Unconventional Monetary Policy According to HANK*

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Current draft: April 26, 2022

Abstract

This paper studies the implications of household heterogeneity for the effectiveness of quantitative easing (QE). We consider a heterogeneous agent New Keynesian (HANK) model with uninsurable household income risk. Financial intermediaries are subject to an endogenous leverage constraint that allows QE to matter. We find that macro aggregates react very similarly to a QE shock in a HANK model compared to a representative agent (RANK) version of the model. This finding is robust across different micro- and macro- distributions of wealth, although these distribution rules have implications for popular inequality metrics as well as the fraction of households that are at the borrowing constraint.

*We are grateful to Drew Creal, Jasmine Xiao for helpful comments. Sims and Wu acknowledge support from the National Science Foundation under Grant No. SES-1949107. Ji acknowledges support from the National Natural Science Foundation of China (No.72003102). Correspondence: esims1@nd.edu, cynthia.wu@nd.edu, zhangji@pbcsf.tsinghua.edu.cn.

1 Introduction

In response to the Great Recession, the Federal Reserve (Fed) lowered its short-term policy rate to zero. With its conventional stabilization tool unavailable, the Fed resorted to unconventional policy to provide support to the US economy. The most prominent of its unconventional policy tools was quantitative easing (QE, also referred to as large scale asset purchases, or LSAPs). Purchasing longer-term Treasuries and agency-backed mortgage backed securities (MBSs), the Fed more than doubled its balance sheet within the span of a few months in late 2008. By the time it halted active asset purchases in the middle part of the 2010s, the Fed's balance sheet had more than quadrupled in size relative to its pre-Great Recession level. The Fed resumed active asset purchases in 2020 in response to the COVID-19 pandemic, once again doubling the size of its balance sheet in the span of less than a year.

There is a large and still growing literature on the effectiveness of QE. Most models in this literature are representative agent dynamic stochastic general equilibrium (DSGE) models – see, for example, Gertler and Karadi (2013), Sims and Wu (2020, 2021a), and Sims, Wu and Zhang (forthcoming). At the same time, there has been significant interest in the implications of micro-level heterogeneity for macroeconomic fluctuations in general, and in particular with respect to the transmission of monetary policy (e.g. Kaplan, Moll and Violante 2018).[1] To date, however, this literature has mainly considered the implications of heterogeneity for conventional monetary policy in the form of movements in short-term interest rates. The the goal of our paper is to marry these two literatures to investigate whether, and to what extent, household heterogeneity matters for the effectiveness of QE.

We develop a quantitative model with household heterogeneity and scope for QE to matter. In our model, households face uninsurable idiosyncratic unemployment risk, which is similar to Krusell and Smith (1998), but with endogenous labor supply. They may save

[1]For other applications of HANK models to the study of monetary policy, see also McKay et al. 2016, Auclert (2019), Acharya and Dogra (2020), Alves, Kaplan, Moll and Violante (2020), and Ravn and Sterk (2020).

via short-term deposits with financial intermediaries, subject to a borrowing constraint, and receive dividends from their ownership in production firms and financial intermediaries. The rest of the model is similar to Sims and Wu (2021a). Financial intermediaries engage in maturity transformation, standing between households who save via short-term deposits, and production firms who float long-term debt to finance investment. Financial intermediaries face an endogenous leverage constraint, and production firms are required to finance a fraction of their investment by issuing long term bonds. Prices and nominal wages are sticky. Central bank purchases of long-term assets can ease the leverage constraint facing intermediaries, resulting in lower interest rate spreads and more investment.

In our benchmark specification, we assume that all households receive the same dividend payout each period. In this specification, the responses of aggregate variables to a QE shock are nearly identical to a representative agent (RANK) version of the model, even though the HANK version features substantial wealth inequality due to uninsurable employment risk and a borrowing constraint. The only noticeable difference is the response of consumption of the very poorest households in the HANK version, which increases sharply after a QE shock. But this distributional difference has little impact on the responses of aggregate variables.

We consider robustness of our results with respect to both micro- and macro- distributions of wealth. First, regarding the micro distribution, we allow a more general dividend distribution rule, in which dividends vary, potentially non-linearly, with household wealth. We explore the implications of this distribution rule for popular inequality metrics, such as the Lorenz Curve and the Gini Coefficient. With a significant gap between dividends received by the richest and poorest households, and sufficient non-linearity in the relationship between dividend receipt and wealth, our model can generate substantially more wealth inequality that closer aligns with what is observed in the data. One might expect that more wealth inequality would matter for the aggregate effects of QE shocks. We find, however, that it does not. Regardless of how we specify the dividend distribution rule, we find that the impulse responses of aggregate variables to a QE shock are remarkably similar, and in turn almost identical to a RANK version of the model.

Next, we investigate the extent to which macro parameterizations related to the wealth distribution influence the aggregate transmission of QE shocks. In our baseline HANK specification, not many households are located at or near the borrowing constraint in the stationary wealth distribution. We focus on two parameters. First, we allow a higher proportion of the population to be unemployed. This results in more households being located at or near the borrowing constraint, and makes the aggregate effects of QE shocks slightly bigger, but these differences are not economically significant. We also consider a parameterization in which the benefit to unemployed households is much larger. With a such parameterization, we are able to generate many more households located at or near the borrowing constraint in the stationary distribution. In order to get a large fraction of households at the borrowing constraint, the unemployment benefit needs to be implausibly high such that unemployment becomes a benefit rather than a risk. With a parameterization that generates about 30 percent of households being constrained, we find that, like with the aggregate unemployment rate, the impact on the aggregate transmission of a QE shock relative to a RANK specification are modest.

There are two potentially important takeaways from our quantitative exercises. The first is that there seems to be little gained by formally modeling household heterogeneity if what one is most interested in is the aggregate effects of a QE shock. Our finding that a RANK specification is a good approximation to a substantially more complicated and numerically more demanding HANK model is consistent with a recent paper by Debortoli and Gali (2022), who argue that idiosyncratic income uncertainty is unimportant for aggregate fluctuations. Another important takeaway from our analysis is that there seems to be little relationship between various inequality metrics and aggregate dynamics in response to a QE shock. Much of the existing literature uses moments such as the Gini coefficient as summary statistics for inequality (e.g. Kaplan, Moll and Violante 2018 and Alves, Kaplan, Moll and Violante 2020). To the extent to which one is only interested in aggregate dynamics, our results suggest that this focus on inequality metrics might be misplaced.

In addition to these substantive issues, our paper also makes a methodological contribution. Though our model features idiosyncratic income risk, wealth inequality, and a borrowing constraint, we are able to solve the model using perturbation methods in Dynare, a popular program for the solution, simulation, and estimation of representative agent DSGE models. Our paper is similar in this respect to Winberry (2018), who also develops a way to solve a heterogeneous agent model with perturbation methods in Dynare. We depart from him in that we follow Young (2010) to approximate the cross-sectional distribution of wealth with a non-parametric histogram, whereas Winberry (2018) follows Algan et al. (2008) to approximate this distribution within a parameteric family. The advantage of our approach is that it does not involve numerical optimization, which slows the calculation when done repeatedly and often does not guarantee convergence. Our method is not limited to the QE application on which we focus in this paper, nor is it specific to the exact form of household heterogeneity.

The remainder of the paper is organized as follows. Section 2 lays out our model. Section 3 discusses our method for solving the model. Section 4 compares and contrasts impulse responses to an exogenous QE shock with and without household heterogeneity. Section 5 discuss our model's implications for popular inequality metrics and considers a more general form of the dividend distribution rule to study how that affects the aggregate dynamics in response to a QE shock. Section 6 considers changing macro parameters to get more households located near the borrowing constraint, and investigates the extent to which this matters for aggregate dynamics in response to a QE shock. The final section concludes.

2 Model

Our model introduces financial frictions, a banking sector, and has scope for central bank asset purchases to matter, as in Sims and Wu (2021a). The main difference is that households are heterogeneous and face uninsurable unemployment risk, similarly to Krusell and Smith (1998). In this section,we provide detail on key elements of the model and relegate remaining details to an appendix.

2.1 Households

The household sector is similar to Krusell and Smith (1998), except that labor supply is endogenous. There are a continuum of households indexed by $j \in [0,1]$. Each maximizes the present discounted value of lifetime flow utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_{j,t} - \chi \frac{l_{j,t}^{1+\eta}}{1+\eta} \right),$$

where $c_{j,t}$ and $l_{j,t}$ are an individual household's consumption and labor supply. $\beta \in (0,1)$ is a discount factor, $\chi > 0$ is a scaling parameter, and η is the inverse Frisch elasticity.

Each household faces an exogenous idiosyncratic labor productivity shock, $\varepsilon_{j,t} \in \{0,1\}$. This productivity shock can be interpreted as an employment shock, where $\varepsilon_{j,t} = 0$ indicates unemployed and consequently $l_{j,t} = 0$, while $\varepsilon_{j,t} = 1$ means that the household will earn a positive market wage and will hence choose to work, $l_{j,t} > 0$. $\varepsilon_{j,t}$ evolves stochastically according to a two-state Markov process with the transition matrix

$$\begin{bmatrix} p(\varepsilon_{j,t+1} = 0 | \varepsilon_{jt} = 0) & p(\varepsilon_{j,t+1} = 1 | \varepsilon_{jt} = 0) \\ p(\varepsilon_{j,t+1} = 0 | \varepsilon_{jt} = 1) & p(\varepsilon_{j,t+1} = 1 | \varepsilon_{jt} = 1) \end{bmatrix} = \begin{bmatrix} p & 1-p \\ \frac{U}{1-U}(1-p) & 1 - \frac{U}{1-U}(1-p) \end{bmatrix}. \quad (2.1)$$

The aggregate unemployment rate is $U = p(\varepsilon_{j,t} = 0)$ and is fixed. Total labor supply is $L_t = \int_0^1 l_{j,t} dj$.

Household j saves via deposits, $d_{j,t}$, with financial intermediaries, which earn the gross interest rate, R_t^d, from t to $t+1$. Deposits are the only asset the household can accumulate. A household faces the following flow budget constraint:

$$d_{j,t} = \frac{R^d_{t-1}}{\Pi_t} d_{j,t-1} + mrs_t[(1-\tau)l_{j,t}\varepsilon_{j,t} + \mu(1-\varepsilon_{j,t})] - c_{j,t} - \mathcal{T}_t + div_{j,t} - X, \quad (2.2)$$

where Π_t is the gross inflation rate, mrs_t is the real wage, τ is a tax rate on labor income, $\mu \times mrs_t$ is an unemployment benefit, $\mathcal{T}_t$ is a lump-sum tax, and X is a real transfer paid out to new financial intermediaries, to be discussed below. $div_{j,t}$ is a dividend transfer from all firms. This dividend is taken as given by households; we will discuss how it is distributed across agents in Section 4 and Section 5. $d_{j,t}$, $c_{j,t}$,and $l_{j,t}$ are endogenous choice variables.

Each household faces a borrowing constraint and a constraint on its time endowment:

$$d_{j,t} \geq \underline{d} \quad (2.3)$$

$$l_{j,t} \leq \bar{l}. \quad (2.4)$$

The first order conditions are

$$c^{-1}_{j,t} \geq \beta R^d_t \mathbb{E}_t \frac{c^{-1}_{j,t+1}}{\Pi_{t+1}} \quad (2.5)$$

$$l^{\eta}_{j,t} \leq \frac{(1-\tau) mrs_t}{\chi c_{j,t}}. \quad (2.6)$$

The equalities hold when the constraints in (2.3) and (2.4) do not bind.

2.2 Labor Market

We introduce wage rigidity into our model via labor unions. Unions are indexed by $h \in [0,1]$. These unions purchase labor from households at real wage mrs_t, repackage it into $L_{d,t}(h)$, and sell it to a representative labor contractor at nominal wage, $W_t(h)$. The labor contractor combines differentiated labor into final labor available for production, $L_{d,t}$ via a CES technology. Final labor is sold to a representative production firm at nominal wage,

W_t.

Unions are subject to a Calvo-style nominal rigidity. Each period, a union can adjust its wage, $W_t(h)$, with probability $1 - \phi_w$, where $\phi_w \in [0, 1]$. Non-updated wages can be indexed to lagged aggregate inflation via the parameter $\gamma_w \in [0, 1]$. This setup gives rise to a conventional wage Phillips curve. See the appendix for details.

2.3 Production

There are four different types of firms in our model. A representative capital goods producer transforms raw investment goods into new physical capital subject to a convex adjustment cost. A competitive final good producer aggregates retail output into final output via a CES technology and sells it at nominal price, P_t. A continuum of retail firms, indexed by $f \in [0, 1]$, repackage wholesale output and sell it to the final good producer at $P_t(h)$. Retailers have market power and are subject to a Calvo-style nominal rigidity, updating their prices with probability $1 - \phi_p$ each period. Non-updated prices can be indexed to lagged inflation via the parameter $\gamma_p \in [0, 1]$. The representative wholesale firm accumulates its own capital, purchases new capital from the capital goods producer, and hires labor from the labor contractor. It produces wholesale output, $Y_{m,t}$, from capital and labor, and sells this output to retail firms at $P_{m,t}$. The wholesale firm can choose the intensity with which it utilizes physical capital, u_t, the cost of which is faster depreciation.

With the exception of the wholesale firm, the production side of the model is reasonably standard, and details are relegated to an appendix. The wholesale firm accumulates its own physical capital, K_t. It purchases new physical capital, $\widehat{I}_t$, from the capital goods producer at nominal price P_t^k. We require that the wholesale firm finance a fraction, $\psi \in [0, 1]$, of its purchases of new physical capital by issuing long-term bonds. As in Woodford (2001), these long-term bonds take the form of perpetuities with decaying coupon payments. Let $\kappa \in [0, 1]$ denote the decay parameter for coupon payments. A bond in period t is sold for Q_t dollars and obligates the issuer to a coupon payment of one dollar in $t + 1$, κ dollars in $t+2$, κ^2 dollars in $t+3$, and so on. The total nominal coupon liability due in t from all past issuances is denoted by $F_{m,t-1}$; new issuance of bonds in t is therefore denoted by $F_{m,t} - \kappa F_{m,t-1}$, which generates Q_t dollars for the issuer. The "loan in advance" constraint facing the wholesale firm is:

$$\psi P_t^k \widehat{I}_t \leq Q_t(F_{m,t} - \kappa F_{m,t-1}). \tag{2.7}$$

(2.7) distorts standard first order conditions related to capital investment and bond issuance. It therefore generates "investment" and "financial" wedges. Fluctuations in these wedges are the mechanism through which QE-type policies transmit to the real economy.

2.4 Financial Intermediaries

Financial intermediaries are structured similarly to Gertler and Karadi (2011, 2013), and Sims and Wu (2021a,b). Each period there is a fixed mass of intermediaries indexed by i. Intermediaries finance themselves with net worth, $N_{i,t}$, and deposits taken from households, $D_{i,t}$. Each period, a fraction $1-\sigma$, with $\sigma \in [0,1]$, stochastically exit and return their net worth to their household owner. They are replaced by an equal number of new intermediaries that begin with real start up funds of X given to them by their household owner.

Intermediaries hold privately issued bonds, $F_{i,t}$; government issued nominal bonds, $B_{i,t}$; and interest-bearing reserves, $RE_{i,t}$, which are held on account with the central bank. Government bonds are structured similarly to private long-term bonds and are priced at $Q_{B,t}$. The balance sheet condition of a typical intermediary is:

$$Q_t F_{i,t} + Q_{B,t} B_{i,t} + RE_{i,t} = D_{i,t} + N_{i,t} \tag{2.8}$$

A financial intermediary accumulates net worth until stochastically exiting. Net worth for surviving intermediaries evolves according to:

$$N_{i,t} = \left(R_t^F - R_{t-1}^d\right) Q_{t-1} F_{i,t-1} + \left(R_t^B - R_{t-1}^d\right) Q_{B,t-1} B_{i,t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right) RE_{i,t-1} + R_{t-1}^d N_{i,t-1} \tag{2.9}$$

R_{t-1}^{re} is the (gross) interest rate on reserves, which is set by the central bank and known at $t-1$. R_{t-1}^d is the deposit rate, which is determined in equilibrium. $(R_t^F - R_{t-1}^d)$, $(R_t^B - R_{t-1}^d)$, and $(R_{t-1}^{re} - R_{t-1}^d)$ are, respectively, the excess returns from holding private bonds, government bonds, and reserves relative to the cost of funding via deposits. The term $R_{t-1}^d N_{i,t-1}$ measures the cost-savings from financing via net worth as opposed to deposits. R_t^F and R_t^B are the realized holding period returns on private and government bonds and satisfy:

$$R_t^F = \frac{1 + \kappa Q_t}{Q_{t-1}} \tag{2.10}$$

$$R_t^B = \frac{1 + \kappa Q_{B,t}}{Q_{B,t-1}} \tag{2.11}$$

The objective of an intermediary is to maximize its expected terminal net worth where discounting is by the stochastic discount factor of the household, $\Lambda_{t,t+1} = \frac{\Pi_{t+1}}{R_t^d}$. Consider the problem of an intermediary continuing after period t. There is a $1-\sigma$ probability that it will exit after $t+1$, a $(1-\sigma)\sigma$ probability that it will exit after $t+2$, and so on. Accordingly, its objective is

$$V_{i,t} = \max \ (1-\sigma)\mathbb{E}_t \sum_{j=1}^{\infty} \sigma^{j-1} \Lambda_{t,t+j} n_{i,t+j}, \tag{2.12}$$

where $n_{i,t} = N_{i,t}/P_t$ is real net worth, with P_t the price of final output.

A financial intermediary faces a costly enforcement constraint as in Gertler and Karadi (2011, 2013). A financial intermediary can choose to abscond with some assets at the end of a period rather than continuing as an intermediary. If an intermediary does this, depositors can recover a fraction of the intermediary's assets, with the intermediary retaining the rest. For depositors to be willing to lend to intermediaries, it must not be optimal for the intermediary to divert funds in this way, which we refer to as going into bankruptcy. Accordingly:

$$V_{i,t} \geq \theta(Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}) \tag{2.13}$$

In (2.13), the right hand side of the inequality represents the (real) funds that a financial intermediary can keep should it choose to enter bankruptcy, while the left hand side is the value of continuing as an intermediary.[2] Should it choose to divert, an intermediary can keep a fraction of its private bonds, θ, and a fraction of government bonds, $\theta\Delta$, where $0 \leq \Delta \leq 1$. We assume that the third type of asset held by intermediaries – reserves – is fully recoverable by depositors in the event of bankruptcy.

All financial intermediaries will behave in the same way with identical optimality conditions. These are

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta \tag{2.14}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^B - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta \Delta \tag{2.15}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_t^{re} - R_t^d \right) = 0, \tag{2.16}$$

where

$$\Omega_t = 1 - \sigma + \sigma \theta \phi_t \tag{2.17}$$

$$\phi_t = \frac{1+\lambda_t}{\theta_t} \mathbb{E}_t [\Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1}] R_t^d. \tag{2.18}$$

(2.14) - (2.16) are the key equilibrium conditions. $\lambda_t \geq 0$ is the multiplier on the costly enforcement constraint. If this constraint does not bind, then to first order expected returns on all three types of assets must equal the cost of funds (i.e. the deposit rate). If the costly enforcement constraint binds, then there will be excess returns of long-term private and

[2] $f_{i,t} = F_{i,t}/P_t$ and $b_{i,t} = B_{i,t}/P_t$ are real private and government bond holdings, respectively.

public bonds over the deposit rate. Because reserves are fully recoverable in the event of liquidation, arbitrage requires that the interest rate on deposits equal the interest rate on reserves. (2.17) is an auxiliary variable introduced to simplify the analysis.

The value of an intermediary satisfies

$$V_{i,t} = \theta \phi_t n_{i,t}. \tag{2.19}$$

When the constraint in (2.13) binds,

$$\phi_t = \frac{Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}}{n_{i,t}}, \tag{2.20}$$

which is an endogenous leverage ratio, whose equilibrium condition is in (2.18). The constraint makes the financial intermediary less levered than it would find optimal. This endogenous leverage constraint is ultimately what can give rise to excess returns.

One can show from (2.18) that:

$$\theta \phi_t \geq 1 + \lambda_t \tag{2.21}$$

If (2.13) does not bind, then $\lambda_{t+j} = 0$ for all j, which implies that $\theta_t \phi_t = 1$.[3] Intuitively, this means that net worth is as valuable to a household as to an intermediary. In this case, returns on all assets would be equal. Hence, whether an intermediary invests in F_t, D_t or RE_t would be irrelevant. When the costly enforcement constraint binds, then $\lambda_t > 0$ and $\theta \phi_t$ is larger than one. In this case, there exist excess returns on holding long term assets (private and government bonds). Hence, net worth is more valuable inside an intermediary as opposed to a household (who cannot hold these assets and hence cannot take advantage of these excess returns).

[3]In this circumstance, $\Omega_t = 1$.

2.5 Fiscal authority

A fiscal authority finances its expenditure by levying lump sum taxes and a labor income tax on households, from receipt of a transfer, $T_{cb,t}$, from the central bank, and by issuing long-term bonds. Its expenditure includes purchases of output, G_t, and unemployment benefits. For simplicity, we assume that the government has a fixed real stock of long-term bonds issued, $\bar{b}_G$. The government's budget constraint is:

$$G_t + \Pi_t^{-1}\bar{b}_G + mrs_t\mu U = \mathcal{T}_t + T_{cb,t} + Q_{B,t}\bar{b}_G(1 - \kappa\Pi_t^{-1}) + \tau mrs_t L_t \tag{2.22}$$

2.6 Monetary policy

The central bank controls the interest on reserves R_t^{re}, which evolves according to a Taylor (1993)-type interest rate rule:

$$\begin{aligned} \ln R_t^{re} = \quad & (1-\rho_r)\ln R^{re} + \rho_r \ln R_{t-1}^{re} + \\ & (1-\rho_r)\left[\phi_\pi\left(\ln \Pi_t - \ln \Pi\right) + \phi_y\left(\ln Y_t - \ln Y_{t-1}\right)\right] + \sigma_r\epsilon_{r,t}, \end{aligned} \tag{2.23}$$

where $0 \leq \rho_r < 1$, $\phi_\pi > 1$, and $\phi_y \geq 0$. $\epsilon_{r,t}$ is a policy shock. We do not formally analyze the implications of the ZLB in our analysis, but it would be straightforward to do so.

In addition to setting the interest rate on reserves, the central bank can buy and sell long-term bonds (either privately issued bonds or government bonds). It finances these holdings via reserves, which are held in the banking system. The central bank's balance sheet, in nominal terms, is:

$$Q_t F_{cb,t} + Q_{B,t} B_{cb,t} = RE_t \tag{2.24}$$

We model a quantitative easing shock as an exogenous and persistent change in central bank bond holdings. In particular, we assume that real central bank holdings of privately issued bonds, $f_{cb,t}$, follow a stationary AR(1) process (and similarly for holdings of real government bonds, $b_{cb,t}$):

$$\ln f_{cb,t} = (1-\rho_f) f_{cb} + \rho_f \ln f_{cb,t-1} + \sigma_f \epsilon_{f,t}, \tag{2.25}$$

where $0 \leq \rho_f < 1$ and f_{cb} denotes steady state real bond holdings.

If financial intermediaries are unconstrained (i.e. (2.13) does not bind), then fluctuations in central bank bond holdings are irrelevant. But if intermediaries are constrained, then increases in central bank bond holdings ease the endogenous leverage constraint, resulting in higher long-term bond prices (equivalently, lower long-term yields). This filters through to the real economy by relaxing the loan in advance constraint on wholesale producers, leading to more investment and higher aggregate demand.

3 Solution Method and Calibration

In this section, we outline our solution method and discuss the parameterization of our model.

3.1 Solution Method

We solve our model using perturbation methods with the popular software Dynare. Perturbation methods can accommodate a large number of state variables and require less computational time. Though it is straightforward to solve a RANK model using perturbation, in general it is not possible to do so when there is substantial household heterogeneity.

Solving the aggregate part of the model via Dynare is straightforward. The key step we take to solve the entire model via perturbation is to approximate the individual's policy functions and the cross-sectional distribution in the space of (ε_t, d_{t-1}).

As we detail in Appendix Appendix B, we approximate the conditional expectation of the right-hand side of (2.5) using Chebyshev polynomials (see, for example, Judd 1998). This gives the inverse of consumption in period t if the borrowing constraint does not bind. We then solve for the policy function for deposits as the maximum of what would be implied by the budget constraint when (2.5) holds with equality and the minimum level of deposits. The policy function for labor is the minimum of what would be implied by (2.6) and the maximum employment level, $\bar{l}$. Given policy functions for labor and deposits, we get a policy function for consumption via the budget constraint. We are left with a system of dynamic equations on the Chebyshev loadings and policy functions over consumption, deposits, and labor supply, i.e. $d_t(d_{t-1}, \varepsilon_t)$, $c_t(d_{t-1}, \varepsilon_t)$, and $l_t(d_{t-1}, \varepsilon_t)$.

Next, We update the cross-sectional distribution from t to $t+1$. We use the non-parametric method proposed by Young (2010) by approximating the joint distribution $p(d_t, \varepsilon_{t+1})$ with a histogram over the grid $\{d_m\}_{m=0}^{M}$ and $\varepsilon_{t+1} = \{0, 1\}$. The transition dynamics for the cross-sectional distribution are given by:

$$p(d_t, \varepsilon_{t+1}) = \sum_{\varepsilon_t} \sum_{d_{t-1}} p(d_t | d_{t-1}, \varepsilon_t) p(\varepsilon_{t+1} | \varepsilon_t) p(d_{t-1}, \varepsilon_t), \tag{3.1}$$

where $p(\varepsilon_{t+1}|\varepsilon_t)$ is given in (2.1).

We know $d_t(d_{t-1}, \varepsilon_t)$ from the policy function. However, even with d_{t-1} on a discrete grid, d_t does not necessarily fall on a grid point. We follow Young (2010) and approximate $p(d_t|d_{t-1}, \varepsilon_t)$ with the d_m grid: find the two neighboring grids $d_{m'}, d_{m'+1}$ that are closest to d_t, where $m' = 0, ..., M-1$. Assign weights to them using

$$\begin{aligned} p(d_t = d_{m'} | d_{t-1} = d_m, \varepsilon_t) &= 1 - \frac{d_t - d_{m'}}{d_{m'+1} - d_{m'}} \\ p(d_t = d_{m'+1} | d_{t-1} = d_m, \varepsilon_t) &= \frac{d_t - d_{m'}}{d_{m'+1} - d_{m'}} \end{aligned} \tag{3.2}$$

With the transition probabilities in (3.2) and (2.1), we can move (3.1) one period forward from $p(d_{t-1} = d_m, \varepsilon_t)$ to $p(d_t = d_m, \varepsilon_{t+1})$. Altogether, we have a system of equations that can be solved using perturbation methods with aggregate shocks in Dynare. See Appendix C for the full set of equilibrium conditions.

For the stationary equilibrium without aggregate shocks, we solve a fixed-point problem over aggregate deposits, D, and labor supply, L (aggregate consumption is then determined given the resource constraint). We proceed in steps. First, given guesses of D and L, we can compute stationary equilibrium values of all aggregate variables. Then we solve for the stationary equilibrium values of the Chebyshev coefficients; this step is also a fixed point problem. Third, we solve the stationary distribution $p(d, \varepsilon)$ by iterating (3.1) forward on the discretized grid until convergence. Once we have the distribution from this step and the policy functions from the Chebyshev step, we can compute updated values of aggregate deposits, $\widetilde{D}$, and labor, $\widetilde{L}$. We then iterate over these steps until $D \approx \widetilde{D}$ and $L \approx \widetilde{L}$. For details of solving the stationary equilibrium, see Appendix D.

Our solution method is closely related to Winberry (2018). He also proposes a methodology to solve a model with heterogeneous agents in Dynare. The main difference relative to our approach is that he approximates the cross-sectional distribution of wealth following Algan, Allais and Haan (2008), who use a parametric family. This methodology requires a large number of numerical optimization procedures. The optimization routines do not guarantee convergence and the computational burden is high. The numerical behavior of their approach becomes problematic when the distribution is far away from a normal distribution, which is the case for our application.

We, instead, follow Young (2010), who uses a histogram over a fixed grid to constructed the cross-sectional distribution of wealth. In his approach, updating the approximate cross-sectional distribution is analytical and does not require numerical optimization. Therefore, it works well with highly skewed distributions like the ones in our paper. One downside of Young's (2010) approach is to accurately approximate the distribution, it requires a large number of grids, which are state variables in the perturbation solution. This makes adding additional idiosyncratic state increasingly difficult if not impossible. Our solution method is also related to Reiter (2009) in the sense that he also approximates the cross-sectional distribution with a histogram and employ a perturbation method.

Table 1: **Calibration of HANK parameters**

Parameters	Value	Target	Description
U	0.05		Fraction unemployed
p	0.5	unemployment duration two quarters	Probability of staying unemployed
χ		$L^{RANK} = 0.95$	Labor disutility scaling parameter
μ	0.4		Unemployment benefit as percentage of wage
τ	0.3		Labor income tax rate
$\underline{d}$	0		Borrowing constraint
$\bar{l}$	1.5		Time endowment

3.2 Calibration

Table 1 lists the parameter values that are unique to the HANK setup. We set $U = 0.05$, which is the fraction of households with low productivity ($\varepsilon = 0$), and hence can be interpreted as the aggregate unemployment rate. Conditional on being unemployed, the probability of remaining unemployed, p, is 0.5. This means that the expected unemployment duration is two quarters. The scaling parameter governing the disutility of labor, χ, is chosen so that steady state labor supply in a representative agent version of the model would be $L^{RANK} = 0.95$.[4] The unemployment benefit is set to 40 percent the wage, so $\mu = 0.4$. This follows Shimer (2005). The labor income tax rate is 30% following Kaplan et al. (2018). $\underline{d} = 0$ implies agents are not allowed to borrow, and $\bar{l} = 1.5$ implies the time endowment for work is 150% of $l^{RANK} = 1$. If we interpret $l^{RANK} = 1$ to eight hours a day, $\bar{l} = 1.5$ means the maximum amount someone can work is 12 hours a day. For other parameters, we follow the calibration from Sims and Wu (2021a); see details in Table E.1 of Appendix E.

[4]This could be interpreted as 95 percent of the population supplying $l^{RANK} = 1$ unit of labor each, which is comparable to the targeted 5 percent unemployment rate in the HANK specification.

4 HANK vs RANK

In this section, we compute impulse responses to a QE shock and compare and contrast those responses under the HANK and RANK specifications.[5]

How taxes and dividends are distributed among agents might be important for how shocks transmit into aggregate variables in HANK models (McKay, Nakamura and Steinsson 2016). We focus on how dividend payments from firms are distributed across households. One can replicate dividend redistribution rules by varying the tax distribution rule. We explore this in more detail in Section 5. But, for the purposes of this section, we simply assume that dividends are distributed evenly across households, where $\int_0^1 div_{j,t} dj = div_t$ and $div_{i,t} = div_{j,t}$ for all i and j.

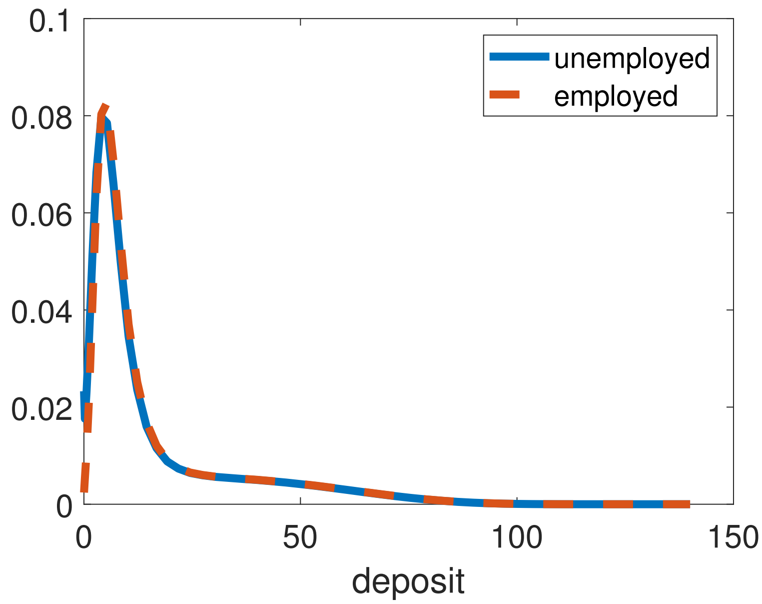
Even with this equal dividend distribution rule, there remains significant household heterogeneity in wealth arising from uninsurable employment/productivity risk. Figure 1 plots the stationary distribution of wealth across households. We do so for both unemployed households (solid blue line) and employed households (dashed red line). With our dividend redistribution rule, few households are located at the borrowing constraint (not shown in the plot) in the stationary distribution. Nevertheless, most households are relatively poor, with the distribution of wealth highly right skewed. There are a decent number of households who are quite wealthy and far away from the borrowing constraint. The distribution of unemployed households is slightly towards the left of the employed households. which implies the former have less wealth than the latter.

Figure 2 plots impulse responses to a persistent, private QE shock. The shock is scaled to represent a four-percent increase in central bank bond holdings.[6] The solid blue lines show responses under a RANK version of the model, while the dashed red lines are responses

[5] The representative household of the RANK model is standard; we discuss the problem facing the household, and the associated first order optimality conditions, in Appendix F.

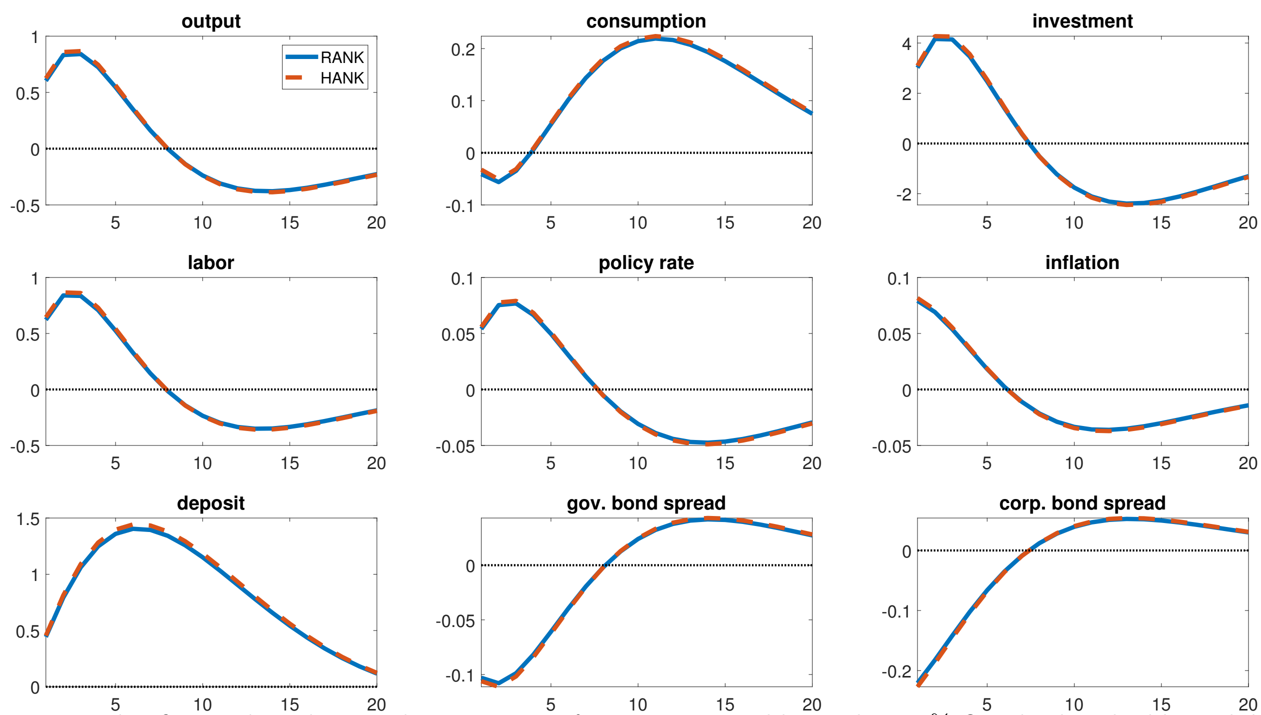
[6] This shock generates a similar-sized response of output as to a conventional shock to the policy rule for the short-term interest rate with a size of 25 basis points; see Figure G.1 in the appendix. As noted in Appendix E, we assume an autoregressive parameter of 0.8. Responses to a public QE shock (i.e. a purchase of long-term government bonds, instead of privately-issued bonds), would produce exactly the same-shaped impulse responses, albeit at a smaller scale.

Figure 1: **Stationary Distribution**



Notes: Blue solid line: unemployed households; red dashed line: employed households. We do not plot the mass for constrained households.

Figure 2: **Impulse responses to a QE shock: RANK vs. HANK**



Notes: This figure plots the impulse responses of aggregate variables under a 4% QE shock. The blue solid lines are for the RANK model, and the red dashed lines are for the HANK model. X-axis is time in quarters, and Y-axis is the percentage change from steady state.

generated from the HANK specification.

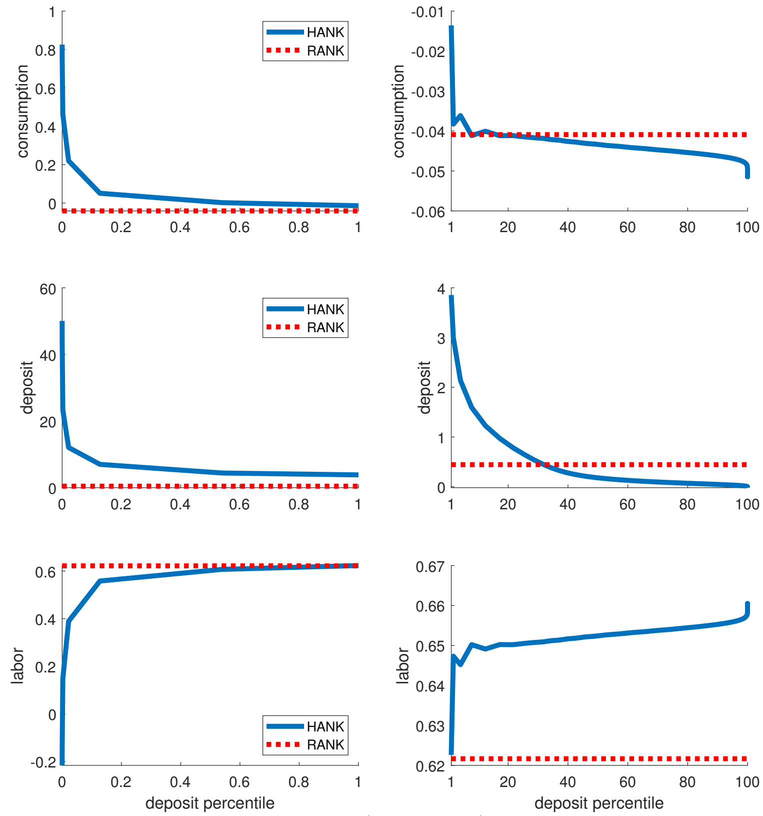
The responses of aggregate variables to a shock to central bank bond-holdings in the RANK specification (blue) are similar to Sims and Wu (2021a). Bond purchases ease the leverage constraint on intermediaries, which results in declining interest rate spreads. This, in turn, results in more investment. Higher aggregate demand results in output, labor, and inflation all rising temporarily.

For all aggregate variables, the responses under the HANK specification (dashed red lines) are qualitatively and quantitatively similar to the RANK version of the model. Consumption, output and labor rise slightly more in the HANK specification on impact, but these differences are small and short-lived.

We next inspect the reasons for the slight differences in aggregate responses in the HANK vs. RANK setups. Figure 3 plots the first-period responses of individual choice variables to a QE shock against the percentile of the wealth distribution. We do so both for a RANK version of the model (dashed red lines) and our baseline HANK version (solid blue). In the RANK version, all households behave identically, so the figures simply show flat lines. To increase readability, the left panel focuses on households in the bottom one percent of the wealth distribution; the right panel shows impact responses for households for the rest of the wealth distribution.

Focus first on the left panel. Households near the bottom of the wealth distribution behave very differently compared to the RANK counterpart. RANK households slightly reduce their consumption on impact, mildly increase savings in the form of deposits, and work more. In the HANK setup, households near the borrowing constraint significantly increase their consumption on impact. They also initially reduce their labor input and save significantly more. What drives these results? Households located near the borrowing constraint are working more and consuming less than they would find optimal absent a borrowing constraint. The extra income from the QE shock allows them to move towards the unconstrained optimal allocation by increasing consumption and reducing labor input. They also take advantage of the income windfall by significantly increasing deposits, which

Figure 3: **First period individual responses to a QE shock: RANK vs. HANK**



Notes: This figure plots the impulse responses of (employed) individual decision variables (consumption, deposits, and labor supply) in the period when a 4% positive QE shock hits. The blue solid lines and red dotted lines are for the baseline HANK and RANK models, respectively. X-axis is the wealth fraction from poor to rich, and Y-axis is the percentage change from steady state.

moves them further from the borrowing constraint.

Importantly, the behavior of individual households is only substantially different from the RANK model for the very poorest. Once we move away from the borrowing constraint, the individual-level decisions are quite similar to what a RANK household would do. For households initially far away from the borrowing constraint (right sides of the right panel), they actually decrease their consumption more, and increase their labor supply more, compared to the RANK household, but these differences are qualitatively small.

We conclude from the analysis in this section that the aggregate responses to an expansionary QE shock are quite similar in the HANK and RANK specifications, at least for the particular equal dividend distribution rule that we have assumed. Though there are micro-level differences in behavior for the poorest households, overall the aggregate responses are similar. This suggests that there may be little to gain from incorporating household heterogeneity into a DSGE model if one is only interested in understanding aggregate effects of a QE shock. In the next section, we investigate whether this conclusion holds up when dividends are distributed differently across households.

5 Implications of the Micro-Distribution of Wealth

In Section 4, we showed that impulse responses of aggregate variables to a central bank asset purchase are qualitatively and quantitatively similar in both HANK and RANK specifications of our model. These results were generated under a particular distribution rule for dividends, which in turn influences the distribution of wealth across households. In this section, we explore further the implications of the wealth distribution for our results. First, we specify several alternative distribution rules for dividends in Subsection 5.1. Next, we show the role of different rules for popular metrics of wealth inequality, such as the Lorenz Curve or the Gini Coefficient in Subsection 5.2. Finally, in Subsection 5.3, we circle back to the main question of interest of the paper and examine how wealth distribution influences the macro responses to a QE shock.

5.1 Distribution Rules

In Section 4, we assumed that dividends from production firms are equally distributed among households. We now assume the following more general dividend distribution rule:

$$\frac{div_{j,t}}{div_t} = \left(a_t + b_t d_{j,t-1}^{\vartheta}\right). \tag{5.1}$$

In (5.1), $d_{j,t-1}$ corresponds to household j's wealth level (in the form of deposits). a_t and b_t are time-varying parameters that govern how household j's share of dividends vary with initial wealth. $\vartheta \geq 1$ allows for a household's dividends to vary non-linearly with its own wealth. Taking ϑ as given, the parameters a_t and b_t are chosen so that the following two equations hold each period:

$$\int_0^1 \left(a_t + b_t d_{j,t-1}^{\vartheta}\right) dj = 1, \tag{5.2}$$

$$\frac{a_t + b_t \bar{d}^{\vartheta}}{a_t + b_t \underline{d}^{\vartheta}} = n. \tag{5.3}$$

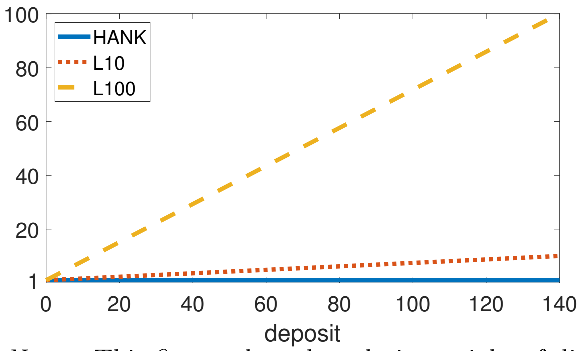
(5.2) simply says that the sum of dividends received by individual households must equal aggregate dividends each period. (5.3) says that the ratio of the dividends received by the richest household (i.e. the household on the largest point on our deposit grid, $\bar{d}$) to the dividends received by the poorest household (i.e. the household on the lowest point on our deposit grid, $\underline{d} = 0$) is equal to n.[7] In Section 4, we implicitly imposed $n = 1$.[8] In this section, we experiment with different values of n and ϑ.

To get a better sense of how different values of n and ϑ impact households in the model, Figure 4 plots the relative weight of dividends received by individuals with different wealth levels in the model's stationary distribution. We take the poorest household, with $\underline{d} = 0$, as the reference point. In the stationary distribution, the weight of household j to the poorest household is $\frac{a+bd_j^{\vartheta}}{a} = \left(1 + \frac{b}{a} d_j^{\vartheta}\right)$. By construction, the poorest household always has a relative weight of unity, and the richest household always has a relative weight of n. The notation we use is that the letter standards for ϑ; we consider three values, L for linear ($\vartheta = 1$), S for squared ($\vartheta = 2$), and C for cubic ($\vartheta = 3$). The number after the letter corresponds to the assumed value of n. We consider values of $n = 10$ and $n = 100$; our baseline case, which we label "HANK," can also be labeled $L1$ (linear, so $\vartheta = 1$, with $n = 1$). The left panel considers linear redistribution schemes ($\vartheta = 1$), while the right panel compares the linear redistribution scheme to the squared and cubic schemes by fixing $n = 100$.

[7] Note that we solve the model with fixed grid points for deposits, so $\bar{d}$ is always the same. In other words, the wealth gap between the richest and poorest household is always fixed. What varies is how dividends are distributed to households away from those end points.

[8] Focusing on (5.3), when $n = 1$, given that $\underline{d} \neq \bar{d}$ it must be the case that $b_t = 0$. But then from (5.2), it follows that $a_t = 1$. When $n \neq 1$, then $b_t \neq 0$ and will be time-varying. This also means that $a_t \neq 1$ and will also be time-varying.

Figure 4: **Relative redistribution weight**



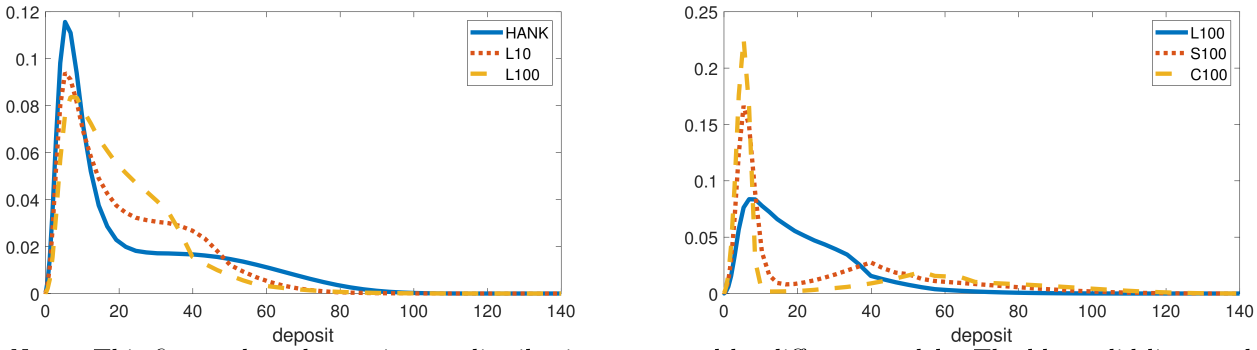
Notes: This figure plots the relative weight of dividend obtained by people at different wealth level at the stationary distribution $(1 + \frac{b}{a} d_j^{\vartheta})$. The blue solid lines, red dotted lines, and the yellow dashed lines on the left(right) column are for the baseline HANK(L100), L10(S100), and L100(C100) models, respectively. X-axis is the wealth level, and Y-axis is the weight.

In our baseline HANK specification (blue line, left panel) all households receive the same dividends, and the relative weights are therefore constant at unity. When we increase n but remain in the linear specification, all other households receive a higher share of dividends relative to the poorest household. The relative weight of a household at a fixed percentile in the wealth distribution is bigger the larger is n.

Interestingly, higher values of n in the linear scheme will have the effect of reducing overall inequality. This implication is illustrated in the left panel of Figure 5, which plots the stationary wealth distribution from our model under the same dividend redistribution rules as the left panel of Figure 4. From Figure 5, we can see that higher values of n shift more mass in the stationary distribution to the right, although this mass remains concentrated away from the far-right tail.

In the right panel of Figure 4, we compare a linear redistribution scheme (solid blue line) to squared and cubic schemes (dotted-red and dashed-orange, respectively). While the endpoints are the same in all cases, more curvature will increase inequality. We show this result with the right panel of Figure 5. Relative to a linear redistribution scheme, more curvature has the effect of concentrating much more mass near the borrowing constraint, which has the effect of increasing overall wealth inequality. For example, in both the squared and cubic specifications, there are about twice as many households at the peak of the stationary

Figure 5: **Stationary distribution**



Notes: This figure plots the stationary distribution generated by different models. The blue solid lines, red dotted lines, and the yellow dashed lines on the left(right) are for the baseline HANK(L100), L10(S100), and L100(C100) models, respectively. X-axis is the wealth level, and Y-axis is the density.

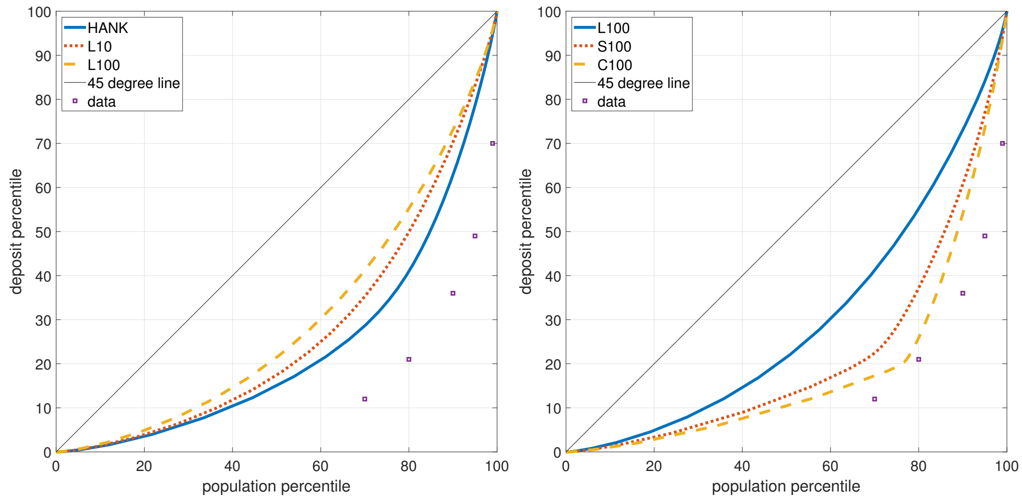
distribution relative to the linear scheme.

5.2 Inequality Measures

The exact form of the dividend distribution rule has important implications for the model's Lorenz curve as well as its Gini coefficient, both of which are important measures of inequality on which the extant literature has often focused. A Lorenz curve plots the cumulative fraction of overall wealth against the population percentile. In our model, household wealth takes the form of deposits. So, points along the 45-degree line represent perfect equality; points below the 45-degree line indicate that wealth is unequally distributed, with more wealth held by a small fraction of households. For example, a point of $(40, 10)$ says that the bottom 40 percent of the population holds 10 percent of aggregate wealth. The Gini coefficient is the ratio of the area between the 45-degree line and the Lorenz curve to the total area under the 45-degree line. A larger Gini coefficient indicates higher inequality.

Figure 6 plots Lorenz curves for different dividend distribution schemes. The purple squares represent the Lorenz Curve measured with US data The left panel plots the Lorenz curves for linear redistribution rules. The blue line corresponds to our baseline specification (labeled HANK, with equal dividend distribution). Qualitatively, our model generates a distribution of wealth that is similar to what is observed in the data, but the distribution is

Figure 6: **Lorenz curve**



Notes: This figure plots the Lorenz curve of stationary wealth distribution when the dividends are distributed according to different functions. The blue solid lines, red dotted lines, and the yellow dashed lines on the left(right) column are for the baseline HANK(L100), L10(S100), and L100(C100) models, respectively, the thin black solid lines are the 45 degree line, and the purple squares are moments in data. X-axis is the fraction of population, and Y-axis is the fraction of wealth.

not sufficiently unequal. The dotted red curve and the dashed orange curve show, respectively, the Lorenz curves associated with linear redistribution schemes with larger values of n (10 and 100). Consistent with the shapes of the stationary distributions, a larger value of n (meaning a bigger gap between the dividends received by the wealthiest and poorest households) reduces observed inequality. In the right panel, we plot Lorenz Curves conditioning on $n = 100$, but for different values of ϑ. Here we observe that larger values of ϑ shift the Lorenz curve down (meaning more inequality), as well as result in more overall curvature. For the cubic specification, the model's Lorenz Curve becomes much closer to the data compared to our baseline HANK case.

Table 2 corroborates the results depicted graphically in Figure 6 by showing Gini coefficients for different dividend distribution rules. The observed Gini coefficient in the data is 0.79. Our baseline HANK specification falls well short of this, with a Gini coefficient of 0.53. Focusing on a linear redistribution scheme with higher values of n, the model's Gini coefficient falls further below what is observed in the data. A squared or cubic specification, in contrast, results in the model better fitting the data. The cubic rule with $n = 100$, for

Table 2: **Gini coefficient**

This table reports the Gini coefficients in the data and generated in different models.

	data	HANK	L10	L100	S100	C100
Gini coeff.	0.79	0.53	0.46	0.40	0.57	0.64

example, results in a Gini coefficient of 0.64.

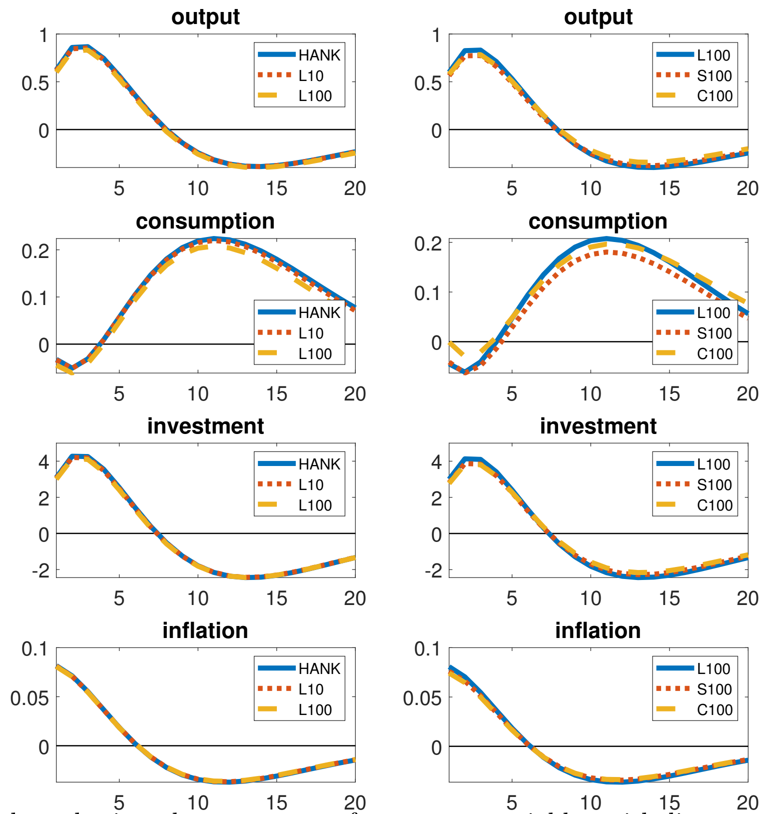
5.3 Macro Aggregates

Our analysis above suggests that our model is capable of generating significant wealth inequality in a way that is similar to what is observed in the data. However, the question we are interested in is not whether the model can match moments of the wealth distribution. Rather, we are interested in whether a model's ability to match inequality moments matters for the transmission of QE shocks into aggregate variables like output. We show in this section that there is little economically meaningful connection between the dividend distribution rule, inequality moments, and aggregate effects of QE shocks.

Figure 7 plots impulse responses to a QE shock under different dividend distribution specifications. The picture is divided into two panels, with responses in the HANK, $L10$, and $L100$ specifications in the left panel, and responses for the $L100$, $S100$, and $C100$ specifications in the right panel. We focus only on responses of output, consumption, investment, and inflation here, but the responses are otherwise generated in the same manner as in Figure 2.

The responses of aggregate variables to a QE shock are remarkably similar under all specifications; this means they are all, in turn, quite similar to the RANK specification as well. There are some small differences for the response of aggregate consumption, but the inflation, output, and investment responses are almost identical across all specifications. These results suggest that the exact form of the dividend distribution rule – though highly relevant for inequality statistics – is unimportant for the aggregate transmission of a QE shock. This, in turn, suggests that the literature's heavy focus on these inequality statistics

Figure 7: **Impulse responses to a QE shock: HANK with different redistribution rules**



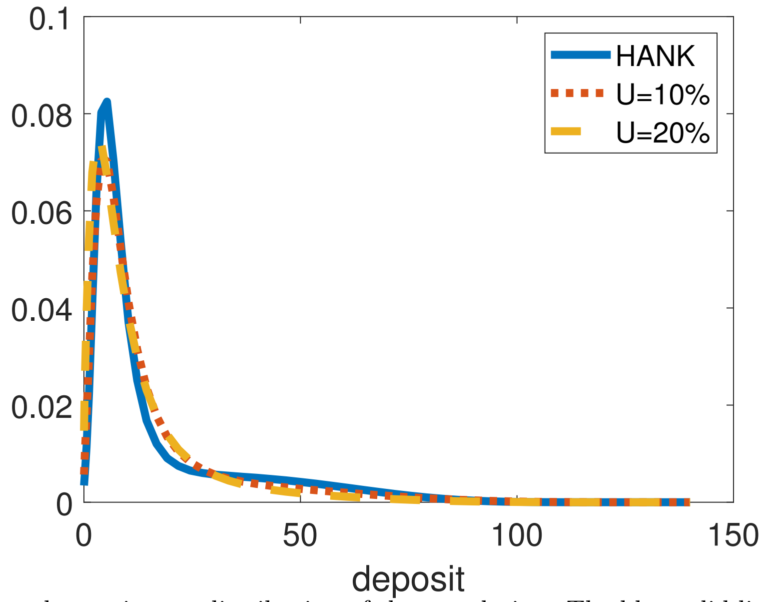
Notes: This figure plots the impulse responses of aggregate variables with linear redistribution under a positive QE shock. The blue solid lines, red dotted lines, and the yellow dashed lines on the left(right) column are for the baseline HANK(L100), L10(S100), and L100(C100) models, respectively. X-axis is time, and Y-axis is the percentage change from steady state.

may be misplaced, at least to the extent to which one focuses on dynamics of aggregate variables.

6 Implications of the Macro-Distribution of Wealth

Section 5 provides robustness checks when we vary the dividend distribution rule, which impacts the micro-distribution of wealth. In this section, we investigate how some parameters related to the macro-distribution of wealth impact our results. In particular, we are interested in how parameters such as the unemployment rate and the unemployment benefit affect the stationary wealth distribution, and in turn how these parameters impact the aggregate

Figure 8: **Stationary Distribution: HANK vs.** $U = 10\%$ **and** $U = 20\%$



*Notes:*This figure plots the stationary distribution of the population. The blue solid lines are for the HANK model, the red dotted lines are for the model with 10% unemployment rate, and the yellow dashed lines are for the model with 20% unemployment rate.

transmission of QE shocks.

In our baseline calibration, we set the unemployment rate to five percent. While consistent with the data, this relatively low unemployment rate means that households face a small probability of job loss. In this section, we allow a higher unemployment rate, which means households face more uninsurable income risk, and study its implications for the behavior of aggregate consumption, and hence other aggregate variables, in response to a QE shock.

Figure 8 plots the stationary wealth distribution in our model with three different targets for the unemployment rate – our baseline specification of five percent (labeled HANK), 10 percent (dotted red lines), and 20 percent (dashed orange lines). Increasing the unemployment rates shifts mass in the stationary distribution towards the borrowing constraint. Table 3 shows the percentage of the population located at the borrowing constraint for different version of the model. In our baseline HANK specification, only 0.025 percent of the population is constrained. This number doubles with an unemployment rate of 10 percent, and increases to 0.16 percent when the unemployment rate is 20 percent.

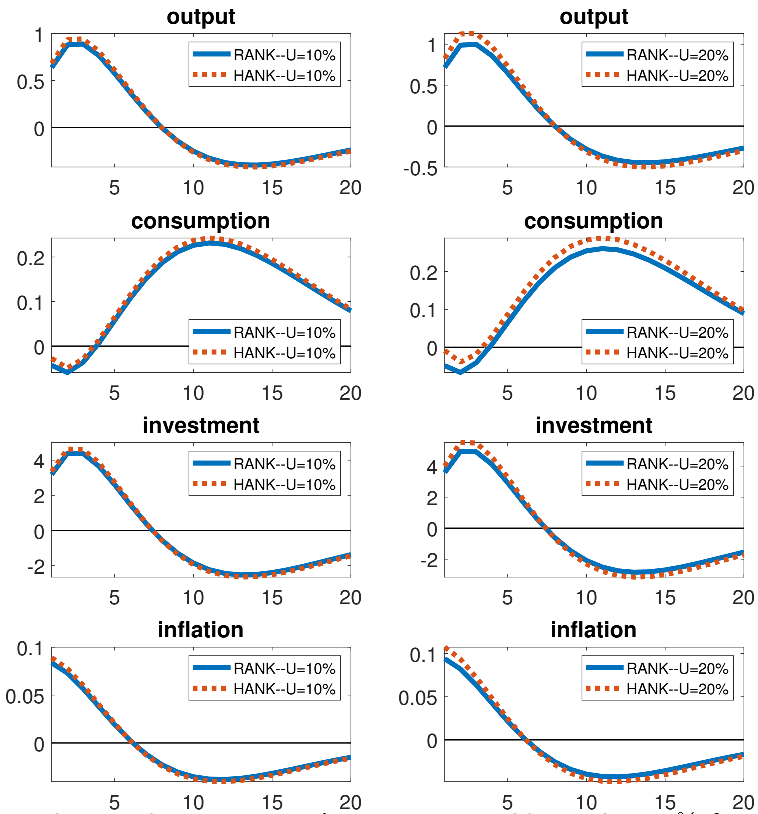
Figure 9 plots impulse responses to a QE shock. The left panel considers our model with

Table 3: **People with binding constraint**

This table reports the fraction of people with binding constraint in different models.

	HANK	$U = 10\%$	$U = 20\%$	$\mu = 69.6\%$
people with binding constraint (%)	0.025	0.055	0.159	28.13

Figure 9: **Impulse responses to a QE shock: HANK vs. RANK for $U = 10\%$ and $U = 20\%$**



Notes: This figure plots the impulse responses of aggregate variables under a 4% QE shock. The blue solid lines and red dotted lines on the left(right) column are for the HANK and RANK models with 10%(20%) unemployment rate, respectively. X-axis is time, and Y-axis is the percentage change from steady state.

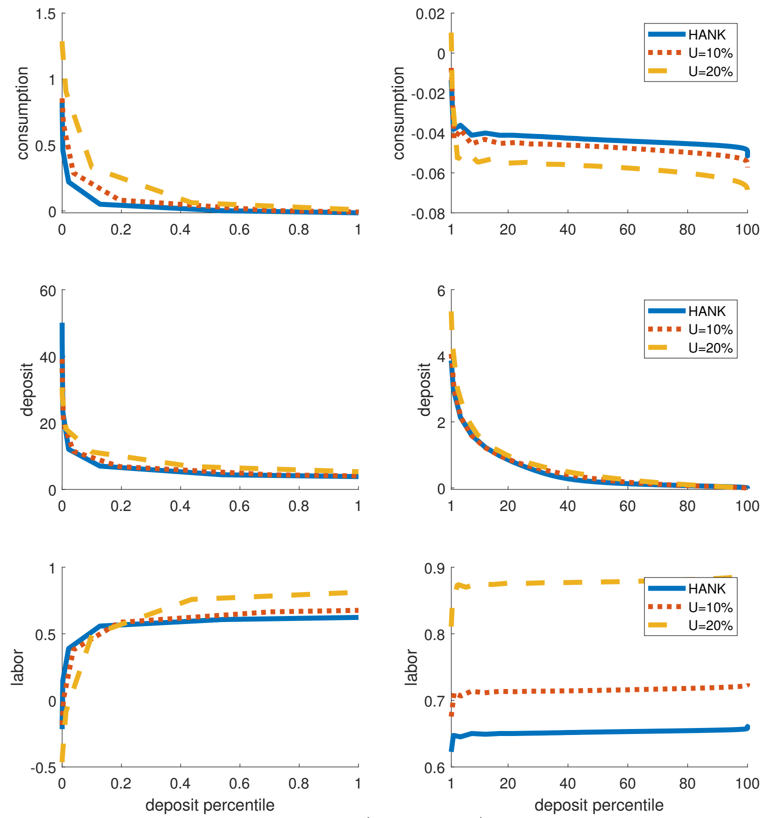
a 10 percent unemployment rate and the right panel with 20 percent. For each panel, we compare the HANK model to RANK version that is re-parameterized to produce comparable average labor input. First, compare the left panel with Figure 2. Consumption, and hence aggregate output, responds more to a QE shock when unemployment is larger, but these differences are quantitatively small. In the right panel, we observe some more noticeable differences between the responses in the HANK and RANK models for the first several periods, but, again, these differences are not quantitatively large. The direction of effects is intuitive – the higher is unemployment, the bigger is the fraction of the population facing the borrowing constraint, and consumption (and hence output) increases more (or decreases less) after a QE shock. Overall, however, the differences between the HANK and RANK models are not economically meaningful whether $U = 10$ or 20 percent.

To provide some intuition, Figure 10 plots the first-period individual choice variable responses to a QE shock as a function of wealth. The figure is analogous to Figure 3, but compares HANK responses with different unemployment rates. The main differences are for the poorest households, who increase their consumption by more, and increase their labor by less, the higher is the unemployment rate. For households away from the borrowing constraint, labor input actually increases more after a QE shock the higher is the unemployment rate. The intuition is as follows: these households try to accumulate more wealth in response to an expansionary shock to insure against a higher probability of potential future job loss.

We next consider the role played by the unemployment benefit, μ in our model. We focus on the case of $\mu = 0.698$, which is substantially higher than our baseline parameterization of 0.4. In the benchmark HANK model, the average steady state individual labor supply is 0.9744. With a tax rate of 30 percent, an average worker would earn 68.2 percent of the market real wage after-tax. Our parameterization of $\mu = 0.696$ implies that unemployed workers make 69.6 percent of the market real wage. This means that unemployment is a benefit rather than a risk. We present the case of $\mu = 0.696$ not because we consider it a reasonable parameterization (in fact, we consider it to be quite extreme), but rather, a parameterization in this range is what is needed to generate a substantial fraction of households at the borrowing constraint.

Figure 11 plots the stationary distribution with $\mu = 0.696$. Most of the population is concentrated around the borrowing constraint. As noted in Table 3, with this parameterization, 28.1 percent of households are located at the borrowing constraint in the stationary distribution. In the HANK literature, the fraction of agents at the constraint is an important metric to generate household heterogeneity and might matter for macro dynamics. However,

Figure 10: **First period individual responses to a QE shock: HANK vs.** $U = 10\%$ **and** $U = 20\%$

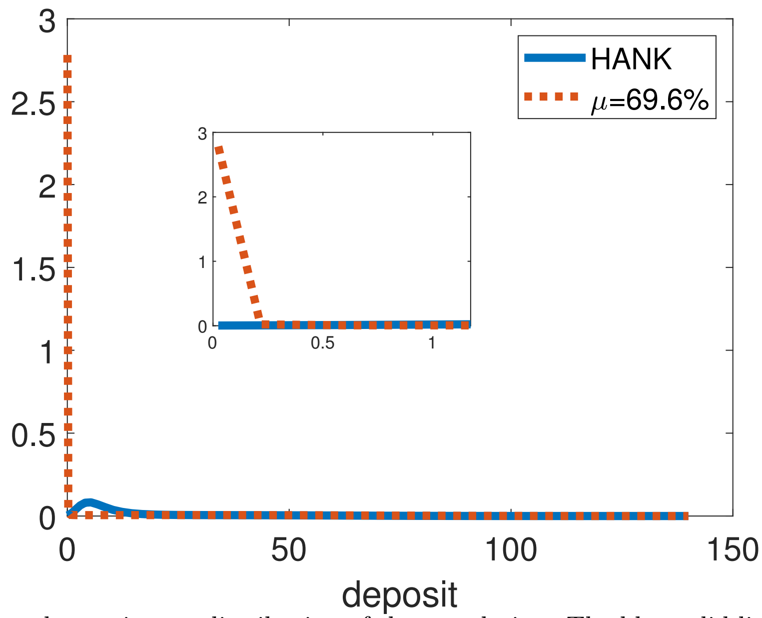


Notes: This figure plots the impulse responses of (employed) individual decision variables (consumption, deposits, and labor supply) in the period when a 4% positive QE shock hits. The blue solid lines and red dotted lines are for the baseline HANK model and the models with $U = 10\%$ and $U = 20\%$, respectively. X-axis is the wealth fraction from poor to rich, and Y-axis is the percentage change from steady state.

in our model, even with 28.1 percent of households at the constraint, the macro implications are similar to our baseline HANK model; see, Figure 12. The only noticeable difference is that consumption increases, instead of decreasing, in response to a QE shock, with this difference lasting for the first several periods. But this difference does not transmits into other aggregate variables.

To conclude, we find that for the aggregate unemployment rate as well as the unemployment benefit, both of which can increase the fraction of constrained households, the transmission of QE shocks into macro aggregates is not very different from a RANK model.

Figure 11: **Stationary Distribution: HANK vs $\mu = 69.6\%$**



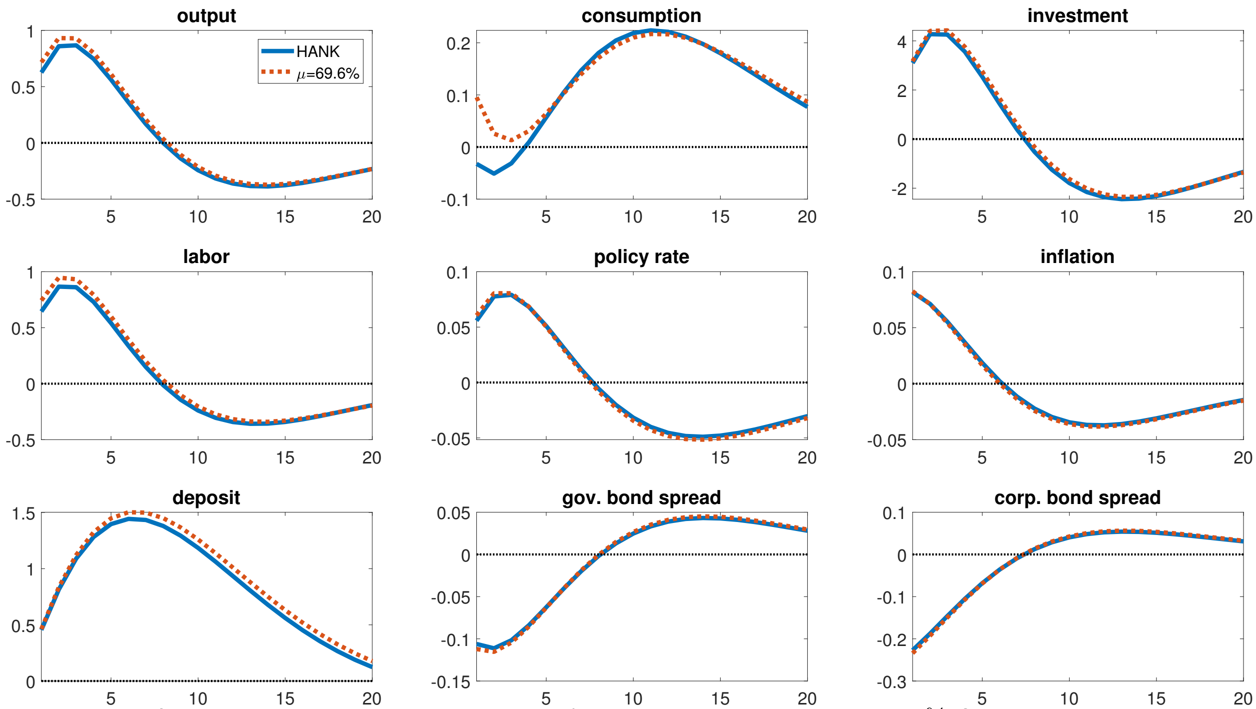
Notes: This figure plots the stationary distribution of the population. The blue solid lines are for the HANK model, the red dotted lines are for the model with 10% unemployment rate, and the yellow dashed lines are for the model with 20% unemployment rate.

7 Conclusion

In this paper, we developed a quantitative DSGE model to study the aggregate implications of central bank asset purchases when there is substantial household wealth heterogeneity. The financial and production sides of the model are similar to Sims and Wu (2021a), which features constrained financial intermediaries and scope for QE to matter. We model households similarly to Krusell and Smith (1998): they are heterogeneous with respect to wealth, face uninsurable unemployment risk, and are subject to a borrowing constraint. We developed a solution method that is compatible with Dynare and uses perturbation methods to solve and simulate the model.

We find that the aggregate responses to a central bank asset purchases are very similar in a HANK specification compared to a RANK version of the model. We consider alternative assumptions about the micro- and macro-distributions of wealth. For the former, we vary how dividends are distributed across households. While different dividend distribution rules can generate different amounts of wealth inequality based on popular metrics, they have

Figure 12: **Impulse responses to a QE shock: HANK vs.** $\mu = 69.6\%$



Notes: This figure plots the impulse responses of aggregate variables under a 4% QE shock. The blue solid lines are for the HANK model, the red dotted lines are for the model with 10% unemployment rate, and the yellow dashed lines are for the model with 20% unemployment rate. X-axis is time in quarters, and Y-axis is the percentage change from steady state.

little discernible impact on how aggregate variables like output react to a QE shock. For the latter, we examine how the aggregate unemployment rate and the unemployment benefit impact the aggregate transmission of QE shocks. While these parameters influence how many households are at or near the borrowing constraint, they do not have much discernible effect on the aggregate effects of a QE shock. We conclude that a RANK specification may provide an adequate approximation for understanding the aggregate implications of QE.

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Appendix A Model Details

This appendix contains details from the model discussed in Section 2.

Appendix A.1 Labor Market

Labor unions, indexed by $h \in [0,1]$ purchase labor from households at price MRS_t, repackage it into $L_{d,t}(h)$, and sell it to a representative labor contractor. A representative labor contractor combines differentiated labor into final labor used in production, via CES technology with elasticity ε_w. The demand curve each labor union faces is

$$L_{d,t}(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon_w} L_{d,t} \tag{A.1}$$

$W_t(h)$ is the wage paid for union h's labor and W_t is the aggregate wage, which satisfies

$$W_t^{1-\epsilon_w} = \int_0^1 W_t(h)^{1-\epsilon_w} dh \tag{A.2}$$

Labor unions are subject to a Calvo-type nominal rigidity. Each period, a union may re-optimize its wage with probability $1-\phi_w$ with $\phi_w \in [0,1]$. Non-reoptimized wages are indexed to lagged inflation at $\gamma_w \in [0,1]$. A labor union chooses a wage to maximize the present discounted value of real profits:

$$\max_{W_t(h)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} \left[\left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{(1-\epsilon_w)\gamma_w} W_t(h)^{1-\epsilon_w} P_{t+j}^{\epsilon_{t}-1} w_{t+j}^{\epsilon_w} L_{d,t+j} - \right.$$
$$\left. mrs_{t+j}\left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{-\epsilon_w\gamma_w} W_t(h)^{-\epsilon_w} P_{t+j}^{\epsilon_w} w_{t+j}^{\epsilon_w} L_{d,t+j} \right] \tag{A.3}$$

where $\Lambda_{t,t+j} = \Lambda_{t,t+1}\cdots\Lambda_{t+j-1,t+j}$.

The first order condition is

$$(\epsilon_w - 1)W_t(h)^{-\epsilon_w} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{(1-\epsilon_w)\gamma_w} P_{t+j}^{\epsilon_w - 1} w_{t+j}^{\epsilon_w} L_{d,t+j} =$$
$$\epsilon_w W_t(h)^{-\epsilon_w - 1} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} mrs_{t+j} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{-\epsilon_w\gamma_w} P_{t+j}^{\epsilon_w} w_{t+j}^{\epsilon_w} L_{d,t+j}$$

The reset wage is the same across all labor unions. Hence, drop the h index, and the optimal price W_t^* can be written as:

$$W_t^* = \frac{\epsilon_w}{\epsilon_w - 1}\frac{F_{1,t}}{F_{2,t}}$$

where $F_{1,t}$ and $F_{2,t}$ are recursive representations of the infinite sums above:

$$F_{1,t} = mrs_t P_t^{\epsilon_w} w_t^{\epsilon_w} L_{d,t} + \phi_w \Lambda_{t,t+1} \Pi_t^{-\epsilon_w\gamma_w} F_{1,t+1}$$
$$F_{2,t} = P_t^{\epsilon_w - 1} w_t^{\epsilon_w} L_{d,t} + \phi_w \Lambda_{t,t+1} \Pi_t^{(1-\epsilon_w)\gamma_w} F_{2,t+1}$$

Written in real terms, $w_t^* = W_t^*/P_t$, it satisfies:

$$w_t^* = \frac{\epsilon_w}{\epsilon_w - 1}\frac{f_{1,t}}{f_{2,t}} \tag{A.4}$$

$$f_{1,t} = mrs_t w_t^{\epsilon_w} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{\Pi_{t+1}}{\Pi_t^{\gamma_w}}\right)^{\epsilon_w} f_{1,t+1} \tag{A.5}$$

$$f_{2,t} = w_t^{\epsilon_w} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{\Pi_{t+1}}{\Pi_t^{\gamma_w}}\right)^{\epsilon_w - 1} f_{2,t+1} \tag{A.6}$$

where $w_t = W_t/P_t$ is the aggregate real wage from (A.2), and $f_{1,t} = F_{1,t}/P_t^{\epsilon_w}$ and $f_{2,t} = F_{2,t}/P_t^{\epsilon_w - 1}$.

Aggregation Integrate (A.1) across h, noting that $\int_0^1 L_{d,t}(h)dh = L_t$. Using the demand function for a union's labor, (A.1), yields

$$L_t = L_{d,t} v_t^w \tag{A.7}$$

where v_t^w is a measure of wage dispersion:

$$v_t^w = \int_0^1 \left(\frac{w_t(h)}{w_t}\right)^{-\epsilon_w} dh$$

Note that this can be written in terms of real wages since it is a ratio. Because of properties of Calvo wage-setting, we can write this as

$$\begin{aligned} v_t^w &= (1-\phi_w)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} + \int_{1-\phi_w}^1 \left(\frac{\Pi_{t-1}^{\gamma_w} W_{t-1}(h)}{W_t}\right)^{-\epsilon_w} dh \\ &= (1-\phi_w)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} + \Pi_{t-1}^{-\gamma_w \epsilon_w} W_t^{\epsilon_w} W_{t-1}^{-\epsilon_w} \int_{1-\phi_w}^1 \left(\frac{W_{t-1}(h)}{W_{t-1}}\right)^{-\epsilon_w} dh \end{aligned}$$

which may be written as

$$v_t^w = (1-\phi_w)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} + \phi_w \Pi_{t-1}^{-\gamma_w \epsilon_w} W_t^{\epsilon_w} W_{t-1}^{-\epsilon_w} v_{t-1}^w$$

Expressing this in real terms gives

$$v_t^w = (1-\phi_w)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} + \phi_w \left(\frac{\Pi_t}{\Pi_{t-1}^{\gamma_w}}\right)^{\epsilon_w} \left(\frac{w_t}{w_{t-1}}\right)^{\epsilon_w} v_{t-1}^w \tag{A.8}$$

From (A.2), we have

$$W_t^{1-\epsilon_w} = (1-\phi_w)\left(W_t^*\right)^{1-\epsilon_w} + \int_{1-\phi_w}^1 \left(\Pi_{t-1}^{\gamma_w} W_{t-1}(h)\right)^{1-\epsilon_w} dh$$

Via a law of large numbers, this is

$$W_t^{1-\epsilon_w} = (1-\phi_w)\left(W_t^*\right)^{1-\epsilon_w} + \Pi_{t-1}^{\gamma_w(1-\epsilon_w)} \phi_w W_{t-1}^{1-\epsilon_w}$$

Dividing both sides by $P_t^{1-\epsilon_w}$ gives

$$w_t^{1-\epsilon_w} = (1-\phi_w)\left(w_t^*\right)^{1-\epsilon_w} + \phi_w \Pi_{t-1}^{\gamma_w(1-\epsilon_w)} \Pi_t^{\epsilon_w - 1} w_{t-1}^{1-\epsilon_w} \tag{A.9}$$

Appendix A.2 Capital Producers

A representative capital producer transfers raw investment goods I_t into new capital $\hat{I}_t$ via:

$$\hat{I}_t = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{A.10}$$

where $S\left(I_t/I_{t-1}\right) = \frac{\kappa_I}{2}\left(I_t/I_{t-1} - 1\right)^2$ is the investment adjustment cost.

The capital producer sells new capital to wholesale firms, and earns real profit in period t:

$$div_{k,t} = p_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t \tag{A.11}$$

where p_t^k is the real price of new capital. The object of a capital producer is to maximize the present value of profit:

$$\max_{I_t} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ p_t^k \left[1 - S\left(\frac{I_{t+j}}{I_{t+j-1}}\right)\right] I_{t+j} - I_{t+j} \right\}$$

The first order condition is

$$1 = p_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^k S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{A.12}$$

Appendix A.3 Output

The final output good, Y_t, is a CES aggregate of retail outputs with elasticity of substitution $\epsilon_p > 1$. Profit maximization by the final good producer generates a demand curve for each retail output and an expression for the aggregate price index:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon_p} Y_t. \tag{A.13}$$

The price of the final goods, P_t, satisfies:

$$P_t^{1-\epsilon_p} = \int_0^1 P_t(f)^{1-\epsilon_p} df. \tag{A.14}$$

Each period, a retailer faces a constant probability, $1 - \phi_p$, of being able to adjust its price. Consider the problem of a retailer given the opportunity to adjust in period t. When it sets a price, it must take into account that the price chosen in t will still be in effect in period $t + j$ with probability ϕ_p^j. Indexation to lagged inflation means that an unupdated price in period $t + j$ will be: $P_t(f)\left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{\gamma_p}$. When choosing a price, the retailer maximizes the present discounted value of real profits returned to the household, where discounting is by the household's stochastic discount factor augmented by the probability of non-adjustment:

$$\max_{P_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} \left[P_t(f)^{1-\epsilon_p} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{(1-\epsilon_p)\gamma_p} P_{t+j}^{\epsilon_p - 1} Y_{t+j} - P_{m,t+j} P_t(f)^{-\epsilon_p} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{-\epsilon_p \gamma_p} P_{t+j}^{\epsilon_p - 1} Y_{t+j} \right]$$

The first order condition is

$$(\epsilon_p - 1)P_t(f)^{-\epsilon_p}\mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{(1-\epsilon_p)\gamma_p} P_{t+j}^{\epsilon_p - 1} Y_{t+j} =$$
$$\epsilon_p P_t(f)^{-\epsilon_p - 1}\mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{m,t+j} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{-\epsilon_p\gamma_p} P_{t+j}^{\epsilon_p} Y_{t+j}$$

Define variables:

$$X_{1,t} = \sum_{j=0}^{\infty} (\phi_p\beta)^j \frac{\mu_{t+j}}{\mu_t} P_{m,t+j} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{-\epsilon_p\gamma_p} P_{t+j}^{\epsilon_p} Y_{t+j}$$
$$X_{2,t} = \sum_{j=0}^{\infty} (\phi_p\beta)^j \frac{\mu_{t+j}}{\mu_t} \left(\frac{P_{t+j-1}}{P_{t-1}}\right)^{(1-\epsilon_p)\gamma_p} P_{t+j}^{\epsilon_p - 1} Y_{t+j}$$

These can be written recursively as:

$$X_{1,t} = p_{m,t} P_t^{\epsilon_p} Y_t + \phi_p \Lambda_{t,t+1} \Pi_t^{-\epsilon_p\gamma_p} X_{1,t+1}$$
$$X_{2,t} = P_t^{\epsilon_p - 1} Y_t + \phi_p \Lambda_{t,t+1} \Pi_t^{(1-\epsilon_p)\gamma_p} X_{2,t+1}$$

Note, all retailers set the same price. We call this reset price P_t^*. Hence, the first order condition can be written as

$$P_t^* = \frac{\epsilon_p}{\epsilon_p - 1}\frac{X_{1,t}}{X_{2,t}}$$

Defining $x_{1,t} = X_{1,t}/P_t^{\epsilon_p}$, $x_{2,t} = X_{2,t}/P_t^{\epsilon_p - 1}$, and $p_t^* = P_t^*/P_t$ gives

$$p_t^* = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}} \tag{A.15}$$

$$x_{1,t} = p_{m,t} Y_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{\Pi_{t+1}}{\Pi_t^{\gamma_p}}\right)^{\epsilon_p} x_{1,t+1} \tag{A.16}$$

$$x_{2,t} = Y_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{\Pi_{t+1}}{\Pi_t^{\gamma_p}}\right)^{\epsilon_p - 1} x_{2,t+1} \tag{A.17}$$

where $p_{m,t} = \frac{P_{m,t}}{P_t}$.

Aggregation Integrate the demand for retail output, (A.13), across retailers, noting that $Y_t(f) = Y_{m,t}(f)$, $\int_0^1 Y_{m,t}(f)df = Y_{m,t}$, where $Y_{m,t}$ is wholesale output. This yields

$$Y_t v_t^p = Y_{m,t} \tag{A.18}$$

where

$$v_t^p = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon_p} df$$

This is a measure of price dispersion. Using properties of Calvo pricing with indexation to lagged inflation yields:

$$v_t^p = (1-\phi_p)(p_t^*)^{-\epsilon_p} + \int_{1-\phi_p}^1 \left(\frac{\Pi_{t-1}^{\gamma_p} P_{t-1}(f)}{P_t}\right)^{-\epsilon_p} df$$
$$= (1-\phi_p)(p_t^*)^{-\epsilon_p} + \Pi_{t-1}^{-\gamma_p\epsilon_p} P_t^{\epsilon_p} P_{t-1}^{-\epsilon_p} \int_{1-\phi_p}^1 \left(\frac{P_{t-1}(f)}{P_{t-1}}\right)^{-\epsilon_p} df$$

Via a law of large numbers, this reduces to

$$v_t^p = (1-\phi_p)(p_t^*)^{-\epsilon_p} + \phi_p \left(\frac{\Pi_t}{\Pi_{t-1}^{\gamma_p}}\right)^{\epsilon_p} v_{t-1}^p \tag{A.19}$$

Similarly, the aggregate price index, (A.14), may be written as:

$$\begin{aligned} P_t^{1-\epsilon_p} &= (1-\phi_p)(P_t^*)^{1-\epsilon_p} + \int_{1-\phi_p}^1 \Pi_{t-1}^{\gamma_p(1-\epsilon_p)} P_{t-1}(f)^{1-\epsilon_p} df \\ &= (1-\phi_p)(P_t^*)^{1-\epsilon_p} + \phi_p \Pi_{t-1}^{\gamma_p(1-\epsilon_p)} P_{t-1}^{1-\epsilon_p} \end{aligned}$$

Divide both sides by $P_t^{1-\epsilon_p}$ to obtain

$$1 = (1-\phi_p)(p_t^*)^{1-\epsilon_p} + \phi_p \Pi_{t-1}^{\gamma_p(1-\epsilon_p)} \Pi_t^{\epsilon_p - 1} \tag{A.20}$$

The representative wholesale firm produces output according to a Cobb-Douglas technology:

$$Y_{m,t} = Z_t (u_t K_t)^\alpha L_{d,t}^{1-\alpha} \tag{A.21}$$

$Y_{m,t}$ is flow output, u_t is the capital utilization rate, and $L_{d,t}$ is labor input. $0 < \alpha < 1$ is the exponent on capital services in the production function. Z_t is an exogenous productivity variable that obeys an exogenous stochastic process. K_t is the stock of physical capital, which the firm owns. Physical capital accumulates according to a standard law of motion:

$$K_{t+1} = \hat{I}_t + (1 - \delta(u_t))K_t \tag{A.22}$$

where $\delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$ is the capital depreciation rate and depends on capital utilization.

The wholesale firm must issue perpetual bonds to finance the purchase of new physical capital, $\hat{I}_t$. Different than them, we only require the firm to finance a constant fraction, $\psi \in [0,1]$, of investment, not the entirety. This gives rise to a "loan in advance constraint" of the form:

$$\psi P_t^k I_t \leq Q_t CF_{m,t} = Q_t (F_{m,t} - \kappa F_{m,t-1}), \tag{A.23}$$

where P_t^k is the price at which the wholesale firm purchases new physical capital.

The wholesale firm hires labor in a competitive spot market at nominal wage W_t. Its nominal dividend is

$$DIV_{m,t} = P_{m,t} Z_t K_t^\alpha L_{d,t}^{1-\alpha} - W_t L_{d,t} - P_t^k I_t - F_{m,t-1} + Q_t (F_{m,t} - \kappa F_{m,t-1}) \tag{A.24}$$

The firm maximizes the present discounted value of real dividends, where discounting is by the stochastic discount factor of households. The first order conditions are

$$w_t = (1-\alpha) p_{m,t} Z_t K_t^\alpha L_{d,t}^{-\alpha} \tag{A.25}$$

$$p_t^k M_{1,t} \delta'(u_t) = \alpha p_{m,t} (u_t K_t)^{\alpha-1} L_{d,t}^{1-\alpha} \tag{A.26}$$

$$p_t^k M_{1,t} = \mathbb{E}_t \Lambda_{t,t+1} \left[\alpha p_{m,t+1} Z_{t+1} K_{t+1}^{\alpha-1} L_{d,t+1}^{1-\alpha} + (1-\delta(u_t)) p_{t+1}^k M_{1,t+1}\right] \tag{A.27}$$

$$Q_t M_{2,t} = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} [1 + \kappa Q_{t+1} M_{2,t+1}] \tag{A.28}$$

$$\frac{M_{1,t} - 1}{M_{2,t} - 1} = \psi \tag{A.29}$$

$w_t = W_t/P_t$ is the real wage, $p_{m,t} = P_{m,t}/P_t$ is the relative price of wholesale output, and $p_t^k = P_t^k/P_t$ is the relative price of new capital. (A.25) is the standard static first order condition for labor demand. $M_{1,t}$ is one plus the product of ψ with the multiplier on the constraint that firms must issue bonds to finance investment, (A.23), while $M_{2,t}$ is simply one plus the multiplier on the constraint. (A.26) is the first-order condition for capital utilization. (A.27) and (A.28) are optimality conditions for capital and bonds, respectively. If the constraint did not bind, then $M_{1,t} = M_{2,t} = 1$ and (A.27)-(A.28) would reduce to standard asset pricing conditions. $M_{1,t}$ serves as an endogenous "investment wedge" and $M_{2,t}$ can be thought of as a "financial wedge." These wedges distort the standard asset pricing decisions and fluctuations in these wedges are the mechanism through which QE type policies transmit to the real economy.

Appendix A.4 Exogenous Processes and Aggregation

Aggregate productivity, A_t; government spending, G_t; and the credit shock follow AR(1) processes in the log:

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \epsilon_{A,t}, \tag{A.30}$$

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + s_G \epsilon_{G,t}, \tag{A.31}$$

$$\ln \theta_t = (1-\rho_\theta \ln \theta + \rho_\theta \ln \theta_{t-1} + s_\theta \epsilon_{\theta,t}. \tag{A.32}$$

Autoregressive parameters are restricted to lie between zero and one, and shocks are drawn from standard normal distributions, with s_A, s_G, and s_θ denoting standard deviations. G and θ are non-stochastic steady state values of government spending and the credits shock, respectively. The non-stochastic steady state value of labor productivity is normalized to unity.

The exogenous process for central bank holdings of private bonds is given in (2.25). There is a similar law of motion for central bank holdings of government bonds:

$$\ln b_{cb,t} = (1-\rho_b)b_{cb} + \rho_b \ln b_{cb,t-1} + \sigma_b \epsilon_{b,t}. \tag{A.33}$$

There are therefore six exogenous shocks in the model: productivity ($\epsilon_{A,t}$), government spending ($\epsilon_{G,t}$), credit ($\epsilon_{\theta,t}$), monetary policy ($\epsilon_{r,t}$), private QE ($\epsilon_{f,t}$), and public QE ($\epsilon_{b,t}$).

Privately-issued and government-issued bonds must be held by either financial intermediaries or the central bank. In real terms, the bond market-clearing conditions are:

$$f_{m,t} = f_t + f_{cb,t} \tag{A.34}$$

$$\bar{b}_G = b_t + b_{cb,t}, \tag{A.35}$$

where $f_t = \sum_i f_{i,t}$ and $b_t = \sum_i b_{i,t}$.

Aggregated across intermediaries, and written in real terms, the aggregate financial intermediary balance sheet condition is:

$$Q_t f_t + Q_{B,t} b_t + re_t = d_t + n_t. \tag{A.36}$$

The aggregate dynamics of intermediary net worth are given by:

$$n_t = \sigma \Pi_t^{-1} \Big[(R_t^F - R_{t-1}^d) Q_{t-1} f_{t-1} + (R_t^B - R_{t-1}^d) Q_{B,t-1} b_{t-1} + (R_{t-1}^{re} - R_{t-1}^d) re_{t-1} + R_{t-1}^d n_{t-1} \Big] + X \tag{A.37}$$

Similarly, aggregating the endogenous leverage constraint across intermediaries yields:

$$Q_t f_t + \Delta Q_{B,t} b_t \leq \phi_t n_t \tag{A.38}$$

The aggregate resource constraint is standard:

$$Y_t = C_t + I_t + G_t \tag{A.39}$$

Appendix B Approximating Individual Policy Functions

Define conditional expectation[9]

$$\mathbb{T}(d_{t-1}, \varepsilon_t) \equiv \beta R_t^d \mathbb{E}_t \left[\frac{c_{t+1}^{-1}}{\Pi_{t+1}} \right] \tag{B.1}$$

Approximate the conditional expectation function $\mathbb{T}(.)$ using Chebyshev polynomials:

$$\mathbb{T}(\varepsilon_t, d_{t-1}) \approx \exp \left\{ \sum_{n=0}^{N} \theta_{n,t}(\varepsilon_t) \mathbb{T}_n(\xi(d_{t-1})) \right\} \tag{B.2}$$

where $\xi(d) = 2\frac{d-\underline{d}}{\bar{d}-\underline{d}} - 1$ transforms the interval $d \in (\underline{d}, \bar{d})$ to $(-1, 1)$, and the Chebyshev polynomials are defined as following:

$$\mathbb{T}_n(x) = \cos(n \arccos x) \tag{B.3}$$

The algorithm aims to fit a set of nodes $\{d_m\}_{m=1}^M$ with an N order polynomial, where $M = N + 1$. The Chebyshev nodes defined on $(-1, 1)$ are

$$x_m = -\cos\left(\frac{2m-1}{2M} \pi \right). \tag{B.4}$$

Adjust nodes to $(\underline{d}, \bar{d})$ interval:

$$d_m = (x_m + 1)\left(\frac{\bar{d} - \underline{d}}{2} \right) + \underline{d}. \tag{B.5}$$

The policy functions can be solved jointly using the following system:

$$d_t = \max \left\{ \frac{R_{t-1}^d}{\Pi_t} d_{t-1} + mrs_t[(1-\tau)l_t\varepsilon_t + \mu(1-\varepsilon_t)] - \mathbb{T}(d_{t-1}, \varepsilon_t)^{-1} - \mathcal{T}_t + div_t - X, \underline{d} \right\} \tag{B.6}$$

$$l_t = \min \left\{ \left[\frac{(1-\tau) mrs_t}{\chi c_t} \right]^{\frac{1}{\eta}}, \bar{l} \right\} \tag{B.7}$$

$$c_t = \frac{R_{t-1}^d}{\Pi_t} d_{t-1} + mrs_t[(1-\tau)l_t\varepsilon_t + \mu(1-\varepsilon_t)] - d_t - \mathcal{T}_t + div_t - X \tag{B.8}$$

For the individual $d_{t-1} = d_m$, the right hand side of (B.1) can be expressed as

$$\mathbb{E}_t \left[\beta \frac{R_t^d}{\Pi_{t+1}} c_{t+1}^{-1} \right] = \mathbb{E}_t \left[\beta \frac{R_t^d}{\Pi_{t+1}} \sum_{\varepsilon_{t+1}} p(\varepsilon_{t+1}|\varepsilon_t) c_{t+1}(d_t(d_{t-1} = d_m, \varepsilon_t), \varepsilon_{t+1})^{-1} \right] \tag{B.9}$$

The next step is to compute the right hand side of (B.9), which requires c_{t+1}. With $d_t(d_{t-1}, \varepsilon_t)$ solved using (B.6) - (B.8), we can compute conditional expectation next period $\mathbb{T}(\varepsilon_{t+1}, d_t)$ by shifting (B.2) one period forward. Then, we can solve c_{t+1} by shifting the system in (B.6) - (B.8) one period forward. Combining (B.1), (B.2), and (B.9), we have $M \times 2$ equations for $M \times 2$ variables $\theta_{n,t}(\varepsilon_t)$:

$$\sum_{n=0}^{N} \theta_{n,t}(\varepsilon_t) \mathbb{T}_n(\xi(d_m)) = \log \mathbb{E}_t \left[\beta \frac{R_t^d}{\Pi_{t+1}} \sum_{\varepsilon_{t+1}} p(\varepsilon_{t+1}|\varepsilon_t) c_{t+1}(\varepsilon_{t+1}, \varepsilon_t, d_{t-1} = d_m)^{-1} \right] \tag{B.10}$$

[9]For brevity, we drop j subscript in the appendices.

Note, $\theta_{n,t}(\varepsilon_t)$ and $\theta_{n,t+1}(\varepsilon_{t+1})$ also enter the right hand side via c_{t+1}.

Appendix C Equilibrium conditions

Our algorithm uses Dynare to solve the system of equations listed below.

Appendix C.1 Individual Households and Their Distribution

- Chebyshev coefficients: $M \times 2$ equations in (B.10) for $M \times 2$ variables $\theta_{n,t}(\varepsilon_t)$ for $m = 1, ..., M$.
- Individual decisions: (B.6) - (B.8) for $d_t(d_m, \varepsilon_t), l_t(d_m, \varepsilon_t), c_t(d_m, \varepsilon_t)$ for $m = 0, ..., M$.. These are $(M+1) \times 2 \times 3$ equations.
- Dynamics of the cross sectional distribution: (3.1) for $p(d_t = d_m, \varepsilon_{t+1})$ for $m = 0, ..., M$, $(M+1) \times 2$ equations for $(M+1) \times 2$ probabilities.

Appendix C.2 Aggregate Variables

Aggregate deposit and labor (2 equations)

$$D_{t-1} = \sum_{\varepsilon_t} \sum_{m=0}^{M} d_m p(d_{t-1} = d_m, \varepsilon_t) \tag{C.1}$$

$$L_t = \sum_{\varepsilon_t} \sum_{m=0}^{M} l_t(\varepsilon_t, d_{t-1} = d_m) p(d_{t-1} = d_m, \varepsilon_t) \tag{C.2}$$

Equations for aggregate variables in different sectors (29 equations)

- Labor market (3 equations): (A.4) - (A.6)
- Production (14 equations): capital producers (A.10) and (A.12), wholesale firms (A.21) - (A.23) and (A.25) - (A.29), retailers (A.15) - (A.17)
- Financial intermediaries (7 equations): (2.10),(2.11), (2.14) - (2.18)
- Monetary policy (1 equation): (2.23)
- Fiscal authority (1 equation): (2.22)
- Central bank (1 equation): $T_{cb,t} = (1 + \kappa Q_t)\,\Pi_t^{-1} f_{cb,t-1} + (1 + \kappa Q_{B,t})\,\Pi_t^{-1} b_{cb,t-1} - R^{re}_{t-1} \Pi_t^{-1} re_{t-1}$.

Aggregation (21 equations)

- Exogenous process

$$\ln Z_t = \rho_z \ln Z_{t-1} + \sigma_z \epsilon_{zt} \tag{C.3}$$

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \sigma_\theta \epsilon_{\theta,t} \tag{C.4}$$

$$\ln G_t = \rho_G \ln G_{t-1} + \sigma_G \epsilon_{G,t} \tag{C.5}$$

$$b_{cb,t} = (1 - \rho_b) b_{cb} + \rho_b b_{cb,t-1} + \sigma_b \epsilon_{b,t} \tag{C.6}$$

$$f_{cb,t} = (1 - \rho_f) f_{cb} + \rho_f f_{cb,t-1} + \sigma_f \epsilon_{f,t} \tag{C.7}$$

- Bond market clearing condition

$$f_{m,t} = f_t + f_{cb,t} \tag{C.8}$$

$$b_{G,t} = b_t + b_{cb,t} \tag{C.9}$$

- Aggregation for retailers (A.18) - (A.20)
- Aggregation for labor market (A.7) - (A.9)
- Aggregate the balance sheet condition of financial intermediaries (2.8)

$$Q_t f_t + Q_{B,t} b_t + re_t = d_t + n_t \tag{C.10}$$

- Aggregate net worth dynamics of the financial intermediaries (2.9)

$$n_t = \sigma \Pi_t^{-1} [(R_t^F - R_{t-1}^d) Q_{t-1} f_{t-1} + (R_t^B - R_{t-1}^d) Q_{B,t-1} b_{t-1} + (R_{t-1}^{re} - R_{t-1}^d) re_{t-1} + R_{t-1}^d n_{t-1}] + X \tag{C.11}$$

- Aggregating (2.20) across intermediaries yields:

$$Q_t f_t + \Delta Q_{B,t} b_t \leq \phi_t n_t \tag{C.12}$$

  where (C.12) holds with equality when (2.13) binds.
- Central bank's balance sheet condition

$$Q_t f_{cb,t} + Q_{B,t} b_{cb,t} = re_t \tag{C.13}$$

- The aggregate resource constraint is standard:

$$Y_t = C_t + I_t + G_t \tag{C.14}$$

- Dividend:

$$div_t = Y_t - w_t L_t - I_t + Q_t \left(f_t - \kappa \frac{f_{t-1}}{\Pi_t} \right) - \frac{f_{t-1}}{\Pi_t} + (1 - \sigma) n_t. \tag{C.15}$$

- Stochastic discount factor:

$$\Lambda_{t-1,t} = \frac{\Pi_t}{R_{t-1}^d} \tag{C.16}$$

These are 49 equations for 49 variables $\Big\{ K_t, L_t, R_t^F, R_t^{re}, R_t^d, Q_t, \Omega_t, \Pi_t, \lambda_t, \phi_t, re_t, n_t, C_t, w_t, p_t^*, x_{1,t}, x_{2,t}, p_{m,t}, Y_t, Y_{m,t}, I_t, p_t^k, f_{m,t}, M_{1,t}, M_{2,t}, I_t, f_{cb,t}, Z_t, \theta_t, f_t, d_t, v_t^p, R_t^B, Q_{B,t}, G_t, T_{cb,t}, b_t, b_{cb,t}, \hat{I}_t, \mathcal{T}_t, div_t, \Lambda_{t-1,t}, mrs_t, L_{d,t}, w_t^*, f_{1,t}, f_{2,t}, v_t^w, u_t \Big\}$.

Appendix D Solving for the Stationary Equilibrium

(Step 1) Given aggregate deposit and labor supply D, L, we compute steady states for all aggregate variables.

(Step 2) Solve for the steady state of the Chebyshev coefficients $\vartheta(\varepsilon) = [\theta_0(\varepsilon), ..., \theta_{N_T}(\varepsilon)]$ that approximate the conditional expectation function. We use the Chebyshev grid d_m for $m = 1, ..., M$, where d_m is computed using (B.5).

(Step 2.1) Compute the Chebyshev polynomial $\mathbb{T}_n(\xi(d_m))$ using (B.3).

(Step 2.2) Initialization

(Step 2.2.1) Initialize by assuming $d_t = d_{t-1} = d_m$, and $l = \bar{l}$, then we can solve consumption $c^{(0)}(\varepsilon, d_m)$ using (B.8).

(Step 2.2.2) Compute the right hand side of (B.10) using $c^{(0)}(\varepsilon, d_m)$ from the previous step.

(Step 2.2.3) For each ε, we have a system of M linear equations in (B.10). We can use them to solve M unknowns in $\vartheta(\varepsilon)$, and call it $\vartheta^{(0)}(\varepsilon)$. The value of the equation computed in the previous step is $\mathbb{T}(d_m, \vartheta^{(0)}(\varepsilon))$.

(Step 2.3) This step outlines how we update from $\vartheta^{(j)}(\varepsilon)$ to $\vartheta^{(j+1)}(\varepsilon)$. Repeat this step until $\vartheta^{(j+1)}(\varepsilon) - \vartheta^{(j)}(\varepsilon)$ is within a specified tolerance, and call it $\vartheta(\varepsilon) = \vartheta^{(j+1)}(\varepsilon)$.

(Step 2.3.1) With $\vartheta_t^{(j)}(\varepsilon_t) = \vartheta^{(j)}(\varepsilon)$, we have computed the conditional expectation $\mathbb{T}(d_{t-1} = d_m, \vartheta_t^{(j)}(\varepsilon_t))$ from the previous iteration.

(Step 2.3.2) Compute current period deposit $d_t(\varepsilon_t, d_{t-1} = d_m)$, consumption $c_t(\varepsilon_t, d_{t-1} = d_m)$, and labor supply $l_t(\varepsilon_t, d_{t-1} = d_m)$ using (B.6) - (B.8) with $\mathbb{T}(d_{t-1} = d_m, \vartheta_t^{(j)}(\varepsilon_t))$. Instead of solving a system of equations with kinks, we implement this step as follows:

- Assume the borrowing constraint is not binding, in this case

$$c_t = \mathbb{T}(d_{t-1} = d_m, \vartheta_t^{(j)}(\varepsilon_t))^{-1} \tag{D.1}$$

  Then we can compute l_t and d_t subsequently.
- Check d_t against its bound $\underline{d}$
  - If $d_t \geq \underline{d}$, then we are done with (Step 2.3.2).
  - If $d_t < \underline{d}$, then $d_t = \underline{d}$, and we can solve c_t and l_t jointly.

(Step 2.3.3) Compute conditional expectation next period $\mathbb{T}(d_t(\varepsilon_t, d_{t-1} = d_m), \vartheta_{t+1}^{(j)}(\varepsilon_{t+1}))$ by shifting (B.2) one period forward, where $\vartheta_{t+1}^{(j)}(\varepsilon_{t+1}) = \vartheta^{(j)}(\varepsilon)$, and $d_t(\varepsilon_t, d_{t-1} = d_m)$ is from the previous step.

(Step 2.3.4) Compute next period's consumption $c_{t+1}(\varepsilon_{t+1}, d_t(\varepsilon_t, d_{t-1} = d_m))$ by shifting (Step 2.3.2) one period forward and using $\mathbb{T}(d_t(\varepsilon_t, d_{t-1} = d_m), \vartheta_{t+1}^{(j)}(\varepsilon_{t+1}))$ from the previous step.

(Step 2.3.5) Compute the right hand side of (B.10) using $c_{t+1}(\varepsilon_{t+1}, d_t(\varepsilon_t, d_{t-1} = d_m))$ from the previous step.

(Step 2.3.6) For each ε_t, we have a system of M linear equations in (B.10). We can use them to solve M unknowns in $\vartheta_t(\varepsilon_t)$ on the left hand side, and call it $\vartheta^{(j+1)}(\varepsilon)$. The value of the equation computed in the previous step is $\mathbb{T}(d_{t-1} = d_m, \vartheta_t^{(j+1)}(\varepsilon_t))$.

(Step 3) Use the histogram approximation of of Young (2010) to approximate the distribution. We approximate over the grid $\{d_m\}_{m=0}^{M}$, which is the same as the Chebyshev grid in (B.5) with one additional point at the borrowing constraint $d_0 = \underline{d}$.

(Step 3.1) Compute conditional expectation and policy functions at the borrowing constraint $d_0 = \underline{d}$. Compute the conditional expectation $\mathbb{T}(d_{t-1} = d_0, \vartheta_t(\varepsilon_t))$ with $\vartheta_t(\varepsilon_t) = \vartheta(\varepsilon)$ from (Step 2.3). Then compute policy function $d_t(\varepsilon_t, d_{t-1} = d_0)$, $c_t(\varepsilon_t, d_{t-1} = d_0)$, $l_t(\varepsilon_t, d_{t-1} = d_0)$ following (Step 2.3.2).

(Step 3.2) Iterate the transition dynamics in (3.1) forward on the $\{d_m, \varepsilon\}$ grid:

(Step 3.3.1) Start with an initial guess for $p(d_0 = d_m, \varepsilon_1)$

(Step 3.3.2) Use the transition probabilities in (3.2) and (2.1) to move one period forward from $p(d_{\tau-1} = d_m, \varepsilon_\tau)$ to $p(d_\tau = d_m, \varepsilon_{\tau+1})$. Iterate until it converges to the stationary distribution, and call it $p(d_{t-1} = d_m, \varepsilon_t)$.

(Step 3.3) Compute the aggregate deposit $\tilde{D}$ and labor $\tilde{L}$ using (C.1) and (C.2) where $p(d_{t-1} = d_m, \varepsilon_t)$ is from the previous step, and $l_t(\varepsilon_t, d_{t-1} = d_m)$ is from (Step 2.3.2).

The above steps compute $\tilde{D}, \tilde{L}$ given D, L. This amounts to a fixed point problem. We solve this fixed point problem by iterating over these steps.

Appendix E Calibration

Table E.1: **Additional Calibration**

Parameters	Value	Target	Description
SW parameters			
κ	$1-40^{-1}$	bond duration $=40$	Coupon decay parameter
ψ	0.81		Fraction of investment from debt
σ	0.95		Intermediary survival probability
θ		$400(R^F-R^d)=3$	Recoverability parameter
X		Leverage $=4$	Transfer to new intermediaries
Δ	1/3		Government bond recoverability
b_{cb}		$\frac{b_{cb}Q_B}{4Y}=0.06$	Steady state central bank Treasury holdings
f_{cb}	0		Steady state central private bond holdings
ρ_θ	0.98		AR credit
ρ_b	0.8		AR central bank Treasury
ρ_f	0.8		AR central bank private bonds
$\bar{b}_G$		$\frac{B_GQ_B}{4Y}=0.41$	Steady state government debt
G		$\frac{G}{Y}=0.2$	Steady state government spending
Standard parameters			
β	0.995		Discount factor
η	1		Inverse Frisch elasticity
α	0.33		Production function exponent on capital
δ_0	0.025		Steady state depreciation
δ_1		$u=1$	Utilization linear term
δ_2	0.01		Utilization squared term
κ_I	2		Investment adjustment cost
Π	1		Steady state (gross) inflation
ϵ_p	11		Elasticity of substitution goods
ϵ_w	11		Elasticity of substitution labor
ϕ_p	0.75		Price rigidity
ϕ_w	0.75		Wage rigidity
γ_p	0		Price indexation
γ_w	0		Wage indexation
ρ_r	0.8		Taylor rule smoothing
ϕ_π	1.5		Taylor rule inflation
ϕ_y	0.25		Taylor rule output growth
ρ_A	0.95		AR productivity
ρ_G	0.95		AR government spending
Shock sizes			
s_A	0.0065		SD productivity
s_G	0.01		SD government spending
s_θ	0.04		SD credit

Appendix F RANK Household

In the representative agent model, the lifetime utility of the household is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \chi \frac{L_t^{1+\eta}}{1+\eta} \right),$$

The flow budget constraint is

$$D_t = \frac{R_{t-1}^d}{\Pi_t} D_{t-1} + mrs_t(1-\tau)L_t - C_t - \mathcal{T}_t + div_t - X, \tag{F.1}$$

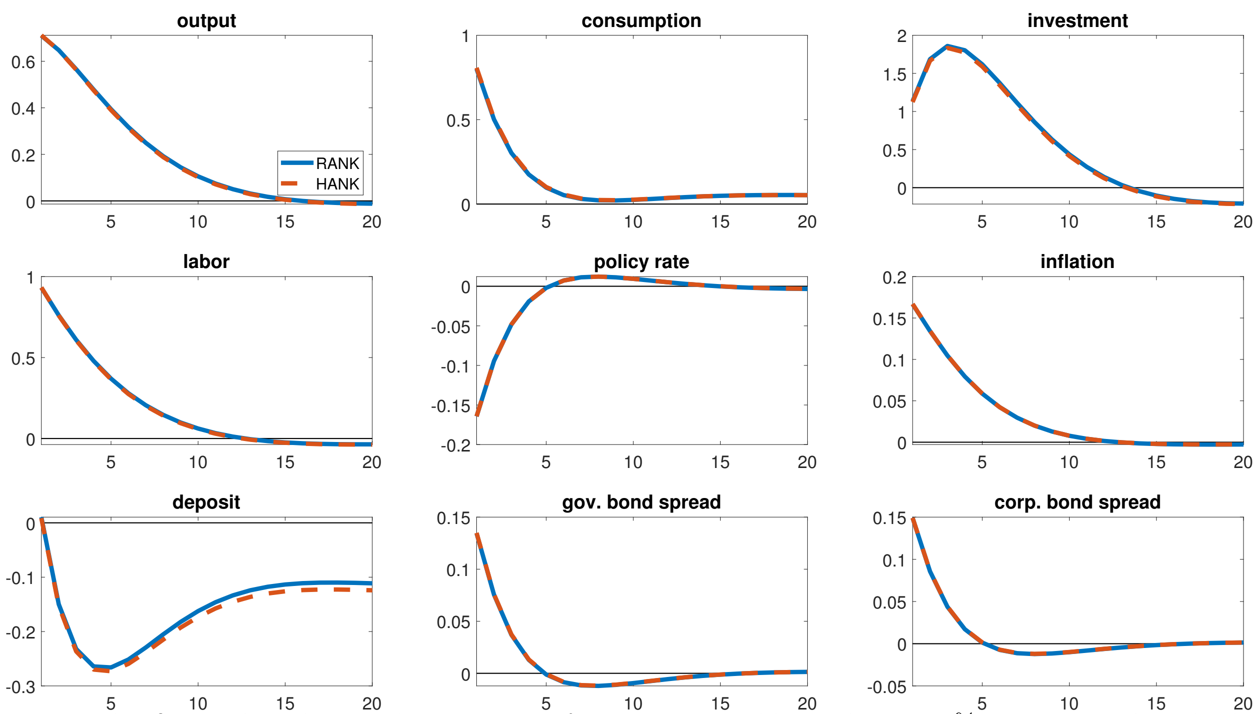
The first order conditions are

$$C_t^{-1} = \beta R_t^d \mathbb{E}_t \frac{C_{t+1}^{-1}}{\Pi_{t+1}} \tag{F.2}$$

$$L_t^{\eta} = \frac{(1-\tau)mrs_t}{\chi C_t}. \tag{F.3}$$

Appendix G Conventional Monetary Policy

Figure G.1: **Impulse responses under a monetary policy shock**



Notes: This figure plots the impulse responses of aggregate variables under a 0.25% monetary policy shock. The blue solid lines are for the RANK model, and the red dashed lines are for the HANK model. X-axis is time in quarters, and Y-axis is the percentage change from steady state.