Cost Coordination*

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Abstract

When firms engage in price discrimination under competition, they can face a trade-off when choosing to collude. In order to maintain price discrimination, upper-level executives may have to involve those lower-level employees with the demand information needed to tailor prices to markets and customers. However, that comes with an enhanced risk of the cartel's discovery. Alternatively, those executives could centralize pricing authority and coordinate on a more uniform price but that means foregoing some of the profits from price discrimination. Here we put forth a third option which is for upper-level executives to coordinate on inflating the cost used in pricing by lower-level employees. Coordinating cost reports is shown to be more profitable than coordinating prices when market heterogeneity is sufficiently great or firms' products are sufficiently differentiated. Recent cartel episodes in which executives coordinated list prices or surcharges are explained to have some of the crucial features of this collusive scheme.

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1 Introduction

Consider a market in which firms engage in price discrimination by pricing to market segments or offering customer-specific discounts. Collusion among firms in such a market is likely to be challenging. Coordinating on many prices could require extensive cartel meetings among senior executives with the inclusion of pricing managers or sales representatives because they have the information necessary to price discriminate. However, expanding the set of employees participating in an unlawful cartel comes with a higher risk of the cartel's discovery. Alternatively, senior executives could centralize pricing authority, make prices more uniform across markets and customers, and coordinate on fewer prices. However, such a scheme means taking pricing authority away from those with the best demand information with the consequent effect of lower profit due to less price discrimination.

This paper proposes a third solution: firms' senior executives coordinate on the "cost" that is used in firms' pricing decisions. If prices are set "as if" cost is higher than it actually is, this inflated cost will permeate all prices and the cartel will be able to achieve higher prices while maintaining some degree of price discrimination, and do so without explicitly involving lower-level employees in the collusive arrangement. A key element of the model which delivers this new collusive scheme is a more realistic representation of the internal pricing process of a firm. Rather than assume the canonical single-actor model of the firm, two levels are assumed: an upper level (such as senior managers) that has better information on cost and a lower level (such as pricing managers) that has better information on demand. For this model, the competitive equilibrium has pricing authority delegated to the lower level. By coordinating to inflate the cost reports given to lower-level employees, those senior managers can raise prices and continue to have price respond to the demand information possessed by those lower-level employees. While the scheme sounds promising, there are two challenges. First, it is unclear how effectively coordinating cost reports can produce profitable price discrimination given prices are set competitively. Second, the collusive outcome - delegating pricing authority to lower-level employees while communicating an inflated cost report to them - is internal to the firm, and that poses a challenge in monitoring for compliance.

While this theory has just been put forth as a new collusive scheme, I believe it captures in a stylized and parsimonious way some features of recently documented collusive practices. Canonically, collusion is with respect to final prices (i.e., the prices faced by customers), as illustrated by the many cartels described in Harrington (2006) and Marshall and Marx (2012). Departing from that scheme, there are instances in which firms coordinated on non-final prices such as list prices (e.g., cement, trucks, and urethane) and surcharges (e.g., air freight, batteries, and rail freight).[1] The model of this paper captures three key elements of those episodes. First, collusion exclusively involved upper-level managers. Second, lower-

[1] "Aggregates: Report on the market study and proposed decision to make a market investigation reference," Office of Fair Trading, OFT1358, August 2011 (cement); Commission Decision of 19.7.2016 relating to a proceeding under Article 101 of the TFEA and Article 53 of the EEA Agreement, AT.39824 - Trucks; *In Re: Urethane Antitrust Litigation*, No. 13-3215 (10th Cir. Sep. 29, 2014); CASE AT.39258 - Airfreight, European Commission, 11 September 2010; "The Belgian Competition Authority imposes fines amount to 3.857.000 Euros for price-fixing in the sector of industrial batteries," Belgian Competition Authority, Press Release No. 4/2016, 23 February 2016; *In re Rail Freight Surcharge Antitrust Litig.*, 587 F.Supp.2d 27 (2008), U. S. District Court, District of Columbia, November 7, 2008. Also see Boshoff and Paha (2021) for a general discussion of collusive practices involving list prices.

level employees likely had some pricing authority. Third, collusion effectively raised cost in the firm's pricing process. For example, a firm's list price is a signal through which cost can be injected into the pricing process. It is well recognized that list prices are sensitive to costs so that when the list price is raised, lower-level employees (as well as buyers when the list price is public information) will infer that cost is higher and that will affect subsequent pricing outcomes. The introduction of a surcharge (e.g., for fuel) can similarly be interpreted as serving to raise perceived cost.

The paper makes two contributions. First, it develops a collusive theory in which firms agree to use an inflated cost in their pricing decisions. In doing so, it delivers an explanation for why certain collusive practices are effective. Second, it provides insight into when firms would choose to coordinate (internal) cost reports rather than collude in the more standard manner of coordinating (external) prices. Coordinating cost reports has the advantage of allowing more tailored pricing because pricing remains delegated to those employees with the best demand information. However, coordinating prices has the advantage that it is easier to monitor for compliance since prices can be directly observed while cost reports are internal to the firm. Our analysis shows that we should expect to see firms coordinating cost reports when firms' products are sufficiently differentiated or there is sufficient market heterogeneity.

Section 2 reviews some related research. The static model is introduced in Section 3 where the competitive equilibrium is characterized, which has the upper level delegating pricing authority to the lower level. Section 4 offers sufficient conditions for the first-best outcome to be more profitable when firms coordinate their cost reports (with decentralized pricing) than when they coordinate final prices (with centralized pricing). The infinitely repeated game is described in Section 5 along with equilibria to sustain the collusive outcomes of interest. Sections 6 and 7 identify market conditions for which upper-level executives prefer to coordinate cost reports and when they prefer to coordinate prices. Section 8 concludes.

2 Related Research

This paper contributes to three literatures. The first is collusion and price discrimination, which include Liu and Serfes (2007), Colombo (2010, 2022), Helfrich and Herwig (2016), Gössl and Rasch (2020), and Peisler, Rasch, and Shekhar (2022). Motivated by the impact of big data in providing more demand information, this work asks whether collusion is more or less difficult when firms segment markets and engage in price discrimination. While that research is motivated by the demand data available to firms, this paper is motivated by the distribution of cost and demand data within the firm and takes account of a firm's internal pricing process. Within the context of my framework, previous research can be seen as investigating collusion when lower-level employees are part of the collusive arrangement, while I focus on when collusion is exclusively among upper-level executives which is consistent with many cartels.

This paper also contributes to the literature examining the interaction between a firm's internal pricing process and strategic conduct between competitors. Beginning with Vickers (1985), Ferhstman and Judd (1987), and Sklivas (1987), contributions encompassing collusion include Lambertini and Trombetta (2002), Rasch and Wambach (2009), and Kim (2022).

This paper's model is unique in assuming that information relevant to pricing is distributed across divisions. Examining common ownership rather than collusion, the model in Antón, Ederer, Giné, and Schmalz (2022) is related in assuming distributed decision making within the firm, which could well be based on distributed information.

Finally, this paper relates to a small literature on collusion with regards to list prices or surcharges; specifically, Harrington and Ye (2019) and Chen (2021). Harrington and Ye (2019) considers firms coordinating their public announcements about cost - such as through list prices - which then affects how buyers bargain. Thus, it shares a feature with this paper's model in exploring how an inflated cost can result in supracompetitive final prices. In Harrington and Ye (2019), the inflated cost affects buyers' beliefs on sellers' costs which causes them to accept higher prices during the buyer-seller bargaining process. By comparison, this paper's model has an inflated cost report affect other agents in the firm who are involved in the pricing process. It is applicable to when list prices are purely internal to a firm, as appears to be the case for the manufacturers involved in the EU trucks cartel.[2,3]

Chen (2021) is motivated by cartel episodes in which firms coordinated surcharges. Similar to this paper, the internal pricing process of the firm is modelled though the upper level chooses a surcharge and the lower level chooses a base price. Under competition, equilibrium has an upper level choosing a positive surcharge and making their manager's compensation more sensitive to base revenue. This is shown to cause a firm's lower-level employee to set a higher final price (= base price + surcharge) and, due to strategic complements, induces the rival firm's lower-level employee to set a higher final price. It is then shown that when upper-level executives coordinate surcharges (while leaving lower-level employees to compete in their setting of base prices), final prices are the same as when upper-level executives coordinate final prices. Given that coordinating surcharges is simpler and does not involve lower-level employees, the theory provides an explanation for why firms would adopt a collusive practice that has them coordinating surcharges. Though relevant to some similar cartel episodes, the model and results of my paper are quite different from those in Chen (2021). First, there are multiple heterogeneous markets in this paper's model and firms are engaging in price discrimination. Second, the rationale for the lower-level employees having pricing authority is that they have private demand information. Finally, this paper's findings are distinct in identifying when firm prefer to coordinate cost reports rather than prices.

[2] *Commission Decision of 19.7.2016 relating to a proceeding under Article 101 of the TFEA and Article 53 of the EEA Agreement, AT.39824 - Trucks* "From 1997 until the end of 2004, the [firms] participated in meetings involving senior managers of all Headquarters [where] the participants discussed and in some cases also agreed their respective gross price increases." [para. 51] There is no mention in the Decision that the gross list prices were shared with customers.

[3] Though more tangential, collusion in list prices is also examined in Gill and Thanassoulis (2016) and Herold (2021). In those models, list prices are paid by some customers and thus act as more than a signal of a firm's cost. There are also a few papers encompassing a two-level model of the firm in which there is an agreement to *exchange* non-transaction prices (such as list prices) rather than *coordinate* those prices; see Andreu, Neven, and Piccolo (2020), Janssen and Karamychev (2021), Harrington (2022), and Klein and Neurohr (2022).

3 Model and Competitive Equilibrium

Consider a symmetric oligopoly setting with $n \geq 2$ firms offering differentiated products. $D_i(p_1, ..., p_n, a) : \Re_+^n \times [\underline{a}, \overline{a}] \rightarrow \Re_+$ is the (symmetric) firm demand function for market (or customer) type a given firms' prices $(p_1, ..., p_n)$. There is a collection of market types which, for analytical ease, is a continuum with a continuously differentiable cdf $G : [\underline{a}, \overline{a}] \rightarrow [0, 1]$, where $\underline{a} < \overline{a}$ and $G'(a) > 0 \; \forall a \in [\underline{a}, \overline{a}]$. Firms have a common cost $c \in \Omega \equiv \{\underline{c}, \underline{c}+\eta, ..., \overline{c}-\eta, \overline{c}\}$ where $\eta > 0$ and, when helpful to the analysis, η is small. $\rho(c)$ is the probability attached to c and $\rho(c) > 0 \; \forall c \in \Omega$.

Each firm's organization has two levels where Ui (Li) denotes the upper (lower) level of firm i, $i = 1, ..., n$. One can think of the upper level as a senior executive and the lower level as a pricing manager or sales representative.[4] Ui has private information of cost and Li has private information of the market type. All other information is common knowledge. Ui chooses whether to have a centralized or decentralized pricing structure. Under centralization, Ui chooses price p_i which does not condition on a. Under decentralization, Ui conveys a (cheap talk) cost report x_i to Li. With knowledge of x_i and a, Li chooses price while conditioning on a.[5]

The extensive form is:

- Stage 1: Ui learns cost c, $i = 1, ..., n$.
- Stage 2: Ui chooses the pricing structure: centralization or decentralization.
- If Ui chose decentralization then:
  - Stage 3: Ui chooses cost report $x_i \in \Omega$.
  - Stage 4: Li observes x_i and a and then chooses price $p_i(a) \; \forall a \in [\underline{a}, \overline{a}]$.
- If Ui chose centralization then:
  - Stage 3: Ui chooses price p_i.

Note that $U1, ..., Un$ make simultaneous pricing structure decisions and thus those decisions are private to the firm when it chooses a cost report and price vector (under decentralization) and a price (under centralization). The payoffs of Ui and Li are assumed to be proportional to a firm's profit based on actual cost and a firm's profit based on reported cost, respectively; hence, there is no agency problem. What is critical to the theory of collusion is that the information relevant to pricing is distributed across divisions within the firm. Allowing for an agency problem would needlessly complicate the analysis.[6]

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[4] There could be many lower-level agents and it is without loss of generality to assume there is just one.

[5] One can think of there being two types of demand variation; that which is observable to the upper (and possibly lower) level and that which is observable only to the lower level. For parsimony, the former is assumed away.

[6] Consistent with the model under decentralization, Antón et al (2022) also assumes full delegation of pricing to the lower level and the lower level maximizes profit. However, an upper level's incentives are endogenously determined (and need not be profit maximizing) and an upper level chooses how much to invest in reducing cost, while an upper level in my model observes cost and decides what to report to the lower level.

Let us begin by characterizing the unique separating perfect Bayes-Nash equilibrium for the one-shot game, which will serve as the competitive solution. Consider the following symmetric strategy profile and beliefs:

- Ui chooses decentralization and $x_i = c \ \forall c \in \Omega$.
- Li chooses $p_i(a) = p^N(a, x_i) \ \forall a \in [\underline{a}, \overline{a}]$ where

$$p^N(a, x_i) \equiv \arg\max_{p_i} (p_i - x_i) D_i \left(p^N(a, x_i), ..., p_i, ..., p^N(a, x_i), a\right).$$

- Li assigns probability one to $c = x_i \ \forall x_i \in \Omega$.

Li's beliefs are correct given Ui's strategy. Note that if $x_i = x'$ then Li believes $x_j = x'$ $\forall j \neq i$ because there is a common cost. Given Lj is then expected to price at $p^N(a, x')$, it is optimal for Li to price at $p^N(a, x')$ given its payoff is proportional to

$$(p_i - x') D_i \left(p^N(a, x'), ..., p_i, ..., p^N(a, x'), a\right).$$

Given c, Ui's payoff from decentralization and reporting x_i is proportional to

$$\int \left(p^N(a, x_i) - c\right) D_i \left(p^N(a, c), ..., p^N(a, x_i), ..., p^N(a, c), a\right) G'(a)da,$$

for which $x_i = c$ optimal. Finally, decentralization is optimal because

$$\int \left(p^N(a, c) - c\right) D_i \left(p^N(a, c), ..., p^N(a, c), a\right) G'(a)da$$
$$> \max_{p_i} \int (p_i - c) D_i \left(p^N(a, c), ..., p_i, ..., p^N(a, c), a\right) G'(a)da.$$

Given there is no conflict of interest between the levels in a firm, the upper level decentralizes pricing and truthfully reports cost to the lower level, and the lower level sets price to maximize firm profit based on the cost report received. Asymmetric information is then the basis for delegating pricing authority which is well recognized in the organizational economics literature.[7] Note that equilibrium pricing is the same as when each firm has only one level that sets price knowing all cost and demand information.[8]

A1-A5 are assumed to hold $\forall (a, c) \in [\underline{a}, \overline{a}] \times \Omega$. These assumptions are standard and hold for the case of linear demand examined in Section 7.

A1 $D_i(c, ..., c, a) > 0$.

[7] See, for example, Lal (1986), Joseph (2001), Mishra and Prasad (2004), and Lo, Dessein, Ghosh, and Lafontaine (2016).

[8] This is the unique separating PBNE because, as the pricing structure decision is private information and the interests of the two levels are fully aligned, Ui always wants to delegate pricing to Li so that price can condition on a. Of course, there are pooling PBNE where cost reports are uninformative. However, it would always be in the mutual interest of a firm's two levels to coordinate on a separating strategy, regardless of what the other firms do.

A2 $(p_i - x_i)\, D_i(p_1, ..., p_n, a)$ is differentiable and strictly quasi-concave in p_i.

A3 $p^N(a, x)$ exists and is twice differentiable and increasing in a and x. (Note: If $D_i(x, ..., x, a) > 0$ then $p^N(a, x) > x$.)

A4 $\pi(p, a, c) \equiv (p - c)\, D_i(p, ..., p, a)$ is differentiable and strictly quasi-concave in p. Hence, $p^M(a, c) \equiv \arg\max (p - c)\, D_i(p, ..., p, a)$ exists. $p^M(a, c)$ is differentiable and increasing in a and c.

A5 $p^M(a, c) > p^N(a, c) > c$.

The specification of market heterogeneity encompasses two substantive restrictions. First, firms' demands are symmetric. While this is a common assumption in the price discrimination literature (see, for example, Holmes 1989), it contributes to firms' competitive profits being higher with price discrimination compared to a uniform price. However, there are alternative demand specifications whereby firms' competitive profits are lower with price discrimination.[9] Second, the change in the monopoly and competitive prices with respect to the market type are of the same sign. As the monopoly price is assumed to be increasing in a, a higher value of a corresponds to a "stronger" market in the sense of a less price-elastic market demand.[10] In assuming the competitive price is also increasing in a, the presumption is that a stronger market also has a firm demand that is less price-elastic. Given that a firm's price elasticity of demand can be decomposed into the sum of the market-price elasticity of demand and the cross-price elasticity of demand, the assumption is that the market variation in the cross-price elasticity of demand is not so great as to offset the variation in the market-price elasticity of demand. In particular, markets with more price-inelastic market demand do not have firms' products being sufficiently more substitutable. While this assumption does rule out some cases, it is still quite general.

4 First-Best Collusion

Suppose firms' upper-level executives collude and do so without involving lower-level employees. Consider a collusive outcome that has them controlling price (centralization) and jointly choosing a common price to maximize each firm's profit; this outcome is referred to as *price coordination*. First-best price coordination has the joint profit-maximizing uniform price:

$$\widehat{p}(c) \equiv \arg\max_p \int (p - c)\, D_i(p, ..., p, a)\, G'(a) da. \tag{1}$$

An alternative outcome - referred to as *cost coordination* - has the upper levels maintaining decentralized pricing (as under competition) while jointly choosing a common cost report

[9] This distinction is discussed in Corts (1998) where properties on the best response function determine whether price discrimination raises or lower prices compared to uniform pricing. In his terminology, our demand specification satisfies best response symmetry.

[10] This discussion is based on Stole (2007), pp. 2234-2235.

and taking as given that the lower level will price competitively based on the cost report it receives.[11] First-best cost coordination has the joint profit-maximizing cost report:

$$\widehat{x}(c) \equiv \arg\max_{x\in\Omega} \int \left(p^N(a,x) - c\right) D_i \left(p^N(a,x), ..., p^N(a,x), a\right) G'(a)da. \tag{2}$$

It is straightforward to establish that $\widehat{x}(c) > c$ (except when $c = \overline{c}$). Generally, both collusive outcomes are second-best. Price coordination is second-best because price does not condition on the demand state. Cost coordination is second-best because price is set competitively conditional on the cost report.

Theorem 1 offers a sufficient condition for first-best cost coordination to be more profitable than first-best price coordination; that is, maximal profit is higher by coordinating on an inflated cost used by pricing managers than by taking control of price and coordinating on a supracompetitive uniform price. Proofs are in the Appendix.

Theorem 1 *If*

$$\frac{\partial p^M(a,c)}{\partial a} > \frac{\partial p^N(a,x)}{\partial a} > 0 \text{ for all } x \geq c, \text{ for all } a \in [\underline{a}, \overline{a}] \tag{3}$$

then first-best cost coordination is more profitable than first-best price coordination:

$$\begin{aligned} &\int \left(p^N(a,\widehat{x}(c)) - c\right) D_i \left(p^N(a,\widehat{x}(c)), ..., p^N(a,\widehat{x}(c)), a\right) G'(a)da \\ > &\int \left(\widehat{p}(c) - c\right) D_i \left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da. \end{aligned} \tag{4}$$

(3) is true if, evaluated at cost, the monopoly price is more sensitive to the market type than is the competitive price - $\partial p^M(a,c)/\partial a > \partial p^N(a,c)/\partial a$ - and the effect of the cost report on the competitive price's sensitivity to the market type, $\partial^2 p^N(a,x)/\partial a \partial x$, is sufficiently bounded. Those conditions hold for the case of linear demand (see Section 7). Furthermore, we generally expect the monopoly price to be more sensitive to demand than the competitive price. For example, when products are highly similar, $p^N(a,x)$ is approximately equal to cost and thus is relatively insensitive to the market type a.[12]

The proof strategy for Theorem 1 is as follows.[13] It is shown there is an inflated cost report $x'(> c)$ which raises the competitive pricing function, as depicted in Figure 1. At $a = a'$, cost coordination with cost report x' delivers the same price and profit as price coordination with uniform price $\widehat{p}(c)$. For stronger demand states $(a > a')$, $p^N(a,x') \in \left(\widehat{p}(c), p^M(a,c)\right)$ so cost coordination yields higher profit (than price coordination) by having price be higher (which follows from the strict quasi-concavity of the profit function with respect to a common

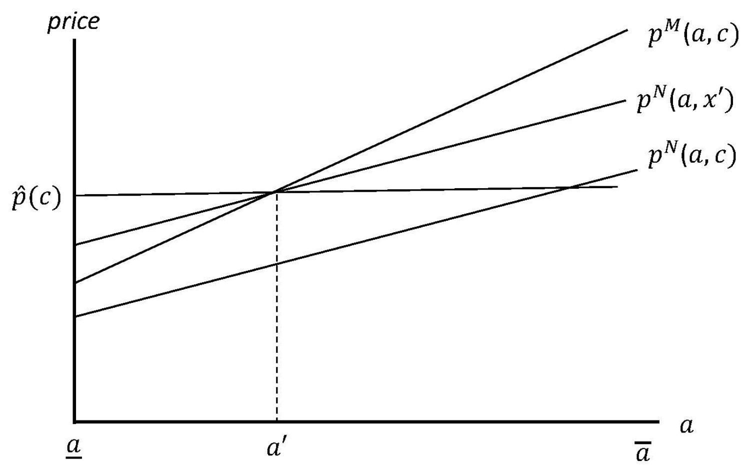
[11] If lower-level employees choose prices to maximize their current compensation and their compensation is proportional to profit based on the reported cost, it is an equilibrium for them to price at $p^N(a,x)$ whether or not they are aware that upper-level executives are colluding.

[12] It is also not literally needed that (3) holds for all $x \geq c$ but rather that it holds up to some upper bound.

[13] The proof strategy is based on initially allowing cost reports to take any non-negative value and then showing that the result can be approximated when Ω is sufficiently fine.

price across firms). For weaker demand states ($a < a'$), $p^N(a, x') \in \left(p^M(a,c), \widehat{p}(c)\right)$ so cost coordination yields higher profit by having price be lower than $\widehat{p}(c)$.

Figure 1



It is interesting that condition (3) does not require that market heterogeneity is sufficiently great even though the relative advantage of cost coordination is allowing price to condition on the market type. Rather, it ensures that competitive pricing with an inflated cost is able to sufficiently approximate first-best price discrimination so that it is more profitable than the first-best uniform price. In a rough sense, the inflated cost is used to raise average price - achieving the same end as a high uniform price - and then there is the added benefit that price is responsive to demand (though how it is responsive is not under the control of the upper-level managers for that is determined by competitive pricing).

The key takeaway from Theorem 1 is that, under fairly general conditions, potential profit is higher when upper-level executives coordinate cost reports compared to coordinating prices. Whether they can realize that higher profit depends on their ability to monitor and enforce a collusive outcome based on inflated cost reports. We turn to that issue in the next section by taking account of incentive compatibility constraints in the context of an infinitely repeated game.

5 Equilibrium Collusion

In order to examine collusion that is exclusive to firms' upper-level executives $Ui, i = 1, ..., n$, an infinitely repeated game is constructed with only those executives as players. Given cost c is realized, the stage game has Ui's strategy comprising a pricing structure, a price (which pertains to centralization), and a cost report (which pertains to decentralization): $(s_i, p_i, x_i) \in \{0,1\} \times \Re_+^n \times \Omega$, where $s_i = 1(0)$ denotes the centralized (decentralized) pricing structure. If Ui chooses (s_i, p_i, x_i) then firm i's price for market type a is specified to be $s_i p_i + (1 - s_i) p^N(x_i, a)$. Thus, an upper level that delegates pricing authority to the lower level anticipates that it will price according to the competitive pricing function.[14]

The stage game payoff to Ui is

$$\begin{aligned} & v_i\left(\left(s_1, p_1, x_1\right), \ldots,\left(s_n, p_n, x_n\right), c\right) \\ \equiv & \int\left(s_i p_i+\left(1-s_i\right) p^N\left(x_i, a\right)-c\right) \times \\ & D_i\left(s_1 p_1+\left(1-s_1\right) p^N\left(x_1, a\right), \ldots, s_n p_n+\left(1-s_n\right) p^N\left(x_n, a\right), a\right) G^{\prime}(a) d a . \end{aligned}$$

By an analogous argument to that provided in Section 3, a Nash equilibrium for the stage game is the symmetric strategy $(s_i, p_i, x_i) = (0, p, c)$ for any p (because p_i is irrelevant when $s_i = 0$). Thus, it has upper-level executives decentralizing and submitting a truthful cost report, which is the same outcome as for the unique separating Bayes–Nash equilibrium characterized when the static game also encompasses the lower-level employees as players.

As of period t, past choices are $\cup_{\tau=1}^{t-1} \cup_{i=1}^{n} (s_i^\tau, p_i^\tau, x_i^\tau)$. Two assumptions are made regarding the publicly observed history. First, (s_i^τ, x_i^τ) is assumed to be private information to Ui. An upper level's decisions regarding the allocation of pricing authority and its cost report are internal to the firm and consequently not part of the public history. Second, a firm's prices are partially observed in that, at the end of each period, other firms observe a finite random sample of its prices. In order to avoid the complications of private monitoring, this sample is common knowledge to all upper levels. Finally, $\delta \in (0, 1)$ is firms' common discount factor.

The analysis will focus on a simple class of perfect public equilibria (PPE) supporting stationary outcomes. A collusive outcome is (s, p, x), and define $\Lambda(s, p, x)$ as the collection of prices across market types induced by (s, p, x). Consider the class of PPE using the following strategy: if past sampled prices have always lied in $\Lambda(s, p, x)$ then an upper level chooses (s, p, x); otherwise, it chooses $(0, p, c)$ (i.e., the stage Nash equilibrium). As this is the grim trigger strategy adapted to this setting, I will refer to this class of PPE as grim PPE. In Section 5.3, I will discuss the rationale for and implications of focusing on grim PPE.

The objective is to characterize the most profitable collusive outcome implementable by grim PPE. The comparison will be conducted when δ is close to 1 so the market is particularly suitable for collusion.[15] Towards that end, Section 5.1 characterizes the best collusive outcome when firms engage in price coordination (so the upper levels centralize and coordinate on a common price), and Section 5.2 characterizes the best collusive outcome when firms engage in cost coordination (so the upper levels decentralize, coordinate on a common cost report, and the lower levels competitively set prices based on those cost reports). Sections 6 and 7 characterize the best collusive outcome by comparing the best outcomes under price coordination and cost coordination.

[14] Those prices are optimal for a lower level if it believes all rival firms have decentralized pricing with the same cost report. For the strategy profiles we will be considering, those beliefs are consistent with other firms' strategies for all histories except in a period for which an upper level deviates from the cooperative outcome. That is potentially problematic when the cooperative outcome is centralization (say, with uniform price p') and the deviation is decentralization. In that case, the lower level with cost report x believes rival firms' prices are $p^N(x, a)$ in market type a when, in fact, prices are p'. In order to provide sufficient conditions for price coordination to be supported by an equilibrium, the proof of Theorem 2 presumes the upper level expects the lower level to best respond to rival firms pricing at p', which provides an upper bound on the deviation payoff.

[15] While the primary reason for $\delta \simeq 1$ is analytical tractability, $\delta \simeq 1$ is also a good approximation for settings where prices are observed on a high-frequency basis such as daily or weekly.

5.1 Price Coordination

Consider the following strategy for the upper level of firm i where the collusive price is the first-best uniform price $\widehat{p}(c)$ (defined in (1)) and c^t is the common cost in period t.

- In period 1, centralize and price at $\widehat{p}(c^1)$.
- In period $t = 2, 3, ...$
  - centralize and price at $\widehat{p}(c^t)$ if all period τ sampled prices equalled $\widehat{p}(c^\tau)$, $\forall \tau = 1, ..., t-1$.
  - decentralize and submit cost report $x_i^t = c^t$ otherwise.

Referred to as the *price coordination strategy*, if all past sampled prices are consistent with compliance then firms charge the uniform price $\widehat{p}(c^t)$; and if any past sampled prices differ from collusive prices then firms charge prices $\left\{p^N(a, c^t)\right\}_{a \in [\underline{a}, \overline{a}]}$.

Assuming centralization with the first-best uniform price is more profitable than decentralization and competitive pricing - which is the case when (5) holds - then the price coordination strategy is a PPE when firms are sufficiently patient.[16]

Theorem 2 *If*

$$\begin{aligned} & \int \left(\widehat{p}(c) - c\right) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a) da \qquad (5) \\ > & \int \left(p^N(a, c) - c\right) D_1\left(p^N(a, c), ..., p^N(a, c), a\right) G'(a) da, \textit{for all } c \in \Omega. \end{aligned}$$

then there exists $\delta' \in (0, 1)$ *such that if* $\delta \in (\delta', 1)$ *then the price coordination strategy is a symmetric perfect public equilibrium.*

Let us consider the challenges to price coordination being stable. As part of the collusive arrangement, upper levels are to centralize and coordinate on a common price $\widehat{p}(c^t)$. An upper level could deviate by continuing to centralize but setting a price below $\widehat{p}(c^t)$ in order to gain more demand. Alternatively, it could deviate by decentralizing and selecting some cost report x^t with the lower levels setting prices $\left\{p^N(a, x^t)\right\}_{a \in [\underline{a}, \overline{a}]}$. That deviation would allow for price discrimination and could also gain more demand by transmitting a low cost report. As either deviation causes firm's prices to (generically) differ from the collusive price $\widehat{p}(c^t)$, a random sampling of its prices will reveal its noncompliance with the collusive outcome. For the usual reasons, the associated incentive compatibility constraints (ICCs) are satisfied as $\delta \rightarrow 1$.

[16] (5) need not always be true, as it is possible that firms do better by competing and engaging in price discrimination than colluding with a uniform price. For the case of linear demand in Section 7, conditions are provided for (5) to hold.

5.2 Cost Coordination

Consider the following strategy for the upper level of firm i. The collusive outcome has decentralization and the upper level using the cost reporting function $\widetilde{x}(\cdot):\Omega\rightarrow\Omega$ when it communicates cost to its lower level. It is not presumed that the cost report is the first-best cost report (2).

- In period 1, decentralize and submit cost report $x_i^1=\widetilde{x}(c^1)$.
- In period t,
  - decentralize and submit cost report $x_i^t=\widetilde{x}(c^t)$ if all period τ sampled prices are in $\left[p^N(\underline{a},\widetilde{x}(c^\tau)),p^N(\overline{a},\widetilde{x}(c^\tau))\right]$, $\forall\tau=1,...,t-1$.
  - decentralize and submit cost report $x_i^t=c^t$ otherwise.

Referred to as the *cost coordination strategy*, compliance with the collusive outcome results in a firm having prices $\left\{p^N(a,\widetilde{x}(c^\tau))\right\}_{a\in[\underline{a},\overline{a}]}$. If all past sampled prices are consistent with compliance then firms continue to delegate pricing authority and report cost $\widetilde{x}(c^t)$. If instead some past sampled prices are inconsistent with compliance - so they do not lie in $\left[p^N(\underline{a},\widetilde{x}(c^\tau)),p^N(\overline{a},\widetilde{x}(c^\tau))\right]$ - then upper levels revert to the static Nash equilibrium so prices are $\left\{p^N(a,c^t)\right\}_{a\in[\underline{a},\overline{a}]}$.

If $\widetilde{x}(c)>c$ then, due to an inflated cost report, prices are supracompetitive at those cost levels. We say that $\widetilde{x}(\cdot)$ is *supracompetitive* if $\widetilde{x}(c)>c$ for some $c\in\Omega$ and $\widetilde{x}(c)\geq c$ for all $c\in\Omega$.

Theorem 3 *There exists $\delta'\in(0,1)$ such that if $\delta\in(\delta',1)$ then there exists a supracompetitive $\widetilde{x}(\cdot)$ such that the cost coordination strategy is a symmetric perfect public equilibrium.*

The collusive outcome involves each upper level delegating pricing authority and conveying cost report $\widetilde{x}(c^t)$ to its lower level. An upper level could deviate by continuing to decentralize but selecting a lower cost report $x^o<\widetilde{x}(c^t)$, which will increase demand while continuing to price discriminate. Given that the firm's prices will be $\left\{p^N(a,x^o)\right\}_{a\in[\underline{a},\overline{a}]}$ instead of $\left\{p^N(a,\widetilde{x}(c^\tau))\right\}_{a\in[\underline{a},\overline{a}]}$, it'll be charging prices in $\left[p^N(\underline{a},x^o),p^N(\underline{a},\widetilde{x}(c^t))\right)$ which, if observed, is evidence of noncompliance. As this off-path deviation is detected with positive probability through the random sampling of prices, the associated ICC is satisfied as $\delta\rightarrow1$. A second form of deviation is for the upper level to take control of pricing authority (centralization) and set some uniform price p^o. If $p^o\notin\left[p^N\left(\underline{a},\widetilde{x}(c^t)\right),p^N\left(\overline{a},\widetilde{x}(c^t)\right)\right]$ then again it is an off-path deviation and the associated ICC is satisfied as $\delta\rightarrow1$. If instead $p^o\in\left[p^N\left(\underline{a},\widetilde{x}(c^t)\right),p^N\left(\overline{a},\widetilde{x}(c^t)\right)\right]$ then that is an on-path deviation. A random sampling of a firm's prices that turns up p^o is consistent with compliance and rival firms having sampled the firm's price in market type a' where $p^N\left(a',\widetilde{x}(c^t)\right)=p^o$. As the off-path ICCs are satisfied as $\delta\rightarrow1$,[17] the essential element of the proof of Theorem 3 is to show there exists a

[17] For the same reasons given for the price coordination strategy, the use of the grim punishment is without loss of generality.

supracompetitive cost report that satisfies the on-path ICC.[18]

For the purpose of characterizing the best outcome when firms coordinate cost reports, define $\Gamma(\delta)$ as the set of supracompetitive $\widetilde{x}(\cdot)$ such that the cost coordination strategy is a symmetric PPE. When δ is close to one, $\Gamma(\delta)$ is non-empty by Theorem 3. $x^*(\cdot) : \Omega \to \Omega$ is defined as the equilibrium collusive cost report:

$$x^*(\cdot) = \lim_{\delta \to 1} \arg \max_{\widetilde{x}(\cdot) \in \Gamma(\delta)} E_c \left[\Pi \left(\widetilde{x}(c), c \right) \right] \tag{6}$$

where

$$\Pi \left(x, c \right) \equiv \int \left(p^N(a, x) - c \right) D_1 \left(p^N(a, x), ..., p^N(a, x), a \right) G'(a) da$$

and $E_c[\cdot]$ is the expectation with respect to cost. As shown in the proof of Theorem 3, the off-path ICCs are satisfied when δ is close to 1 so it is the on-path ICC that is the binding constraint. Consequently, $x^*(\cdot)$ is the solution to the following problem: for all $c \in \Omega$,

$$x^*(c) = \arg \max_{x \in \Omega} \Pi \left(x, c \right)$$

subject to

$$\Pi \left(x, c \right) \ \geq \max_{p_1 \in [p^N(\underline{a}, x), p^N(\overline{a}, x)]} \int \left(p_1 - c \right) D_1 \left(p_1, p^N(a, x), ..., p^N(a, x), a \right) G'(a) da, \tag{7}$$

(33) is the on-path ICC where the LHS is the collusive profit and the RHS is the maximal profit from setting a uniform price consistent with the equilibrium price for some market type.

The main results of the paper are in Sections 6 and 7 where market conditions are identified for which it is more profitable for upper-level executives to centralize and coordinate on charging $\widehat{p}(\cdot)$ and for which it is more profitable for them to decentralize and coordinate on communicating $x^*(\cdot)$ to lower-level employees who then charge prices $\left\{ p^N(a, x^*(\cdot)) \right\}_{a \in [\underline{a}, \overline{a}]}$.

5.3 Rationale for Focusing on Grim PPE

There is a fundamental trade-off between coordinating prices and coordinating cost reports. By allowing for price discrimination, coordinating cost reports (with decentralized pricing) has the potential to be more profitable than coordinating prices (with centralized pricing). While the prospect of price discrimination favors cost coordination, monitoring for compliance with the collusive outcome favors price coordination. With price coordination, upper-level executives are agreeing to prices, and prices are directly observed by competitors. With cost coordination, upper-level executives are agreeing to the delegation of pricing authority

[18] If a firm is complying then it is a probability zero event to observe the same price more than once. Thus, one might imagine this deviation could be deterred with a punishment whenever a price, such as p^o, is observed multiple times. First note that this is an artifact of having an infinity of values of a. However, even under that assumption, a deviating upper level could easily avoid that punishment by adding a small amount of noise to its deviation price which is randomly assigned to different market types. To avoid appearing as if it has set a uniform price, an upper level could also cheat by randomly selecting a small subset of markets for which to set a uniform price. The analysis is robust to these alternative deviations.

and cost reports, and those are *not* directly observed by competitors because they are internal to the firm. Consequently, compliance with the collusive outcome can only be indirectly inferred through prices. This poses a monitoring challenge that manifests itself in a possible on-path deviation in which an upper-level executive intervenes in the pricing process and sets a low (uniform) price. Note that an on-path ICC is present with any PPE supporting cost coordination and is not special to grim PPE. The appeal of focusing on the optimal grim PPE is that it offers an analytically tractable and plausible approach to exploring the implications of this trade-off involving price discrimination and monitoring.

A concern is that results based on the optimal equilibrium from the set of grim PPE may not be robust to considering the set of all PPE. First note that the best equilibrium outcome when upper levels are coordinating prices would not change because the first-best uniform price is sustainable with a grim PPE (Theorem 2). However, it has not been shown the first-best cost report is implementable with a grim PPE and, in fact, we will see that it is generally not implementable because of the on-path ICC in (7). Thus, results showing when coordinating cost reports is more profitable are robust but it is possible that results showing when coordinating prices is more profitable are not. Let me explain why I think robustness is likely and, even if that is not the case, why the results of this analysis deliver insight into actual cartels.

A grim PPE with cost coordination specifies the grim punishment with probability one if the history includes an off-equilibrium path event, and a punishment with probability zero if the history does not include an off-equilibrium path event. The grim punishment is without loss of generality because the off-path ICCs are not binding as $\delta \to 1$. Where the grim PPE may be less than optimal is in not allowing for a punishment with positive probability for some on-path histories. A probabilistic punishment for certain on-path prices (specifically, those that are especially profitable as a uniform price deviation) could loosen the on-path ICC and allow for a more profitable cost report to be sustained. Before addressing the issue of robustness, note that this change significantly complicates the problem because the static ICC in (7) is replaced with a dynamic ICC (as an on-path deviation will affect the probability of a punishment and thus the continuation payoff). Focusing on grim PPE delivers a lot in terms of analytical tractability. Returning to the issue of robustness, even if the first-best collusive cost report were to be implementable with an optimal punishment,[19] the equilibrium payoff would be less than the first-best payoff because a punishment is occurring with positive probability in equilibrium. Thus, an optimal equilibrium from the set of all PPE will still mean the monitoring advantage to coordinating prices could outweigh the price discrimination advantage from coordinating cost reports. Hence, the tension present with grim PPE exists for all PPE.

While the forces at play in grim PPE are then robust, it is still possible that the specific results are not. As we later show, there are market conditions for which coordinating cost reports is more profitable and other market conditions for which coordinating prices is more profitable. Upon allowing for optimal punishments, it is possible that coordinating cost reports could be found to be more profitable under all market conditions. However, such a finding would be counterfactual - as, in practice, many cartels coordinate prices - and

[19]That is actually unlikely because, as is typical with imperfect monitoring, the on-path ICC will be binding even as $\delta \to 1$.

that would undermine the validity of assuming optimal punishments. Allowing for optimal punishments could instead change the set of market conditions under which one collusive arrangement is preferred over the other. However, as we'll see in Sections 6 and 7, those market conditions are highly intuitive and thus robustness seems likely. Finally, even if that were not true, I would argue that results based on grim PPE still advance our understanding of cartels. Assuming optimal punishments is appropriate to the extent one believes they are a plausible property of actual collusive strategies. A strategy that assigns a punishment to some but not all equilibrium prices (or varies the probability with the price) requires conscious design by cartel members and an explicit agreement among them. However, there is little evidence to support such a claim. As documented in Harrington (2006) and Marshall and Marx (2012), cartelists give considerable attention to selecting the collusive outcome and structuring the monitoring protocol, but discussions about punishments are highly incomplete and often entirely absent. In light of the absence of documentary evidence to support complex punishments on the equilibrium path, there is a certain appeal to the simple and focal structure of grim PPE: punish when there is evidence of non-compliance, and continue with the collusive outcome otherwise. In sum, grim PPE are plausible, tractable, and likely to deliver robust insight into actual cartel conduct.

6 Relative Performance of Cost Coordination and Price Coordination

In exploring when colluding firms will coordinate cost reports and when they will coordinate prices, two market traits are considered: product differentiation and market (or customer) heterogeneity. To make the analysis tractable while maintaining the key elements that distinguish cost coordination and price coordination, I will restrict the probability distribution on cost. A range of possible cost levels is essential to the mechanism as that gives an upper level the option of inflating the cost report above the actual cost. At the same time, the degree of cost variability is not relevant to the functioning of the different collusive schemes. For that reason, it will be assumed that the probability distribution on cost ρ places almost all mass in a neighborhood of some cost which, with an abuse of notation, is denoted c. Thus, when comparing the relative performance of coordinating prices and coordinating cost reports, it is sufficient (by continuity) to perform our evaluation only for the cost c. By this simplification, *cost coordination is more profitable than price coordination* when:

$$\begin{aligned} &\int \left(p^N(a, x^*(c)) - c\right) D_1 \left(p^N(a, x^*(c)), ..., p^N(a, x^*(c)), a\right) G'(a)da \\ > &\int \left(\widehat{p}(c) - c\right) D_1 \left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da. \end{aligned} \tag{8}$$

Recall that, in equilibrium as $\delta \to 1$, $\widehat{p}(c)$ is the best price sustainable by the price coordination strategy and $x^*(c)$ is the best cost report sustainable by the cost coordination strategy.

6.1 Effect of Product Differentiation

The variable $\gamma \in [0,1]$ is introduced to represent the degree of product similarity where $\gamma = 0(1)$ is independent (homogeneous) products. Recall that $\widehat{p}(c)$ and $p^M(a,c)$ is the joint profit-maximizing price when pricing is uniform and when it involves third-degree price discrimination, respectively. $\widehat{p}(c)$ and $p^M(a,c)$ are assumed to be independent of γ. The symmetric Nash equilibrium (or competitive) price is

$$p^N(a,x,\gamma) \equiv \arg\max_{p_1} (p_1 - x)\, D_1\left(p_1, p^N(a,x,\gamma), ..., p^N(a,x,\gamma), a, \gamma\right),$$

and assume: if $p > (<) p^N(a,x,\gamma)$ then $\psi_1(p,...,p,a,x,\gamma) < (>) p$ where $\psi_1(p_2,...,p_n,a,x,\gamma)$ is firm 1's static best response function.[20] That higher values of γ correspond to less differentiated products is reflected in the following three assumptions. First, $p^N(a,x,\gamma)$ is continuously decreasing in γ. Second, the competitive price approaches cost as products become homogeneous: $\lim_{\gamma \to 1} p^N(a,x,\gamma) = x$. Third, the competitive price approaches the joint profit-maximizing price when products become independent: $\lim_{\gamma \to 0} p^N(a,x,\gamma) = p^M(a,x)$.

Theorem 4 shows if products are sufficiently differentiated then colluding firms find it more profitable to inject an inflated cost into their decentralizing pricing process than coordinate on a uniform price.

Theorem 4 *There exists $\gamma' > 0$ such that if the degree of product similarity $\gamma \in (0,\gamma')$ then cost coordination is more profitable than price coordination.*

Theorem 4 is proven by the following argument. As products become maximally differentiated, the profits from competition converge to those from first-best collusion. At the same time, the profits from third-degree price discrimination are bounded above the profits from a uniform price. Thus, when products are sufficiently differentiated, competition with third-degree price discrimination is more profitable than collusion with a uniform price. Hence, price coordination is inferior to competition. At the same time, there exists an inflated cost report for which cost coordination is incentive compatible and more profitable than competition. Though this proof strategy is based on deriving sufficient conditions for price coordination to be less profitable than competition, the result is more general in that an intermediate level of product differentiation can make cost coordination more profitable than price coordination even when price coordination is more profitable than competition. Though this is difficult to generally prove, it is established for the case of linear demand in Section 7.

The next result shows price coordination is more profitable than cost coordination when products are sufficiently similar. For example, if firms are offering commodities then they will prefer the more standard method of coordinating the prices they charge to buyers.

Theorem 5 *There exists $\gamma' < 1$ such that if the degree of product similarity $\gamma \in (\gamma',1)$ then price coordination is more profitable than cost coordination.*

[20] $\psi_1(p_2,...,p_n,x_1,a) \equiv \arg\max_{p_1} (p_1 - x_1)\, D_1(p_1,p_2,...,p_n,a)$.

When products are near homogeneous, price is close to cost and thus highly insensitive to market type. The advantage of price discrimination from cost coordination is then small when product differentiation is low. Furthermore, for cost coordination to be at least as profitable as price coordination, the inflated cost report must result in a price in the neighborhood of the optimal uniform price $\widehat{p}(c)$. As the competitive price is close to cost when products are near homogeneous, this means the cost report $x^*(c,\gamma)$ must be close to $\widehat{p}(c)$. However, as shown in the proof, this leaves room for a profitable on-path deviation. If firms are coordinating on a common cost report of $x^*(c,\gamma)$, a firm can profitably deviate by centralizing pricing authority and setting a uniform price of $p^N(\underline{a}, x^*(c,\gamma), \gamma)$; that is, pricing as if $a = \underline{a}$. This on-path deviation is shown to yield higher profit for all values of a when products are sufficiently similar. Even for markets with strong demand (i.e., high values of a), a firm's profit is higher by undercutting with price $p^N(\underline{a}, x^*(c,\gamma), \gamma)$ rather than charging the market-specific collusive price $p^N(a, x^*(c,\gamma), \gamma)$.

This preference for price coordination can also be described as follows. When products are sufficiently similar, prices are almost uniform under cost coordination so the first-best outcome under cost coordination is only slightly better than under price coordination. Still, it is better, so cost coordination would be chosen but for incentive compatibility issues. The problem is that monitoring with cost coordination is difficult because an upper-level executive can intervene in pricing in an undetected manner and lower the average price. That restricts how much the cost report can be inflated and thereby limits the collusive markup. In comparison, monitoring is more effective under price coordination and, by being able to sustain a higher average price, price coordination more than compensates for the lack of price discrimination.

6.2 Effect of Market Heterogeneity

Here it is established that price coordination is preferred when market heterogeneity is sufficiently small. The next section shows, under the assumption of linear demand, that firms prefer cost coordination when market heterogeneity is sufficiently large.

In order to consider when market heterogeneity is small, define an extreme distribution that puts all mass on market type $\widehat{a}$:

$$\widehat{G}(a) = \begin{cases} 0 & \text{if } a \in [\underline{a}, \widehat{a}) \\ 1 & \text{if } a \in [\widehat{a}, \overline{a}] \end{cases} .$$

Low market heterogeneity is represented by distributions that are close to $\widehat{G}$. For this purpose, let $a_k \sim G_k : [\underline{a}, \overline{a}] \to [0,1]$ where G_k is continuously differentiable and $G'_k(a) > 0$ $\forall a \in [\underline{a}, \overline{a}]$.

Theorem 6 *If $\{a_k\}_{k=1}^{\infty}$ converges in distribution to $\widehat{a}$ then there exists k' such that if $k > k'$ then price coordination is more profitable than cost coordination.*

When G puts sufficient mass in a sufficiently small neighborhood around $\widehat{a}$ then, in order to be as profitable as price coordination, cost coordination must price close to $p^M(\widehat{a}, c)$. However, a firm's upper level can then engage in an on-path deviation by setting a uniform price just below $p^M(\widehat{a}, c)$. As in the case with minimal product differentiation, price coordination does not forego much potential profit with its uniform price but is superior in terms of monitoring.

7 Linear Demand

Towards more fully understanding when a cartel would choose to coordinate cost reports rather than prices, this section considers the duopoly case with linear demand. A firm's demand is

$$D_1(p_1, p_2, a) = a - bp_1 + dp_2, \tag{9}$$

where $b > d > 0$. $a \sim G : [\underline{a}, \overline{a}] \rightarrow [0, 1]$ and let μ_a and σ_a^2 denote the mean and variance of a, respectively. σ_a^2 measures the degree of market heterogeneity. It can be shown that a stronger market (i.e., a higher value of a) has a more price-inelastic firm demand function. As a reminder, it is assumed that almost all mass is put on a particular cost level which is denoted c. By continuity, the analysis can then be conducted assuming mass one is placed on c. In deriving the closed-form solutions in Section 7.1, it is presumed that firms' demands are interior for all relevant prices. I will return to this qualification later.[21]

7.1 Analytical Results

If firms decentralize pricing and have a common cost report x then the symmetric Nash equilibrium price is

$$p^N(a, x) = \frac{a + bx}{2b - d}. \tag{10}$$

Under competition, the cost report is truthful ($x = c$) and profit is

$$\frac{b\left(\mu_a - (b-d)c\right)^2 + b\sigma_a^2}{(2b-d)^2}. \tag{11}$$

The first-best price under price coordination is

$$\widehat{p}(c) = \frac{\mu_a + (b-d)c}{2(b-d)} \tag{12}$$

with profit

$$\frac{\left(\mu_a - (b-d)c\right)^2}{4(b-d)}. \tag{13}$$

It is then more profitable for firms to centralize and coordinate on $\widehat{p}(c)$ than to decentralize and set competitive prices $p^N(a, c)$ when (13) exceeds (11) or, equivalently,

$$\sigma_a^2 < \frac{d^2\left(\mu_a - (b-d)c\right)^2}{4b(b-d)}. \tag{14}$$

[21] The supporting analysis for the results in Section 7 are available in the Online Appendix.

Thus, if market heterogeneity is not too great then firms prefer collusion with a uniform price to competition with price discrimination. (14) corresponds to (5) in Theorem 2.

Turning to cost coordination, the first-best collusive cost report is

$$\widehat{x}(c) = c + \frac{d(\mu_a - (b-d)c)}{2b(b-d)} \tag{15}$$

which results in a price for market type a of

$$\frac{\mu_a + (b-d)c}{2(b-d)} + \frac{a - \mu_a}{2b-d}. \tag{16}$$

Taking the expectation of (16) with respect to a, note that it equals (12). Hence, at the first best, the average price under cost coordination is the same as the uniform price under price coordination.

The equilibrium collusive cost report (6) is[22]

$$x^*(c) = c + \left(\frac{2}{2b-d}\right)\sigma_a. \tag{17}$$

Thus, the extent to which cost is inflated is increasing in the degree of market heterogeneity. Inserting (17) into (10), the price for market type a is

$$\frac{a+bc}{2b-d} + \frac{2b\sigma_a}{(2b-d)^2}. \tag{18}$$

One can show that the expectation of (18) is less than (12) so, in equilibrium, average price is lower with cost coordination than with price coordination.

It is shown in the Online Appendix that, at the optimal PPE, there exists $\omega_1, \omega_2, \omega_3$ - where $\omega_1 > \omega_2 > \omega_3$ - such that:

- If $\sigma_a^2 \in (0, \omega_3)$ then price coordination is more profitable than cost coordination and competition.
- If $\sigma_a^2 \in (\omega_3, \omega_2)$ then cost coordination is more profitable than price coordination (which is more profitable than competition).
- If $\sigma_a^2 \in (\omega_2, \omega_1)$ then cost coordination is more profitable than price coordination (which is less profitable than competition).

The key finding is that if market heterogeneity is sufficiently great - $\sigma_a^2 > \omega_3$ - then firms prefer to coordinate cost reports.[23] Though intuitive, the result is not as immediate as one might suppose. Note that the case of linear demand satisfies the conditions in Theorem 1 which means the first-best outcome under cost coordination is always more profitable than the first-best outcome under price coordination, regardless of the value of σ_a^2 (as long as it is positive). However, if σ_a^2 is low then equilibrium constraints imply that price coordination

[22] This solution applies when σ_a^2 is not too high so that the on-path ICC is binding.

[23] The analysis focuses on when the on-path ICC is binding for cost coordination which requires $\sigma_a^2 < \omega_1$.

is more profitable than cost coordination. In order to make cost coordination immune to an upper-level executive centralizing pricing authority and lowering the average price, the collusive cost report must be set below the first-best cost report. That results in average price being lower than under price coordination. The higher profit from the higher average price under price coordination is balanced against the higher profit from price discrimination under cost coordination. As the latter effect is small when σ_a^2 is low, firms prefers to coordinate on a higher uniform price; they are willing to forego price discrimination in order to be able to sustain a higher average price. When instead σ_a^2 is high, the additional profit from price discrimination more than offsets the lower average price under cost coordination so firms prefer to coordinate cost reports.

In concluding this section, it must be emphasized that these closed-form solutions are correct if firms' demands are interior for all prices relevant to the derivations of those solutions. Where that may be problematic is when price is uniform - as with price coordination or a uniform deviation price under cost coordination - because both firms' demands may not be positive when there is sufficient variation in a and the realized value of a is low. Thus, requiring σ_a^2 to be sufficiently great could imply that firms' demands are not positive for some prices and demand realizations, which would then invalidate the analysis. To allay that concern, numerical analysis is conducted in the next sub-section which does not suppose that firms' demands are always interior. That analysis supports the preceding findings.

7.2 Numerical Results

In performing the numerical analysis, it is necessary to begin with a representative agent's utility function:

$$\theta\left(q_1+q_2\right)-\left(\frac{1}{2}\right)\left(\beta\left(q_1^2+q_2^2\right)+2\gamma q_1 q_2\right), \tag{19}$$

so that we may derive the firm demand function that encompasses corner solutions:

$$D_1\left(p_1,p_2,\theta\right) \tag{20}$$

$$\equiv\begin{cases}\frac{\theta}{\beta}-\left(\frac{1}{\beta}\right)p_1 & \text{if } p_1\leq\left(\frac{1}{\gamma}\right)\left(\beta p_2-(\beta-\gamma)\theta\right)\\ \frac{\theta}{\beta+\gamma}-\left(\frac{\beta}{\beta^2-\gamma^2}\right)p_1+\left(\frac{\gamma}{\beta^2-\gamma^2}\right)p_2 & \text{if } \left(\frac{1}{\gamma}\right)\left(\beta p_2-(\beta-\gamma)\theta\right)\leq p_1\leq\left(\frac{1}{\beta}\right)\left(\gamma p_2+(\beta-\gamma)\theta\right)\\ 0 & \text{if } \left(\frac{1}{\beta}\right)\left(\gamma p_2+(\beta-\gamma)\theta\right)\leq p_1.\end{cases}$$

When firms' demands are interior for all relevant prices then firm demand is

$$\frac{\theta}{\beta+\gamma}-\left(\frac{\beta}{\beta^2-\gamma^2}\right)p_1+\left(\frac{\gamma}{\beta^2-\gamma^2}\right)p_2.$$

which is the same as (9) where:

$$a\equiv\frac{\theta}{\beta+\gamma},b\equiv\left(\frac{\beta}{\beta^2-\gamma^2}\right),d\equiv\left(\frac{\gamma}{\beta^2-\gamma^2}\right). \tag{21}$$

Referring to (19), the degree of product differentiation is decreasing in γ where products are independent at $\gamma=0$ and identical at $\gamma=\beta$. The market type is represented by θ

which is uniformly distributed on $[\mu_\theta - (r/2), \mu_\theta + (r/2)]$. r is the range of θ and captures market heterogeneity; an increase in r raises the variance $\sigma_\theta^2 = r^2/12$ but leaves the mean unchanged. Assume ρ puts (almost) all mass on one cost level, which is denoted c, and Ω is sufficiently fine so that $\widetilde{x}(c) \in \Omega$. The model's parameters are $(\mu_\theta, r, \beta, \gamma, c)$ and are chosen so that $\gamma \in [0, \beta]$ and $\mu_\theta - (r/2) > c \geq 0$.[24]

Results are reported in Figures 2 and 3 where the horizontal axis measures the degree of market heterogeneity r and the vertical axis measures the degree of product similarity γ. Figure 2 assumes $(\mu_\theta, \beta) = (10, 1)$ where $(r, \gamma) \in \{.1, .2, ..., 4.9, 5.0\} \times \{.00, .01, ..., .98, .99\}$. For $c = 1$, the solid line partitions the space between values of (r, γ) where cost coordination is more profitable (below the line) and price coordination is more profitable (above the line). Also shown are the thresholds when $c = 3$ (dashed line) and $c = 5$ (dotted line). Consistent with the results in Sections 6 and 7.1, cartels facing greater market heterogeneity and more product differentiation are more likely to coordinate cost reports than prices. This finding is confirmed in Figure 3 for $(\mu_\theta, \beta) = (15, 1)$ and $c \in \{1, 3, 5, 7\}$ where $(r, \gamma) \in \{.1, .2, ..., 9.9, 10.0\} \times \{.00, .01, ..., .98, .99\}$.

Figure 2

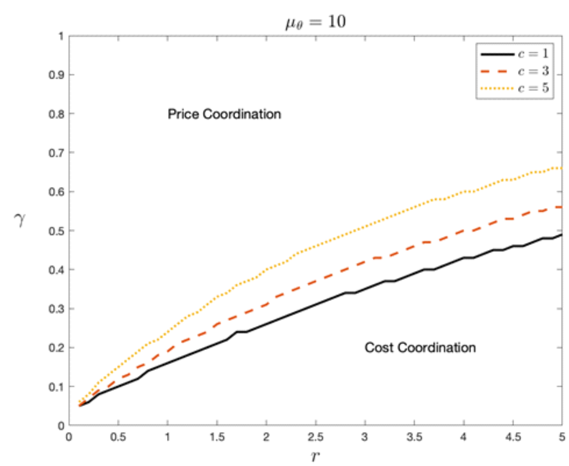
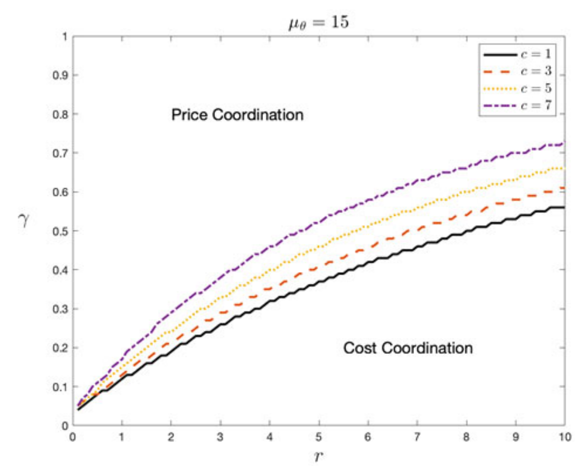


Figure 3



8 Concluding Remarks

This paper developed a theory of collusion in which senior managers delegate pricing authority and coordinate on the cost that is given to lower-level pricing managers and sales representatives. With this theory, we compared the relative performance of coordinating on an inflated cost report - in order to induce those with pricing authority to set higher market-specific prices - with the more traditional method in which senior managers control and coordinate final prices. The trade-off between the two schemes is that monitoring for compliance is less effective with cost coordination (because a colluding executive could secretly intervene in the internal pricing process and lower price) which results in a lower average price than with price coordination, but the potential profit that can be earned is higher because of greater price discrimination from delegating pricing authority to those with the best demand information. When market heterogeneity is high, the profit gain from price

[24] Details on the numerical analysis are provided in the Online Appendix.

discrimination is sufficient to offset the lower average price so the cartel coordinates cost reports. With low market heterogeneity, the cartel chooses the more standard method of coordinating prices. When products are highly substitutable, the extent of price discrimination under cost coordination is low because, given prices are set competitively, they will tend to be close to firms' (inflated) cost. Thus, price is close to being uniform even when firms delegate pricing authority. Furthermore, the average price with cost coordination is below the uniform price with price coordination because less effective monitoring constrains how high a cost report the cartel can sustain. We then find that a cartel will tend to coordinate prices when products are commodities and coordinate cost reports when products are highly differentiated.

There are a number of promising research directions. Motivated by the development of Big Data, one could allow for the upper-level executives to have some demand information that is not available to lower-level employees (e.g., information acquired by collecting market data), while still allowing for the latter to have some private demand information (e.g., information acquired by personally interacting with customers). A collusive arrangement might then involve coordinating cost reports and delegating pricing authority but, at the same time, upper levels using their private demand information to constrain the pricing of lower levels. Another research direction is to expand the set of collusive arrangements so that upper levels can decide whether to invite lower levels to participate in the cartel. In deciding to include lower-level employees, upper-level executives would face a trade-off between more effective collusive pricing and a greater risk of antitrust penalties. Such an extension would require some innovative modelling to capture how the set of involved employees affects the cartel's discovery and conviction.

The key starting point to the paper's analysis was recognizing the importance of taking account of a firm's internal pricing process. That recognition was reached while puzzling over how certain collusive practices could be effective. How could a cartel that coordinates list prices be effective when it does not coordinate discounts off of list prices? How could a cartel that coordinates on introducing a surcharge be effective when it does not coordinate on fixing other components of the final price? In addressing these questions, the insight delivered in Harrington and Ye (2019), Chen (2021), and this paper runs contrary to the canonical understanding which is that collusion is less effective when the cartel does not fully control prices as, for example, arises under imperfect monitoring (Green and Porter, 1984). Here we see that some practices are effective only because the colluding executives do *not* fully control prices. The general takeaway is that some collusive conduct can be better understood by taking account of how prices are set within the firms comprising the cartel.

9 Appendix

Proof of Theorem 1. I begin with some preliminary results (where the dependence on c is dropped to avoid extraneous notation). Let us show $\widehat{p} \in \left(p^M(\underline{a}), p^M(\overline{a})\right)$ where $\widehat{p}$ is defined by:

$$\int \left(\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p}\right) G'(a)da = 0. \tag{22}$$

Suppose instead $\widehat{p} \leq p^M(\underline{a})$. By strict quasi-concavity and $p^M(a)$ is increasing in a,

$$\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p} > 0 \ \forall a \in (\underline{a}, \overline{a}]$$

and, therefore,

$$\int \left(\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p}\right) G'(a)da > 0$$

which contradicts (22). If instead $\widehat{p} \geq p^M(\overline{a})$ then, by strict quasi-concavity and $p^M(a)$ is increasing in a,

$$\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p} < 0 \ \forall a \in [\underline{a}, \overline{a})$$

and, therefore,

$$\int \left(\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p}\right) G'(a)da < 0$$

which contradicts (22). Hence, $\widehat{p} \in \left(p^M(\underline{a}), p^M(\overline{a})\right)$.

Define a' as the market type such that the monopoly price when conditioning on a equals the monopoly price when not conditioning on a: $p^M(a') = \widehat{p}$. Note that $\widehat{p} \in \left(p^M(\underline{a}), p^M(\overline{a})\right)$ implies $a' \in (\underline{a}, \overline{a})$. Allowing cost reports to take any non-negative value, define the cost report x' that equates the competitive price to the price under price coordination at market type a': $p^N(a', x') = \widehat{p}$. To show that x' exists, first note that $D_i(c, ..., c, a') > 0$ implies $D_i\left(p^M(a'), ..., p^M(a'), a'\right) > 0$. Therefore, if $x = p^M(a')$ then $p^N(a', p^M(a')) > p^M(a')$. We then have:

$$p^N(a', p^M(a')) > p^M(a') > p^N(a', c).$$

By continuity of $p^N(a, x)$ in x, $\exists x' \in \left(c, p^M(a')\right)$ such that $p^N(a', x') = p^M(a')$ and thus $p^N(a', x') = \widehat{p}$.

Based on the definitions of a' and x' and (3), we have Figure 1. Note that $p^N(a, x') \in \left(p^M(a), \widehat{p}\right) \ \forall a \in [\underline{a}, a')$ and $p^N(a, x') \in \left(\widehat{p}, p^M(a)\right) \ \forall a \in (a', \overline{a}]$. By strict quasi-concavity of $(p - c) D_i(p, ..., p, a)$ in p and $p^M(a) \equiv \arg\max_p (p - c) D_i(p, ...p, a)$ then

$$\left(p^N(a, x') - c\right) D_i\left(p^N(a, x'), ..., p^N(a, x'), a\right) > (\widehat{p} - c) D_i(\widehat{p}, ..., \widehat{p}, a) \,\forall a \in [\underline{a}, a')$$

and

$$\left(p^N(a, x') - c\right) D_i\left(p^N(a, x'), ..., p^N(a, x'), a\right) > (\widehat{p} - c) D_i(\widehat{p}, ..., \widehat{p}, a) \,\forall a \in (a', \overline{a}].$$

Therefore,

$$\begin{aligned} &\int \left(p^N(a,x') - c\right) D_i \left(p^N(a,x'), ..., p^N(a,x'), a\right) G'(a)da \\ > &\int \left(\widehat{p} - c\right) D_i \left(\widehat{p}, ..., \widehat{p}, a\right) G'(a)da. \end{aligned} \tag{23}$$

Assuming Ω is sufficiently fine then, given

$$\widehat{x} = \arg\max_{x \in \Omega} \int \left(p^N(a,x) - c\right) D_i \left(p^N(a,x), ..., p^N(a,x), a\right) G'(a)da,$$

it follows from (23) that (4) is true. ■

Define $\widehat{\Pi}(c)$ as a firm's profit under centralization with price $\widehat{p}(c)$ and cost c:

$$\widehat{\Pi}(c) \equiv \int \left(\widehat{p}(c) - c\right) D_1 \left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da.$$

Define $\Pi(x,c)$ as a firm's profit under decentralization with cost report x and cost c:

$$\Pi(x,c) \equiv \int \left(p^N(a,x) - c\right) D_1 \left(p^N(a,x), ..., p^N(a,x), a\right) G'(a)da.$$

Note that $\Pi(c,c)$ is the stage game Nash equilibrium profit.

Proof of Theorem 2. The public history in period t are costs and sampled prices over periods $1, ..., t-1$. If there is some $\tau < t$ such that a sampled price does not equal $\widehat{p}(c^\tau)$ then the price coordination strategy profile is a Nash equilibrium for the infinitely repeated game, given it calls for the stage game Nash equilibrium. If the public history has the sampled prices equalling $\widehat{p}(c^\tau)$ $\forall \tau = 1, ..., t-1$ then the price coordination strategy has an upper level centralizing and pricing at $\widehat{p}(c^t)$. Equilibrium requires that the associated payoff is at least as great as the payoff from: 1) centralization with a price different from $\widehat{p}(c^t)$; and 2) decentralization and choosing a cost report from Ω. I will consider each of the associated incentive compatibility constraints (ICCs). By symmetry, it is without loss of generality to conduct the analysis from the perspective of firm 1.

Consider some period and let c' be the current period's cost. The ICC associated with maintaining centralization and deviating by charging a price different from $\widehat{p}(c')$ is:

$$\begin{aligned} &\widehat{\Pi}(c') + \left(\frac{\delta}{1-\delta}\right) E_c \left[\widehat{\Pi}(c)\right] \\ \geq\ &\max_{p_1} \int \left(p_1 - c'\right) D_1 \left(p_1, \widehat{p}(c'), ..., \widehat{p}(c'), a\right) G'(a)da + \left(\frac{\delta}{1-\delta}\right) E_c \left[\Pi(c,c)\right] \\ \forall c' \in\ &\Omega, \end{aligned} \tag{24}$$

where $E_c[\cdot]$ is the expectation with respect to cost. Next consider when deviation involves decentralization. So as to derive a sufficient condition for the ICC to hold, an upper bound on the deviation payoff will be used by assuming the firm's prices are best responses to rival firms' prices. For this purpose, define:

$$\psi_1(p_2,...,p_n,x_1,a) \equiv \arg\max_{p_1}(p_1-x_1)\,D_1(p_1,p_2,...,p_n,a).$$

A sufficient condition for the ICC to hold is

$$\begin{aligned}&\widehat{\Pi}(c')+\left(\frac{\delta}{1-\delta}\right)E_c\left[\widehat{\Pi}(c)\right] \qquad (25)\\ \geq\ &\max_{x_1}\int(\psi_1(\widehat{p}(c'),...,\widehat{p}(c'),x_1,a)-c')\,D_1(\psi_1(\widehat{p}(c'),...,\widehat{p}(c'),x_1,a),\widehat{p}(c'),...,\widehat{p}(c'),a)\,G'(a)da\\ &+\left(\frac{\delta}{1-\delta}\right)E_c\left[\Pi(c,c)\right]\\ \forall c'\ \in\ &\Omega.\end{aligned}$$

Referring to (24) and (25), if the future expected profit on the LHS exceeds the future expected profit from on the RHS,

$$E_c\left[\widehat{\Pi}(c)\right] > E_c\left[\Pi(c,c)\right], \qquad (26)$$

then (24) and (25) are satisfied as $\delta \to 1$. As (5) implies (26) then this proves Theorem 2. ■

Proof of Theorem 3. Consider a supracompetitive $\widetilde{x}(\cdot)$ that produces higher profit than reporting the actual cost:

$$\Pi(\widetilde{x}(c),c) > \Pi(c,c),\ \forall c\in\Omega-\{\overline{c}\}. \qquad (27)$$

That such an $\widetilde{x}(\cdot)$ exists follows from A3-A5. By A4-A5, $(p-c)\,D_1(p,...,p,a)$ is increasing in p in a neighborhood of $p=p^N(a,c)$. By A3, $p^N(a,x)$ is increasing in x. Thus, (27) holds for $\widetilde{x}(c)=c+\eta$ for $\eta>0$ and small. Recall that $\Omega\equiv\{\underline{c},\underline{c}+\eta,...,\overline{c}-\eta,\overline{c}\}$ and we are allowing Ω to be sufficiently fine as needed.

Given cost c', there are two forms of deviations. First, an upper level continues to decentralize and chooses a cost report different from $\widetilde{x}(c')$. Second, an upper level centralizes and chooses a uniform price. First note that the associated ICCs are trivially satisfied when $\widetilde{x}(c')=c'$ as then the collusive outcome coincides with the static Nash equilibrium. Thus, from hereon suppose $\widetilde{x}(c')>c'$.

Consider an upper level maintaining decentralization and deviating with cost report $x_1<\widetilde{x}(c')$.[25] This deviation results in some prices below $p^N(\underline{a},\widetilde{x}(c'))$ which is evidence of noncompliance with the collusive cost report. Hence, there is a probability $\beta(x_1)>0$ of sampled prices not lying in $\left[p^N(\underline{a},\widetilde{x}(c')),p^N(\overline{a},\widetilde{x}(c'))\right]$ so the continuation payoff is the grim

[25] Clearly, it would not be optimal to choose $x_1>\widetilde{x}(c')$ as that lowers current profit and does not raise future profit.

punishment.[26] The ICC is:

$$\begin{aligned}
& \Pi\left(\widetilde{x}(c'), c'\right) + \left(\frac{\delta}{1-\delta}\right) E_c\left[\Pi\left(\widetilde{x}(c), c\right)\right] \\
\geq\ & \max_{x_1<\widetilde{x}(c')} \int \left(p^N(a, x_1) - c'\right) D_1\left(p^N(a, x_1), p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a)da \\
& +\beta(x_1)\left(\frac{\delta}{1-\delta}\right) E_c\left[\Pi\left(c, c\right)\right] + \left(1-\beta(x_1)\right)\left(\frac{\delta}{1-\delta}\right) E_c\left[\Pi\left(\widetilde{x}(c), c\right)\right] \\
\forall c' \ \in\ & \Omega.
\end{aligned}$$

Re-arranging the ICC, we have:

$$\begin{aligned}
& \left(\frac{\delta}{1-\delta}\right) \beta(x_1)\left(E_c\left[\Pi\left(\widetilde{x}(c), c\right)\right] - E_c\left[\Pi\left(c, c\right)\right]\right) \\
\geq\ & \max_{x_1<\widetilde{x}(c')} \int \left(p^N(a, x_1) - c'\right) D_1\left(p^N(a, x_1), p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a)da \\
& -\Pi\left(\widetilde{x}(c'), c'\right),\ \forall c' \in \Omega.
\end{aligned} \tag{28}$$

If

$$E_c\left[\Pi\left(\widetilde{x}(c), c\right)\right] - E_c\left[\Pi\left(c, c\right)\right] > 0 \tag{29}$$

then the LHS of (28) goes to $+\infty$ as $\delta \to 1$. As $\widetilde{x}(\cdot)$ is assumed to satisfy (27) then (29) is true by construction. Given the RHS of (28) is bounded then (28) holds as $\delta \to 1$.

Next consider a deviation in which an upper level centralizes and charges a uniform price. As a uniform price outside of $\left[p^N(\underline{a}, \widetilde{x}(c)), p^N(\overline{a}, \widetilde{x}(c))\right]$ is an off-path deviation, it is straightforward to show the associated ICC is satisfied as $\delta \to 1$, as long as (29) holds. If instead the uniform price lies in $\left[p^N(\underline{a}, \widetilde{x}(c')), p^N(\overline{a}, \widetilde{x}(c'))\right]$ then it is an on-path deviation and the ICC is:

$$\begin{aligned}
& \Pi\left(\widetilde{x}(c'), c'\right) \\
\geq\ & \max_{p_1 \in [p^N(\underline{a}, \widetilde{x}(c')), p^N(\overline{a}, \widetilde{x}(c'))]} \int \left(p_1 - c'\right) D_1\left(p_1, p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a)da,\ \forall c' \in \Omega.
\end{aligned} \tag{30}$$

To begin, I will show that (30) holds with strict inequality for $\widetilde{x}(c') = c'$; that is,

$$\begin{aligned}
& \left(\Pi\left(c', c'\right) =\right) \int \left(p^N(a, c') - c'\right) D_1\left(p^N(a, c'), ..., p^N(a, c'), a\right) G'(a)da \\
>\ & \max_{p_1 \in [p^N(\underline{a}, c'), p^N(\overline{a}, c')]} \int \left(p_1 - c'\right) D_1\left(p_1, p^N(a, c'), ..., p^N(a, c'), a\right) G'(a)da,\ \forall c' \in \Omega.
\end{aligned} \tag{31}$$

Given rival firms are pricing at $p^N(a, c')$, the integrand on the LHS is profit from choosing the best reply $p^N(a, c')$ and the integrand on the RHS is profit from choosing some p_1 which

[26] $\beta(x_1)$ can be derived as follows. Given p^N is strictly increasing in x then $x_1 < \widetilde{x}(c')$ implies $p^N(\underline{a}, x_1) < p^N(\underline{a}, \widetilde{x}(c'))$. If $p^N(\overline{a}, x_1) > p^N(\underline{a}, \widetilde{x}(c'))$ then, given p^N is continuous and strictly increasing in a, $\exists \kappa(x_1) \in (\underline{a}, \overline{a})$ such that $p^N(\kappa(x_1), x_1) = p^N(\underline{a}, \widetilde{x}(c'))$. If $p^N(\overline{a}, x_1) \leq p^N(\underline{a}, \widetilde{x}(c'))$ then set $\kappa(x_1) = \overline{a}$. Price monitoring reveals evidence of cheating when price is below $p^N(\underline{a}, \widetilde{x}(c'))$ which occurs when price is collected from market type $a < \kappa(x_1)$; that event occurs with probability $G(\kappa(x_1))$. If w prices are collected as part of price monitoring then the probability of detection of cheating is $\beta(x_1) \equiv 1 - \left(1 - G(\kappa(x_1))\right)^w$.

is, generically, not the best reply. As then the integrand on the LHS exceeds the integrand on the RHS for almost all a, (31) is true. Given (30) holds strictly for $\widetilde{x}(c') = c'$, it follows by continuity that (30) holds for $\widetilde{x}(c') = c' + \eta$ for $\eta > 0$ and small. Thus, there exists a supracompetitive $\widetilde{x}(\cdot)$ such that all ICCs hold as $\delta \rightarrow 1$. ∎

Proof of Theorem 4. If $c < \overline{c}$, I will show that if products are sufficiently differentiated then $\exists x > c$ such that cost coordination with x is more profitable than price coordination - (32) holds - and x is incentive compatible - (33) holds:

$$\int_{\underline{a}}^{\overline{a}} (p^N(a,x) - c) D_1 \left(p^N(a,x), ..., p^N(a,x), a \right) G'(a) da \tag{32}$$
$$> \int_{\underline{a}}^{\overline{a}} (\widehat{p}(c) - c) D_1 \left(\widehat{p}(c), ..., \widehat{p}(c), a \right) G'(a) da,$$

$$\int \left(p^N(a,x) - c \right) D_1 \left(p^N(a,x), ..., p^N(a,x), a \right) G'(a) da \tag{33}$$
$$\geq \max_{p_1 \in [p^N(\underline{a},x), p^N(\overline{a},x)]} \int (p_1 - c) D_1 \left(p_1, p^N(a,x), ..., p^N(a,x), a \right) G'(a) da, \ \forall c \in [\underline{c}, \overline{c}].$$

Let us start with $x = c$. $x = c$ implies (33) holds strictly because $p^N(a,x)$ is the best response to other firms pricing at $p^N(a,x)$ given a. Hence, the integrand on the LHS exceeds the integrand on the RHS for almost all a. Given $\lim_{\gamma \rightarrow 0} p^N(a,c,\gamma) = p^M(a,c)$ then, for γ close to 0, the LHS of (32) is close to the monopoly profit from third-degree price discrimination and the RHS of (32) is close to the monopoly profit from a uniform price. Hence, (32) holds strictly as $\gamma \rightarrow 0$.

Thus far, it has been shown $\exists \gamma' > 0$ such that if $\gamma \in (0, \gamma')$ then (32)-(33) hold strictly for $x = c$. By continuity, if $\gamma \in (0, \gamma')$ then (32)-(33) hold for $x = c + \eta$ when η is sufficiently small. Also note that cost coordination with $c + \eta$ (as long as η is sufficiently small) is more profitable than competition (i.e., $x = c$):

$$\int (p^N(a, c+\eta) - c) D_1 \left(p^N(a, c+\eta), ..., p^N(a, c+\eta), a, \gamma \right) G'(a) da \tag{34}$$
$$> \int (p^N(a,c) - c) D_1 \left(p^N(a,c), ..., p^N(a,c), a, \gamma \right) G'(a) da.$$

The preceding condition follows from the strict quasi-concavity of $(p-c)D_1(p, ..., p, a, \gamma)$ and that $p^N(a,x)$ is increasing in x which then implies the integrand of the LHS of (34) exceeds the integrand of the RHS for $\eta > 0$ and close to zero.

In sum, when products are sufficiently differentiated, cost coordination with $x = c + \eta$, where $\eta > 0$ and small, is preferable to price coordination. ∎

Proof of Theorem 5. Let us show $\exists \gamma' < 1$ such that if $\gamma \in (\gamma', 1)$ then $\nexists x > c$ such that (32)-(33) hold. (It will also be true that (32)-(33) does not hold for $x = c$.) To prove this claim, let us suppose the contrary and derive a contradiction. Thus, suppose $\exists x^o(\gamma) > c$ satisfying (32)-(33). Recall that $\widehat{p}$ is the uniform price charged under price coordination (which, by assumption, is independent of γ).

Given $\lim_{\gamma\to 1} p^N(a,x,\gamma) = x\ \forall a$ then, for γ close to one, $p^N(a,x,\gamma)$ is close to a uniform price in that it is in a small neighborhood around x. Given that $\widehat{p}$ is the optimal uniform price then, for (32) to hold when γ is close to one, x must be close to $\widehat{p}$ so that $p^N(a,x,\gamma)$ is close to $\widehat{p}$. We then have: $\lim_{\gamma\to 1} x^o(\gamma) = \widehat{p}$.

Next consider (33). Given $\lim_{\gamma\to 1} p^N(a,c,\gamma) = c$ and it was just shown (32) implies $\lim_{\gamma\to 1} p^N(a,x^o(\gamma),\gamma) = \widehat{p}(> c)$ then $p^N(a,x^o(\gamma),\gamma)$ is bounded above $p^N(a,c,\gamma)$ as $\gamma \to 1$. By assumption, if $p > p^N(a,c,\gamma)$ then $\psi_1(p,...,p,a,c,\gamma) < p$. It then follows:

$$\lim_{\gamma\to 1} p^N(a,x^o(\gamma),\gamma) > \lim_{\gamma\to 1} \psi_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a,c,\gamma\right).$$

Given $\lim_{\gamma\to 1} p^N(a,x,\gamma) = x$ then, as $\gamma \to 1$, $p^N(\underline{a},x,\gamma)$ converges to $p^N(a,x,\gamma)\ \forall a \in (\underline{a},\overline{a}]$ and does so from below (because $p^N(a,x,\gamma)$ is increasing in a). Combining the previous two results: $\exists \gamma' < 1$ such that if $\gamma \in (\gamma',1)$ then

$$p^N(\underline{a},x^o(\gamma),\gamma) \in \left(\psi_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a,c,\gamma\right), p^N(a,x^o(\gamma),\gamma)\right)\ \ \forall a \in (\underline{a},\overline{a}].$$

By strict quasi-concavity of $(p_1 - c)D_1(p_1,...,p_n,a)$ in p_1, the previous condition implies

$$\begin{aligned} & (p^N(\underline{a},x^o(\gamma),\gamma) - c)D_1\left(p^N(\underline{a},x^o(\gamma),\gamma), p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right) \\ > \ & (p^N(a,x^o(\gamma),\gamma) - c)D_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right)\ \ \forall a \in (\underline{a},\overline{a}]. \end{aligned}$$

Taking the integral of each side of the preceding equation, we have:

$$\begin{aligned} & \int (p^N(\underline{a},x^o(\gamma),\gamma) - c)D_1\left(p^N(\underline{a},x^o(\gamma),\gamma), p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right)G'(a)da \\ > \ & \int (p^N(a,x^o(\gamma),\gamma) - c)D_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right)G'(a)da, \end{aligned}$$

which contradicts (33). ■

Proof of Theorem 6. There are two properties associated with convergence in distribution that will be used. First, given $(p^N(a,x) - c)D_1\left(p^N(a,x),...,p^N(a,x),a\right)$ is bounded and continuous in a then

$$\begin{aligned} & \lim_{k\to\infty} \int (p^N(a,x) - c)D_1\left(p^N(a,x),...,p^N(a,x),a\right)G_k'(a)da \\ = \ & (p^N(\widehat{a},x) - c)D_1\left(p^N(\widehat{a},x),...,p^N(\widehat{a},x),\widehat{a}\right). \end{aligned}$$

Second, given $(p-c)D(p,...,p,a)$ is bounded and continuous in a then

$$\lim_{k\to\infty} \int (p-c)D(p,...,p,a)G_k'(a)da = (p-c)D_1(p,...,p,\widehat{a}).$$

Defining

$$\widehat{p}(G_k) \equiv \arg\max \int_{\underline{a}}^{\overline{a}} (p-c)D(p,...,p,a)G_k'(a)da,$$

it follows:

$$\lim_{k\to\infty} \widehat{p}(G_k) = p^M(\widehat{a}).$$

It will be shown: if $\{a_k\}_{k=1}^{\infty}$ converges in distribution to $\widehat{a}$ then $\exists k'$ such that $\nexists x$ satisfying (32)-(33) $\forall k > k'$. To prove it, suppose the contrary - $\exists \{\widehat{x}_k\}_{k=1}^{\infty}$ and k' such that (32)-(33) is satisfied $\forall k > k'$ - and let us derive a contradiction.

(32) is reproduced here:

$$\begin{aligned} &\int (p^N(a,x) - c) D_1\left(p^N(a,x), ..., p^N(a,x), a\right) G_k'(a) da \\ > &\int (\widehat{p}(G_k) - c) D_1\left(\widehat{p}(G_k), ..., \widehat{p}(G_k), a\right) G_k'(a) da, \end{aligned} \tag{35}$$

Considering the RHS of (35), $\{a_k\}_{k=1}^{\infty} \to \widehat{a}$ implies: $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then

$$\begin{aligned} &\int (\widehat{p}(G_k) - c) D_1\left(\widehat{p}(G_k), ..., \widehat{p}(G_k), a\right) G_k'(a) da \\ \in &\left((p^M(\widehat{a}) - c) D_1\left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon, (p^M(\widehat{a}) - c) D_1\left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + \varepsilon\right). \end{aligned} \tag{36}$$

Considering the LHS of (35), $\{a_k\}_{k=1}^{\infty} \to \widehat{a}$ implies: $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then

$$\begin{aligned} &\int (p^N(a,\widehat{x}_k) - c) D_1\left(p^N(a,\widehat{x}_k), ..., p^N(a,\widehat{x}_k), a\right) G'(a) da \\ \in &\left((p^N(\widehat{a},\widehat{x}_k) - c) D_1\left(p^N(\widehat{a},\widehat{x}_k), ..., p^N(\widehat{a},\widehat{x}_k), \widehat{a}\right) - \varepsilon,\right. \\ &\left.(p^N(\widehat{a},\widehat{x}_k) - c) D_1\left(p^N(\widehat{a},\widehat{x}_k), ..., p^N(\widehat{a},\widehat{x}_k), \widehat{a}\right) + \varepsilon\right). \end{aligned} \tag{37}$$

Given (35) is assumed to be satisfied and (36) provides a lower bound on the RHS of (35) and (37) provides an upper bound on the LHS of (35), we then have: $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then

$$(p^N(\widehat{a},\widehat{x}_k) - c) D_1\left(p^N(\widehat{a},\widehat{x}_k), ..., p^N(\widehat{a},\widehat{x}_k), \widehat{a}\right) + \varepsilon > (p^M(\widehat{a}) - c) D_1\left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon$$

or

$$(p^N(\widehat{a},\widehat{x}_k) - c) D_1\left(p^N(\widehat{a},\widehat{x}_k), ..., p^N(\widehat{a},\widehat{x}_k), \widehat{a}\right) + 2\varepsilon > (p^M(\widehat{a}) - c) D_1\left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right). \tag{38}$$

As the RHS of (38) is the unique maximum of $(p - c) D_1(p, ..., p, a)$ then, for (38) to hold $\forall \varepsilon > 0$, it must be true:

$$\lim_{k\to\infty} p^N(\widehat{a},\widehat{x}_k) = p^M(\widehat{a}).$$

Next consider (33) which is reproduced here:

$$\begin{aligned} &\int (p^N(a,x) - c) D_1\left(p^N(a,x), ..., p^N(a,x), a\right) G_k'(a) da \\ \geq &\max_{p_1 \in [p^N(\underline{a},x), p^N(\overline{a},x)]} \int (p_1 - c) D_1\left(p_1, p^N(a,x), ..., p^N(a,x), a\right) G_k'(a) da. \end{aligned} \tag{39}$$

Given it has been shown $\lim_{k\to\infty} p^N(\widehat{a},\widehat{x}_k) = p^M(\widehat{a})$ then $\{a_k\}_{k=1}^{\infty} \to \widehat{a}$ implies: $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then, referring to the LHS of (39),

$$\begin{aligned} &\int (p^N(a,\widehat{x}_k) - c) D_1\left(p^N(a,\widehat{x}_k), ..., p^N(a,\widehat{x}_k), a, \gamma\right) G_k'(a) da \\ \in &\left((p^M(\widehat{a}) - c) D_1\left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon, (p^M(\widehat{a}) - c) D_1\left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + \varepsilon\right); \end{aligned} \tag{40}$$

and, referring to the RHS of (39),

$$\int (p_1 - c) D_1 \left(p_1, p^N(a, \widehat{x}_k), ..., p^N(a, \widehat{x}_k), a, \gamma\right) G'_k(a) da$$
$$\in \left((p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon, (p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + \varepsilon\right) \ \forall p,$$

which implies:

$$\max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} \int (p_1 - c) D_1 \left(p_1, p^N(a, \widehat{x}_k), ..., p^N(a, \widehat{x}_k), a\right) G'(a) da \tag{41}$$
$$\in \Bigg(\max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} (p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon,$$
$$\max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} (p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + \varepsilon \Bigg).$$

It follows from (40)-(41): $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then

$$(p^M(\widehat{a}) - c) D_1 \left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + \varepsilon > \int (p^N(a, \widehat{x}_k) - c) D_1 \left(p^N(a, \widehat{x}_k), ..., p^N(a, \widehat{x}_k), a\right) G'_k(a) da \tag{42}$$

and

$$\max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} \int (p_1 - c) D_1 \left(p_1, p^N(a, \widehat{x}_k), ..., p^N(a, \widehat{x}_k), a\right) G'_k(a) da \tag{43}$$
$$> \max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} (p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon.$$

As the LHS of (42) is greater then the LHS of (39) and the RHS of (43) is less than the RHS of (39), (39) holding $\forall k$ implies: $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then

$$(p^M(\widehat{a}) - c) D_1 \left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + \varepsilon > \max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} (p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) - \varepsilon. \tag{44}$$

Let us show (44) does not hold; that is, $\exists \varepsilon > 0$ such that $\forall k$,

$$\max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} (p_1 - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) > (p^M(\widehat{a}) - c) D_1 \left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) + 2\varepsilon, \tag{45}$$

which will be our contradiction. As $p^N(a, x)$ is increasing in a and $\lim_{k \to \infty} p^N(\widehat{a}, \widehat{x}_k) = p^M(\widehat{a})$ then $\widehat{a} \in (\underline{a}, \overline{a})$ implies $p^M(\widehat{a}) \in \left(p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)\right)$ as $k \to \infty$. Also note that $p^N(\underline{a}, \widehat{x}_k)$ is bounded below $p^M(\widehat{a})$. Hence, for $\eta > 0$ and small, $p^M(\widehat{a}) - \eta \in \left[p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)\right]$ as $k \to \infty$ which implies

$$\max_{p_1 \in [p^N(\underline{a}, \widehat{x}_k), p^N(\overline{a}, \widehat{x}_k)]} (p - c) D_1 \left(p_1, p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right) > (p^M(\widehat{a}) - c) D_1 \left(p^M(\widehat{a}), ..., p^M(\widehat{a}), \widehat{a}\right).$$

Therefore, (45) holds for ε small. ■

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Online Appendix to "Cost Coordination"

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Assuming two single-product firms, a representative agent's utility function is specified to be

$$\theta\left(q_1+q_2\right)-\left(\frac{1}{2}\right)\left(\beta\left(q_1^2+q_2^2\right)+2\gamma q_1 q_2\right),$$

which results in the following firm demand function:

$$D_1\left(p_1,p_2,\theta\right) \tag{1}$$
$$\equiv \begin{cases} \frac{\theta}{\beta}-\left(\frac{1}{\beta}\right)p_1 & \text{if } p_1\leq\left(\frac{1}{\gamma}\right)\left(\beta p_2-(\beta-\gamma)\theta\right) \\ \frac{\theta}{\beta+\gamma}-\left(\frac{\beta}{\beta^2-\gamma^2}\right)p_1+\left(\frac{\gamma}{\beta^2-\gamma^2}\right)p_2 & \text{if } \left(\frac{1}{\gamma}\right)\left(\beta p_2-(\beta-\gamma)\theta\right)\leq p_1\leq\left(\frac{1}{\beta}\right)\left(\gamma p_2+(\beta-\gamma)\theta\right) \\ 0 & \text{if } \left(\frac{1}{\beta}\right)\left(\gamma p_2+(\beta-\gamma)\theta\right)\leq p_1. \end{cases}$$

As a reminder, it is assumed that almost all mass is put on a particular cost level which is denoted c. By continuity, the analysis is conducted assuming mass one is placed on c.

1 Analysis for Section 7.1

In deriving the closed-form solutions of this sub-section, it is presumed that firms' demands are interior for all relevant prices. I will return to this qualification later. Referring to (1), the implication of that presumption is that firm demand is

$$\frac{\theta}{\beta+\gamma}-\left(\frac{\beta}{\beta^2-\gamma^2}\right)p_1+\left(\frac{\gamma}{\beta^2-\gamma^2}\right)p_2.$$

To economize on notation, define

$$a\equiv\frac{\theta}{\beta+\gamma}, b\equiv\left(\frac{\beta}{\beta^2-\gamma^2}\right), d\equiv\left(\frac{\gamma}{\beta^2-\gamma^2}\right) \tag{2}$$

so firm demand is

$$D_1\left(p_1,p_2,a\right)=a-bp_1+dp_2, \tag{3}$$

where $b>d\geq 0$. $a\sim G:[\underline{a},\overline{a}]\rightarrow[0,1]$ and let μ_a and σ_a^2 denote the mean and variance of a, respectively. σ_a^2 measures the degree of market heterogeneity. It can be shown that a stronger market (i.e., a higher value of a) has a more price-inelastic firm demand function.

If firms decentralize pricing and have a common cost report x then the symmetric Nash equilibrium price is

$$p^N(a,x)=\frac{a+bx}{2b-d}. \tag{4}$$

Under competition, the cost report is truthful ($x=c$) and profit is

$$\frac{b\left(\mu_a-(b-d)c\right)^2+b\sigma_a^2}{\left(2b-d\right)^2}. \tag{5}$$

The first-best price under price coordination is

$$\widehat{p}(c)=\frac{\mu_a+(b-d)c}{2(b-d)} \tag{6}$$

and profit is

$$\frac{\left(\mu_a-(b-d)c\right)^2}{4(b-d)}. \tag{7}$$

It is more profitable for firms to centralize and coordinate on a common price (price coordination) than to decentralize and compete if and only if (7) exceeds (5):

$$\frac{\left(\mu_a-(b-d)c\right)^2}{4(b-d)}>\frac{b\left(\mu_a-(b-d)c\right)^2+b\sigma_a^2}{\left(2b-d\right)^2}\Leftrightarrow\sigma_a^2<\frac{d^2\left(\mu_a-(b-d)c\right)^2}{4b\left(b-d\right)}\equiv\omega_2. \tag{8}$$

Thus, if market heterogeneity is not too great then firms prefer collusion with a uniform price to competition with price discrimination.

Turning to cost coordination, the profit from cost report x is

$$\begin{aligned}V\left(x\right)&\equiv\int\left(\frac{a+bx}{2b-d}-c\right)\left(a-(b-d)\left(\frac{a+bx}{2b-d}\right)\right)G'(a)da\\&=\frac{b\left(\mu_a-(b-d)x\right)\left(\mu_a-(b-d)c+b(x-c)\right)}{\left(2b-d\right)^2}+\frac{b\sigma_a^2}{\left(2b-d\right)^2}.\end{aligned}$$

The equilibrium collusive cost report is the solution to the following constrained optimization problem:

$$x^*(c)\equiv\arg\max_{x\in\Omega}\frac{b\left(\mu_a-(b-d)x\right)\left(\mu_a-(b-d)c+b(x-c)\right)}{\left(2b-d\right)^2}+\frac{b\sigma_a^2}{\left(2b-d\right)^2} \tag{9}$$

subject to

$$\frac{b\left(\mu_a-(b-d)x\right)\left(\mu_a-(b-d)c+b(x-c)\right)}{(2b-d)^2}+\frac{b\sigma_a^2}{(2b-d)^2} \tag{10}$$
$$\geq \max_{p_1\in\left[\frac{\underline{a}+bx}{2b-d},\frac{\overline{a}+bx}{2b-d}\right]}(p_1-c)\left(\mu_a-bp_1+d\left(\frac{\mu_a+bx}{2b-d}\right)\right).$$

As a benchmark, the unconstrained optimum of (9) is[1]

$$\widehat{x}(c)\equiv c+\frac{d(\mu_a-(b-d)c)}{2b(b-d)} \tag{11}$$

with a price of

$$\frac{a+b\left(c+\frac{d(\mu_a-(b-d)c)}{2b(b-d)}\right)}{2b-d}=\frac{\mu_a+(b-d)c}{2(b-d)}+\frac{a-\mu_a}{2b-d}.$$

Lemma 1 *If $\sigma_a^2\leq(\mu_a-\underline{a})^2$ and*

$$\sigma_a^2\leq\frac{d^2(2b-d)^2(\mu_a-(b-d)c)^2}{16b^2(b-d)^2}\equiv\omega_1 \tag{12}$$

then the solution to (9)-(10) is

$$x^*(c)=c+\left(\frac{2}{2b-d}\right)\sigma_a. \tag{13}$$

Proof. In solving (9)-(10), the analysis will be conducted supposing x can take any non-negative number. If $x^*(c)\in\Omega$ then that is the solution and if $x^*(c)\notin\Omega$ then it is a close approximation when Ω is sufficiently fine.

The proof strategy is as follows. First, it is shown that if the constraint on the deviation price in the ICC is binding at the optimal solution - that is, the unconstrained optimal deviation price does not lie in $\left[\frac{\underline{a}+b\widetilde{x}(c)}{2b-d},\frac{\overline{a}+b\widetilde{x}(c)}{2b-d}\right]$ - then $\sigma_a^2>(\mu_a-\underline{a})^2$. Hence, if $\sigma_a^2\leq(\mu_a-\underline{a})^2$ then a solution must have the unconstrained optimal deviation price lying in $\left[\frac{\underline{a}+b\widetilde{x}(c)}{2b-d},\frac{\overline{a}+b\widetilde{x}(c)}{2b-d}\right]$. Second, under that assumption, transform the ICC into a constraint on x. Third, necessary and sufficient conditions are derived for the ICC to be binding at the optimal solution. Given the strict quasi-concavity of the objective function, $x^*(c)$ is then the highest value of x satisfying the ICC.

To implement the first step, suppose the optimal deviation price is constrained at the solution to (9)-(10). Maximizing the LHS of (10), the unconstrained optimal deviation price is

$$p_1=\frac{2(\mu_a+bc)+d(x-c)}{2(2b-d)}. \tag{14}$$

Suppose it was to exceed the upper bound to the choice set for p_1:

$$\frac{2(\mu_a+bc)+d(x-c)}{2(2b-d)}>\frac{\overline{a}+bx}{2b-d}.$$

[1] As x must lie in Ω then this requires $c+\frac{d(\mu_a-(b-d)c)}{2b(b-d)}\in\Omega$.

From this condition is derived:

$$\begin{aligned}
\frac{2(\mu_a + bc) + d(x-c)}{2(2b-d)} &> \frac{\overline{a} + bx}{2b-d} \Leftrightarrow 2(\mu_a + bc) + d(x-c) > 2\overline{a} + 2bx \\
2(\mu_a - \overline{a}) + (2b-d)c &> (2b-d)x \Leftrightarrow c - \frac{2(\overline{a} - \mu_a)}{2b-d} > x.
\end{aligned}$$

$x < c$ is inconsistent with a solution to (9)-(10) because it would deliver profit lower than that from $x = c$ which is assured of satisfying the ICC. The relevant constraint is the lower bound to the choice set for p_1, which is violated by the unconstrained optimal deviation price iff

$$\frac{2(\mu_a + bc) + d(x-c)}{2(2b-d)} < \frac{\underline{a} + bx}{2b-d},$$

which gives us this constraint on x:

$$\begin{aligned}
\frac{2(\mu_a + bc) + d(x-c)}{2(2b-d)} &< \frac{\underline{a} + bx}{2b-d} \Leftrightarrow 2(\mu_a + bc) + d(x-c) < 2\underline{a} + 2bx \\
2(\mu_a - \underline{a}) + (2b-d)c &< (2b-d)x
\end{aligned}$$

$$\frac{2(\mu_a - \underline{a})}{2b-d} + c < x. \tag{15}$$

If x satisfies (15) then the (constrained) optimal deviation price is $\frac{\underline{a}+bx}{2b-d}$ and the associated deviation profit is

$$\left(\frac{\underline{a} + bx}{2b-d} - c\right)\left(\mu_a - b\left(\frac{\underline{a} + bx}{2b-d}\right) + d\left(\frac{\mu_a + bx}{2b-d}\right)\right).$$

Consequently, the ICC (10) is

$$\begin{aligned}
&\frac{b\left(\mu_a - (b-d)x\right)\left(\mu_a - (b-d)c + b(x-c)\right)}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2} \\
\geq\; &\left(\frac{\underline{a} + bx}{2b-d} - c\right)\left(\mu_a - b\left(\frac{\underline{a} + bx}{2b-d}\right) + d\left(\frac{\mu_a + bx}{2b-d}\right)\right)
\end{aligned}$$

$$\begin{aligned}
\frac{b\sigma_a^2}{(2b-d)^2} \geq\; &\left(\frac{\underline{a} + bx}{2b-d} - c\right)\left(\mu_a - b\left(\frac{\underline{a} + bx}{2b-d}\right) + d\left(\frac{\mu_a + bx}{2b-d}\right)\right) \\
&- \frac{b\left(\mu_a - (b-d)x\right)\left(\mu_a - (b-d)c + b(x-c)\right)}{(2b-d)^2}
\end{aligned}$$

$$x \leq c + \frac{(\mu_a - \underline{a})^2 + \sigma_a^2}{(\mu_a - \underline{a})(2b-d)}. \tag{16}$$

In sum, a value for x results in the optimal deviation price being constrained and the ICC being satisfied iff (15) and (16) hold:

$$\frac{2(\mu_a - \underline{a})}{2b-d} + c < x \leq c + \frac{(\mu_a - \underline{a})^2 + \sigma_a^2}{(\mu_a - \underline{a})(2b-d)}. \tag{17}$$

A necessary condition for (17) to hold is:

$$\frac{2(\mu_a - \underline{a})}{2b-d} + c < c + \frac{(\mu_a - \underline{a})^2 + \sigma_a^2}{(\mu_a - \underline{a})(2b-d)} \Leftrightarrow 2(\mu_a - \underline{a})^2 < (\mu_a - \underline{a})^2 + \sigma_a^2 \Leftrightarrow (\mu_a - \underline{a})^2 < \sigma_a^2.$$

Thus, if $(\mu_a - \underline{a})^2 \geq \sigma_a^2$ then the optimal deviation price must not be constrained at a solution to (9)-(10); that is, the unconstrained optimal deviation price must lie in $\left[\frac{\underline{a}+b\widetilde{x}(c)}{2b-d}, \frac{\overline{a}+b\widetilde{x}(c)}{2b-d}\right]$. From hereon, this assumption is made.

Let us consider the ICC with the unconstrained optimal deviation price. Evaluating the RHS of (10) at the price in (14), the deviation profit is

$$\frac{b\left(2(\mu_a - (b-d)c) + d(x-c)\right)^2}{4(2b-d)^2}$$

which results in (10) taking the form:

$$\begin{aligned} & \frac{b(\mu_a - (b-d)x)(\mu_a - (b-d)c + b(x-c))}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2} \\ \geq \quad & \frac{b\left(2(\mu_a - (b-d)c) + d(x-c)\right)^2}{4(2b-d)^2} \\ \Leftrightarrow \quad & x \leq c + \left(\frac{2}{2b-d}\right)\sigma_a. \end{aligned}$$

Note that the unconstrained optimal value of x in (11) exceeds the RHS in the preceding condition when:

$$c + \frac{d(\mu_a - (b-d)c)}{2b(b-d)} \geq c + \left(\frac{2}{2b-d}\right)\sigma_a \Leftrightarrow \sigma_a^2 \leq \frac{d^2(2b-d)^2(\mu_a - (b-d)c)^2}{16b^2(b-d)^2} \equiv \omega_1.$$

Under that condition on σ_a^2 and given the strict concavity of $V(x)$, the solution to (9)-(10) is $x^*(c) = c + \left(\frac{2}{2b-d}\right)\sigma_a$.

To verify the conjecture that the optimal deviation price lies in the choice set, we need to show:

$$\frac{2(\mu_a + bc) + d(x-c)}{2(2b-d)} \in \left[\frac{\underline{a} + bx}{2b-d}, \frac{\overline{a} + bx}{2b-d}\right]$$

when $x = x^*(c)$. That condition is equivalent to

$$x \in \left[c + \frac{2(\mu_a - \overline{a})}{2b-d}, c + \frac{2(\mu_a - \underline{a})}{2b-d}\right].$$

Given $x = c + \left(\frac{2}{2b-d}\right)\sigma_a$, we then need

$$\frac{2(\mu_a - \overline{a})}{2b-d} \leq \left(\frac{2}{2b-d}\right)\sigma_a \leq \frac{2(\mu_a - \underline{a})}{2b-d} \Leftrightarrow \mu_a - \overline{a} \leq \sigma_a \leq \mu_a - \underline{a}.$$

Clearly, the LHS inequality holds since $\mu_a - \overline{a} < 0$, and the RHS inequality is equivalent to $\sigma_a^2 \leq (\mu_a - \underline{a})^2$.

In sum, if

$$\sigma_a^2 \le (\mu_a - \underline{a})^2 \text{ and } \sigma_a^2 \le \frac{d^2(2b-d)^2(\mu_a-(b-d)c)^2}{16b^2(b-d)^2}$$

then the cost coordination solution is

$$x^*(c) = c + \left(\frac{2}{2b-d}\right)\sigma_a.$$

■

$\sigma_a^2 \le (\mu_a - \underline{a})^2$ is a relatively weak condition; for example, it strictly holds for any G with a symmetric density. When market heterogeneity is sufficiently low - as specified in (12) - the ICC is binding at the optimal cost report and, consequently, (13) is the highest cost report satisfying (10).

Given cost report (13), the cost coordination price for market type a is

$$\frac{a+bc}{2b-d} + \frac{2b\sigma_a}{(2b-d)^2}. \tag{18}$$

It is straightforward to show, under (12), that average price under cost coordination is lower than the uniform price under price coordination. The associated profit from cost coordination is

$$V\left(c + \left(\frac{2}{2b-d}\right)\sigma_a\right) = \frac{b\left(2b\mu_a + d\sigma_a - d\mu_a - 2b^2c - cd^2 + 3bcd\right)^2}{(2b-d)^4}. \tag{19}$$

Using (19) and (7), cost coordination is more profitable than price coordination iff

$$\frac{b\left(2b\mu_a + d\sigma_a - d\mu - 2b^2c - cd^2 + 3bcd\right)^2}{(2b-d)^4} > \frac{\left(\mu_a - bc + cd\right)^2}{4(b-d)} \Leftrightarrow$$

$$\sigma_a^2 > \frac{(2b-d)^2(\mu_a-(b-d)c)^2\left((2b-d)-\sqrt{4b(b-d)}\right)^2}{4b(b-d)d^2} \equiv \omega_3. \tag{20}$$

To summarize, the solution to (9)-(10) is (13) when $\sigma_a^2 \le (\mu_a - \underline{a})^2$ and $\sigma_a^2 \le \omega_1$. Price coordination is more profitable than competition iff $\sigma_a^2 < \omega_2$, and price coordination is more profitable than cost coordination iff $\sigma_a^2 < \omega_3$.

Lemma 2 *If $d > 0$ then $\omega_1 > \omega_2 > \omega_3$.*

Proof. We have that $\omega_1 > \omega_2$,

$$\omega_1 \equiv \frac{d^2(2b-d)^2(\mu_a-(b-d)c)^2}{16b^2(b-d)^2} > \frac{d^2\left(\mu_a-(b-d)c\right)^2}{4b\,(b-d)} \equiv \omega_2$$

$$(2b-d)^2 > 4b(b-d) \Leftrightarrow d^2 > 0;$$

and $\omega_2 > \omega_3$,

$$\omega_2 \equiv \frac{d^2\left(\mu_a-(b-d)c\right)^2}{4b\,(b-d)} > \frac{(2b-d)^2(\mu_a-(b-d)c)^2\left((2b-d)-\sqrt{4b(b-d)}\right)^2}{4b(b-d)d^2} \equiv \omega_3$$

$$\begin{aligned}
d^4 &> (2b-d)^2\left((2b-d)-\sqrt{4b(b-d)}\right)^2 \\
d^2 &> 4b^2-4bd+d^2-(2b-d)\sqrt{4b(b-d)} \\
(2b-d)\sqrt{4b(b-d)} &> 4b(b-d) \Leftrightarrow (2b-d)^2 > 4b(b-d) \\
4b^2-4bd+d^2 &> 4b^2-4bd \Leftrightarrow d^2 > 0
\end{aligned}$$

■

2 Analysis for Section 7.2

Using the expressions in Section 7.1 and substituting with (2), price under price coordination is:

$$\widehat{p} = \frac{\mu_a + (b-d)c}{2(b-d)} = \frac{\frac{\mu_\theta}{\beta+\gamma} + \left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)c}{2\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)} = \frac{\frac{\mu_\theta}{\beta+\gamma} + \left(\frac{1}{\beta+\gamma}\right)c}{2\left(\frac{1}{\beta+\gamma}\right)} = \frac{\mu_\theta + c}{2}.$$

Parameterizations are considered such that firm demand is positive for all market types:

$$\frac{\underline{\theta}}{\beta+\gamma} - \left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)\left(\frac{\mu_\theta + c}{2}\right) > 0 \Leftrightarrow \underline{\theta} - \frac{\mu_\theta + c}{2} > 0.$$

In that case, profit is

$$\frac{(\mu_a - (b-d)c)^2}{4(b-d)} = \frac{\left(\frac{\mu_\theta}{\beta+\gamma} - \left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)c\right)^2}{4\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)} = \frac{(\mu_\theta - c)^2}{4(\beta+\gamma)} \equiv V^{pc}. \tag{21}$$

The symmetric Nash equilibrium price is

$$p^N(a,x) = \frac{a+bx}{2b-d} = \frac{\frac{\mu_\theta}{\beta+\gamma} + \frac{\beta}{\beta^2-\gamma^2}x}{\frac{2\beta-\gamma}{\beta^2-\gamma^2}} = \frac{(\beta-\gamma)\theta + \beta x}{2\beta-\gamma}$$

and demand is positive if and only if

$$\frac{\theta}{\beta+\gamma} - \left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)\left(\frac{(\beta-\gamma)\theta + \beta x}{2\beta-\gamma}\right) > 0\ \forall\theta \Leftrightarrow \left(\frac{\beta(\theta - x)}{2\beta-\gamma}\right) > 0\ \forall\theta \Leftrightarrow \underline{\theta} > x.$$

If $\underline{\theta} > c$ then profit under competition ($x = c$) is

$$V^{comp} \equiv \left(\frac{\beta(\beta-\gamma)}{(\beta+\gamma)(2\beta-\gamma)^2}\right)\left((\mu_\theta - c)^2 + (\beta+\gamma)^2\sigma_\theta^2\right). \tag{22}$$

Price coordination is more profitable than competition iff

$$V^{pc} > V^{comp} \Leftrightarrow \sigma_\theta^2 < \frac{(\mu_\theta - c)^2\gamma^2}{4\beta(\beta-\gamma)(\beta+\gamma)^2}.$$

Turning to cost coordination, if $\underline{\theta} > x$ then the profit from cost coordination is

$$\frac{\beta\left(\mu_\theta - x\right)\left((\beta-\gamma)(\mu_\theta - c) + \beta(x-c)\right) + \beta\left(\beta-\gamma\right)\sigma_\theta^2}{\left(2\beta-\gamma\right)^2\left(\beta+\gamma\right)}. \tag{23}$$

Substituting (13) and $\sigma_\theta^2 = r^2/12$ in (23), the profit from cost coordination is:

$$V^{cc} \equiv \frac{\beta\left(\mu_\theta - \widetilde{x}\right)\left((\beta-\gamma)(\mu_\theta - c) + \beta(\widetilde{x}-c)\right) + \beta\left(\beta-\gamma\right)\left(r^2/12\right)}{\left(2\beta-\gamma\right)^2\left(\beta+\gamma\right)}. \tag{24}$$

Cost coordination is more (less) profitable than price coordination when $V^{cc} > (<) V^{pc}$.