THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

January 2021

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section (precise instructions are below)—100 points in each section—for a total of 200 points. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are five (5) pages, including this one.

SECTION I

Please answer any **two** of the three questions from this section.

I.1 (50 points) Suppose that *two* individuals have *indirect utility* functions of the form:

$$v_i(\mathbf{p}, w_i) = \phi_i(\mathbf{p}) w_i$$

where $\phi_i : \mathbb{R}_+^L \to \mathbb{R}$ and w_i is individual i's wealth level. Suppose also that each individual's wealth is a constant share $\delta_i > 0$ of aggregate wealth w so that $w_i = \delta_i w$ where $\delta_1 + \delta_2 = 1$.

(a) Find the individual Walrasian demand functions $\mathbf{x}_i(\mathbf{p}, w_i)$.

(b) What is the aggregate demand $\mathbf{x}(\mathbf{p}, w) = \mathbf{x}_1(\mathbf{p}, \delta_1 w) + \mathbf{x}_2(\mathbf{p}, \delta_2 w)$?

(c) Show that aggregate demand can be generated by the indirect utility function

$$v(\mathbf{p}, w) = \phi_1(\mathbf{p})^{\delta_1} \phi_2(\mathbf{p})^{\delta_2} w$$

I.2 (50 points) Consider a production economy $\mathcal{E} = \left((u_i, \mathbf{w}_i)_{i \in I}, (Y_j)_{j \in J}, (\theta_{ij})_{i \in I, j \in J}\right)$ with two consumers and two firms. There are three goods: leisure (ℓ) and two produced goods (x and y). The utility functions of the two consumers are

$$u_1(\ell_1, x_1, y_1) = \ell_1 + x_1 + y_1 \text{ and } u_2(\ell_2, x_2, y_2) = \ell_2 (x_2)^2 (y_2)^2$$

respectively, and the production functions of the two firms are

$$x = 2\sqrt{L} \text{ and } y = 2\sqrt{L}$$

respectively. Thus, the first firm makes x using only labor as an input, while the second firm makes y using only labor as an input. Each consumer is endowed with 2 units of leisure and none of x and y. Consumer 1 is the sole owner of the first firm (that makes x) and consumer 2 is the sole owner of the second firm (that makes y)

(a) Find a Walrasian equilibrium for this economy.

(b) Is there a unique Walrasian equilibrium?

I.3 (50 points) Consider an economy $\mathcal{E} = (U_i, \mathbf{w}_i)_{i=1}^{i=n}$ in which there S states of nature at date 1 and only one physical good. The consumers' utility functions are of the expected utility form; that is,

$$U_i(\mathbf{x}_i) = \sum_{s=1}^{S} \pi_s v_i(x_{is})$$

where x_{is} denotes i's consumption of the good in state s and π_s is the probability of state s (for all s, $\pi_s > 0$ and $\sum_s \pi_s = 1$). Note all consumers agree on the probabilities of the different states. Further, assume that all v_i are increasing, strictly concave and satisfy $v_i(0) = 0$. Suppose that there is no aggregate uncertainty. In other words, for any two states s and s', the total endowment of the economy in state s, $\sum_i w_{is} = \sum_i w_{1s'}$, the total endowment in state s'.

Let $(\mathbf{p}^*, \mathbf{x}^*)$ be a Walrasian equilibrium (with complete markets at date 0) for $\mathcal{E}$ such that the price of the good in each state $p_s^* > 0$ and $\sum_s p_s^* = 1$. Moreover, each $x_{is}^* > 0$.

(a) Show that in such an equilibrium every consumer insures completely—that is, he or she consumes the same amount of the good in every state.

(b) Show that the price in state s, $p_s^* = \pi_s$, the probability of state s.

SECTION II

Please answer **II.1** and any **one** of the other two questions (**II.2 or II.3**) from this section.

II.1 Two firms can produce any quantity of a good at cost 0. Firm 1 is the incumbent operating in the market. Firm 2 is considering entering the market. Entry is costly for firm 2 and the cost is some known $c \geq 0$. Market demand is given by $p = A - q$, where A is the level of advertising and $q = q_1 + q_2$ is the aggregate amount of the good produced (the sum of the amounts both firms produce). If the firms produce more than A, that is, if $q > A$, then the price is $p = 0$.
The game proceeds as follows. First, firm 1 decides on the level of advertising $A \geq 0$ and pays $2A^3/54$. Firm 2 observes A and decides whether to enter the market. If firm 2 enters, it pays the entry cost $c \geq 0$ and the two firms engage in Cournot competition. If firm 2 does not enter, then firm 1 produces as a monopolist in the market.

(a) (5 points) Draw the game in extensive form.

(b) (5 points) What is each firm's set of strategies?

(c) (10 points) Suppose that $c = 0$. Solve for a subgame-perfect equilibrium.

(d) (10 points) Suppose that c is small but positive. Solve for a subgame-perfect equilibrium.

(e) (10 points) Suppose that c is small but positive. Find another Nash equilibrium that is not subgame perfect.

(f) (10 points) Suppose that c is large. Find another subgame-perfect equilibrium

II.2 Two players are about to play either a first-price auction or an all-pay auction. In both auctions the highest bidder wins, and in case of a tie between the players, each player wins with probability $1/2$. Player 1's known value is $v > 0$ and player 2's privately-known value is v with probability p and V with probability $1 - p$, where $V > v$.

Suppose that $p = 1$.

(a) (5 points) Solve for an equilibrium of the all-pay auction.

(b) (5 points) Solve for an equilibrium of the first-price auction.

Suppose that $p = 0$.

(c) (5 points) Solve for an equilibrium of the all-pay auction.

(d) (5 points) Solve for an equilibrium of the first-price auction.

Suppose that $p \in (0, 1)$.

(e) (15 points) Solve for an equilibrium of the all-pay auction.

(f) (15 points) Show that the first-price auction does not have an equilibrium.

or

II.3 There are two partners in a firm. Each partner i chooses an effort $x_i \in [0, 4]$ and incurs a disutility of $x_i^2/2$ by doing so. The partners make their choices simultaneously and independently. This leads to firm revenue

$$4\left(x_1 + x_2 + cx_1x_2\right),$$

where the known parameter $c \in [-2, 2]$ measures the degree of complementarity between the partners' efforts. The partners split the revenue equally, and each partner's utility is his share of the revenue minus his disutility from his effort.

(a) (20 points) For all possible values of c, solve for the Nash equilibria.

(b) (30 points) For all possible values of c, solve for the rationalizable strategies (recall that a strategy is undominated if and only if it is not a best response to any belief).