Uncertainty, firm life-cycle growth, and aggregate productivity

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Abstract

This paper studies how changes in firm-level uncertainty affect firm owners' incentive to invest in productivity, and, thereby, impact firm life-cycle growth, aggregate productivity and aggregate output. Using a version of a dynamic general equilibrium Beweley-Aiyagari-Hugget model, it finds that an increase in riskiness of returns to productivity investment translating into a one percentage point increase in the job turnover rate reduces average firm productivity by about 0.92 percent, aggregate output by 0.73 percent, the measured TFP by 0.55 percent, and the firm growth rate over 20 years by 5.5 percent. These effects are considerably magnified if the firms have limited excess to external capital financing. These results suggest that empirically-relevant changes in uncertainty associated with returns to investment in intangible capital can be an important factor in understanding the differences in cross-country income and firm life-cycle growth.

Keywords: innovative investment, firm growth, volatility

JEL codes: E22, L11, L26, D14, D24, O16, O3

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1 Introduction

Investment in intangible capital are undoubtedly an important engine of firm productivity growth. According to various estimates, the share of such investment in output ranges from 6 to 15 percent.[1] Naturally, various features of the economic environment and policies affects firms' incentives to invest in intangible capital and, consequently, have aggregate implications for overall productivity, output, and the allocation of resources. This paper draws attention to one such feature – the extent of uncertainty in returns to innovative investment. Intuitively, if the firm owners are risk averse, either because ownership is concentrated in the hands of a few individuals, or because the firms face financing constraints, higher uncertainty discourages firms from investing in intangible capital. At the microeconomic level, this may result in slower firm growth over time. At the macroeconomic level, this could lead to lower productivity and per capita output. The objective of this paper is to quantify these effects of uncertainty in returns to intangible investment on the firm life-cycle productivity growth and aggregate economic activity.

Uncertainty in returns to investment in intangible capital may come from many different sources. It could arise because firms pursue riskier activities, or it can stem from the higher inherent risk in the environment in which the firms operate. Some examples of the latter include policy uncertainty, poor property rights protections (e.g., firms expropriating each other's innovations or newly introduced products), or weak law enforcement (e.g., ad hoc application of tax and business regulations, subjective permit issuance, etc.). Empirically, firm-level uncertainty translates into volatility of firm growth rates, as well as job turnover rates. Existing empirical studies have documented that economic development is associated with lower firm-level volatility (Moscoso-Boedo (2018)) and lower levels of job flows (e.g., Haltiwanger et al. (2006) and Donovan et al. (2022)).[2] At the same time, it has been widely documented that firms in developing countries grow slower over the life-cycle (e.g., Hsieh and Klenow (2014)), have less excess to external financing, and invest less in intangible capital. This paper argues that firms' idiosyncratic risk and excess to external financing are important determinants of their investment in intangible capital and the resulting life-cycle growth, and attempts to assess this mechanism quantitatively.

To study the relationship between firm-level uncertainty, firm productivity growth, and ag-

[1] See, for example, McGrattan and Prescott (2010) or Corrado et al. (2018).

[2] The latter relationship is not present when comparing the U.S. and Western Europe, which is typically attributed to more prominent labor market protection policies in many European countries.

gregate economic activity, I use a variation of a Bewley-Aiyagari-Huggett[3] model with uninsured labor and production risks, in which, in addition to providing labor services and saving in a risk-free bond, risk-averse individuals may also operate productive ideas. These ideas arrive and die exogenously[4], but their evolution is affected by the firm owner's innovative investment that is subject to the idiosyncratic firm-level shocks. An increase in the risk associated with innovative investment discourages firm owners from making productivity investment, which, in turn, leads to slower firm growth over time, and reduces aggregate productivity. In contrast, if in this model the firm productivity were to follow an exogenous process (as, for example, it is standard in firm-dynamics models based on Hopenhayn (1992) or Lucas (1978)), mean-preserving spread in the firm-specific productivity shocks would have no impact on the aggregate productivity and output.

The model is parameterized to match the key features of the firm dynamics, income distribution, and wealth distribution. In particular, the parameters of the function describing how firms' innovative investments translate into firm productivity growth over time are pinned down by the features of the firm life-cycle growth profile, the aggregate job destruction rate, and the share of intangible capital investment in output. Then the model is used to analyze how aggregate productivity and firm life-cycle growth are impacted by the amount of uncertainty embedded in the technology describing the returns to intangible investment. Specifically, I vary the parameters of the distribution of shocks affecting the returns to innovative investment, while keeping the mean of these shocks constant, recompute the stationary equilibrium outcome in the economy, and analyze how the key aggregate indicators vary with the job destruction rate which, as mentioned above, is one of plausible measures of firm-level uncertainty. Under a conservative calibration strategy, the model predicts that a one percentage point increase in the job destruction rate (e.g., from 10 to 11 percent) is associated with about a 0.92 percent drop in average firm productivity, a 0.73 percent drop in aggregate output and capital, and a 0.55 percent drop in the measured TFP. These effects imply that an increase in firm-level uncertainty resulting in the job destruction rate three times as high as in the benchmark model (Donovan et al. (2019) find that job flows

[3]See Aiyagari (1994) and Hugget (1993).

[4]Abstracting from endogenous entry / exit allows to cleanly zoom in on the main mechanism studied in the paper. In this sense, the paper departs from dynamic occupational choice models combining Lucas (1978) and Hopenhayn (1992) analyzing the effects of borrowing constraints (e.g., Quadrini (2000), Cagetti and De Nardi (2006), Buera et al. (2011) and references therein), tax distortions (Guner et al. (2008)) or skill-biased technical change (Poschke (2018). At the same time, examples of the papers abstracting from selection through entry and exit to highlight the main driving forces are abundant, e.g. Angeletos (2007), Caggese (2012), or Vereshchagina (2022).

in poor countries are 2-5 times higher than in the U.S.) reduces the average firm productivity by about 18.2 percent, aggregate output by 14.6 percent and the measured TFP by 11 percent. At the same time, while these effects are rather modest, the impact on the firm life-time growth is more significant: Each percentage point increase in job destruction rate shaves about 5.5 percentage points off the firm growth over the first 20 years of life; so an increase in the job destruction rate from 0.1 to 0.3 is associated with the drop in employment growth over 20 years from 300 percent (i.e., four times increase in employment) to 180 percent (i.e., 2.8 times increase in employment). This suggests that variations in the degree of uncertainty across countries may be an important contributor to differences in firms' life cycle profiles documented, for example, by Hsieh and Klenow (2014).

In addition, I also demonstrate that the effects of uncertainty are considerably amplified by the presence of firm financing constraints – a friction commonly associated with poor economic development. Intuitively, if firm owners have limited excess to external financing, they might not be able to finance the firm efficiently if a high productivity shock arrives; this implies that mean-preserving spreads in productivity shocks might be associated with a decline in expected payoff, which has an additional adverse effect on firm owners' incentives to make innovative investment. In the model, an introduction of a firm financing constraint which reduces the amount of externally financed operational capital from 74 percent to 17 percent and lowers the capital-output ratio by 46 percent,[5] magnifies the effects of higher uncertainty on key aggregate variables up to three times compared to the benchmark: a mean-preserving spread in productivity shocks which increases the job destruction rate from 0.1 to 0.3, results in an 18-38 percent drop in aggregate productivity, a 20-50 percent drop in output, a 25-35 percent drop in measured TFP,[6] and shaves off $70-90\%$ of firm growth over 20 years.

This paper contributes to three strands of literature. First, it is related to a large and growing literature studying the negative effects of uncertainty on economic activity. For example, a series of papers emphasized the impacts of uncertainty on firm's tangible investment in the presence of adjustment costs (e.g., Bloom (2009) and Fernandez-Villaverde et al. (2015)) or costly

[5]For comparison, low-income countries (according to the World Bank classification) have the private credit-to-GDP ratio of 16 percent, and their capital-to-output ratio is about 45 percent lower than the capital-to-output ratio in the high income countries. See Qian and Vereshchagina (2022) for more details.

[6]In the presence of the borrowing constraint, the effects depend on how skewed the mean-preserving spread is, so I report the range for each effect.

bankruptcy (Arellano et. al. 2018). Others emphasize the role of one-time uncertainty shocks on investment (e.g., Stokey (2016) or Julio and Yook (2012)). Differently from these paper, this work focuses on the effect on intangible investment, and relates these effects to the cross-country differences on firm life-cycle growth and productivity. On the empirical side, Michelacci and Schivardi (2013), using the stock market data for 42 countries, demonstrate that countries with low level of diversification opportunities perform relatively worse in sectors characterized by high idiosyncratic risk, and Caggese (2012), using a panel of Italian manufacturing firms, shows that an increase in uncertainty has a large negative effect on risky innovation of entrepreneurial firms. Caggese (2012) also argues that such empirical findings would be consistent with the predictions of a model in which risk-averse entrepreneurs decide whether to invest in risky innovation. While the model in this paper zooms in on a mechanism similar to the one in Caggese (2012), there are several important differences. First, the model in that paper is a partial equilibrium model, while I account for the general equilibrium effects, and show that they play an important role. Second, Caggese (2012) models innovation process very differently: the firm owners first decide whether to invest in innovation which delivers certain level of productivity in the future, and then an ex post shock to the cost of investment is realized; this means that all the costs of uncertainty must be borne in the current period and cannot be smoothed over time, which makes uncertainty very expensive. In contrast, in this paper, the cost of innovation decision is deterministic while returns on the investment are stochastic, consistently with the empirical findings studying the evolution of firm productivity (see, for example, Doraszelski and Jaumandreu (2013) and references therein), which allows firm owners to smooth the adverse effects of uncertainty in productivity over time. Finally, and most importantly, while Caggese (2012) provides an illustrative quantitative example, this paper attempts to quantify the effects of uncertainty on firm life cycle growth and aggregate variables by using the data on firm dynamics, income distribution and wealth distribution to discipline the parameters of the model.

Second, this work contributes to the literature on cross-country differences in TFP and GDP. It has been well-recognized that various features of economic environments impacting firm-level decisions, for example higher start-up costs (Barseghyan and DiCecio 2011), financing constraints (e.g., Levine (2005) and references therein, Buera et. al. 2011, Midrigan and Xu 2014 or Cole et. al. 2016) or various firm-level distortions (e.g., Restuccia and Rogerson (2008), Hsieh and Klenow (2009), and many follow-up papers), may have negative effects on the allocation of resources. To my knowledge, the role of firm-level uncertainty in reducing the aggregate productivity and GDP, via its effect on intangible investment, has not been emphasized in the existing literature.

Third, large and growing misallocation literature has emphasized the importance of accounting for firm endogenous productivity growth / investment in intangible capital for understanding the aggregate patterns (McGrattan and Prescott (2010)), the effects of various policy distortion (e.g., size-related taxes in Bhattacharyaa et al. (2013) or Bento and Restuccia (2017)), or borrowing constraints (e.g., Vereshchagina (2022)). However, the relationship between the incentives to invest in productivity / intangible capital and the level of firm-level uncertainty has not been discussed in any of these papers.

The rest of the paper is organized as follows. Secion 2 discusses existing empirical evidence documenting the cross-country differences in firm-level volatility. Section 3 sets up the model, Section 4 describes how the benchmark model is parameterized, Section 5 summarizes the main numerical results, and Section 6 outlines the next steps in this research project.

2 Motivating Evidence: Firm-level risk across countries

Firms in poorer countries operate in business environments characterized by weaker institutions and less predictability. For example, the data collected by World Bank Enterprise Survey[7] indicate that firms in poorer countries pay bribes more frequently, are more likely to view corruption or permit issuance as major obstacles, and are more concerned with political instability. All these factors create additional risks for firms. However, the extent of these specific risks, as well as their implications for firms' growth, is hard to quantify.

To quantitatively assess differences in firm-level risk across countries, one needs to utilize firm-level cross-country panel data, which is very limited. Bureau van Dijk's (BvD) ORBIS database for 2005-2011 is one such data source. Moscoso-Boedo (2018) analyzes this firm-level panel data from 2005-2011. Specifically, following the approach of Castro et al. (2009), it

[7]World Bank Enterprise Survey is a publicly available survey of more than 150 countries in which firms are surveyed during 2009-2015. See https://www.enterprisesurveys.org/en/enterprisesurveys for more details.

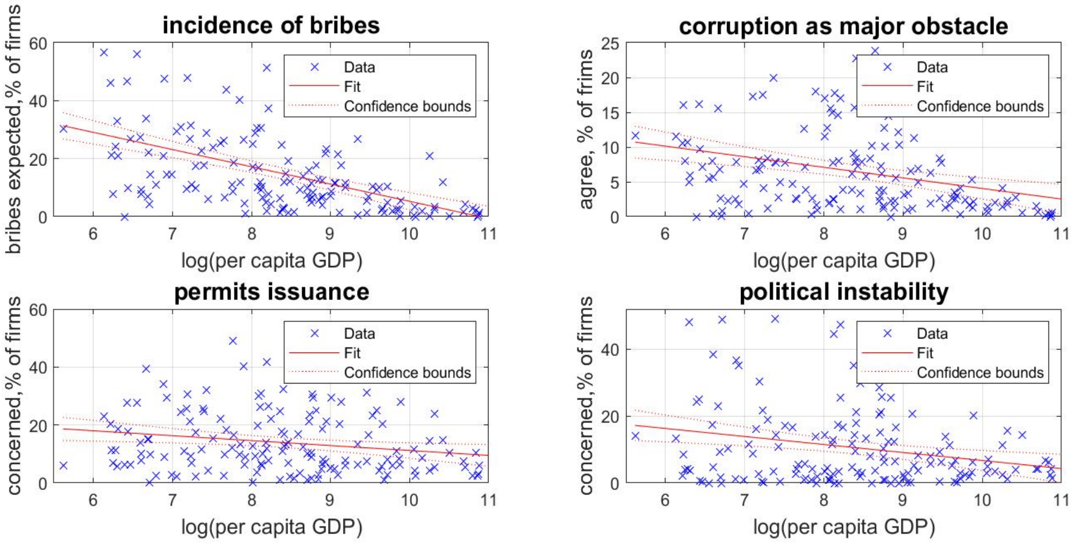

Figure 1: Percentage of firms reporting that they expect to pay bribes, view corruption as a major obstacle, are concerned about getting a permit, and are concerned about political instability vs. per capita GDP in the country. *Source:* World Bank Enterprise Survey.

estimates the regression

$$\Delta ln(sales_{ijct}) = \mu_i + \delta_{jt} + \gamma_{ct} + \beta_{1j} \ln(size_{ijct}) + \epsilon_{ijct},$$

where $\Delta \ln(sales_{ijct})$ is the growth of real sales for firm i, in industry j, in country c between period t and period $t+1$. The variable μ_i, δ_{jt} and γ_{ct} are the firm-specific, time- and industry-specific, and time- and country-specific fixed effects, respectively. It then reports the values for the cross-sectional standard deviation of ϵ_{ijct} country by country, for more than fifty countries. I combine these estimates with the real per capita GDP from the Penn World Tables 8.0 to produce Figure 2. As can be seen, there is a negative correlation between the measure of firm-level uncertainty reported by Moscoso-Boedo (2018) and per capita GDP across countries.[8]

Another statistics related to firm-level idiosyncratic volatility is the measure of job flows. In particular, in macroeconomic models of firm dynamics (e.g., Lucas (1978), Hopenhayn (1992) or Hopenhayn and Rogerson (1993)), reallocation of workers between the firms is driven by the

[8]The correlation coefficient between the two variables reported in Figure 2 is -0.48, implying that one percent increase in GDP per capita is associated with almost half a percent decline in the mean standard deviation of ϵ_{ijct}. Moscoso-Boedo (2018) also finds a statistically significant negative relationship between the values of the cross-sectional standard deviation of ϵ_{ijct} and measured TFP across countries.

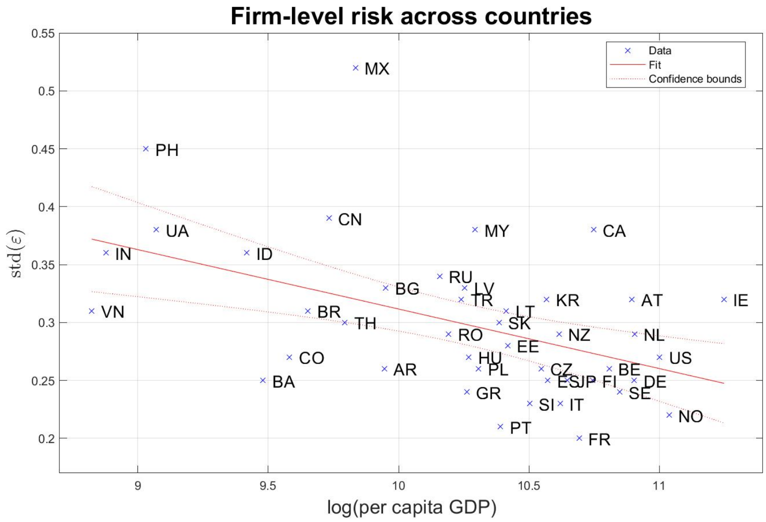

Figure 2: Cross-sectional standard deviation of ϵ_{ijct} (Moscoso-Boedo (2018)) and natural log of real per capita GDP across countries (Penn World Table 8.0).

evolution of firm-level productivity shocks. Several studies document that job flows are higher in developing countries than in the developed ones. For instance, Donovan et al. (2022) use the rotating panel labor surveys of 49 countries around the world and find that job flows are negatively correlated with development. In particular, they document that employment-exit rates are two to three times higher, and job-job and occupational switching rates are five times higher in poorer countries than in richer countries. In another study, Haltiwanger et al. (2006) document that the job flows in the Latin American countries and Eastern European transition economies are 30-70 percent higher than in the OECD countries.

These observations are consistent with higher levels of firm idiosyncratic risk in developing countries. Such higher risk levels could be attributed to poor property rights protection, ad hoc application of tax rules and various regulations, limited access to insurance, corruption, etc. In addition, it is well documented that firm ownership in developing economies is poorly diversified and firms have less access to external financing, which increases the relevance of firm exposure to idiosyncratic risk. In what follows, I develop a firm dynamics model to quantify the impact of firm-level risk on firms' incentives to invest in intangible capital, and, via it, firm life-cycle growth, as well as aggregate productivity and output.

3 The model setup

The modeling environment is a variation of the Bewley-Aiyagari-Hugget[9] model with production risk. The economy is populated by a continuum of agents of mass one who receive stochastic labor productivity shocks, supply labor to the competitive labor market, and can save, subject to a borrowing constraint, in a risk-free asset traded on a competitive market. In addition, with some probability, agents receive productive ideas and can operate them, thereby becoming firm owners. Productive ideas generate output using labor and capital inputs, and the evolution of their productivity over time is impacted by the firm owner's innovative investment. Such investment is, however, risky, and the associated level of risk is exogenously given.[10] The objective of the analysis is to study how the changes in the exogenous variance of returns to innovative investment affect firm owners' innovative investment, firm life-cycle growth, and aggregate productivity.

There is a continuum of consumers of mass one, with preferences

$$E\sum_{t=0}^{\infty}\beta^t u(c_t), \quad 0<\beta<1,$$

where $u(\cdot)$ is a strictly concave, strictly increasing utility function on $(0,+\infty)$ which satisfies the Inada conditions and c_t is the level of consumption in period t. In the beginning of period t, a consumer has total assets a_t, and may have a productive idea of quality z_t which generates proprietary income $\pi(a_t, z_t)$ specified below. If the agent has no idea in the beginning of the period, with probability η he may draw one from the distribution $G(z)$, i.i.d. across agents and over time. Existing ideas disappear in the following period with probability q, i.i.d across agents and over time. Additionally, in every period, each consumer receives labor productivity shock ξ_t and earns income $w\xi_t$ by supplying labor to the competitive labor market, where w is the wage per unit of effective labor. The analysis focuses on a stationary equilibrium, so all prices are constant. The shock ξ_t is assumed to be i.i.d. across agents and over time and is drawn from the distribution $F_W(\xi)$.[11] After the income in the current period is realized, a consumer chooses how

[9]See Aiyagari (1994) and Hugget (1993).

[10]Vereshchagina and Hopenhayn (2009) analyze firm's risk choice in the Lucas span-of-control model.

[11]As explained below, the main objective of introducing the labor productivity shock into the model is to avoid high concentration at the bottom of the wealth distribution. The assumption that the labor productivity shocks are i.i.d. is obviously unrealistic, and the model will not be able to generate the correlation between labor earnings and wealth consistent with the data. However, such relationship is outside the scope of the model, since the focus is

much to consume, how much to save in a risk-free asset that generates the rate of return r, subject to the borrowing constraint $a_{t+1} \geq \underline{a}$, and, if the agent has a productive idea, how much to invest in firm productivity growth. In order to continue operating the idea, the firm owner must pay a fixed overhead cost ϕ.

A consumer operating an idea z_t has access to production technology

$$y(z_t, k_t, n_t) = z_t^{1-\gamma} \left(k_t^{\alpha} n_t^{1-\alpha}\right)^{\gamma}, \quad \gamma, \alpha \in (0,1) \tag{1}$$

where k_t and n_t are capital and labor inputs, respectively. The capital used in the firm may be subject to a capital financing constraint, $k_t \leq \overline{k}(a_t, z_t)$. Thus, the proprietary income generated by a firm of type z_t is

$$\begin{aligned} \pi(z_t, a_t) = \max_{k_t, n_t} \quad & z_t^{1-\gamma} \left(k_t^{\alpha} n_t^{1-\alpha}\right)^{\gamma} - w n_t - (r+\delta) k_t \\ \text{s.t.} \quad & k_t \leq \overline{k}(a_t, z_t) \end{aligned} \tag{2}$$

If no financing constraint is imposed, the proprietary income does not depend on the agent's assets a_t. The model allows for the presence of the financing constraint because such constraints may affect the firm owners' risk preferences and influence their incentives to invest in firm productivity growth. Thus, the model could also be used to study the effects of financial frictions and idiosyncratic uncertainty, as well as their combination, on the firm productivity growth over time.

The evolution of firm productivity process is endogenous. The productivity z_{t+1} in the next period depends on the productivity z_t in the current period, and on innovative investment x_t:

$$z_{t+1} = g(z_t, x_t) \cdot \varepsilon_{t+1}, \tag{3}$$

where ε_{t+1} is an i.i.d. shock drawn from the distribution $F_{\varepsilon}(\cdot)$ defined on $(0, +\infty)$. Without loss of generality, it is assumed that $\mathbb{E}\left\{\varepsilon_{t+1}\right\} = 1/(1-q)$. The function $g(z_t, x_t)$ is increasing and

on the evolution of the firm productivity levels. Adding persistency to the labor productivity would considerably complicate the computation of the model because it would require introducing an additional state variable. Thus, to save on computational time, I impose a simplifying assumption that the labor productivity shocks are independent over time.

concave in each argument. In particular, in what follows, I assume that

$$g(z_t, x_t) = (1-\delta_z)z_t + Bz_t^{\theta}x_t^{1-\theta} = \left(1-\delta_z + B\left(\frac{x_t}{z_t}\right)^{1-\theta}\right)\cdot z_t, \tag{4}$$

where $\theta \in [0,1]$. This functional form is borrowed from Bhattacharyaa et al. (2013). If no investment in intangible capital is made, the quality of the idea, on average, depreciates at rate δ_z. As seen from the second term in parenthesis, the expected rate of growth of firm productivity increases with x_t at a decreasing rate, but is smaller for higher z_t, encompassing the idea that bigger firms should invest more in intangible capital to achieve a particular rate of growth.[12] Parameters δ_z, B and θ affect how much the firms grow over time, how much innovative investment the firms needs to make to achieve a certain level of productivity growth, and how the marginal benefit from productivity investment varies with firm size, while the variance of ε_t impacts the volatility of firm growth rates as well as the job turnover rate in equilibrium. This determines the natural empirical moments used to calibrate these parameters.

Note also that the law of motion for firm productivity defined by (3)-(4) is consistent with empirical findings. For example, Doraszelski and Jaumandreu (2013) report that the evolution of firm productivity is subject to the high degree of uncertainty and that the average log of next-period productivity is a non-separable function of the log of current productivity and the log of measured R&D investment. They also use and estimate the innovation production function that, like (3), is multiplicatively separable in $g(z_t, x_t)$ and shock ε_{t+1}. Note also that, with $\theta = 0$, (4) becomes a familiar law of motion of productivity from the knowledge capital model used in many papers dating back to Griliches (1979). While (4) allows for $\theta = 0$, it also accommodates $\theta > 0$, which is consistent with the aforementioned non-separability reported by Doraszelski and Jaumandreu (2013).

The total disposable income of a firm owner who operates the idea of quality z_t, invests x_t in intangible assets, and experiences a labor productivity shock ξ_t is

$$\pi(z_t, a_t) - x_t + w\xi_t. \tag{5}$$

[12]Law of motion (4) implies that if the firm's profit is linear in z_t and the firm owners are risk-neutral, the optimal growth rate is independent of firm size. With concave utility, the growth rate might vary with z_t or the firm owner's assets.

The assumptions that the firm owners do not need to forego wage income and have the option to discontinue the firm without paying the overhead costs ensures that all the new productive ideas are started, though some of them may be discontinued at the later stage. This distinguishes this modeling environment from the span-of-control model in Lucas (1978), where agents choose between operating their productive ideas and being workers, and only the most productive ones become firm owners. Abstracting from the selection channel allows to isolate the effect of uncertainty on the innovative investment of firms incurred during their lifetime. Potentially, it would also be interesting to analyze how uncertainty at the moment of firm origination can affect the decisions to start a firm and the choice of technology at the moment of origination, but such questions are outside the scope of this paper.

Now let us set up an agents' decision problems recursively. Since the analysis focuses on a stationary equilibrium, prices (w, r) are omitted from the set of the state variables. Let a be the agent's asset level in the beginning of the period. Denote by $V_W(a, \xi)$ the expected value of the agent pursuing no productive ideas in the current period and by $V_E(a, z, \xi)$ the expected value of the agent operating an idea of quality z in the current period, both conditional on the agent drawing productivity shock ξ in the current period. It is also convenient to introduce an auxiliary value function, $V_0(a)$, denoting the value of the agent who has no idea in the beginning of the period but still has a chance to draw one. Such value is determined as

$$V_0(a) = \int \left\{(1-\eta)V_W(a,\xi) + \eta \int V_E(a,z,\xi)dG(z)\right\} dF_W(\xi). \tag{6}$$

Note that, in general, the agent could also decide not to pursue an idea if he draws one. Formally, this would be described by including $V_W(a, \xi)$ as an additional options under the max operator in the second term. However, since, as discussed above, $\pi(a, z) \geq 0$ all new ideas will be pursued, and, therefore, the choice between remaining a worker or continuing as firm owner can be omitted. The expected value of the worker $V_W(a, \xi)$ is given by

$$V_W(a,\xi) = \max_{a' \geq \underline{a}}\{u(w\xi + (1+r)a - a') + \beta V_0(a'), \tag{7}$$

since the worker receives labor income, makes the savings decision and starts the next period without a productive idea. The expected values of the firm owner is defined as

$$V_E(a,z,\xi) = \max_{a' \geq \underline{a}, x \geq 0} \Big\{ u(w\xi + \pi(a,z) + (1+r)a - a' - x) \\ + \beta \left(qV_0(a') + (1-q) \int V_E(a', z'(z,x,\varepsilon')) dF_\varepsilon(\varepsilon') \right) \Big\}, \tag{8}$$

where $z'(z,x,\varepsilon')$ is specified by (3)-(4). The firm owner receives proprietary and employment income, makes a savings decision and innovative investment, and, in the next period, loses the idea with probability q or continues with probability $1-q$, in which case he receives the return-to-innovation shock ε'.

The optimal policies in (6)-(8), together with the evolution of idea and labor productivity shocks, induce a stationary distribution over agents' asset levels, idea ownership and idea quality. Denote by $\mu_W(a)$ and $\mu_E(a,z)$ the stationary distributions across assets for the workers, and stationary distribution across asset and idea productivitites the firm owners. Then, in a stationary competitive equilibrium, prices w and r clear the labor and capital market, respectively:

$$\int n(a,z)\, d\mu_E(a,z) = 1 \tag{9}$$

$$\int k(a,z)\, d\mu_M(a,z) = \int a\, d\mu_W(a) + \int a\, d\mu_E(a,z), \tag{10}$$

where $n(a,z)$ and $k(a,z)$ represent the labor and capital demand of the firm of quality z whose owner has asset level a. In the benchmark unconstrained economy, these quantities depend only on firm productivity z, but in the constrained economy they may vary with the assets of the firm owner.

The objective of the analysis is to study the implications of the changes in the degree of uncertainty in returns to innovative investment. For this purpose, I analyze how the individual decisions and the aggregate outcomes respond to mean-preserving changes in the distribution $F_\varepsilon(\cdot)$. A standard recursive argument implies that the value of the firm owner $V_E(a,z)$ is concave not only in wealth a but also in the firm's productivity z as long as $\pi(a,z)$ and $g(z,x)$ defined in (2) and (4), respectively, are weakly concave. This suggests that an increase in uncertainty respect to returns on innovative investment would reduce incentives to undertake such a risky activity, and result in lower firm productivity growth over time. In what follows, I quantify the magnitude of this effect.

4 Calibration

The model is parameterized to match the relevant features of the U.S. firm dynamics, earnings distribution, and wealth distribution in the steady state equilibrium allocation without any firm financing constraints. The parameters are chosen in such a way that the labor and capital markets clear at the normalized wage $w = 1$ and the annual interest rate $r = 0.03$. There are fours sets of parameters to be calibrated – the parameters affecting preferences, labor productivity, production technology, as well as arrival and evolution of productive ideas. Tables 4 and 4 list all the parameters, their values in the benchmark specification, as well as the calibration targets.

Table 4 summarizes the parameter values that are set outside the model. It is assumed that the consumers have CRRA utility function, $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The risk aversion parameter is $\sigma = 2$ in the benchmark case; the comparative statics with respect to this parameter will be performed later since consumers' risk preferences play key role in quantifying the effects of uncertainty. The borrowing limit $\underline{a}$ is set to be equal to the mean annual income of the workers, which is normalized to 1. Turning to the firm evolution and technology, the capital depreciation rate is set to $\delta = 0.06$ following Hugget (1996). The parameter α in the production technology (1) is set to match the share of capital to labor cost ratio.

The probability η of getting productive ideas by the agents who start a period without such an idea is pinned down by the average firm size and the firm exit rate. According to the Business Dynamics Statistics (BDS) data for 1998-2006, the average annual firm exit rate is 7.38% and the average firm size measured by the number of employees is 22.7.[13] Denote by m the mass of firm owners operating ideas in the stationary equilibrium. Since all the agents (with a total mass of one) provide labor services, the average firm size is $1/m$, implying that, to match the average size of 22.7 employees, $m = 1/22.7$ must hold. The stationarity condition on the number of active firm owners stipulates that

$$(1 - m + m \cdot q) \cdot \eta = m \cdot q,$$

[13]BDS is publicly available at https://www.census.gov/ces/dataproducts/bds/data.html. I use the averages across 1998-2006 to construct all the relevant statistics. See Appendix for details.

Table 1: Parameters set outside of the model or calibrated individually

Parameter	Value	Target and/or Source
σ , risk aversion	2	standard value
$\underline{a}$, borrowing limit	1	mean workers's annual income
δ, capital depreciation rate	0.06	Hugget (1996)
α, capital vs. labor returns in (2)	0.3264	share of capital to labor cost
q, prob. of loosing idea	0.074	mean firm exit rate in BDS
η, prob. of getting ideas	0.0034	mean firm employment of 22.7 in BDS

where $q = 0.0738$ is the exit rate. The left-hand side measures the number of new firms started every period: with probability η the agents who had no ideas or chose to abandon their ideas in the previous period, receive new ideas and operate them. The right-hand side measures the mass of firms exiting every period. This implies that setting

$$\eta = \frac{m \cdot q}{1 - m + m \cdot q} \approx 0.0034$$

guarantees that the model matches the mean firm employment.

The remaining parameters (β, $F_W(\xi)$, γ, $G(z)$, n_0, B, δ_z, θ, and $F_\varepsilon(\varepsilon)$) are calibrated jointly to match the following moments: the equilibrium interest rate, the percentage of population with negative asset holdings, the variance of log earnings, the share of the top 5% income earners, the size distribution of age-0 firms, the share of total revenues spent on intangible investment, the average growth of firms over the first 15 years of life, and job destruction rate. Table 4 summarizes these parameters, along with the targeted moments.

The labor productivity shock is assumed to be binary,[14] taking values ξ_H with probability p and ξ_L with probability $1 - p$, normalized such that $p\xi_H + (1 - p)\xi_L = 1$. The values of ξ_H and

[14]Since the sole purpose of the uncertainty in labor productivity is to generate less concentrated wealth distribution, a more dense grid for labor productivity offers little advantage, but makes the already intense computational procedure more time consuming.

Table 2: Parameters calibrated jointly

Parameter	Value	Target / Source	Data	Model
ξ_H, earnings shock	1.209	fract. of pop. with negative assets Diaz-Gimenez et al. (2011)	0.10	0.125
p, prob. of ξ_H	0.5	var. of log earnings Hugget (1996)	0.045	0.045
γ, returns to scale in (1)	0.77	top 5% income share Piketty and Saez (2003)	0.26	0.35
$G(z)$, new ideas distr.		BDS data, see Table 4		
B, productivity in (4)	1.55	normalized equilibrium wage	1	1
δ_z, depreciation in (4)	0.18	intangible investment/output share Corrado et al. (2018)	0.09	0.087
θ, returns to scale in (4)	0.10	mean size of 10-15 y.o. firms, BDS	15.8	15.7
Δ_L, low productivity shock	0.0134	job destruction rate Davis et al. (2006)	0.1	0.1
β, time discount factor	0.9425	equilibrium interest rate	0.03	0.03

p affect the fraction of the population with assets below 0, and the volatility of the idiosyncratic component of the earnings process, and are pinned down to match their empirical counterparts.

The returns to scale parameter γ affects the profit-to-output ratio of the firms, and thus controls the income share of the firm owners who, given the average firm size of 22.7 employees, constitute in the model about 4.4% of the population. Firm ownership is also highly correlated with wealth holding and, therefore, with capital income. Thus, the choice of γ affects the income share of the top 5% of population which, according to Piketty and Saez (2003) amounts to 26%.Once the technology parameters α and γ are calibrated, the distribution of the productivity shocks for the entering firms $F_M(z)$ is pinned down to match the distribution of age 1 firms in the BDS data (reported in Table 4 in the Appendix), assuming that the equilibrium wage is $w = 1$

and the interest rate is $r = 0.03$.

The rest of the parameters are tightly linked together. The time discount rate β impacts consumers' incentives to save and its choice ensures that the asset market clears at $r = 0.03$. Parameter B affects the firm growth and, via it, the average firm size. The choice of η described above guarantees that the labor market would clear at $w = 1$ if the modern firm, on average hire 22.7 workers. Changes in B impact the average firm size and, hence, impact the aggregate labor demand. Thus, adjusting B to match the average firm size also ensures that the labor market is in equilibrium at $w = 1$. Increasing δ_z implies that the productive ideas depreciate faster and, therefore, more innovative investment is required in order to match the targeted average firm size. Existing literature provides a wide range of estimates for this variable. The relevant measure of intangible investment is one that includes not only R&D investment, but investment in other goods that improve future productivity, such as software, databases, marketing research, organizational capital, etc. According to various estimates (see Corrado et al. (2018) for an excellent summary), the lower bound on such investment is about 9 percent for the US.[15] In order to stay on the conservative side, I target the lower bound of this interval. Parameter θ controls how much harder it is to grow high- versus low- productivity firms, and, thereby, affects how much of the firm's life-time growth is achieved over the first 15 years of life.

The distribution of shocks affecting the firm growth rate affects firm volatility and, in turn, the turnover of jobs in the economy. I assume that ε' can take one two values, $\frac{1}{1-q} - \Delta_L$ and $\frac{1}{1-q} + \Delta_H$, with probabilities p_H and p_L, respectively, such that $p_L + p_H = 1$ and $p_H \Delta_H = p_L \Delta_L$, so the mean ε equals to $\frac{1}{1-q}$. Given these restrictions, only two parameters, p_L and Δ_L are to be chosen. In the benchmark simulation, I assume this distribution is symmetric, set $p_L = 0.5$ and calibrate Δ_L to match the mean job destruction rate of about 10 percent in the US economy.[16] The same job destruction rate can be generated under different values of p_L and Δ_L. Thus in section 5.3 I recalibrate the model using different values of p_L and illustrate that the results are not sensitive to

[15]Corrado et al. (2018) report that, averaged over 2000-2013, this ratio is about 14 percent in industry and 12 percent in services, and has been growing faster than its tangible investment counterpart.

[16]Another moment impacted by the variance of productivity shocks ε is the volatility of firm growth rates. The publicly available BDS data is not a panel, and cannot be used to characterize the moments of the evolution of firm size over time. However, Davis et al. (2006) use the Longitudinal Business Database and compute the employment-weighted volatility of firm employment growth rates; the same moment computed in the benchmark calibrated model is equal to 0.3, within the range of the values reported over time by Davis et al. (2006) (see Figure 2.6 in their paper, and pp. 122-123 for the details on how to compute the "modified' volatility measure that uses a standard degree-of-freedom correction as described).

this parameter. The main goal of the quantitative analysis is to assess how changes in p_L and Δ_L, which increase the dispersion of productivity shocks while keeping their mean constant, affect the firm owner's incentives to invest in firm productivity, and, via it, the firm life-cycle growth and aggregate productivity.

5 Results

The objective is to analyze how an increase in the risk associated with firm owners' innovative investment affects the firm's incentives to invest in productivity growth, and, as a result, impacts the aggregate productivity and the firms' life-cycle employment profile. Within the model, such increased uncertainty can arise due to changes in p_L, Δ_L, or both. Recall that, since $p_L\Delta_L = p_H\Delta_H$, changes in (p_L, Δ_L) have no impact on the mean of the productivity shock ε. Thus, I perform a series of exercises varying (p_L, Δ_L) and report how the key aggregate variables adjust as the degree of uncertainty in returns to innovative investment changes. Note also that if in (4) parameter B were equal to 0, i.e., the expected productivity growth in the model were exogenous, the mean-preserving changes in the distribution of productivity shocks ε would have no impact on the average firm productivity or measured TFP.

5.1 Steady state effects of a change in (p_L, Δ_L)

I start by discussing in details one particular experiment, switching from the benchmark value of $p_L = 0.5$ and $\Delta_L = 0.1$ to $p_L = 0.25$ and $\Delta_L = 0.9$, to which I refer to as 'high uncertainty' scenario hereafter. As shown below, for this combination of parameters, the equilibrium job destruction rate rises from 0.1 to 0.3, and the employment-weighted volatility of firm growth rates increases from 0.3 to 0.75. Targeting the job destruction rate of 0.3 in the first experiment is motivated by the recent empirical work by Donovan et al. (2019) which documents that in the poorest countries the job flows are about three times as high as in the U.S.

To outline the key driving forces, I first discuss the partial equilibrium effects (keeping prices w and r constant) arising from this change, and then summarize how the resulting changes in prices further affect the economy. Figure 3, Table 3 and Figure 4 accompany this discussion.

Figure 3 depicts how the aforementioned change in p_L and Δ_L affects the firm owner's in-

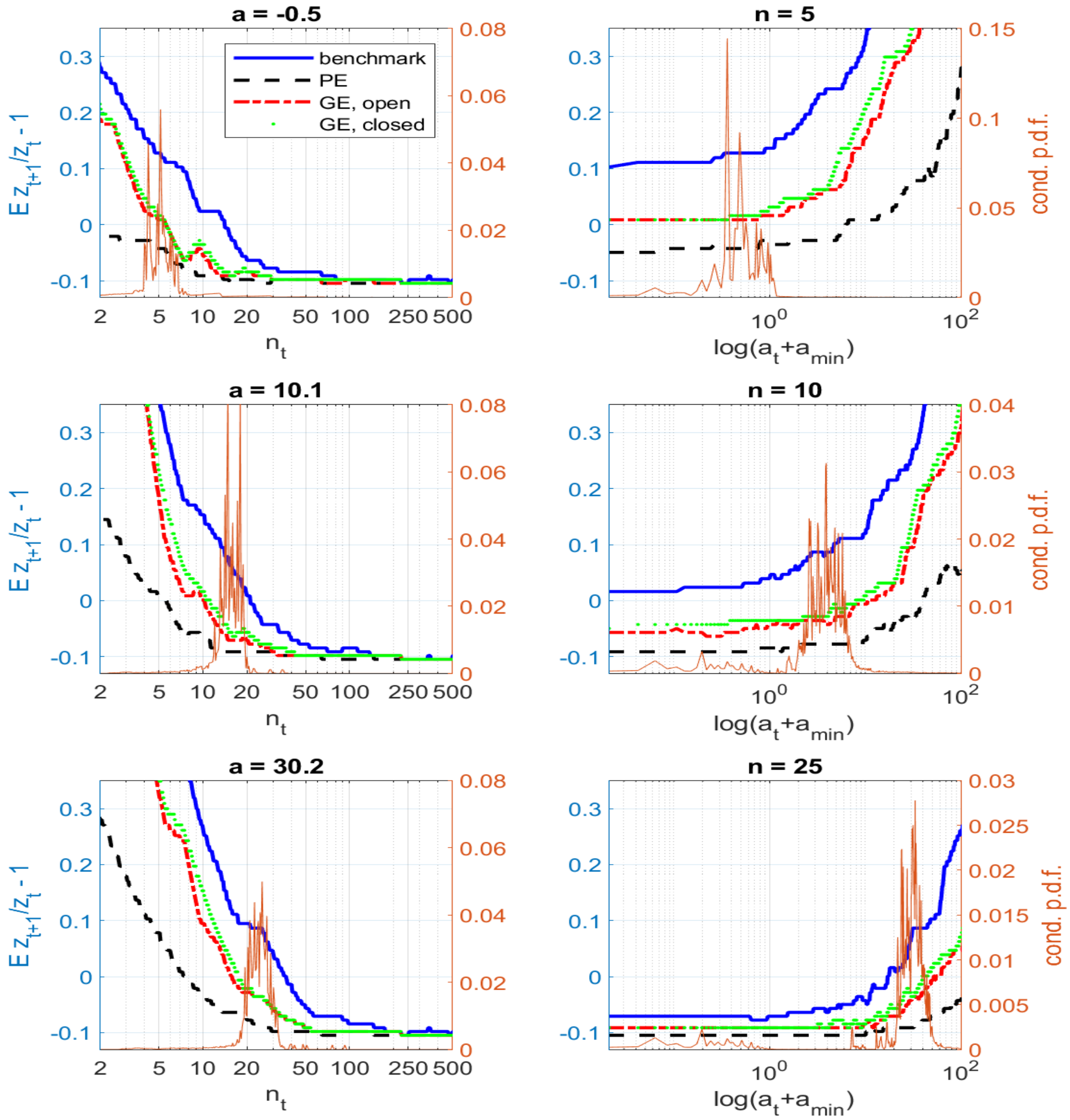

Figure 3: The effects of a switch from the benchmark ($p_L = 0.5$ and $\Delta_L = 0.0134$) to the high uncertainty ($p_L = 0.25$ and $\Delta_L = 0.9$) scenario on firm's mean growth rates, conditional on survival, measured as $\mathbb{E}z_{t+1}/z_t - 1 = -\delta_z + B\mathbb{E}_\xi(x_t(a_t, z_t, \xi)/z_t)^{1-\theta}$. The plots in the left column illustrate how growth rates vary with current productivity shock (proportionate to employment) for a given current wealth level; the plots in the right column illustrate how the growth rates vary with the current wealth level for a given level of current productivity (for tractability, the productivity is measured by the employment level at the benchmark wage $w = 1$). The plots illustrate the optimal policies in the benchmark economy (blue solid lines), the partial equilibrium effects (black dashed lines), the general equilibrium effects in the open economy (red dash-an-dotted lines), and the general equilibrium effects in the closed economy (green dotted lines). The optimal policies are juxtaposed against the benchmark steady state distribution of types with respect to productivity levels (assets) conditional on a given level of assets (productivity).

novative investment $x(a, z, \xi_H)$. It plots the firm's expected growth rate in terms of employment for different levels of current wealth (in the left column) and current productivity shock (in the right column). Recall that, by (4), the expected employment growth rate is directly related to the share of innovative investment relative to the firm's productivity z_t, and hence employment n_t, via $\mathbb{E}n_{t+1}/n_t = \mathbb{E}z_{t+1}/z_t = \frac{1-\delta_z}{1-q} + \frac{B}{1-q}(x_t/z_t)^{1-\theta}$, where the former equality follows from the fact that, in the absence of firm financial constraints, employment is proportionate to the current productivity level. First, let's focus on the blue solid line illustrating the innovative investment in the benchmark economy. Naturally, due to concavity of the firm owner's value function, the share of innovative investment increases with the firm owner's assets and decreases with the current productivity level (and, hence, firm's current employment).

Keeping all the variables constant, an increase in the volatility of returns to innovative investment increases firm owners' risk exposure, reduces their incentives to invest in productivity growth, and results in smaller firm growth rates. The changes in the optimal policies in a partial equilibrium environment are depicted on Figure 3 using black dashed lines. This decline is most pronounced at the small and medium-range levels of z, since firm owners with high levels of productivity generating a lot of profit in the current period save a lot for the future and are less risk-averse than the firm owners who save little.[17] If the distribution over (a, z) were to remain unchanged, the total decline in innovative investment would shed about 11% in the next period's firm productivity $z^{1-\gamma}$. This, of course, implies that the joint distribution over firm productivity and assets must also adjust. On the one hand, the reduction in optimal innovative investment shifts the distribution of firm productivities to the left; this effect is further amplified by the fact the effect of uncertainty on innovative investment is bigger at the lower productivity levels, as shown in Figure 3. On the other hand, as the firm owners operate less productive firms, their incomes fall, and they accumulate less assets.[18] This further amplifies the negative effects on productivity since firm owners with less assets make less innovative investment.

Figure 4 illustrates how these changes affect the firm growth over lifecycle (on the left plot)

[17]Note that there is no corporate risk-neutral sector in the economy. However, the biggest firms in the model exhibit nearly risk-neutral behavior and respond very little to changes in the level of uncertainty.

[18]There is also an opposing force impacting the saving behavior: when productivity investment are riskier, the firm owners have incentives to reallocate their investment to safe assets, both due to changes in risk / return composition, and due to precautionary motive. However, in the quantitative analysis, this force appears weak in comparison with the direct impact of lower aggregate productivity, and the aggregate asset distribution shifts to the left.

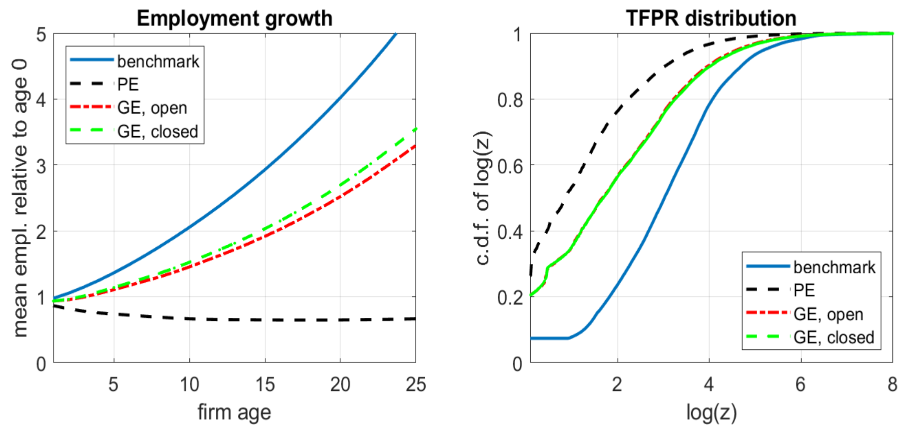

Figure 4: The effects of a switch from the benchmark ($p_L = 0.5$ and $\Delta_L = 0.0134$) to the high uncertainty ($p_L = 0.25$ and $\Delta_L = 0.9$) scenario on mean firm size over age measured by employment, relative to the mean size entrants (left plot) and the steady state c.d.f. of $\log(z)$ (right). The plots illustrate the outcomes in the benchmark economy (blue solid lines), the partial equilibrium effects (black dashed lines), the general equilibrium effects in the open economy (red dash-an-dotted lines), and the general equilibrium effects in the closed economy (green dotted lines).

and the steady state distribution of firm productivities (on the right plot). The blue solid line on the left plot illustrates that in the benchmark scenario the firms' mean employment grows by about four times over the first twenty years.[19] The black dotted line demonstrates that, keeping wage and interest rate constant, results in so much decline in innovative investment that all of the lifecycle firm grows would be erased. The drastic leftward shift in the steady state distribution of firm productivities results in a considerable decline in the demand for production inputs.

The aggregate steady state implications of these changes are summarized in the second column of Table 3 (column PE). All the variables, except for the interest rate, the two measures of uncertainty (the job destruction rate and the volatility of firm growth rates), and the share of innovative investment in output are normalized to be equal to 1 in the benchmark model. Switching to the high uncertainty environment results in a decline of the share of innovative investment in output from 8.7 percent to 4.3 percent. As a result, the mean TFPR, measured as the average of firm productivities $z^{1-\gamma}$ in (2), reduces by 34 percent,while the aggregate labor demand,

[19] Since the model is calibrated to match the average size of the entrants, the average size at age 15, and the mean firm size overall, the firm lifecycle profile generated in the model is very close to the one observed in the BDS data. It is also consistent with the lifecycle profile reported in Hsieh and Klenow (2014).

capital demand, and output fall by 80 percent.[20] Recall that, as mentioned earlier, the direct one-period impact of a reduction in optimal innovative investment on average firm productivity is only 11 percent, so the remaining 25 percent of the decline in aggregate productivity occurs due to changes in the distribution over (z, a). Since the firm owners generate less profit, their owners receive less income, accumulate less assets, and the whole wealth distribution shifts to the left. The total supply of assets drops by 79 percent, while the aggregate labor supply (which is not reported in the Table because it is exogenous) remains unchanged. In the steady state, the job destruction rate and the employment-weighted volatility of firm growth rates increase to 0.3 and 0.75, respectively.

Next, let us turn attention to the general equilibrium effects arising in the open economy in which the interest rate remains unchanged but the wage adjusts to clear the labor market. These effects are summarized in the third column of Table 3 (GE, open). In order to clear the labor market in the high uncertainty scenario, the equilibrium wage must fall to 0.8477. This wage decline benefits firm owners and affects their incentives to invest in productivity in two ways. First, it directly increases the returns to productivity investment because a foregone unit of consumption in the current period results in higher future expected profit. As shown by the red dash-and-dot policies on Figure 3, the effects of increased uncertainty on firm's productivity investment are considerably muted once the wage drops to the level that clears the labor market. Second, since firm owners now receive more income, they accumulate more assets, which further promotes productivity investment. Despite this, as reported in the third column of Table 3, the equilibrium aggregate effects are still significant: the average firm productivity falls by 20 percent, and the aggregate output falls by 16 percent. The equilibrium decline in wage also dampens the effects increased uncertainty on the firm life-cycle growth: over the first 20 years of life, the firms now on average grow by 2.51 times, compared to no growth in the partial equilibrium. At the same time, this lifecycle growth is considerably smaller than the one observed in the benchmark economy, suggesting that uncertainty can be an important factor in understanding why firms grow slower in poorer countries, a well documented pattern dating back to Hsieh and Klenow (2014).

Note that, by construction of the model, the labor supply in this environment is inelastic: all the agents provide one unit of labor regardless of the wage level. Had the labor supply been

[20] Since prices are constant in the partial equilibrium, the optimal capital, labor and output all decline by the same amount since they all are proportionate to the firm's productivity level z.

Table 3: The effects of a switch from the benchmark ($p_L = 0.5$ and $\Delta_L = 0.0134$) to the high uncertainty ($p_L = 0.25$ and $\Delta_L = 0.9$) scenario on the aggregate variables.

Aggregate variable	Benchmark $p_L = 0.5$ $\Delta_L = 0.1$	High uncertainty $p_L = 0.21$, $\Delta_L = 0.9$		
		PE	GE,open	GE, closed
interest rate, r	0.03	0.03	0.03	0.0215
wage, w	1	1	0.8477	0.8902
share of innov. inv. in output	0.087	0.043	0.069	0.071
firm growth, n_{20}/n_0	4	0.64	2.5	2.7
mean TFPR	1	0.66	0.80	0.80
labor demand	1	0.2	1	1
capital demand	1	0.2	0.84	0.98
output	1	0.2	0.84	0.88
measured TFP	1	0.72	0.88	0.89
capital supply	1	0.21	1.07	0.98
job destruction rate	0.1	0.3	0.3	0.3
vol. of firm growth rates	0.3	0.75	0.73	0.75

elastic, the equilibrium wage would not fall as much, and the equilibrium aggregate and life-cycle implications of the increase in uncertainty would be greater than described above. In other words, by assuming that the labor supply is inelastic, the model aims to provide a lower bound on the effects arising due to increased riskiness of innovative investment.[21]

[21] One easy way to accommodate elastic labor supply in this model would be to assume that agents to draw alternative production opportunities (e.g., home production of low-tech traditional sector), and choose whether they want to work in the labor market or pursue such opportunities. A decline in the labor market wage would then push more workers into a traditional / home production sector, thereby making the labor supply elastic. The elasticity of the labor supply would then depend on the distribution of these alternative production opportunities. One can

Turning to the closed economy equilibrium, the interest rate must decline in order to clear the asset market. This decline in the interest rate encourages innovative investment since, on the one hand, it increases firms' profits and, hence, returns to investment in productivity, and, on the other hand, reduces the returns on risk-free investment. In the equilibrium, the interest rate drops to 0.215 percent, which slightly mitigates the adverse aggregate effects: as shown in the last column of Table 3, the average firm productivity, aggregate output, capital and measured TFP fall by 20, 12, 2 and 11 percent, respectively, slightly less than their counterparts in the open economy.

5.2 The aggregate effects across many (p_L, Δ_L)

The numerical exercise summarized above computes the implication of increased uncertainty for one particular change in parameters (p_L, Δ_L) guiding the evolution of firm-specific productivity shocks. As mentioned earlier, the same amount of volatility in the firm growth rates can be generated by different combinations of (p_L, Δ_L). Thus, at the next step, I recompute the steady state outcomes for a variety of other values of these parameters for which the volatility is higher than in the benchmark model, and illustrate the key aggregate variables on Figure 5.[22] All the plots illustrate how these variables change with the job destruction rate obtained in the economy as p_L and Δ_L adjust. The dots correspond to the steady state outcomes in the partial equilibrium economy, and the stars – to the steady state outcomes in the general equilibrium of the open economy.[23] The dots / stars of the same color correspond to the experiments in which p_L remains constant and Δ_L varies (obviously, higher Δ_L leads to a higher job destruction rate). The point labeled by the circle marks the benchmark allocation.

The main conclusions that can be drawn by observing the plots on Figure 5 are as follows. First, in a partial equilibrium, changes in (p_L, Δ_L) leading to small changes in job destruction rate can result in large changes in aggregate variables. These effects, however, are considerably mitigated in the equilibrium setting in the open economy, in which the wage rate adjusts to clear the labor market. This suggests, as mentioned earlier, that the aggregate effects of uncertainty might be magnified if the labor supply in the model is elastic (see footnote 21 for further discussion).

obviously construct such a distribution so that the presence of these production opportunities has no impact on the calibrated benchmark economy (e.g., the support of this distribution is bounded away by 1) and the labor market equilibrium approximates the steady state of the partial equilibrium allocation in the high uncertainty environment.

[22]Namely, I vary p_L and Δ_L between 0 and 1, and recompute the steady state allocation.

[23]The general equilibrium in the closed economy is still being computed.

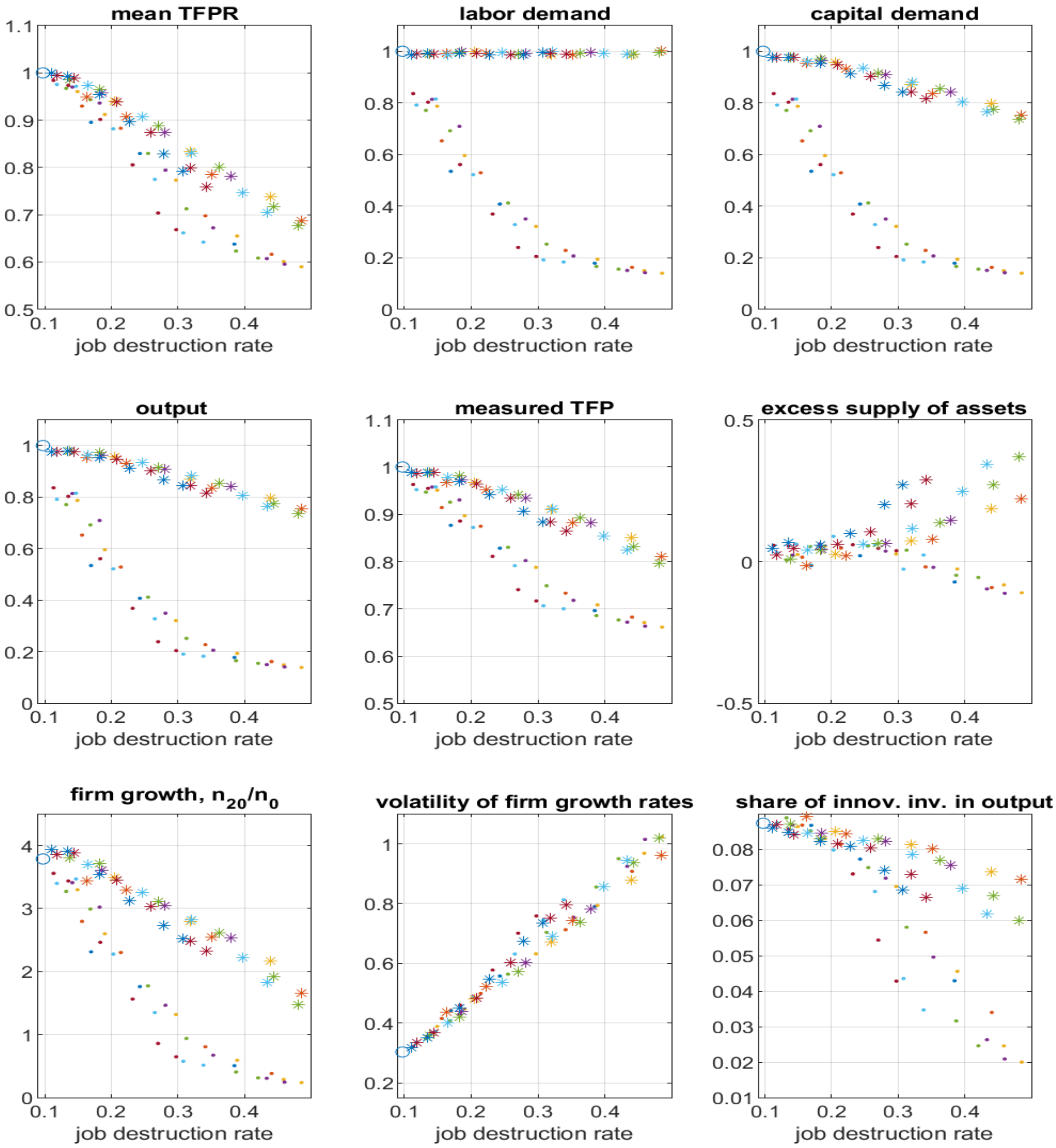

Figure 5: The relationship between the job destruction rate and other aggregate variables arising due to changes in p_L and Δ_L in the partial equilibrium (dots) and the general equilibrium of the open economy (stars); the benchmark economy in which the job destruction rate equals to 0.1 is labeled by the circle. The mean TFPR, labor and capital demand, output, and measured TFP are measured relative to their values in the benchmark economy; the access supply of assets is measured relative to the capital demand, the firm growth rate is the employment at age 20 relative to employment at age 1, and the volatility of employment growth rates is computed as in Davis et al. (2006).

Second, the same changes in the job destruction rate are associated with a range of changes in aggregate variables, depending on what combination of (p_L, Δ_L) induced a particular change in the job destruction rate. Not surprisingly, such a range is wider in the partial equilibrium than in the general equilibrium setting; this is because a bigger drop in the labor demand in the partial equilibrium necessitates a bigger wage decline in the general equilibrium. For example, if the job destruction rate tripples from 0.1 to 0.3, mean TFPR falls by 22% to 34% in the partial equilibrium, and by 15% to 20% in the equilibrium of the open economy.

Third, within the reasonable range of equilibrium job destruction rates, the relationships between the change in the job destruction rate and the average change in key aggregate variables[24] are reasonably well approximated by linear functions. Such approximation suggests that a change in (p_L, Δ_L) leading to a one percentage point increase in the job destruction rate (e.g., from 10 percent to 11 percent) results in a 0.92 percent drop in mean TFPR, 0.73 percent drop in aggregate output and capital, and 0.55 percent drop in the measured TFP.[25]

Notably, while the effects of increased uncertainty on measured TFP are rather modest, the impact on the firm life-time growth is more significant: each percentage point increase in job destruction rate shaves about 5.5 percentage points off the firm growth over the first 20 years of life (e.g., an increase in the job destruction rate from 0.1 to 0.3 is associated, on average, with the drop in employment growth over 20 years from 400 percent to 280 percent). This suggests that variations in the degree of uncertainty across countries may be an important contributor to differences in firms' life cycle profiles documented by Hsieh and Klenow (2014). Finally, it is worth pointing out that the changes in job destruction rate induced by varying (p_L, Δ_L) are closely related to the changes in the employment-weighted volatility of firm growth rate used in Davis et al. (2006): each percentage point increase in the job destruction rate is associated with about 6 percent increase in such a volatility measure.

[24]For this purpose, the aggregate effects are averaged across uniformly drawn (p_L, Δ_L).

[25]Since there is no change in equilibrium employment, the 0.73 percent change in output can be decomposed into the 0.18 percent change coming from the capital contribution (since the aggregate capital declines by 0.73 percent, its contribution to output declines by $\alpha \cdot \gamma \cdot 0.73 \approx 0.18$ percent) and 0.55 percent change in measured TFP.

5.3 Comparative statics

In this section, I analyze whether the model's quantitative results are sensitive to the changes in some of the parameters. In the benchmark model, the probability of drawing low productivity shock was arbitrarily set to $p_L = 0.5$. Now I adjust this parameter to $p_L = 0.1$, recalibrate the rest of the parameters to match the same moments (see the second column of Table 7.3 in the Appendix for the new values of the calibrated parameters), and recompute the steady state effects of changes in (p_L, Δ_L). Figure 7 in the Appendix illustrates these results. As can be seen, the new calibrated parameter values differ little from those in the benchmark economy. This is not surprising: as can be seen from Figure 5, there is little variation across aggregate equilibrium outcomes for different values of (p_L, Δ_L) for which the job destruction rate is equal to 10 percent. As a result, the effects of changes in (p_L, Δ_L) resulting in higher job destruction rates are similar across all the models in which 10 percent job destruction rate is obtained in the benchmark case. Indeed, the effects illustrated on Figure 7 in the Appendix look very similar to the effects earlier summarized in Figure 5. The approximation by a linear function yields very similar predictions: a one percentage point increase in the job destruction rate is associated with a 0.96 percent decline in the average firm productivity, a 0.66 percent decline in output, a 0.64 percent decline on the aggregate capital and 0.5 percent decline in measured TFP (compared to 0.92 percent, 0.73 percent, 0.73 percent and 0.55 percent, respectively, in the benchmark model). In terms of the effects on the firm life cycle growth, an increase in job destruction rate from 0.1 to 0.3 results in a reduction of firm growth over 20 years from 4 to 2.8, same as in the benchmark calibration.

Next, I study how changes in θ affect the model's predictions. Recall that the benchmark $\theta = 0.15$ is pinned down to match the average size of the firms that are 10-15 years old. Changing θ to $\theta = 0.5$ and recalibrating the rest of the parameters to match the same targeted moments (see column 3 of Table 7.3 in the Appendix for the new calibrated values of the rest of the parameters) has relatively small effect on the average size of the 10-15 years old firms: it drops from the 15.9 workers in the benchmark calibration to 13.7.[26] Thus, it is useful to perform a comparative statics exercise with respect to this parameter. Figure 8 illustrates how changes in (p_L, Δ_L) affect

[26]This change in θ also results in the lower employment growth over the first 20 years, which drops from 300 percent to about 250 percent.

the relationship between the aggregate variables and the job destruction rate for $\theta = 0.5$. As can be seen, the effects are slightly higher than in the benchmark economy: in the equilibrium of the open economy (labeled by stars), a one percentage point increase in the job destruction rate leads to a 1.1 percent decline in average firm productivity, a 0.89 percent decline in output, 0.8 percent decline in capital, and 0.7 percent decline in measured TFP. Intuitively, the higher is θ, the more important is the accumulated intangible capital in the production of new intangible capital. Since higher uncertainty has biggest effects for firms with relatively low productivity levels, and entrants tend to have low productivity levels, uncertainty has large effects on the innovative investment in the beginning of firm life-cycle. When θ is high, these effects have a bigger impact on future productivity resulting in bigger aggregate effects of uncertainty.

Finally, I also analyze how the model's predictions change if the risk aversion coefficient adjusts from $\sigma = 2$ to $\sigma = 3.5$. One the one hand, one may expect that higher risk aversion might increase the effects of uncertainty. On the other, when σ increases, the rest of the parameters are recalibrated (see column 3 of Table 7.3 in the Appendix) to generate the same amount of risky innovative investment in the benchmark allocation. As a result, the effects of increasing uncertainty in the recalibrated model with high $\sigma = 3.5$ are slightly smaller than in the benchmark scenario: as shown in Figure 9 in the Appendix, a change in (p_L, Δ_L) leading to one percentage point increase in the job destruction rate results in about 0.87 percent drop in mean firm productivity, 0.66 percent drop in output, 0.62 percent drop in capital, and 0.46 percent drop in measured TFP. The firm growth over 20 years falls from 4 to 2.8, same as in the benchmark model.

5.4 The effects of uncertainty in the presence of firm financing constraints

In this Section I investigate whether the effects of higher uncertainty in returns to innovative investment are impacted by the presence of firm financing constraints, which are commonly associated with less developed economies. Intuitively, if the firm has limited access to external financing, it might not be able to efficiently finance its operations if it draws a high level of productivity shock, which would reduce the expected return on making innovative investment.

More formally, in the presence of the firm financing constraint, the profit function (2) becomes

$$\pi(z,a) = \begin{cases} \Pi \cdot z, & \text{if } \frac{\gamma}{1-\gamma}\frac{\alpha}{1-\alpha}\Pi z \leq \overline{k}(z,a) \\ \overline{\Pi} \cdot z^{\frac{1-\gamma}{1-(1-\alpha)\gamma}} \cdot \overline{k}(z,a)^{\frac{\alpha\gamma}{1-(1-\alpha)\gamma}} - (r+\delta)\overline{k}(z,a), & \text{otherwise,} \end{cases} \tag{11}$$

where Π and $\overline{\Pi}$ are the coefficients that depend only on the technology parameters and the prices. In the absence of the firm financing constraint (i.e., when $\overline{k}(a,z) = +\infty$), this profit function is linear in z. If the firm financing constraint binds for some z, this function changes its shape: it remains equal to Πz for the values for which $k \leq \overline{k}(z,a)$ is slack, and falls below Πz otherwise, giving rise to regions in which mean-preserving spreads in z reduces the expected profit. The shape of $\pi(z,a)$ in the region in which the firm financing constraint binds obviously depends on the functional form of the constraint. In particular, if

$$\overline{k}(z,a) = a + \phi z + b_0, \quad \phi > 0$$

the profit function is concave as long as ϕ is sufficiently small. When $\phi = 0$, the firm owners must rely on own accumulated assets for firm financing and, in addition, can borrow a fixed amount b_0. When $\phi > 0$, the firm owners can also borrow up to a fraction of the firm's optimal operational capital. The firm financing constraints of this form have been extensively used in the development literature (e.g., Buera et al. (2015), Midrigan and Xu (2014), and many others), and can be rationalized by the presence of limited commitment on the side of the firm owner. The concavity of the profit function (11) makes the firm owners more risk averse. As a result, mean-preserving spreads in productivity shock ε have larger effects on firms' innovative investment and the economy's aggregate variables.

In the main exercise in this Section, I assume that the firm financing constraint is

$$k \leq \overline{k}(z,a) = a + b_0,$$

where b_0 is an exogenous borrowing limit. I compute the equilibrium outcome of the model in the presence of this constraint, and then evaluate the aggregate effects of changes in (p_L, Δ_L). The experiment is computed for $b_0 = 6$, for which 17% of the operational capital in the equilibrium is externally financed. It is well understood that the financing constraint per se has a negative impact on the economy in this environment: it discourages innovative investment (e.g., Vereshchagina (2022)) and leads to misallocation of capital and labor across the firms (e.g., Midrigan and Xu (2014)). In the calibrated benchmark model, the aforementioned borrowing results in lower innovative investment (3.2 percent of output), a 14 decline in average firm productivity, a 50 percent drop in output, a 77 percent decline in capital used in production, and a 26 percent decline in measured TFP.[27]

Figure 6, analogously to Figure 5, illustrates he relationship between the aggregate variables and the job destruction rate arising in the economy as parameters p_L and Δ_L adjust. All the variables in the plots in the two top rows are measured relative to their values in the equilibrium of the economy in which the aforementioned firm financing constraint is imposed. Comparison of Figures 5 and 6 yields the following observations. First, the difference between the partial and general equilibrium effects is smaller in the setting with the firm financing constraints. Second, the relationship between the aggregate variables and the job destruction rate now is more dependent on what combination of parameters (p_L, Δ_L) generated a particular job destruction rate (this is reflected in the plots by a bigger range of aggregate variables that can be observed at a given job destruction rate); the effects are biggest when p_L is low and Δ_L is large. Finally, and most importantly, an increase in the job destruction rate is associated with much bigger effects on the aggregate variables in the environment in which firms face financing constraints.

Quantitatively, if (p_L, Δ_L) change in such a way that the job destruction rate tripples from 0.1 to 0.3, the mean average productivity falls by $18\% - 38\%$, the output falls by $20\% - 50\%$, the capital used in production falls by $25\% - 65\%$ and the measured TFP falls by $15\% - 35\%$. Such increase in the job destruction rate is also associated with a large decline in the firm growth over lifecycle: the 20-years old firms are only 1.2-2 times bigger than the entrants when the job destruction rate is 0.3. As in the benchmark model, the relationship between the aggregate variables and the job destruction rate are well approximated by the linear function, implying that a one percentage point increase in the job destruction rate reduces average firm productivity by

[27]The effects on innovative investment and aggregate productivity are considerably bigger than the ones found in Vereshchagina (2022) because in this model the innovative investment is risky even under the benchmark calibration since it is made prior to the realization of the exit shock and the productivity shock. In contrast, Vereshchagina (2022) abstracts from these risks because it aims to provide a lower bound on the effects arising due to endogeneity of firm productivity growth over lifecycle.

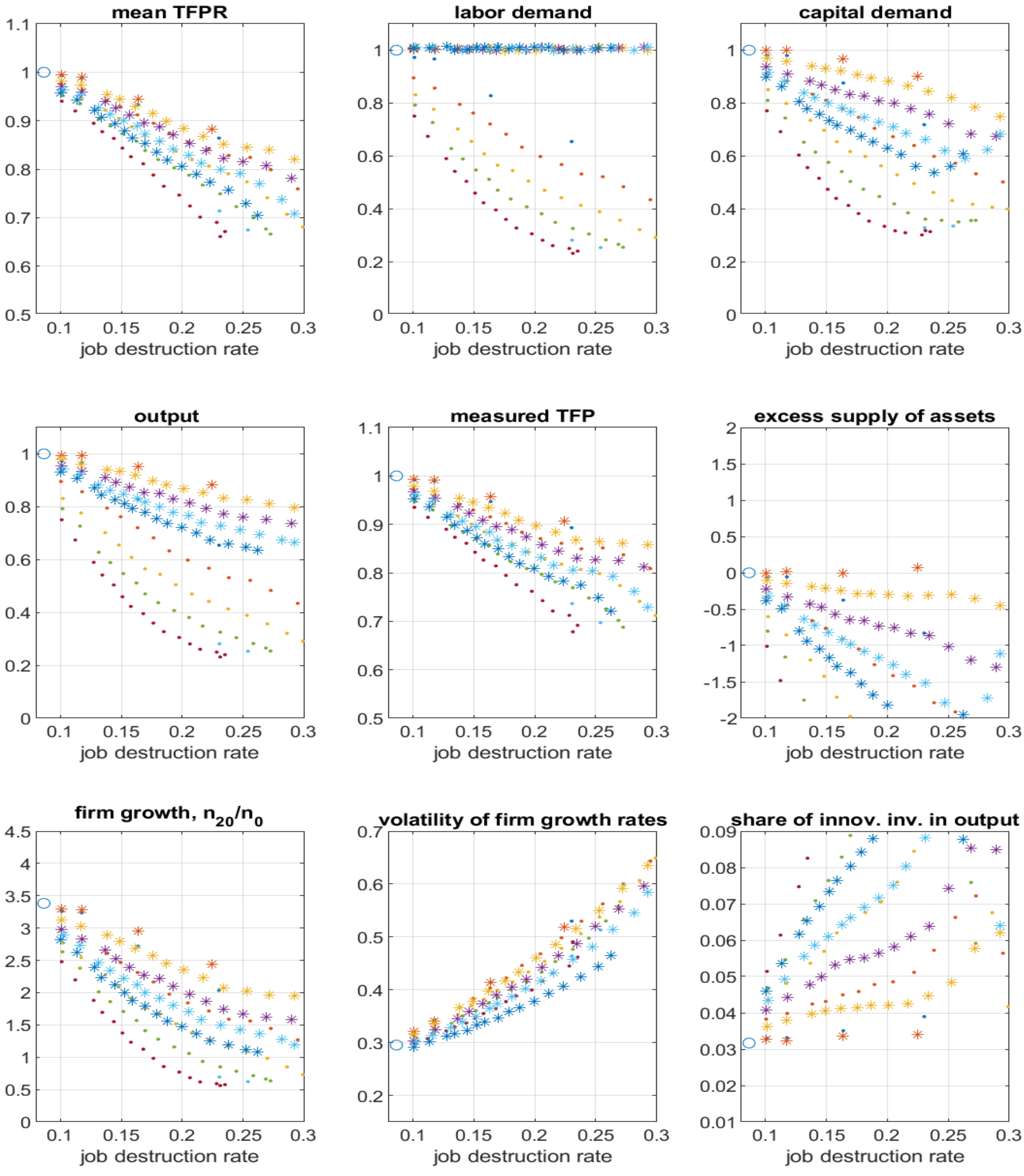

Figure 6: The relationship between the job destruction rate and other aggregate variables arising in the presence of the borrowing constraint $k \leq a + b_0$, where $b_0 = 6$. This figure is an analogue of Figure 5.

$0.9\% - 1.9\%$ (compared to 0.92% in the benchmark model), the output by $1\% - 2.5\%$ (compared to 0.73% in the benchmark model), the capital by $1.25\% - 3.25\%$ (compared to 0.73% in the benchmark model), and measured TFP by $0.75\% - 1.75\%$ (compared to 0.55% in the benchmark model). These results suggest that the effects of uncertainty on aggregate productivity and output can indeed be considerably amplified by financing constraints commonly present in the developing economies.

6 Final remarks

The analysis in the paper suggests that entrepreneurial firms (owned by a risk-averse individual) may considerably reduce their innovative investment in face of higher uncertainty, and that these adverse effects can be substantially magnified in the presence of firm financing constraint. One shortcoming of this model is that, in reality, many large firms are not owned by individuals, and perhaps would respond differently to higher levels of uncertainty. In the model, however, the owners of large firms also hold large amounts of wealth and have lower risk aversion. As a result, as discussed in Section 5, the effects of higher uncertainty are much smaller for high productivity firms, and the large aggregate effects arise because the young / small firms respond a lot to higher levels of uncertainty, which considerably slows down firm productivity growth in the beginning of firm lifecycle. At the same time, it would be interesting to embed this mechanism into a richer model, in which firms start as private entities, grow over time, and may go public through an IPO as their level of productivity rises. In such a model, higher uncertainty would primarily affect privately owned firms, reduce their growth, delay IPOs, and adversely affect the inflow of publicly-traded firms in the economy.

The model in the paper is rather stylized, and is constructed in such a way that mean-preserving spreads in productivity shocks would have no impact on aggregate productivity if the firms' expected growth rates were exogenous. For this purpose, the model abstracts from the selection mechanism, which occurs if the firms need to pay fixed operational costs and may prefer to exit if a sufficiently low productivity shock is drawn. Such a selection mechanism would automatically imply that mean-preserving spreads in productivity shock would push more firms below the exit cutoffs, thereby increasing the average productivity of survivors. In this environment, the aggregate effects on productivity would manifest through changes along the intensive margins – a decline in the number of firms in the partial equilibrium, and a potential entry of less productive firms (in the Lucas span-of-control model, Lucas (1978)) in the general equilibrium. To quantify the latter channel, the model would have to make assumptions about the productivity distribution of such potential entrants who remain inactive in the benchmark calibrated model. Thus, to focus on and quantify the potential importance of the mechanism at the heart of the paper, I abstract from the endogeneity along the extensive entry / exit margin, and focus exclusively on the role of intensive margin arising due to the effects of uncertainty on the innovative investment of existing firms. It would be interesting, to analyze to what extent the extensive margin can weaken / magnify the effects arising in the model presented in the paper.

The model also assumes that the technology (4) guiding the evolution of firm productivity is a constant-returns-to-scale function of the current productivity and the innovative investment. While under this parameterization the model reproduces the firm life-cycle profile observed in the data, this is an ad hoc assumption. The advantage of using such a function is that the resulting profit function (in the absence of firm financing constraints) is linear in firm productivity and, on its own, does not affect the firm owners' incentives to adjust innovative investment in reponse to the changes in the level of uncertainty. If such function has decreasing (increasing) returns to scale, one may suspect that the economy's response to changes in the level of uncertainty would be bigger (smaller). However, with this modification the model would have to be re-parameterized to match the same set of moments in the benchmark allocation. In Section 5.3, it was shown that adopting higher risk aversion parameter has little effect on the model's predictions since other parameters of the model must be recalibrated. I expect, and my preliminary calculations support this, that the same occurs if the functional form of (4) changes. At the same time, it would be interesting to think about how to pin down the exact shape of this function from the data, and what kind prediction may crucially depend on this shape.

Table 4: Size distribution of age 1 firms, own calculations on BDS data, 1998 - 2006

num. of workers	1-4	5-9	10-19	20-49	50-99	100-249	250 and more
fraction of firms	0.7269	0.1506	0.0711	0.0365	0.0094	0.0042	0.0014
mean employment	1.859	6.45	13.27	29.47	67.96	147.53	491.45

7 Appendix

7.1 Calibration details

7.1.1 BDS data

To compute all the facts related to the dynamics of modern firms, I use publicly available BDS data, available at https://www.census.gov/ces/dataproducts/bds/data.html. I use the averages across 1998-2006 to construct all the relevant statistics. To parameterize the distribution $G(z)$ from which new modern ideas are drawn, I use the distribution of age 1 firms reported in Table 4.[28] In the numerical analysis, to match the moments reported in Table 4, I approximate $G(x)$ within each size interval by the linear combination of the two uniform distributions, one with the bounds equal to the employment cutoffs, and another with the bounds equal to the lower cutoff and the mean employment. This is done because the mean employment in each region is smaller than the midpoint between the corresponding cutoff levels.

7.1.2 Numerical approach

There are two main parts in the numerical solution of the model: solving the dynamic decision problems (6)-(8), and finding the invariant stationary distribution. The individual decision problems are computed using the value function iteration approach. Since the decision problems have two continuous state variables, a and z, and two continuous control variables, a' and x, as well as occasionally-binding borrowing constraints, the numerical algorythm is time consuming. To optimaze the computational time, I compute the $\max$ operator over the sparse grid for (a, z), with

[28] To replicate these derivations using the BDS data, one needs to use the data organized by *Age* and *Initial Size* (as opposed to just *Size* which averages the firms' sizes across two consecutive years).

Table 5: The effects of a switch from the benchmark ($p_L = 0.5$ and $\Delta_L = 0.0134$) to the high uncertainty ($p_L = 0.25$ and $\Delta_L = 0.9$) scenario on firm size distribution

Fraction of firms by employment					
num. of workers:	1-9	10-20	20-50	50-100	100+
Benchmark:	0.56	0.20	0.15	0.05	0.03
High uncertainty, open economy:	0.69	0.13	0.11	0.04	0.03
High uncertainty, closed economy:	0.69	0.13	0.11	0.04	0.03
Share of employment by firm size					
num. of workers:	1-9	10-20	20-50	50-100	100+
Benchmark:	0.10	0.13	0.21	0.15	0.41
High uncertainty, open economy:	0.08	0.08	0.15	0.13	0.56
High uncertainty, closed economy:	0.08	0.08	0.14	0.13	0.57

the choice variables $(a', g(z, x))$ taken from the dense grid, and then at extrapolate the resulting sparse value function on the dense grid to be used at the next step of the iteration. In the process, to account for the fact that the next period shock ε' may increase the future productivity, and the firms with relatively high productivity may receive the productivity level above the upper ground of the grid on z, I extrapolate the value function to the outside of the grid at every iteration. To compute the stationary distribution, I use the Monte Carlo method.[29] The model with 500,000 individuals for at least 1000 periods till the distribution over (z, a) converges. In the resulting stationary equilibrium of the benchmark economy, there are more than 22,000 firm owners. To test that the simulated economy is in the stationary distribution, I verify that the mean and standard deviation of the employment and asset distribution remain within the tolerance criteria (e.g., 0.1 percent relative to the mean) for twenty five consecutive periods.

7.2 The effects of increased uncertainty on firm size distribution

An increase in the uncertainty has empirically relevant implications for the firm size distribution summarized in Table 5. It describes the effects on the firm size distribution resulting from a switch from the benchmark to 'high uncertainty' environment discussed in Section 5.1. Not surprisingly, higher variance in returns to innovative activity leads to heavier tails of the firm size distribution. At the left tail, the fraction of small firms (hiring 1-9 employees) rises from 56 percent to 69 percent, while the fraction of the labor force they hire slightly reduces from 10 percent to 8 percent. At the right tail, the fraction of large firms (100 or more employees) remains unchanged. This is because, on the one hand, firms productivity growth slows down but, on the other hand, firms for a given productivity level hire more workers since the wages are lower, and the forces balance each other out. However, the large firms get larger, the fraction of the labor force they hire increases from 41 percent to 56 percent. These effects are very similar in the equilibrium open and close economies, being slightly more pronounces in the latter case due to a smaller interest rate, resulting in even bigger firms in the right tail of the distribution. These predictions are consistent with the well-documented evidence that in less developed countries, compared to the US, there are more small firms, but the large firms are larger.

7.3 Supplementary tables and figures for the comparative statics analysis summarized in Section 5.3

[29]Direct computation of the invariant distribution as the solution to the equation $\lambda Q = \lambda$, where λ is the distribution and Q is the transition matrix, requires operating with very large matrices. The grid over (z, q) has more than 850×700 elements, so Monte Carlo approach results in faster computations.

Table 6: Recalibrated parameters for the comparative statics exercises summaried in Section 5.3

Table 7: Recalibrated parameters

Parameter	Benchmark	change p_L	change θ	change σ
σ, risk aversion	2	2	2	**3.5**
B, productivity in (4)	1.55	1.71	0.9846	2.6125
δ_z, depreciation in (4)	0.18	0.18	0.26	0.25
θ, returns to scale in (4)	0.15	0.15	**0.5**	0.15
p_L, prob. of low productivity shock	0.46	**0.1**	0.46	0.46
Δ_L, low productivity shock	0.0134	0.19	0.0134	0.0034
β, time discount factor	0.9425	0.9425	0.9425	0.9180

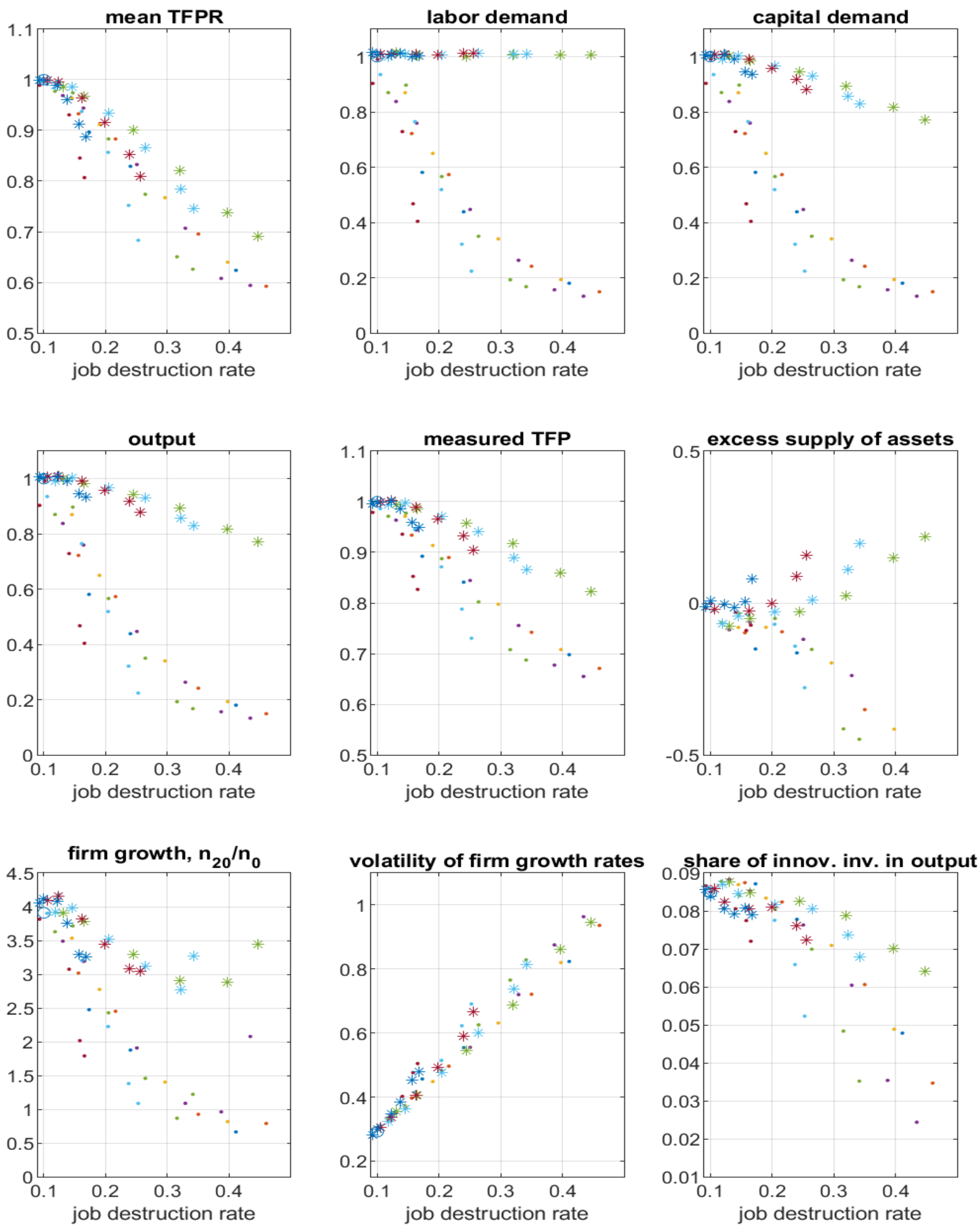

Figure 7: The relationship between the job destruction rate and other aggregate variables arising when $p_l = 0.5$ is replaced with $p_L = 0.1$ in the benchmark calibration. This figure is an analogue of Figure 5 under the new parameter values.

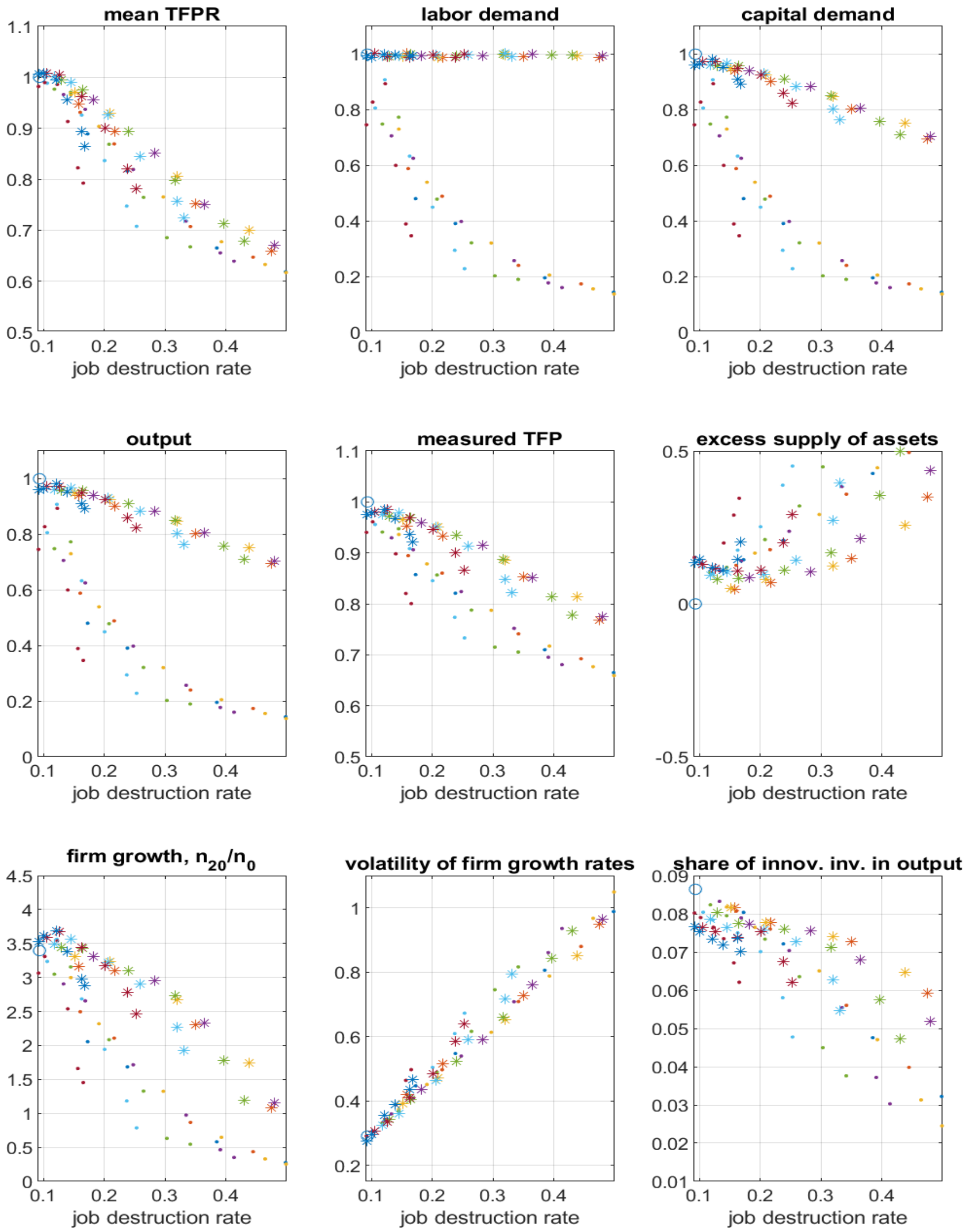

Figure 8: The relationship between the job destruction rate and other aggregate variables arising when $\theta = 0.15$ is replaced with $\theta = 0.5$ in the benchmark calibration. This figure is an analogue of Figure 5 under the new parameter values. Note that when θ changes, the firm life-cycle profile is affected, and the model generates different firm growth rate over the first 20 years.

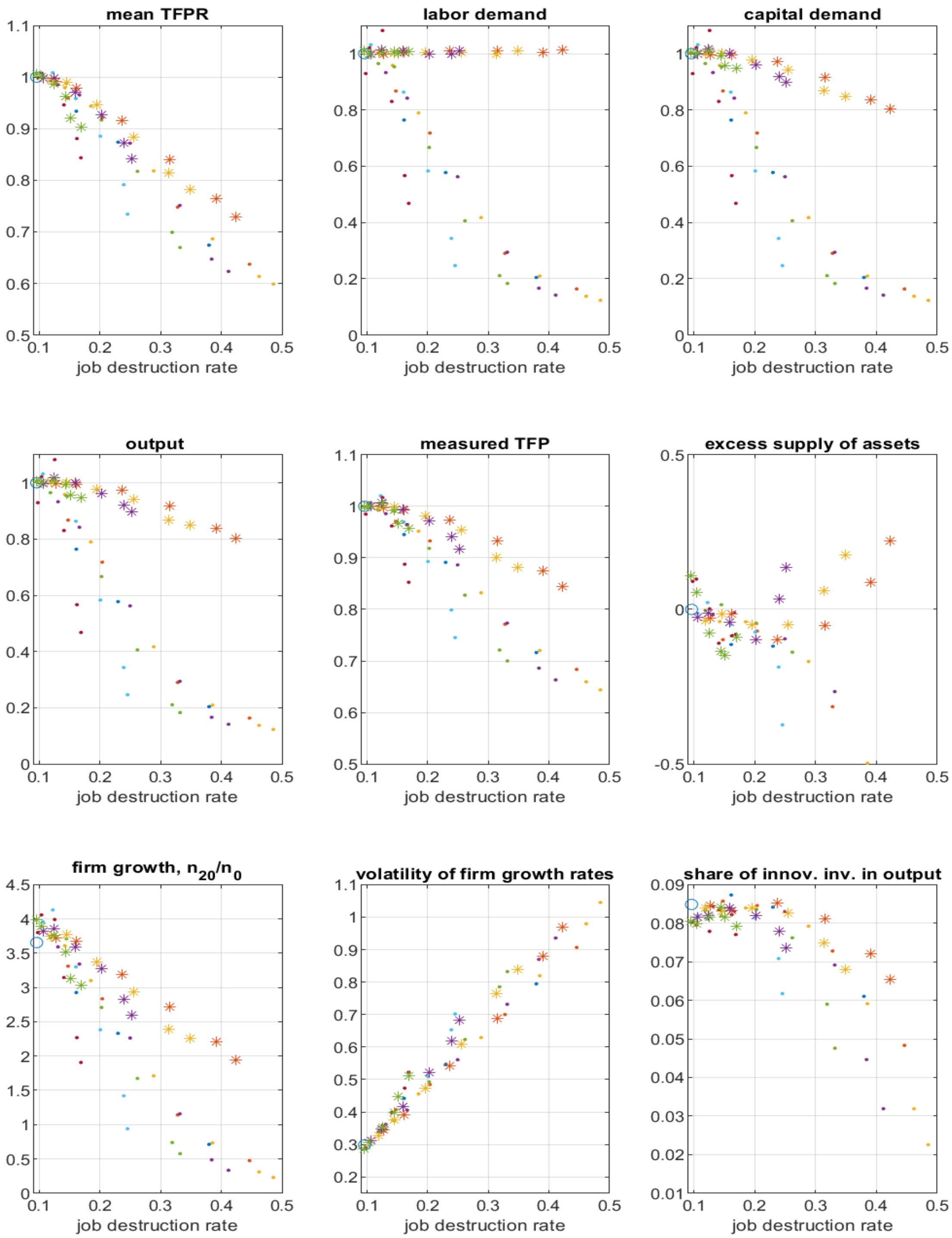

Figure 9: The relationship between the job destruction rate and other aggregate variables arising when $\sigma = 2$ is replaced with $\sigma = 3.5$ in the benchmark calibration. This figure is an analogue of Figure 5 under the new parameter values.

References

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

Angeletos, George-Marios, "Uninsured idiosyncratic investment risk and aggregate saving," *Review of Economic Dynamics*, 2007, *10* (1), 1 – 30.

Bento, Pedro and Diego Restuccia, "Misallocation, Establishment Size, and Productivity," *American Economic Journal: Macroeconomics*, July 2017, *9* (3), 267–303.

Bhattacharyaa, Dhritiman, Nezih Guner, and Gustavo Ventura, "Distortions, endogenous managerial skills and productivity differences," *Review of Economic Dynamics*, 2013, *16*, 11–25.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009, *77* (3), 623–685.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, August 2011, *101* (5), 1964–2002.

Buera, Francisco, Joe Kaboski, and Yongseok Shin, "Entrepreneurship and Financial Frictions: A Macrodevelopment Perspective," *Annual Review of Economics*, 2015, *7*.

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 2006, *114* (5), 835–870.

Caggese, Andrea, "Entrepreneurial risk, investment, and innovation," *Journal of Financial Economics*, 2012, *106* (2), 287 – 307.

Castro, Rui, Gian Luca Clementi, and Glenn Macdonald, "Legal Institutions, Sectoral Heterogeneity, and Economic Development," *The Review of Economic Studies*, 04 2009, *76* (2), 529–561.

Corrado, Carol, Jonathan Haskel, Cecilia Jona-Lasinio, and Massimiliano Iommi, "Intangible investment in the EU and US before and since the Great Recession and its contribution to productivity growth," *Journal of Infrastructure, Policy and Development*, 02 2018, *2*.

Davis, Steven J., John Haltiwanger, Ron Jarmin, Javier Miranda, Christopher Foote, and Eva Nagypal, "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms [with Comments and Discussion]," *NBER Macroeconomics Annual*, 2006, *21*, 107–179.

Diaz-Gimenez, Javier, Andy Glover, and Jose-Victor Rios-Rull, "Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update," *Federal Reserve Bank of Minneapolis Quarterly Revie*, 2011, *34.*

Donovan, Kevin, Will Jianyu Lu, and Todd Schoellman, "Labor Market Flows and Development," *mimeo*, 2019.

_ , _ , and _ , "Labor Market Dynamics and Development," *Manuscript*, 2022.

Doraszelski, Ulrich and Jordi Jaumandreu, "RandD and Productivity: Estimating Endogenous Productivity," *The Review of Economic Studies*, 2013, *80* (4 (285)), 1338–1383.

Fernandez-Villaverde, Jesus, Pablo Guerron-Quintana, Keith Kuester, and Juan Rubio-Ramirez, "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, November 2015, *105* (11), 3352–84.

Griliches, Zvi, "Issues in Assessing the Contribution of Research and Development to Productivity Growth," *Bell Journal of Economics*, 1979, *10* (1), 92–116.

Guner, Nezih, Gustavo Ventura, and Yi Xu, "Macroeconomic implications of size-dependent policies," *Review of Economic Dynamics*, 2008, *11* (4), 721 – 744.

Haltiwanger, John, Stefano Scarpetta, and Helena Schweiger, "Assessing Job Flows across Countries: The Role of Industry, Firm Size and Regulations," *Manuscript*, 2006.

Hopenhayn, Hugo, "Entry, Exit, and firm Dynamics in Long Run Equilibrium," *Econometrica*, 1992, *60*, 1127–1150.

_ and Richard Rogerson, "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 1993, *101* (5), 915–938.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India*," *The Quarterly Journal of Economics*, 11 2009, *124* (4), 1403–1448.

_ **and** _ , "The Life Cycle of Plants in India and Mexico *," *The Quarterly Journal of Economics*, 2014, *129* (3), 1035–1084.

Hugget, Mark, "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, 1993, *17*, 953–969.

_ , "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 1996, *38*, 469–494.

Julio, Brandon and Youngsuk Yook, "Political Uncertainty and Corporate Investment Cycles," *The Journal of Finance*, 2012, *67* (1), 45–83.

Levine, Ross, "Chapter 12 Finance and Growth: Theory and Evidence," in Philippe Aghion and Steven N. Durlauf, eds., *Philippe Aghion and Steven N. Durlauf, eds.*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, 2005, pp. 865 – 934.

Lucas, Robert, "On the Size Distribution of Business Firms," *The Bell Journal of Economics*, 1978, *9* (2), 508–523.

McGrattan, Ellen R. and Edward C. Prescott, "Unmeasured Investment and the Puzzling US Boom in the 1990s," *American Economic Journal: Macroeconomics*, October 2010, *2* (4), 88–123.

Michelacci, Claudio and Fabiano Schivardi, "DOES IDIOSYNCRATIC BUSINESS RISK MATTER FOR GROWTH?," *Journal of the European Economic Association*, 2013, *11* (2), 343–368.

Midrigan, Virgiliu and Daniel Yi Xu, "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 2014, *104*, 422–58.

Moscoso-Boedo, Hernan, "NEW FACTS ABOUT FIRM RISK ACROSS COUNTRIES AND OVER THE BUSINESS CYCLE," *Economic Inquiry*, 2018, *56* (1), 510–529.

Piketty, Thomas and Emmanuel Saez, "Income Inequality in the United States, 1913 to 1998," *The Quarterly Journal of Economics*, 2003, *118* (1), 1–41.

Poschke, Markus, "The Firm Size Distribution across Countries and Skill-Biased Change in Entrepreneurial Technology," *American Economic Journal: Macroeconomics*, July 2018, *10* (3), 1–41.

Qian, Long and Galina Vereshchagina, "Capital
nancing constraints, size-dependent distortions, and aggregate productivity," *mimeo*, 2022.

Quadrini, Vincenzo, "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics*, 2000, *3* (1), 1 – 40.

Restuccia, Diego and Richard Rogerson, "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 2008, *11* (4), 707 – 720.

Stokey, Nancy L., "Wait-and-see: Investment options under policy uncertainty," *Review of Economic Dynamics*, 2016, *21*, 246–265.

Vereshchagina, Galina, "Financial constraints and economic development: the role of firm productivity investment," *mimeo*, 2022.

_ **and Hugo A. Hopenhayn**, "Risk Taking by Entrepreneurs," *American Economic Review*, December 2009, *99* (5), 1808–30.