THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

August 2020

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section (precise instructions are below)—100 points in each section—for a total of 200 points. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are six (6) pages, including this one.

SECTION I

Please answer any **two** of the three questions from this section.

I.1 (50 pts) There are two goods. The price of good 1 is uncertain: with probability π, it will be p_1' and with probability $1-\pi$, it will be p_1''. The price of good 2 is $\overline{p}_2$ for sure. The government is considering a *price-stabilization* program which will ensure that the price of good 1 equals $\overline{p}_1 = \pi p_1' + (1-\pi) p_1''$ for sure. Thus, if the program goes into effect the consumer will face certain prices $(\overline{p}_1, \overline{p}_2)$.

(a) Suppose that utility-maximizing consumers will make their consumption choices *after* the prices are realized, that is, after it is known whether the price of good 1 is p_1' or p_1''. Would such a price-stabilization program benefit consumers?

(b) Suppose that a profit-maximizing and price-taking firm produces good 2 using good 1 as an input. The production set of the firm is Y and this convex, closed and $0 \in Y$ and production choices are made after prices are realized. Would the price stabilization program benefit the firm?

I.2 (50 pts) Let $\mathcal{E} = (u_i, \mathbf{w}_i)_{i=1,2}$ be a two-person exchange economy with two goods denoted by x and y. The two consumers have identical utility functions $u_1(x,y) = u_2(x,y) = \sqrt{xy}$ and but differing endowments $\mathbf{w}_1 = (99,1)$ and $\mathbf{w}_2 = (1,99)$.

(a) Argue that the allocation $\mathbf{x}_1 = (x_1, y_1) = (10,10)$ and $\mathbf{x}_2 = (x_2, y_2) = (90,90)$ is in the *core* of the economy $\mathcal{E}$.

(b) Let $\mathcal{E}^{(r)}$ denote the r-replica economy of $\mathcal{E}$ in which there are r replicas ("clones") of each consumer. Suppose $r=2$ and argue that the allocation in which each consumer of type 1 gets $\mathbf{x}_1 = (10,10)$ and each consumer of type 2 gets $\mathbf{x}_2 = (x_2, y_2) = (90,90)$ is not in the core of the economy $\mathcal{E}^{(2)}$.

(c) Find an allocation that is in the core of $\mathcal{E}^{(r)}$ for all r.

I.3 (50 pts) Consider a two-person exchange economy $\mathcal{E}$ with uncertainty: there are two equally likely states of nature S, "rain" (r) and "shine" (s). There is only one physical good, denoted by x. Agent 1 is risk-neutral and his expected utility is

$$u_1(x_{1r}, x_{1s}) = \frac{1}{2}x_{1r} + \frac{1}{2}x_{1s}$$

whereas agent 2 is risk-averse and her expected utility is

$$u_2(x_{2r}, x_{2s}) = \frac{1}{2}\ln(1 + x_{2r}) + \frac{1}{2}\ln(1 + x_{2s})$$

where x_{iS} denotes i's consumption of the good in state S. The initial endowments are

$$(w_{1r}, w_{1s}) = (2, 0) \text{ and } (w_{2r}, w_{2s}) = (0, 2)$$

where w_{iS} denotes i's endowment of the good in state S.
There is a complete set of contingent markets.

(a) Find a Walrasian equilibrium for this economy.

(b) Now suppose that both agents learn the state of nature prior to any trade, say as a result of a perfect forecast. For each of the two agents, determine if he or she is better off, worse off or indifferent relative to the situation in which the state was not known prior to trade.

SECTION II

Please answer any **two** of the three questions from this section.

II.1 Two firms with no fixed costs face a linear demand curve of $q = V - p$, where p is the price, $V > 1$ is a known value, and q is the quantity. The firms engage in Bertrand competition. Firm i's privately-known marginal cost c_i is drawn independently from the uniform distribution on $[0, 1]$.

(a) (10 pts) Assuming that a strictly increasing, differentiable, symmetric Bayesian Nash equilibrium exists, write the first-order condition (that is, the differential equation) that characterizes the equilibrium.

Now suppose that a regulator learns both firms' costs and makes them public before the firms compete.

(b) (20 pts) Consider the subgames that follow the regulator's revelation, and recall that if the firms set the same price, then they split the market. Solve for all the pure-strategy Nash equilibria in each such subgame. If no pure-strategy equilibrium exists for a subgame, describe one mixed-strategy equilibrium.

Now suppose that the regulator wants to guarantee the existence of a pure-strategy equilibrium in every such subgame.

(c) (10 pts) Can the regulator guarantee the existence of a pure-strategy equilibrium in every such subgame by changing the tie-breaking rule? Explain.

(d) (10 pts.) Can the regulator guarantee the existence of a pure-strategy subgame-perfect equilibrium in every such subgame by having the players set prices sequentially instead of simultaneously? Explain.

II.2 A worker whose productivity p is 0 or 1, each with probability $1/2$, is about to be hired by the market. We model the market as a single player who offers the worker a wage w. The worker's payoff is equal to the wage the market offers, w. The market's payoff is $|w-p|^2$, so if the market's expectation of the worker's productivity given all the information available to the market is $\bar{p}$, it optimally offers a wage of $w = \bar{p}$, which the worker accepts.

Suppose first that the worker knows his productivity (and the market does not) and the worker can credibly report his productivity to the market. The worker cannot lie, that is, he cannot report that his productivity is 1 when it is 0 or vice versa, but he can choose to stay silent and not report his productivity. The market observes the worker's report (or silence) and then offers a wage.

(a) (5 pts) Write the game in extensive form.

(b) (15 pts) Solve for all the pure-strategy WPBE of the game. (5/15 pts: solve for any mixed-strategy WPBE of the game.)

Let us now change the game: suppose that the worker does not know his productivity. Instead of reporting his productivity (which the worker cannot do), before approaching the market the worker can pay a small amount $\varepsilon > 0$ for a test that accurately reveals his productivity to the market. If the worker does not pay the fee, the market does not learn the worker's productivity before offering a wage to the worker.

(c) (15 pts) Solve for all the pure-strategy WPBE of the game. (5/15 pts: solve for any mixed-strategy WPBE of the game.)

(d) (15 pts) Solve for all the pure-strategy WPBE of the game if the test is free, that is, $\varepsilon = 0$. (5/15 pts: solve for any mixed-strategy WPBE of the game.)

II.3 An entrepreneur has an idea that he will sell to an investor. The idea can be good or bad, each with probability $1/2$. If the idea is good, the investor will pay 1 for it; if it is bad, the investor will pay 0 for it. We denote the value of the idea by $v \in \{0, 1\}$. If the investor believes that the idea's expected value is $\bar{v}$ he will pay the entrepreneur $s = \bar{v}$. The entrepreneur's payoff is equal to s. The entrepreneur does not know whether the value of his idea is 0 or 1. A profit-maximizing intermediary evaluates the idea accurately to determine its value, and then provides the evaluation to the entrepreneur. The entrepreneur then chooses whether to disclose the accurate evaluation to the investor, and if he does he pays the intermediary a fee of c. (The entrepreneur does not have to disclose the evaluation, and does not pay the fee in this case.)

The game proceeds as follows. First, the intermediary announces a fee c in $[0, 1]$, which both the entrepreneur and the investor observe. The intermediary then evaluates the idea and provides the evaluation to the entrepreneur. The entrepreneur then decides whether to disclose the evaluation to the investor, and if he does so he pays c to the intermediary. Finally, the investor pays the entrepreneur the amount $s = \bar{v}$, which is the entrepreneur's expectation of the value of the idea given all the information the investor has.

(a) (30 pts) For every fee c, solve for all the pure-strategy WPBE of the subgame that follows the announcement of c. (10/30 pts: Bonus: solve for any mixed-strategy WPBE of the game.)

(b) (10 pts) If the intermediary could choose the equilibrium of the subgame, which fee c would he announce and what will be his profit?

(c) (10 pts) If the intermediary cannot choose the equilibrium of the subgame and is worried that in every subgame the worst equilibrium (from the intermediary's point of view) will be played, which fee c would he announce and what will be his profit?