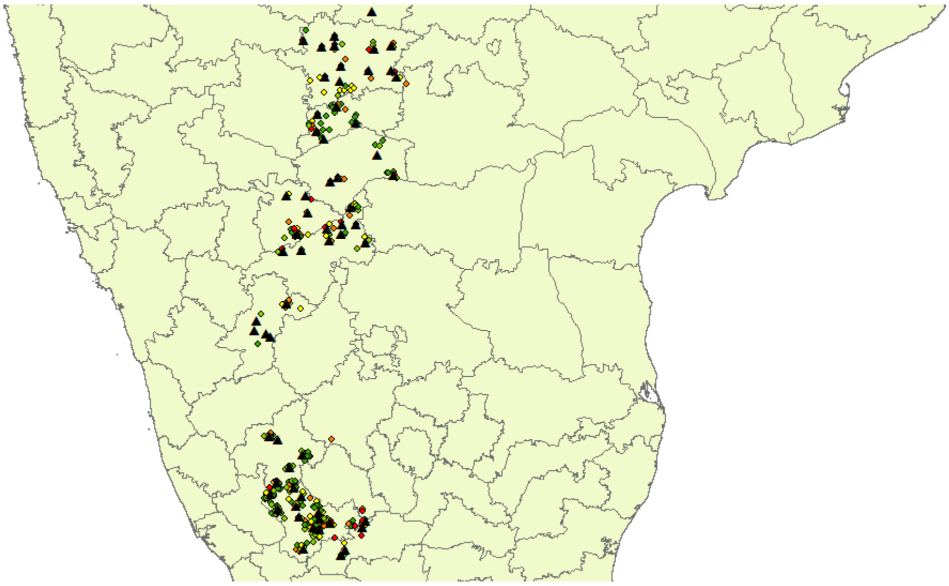
2.1 Data description

We study agricultural rental markets in rural India. The study focuses on the state of Karnataka, one of the least mechanized states in India (Satyasai and Balanarayana, 2018), and where the majority of agriculture occurs on small farms (less than 4 acres). Our own census of farming households in 150 villages across the state shows that equipment ownership rates are low and that farmers rely on informal equipment rental markets within each village.

In 2016, the state government partnered with the largest manufacturer of agricultural equipment in India to design and manage a platform through which farmers could rent equipment. They generated a subsidy scheme for equipment purchases to create custom hiring centers (CHCs, also known as "hubs") in 25 districts throughout the state. In exchange for these government subsidies, the service provider committed to rental rates of about 10% below their market counterparts. Figure 1 plots currently active hubs in space. These CHCs provide rental services in nearby areas and farmers can access these services through a call-center, via an app on a smart-phone, or in person at the CHC.

Our first source of data comes from the universe of transaction-level data maintained electronically by these hubs. This administrative data consists of all the transactions completed through that platform since 2016 for about 60 hubs in Karnataka (27 hubs entered in 2016, 29 in 2018 and 10 in 2019). Over the time period covered by the data (October 2016-May 2019), over 17,000 farmers from 840 villages rented equipment from these CHCs. The data contains information on number of hours requested, acreage, implement type, as well as farmer identifiers (such as their name, village, and phone number). Equipment available varies across CHCs (hubs) but the median hub provides equipment that ranges from sprayers

Figure 1: Locations of CHCs and Demand



Triangles indicate CHCs. We aggregate demand following the village where each farmer is registered. Green dots correspond to demand for the smallest plots (1 acre or less), red dots correspond to demand for the largest plots (4 acres or more).

to Rotavators or ploughs. Rental rates in the platform are about 10% below market prices on average, as part of the government's initiative to increase mechanization access. The service provision is first-come-first-served. When a service is fulfilled, a professional driver brings and operates the equipment at the farmer's plot. Equipment arrives within a lapse of two days in most cases. If delays are longer, farmers are informed at the moment of booking (and may choose to cancel). Table 1 reports the 10 most commonly rented implements for the years 2017 and 2018, the number of transactions recorded for each implement, their per-hour rental price, and month where the implement is most commonly rented (has the highest number of transactions).

We use two additional sources of data in Karnataka. The first is survey data collected in June-July 2019 around a representative sample of the equipment hubs in the main sample. The survey includes approximately 7,000 farmers, and asks for detailed information on input and output per plot which allow us to generate measures of productivity at the plot level. We also ask about their engagement in rental markets, rental rates, and perceptions on barriers to participation in the market. The second is a census of agricultural farmers including more than 40,000 farmers in the villages covered by the survey data. This census allows us to

Table 1: Summary Statistics of Commonly Rented Implements from Rental Database

	Commonly Rented Implements		
	Number of Transactions	Median Price Per Hour	Peak Month
Rotavator 6 Feet	11,239	770	July
Cultivator Duckfoot	7,287	550	April
Cultivator 9 Tyne	5,245	525	May
Plough 2MB Hydraulic Reversible	3,716	450	February
Trolley 2 WD	2,436	250	January
Harvester Tangential Axial Flow (TAF)-Trac	2,048	1800	May
Rotavator 5 Feet	1,811	700	September
Blade Harrow Cross	1,793	360	March
Knapsack Sprayer 20 Litres	1,688	22.5	October
Blade Harrow 5 Blade	1,600	360	June

validate the characteristics of the farmers in the survey relative to the population. It also allows us to measure inventories of equipment potentially available to farmers from nearby villages.

Finally, we use data from a broader set of Indian states as recorded in ICRISAT's household-level panel data. This data which contains detailed agricultural information, including, season-level agricultural operations, their timing, costs and total revenues. The data covers eighteen villages over 2009-2014 in Andhra Pradesh, Gujarat, Karnataka, Maharashtra, and Madhya Pradesh.

2.2 Motivating facts

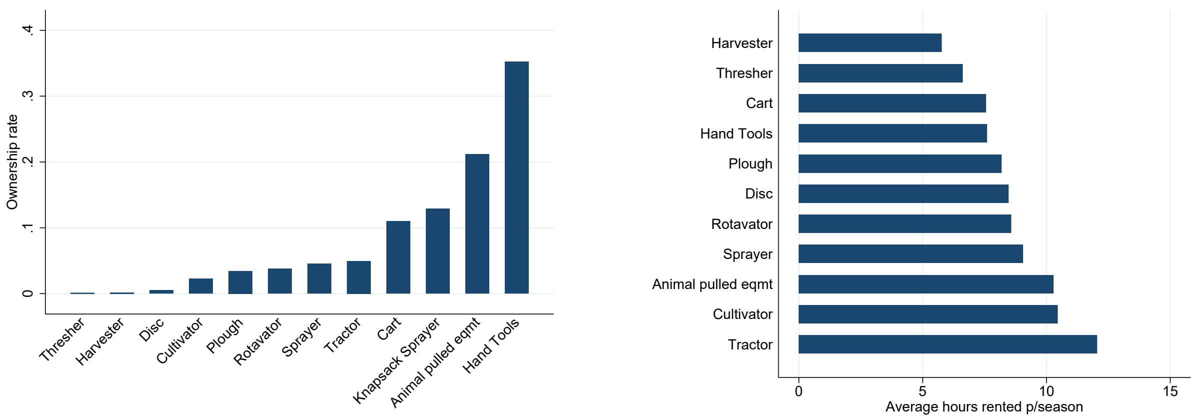
We start by describing the characteristics of the service demand and farmers equipment supply. Then, we focus on a handful of outcomes that are informative to the theory that we describe in Section 3. First, because agricultural activities are highly time sensitive, the timing of demand is synchronous leading to endogenous waiting times as a function of service capacity. The service capacity includes farmers' ownership as well as CHCs capacity. Second, because equipment needs to travel for transactions to take place, the joint distribution between travel times and the scale of demand, i.e. equipment-hours per request, is a key input when optimizing service provision. Third, we document substantial price dispersion in rental rates after controlling for observable household characteristics and village/market characteristics, consistent with frictional rental markets. Fourth, delays in service provision are costly to farmers, because they affect field productivity. In what follows we document each of these features.

2.2.1 Service Capacity and Service Demand.

We start by reporting patterns of ownership (service capacity by farmers) and rentals of equipment across the farmers in our survey (see Figure 2).[3] Most farmers report to own hand tools and animal pulled equipment. Less than 10% of the farmers report to own larger equipment such as tractors, or rotavators and cultivators. At the same time, tractors and cultivators are among the pieces of equipment with the highest equipment-hours rented. The average hours rented in a season per farmer is 12 hours for tractors and 10 hours for cultivators. These rental transactions mostly entail relational contracts. We collect information on the typical customer for a farmer that rents out his/her equipment. We find that 72% of owners report to rent out to people they know from the village or who they have worked with in the past.

Delays are the most common issue faced by farmers when renting equipment, with 78% of farmers reporting it as an issue. Importantly, larger farmers (cultivating at the 75th percentile of the land size distribution, i.e. larger landholdings) are nearly 5 percentage points less likely to report delays as an issue. Hence, delays in accessing mechanization are more pervasive among smaller farmers.

Figure 2: Ownership and rentals by implement.



Owned Rentals

The ownership rate is the share of farmers that report to own a given implement relative to the total population surveyed. Rental hours correspond to the average hours reported for the whole season.

[3]Appendix D reports similar statistics using data from the Census.

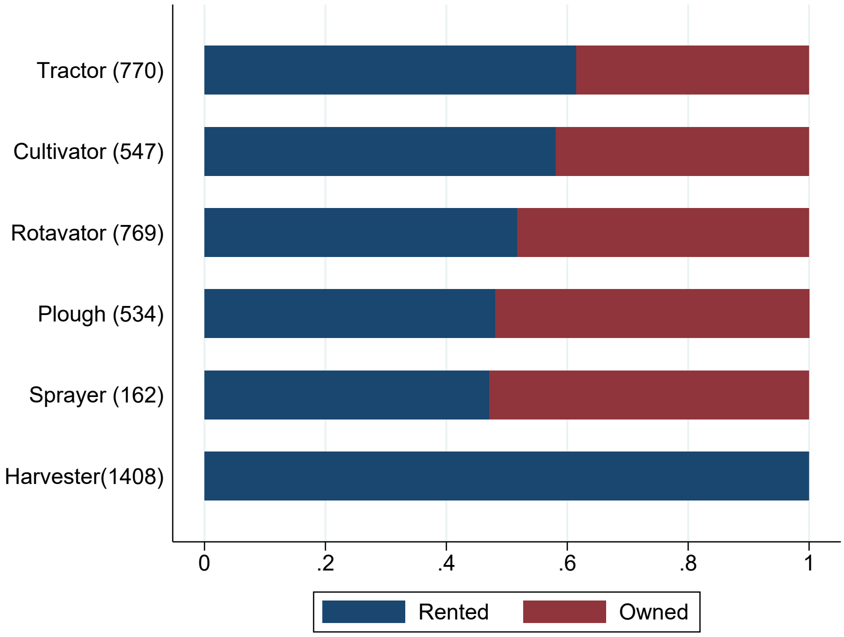
Given the disparities in value of agricultural implements as well as their contribution to production, it is useful to construct a measure of equipment services from rentals and owned equipment. We measure these services as the product of average hours of usage during a season h_i, market rental rates, r_i and the number of implements i owned or rented, N_i. Hence, equipment services in a farm k are

$$k = \sum_i N_i r_i h_i$$

The main hypothesis behind this measure is that differences in rental rates across implements shall reflect differences in the services they provide, and that therefore, more expensive equipment provides higher services to production. The main challenge in constructing such a measure is the availability of data on market rental rates. We exploit our transaction level dataset to construct mean rental rates per implement at the village level. Figure 3 displays log owned and rented services. Harvesters (the most expensive implement in our bundle) is reported to be only rented. For those farmers using tractors, more than 60% of the services available in the farming sector come from rentals whereas the remaining 40% stem from ownership. Services associated to smaller and cheaper equipment, such as sprayers, are equally accounted for by rentals and ownership.

It is worth noting that given land holdings, ownership of equipment is not cost-effective for most farmers. For instance, the rental price of a rotavator is between ₹750 and ₹1,000 per hour (including tractor, a driver and fuel) and the average farmer demands about 6 hours of rotavator services in the season or between ₹4500 and ₹6000 in services. The purchase price of a new rotavator is over ₹110,000 which means that, absent maintenance costs (which are certainly non-negligible), the average farmer needs 19 years to amortize the investment. The rental rate for an inferior technology that serves a similar purpose, i.e. a harrow, is half of the rental rate of the rotavator (₹360) and the cost of purchase is about ₹50000. Overall, these price differentials are consistent with the observed extensive engagement in rental markets for equipment.

Figure 3: Capital services from ownership and rentals, by implement.



Shares of log capital services by implement and ownership/rental status. Average rental rates for an hour of service (in ₹) are reported next to each implement.

2.2.2 Heterogeneous Queuing by Production Scale

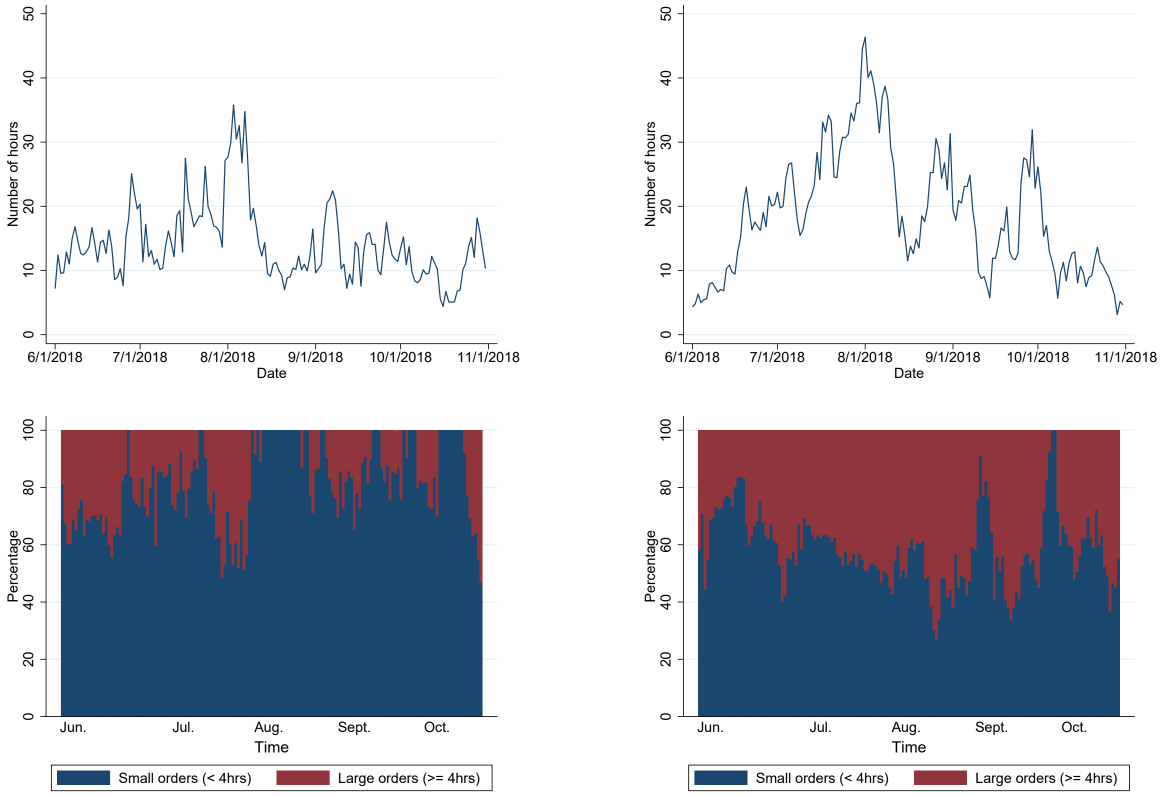
The demand for equipment rental services vary by agricultural process and therefore throughout the agricultural season. The synchronous nature of many of these processes across farmers induces queuing in the market. Our transaction level data allows us to measure demand fluctuations by computing hours outstanding for service at a daily frequency. We focus on two commonly rented implements for land preparation, rotavators and cultivators. Indeed, our survey data indicates that farmers are most likely to engage in the rental market for land preparation.

Figure 4 shows hours of unfulfilled orders for each of these implements over the 2018 kharif season. Queueing peaks by the end of July for rotavators and beginning of August for cultivators. At the peak of the season, the average provider faces 40 hours of demanded services in queue, which account for over 12 orders on average at a point in time.

Demand moves distinctively between large and small requests, measured in service hours (Figure 4). A large portion of hours outstanding are accounted for by small orders (less than

Figure 4: Hours outstanding in the queue.

Cultivator | Rotavator



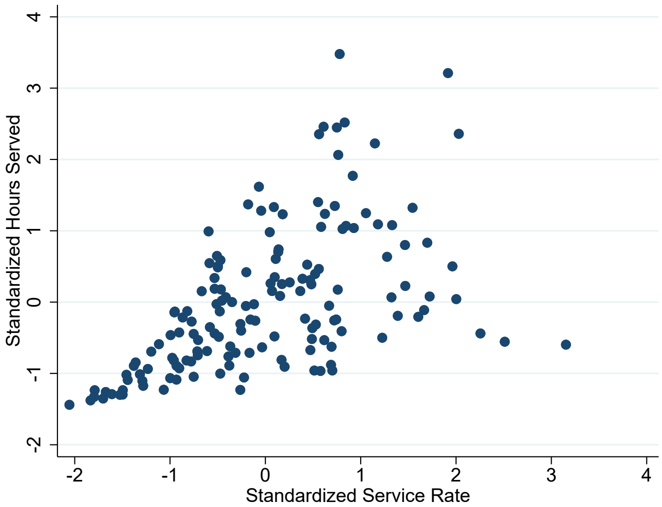
by size | by size

Averages hours outstanding in the queue across hubs in Kharif 2018, overall (top panel) and by order size (bottom panel).

4 hours of service), although at pick time the share of hours accounted for by large and small holder farmers equalizes. This is not explained by a higher number of large orders but rather by larger orders altogether.

As demand fluctuations over the season in a somehow predictable manner, it is expected that service supply may adjust. To explore these movements we compute service rates as the fraction of serviced hours within a day divided by the number of service hours outstanding. On average, we see up to 3 orders being fulfilled during a day per equipment piece. We find that service rates move during the season, and that they positively correlate with hours demanded (see Figure 5). However, they do not move enough to avoid queueing and congestion in service provision, likely due to capacity constraints or frictions related to finding alternative providers, as we discuss in Section 2.2.4.

Figure 5: Service rates.



Averages across hubs (rotavator) Kharif 2018.

2.2.3 Spatial distribution of rental services

We first document that service delays are negatively associated with cultivated size suggesting that even if the productivity costs of delays are of same magnitude between small- and large-holder farmers, the incidence of those delays is disproportionally bear by those with small plot sizes, columns (1) and (3) in Table 2. It is possible that these delays are explained by the geographical location of plots since equipment needs to travel to generate services. Columns (2) and (4) Table 2 show that delays have an important spatial dimension–adding village fixed effects substantially attenuates the coefficient on the log of land size, and increases the r-squared by eight or nine times (depending on whether only positive delays are considered, or all delays are included in the regression). That is, in the surroundings to a particular village, small and large farmers face similar delays, but if this clustering is not accounted for, smaller farmers face longer delays.

Table 2: Delays as a Function of Land Area and Location Fixed Effects

	Delays (Sum of Average Delays Over the Season)			
Log(Area)	-0.215*	-0.144	-0.319**	-0.128
	(0.115)	(0.0926)	(0.145)	(0.108)
Observations	5,615	5,615	4,345	4,345
R-squared	0.002	0.182	0.003	0.252
Village Fixed Effects	No	Yes	No	Yes
Mean Delays	2.158	2.158	2.789	2.789

Estimated coefficients from a regression of reported delays in service provision and the log(area) owned. The first two columns include those that report zero delays whereas the last two columns only focus on those that report positive delays.

2.2.4 Frictional Rental Markets

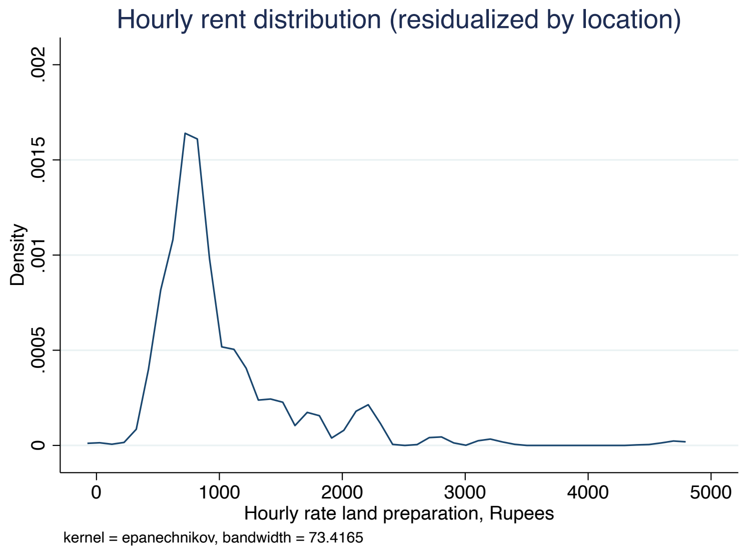
But why are there delays to begin with? Is this a consequence of low ownership rates and service capacity, or rather the consequence of frictions in the rental market that prevent farmers and providers to contract services when desired? There are two features of the market that indicate the presence of frictions in the rental market.

The first one is that the current supply of equipment seem adequate to serve market demand. To compute supply we turn to a Census of 150 villages from the same area, which includes information on over 40,000 farmers. We assume that the equipment has a catchment area of about 10km, since transporting equipment over large distances is time-consuming and expensive (particularly for farmers whose main activity are not rentals). For a median village, the number of available cultivators within a 10 km radius can serve 2016 orders per season, while average demand is 1190 orders. For rotavators, available supply can serve 1008 orders in the season while market demand is 450 for the median village.[4] Hence, the observation of pervasive delays in service provision paired with adequate supply within each geographical market suggest that congestion does not necessarily arise due to supply shortages.

The second one is the presence price dispersion in rental prices of equipment within a 10km catchment area of each village. Using information from our survey of 7,000 farmers, Figure ?? shows the distribution of residualized rental rates during land preparation from

[4]We assume a six-week season, and that each piece of equipment serves three orders a day. The number of orders served in the day is consistent with services per equipment per driver at peak utilization that we observe in the transaction dataset. Even when assuming a shorter 4 week season, supply would account for 1344 per season for cultivators and 672 orders per season for rotavators, well under seasonal demand for each equipment.

village effects, i.e. that is the variation in rental rates per hour serviced across farmers within a village. The interquartile range is 1.7 while the coefficient of variation is 0.7. Burdett and Judd (1983) were the first to show that price dispersion can arise in an environment with identical agents where consumers/farmers find it costly to search for providers. Price dispersion can also be related to informational asymmetries Varian (1980) or to consumer preferences for certain providers over others Rosenthal (1980). Overall, the exchange of identical goods for heterogeneous prices is typically a sign of frictions in the market, which we entertain through the structural model that we study next.



3 A frictional model of capital rental services in space

3.1 Environment

Consider an economy populated by F farmers that use capital services for production. Capital services are available for rent in frictional rental markets.

Farmers differ in their demand for equipment-hours and in their geographic location. A fraction s of them are "large-scale" farmers and demand k_s hours, while the remaining $(1-s)$ fraction are "small-scale" farmers, and demand k_{s^-} hours. We think of the demand size as being determined by land-holdings. Farmer's productivity gets realizes once inputs have been committed, e.g., weather shocks or delays in equipment rental service provision that affect the output from the farm. These shocks induce revenue costs to farmers. For simplicity, we focus on ex-post shocks related to delays in service provision, which are idiosyncratic to the farm.

There are H rental service providers that can serve up to o orders a day. Providers j differ in their service capacity, up to $\bar{k}_j$ machine-hours a day, and in their technology for service provision. A fraction h of providers use a first-come-first-served (fcfs) technology, while the remaining fraction $1 - h$ has access to a selection technology that allows them to prioritize certain type of service requests (mkt).[5] We assume no systematic differences in providers' geographical location, i.e. the expected travel time for service provision conditional on the machine-hours demanded is the same for both providers.

Denote the ratio of farmers to service providers, $f = \frac{F}{H}$, and focus on the case where the market is large, i.e. $F, H \rightarrow \infty$ and neither side is infinitely larger than the other, $f \in (0, \infty)$. Providers post prices r_{ij} and a selection criteria (with commitment) simultaneously at the beginning of each period. Geographical considerations for service provision are included into the opportunity cost of moving equipment from a provider to the plot. Then farmers decide whether and which provider to approach, generating queues for each available provider. Finally, providers decide which orders to serve given the selection criteria and farming production takes place. Given the large number of providers and farmers we focus on a symmetric mixed-strategy equilibrium where ex ante identical providers and farmers use the same strategy and farmers randomize over the set of preferable providers. The key assumption is that agents find it difficult to coordinate their decisions in a large market.

A type i-farmer's strategy is a vector of probabilities $P_i \equiv \left(p_{i,\text{fcfs}}, ...; p_{i\text{mkt}}, ...\right)$ where p_{ij} is the probability of applying to each type j-provider. A type-j provider's strategy consists of rental rates per hour serviced, r_{ij} and a selection rule $\chi_j \in [0, 1]$ for the market provider. The selection rule applies only when a provider receives requests from both type of farmers, in which case the provider prefers the large scale farm if $\chi_j = 1$, prefers a small scale farm if $\chi_j = 0$, and he is indifferent between them for $\chi_j \in (0, 1)$. When the provider receives requests from a single type of farmer type, he randomly selects one farmer for service. Those that request a service from the fcfs provider face the same probability of being first in line

[5]Albeit h and H are assumed exogenous, both of them can be easily endogenized with a costly set up of providers and an associated free-entry condition.

irrespective of the machine-hours demanded.

3.2 Queue lengths as strategies

Each farmer maximizes expected profits from farming trading off the probability of obtaining a rental service and the cost of such a service. The characteristics of their demand (or ex-ante productivity) and geographical locations are as in Assumption 1.

Assumption 1: *Equipment-hours demanded by small and large scale farmers satisfy* $k_s > k_{s^-}$. *The expected travel time to servicing small-scale farmers is weakly higher than that for large-scale farmers,* $E(d_s) \leq E(d_{s^-})$.

A convenient object for analysis is the *queue length*, i.e., the expected number of farmers requesting a service from a given provider.[6] Let q_{ij} be the queue length of type i farmers that apply to a type j provider, where $i \in \{s, s^-\}$ and $j \in \{\text{fcfs}, \text{mkt}\}$. Then, $q_{sj} = sFp_{sj}$ and $q_{s^-j} = (1-s)Fp_{s^-j}$. The probabilities of approaching different providers for a single farmer should add up to one, which leads to

$$H\left(hq_{s,\text{fcfs}} + (1-h)q_{s,\text{mkt}}\right) = Fs \tag{1}$$

$$H\left(hq_{s^-,\text{fcfs}} + (1-h)q_{s^-,\text{mkt}}\right) = F(1-s) \tag{2}$$

Assumption 2: *The demand for machine-hours satisfies,*

$$o(k_{s^-} + E(d_{s^-})) <= \bar{k}_j \quad \text{and} \quad (o+1)(k_{s^-} + E(d_{s^-})) > \bar{k}_j$$

$$(o-1)(k_s + E(d_s)) <= \bar{k}_j \quad \text{and} \quad (o-1)(k_s + E(d_s)) + k_{s^-} + E(d_{s^-}) > \bar{k}_j$$

$$(o-1)(k_{s^-} + E(d_{s^-})) + k_s + E(d_s) <= \bar{k}_j \quad \text{and} \quad (o-1)(k_{s^-} + E(d_{s^-})) + 2(k_s + E(d_s)) > \bar{k}_j$$

In other words, for tractability we assume that the service capacity $\bar{k}_j$, is enough to serve at most "o" orders; and that if the provider serves only large-scale orders, it can serve "o-1" orders. Finally, we assume enough capacity to serve $o-1$ small scale orders and 1 large scale

[6] From a theory standpoint, when the number of providers and firms grow large, the probability of requesting a service to a given provider approaches zero and it is inconvenient to work with. From an empirical standpoint, queues are directly observable from our rental requests data while probabilities are not.

order.

A farmer of scale i that requests service from provider j gets served with probability Δ_{ij}. This conditional probability depends on the provider's selection criteria and the number of orders it can potentially serve within each period, i.e. its capacity $\bar{k}_j$; as well as on the machine-hours demanded k_i and the expected travel time for service $E(d_i)$. We assume the empirically relevant service capacity which implies that, on average, $o = 3$ orders can be fulfilled within the period.[7] Hence, Δ_{ij} is the sum of the probabilities of servicing $\bar{o}_i$ type i farmers, $\phi_{ij}(\bar{o}_i)$, times the probability that a certain farmer of type i is chosen, $\tilde{\Delta}_{ij}(\bar{o}_i)$, across all possible number of orders of type i being served, $\bar{o}_i$.

$$\Delta_{ij} = \sum_{\bar{o}_i \in \{1,2,3\}} \phi_{ij}(\bar{o}_i)\tilde{\Delta}_{ij}(\bar{o}_i).^{8} \tag{3}$$

Using the definition of the probability of service, equation 3, it is possible to show that the probability of type i being served (weakly) declines in the queue length of type $i' \neq i$ farmers. For the first-come-first-served provider the result is straightforward because service probabilities decline with the number of machine-hours in the queue, irrespective of their type. For the market provider, when the selection criteria favors type i farmers, the decline in the probability of service for type $i' \neq i$ farmers is strict as the number of type i farmers in the queue increase, while the service probability for type i farmers is independent of the queue length of type $i' \neq i$ farmers.

3.3 Market cost of provision.

We follow Burdett et al. (2001) and describe a farmer's decision as a function of the market price it would get for the rental service, r_{ij}, which in turn determines its expected "market" profits, U_i. The agents take this value as given when the number of agents in the economy is large, $F, H \to \infty$. Let $q_j \equiv \{q_{sj}, q_{s-j}\}$ be the queue at provider j. Each farmer chooses the service provider to minimize costs given U_i and the production technology: [9]

[7] This is consistent with the median number of orders served within a day in our administrative data.

[8] The full derivation can be found in Appendix A.

[9] Notice that the farmer takes the capital demand as given for convenience. We could trivially model the link between land-holdings and capital demand through a Leontief production function between capital and land.

$$C_i(r_{ij}, q_j, k_i) = \min_{j'} C_i(k_i, r_{ij'}, q_{j'}) \equiv \min_{j'} r_{ij'} k_i$$

subject to

$$\Delta_{ij}\pi_{ij}(k_i, r_{ij}, q_j) \equiv \Delta_{ij}\left(E(z(\Delta_{ij}))k_i^\alpha - r_{ij}k_i\right) \geq U_i,$$

where $E(z(\Delta_{ij}))$ is the expected productivity in the farm.

The farm's productivity depends on the realization of a random shock related to mistakes in the timing of agricultural activities. To ease the exposition we summarize the optimal timing for agricultural activities as the optimal "land preparation" date, $\theta^\star$, and relate deviations from this optimal timing to productivity costs. The realized land preparation date in a season is a random draw, θ, from a known distribution $G(\bar{\theta}(\Delta_{ij}))$ with mean $\bar{\theta}(\Delta_{ij})$ that depends on the probability of service given the provider choice. If the realization of the preparation date differs from the optimal, the farmer faces a productivity cost proportional to the delay relative to the optimal date, $\theta^\star$ as follows,

$$z = 1 - \eta(\theta - \theta^\star)I_{\theta^\star \leq \theta}$$

where η is the productivity cost associated to service delays. Then, expected productivity is $E(z(\Delta_{ij})) = 1 - \eta(\bar{\theta}(\Delta_{ij}) - \theta^\star)I_{\theta^\star \leq \bar{\theta}(\Delta_{ij})}$. We assume that $\frac{\partial\bar{\theta}(\Delta_{ij})}{\partial\Delta} < 0$ so that a high probability of service induces shorter average wait time to service. Importantly, given that the productivity function is asymmetric, the expected productivity is independent of the choice of provider whenever the expected wait times are relatively low, i.e. the probability of service is high. We assume this is the case in the reminder of the analysis so that $E(z(\Delta_{ij})) = 1$. Appendix B includes the analysis when there are expected losses from timing at the beginning of the season. Finally, because the draw of the service provision is idiosyncratic, there is no aggregate uncertainty in the economy and factor prices are time independent.

A type i farmer requests a service from a type j firm with positive probability if the expected profits are larger than or equal than U_i. The strict inequality cannot hold because then a type i farmer would apply to that provider with probability 1, yielding $q_{ij} \to \infty$ as their number grows large. Then, $\Delta_{ij} \to 0$ which contradicts, $\Delta_{ij}\pi_{ij}(r_{ij}, k_i) > \tilde{U}_i$. The farmers' strategy is

$$q_{ij} \in (0, \infty) \qquad \text{if} \qquad \Delta_{ij}\pi_i(r_{ij}, k_i) = \tilde{U}_i \tag{4}$$

$$q_{ij} = 0 \qquad \text{if} \qquad \Delta_{ij}\pi_i(r_{ij}, k_i) < \tilde{U}_i \tag{5}$$

This expression summarizes the tradeoff between potentially lower provision cost and higher profits, $\pi_i(r_{ij}, k_i)$ and a lower probability of service, Δ_{ij}. Given the shape of the probability function, there exist a unique queue length $q(r_{ij}, U_i)$ that satisfies the problem of the farmer. The farmer decides his queueing strategy as a function of his capital demand, k_i and market prices r_{ij}.

3.4 Service Providers

A service provider with capacity j maximizes expected returns. The stocks of machine-hours available to a provider are exogenously given at $\bar{k}_j$. For simplicity, we assume no depreciation or capital accumulation and no maintenance costs. The cost of servicing a farmer depends on its location relative to the provider. The location of each plot is indexed by d_i and $\mathbf{d}_{\hat{q}_t}$ is a vector collecting the locations of the orders completed within a period, $\hat{q}_t$. Providers choose the cost of service r_{ij} taking the size of the machine-hours demand for each type of farmer as given. As in Burdett et al. (2001), the provider takes the functional relationship 4 as given to solve his problem. Given U_i, k_i, he chooses the queue lengths by picking the cost of service and its service strategy. The cost of servicing a plot depends on the traveling time, as accounted by the expected travel time $E(d_i)$. This cost includes the foregone services that could have been provided as well as the opportunity cost of the driver, which commands a wage w per hour. Finally, we assume that the queue length is reset each period and therefore the service provision problem is static.

Consider the problem of a first-come-first-served provider. His value is the expected return from servicing at most $o = 3$ orders within each period. Let $\bar{o}_i$ be the number of orders of type i being served within the period. The per period return , $\tilde{V}$ from facing queue $\mathbf{q_{\text{fcfs},t}}$ depends on the type of orders being served and their quantity, as well as on the revenue per type net of the cost of labor and transportation.[10]

The value for a first-come-first-served provider is

$$V(\mathbf{q}_{\text{fcfs},t}) \quad = \quad \max_{\{r_{i,\text{fcfs}}\}_{i=s,s^-}} \tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{\mathbf{q}_{\text{fcfs}}}, \{(r_{i,\text{fcfs}} - w)k_i - wE(d_i)\}_{i=s,s^-}\right) \tag{6}$$

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in \hat{q}_{\text{fcfs}}} k_s(i) + E(d_s(i)) \leq \bar{k}_{\text{fcfs}}$$

Consider now the problem of a market provider. In addition to choosing the cost of provision, r_{mkt} the provider chooses its optimal selection criteria χ. This choice in turn determines the type of orders being served and their quantity, given service capacity. We can characterize the problem of a market provider as

$$V(\mathbf{q}_{\text{mkt},t}) \quad = \quad \max_{\chi, \{r_{i,\text{mkt}}\}_{i=s,s^-}} \tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{(\mathbf{q}_{\text{mkt}},\chi)}, \{(r_{i,\text{mkt}} - w)k_i - wE(d_i)\}_{i=s,s^-}\right) \tag{7}$$

subject to farmers' strategies 4 and feasibility

$$\sum_{i \in \hat{q}_{\text{mkt}}} k_s(i) + E(d_s(i)) \leq \bar{k}_{\text{mkt}}$$

4 Symmetric Equilibrium

Consider the ratio of farmers to hubs, $\frac{F}{H}$, as well as the fraction of providers that serve on a first-come-first-served basis, h, as exogenously given. A symmetric equilibrium consists of farmers expected profits U_s, U_{s^-}, provider strategies r_{ij}, χ, and farmer strategies, q_{ij} for $i = \{s, s^-\}$ and $j = \{\text{fcfs}, \text{mkt}\}$, that satisfy:

1. given U_s, U_{s^-} and other providers' strategies, each type j provider solves 6 or 7;

2. observing the providers' decision, farmers of productivity z_i choose who to queue with,

[10] For tractability, we assume the revenue is separable in the number of orders, and relax this assumption in the quantitative exercise when we allow the provider to optimize service provision in space, i.e. minimize transportation cost across orders.

i.e. solves 4; and

3. the values U_s, U_{s^-}, through q_{ij}, are consistent with feasibility, equations 1 and 2.

Proposition 1. *In all symmetric equilibria, the selection process is $\chi = 1$ and the per period profit for a provider that serves both types of farmers from farmers of type i is V_i^j, characterized by:*

$$V_i^j = \gamma_{1i}^j (zk_s^\alpha - wk_s - wE(d_s)) + \gamma_{2i}^j (zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))$$

where $\gamma_{1,i}^j, \gamma_{2,i}^j$ are non-linear functions of the queue lengths and the elasticity of the service probabilities with respect to the length of the queues, ζ.

The expected per period profit of servicing large-scale farmers is higher than for low-scale farmers, $V_s^j > V_{s^-}^j$. If the relative surplus from trade of large vs. small scale orders is large enough the expected profits from service as larger for large-scale farmers than for low-sale ones $U_s > U_{s^-}$.[11]

A few characteristics are worth highlighting. First, differences in location and the cost of travel explain disparities in the incentives to serve farmers operating different scales. In other words, for two plots located at the same distance to the provider, the marginal cost of service is lower for larger scale farmers. On the farmer side, his expected profits depend on its own demand for services and on the rental rate of service.

In equilibrium, farmers that get served from both providers shall be indifferent between either provider type. If queues were identical across providers, equalizing expected profits to the farmers would imply that the cost of service, r, was the same across farmers of different scales in equilibrium. But this cannot be the case since the marginal cost of service for large-scale farmers is smaller than for small-scale farmers, ceteris paribus.

[11] The ratio of $\frac{zk_s^\alpha - wk_s - wE(d_s))}{zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))}$ must be larger than a constant that depends on the elasticity of the probability of service, see Appendix B.

5 The role of government regulation in the rental market for equipment

In this section, we bring the model to the data to characterize how allocations change with the presence of a first-come-first-served dispatch system relative to the market dispatch system, both in space and across farmers of different production scales. The key outcomes of interest are the selection of farmer types across providers, the expected delays and therefore farming productivity costs, as well as provider profitability. We later study how these outcomes change under two counterfactual market arrangements: first, we allow first-come-first-serve providers to have access to a service selection technology, i.e., behave as market providers and mimicking a market deregulation; second, we characterize a long-run equilibrium where in response to market deregulation providers' are also allowed to exit the market.

5.1 Bringing the model to the data.

The quantitative assessment of the impact of the government intervention in the rental markets for equipment consists of two blocks. The first ones solves the model in Section **??** for the equilibrium market rental rates and queue lengths, given the empirical supply and demand for equipment services. The second ones simulates queues and service provision to farmers with different scale of production and distribution in space.

Solving the model requires taking a stand on the heterogeneity in machine-hours demanded. We construct two groups of farmers following their average machine-hours requests in the transaction data: those with more than 3.5 machine-hours requests per order are denominated large-scale while those with less than 3.5 machine-hours are denominated small-scale. Then, we solve for an equilibrium in which both type of farmers are served by both types of providers, as in the data. We call this equilibrium the "status quo".

The second block involves finding the expected delay and subsequent productivity costs as well as provider profitability under the first-come-first-served and market dispatch systems using rental rates and queue lengths from the status quo equilibrium. In theory, the queue length itself yields the expected wait time by farm type. However, we recognize that empirically, farmer heterogeneity is richer than the one accommodated by the stylized theoretical model both in terms of machine-hours demanded and in the spatial allocation of demand. By simulating queues that are consistent with the equilibrium predictions for the first block, we can quantify expected delay, productivity costs and provider profitability for a more granular disaggregation of farmers. The simulation exercise also allow us to consider the implications of optimizing service provision in space (by minimizing travel time), a consideration that is abstracted away in the theory to make the problem tractable. The optimization of service provision in space is effectively the solution to a travelling salesman problem, conditional on the set of orders in the queue.

Summarizing, we simulate 300 paths of queues of length $q^\star$ and composition $(q^\star_s, q^\star_{s^-})$ as dictated by the equilibrium of the selection model. Then, given the equilibrium rental rates we let the provider optimize service delivery given their dispatch system. The actual sample paths for queues $(q^\star_s, q^\star_{s^-})$ are drawn from the joint empirical distribution of machine-hours and geographical location.

5.1.1 Data

Consistently with the evidence in Section 2 we use data for the Kharif season (June to October) in year 2018. We exploit three sources of data: (1) detailed transaction data from the government subsidized service provider, (2) our own survey of farmers, and (3) our own census of farming households in the catchment area of the subsidized service providers.

The first source of data contains information on machine-hours requested by service, the acreage serviced, the total cost of the service, the implement rented, and the nearest village to the farmer's service request. We focus again on commonly used implements, i.e. rotavators and cultivators; and we narrow the set of provider hubs to those with more than a 100 transactions within the season We use information on the closest village to each request as well as service hours requested to compute the empirical joint distribution of machine-hours demanded and service travel time in the catchment area of each hub. We geolocate villages and hubs and compute travel times using information from a commonly used Application Programming Interface (API), which factors in road connectivity across locations. Rural areas are not always well connected and while geographical distances may seem short, travel times increase rapidly.

The second source of data allows us to characterize the productivity of the farmers being served in the vicinity of these hubs. We compute the empirical joint distribution of productivity and production scale (i.e., cultivated area) within the catchment area of each provider hub. Productivity is measured as output per acre.

The third source of data allows us to characterize overall equipment supply, including equipment owners' supply of rental services, other than those in the transaction dataset. It also allow us to measure the total demand of services by production scale within the catchment area of a hub, including demand by those sourcing services from market providers rather than the government subsidizes hubs.

5.1.2 Parameterization

There are 10 parameters per hub that need to be calibrated, as shown in Table 3. A total of 8 of these parameters are calibrated directly from the data while the remaining 2 are calibrated internally by solving the model. From those parameters measured in the data, 4 of them are common across hubs: the providers' discount factor β, and their opportunity cost of moving equipment in space w, the cost of delays for farmers, η and the curvature of the profit function for farmers, α. The remaining 6 parameters are hub specific and include the share of first-come-first-serve providers relative to the total supply of equipment in the catchment area of a hub h, the parameters characterizing the joint distribution of productivity and machine-hours demand within the catchment area of the hub (i.e., mean and standard deviation of productivity and the correlation between productivity and machine-hours), the ratio of farmers demanding service to the providers in the catchment area of each hub f, as well as the share of large farmers in the population of farmers demanding equipment in the catchment area of the hub s. The latter two model-calibrated parameters are chosen to match the queue length of small-scale farmers at first-come-first-served providers, and to make sure the equilibrium displays positive queues of small and large-scale farmers with both providers, as we observe in the data. In addition to these 10 parameters, we feed the distribution of plots in space (and their corresponding travel-time) as measured from the platform data.

We set the discount factor to $\beta = 0.96$ with an implied daily discount rate of 4%. The opportunity cost of travel time equals the hourly wage of a driver which is directly observed from the platform data, at $w =$ ₹75. The curvature of the profit function is set to 0.6, as estimated from our own survey data. We exploit the fact that farming profits are proportional to this parameter, i.e. $\pi_i = (1-\alpha)y_i$ and estimate α from the average ratio of profits to value added as reported by farming households.

To discipline the productivity costs of delays, η, we use high frequency data from ICRISAT. We study profitability and value-added per acre of about 6200 plots in 18 villages in India during 2009-2014. We define an optimal planting time as the date that maximizes the profits per acre in a given village year. Then, we define the cost of the delay as the difference in average value added per acre or profit per acre (depending on the variable of interest) as we move away from the optimal planting date.

Formally, we estimate

$$Y_{i,year} = \beta_0 + \beta_1^+ (_{\text{Planting Date-Optimal}})_{>0} + \beta_1^- (_{\text{Planting Date-Optimal}})_{\leq 0} + \alpha X_{i,year} + \epsilon_{i,year}$$

where X are controls for plot characteristics, farmer, village and time fixed effects. Standard errors are clustered at the village level. Our estimates for the costs in value added per acre are reported in Table 8. They indicate that within a 5-day windows, each additional day away from the profit maximizing date entails a cost of 3.4% in terms of value added per acre. If we compute the cost at a 10-day window, the cost raises to 8.5% per day. For our assessment we focus on the former, more conservative estimate of the cost of delay, $\eta = 3.4\%$.

Then, we calibrate hub-specific parameters. Using our census of more than 40000 households across 150 villages that overlap with the location of hubs, we compute the share of machinery available from government-subsidized hubs and that available from machine-owners (i.e. we count inventory of implements per hub and implements owned by farmers within the catchment area of each hub).

To characterize the productivity of farmers requesting different machine-hours we use the subsample of transactions that overlaps with the survey data (approximately, 1300 observations) and compute the underlying correlation between farm productivity, measured as output per acre, and machine-hours requested. Their correlation ranges from -0.28 to 0.35 displaying the wide-heterogeneity in demand characteristics across hubs, column (5) in Table 4. When machine-hours requested are proportional plot size a negative correlation

Table 3: Parameterization

Parameter	Description	Value	Source/Moment
Measured directly in the data			
common across hubs			
α	Curvature of the profits function	0.6	Survey data
β	Discount factor	0.99	Interest rate
w	Travel/op. cost (INR/hr)	75	Platform data
η	Productivity loss/day	3%	ICRISAT sample
hub specific			
h	Share of fcfs providers	.	Census data
μ	Log-normal mean of productivity	.	Survey data
σ	Log-normal s.d. of productivity	.	Survey data
ρ	Correlation order size and productivity	.	Survey + Platform data
k_i	Distribution of order size	.	Platform data
$E(d_i)$	Distribution of travel time	.	Platform data
Measured using the model			
s	Share of large farmers	.	Large-scale queue, fcfs
f	No. of farmers/No. of equipment	.	Small-scale queue, fcfs
z	Level of profitability	.	Status-quo profitability, 10%

between output per acre and machine-hours requested follows from the negative correlation between productivity and farm size, as has been widely documented in the literature Foster and Rosenzweig (2017). A positive correlation on the other hand is consistent with having more mechanized farms being more productivity.

At the same time, we assume that the distribution of productivity is log-normal, $ln(z) \sim \mathcal{N}(\mu, \sigma)$ and fit the empirical distribution of value-added per acre for survey farmers in the catchment area of each hub via maximum likelihood. The estimated mean of productivity suggests differences in log productivity across hubs of 36% (from 7.4 to 9.3) on average, and a log-variance ranging from 1.1 to 2.9, columns (3-4) in Table 4. Finally, we fit the joint distribution of machine-hours demanded and travel time to services from the platform data for each hub using B-splines, akin to a non-parametric estimation of the distribution. On the travel dimension, the distribution is typically bimodal, with orders bunching at less than 10-minutes travel time from the hub and 30-minutes travel time.[12]

We take the stand that small-scale farmers are those with 4 or less machine-hours per

[12]The relatively concentrated distribution of services in space is also consistent with the fact that there are no transportation charges for plots located within 10km of the hub, and that plots farther away face a price surcharge proportional to the distance travelled. Empirically, it is in rare occasions that farmers far away from the hub use the dispatch system.

Table 4: Parameterization, hub specific characteristics

Supply		Demand				*Farmers*
Hub	share	Productivity		Correlation	*share*	*per provider*
	fcfs	mean	variance	prod - hours	*large-scale*	*f*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.86	9.35	1.08	-0.20	*0.35*	*3.22*
2	0.80	8.82	1.50	-0.07	*0.30*	*3.95*
3	0.50	9.83	1.12	-0.01	*0.25*	*4.30*
4	0.86	7.43	1.57	-0.16	*0.30*	*5.21*
5	0.86	8.85	1.07	0.00	*0.30*	*4.60*
6	0.86	8.85	1.07	0.00	*0.45*	*7.48*
7	0.86	8.89	1.33	-0.12	*0.25*	*5.32*
8	0.86	8.89	1.33	-0.12	*0.30*	*4.45*
9	0.86	8.89	1.33	-0.12	*0.25*	*5.33*
10	0.75	8.67	1.49	0.07	*0.35*	*3.35*
11	0.75	9.08	1.52	0.07	*0.25*	*3.58*
12	0.86	8.85	2.56	0.16	*0.35*	*3.22*
13	0.63	8.85	2.56	0.16	*0.45*	*3.82*
14	0.86	9.08	1.52	0.07	*0.30*	*4.57*
15	0.86	9.08	1.52	0.07	*0.30*	*5.23*
16	0.67	8.15	2.89	0.01	*0.30*	*4.72*
average	0.79	8.85	1.59	-0.01	*0.31*	*4.5*

Notes: Hub-specific parameters for each hub-implement combination, "Hub" in Column (1). Hubs labeled 6 and 7 correspond to Cultivators while the remaining hubs contain information for Rotavators. Hubs labeled 7,8,9 correspond to a single government subsidized hub, and therefore demand characteristics are the same. Column (2) reports the share of first-come-first-serve providers in the total equipment supply within each catchment area. Columns (3)-(6) report demand characteristics for each hub. Column (6) and (7) present the calibrated share of large-scale farmers and the ratio of farmers to providers for the catchment area of each hub.

order, and large-scale farmers are those with more than 4 machine-hours per order. Then we report the ratio of farmers to providers that makes the model predicted queue of small-scale farmers requesting service from the first-come-first-servers consistent with the data, column (7) in Table 4. This ratio ranges from 2.3 to 4.9 and generates queues that are broadly in line in the data, Table ??.[13]We also report the share of large-scale requests in the catchment area of each hub that is consistent with an equilibrium, column (6) in Table 4. Indeed, given the parameterization of the model and the targeted queue there are multiple shares that are consistent with an equilibrium where both types of firms request service from both type of providers. We pick the share that is closest to its empirical counterpart, measured from our Census data, see Table ??.[14]In most hubs, the observed share of large-scale farmers is too small to sustain an equilibrium where both type of farmers request service from both providers, and where the queue length for small-scale request in the first-come-first-served provider are consistent with the data. The flip-coin of this feature is that untargetted queue length of large-scale farmers are relatively low relative to their data-counterpart, with an average difference between the data and the model of 0.3 orders.

5.2 Status quo equilibrium

We solve for the rental rates and queue lengths when both types of farmers have access to both types of providers, i.e. the status quo equilibrium. Each type of farmer is indifferent between being served by a fcfs provider or by a market provider (see equation ?? in Appendix B for details). Table 6 summarizes the equilibrium outcome.

The rental rates charged by the first-come-first-served provider are lower for both types of farmers compared to a market provider. These differences in rental rates partly explain why we observe longer queue lengths at the market provider for both types of farmers. While queue lengths for small-scale farmers are slightly higher at the fcfs providers, queue lengths are actually lower for large-scale farmers. Yet, the ability of the market provider to

[13]We truncate the queue length for hubs 5 and 8, to 4.3, the highest queue length consistent with the status quo equilibrium given the parameterization. We also truncate the queue lengths of hubs 2, 9 and 10 from below at 2 orders.

[14]Alternatively, we could have targeted the queue of large-scale farmers in the first-come-first-serve providers which we currently use as an untargeted moment. Results are qualitatively similar to those reported here and available upon request.

Table 5: Moments

	Share of large-scale		Queues, first-come-first-served			
					untargeted	
	s		small-scale		large/small-scale	
	data	model	data	model	data	model
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.06	0.35	1.3	2.0	1.3	1.9
2	0.11	0.40	1.5	2.0	1.5	1.9
3	0.08	0.30	1.5	3.0	1.5	3.1
4	0.21	0.25	2.3	3.5	2.3	3.5
5	0.16	0.30	2.7	4.0	2.7	3.5
6	0.09	0.30	2.3	3.5	2.3	3.2
7	0.09	0.45	4.3	4.3	4.3	1.2
8	0.07	0.25	4.3	4.3	4.3	3.8
9	0.07	0.30	3.3	3.3	3.3	2.9
10	0.07	0.25	7.3	4.3	4.4	3.8
11	0.17	0.35	0.7	2.0	0.7	1.9
12	0.10	0.25	1.0	2.0	1.0	1.7
13	0.12	0.35	2.0	2.0	2.0	1.9
14	0.12	0.00	2.3	2.3		
15	0.12	0.45	1.0	2.5	1.0	2.4
16	0.16	0.00	3.2	4.3		
17	0.10	0.30	2.3	3.5	2.3	3.3
18	0.10	0.30	4.0	4.0	4.0	3.4
19	0.14	0.30	2.0	4.0	2.0	4.1

Notes: Calibration moments, data and model counterparts, Columns (2-5). Untargeted queue length for large-scale farmers relative to small-scale farmers, Columns (6-7). Correlation between Columns (6) and (7) is 0.39.

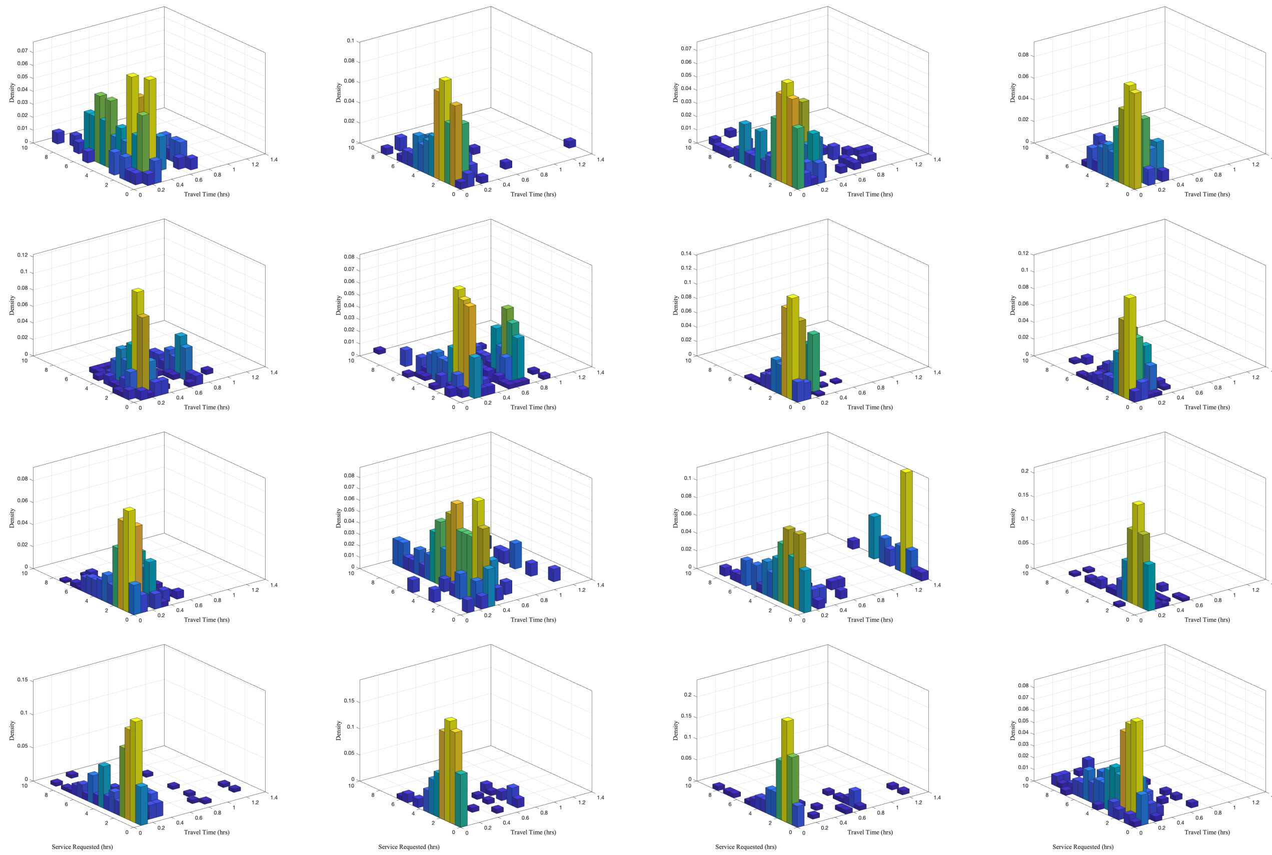


Figure 6: Order distribution by service requested and travel time

Each panel corresponds to the distribution of hours of service requested and travel time in the catchment area of each hub.

prioritize large-scale orders makes them attractive for large-scale farmers, and these providers can charge higher prices in response. Interestingly, small-scale orders are charged a higher price per hour consistently with higher marginal provision costs as measured by travel time per unit serviced. This feature is also true for the fcfs in most hubs, with the exception of 4 of them. In these four hubs the queue length of large-scale farmers is substantially lower than in other hubs and therefore the fcfs provider finds it optimal to lower prices for small-scale farmers and attract them to maximize capacity utilization.

Table 6: Status Quo Equilibrium Outcome

(a) Average Queue Length

	small		large	
	fcfs	mkt	fcfs	mkt
1	2.0	2.6	1.0	1.7
3	3.0	1.8	1.0	2.1
4	3.5	2.9	1.0	1.1
5	4.0	1.5	1.1	4.2
6	3.5	1.5	1.1	3.0
7	4.3	2.8	3.6	2.2
8	4.3	1.9	1.1	2.6
9	3.3	1.8	1.2	2.4
10	4.3	2.0	1.1	2.5
11	2.0	2.7	1.0	1.6
12	2.0	4.8	1.2	0.1
13	2.0	2.6	1.0	1.6
15	2.5	1.4	1.1	2.8
17	3.5	1.4	1.1	3.3
18	4.0	1.6	1.2	3.9
19	4.0	1.9	1.0	2.3

(b) Rental Rates

	small		large	
	fcfs	mkt	fcfs	mkt
1	0.69	0.69	0.51	0.51
2	0.56	0.61	0.47	0.49
3	0.63	0.67	0.49	0.50
4	0.67	0.71	0.55	0.55
5	0.67	0.70	0.53	0.53
6	0.75	0.77	0.54	0.54
7	0.70	0.73	0.54	0.54
8	0.66	0.69	0.52	0.53
9	0.69	0.71	0.54	0.54
10	0.70	0.70	0.51	0.51
11	0.73	0.74	0.53	0.54
12	0.74	0.74	0.52	0.52
13	0.65	0.70	0.52	0.52
14	0.61	0.67	0.54	0.54
15	0.71	0.74	0.54	0.54
16	0.61	0.64	0.47	0.49

Panel (a) reports equilibrium average queue lengths by hub for farmers of different scale and different providers. Panel (b) reports the equilibrium rental rates per hour for the relevant implement.

5.3 Accommodating additional heterogeneity

Given the heterogeneity in location and demand characteristics (machine-hours of service) that is not directly captured by the stylized equilibrium queueing model, we recur to simulation exercises to assess the cost of delayed rental service provision to farmers as well as to further explore heterogeneity in the value of service and incentives to service provision.

Using the equilibrium queue lengths by provider and demand scale we accommodate additional heterogeneity within each scale category by simulating queues of orders following the empirical distribution of machine-hours demanded and location within a scale category.[15]On the supply side, we solve for service dispatch system through two possible delivery routes. One follows a "hub and spoke" pattern, under which the equipment must return to the CHC between two orders. The other route solves a "Traveling Salesman Problem (TSP)" where the implement travels optimallly from order to order within the day. Under fcfs, this means that the provider follows the route that minimizes the travel time within a given day for a given set of requests (and their order) in the queue. Under the market allocation, the set of requests being served is jointly determined with the best service route. The value of an order in the market allocation depends on the density of orders around them, and the size of the order relative to the CHC serving capacity.

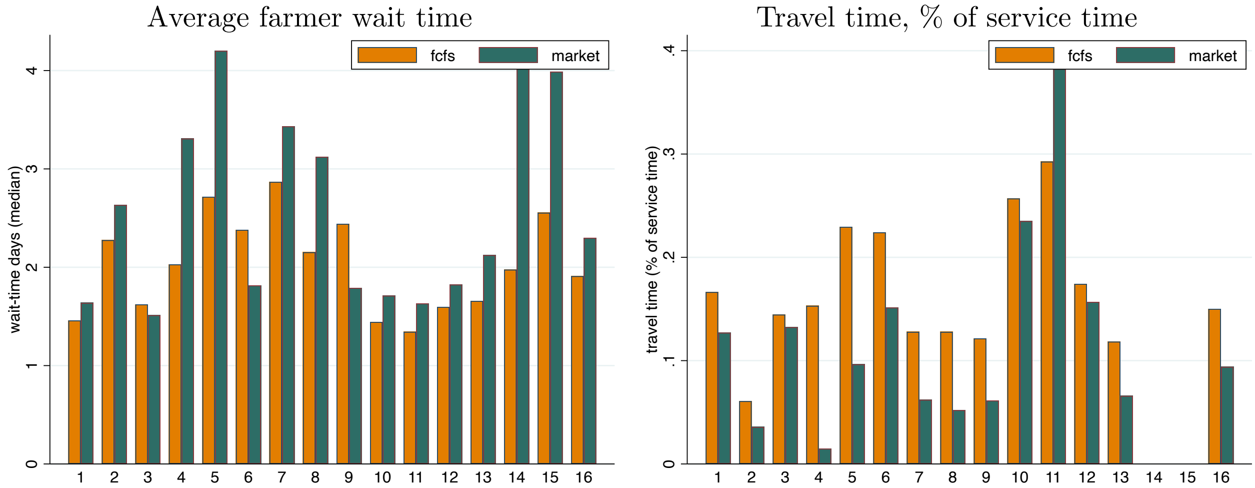
We fix the number of orders each provider can serve in a day to 3, in line with the maximum number of orders that we observe being served by a driver in the administrative data.[16] Then we estimate the value function for each provider, i.e. a function that maps any queue of orders to their service value, conditional on the dispatch system and the delivery route.

Using the empirical joint distributions of machine-hours and location we sample (with replacement) 1000 queues per provider. Each order in the queue is a three dimensional object that includes the machine-hours demanded, the location of the plot (or the travel distance to the hub), and the productivity of the farmer that requested service. We use the empirical correlation between farmer productivity and machine-hours demanded to discipline this third

[15]We adjust the observed ratio of orders from large and small scale farmers to match the ratio of the equilibrium queue lengths.

[16]As we explain in Appendix C, this is a high dimensional problem, and the number of possible combinations of orders to be served within a period grows exponentially with the number of orders in the queue and its characteristics (including hours serviced and location, i.e., latitude and longitude).

Figure 7: Time management by hub



dimension of heterogeneity, see Table 4.[17] Finally, in reporting results, we discretize the space of machine-hours demanded in five bins and the space of travel times in two bins, far and near.

5.3.1 The cost of delays

Figure 7 compares waiting times for different dispatch system and demand characteristics.

5.4 Market Deregulation

We now study the effect of market deregulation through counterfactuals. We first explore a scenario in which the technology that allows the market providers to prioritize large farmers is made available to the fcfs providers. Because the prioritization is costless and the fcfs providers are at least as well off as before (i.e. they can now prioritize the high marginal return orders), a profit driven fcfs provider would choose to adopt the technology, i.e. $h = 0$.[18] In other words, there is no longer any differentiation between these two types of providers.

[17] As robustness, we simulate outcomes when we assume no correlation between farm productivity and order sizes, and when we flip the sign of the empirical correlation between machine-hours and productivity within the catchment area of a hub.

[18] As we pointed out before, the value of service for market providers is always above the one for first-come-first-served providers, and therefore each provider has incentives to adopt a dispatch system that prioritizes large orders.

Figure 8: Demand characteristics by provider

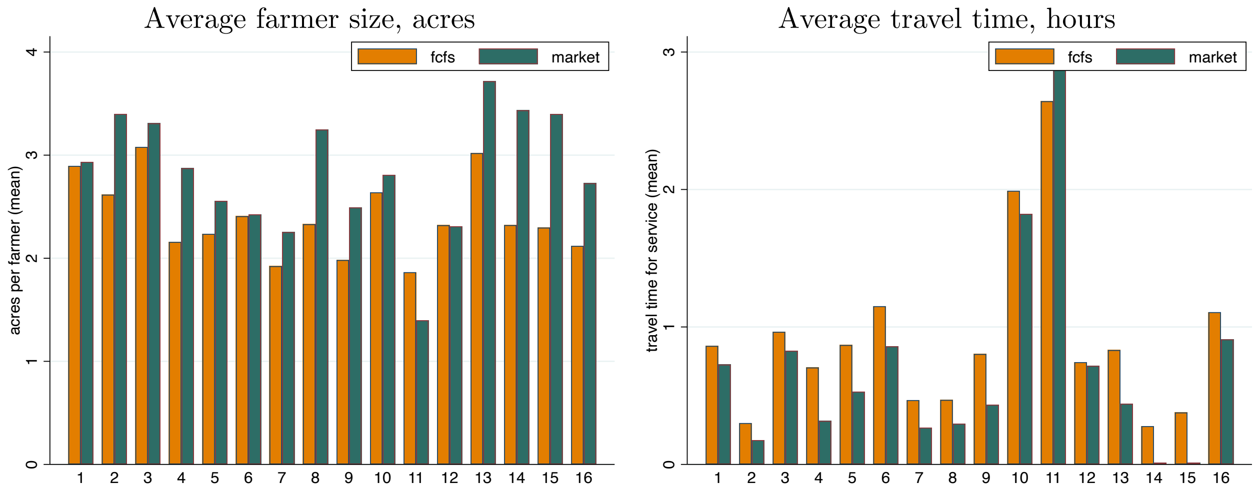
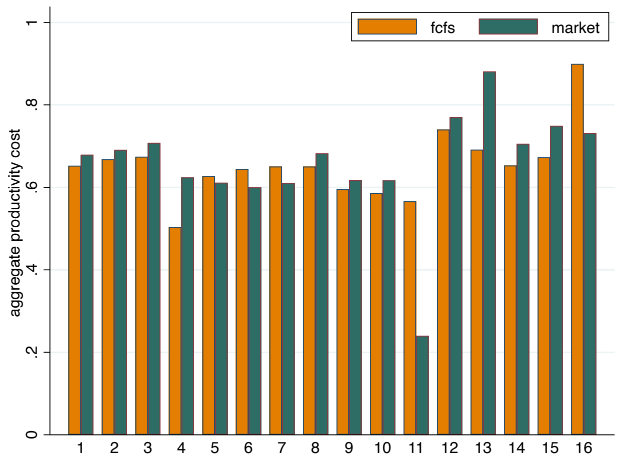


Figure 9: Productivity costs of delays



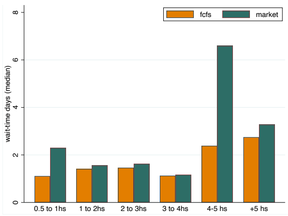


fig1: Hub 1

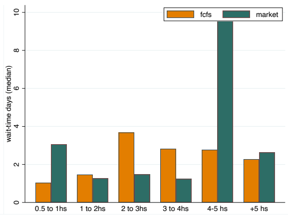


fig2: Hub 2

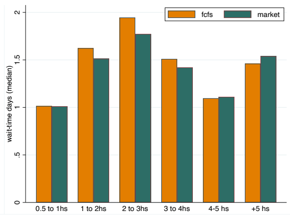


fig3: Hub 3

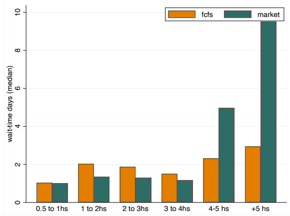


fig4: Hub 4

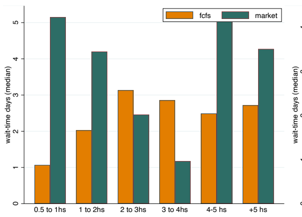


fig5: Hub 5

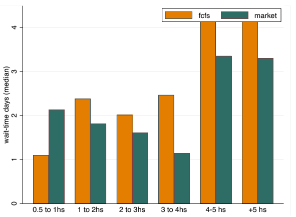


fig6: Hub 6

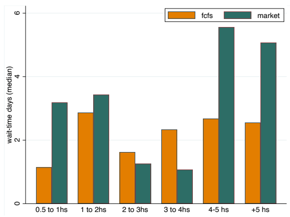


fig7: Hub 7

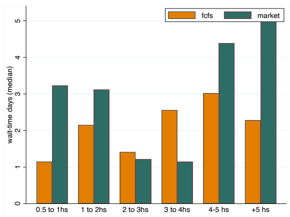


fig8: Hub 8

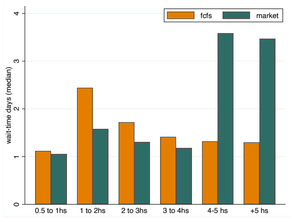


fig9: Hub 9

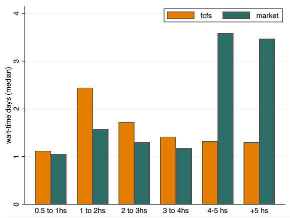


fig10: Hub 10

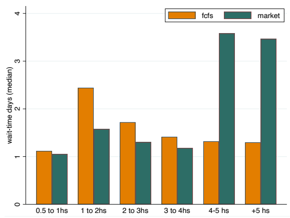


fig11: Hub 11

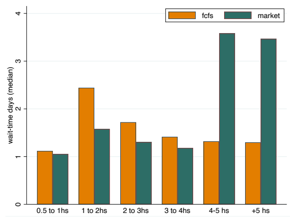


fig12: Hub 12

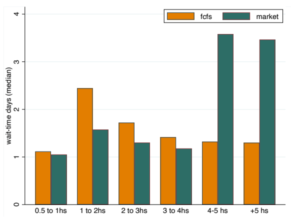


fig13: Hub 13

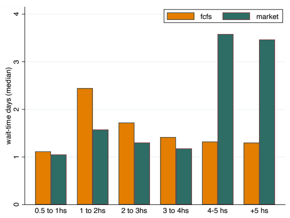


fig14: Hub 14

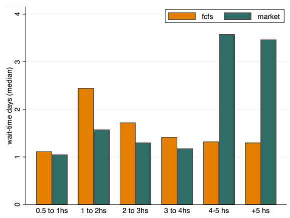


fig15: Hub 15

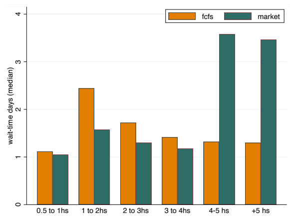


fig16: Hub 16

Figure 10: Distribution of delays by demand category for each hub.

The nature of the equilibrium may however change. We study the equilibrium effects in two scenarios: first we do not allow providers to exit or enter the market, i.e. the short-run; and second, we allow this margin to change until equilibrium expected profits are exhausted. [19]

Since the expected per period profit of serving a small farmer is negative as demonstrated in the first case, providers will choose to price the small farmers out in the absence of government intervention. However, given the small number of large farmers, there will be

[19] In a competitive search model entry and exit occurs until firms' net profit is zero. Because we have assumed no entry costs, net profits are normalized to zero. We could alternatively estimate entry costs given the calibrated model from the observed queues.

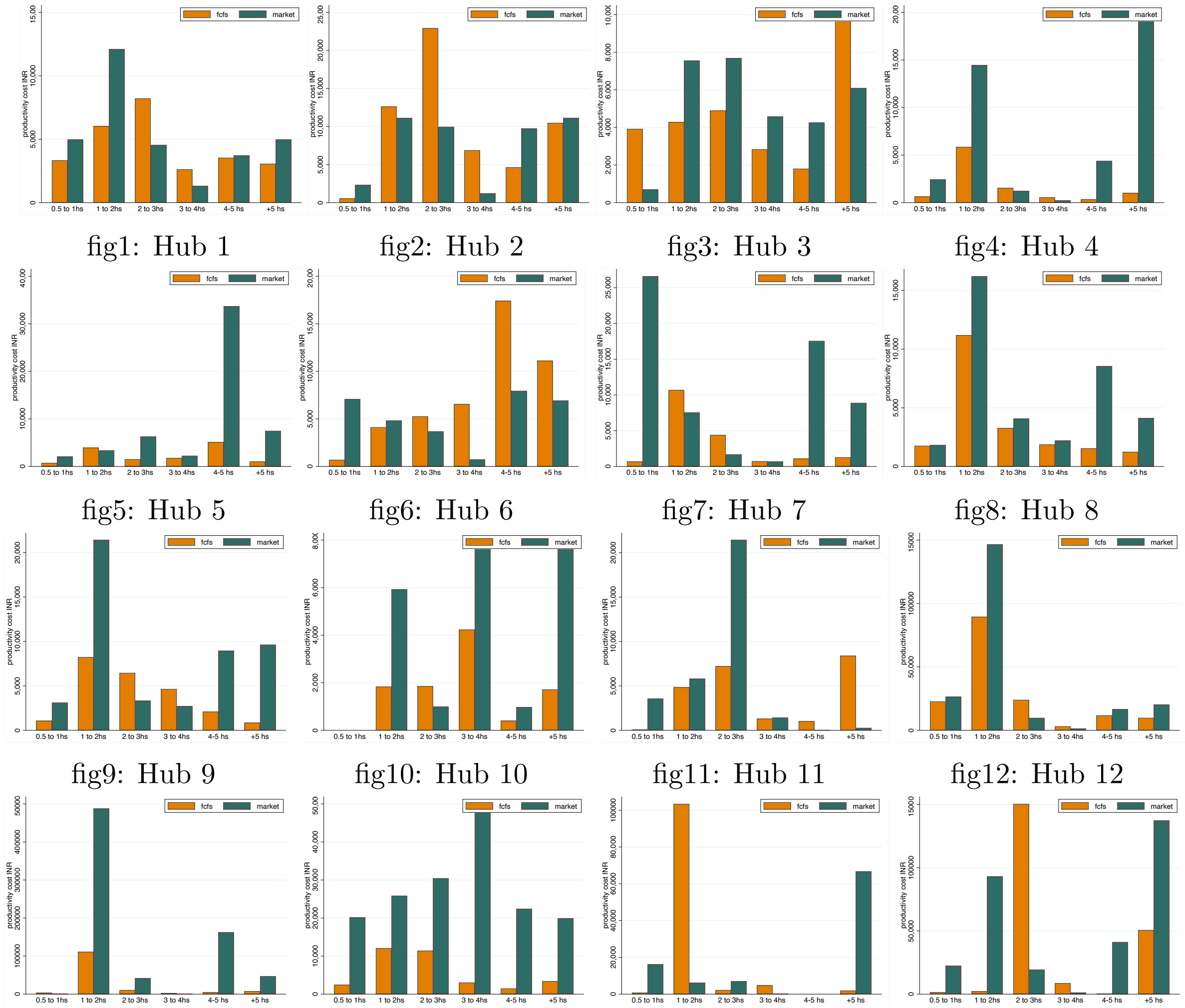


fig1: Hub 1 fig2: Hub 2 fig3: Hub 3 fig4: Hub 4

fig5: Hub 5 fig6: Hub 6 fig7: Hub 7 fig8: Hub 8

fig9: Hub 9 fig10: Hub 10 fig11: Hub 11 fig12: Hub 12

fig13: Hub 13 fig14: Hub 14 fig15: Hub 15 fig16: Hub 16

Figure 11: Distribution of productivity costs per acre by demand category for each hub.

over capacity in the market which leads to negative profits.

In the long run, exits take place until the expected profit increases to zero. Panel 3 of Table ?? shows that after the capacity reduction caused by exits, the capacity per farmer (measured by $\frac{1}{f}$) falls by 88% from 0.26 to 0.03. The dramatic reduction in capacity causes the queue length to increase by over 5 times for the large farmers who continue to be served. As a result, they experience much longer waiting time (see Fig. ??).

Figure 12 plots the change in average farm sizes served and the travel time for service, i.e. the change in the distribution of served location, as the market deregulates. Exit of

Table 7: Long-run impact on service capacity

Farmers per provider, f		
status quo	*adjusted*	*long-run*
2.29	*0.46*	*3.42*
2.27	*0.45*	*3.42*
3.56	*0.71*	*4.10*
4.57	*0.69*	*6.83*
4.88	*0.73*	*6.83*
4.90	*0.74*	*6.83*
3.83	*0.57*	*6.83*
4.89	*1.22*	*6.83*
2.27	*0.57*	*4.10*
2.27	*0.79*	*4.10*

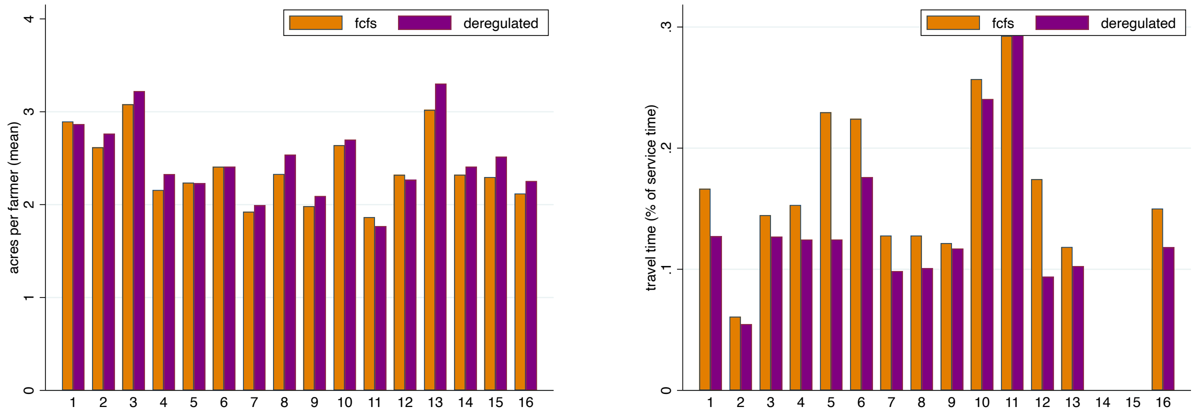
Column (1) presents the calibrated ratio of farmers to providers. Column (2) adjust the ratio to only account for large-scale farmers, which are the only ones served in the long-run. Column (3) presents the results ratio after exit is allowed so that net profits are eliminated.

providers induces an increase in the average size of the farm served by each provider, and a reduction in the travel time to service, consistently with providers prioritizing services with low marginal cost of provision. For most hubs, the travel time as a proportion of the service time more than halves.

The first two panels of Table **??** compare the equilibrium queue lengths, rental rates and expected per period profit of serving one farmer ($V_{i,\text{mkt}}$) under the status quo equilibrium and under the short-run market equilibrium. The rental rates charged to both types of farmers under short-run market equilibrium are significantly lower. This is primarily due to the fact that without any differentiation in services, providers compete heads-on with one another, which significantly diminishes their pricing power. Note that the expected per period profit of serving a small farmer is now negative. This suggests that a profit maximizing provider will not serve small farmers.

On the flip side of the service efficiency gains from the provider we can assess the cost to farmers. Wait times raise by 10% relative to the short run equilibrium, on average across hubs, while the productivity cost per acre, relative to the mean productivity in the catchment area of a hub declines by 8.6%. The reason is that due to the shift of service to larger plots, a slight increase in wait times can be associated with a decline in the cost of service per acre on average.

Figure 12: Impact of Deregulation



Farmers' size, acres | Travel time, % service time

The counterfactual analysis appears to suggest that the impact from the Indian government's intervention in setting up an fcfs equipment rental system is mixed. On the one hand, by setting up a system that follows a different dispatching system, small farmers that would not otherwise have access to the market are now served. Further, both large and small farmers benefit from the product differentiation. In fact, the product differentiation significantly increases the farmers' willingness to pay, leading to a higher profit level for both the fcfs and the market providers. On the other hand, once the government stops regulating the fcfs providers and allows them to become profit driven, they will adopt the dispatching system of the market providers. This will lead to strong competition among all providers and suppress prices to small farmers to the point that it is no longer profitable to serve them. The low level of demand will cause providers to exit in the long run, leading to a significant lower market capacity. Small farmer lose access to the rental services. Large farmers do pay a lower price, but suffer from a longer wait time and greater productivity losses.

TO BE COMPLETED

6 Conclusion

Rental markets hold considerable promise in expanding mechanization access and increasing productivity in the farming sector. However, the spacial distribution of demand in space and its synchronous nature, as well as the fixed supply capacity, pose interesting trade offs between efficiency and market access. The returns to these rental markets depend crucially on factors such as density, i.e. the proximity of suppliers to farmers, the overall supply capacity, and the ability to optimize traveling equipment time. In this paper, we document and quantify how these factors determine the allocative efficiency and distributional effects of rental markets.

We find that when the government increases service capacity by subsidizing the purchase of equipment from rental service provision, and at the same time imposes a first-come-first-serve dispatch system to allocate services, it induces misallocation in service provision. Indeed, when equipment owners are allowed to behave optimally, by prioritizing larger scale orders (which are the most cost effective) the equilibrium returns to one where smallholder farmers are rationed out of the market.

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A Characterization of the probability of service.

A.1 Probability of service conditional on queueing.

Consider a farmer of type i that requests a service from a provider j given that other farmers request services with probability $p_{i,j}$. The probability that a given farmer is being served given that the provider chooses one farmer of type i and he is queuing with this provider is $\tilde{\Delta}_{ij}(1)$, i.e.

$$\tilde{\Delta}_{ij}(1) = \sum_{n=0}^{f_i-1} \binom{f_i - 1}{n} p_{ij}^n (1 - p_{ij})^{f_i-1-n} \frac{1}{n+1},$$

where f_i is the number of farmers of type i searching for a provider, $f_s = sF$ and $f_{s^-} = (1-s)F$ and $\binom{f_i-1}{n} = \frac{f_i-1!}{n!(f_i-1-n)!}$. Hence,

$$\tilde{\Delta}_{ij}(1) = \frac{1-(1-p_{ij})^{f_i}}{f_i p_{ij}}.$$

As the number of agents in the economy gets large, and using the definition of queue lengths above, the service probability simplifies to

$$\tilde{\Delta}_{ij}(1) = \frac{1-e^{-q_{ij}}}{q_{ij}}.$$

That is, a given farmer of type i is served if at least one farmer of type i has requested a service, which occurs with probability $1-e^{-q_{ij}}$, divided by the number of requests of a given type, q_{ij}.

Next, consider the probability of a given farmer being served when the provider serves $\bar{o} = 2$ orders of type i, $\tilde{\Delta}_{ij}(2)$. Similar computations to those above yield a service probability as follows

$$\tilde{\Delta}_{ij}(2) = 2(\frac{1-e^{-q_{ij}}}{q_{ij}}) - e^{-q_{ij}}.$$

Finally, consider the probability of a given farmer being served when the provider serves $\bar{o} = 3$ orders of type i, $\tilde{\Delta}_{ij}(3)$, which follows

$$\tilde{\Delta}_{ij}(3) = 3(\frac{1-e^{-q_{ij}}}{q_{ij}}) - 2e^{-q_{ij}} - e^{-q_{ij}}q_{ij}$$

In what follows we characterize the probability that a provider of type “j” services a farmer of type “i” given that a farmer of type “i” is standing in the queue.

First-come-first-served. The fcfs provider only considers feasibility and the position in the queue. Let the probability of serving $\bar{o}$ farmer of type i be $\phi_{i,fcfs}(\bar{o})$. Given the queue lengths at this provider, there are ${}^{q_s+q_{s^-}}P_o = \frac{q_s+q_{s^-}!}{(q_s+q_{s^-}-o)!}$ possible permutations for the o-tuple, (the provider identifier has been dropped for notational convenience). Under Assumption 1, a fcfs provider serves a single large-scale farmer if one of the large-scale farmers are among the first three positions in the queue and at least one has applied. Let this probability be $\hat{\phi}_{s,fcfs}(1) \equiv 3q_s \frac{{}^{q_{s^-}}P_2}{{}^{q_s+q_{s^-}}P_3}$.

$$\phi_{s,fcfs}(1) = \psi_{s,fcfs}(1)\hat{\phi}_{s,fcfs}(1),^{20}$$

where $\psi_{s,fcfs}(1) \equiv (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$ is the probability of having at least three orders in the queue of which at least two are of type s^-, when a single farmer of type s has requested service. To this probability we should add the probability of service when less than $o = 3$ farmers apply for service, $\hat{\psi}_{s,fcfs}(1) \equiv \left(e^{-q_{s,fcfs}}(e^{-q_{s^-,fcfs}} + q_{s^-,fcfs}e^{-q_{s^-,fcfs}})\right)$ which is the probability of service of large scale order when there are no other service request or there is exactly one additional order requested.

A fcfs provider services 2 large-scale farmers if there are two or more large-scale orders in the first o positions of the queue and at least two large scale farmers have applied . Let the first probability be $\hat{\phi}_{s,fcfs}(2) \equiv 3q_s(q_s-1)\frac{{}^{q_s-2+q_{s^-}}P_1}{{}^{q_s+q_{s^-}}P_3}$

$$\phi_{s,fcfs}(2) = \psi_{s,fcfs}(2)\hat{\phi}_{s,fcfs}(2),$$

where $\psi_{s,fcfs}(2) = (1 - e^{-q_{s,fcfs}} - e^{-q_{s^-,fcfs}}q_s e^{-q_{s,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s requesting service, of which at least two are of type s (including the one requesting service). [21] To this probability we should add the probability that there are only two large-scale farmers in the queue $\hat{\psi}_{s,fcfs}(2) \equiv \left(q_{s,fcfs}e^{-q_{s,fcfs}}e^{-q_{s^-,fcfs}}\right)$.

Given feasibility, the fcfs provider never serves 3 large-scale orders, $\phi_{s,fcfs}(3) = 0$.

A fcfs provider serves a single small-scale farmer if there is one of them in the first o positions of the queue. This probability is defined analogously to its counterpart for large scale orders, exchanging indexes,

$$\phi_{s^-,fcfs}(1) = \psi_{s^-,fcfs}(1)\hat{\phi}_{s^-,fcfs}(1),$$

[20]Note that $\hat{\phi}_{s,fcfs}(i)$ are not the expected probabilities, but rather the probability conditional on the observed queue length. We can numerically show that when $F, H \to \infty$ these two are arbitrarily close.

[21]This is the probability that at least another large scale and at least one small scale farmer request service, or at least two other large scale farmers request service.

and adding the probability $\hat{\psi}_{s^-,fcfs}(1)$ when there are less than three orders.

A fcfs provider services 2 small-scale farmers if at least two small-scale orders in the first o positions of the queue, $\hat{\phi}_{s^-,fcfs}(2) \equiv 3\frac{q_{s}-(q_{s^-}-1)q_s}{q_s+q_{s^-}P_3}$

$$\phi_{s^-,fcfs}(2) = psi_{s^-,fcfs}(2)\hat{\phi}_{s^-,fcfs}(2),$$

where $\psi_{s^-,fcfs}(2) = (1 - e^{-q_{s,fcfs}})(1 - e^{-q_{s^-,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s^- requesting service, of which at least one is of type s and at least two are of type s^- (including the one requesting the service). To this probability we should add the probability that there are only two orders in the queue, $\hat{\psi}_{s^-,fcfs}(2)$ defined analogously than for large-scale farmers.

A fcfs provider services 3 small-scale farmers if there are three small-scale orders in the first o positions of the queue. This probability is defined as $\hat{\phi}_{s^-,fcfs}(3) = \frac{q_{s^-}P_3}{q_s+q_{s^-}P_3}$

$$\phi_{s^-,fcfs}(3) = \psi_{s^-,fcfs}(3)\hat{\phi}_{s^-,fcfs}(3),$$

where $\psi_{s^-,fcfs}(3)$ is the probability of having at least two other small scale requests, i.e. $\psi_{s^-,fcfs}(3) = (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$.

The general form for the probability of service is,

$$\Delta_{i,fcfs} = \sum_{\bar{o}=1}^{3} \hat{\psi}_{i,fcfs}(\bar{o}) + \phi_{i,fcfs}(\bar{o})\tilde{\Delta}_{i,fcfs}(\bar{o}), \tag{8}$$

where we have defined $\hat{\psi}_{s,fcfs}(3) \equiv 0$ to ease notation.

The main difference in the probability of service for large and small relies on the queue lengths. If the queue lengths are identical, then a first-come-first-served provider serves both types of farmers with the same probability, $\sum_{\bar{o}=2}^{3} \phi_{s^-,fcfs}(\bar{o}) = \sum_{\bar{o}=2}^{3} \phi_{s,fcfs}(\bar{o})$.

Market. The market provider has a technology that allows him to prioritize farmers of either type. The probability of interest is the probability that exactly $\bar{o}$ farmers of type i are served conditional on the farmer under consideration having applied and at least 3 farmers of either type requesting service to the provider.

Conditional on a large farmer having applied, a single large-scale farmer is served by a market provider if the provider does not prioritize large scale farmers and there is one large-scale order among the first o available positions, which happens with probability $(1 - \chi)\tilde{\phi}_{s,mkt}(1) = (1-\chi)\phi_{s,fcfs}(1)$; or if the provider prioritizes large scale farmers and no other large-scale farmer requested service, $\chi\psi_{s,mkt}(1) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$.

These service probabilities add up to,

$$\phi_{s,mkt}(1) = \chi(\psi_{s,mkt}(1)) + (1-\chi)\tilde{\phi}_{s,mkt}(1),$$

where we should the events when there are less than three orders $\hat{\psi}_{i,mkt}(1) = \hat{\psi}_{i,fcfs}(1)$ for any i=s, s^- by definition.

Two large-scale farmers are served by a market provider if he does not prioritize large orders and they stand in the first 3 positions, which happens with probability $(1-\chi)\tilde{\phi}_{s,mkt}(2) = (1-\chi)\phi_{s,fcfs}(2)$; or if the provider prioritizes those orders and there is at least one additional large-scale service request, which happens with probability $\chi\psi_{s,mkt}(2) = \chi(1 - e^{-q_{s,mkt}} - e^{-q_{s^-,mkt}}q_s e^{-q_{s,mkt}})$.[22] These service probabilities add up to

$$\phi_{s,mkt}(2) = \chi(\psi_{s,mkt}(2)) + (1-\chi)\tilde{\phi}_{s,mkt}(2).$$

to this probability we add $\hat{\psi}_{s,mkt}(2)$ defined analogously than for the fcfs provider.

Feasibility prevents three large-scale orders to be served within the period and therefore, $\phi_{s,mkt}(3) = 0$.

Analogous arguments can be used to describe the probabilities of service of small scale farmers. A single small-scale farmer is always served by a market provider (conditional on a request) if it prioritizes high-scale requests and at least two large scale farmers have requested service, which occurs with probability $\chi\psi_{s^-,mkt}(1) = \chi(1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}}$; or if the provider does not prioritize high-scale requests and there is a single small-scale order among the first three orders in the queue, $(1-\chi)\tilde{\phi}_{s^-,mkt}(1)$, where $\tilde{\phi}_{s^-,mkt}(1) = \phi_{s^-,fcfs}(1)$. The reason for always serving a small scale order even when prioritizing large scale is that capacity constraints allow the provider to served at most $o - 1$ orders leaving always and idle slot. To these probabilities we add those associated to the event when there are strictly less than two orders in the queue.

$$\phi_{s^-,mkt}(1) = \chi(\psi_{s^-,mkt}(1)) + (1-\chi)\tilde{\phi}_{s^-,mkt}(1),$$

to what we add $\hat{\psi}_{s^-,mkt}(1)$.

Two small-scale farmers are served by a market provider if it prioritizes high-scale requests and exactly one large-scale farmer requests service and at least another small scale farmer requests service, which occurs with probability $\chi\psi_{s^-,mkt}(2) = \chi q_{s,mkt}e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}})$. Alternatively, two small-scale farmers are served if the provider does not prioritize large-scale

[22] If there are more large-scale orders the provider still serves two because of its capacity constraints.

orders and there are two small-scale orders among the first three orders in the queue.

$$\phi_{s^-,mkt}(2) = \chi(\psi_{s^-,mkt}(2)) + (1-\chi)\tilde{\phi}_{s^-,mkt}(2).$$

To these probabilities we add those associated to the event when there are strictly less than two orders in the queue, $\hat{\psi}_{s^-,mkt}(2)$.

Three small-scale farmers are served by the market provider if it prioritizes high-scale requests and no large-scale farmer requests service and there are at least three small requests, which occurs with probability $\chi\psi_{s^-,mkt}(3) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$, or if it does not prioritize them and there are three small-scale orders among the first three in the queue,

$$\phi_{s^-,mkt}(3) = \chi\psi_{s^-,mkt}(3) + (1-\chi)\tilde{\phi}_{s^-,mkt}(3).$$

In sum, the probability of service for a market provider follows

$$\Delta_{i,mkt} = \sum_{\bar{o}=1}^{3} \hat{\psi}_{i,mkt}(\bar{o}) + \phi_{i,mkt}(\bar{o})\tilde{\Delta}_{i,mkt}(\bar{o}). \tag{9}$$

Following equations 8 and 9, the probability of being served for a given type i (weakly) declines in the queue length of the other type of farmers. In the first-come-first-served provider the result is straightforward. For the market provider, the decline in the probability of service is strict for the small scale farmers and independent of the queue length of small-scale orders when the provider prioritizes large-scale orders.

A.2 Unconditional probabilities of service.

The unconditional probabilities of service are important in characterizing the value of service for each provider. We consider alternative scenarios, i.e. when the provider serves at capacity ($o = 3$ orders) and when the provider serves less than capacity.

The probability of serving at least an order is

$$\hat{\Phi} = (1 - e^{-q_{sj}}) + (1 - e^{-q_{s^-j}})$$

The first-come-first-served provider can serve three orders of small scale (given feasibility) with a service probability of

$$\Phi_{s^-,fcfs}(3) = \left(1 - e^{-q_{s^-,j}}(1 + q_{s^-,j} + \frac{1}{2}q_{s^-,j}^2)\right)\frac{q_{s^-,j}P_3}{q_{s,j}+q_{s^-,j}P_3};$$

or to serve two orders of one type and one of another, with probability

$$\bar{\Phi}_{i,fcfs}(1) = (1 - e^{-q_{i,j}})(1 - e^{-q_{i',j}} - q_{i',j}e^{-q_{i',j}})\frac{3q_{i,j}q_{i',j}(q_{i',j} - 1)}{{}_{q_{i,j}+q_{i',j}}P_3},$$

and $\bar{\Phi}_{i',fcfs}(2) = \bar{\Phi}_{i,fcfs}(1)$ for $i' \neq i$.

The provider can also serve two orders of large size, (either because he received only two orders, or because all orders in the queue are of large scale)

$$\tilde{\Phi}_{s,fcfs}(2) = \left(1 - e^{-q_{s,fcfs}} - q_{s,fcfs}e^{-q_{s,fcfs}}\right) e^{-q_{s-,fcfs}},$$

or it can receive exactly two orders of small size and serve those,

$$\tilde{\Phi}_{s^-,fcfs}(2) = \frac{1}{2}q_{s^-,j}^2 e^{-q_{s^-,j}}(e^{-q_{s,j}}).$$

Finally, the provider can serve two orders, one of each type

$$\tilde{\Phi}_{i,fcfs}(1_2) = \left(q_{i,j}e^{-q_{i,j}}q_{i',j}e^{-q_{i',j}}\right).$$

or only one order, with occurs with probability

$$\tilde{\Phi}_{i,fcfs}(1_1) = \left(q_{i,j}e^{-q_{i,j}}e^{-q_{i',j}}\right).$$

The probabilities for the market provider are similar to the ones above, except that we need to account for the market provider's ability to select large scale orders.

The market provider can serve three orders of small scale,

$$\bar{\Phi}_{s^-,mkt}(3) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}} - \frac{1}{2}q_{s^-,mkt}^2 e^{-q_{s^-,mkt}}) + (1-\chi)\Phi_{s^-,fcfs}(3), \tag{10}$$

or the orders of large scale and one small,

$$\bar{\Phi}_{s,mkt}(2) = \chi((1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}})(1 - e^{-q_{s^-,mkt}}) + (1-\chi)\bar{\Phi}_{s,fcfs}(2), \tag{11}$$

or two orders of small scale and on large,

$$\bar{\Phi}_{s^-,mkt}(2) = \chi q_{s,mkt}e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}}) + (1-\chi)\bar{\Phi}_{s^-,fcfs}(2). \tag{12}$$

When there are less than three orders in the queue there is no need to prioritize orders, and therefore the probabilities of service are identical to those characterized for the FCFS problem, i.e. $\tilde{\Phi}_{i,mkt} = \tilde{\Phi}_{i,fcfs}$.

Expected value of service provision The characterization of the probability allows us to compute the expected value of service provision:

$$
\begin{aligned}
\tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{\mathbf{q}_{\text{fcfs}}}, \{(r_{i,\text{fcfs}} - w)k_i - wE(d_i)\}_{i=s,s^-}\right) \equiv \\
\hat{\Phi}^{-1}\{\sum_{i=s,s^-} \bar{\Phi}_{i,fcfs}(2)\left[2\left((r_{i,fcfs} - w)k_i - wE(d_i)\right) + (r_{i',fcfs} - w)k_{i'} - wE(d_{i'})\right] + \\
\Phi_{s^-,fcfs}(3)3((r_{s^-,fcfs} - w)k_{s^-} - wE(d_{s^-})) + \\
\tilde{\Phi}_{i,fcfs}(1_1)\left((r_{i,fcfs} - w)k_i - wE(d_i)\right) + \\
+\tilde{\Phi}_{i,fcfs}(1_2)\left((r_{i,fcfs} - w)k_i - wE(d_i) + (r_{i',fcfs} - w)k_{i'} - wE(d_{i'})\right) + \\
\tilde{\Phi}_{i,fcfs}(2)2\left((r_{i,fcfs} - w)k_i - wE(d_i)\right)\}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{(\mathbf{q}_{\text{mkt}},\chi)}, \{(r_{i,\text{mkt}} - w)k_i - wE(d_i)\}_{i=s,s^-}\right) \equiv \\
\hat{\Phi}^{-1}\{\sum_{i=s,s^-} \bar{\Phi}_{i,mkt}(2)\left[2\left((r_{i,mkt} - w)k_i - wE(d_i)\right) + (r_{i',mkt} - w)k_{i'} - wE(d_{i'})\right] + \\
\Phi_{s^-,mkt}(3)3((r_{s^-,mkt} - w)k_{s^-} - wE(d_{s^-})) + \\
\tilde{\Phi}_{i,mkt}(1_1)\left((r_{i,mkt} - w)k_i - wE(d_i)\right) + \\
+\tilde{\Phi}_{i,mkt}(1_2)\left((r_{i,mkt} - w)k_i - wE(d_i) + (r_{i',fcfs} - w)k_{i'} - wE(d_{i'})\right) + \\
\tilde{\Phi}_{i,mkt}(2)2\left((r_{i,mkt} - w)k_i - wE(d_i)\right)\}.
\end{aligned} \tag{14}
$$

where the first two terms in either expression correspond to the expected value of serving three orders or different types, while the remaining terms correspond to the expected returns of serving strictly less than three orders. Also note that the scaler $\hat{\Phi}^{-1}$ is inconsequential to the optimality conditions of the problem (and therefore omitted in the derivations that follow).

B Proofs

B.1 Proposition 1

First, we solve for the equilibrium value of service when both farmers queue with both providers. Then, we show the value when the service provider serves a single type of farmers. Then we show that the guess that the selection criteria for the market provider should be to prioritize large scale orders. Finally, we show that the expected value of service is higher for large scale farmers.

Value of service when serving both type of farmers.

Proof. Using the definition of the expected value of service provision (equations 13 and 14) and rearranging terms, as well as the participation constraint for the farmers, equation 4, the problem of the provider is

$$\max_{q_{s,j},q_{s^-,j}} \sum_{i=s,s^-} \Phi_{i,j}(2)[zk_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i + E(d_i))] +$$
$$\sum_{i=s,s^-} \Phi_{i,j}(1)[zk_{i'}^\alpha - \frac{U_{i'}}{\Delta_{i'j}} - w(k_{i'} + E(d_{i'}))] +$$
$$\Phi_{s^-,j}(3)3[zk_{s^-}^\alpha - \frac{U_{s^-}}{\Delta_{s^-j}} - w(k_{s^-} + E(d_{s^-}))],$$

where $\Phi_{i,j}(2) = (\bar{\Phi}_{i,j}(2)2 + \tilde{\Phi}_{i,j}(2)2 + \tilde{\Phi}_{i,j}(1_1) + \tilde{\Phi}_{i,j}(1_2))$ and $\Phi_{i,j}(1) = (\bar{\Phi}_{i,j}(2) + \tilde{\Phi}_{i,j}(1_2))$.

Let $\bar{V}_{ij}$ be the profit per order of type i for provider j, i.e. $\bar{V}_{ij} \equiv \big((r_{i,j} - w)k_i - wE(d_i)\big)$. The optimality condition swith respect to the queue length of large-scale and small-scale farmers are

$$\sum_{i=s,s^-}\left(\frac{\partial \Phi_{i,j}(2)}{\partial q_{s,j}}\bar{V}_i + \frac{\partial \Phi_{i,j}(1)}{\partial q_{s,j}}\bar{V}_{i'}\right) + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}}\bar{V}_{s^-} +$$
$$\sum_{i=s,s^-}\left(\Phi_{i,j}(2)\frac{\partial \bar{V}_i}{\partial q_{s,j}} + \Phi_{i,j}(1)\frac{\partial \bar{V}_{i'}}{\partial q_{s,j}}\right) + \Phi_{s^-,j}(3)3\frac{\partial \bar{V}_{s^-}}{\partial q_{s,j}} = 0, \tag{15}$$

$$\sum_{i=s,s^-}\left(\frac{\partial \Phi_{i,j}(2)}{\partial q_{s^-,j}}\bar{V}_i + \frac{\partial \Phi_{i,j}(1)}{\partial q_{s^-,j}}\bar{V}_{i'}\right) + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}}\bar{V}_{s^-} +$$
$$\sum_{i=s,s^-}\left(\Phi_{i,j}(2)\frac{\partial \bar{V}_i}{\partial q_{s^-,j}} + \Phi_{i,j}(1)\frac{\partial \bar{V}_{i'}}{\partial q_{s^-,j}}\right) + \Phi_{s^-,j}(3)3\frac{\partial \bar{V}_{s^-}}{\partial q_{s^-,j}} = 0, \tag{16}$$

where

$$\frac{\partial \bar{V}_i}{\partial q_{i,j}} = -\left[\frac{\bar{V}_i + w(k_i + E(d_i)) - zk_i^\alpha}{\Delta_{ij}}\right]\left(\frac{\partial \Delta_{i,j}}{\partial q_{ij}}\right).$$ [23]

[23] If we account for the cost of expected delays, then the envelope condition is $-\frac{\partial \Delta_{i,j}}{\partial q_{ij}}\frac{1}{\Delta_{ij}}\left[\frac{U_i}{\Delta_{ij}} - \frac{\partial z}{\partial \Delta_{ij}}\frac{\Delta_{ij}}{z}zk_i^\alpha\right]$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q}\frac{q_{sj}}{\Delta_{sj}(o)}$, let the elasticity of the value of service to the queue length be $\zeta_{\bar{V}q} \equiv \frac{\partial \bar{V}}{\partial q_{ij}}\frac{q_{ij}}{\bar{V}}$ and that of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q}\frac{q}{\psi(o)}$.

Then, the envelope condition indicates that the elasticity of the value of service to the queue length is an inversely proportional function of the elasticity of the probability of service to the queue length, $\zeta_{\Delta ij}$.

$$\zeta_{\bar{V}_i} = \left(1 + \frac{w(k_i + E(d_i)) - zk_i^{\alpha}}{\bar{V}_i}\right)\zeta_{\Delta ij}.$$

Equations 15 and 16 form a system of linear equations that can be solved for the two unknowns $\bar{V}_{s^-}, \bar{V}_s$ as a function of the queue lengths.[24]

$$\Gamma \begin{bmatrix} \bar{V}_s \\ \bar{V}_{s^-} \end{bmatrix} = a \begin{bmatrix} zk_s^{\alpha} - w(k_s + E(d_s)) \\ zk_{s^-}^{\alpha} - w(k_{s^-} + E(d_{s^-})) \end{bmatrix}$$

where $\Gamma = \begin{bmatrix} \Gamma_1 \Gamma_2 \\ \Gamma_3 \Gamma_4 \end{bmatrix}$, for

$$\Gamma_1 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s,j}} + \zeta_{\Delta_s q_s}\left(\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s^-,j}(1)}{q_{s,j}}\right)$$

$$\begin{aligned}\Gamma_2 \quad = \quad & \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s,j}} + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}} + \\ & \zeta_{\Delta_{s^-} q_s}\left(\frac{\Phi_{s,j}(1)}{q_{s,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s,j}}\right)\end{aligned}$$

$$\Gamma_3 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s^-,j}} + \zeta_{\Delta_s q_{s^-}}\left(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s,j}(2)}{q_{s^-,j}}\right)$$

$$\begin{aligned}\Gamma_4 \quad = \quad & \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s^-,j}} + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}} + \\ & \zeta_{\Delta_{s^-} q_{s^-}}\left(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s^-,j}}\right)\end{aligned}$$

[24]Note that equation 15 reduces to 19 when there are no small-scale orders.

and in the LHS

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \zeta_{\Delta_s q_s}(\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s^-,j}(1)}{q_{s,j}}) & \zeta_{\Delta_{s^-} q_s}(\frac{\Phi_{s,j}(1)}{q_{s,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s,j}}) \\ \zeta_{\Delta_s q_{s^-}}(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s,j}(2)}{q_{s^-,j}}) & \zeta_{\Delta_{s^-} q_{s^-}}(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s^-,j}}) \end{bmatrix}$$

The last vector on the LHS corresponds to the surplus from trade for each farmer type.

Standard matrix algebra implies that expected value to the providers satisfies

$$\bar{V}_s^j = \gamma_{1s}^j(zk_s^\alpha - wk_s - wE(d_s)) + \gamma_{2s}^j(zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-})) \tag{17}$$

$$\bar{V}_{s^-}^j = \gamma_{1s^-}^j(zk_{s^-}^\alpha - wk_{s^-} - E(d_{s^-})) + \gamma_{2s^-}^j(zk_s^\alpha - wk_s - wE(d_s)) \tag{18}$$

for $\gamma_{1s^-}^j = \frac{\Gamma_1 a_{22} - a_{12}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ and $\gamma_{2,s^-}^j = \frac{a_{21}\Gamma_1 - a_{11}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ while $\gamma_{1,s}^j = \frac{a_{11}\Gamma_4 - \Gamma_2 a_{21}}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ and $\gamma_{2s}^j = \frac{a_{12}\Gamma_4 - \Gamma_2 a_{22}}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$. Notice that the denominator of each of the γ parameters shifts depending on the provider as a function of the probability of service. This heterogeneity changes the value for the derivatives in Γ. □

Value of service when serving only large scale farmers.

If a ***provider j attracts only large-scale farmers****, i.e.* $q_{s^-j} = 0$*, then the expected per period profit of the provider satisfies*

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(zk_s^\alpha - w(k_s + E(d_s)))$$

where the second term corresponds to the surplus associated to the transaction and $\gamma \in (0, 1)$ *is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue,* ζ*, and the share of capital in farming production.*

Proof. The problem of the supplier when it only receives large scale orders is

$$\max_{q_{sj}, r_{sj}} \psi \bar{V}_s$$

subject to

$$\tilde{\Delta}_{sj}\pi_s(r_{sj}, k_s) \geq U_s$$

$$\sum_{i \in \hat{q}_j} k_s(i) + E(d_s(i)) \leq \bar{k}_j$$

where $\psi = 2\left(1 - e^{-q_{s,\text{mkt}}}(1 + q_{s,\text{mkt}})\right) + e^{-q_{s,\text{mkt}}} q_{s,\text{mkt}}$ because there are no small-scale orders and either the supplier serves one or two orders of large scale.

Using the definition of profits to the farmers, equation **??**, we can replace the cost of capital into the objective function. Replacing the rental price of capital as a function of the expected profits, the provider solves

$$\max_{q_{sj}} \psi \left[z k_s^{\alpha} - \frac{U_s}{\Delta_{sj}} - w(k_s + E(d_s)) \right]$$

Note that the properties of the probabilities ψ and Δ (decreasing and convex in the queue length) imply that the first order conditions to the problem are necessary and sufficient for an optimum. The optimality condition for the queue length is

$$\frac{\partial \psi}{\partial q} \bar{V}_s - \psi \left[\frac{\bar{V}_s + w(k_s + E(d_s)) - z k_s^{\alpha}}{\tilde{\Delta}_{sj}(2)} \right] \frac{\partial \tilde{\Delta}_{sj}(2)}{\partial q} = 0. \tag{19}$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q} \frac{q_{sj}}{\Delta_{sj}(o)}$ and let the elasticity of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$. Finally, let $\gamma(q_{s,j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) \equiv \frac{\zeta_{q\Delta}}{\zeta_{q\psi} + \zeta_{q\Delta}}$,

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(z k_s^{\alpha} - w(k_s + E(d_s))) \tag{20}$$

which proves the result. For the value to be positive we require $\gamma > 0$ which is true by construction. The provider takes a fraction of the surplus from the transaction.

□

*If a **provider j attracts only small-scale farmers**, i.e. $q_{s,j} = 0$, then the expected per period profit of the provider satisfies*

$$\bar{V}_{s^-} = \gamma(q_{s^- j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(z k_{s^-}^{\alpha} - w(k_{s^-} + E(d_{s^-})))$$

where $\gamma \in (0, 1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, ζ, and the share of capital in farming production.

The derivations when the provider serves only small-scale providers follow the same steps as the ones above, so we omit them for brevity.

The market provider wants to prioritize large scale orders: Compute $\frac{\partial \Pi_{\text{mkt}}}{\partial \chi}$, which are strictly positive, given the definition for the unconditional probabilities of service, 10 to 12, and the value of the provider, equations 7 and 14. Then, the optimal selection rule is at the corner, $\chi = 1$.

Expected profits to the farmers The expected profits to the farmers depend on the equilibrium being realized, i.e. whether providers serve both type of farmers or providers specialize in a single type. The reason is that the expected profits to the farmer depend on the cost of service, which can be in turn expressed as a function of the value of service using the definition of the per period value, $zk_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i + E(d_i)) = V_i^j$

$$U_i = (zk_i^\alpha - w(k_i + E(d_i)) - V_i^j)\Delta_{ij}$$

Replacing the values of expected profits for the providers we obtain alternative specification

1. If providers serve both type of farms,

$$U_s = \Delta_{sj}((1-\gamma_{1s}^j)(zk_s^\alpha - wk_s + wE(d_s)) - \gamma_{2s}^j(zk_{s^-}^\alpha - wk_{s^-} + wE(d_{s^-})))$$

$$U_{s^-} = \Delta_{s^-j}((1-\gamma_{1s^-}^j)(zk_{s^-}^\alpha - wk_{s^-} + wE(d_{s^-})) - \gamma_{2s^-}^j(zk_s^\alpha - wk_s + wE(d_s)))$$

2. If a provider serves only large scale farmers,

$$U_s = \tilde{\Delta}_{sj}(1-\gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(zk_s^\alpha - wk_s + wE(d_s))$$

3. If a provider serves only small scale farmers,

$$U_{s^-} = \tilde{\Delta}_{s^-j}(1-\gamma(q_{s^-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(zk_{s^-}^\alpha - wk_{s^-} + wE(d_{s^-}))$$

When the providers specialize in service provision, they determine the expected profits to the farmer.

Equilibrium queue lengths. In an equilibrium where farmers reach out to both providers, they should be indifferent across them and the feasibility constraints of the economy should be satisfied.[25] We describe the indifference condition for large scale farmers, the ones for small scale farmers are analogous.

$$\frac{\Delta_{\text{smkt}}}{\Delta_{\text{sfcfs}}} = \frac{(1-\gamma_{1s}^{\text{fcfs}})(zk_s^\alpha - wk_s - wE(d_s)) - \gamma_{2s}^{\text{fcfs}}(zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))}{(1-\gamma_{1s}^{\text{mkt}})(zk_s^\alpha - wk_s - wE(d_s)) - \gamma_{2s}^{\text{mkt}}(zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))}$$

[25] If they choose to reach out to a single provider, then the equilibrium queue length is determined by feasibility only, which in turn determines the expected value for farmers.

When small scale farmers queue only with fcfs providers, the indifference condition for the large farmer is

$$\frac{\Delta_{smkt}}{\Delta_{sfcfs}} = \frac{(1-\gamma_{1s}^{\text{fcfs}})(zk_s^\alpha - wk_s - wE(d_s)) - \gamma_{2s}^{\text{fcfs}}(zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))}{(1-\gamma_s^{\text{mkt}})(zk_s^\alpha - wk_s - wE(d_s))}$$

These indifference conditions jointly with the feasibility constraints of the economy, equations 1 and 2, yield the optimal queue lengths by provider and type.

Rental rates The rental rates can be computed from the definition of U once the optimal queues have been solved for.

Value of service for farmers $U_s \geq U_{s^-}$ whenever the different in the surplus of service for large-scale providers is large enough. Because in equilibrium the market value of service is the same irrespective of the provider, it is w.l.o.g. to use the values from the fcfs providers.

$$\begin{aligned} U_s - U_{s^-} &\geq 0 \\ (\Delta_{sj}(1-\gamma_{1s}^j) + \Delta_{s^-j}\gamma_{2s^-}^j)(zk_s^\alpha - wk_s - wE(d_s)) &- \\ (\Delta_{sj}\gamma_{2s}^j + \Delta_{s^-j}(1-\gamma_{1s^-}^j))(zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))) &\geq 0 \end{aligned}$$

If the surplus is weakly higher for large scale farmers, $zk_s^\alpha - wk_s - wE(d_s) \geq zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-})$, then it is sufficient that

$$\frac{(\Delta_{sj}(1-\gamma_{1s}^j) + \Delta_{s^-j}\gamma_{2s^-}^j)}{(\Delta_{sj}\gamma_{2s}^j + \Delta_{s^-j}(1-\gamma_{1s^-}^j))} \geq \frac{(zk_{s^-}^\alpha - wk_{s^-} - wE(d_{s^-}))}{(zk_s^\alpha - wk_s - wE(d_s))}$$

The above is a condition on the elasticities of the probability of service to the queue lengths (in γ) relative to the values of the surplus.

Value of service for providers The value of serving large farmers is higher than the value of serving small-farmers for any provider For $\bar{V}_s > \bar{V}_{s^-}$ it is sufficient that $(\gamma_{1s} - \gamma_{2s^-}) > 0$ and $(\gamma_{2s} - \gamma_{1s^-}) > 0$, see equations 17 and . This is the same as

$$a_{11}(\Gamma_4 + \Gamma_3) - a_{21}(\Gamma_2 + \Gamma_1)$$

$$a_{12}(\Gamma_4 + \Gamma_3) - a_{22}(\Gamma_2 + \Gamma_1)$$

If the queue lengths are the same $\Gamma_4 + \Gamma_3 = \Gamma_2 + \Gamma_1$ and also $a_{11} > a_{21}$ and $a_{12} > a_{22}$ because the elasticity of the probability of service to the queue large farmers is higher than for the queue of small farmers. By continuity, if the queue lengths are not too different the above result holds. Intuitively, the reason is that if there are no systematic differences in travel time across farmers, then the provider's marginal cost of provision is higher for the smaller farmers and therefore the provider finds them less valuable.

C Numerical Solution and Output

C.1 Value function computation

The value function maps an ordered queue to the expected present value of this queue. Each order i in the queue comprises two dimensions: h_i, the number of hours demanded discretized to 6 bins, and d_i the travel hours to and from the hub that represents a variable cost of service. For a queue length equal to 3, the value function is a mapping from R^6 to R^1.

$$V(\{(k_1, \nu_1), (k_2, \nu_2), (k_3, \nu_3)\}) : R^6 \to [0, \infty]$$

The relatively high dimensionality of the problem prompts us to implement the sparse-grid method proposed by Smolyak (1963) (see Judd et al. (2014) for details). The grid points are selected for an approximation level of 2, which results in 85 grid points. We then construct a Smolyak polynomial consisting 85 orthogonal basis functions, which belong to the Chebyshev family. The integration nodes are selected by applying the tensor product rule to the one-dimensional Smolyak grid points at the approximation level of 2. Integration is carried out using Newton-Cotes quadrature.

C.2 Simulations

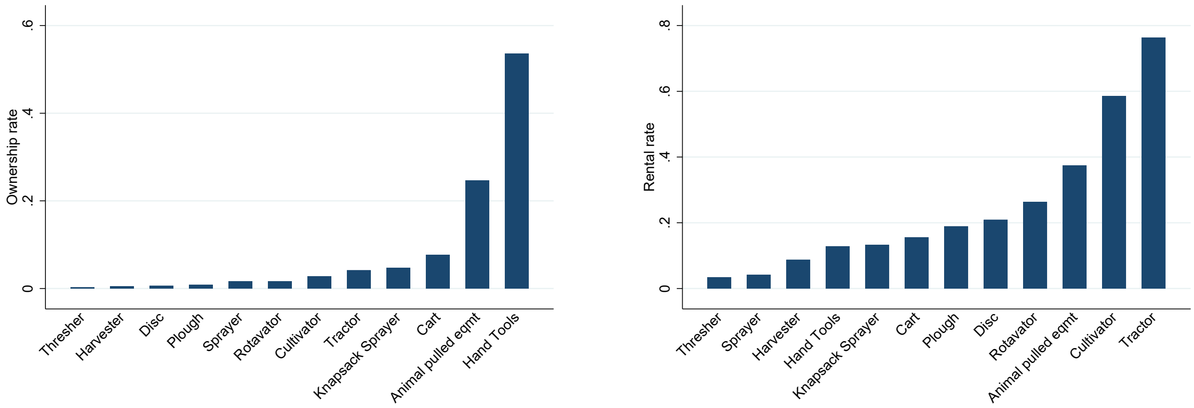
We simulate the expected wait time and productivity cost under the fcfs arrangement and the market arrangement respectively for three cases: when productivity is uncorrelated, negatively correlated or positively correlated to the number of hours demanded. Productivity, measured in revenue per acre, is simulated and assigned to each order observed in the actual data. We make a large number of draws of productivity sequences, each with length equal to the number of actual orders, from a log normal distribution where the parameters are obtained by fitting the actual productivity information to a log normal distribution by hub. We then choose a sequence for each simulation case that produces a correlation with hours demanded in the data that is the closest to a target correlation for that case, and assign that sequence to the actual orders. In the uncorrelated case, the target is zero; in the negatively correlated case, the target is the actual correlation for that hub; in the positively correlated case, the target is symmetric to the negative correlation.

We use bootstrap sampling of the actual orders for the simulation and assume each bootstrap sample represents an actual queue.

We compute the wait time for the first three orders in each bootstrap sample under the fcfs arrangement and the market arrangement respectively. In any period if one or more out of these three orders are not served, the queue is filled going through the bootstrap sample. To avoid having a large order "jamming" the queue, we assume every order is feasible. This implies that the maximum wait time under the fcfs arrangement is 3. We cap the wait time at 5 for the market arrangement. The productivity cost is then calculated by multiplying the simulated productivity by a percentage loss as described in the table 3.

D Additional Tables and Figures

Figure 13: Ownership and rentals by implement.



Owned Rentals

The ownership (rental) rate is the share of farmers that report to own a given implement relative to the total population surveyed.

Table 8: Costs of Delays Relative to Optimal Planting Time, Value Added per Acre

	Cost per day, value added per acre				
	Whole Sample	5-day around optimal		10-day around optimal	
		Before	After	Before	After
β_1	-41.97	391.1**	-215.7	1,166***	-931.1***
	(26.33)	(140.2)	(146.9)	(338.7)	(298.5)
Observations	6,034	1,461	1,882	1,010	1,221
R-squared	0.408	0.625	0.584	0.706	0.659
Mean of Productivity	10228	11425	10694	11921	10998

Figure 14: Value Functions