

Electoral Systems and Inequalities in Government Interventions

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


ABSTRACT

This paper studies the political determinants of inequalities in government interventions under majoritarian (MAJ) and proportional representation (PR) systems. We propose a model of electoral competition with highly targetable government interventions and heterogeneous localities. We uncover a novel *relative electoral sensitivity effect* that affects government interventions only under the majoritarian (MAJ) systems. This effect tends to reduce inequality in government interventions under MAJ systems when districts are composed of sufficiently homogeneous localities. This effect goes against the conventional wisdom that MAJ systems are necessarily more conducive to inequality than PR systems. We illustrate the empirical relevance of our results with numerical simulations on possible reforms of the U.S. Electoral College.

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1 Introduction

Government interventions are fraught with inequality. Substantial geographic disparities have been documented both in terms of the quantity and quality of public goods and services (Alesina et al. 1999, World Bank 2004, Banerjee et al. 2008) and in terms of taxation (Albouy 2009, Troiano 2017). A large literature on distributive politics highlights the importance of political factors (Brams and Davis 1974 and Weingast et al. 1981). Particularly significant dimensions to this regard include the apportionment of constituencies (Ansolabehere et al. 2002); their electoral contestability (Strömberg 2008); and voter characteristics such as turnout (Martin 2003, Strömberg 2004), information (Besley and Burgess 2002, Strömberg 2004), the presence of core supporters or co-ethnics (Cox and McCubbins 1986, Dixit and Londregan 1996, Schady 2000, Hodler and Raschky 2014) and their responsiveness to electoral promises (Johansson 2003, Strömberg 2008). Overall, the political distortions of government interventions appear sizable. For instance, using Brazilian data, Finan and Mazzocco (2016) estimate that 25 percent of the public funds allocated by legislators are distorted relative to the socially optimal allocation.

This paper revisits a classic question in the literature: the effects of electoral systems on government interventions. We focus on the *Majoritarian* (MAJ) and the *Proportional Representation* (PR) systems.¹ Both systems are ubiquitous² and debate over which one to use is common, both during democratic transitions and in established democracies. In the latter case, such debates have relatively frequently led to actual electoral reforms.³ Moreover, our analysis is of direct relevance for discussions over the potential reform of

¹In MAJ systems, a multitude of electoral districts each select a limited number of representatives using some winner-takes-all method. The archetype of such systems is the version with single-member districts and first-past-the-post voting. In PR systems, there are fewer electoral districts that each select at least two representatives, more or less in proportion to the vote shares of each party. The epitome of PR systems is the version with a single nationwide electoral district. For further discussion of the differences between these two systems, see, e.g., Shugart and Taagepera (2017).

²82% of legislative elections held in the 2000s took place under either a MAJ or PR system (Bormann and Golder 2013).

³Historically, the evolution of electoral systems has largely concerned countries that initially adopted MAJ and later decided to adopt PR (see the discussion in, e.g., Rokkan 1970, Boix 1999 and Leemann and Mares 2014). Yet, this is far from the rule: Colomer (2004) (p. 55) counts “82 major electoral system changes for assemblies [...] in 41 countries” between the early nineteenth century and 2002. Of these reforms, in 40 cases a MAJ system was replaced by a PR (or a mixed) system, and 13 cases consisted of a reform from a PR to MAJ (or mixed) system. The example of Italy is particularly striking: its electoral system was reformed three times between 1993 and 2015.

the U.S. Electoral College, a majoritarian electoral system.⁴

The conventional wisdom is that MAJ systems are more conducive to inequality, as they provide steeper incentives for targeting government interventions onto specific groups (Persson and Tabellini 1999, 2000; Persson 2002; Lizzeri and Persico 2001; Milesi-Ferretti et al. 2002; Grossman and Helpman 2005; Strömberg 2008). This view is based on multiple theoretical arguments, which we detail in Section 2. One of the most powerful is that of Lizzeri and Persico (2001): in MAJ systems, parties only need *fifty* percent of the votes in *fifty* percent of the electoral districts to win a majority of seats in the national assembly. By contrast, they need fifty percent of *all* votes in PR systems, doubling the number of votes necessary to hold a majority of seats. A related mechanism underlines the importance of district contestability (the likelihood that electoral promises change which party wins a district). In MAJ systems, parties target the most contestable districts (Persson and Tabellini 1999, 2000 and Persson 2002), or the district most contestable when the national assembly is close to changing hand (Strömberg 2008).

Yet, this view overlooks another important difference: the geographic distribution of voters matters differently in the two systems. In MAJ systems, parties must win in different electoral districts in order to win multiple seats. Paraphrasing Lizzeri and Persico (2001), they need to win fifty percent of the votes in *at least* fifty percent of the districts. This geographical constraint is largely absent in PR systems. Indeed, additional votes from any location help a party win more seats in the national assembly.

To take into account the geographical distribution of voters, we develop a model of electoral competition in which two parties compete by targeting governmental resources (cash transfers, goods, or services) to groups of voters, called *localities*. The first key feature is that localities are heterogeneous along several dimensions. Specifically, they may differ in population size, turnout rate, and swingness. Together, these characteristics define what we call the *electoral sensitivity* of a locality: a measure of the electoral responsiveness of that group of voters to government intervention. A large empirical literature shows

⁴As discussed in Whitaker and Neale (2004), this is a recurrent debate: “Since the adoption of the Constitution, [...] in almost every session of Congress, resolutions have been introduced proposing Electoral College reform. Indeed, more proposed constitutional amendments have been introduced in Congress regarding Electoral College reform than on any other subject.” (p. CRS-17) A current initiative among states to award all their electoral votes to the candidate that wins the popular vote, the National Popular Vote Interstate Compact (www.nationalpopularvote.com), is gaining momentum.

that each component of the electoral sensitivity influences the allocation of governmental resources (see, e.g., Martin 2003, Strömberg 2004, 2008, and Golden and Min 2013 for a survey), and we find substantial heterogeneity in sensitivity across U.S. counties and states (see Section 6.2).

The second key feature of our model is that interventions can target a locality; a level that is typically finer than the electoral district. As we discuss at length in Appendix A, such targetability is relevant for a wide array of government structures and interventions. Substantial empirical evidence for both the U.S. and other countries shows that political factors affect the allocation of various types of government interventions at the sub-district level (see, e.g., Ansolabehere et al. 2002 and Berry et al. 2010, Hsieh et al. 2011, Kriner and Reeves 2015, and De Luca et al. 2018).

With these two features, our model uncovers a *relative electoral sensitivity effect* present only in MAJ systems. In these systems, the political attractiveness of a locality depends positively on its electoral sensitivity, but *negatively* on the aggregate sensitivity of the localities in the same districts (its neighbors). The positive effect of sensitivity is present in both MAJ and PR systems: targeting resources to more sensitive localities generates a larger increase in expected vote share, making them more attractive politically. The negative effect of the neighbors' sensitivity in MAJ systems is novel: when aggregate sensitivity is high, the probability that the district swings from one party to the other decreases. Therefore, gaining extra votes in that district becomes less relevant. In contrast, in PR systems the geographic distribution of the votes collected by each party is irrelevant: there are no districts to swing. This creates an incentive to allocate greater resources to localities that are electorally more sensitive, independently of their neighbors.

Two recent empirical papers use our model and test the relative sensitivity effect in the U.S. Stashko (2020) focuses on U.S. state governments and legislative elections. She finds that our relative electoral sensitivity effect influences significantly (both statistically and economically) the allocation of state expenditures to counties. That is, the amount received by a county depends not only on its electoral sensitivity, but also on that of the other counties in the same district. Naddeo (2020) focuses on the allocation of FEMA funds to counties by state governments, similarly finding that the relative sensitivity of a

county influences significantly the allocation of funds.

We then explore the consequences of the relative electoral sensitivity effect for the comparison of government interventions under MAJ and PR systems. We find that the relative sensitivity effect may induce parties to spread resources more evenly across districts under MAJ than PR systems. Hence, electoral reforms may deliver outcomes that are at odds with common theoretical predictions. Consider for instance a hypothetical country in which low sensitivity localities are grouped in one district and high sensitivity localities are located in a second district. For the sake of the argument, assume this is the only difference between the two districts. Then, under MAJ systems, all localities receive similar allocations; there is low inequality in government interventions. This is because localities are relatively homogeneous within each district, implying that all *relative* sensitivities are close to 1. A switch to PR would instead induce a reallocation of resources towards high sensitivity localities in the second district; inequality in government interventions would increase. Only *absolute* sensitivity matters under PR.

For the specific case of one locality per district (i.e., no sub-district targeting), we formally identify under which conditions MAJ systems produce government allocations socially superior to PR. These conditions depend, on the one hand, on the comparison of the heterogeneity of aggregate sensitivities across districts and, on the other hand, the heterogeneity of district contestabilities. Yet, by construction, this specific case overlooks within-district inequalities. We show that taking these inequalities into account may reverse the conclusion that one system generates more inequality than the other.

We then use our model to assess, both theoretically and numerically, the effects of several possible reforms of the U.S. Electoral College: (i) the National Popular Vote (NPV) and (ii) a version of the Electoral College in which the electoral votes of each state are allocated proportionally (PR-EC). The data show substantial heterogeneity in sensitivity both within and across states. As a result, our numerical exercise predicts an increase in geographical targeting (measured by a Gini coefficient of the allocations) following a reform to NPV or PR-EC. However, the lion's share of the heterogeneity in sensitivity is driven by differences in population size, which can be socially desirable. For this reason, the comparison between the systems reverses when focusing on socially undesirable in-

equalities (measured by an Atkinson-like index). Interestingly, we find that PR-EC would be preferable for the U.S. Moreover, sub-district targeting proves to be central to the assessment of these reforms. We find that the predicted magnitudes of a state's gains or losses following a reform are substantially different when counties in a given state do not need to be treated uniformly.

Finally, we explore two additional extensions. We consider a version of our model in which politicians can target socio-demographic groups that span multiple geographical districts, and derive a modified version of the relative sensitivity effect for this context. A natural follow-up question is then how the electoral system modifies the incentive to continue targeting specific localities or groups, or to instead focus on public goods that benefit the population more broadly. We show at what point heterogeneity reverses the usual finding that PR would favor public good provision (Persson et al. 2000, Lizzeri and Persico 2001).

2 Related Literature

Our paper contributes to the literature on distributive politics, which assesses the allocation of governmental resources to various subsets of the population. Within this body of work, our analysis relates most closely to studies on the effects of electoral systems. As mentioned, a recurrent theme in this literature is that parties want to target a smaller fraction of the population in MAJ systems than in PR ones. Various mechanisms produce this outcome, including the already discussed fifty-of-fifty percent mechanism highlighted by the so-called Colonel Blotto literature (see, e.g., Lizzeri and Persico 2001). While this literature also allows for sub-district targeting (even voter-level targeting), it does not consider heterogeneity at that level. In the presence of such heterogeneity, the fifty-of-fifty percent mechanism could become fifty-of-*at-least*-fifty percent.

Another mechanism underlines the importance of district contestability (the likelihood that electoral promises change which party wins a district) in MAJ systems. Persson and Tabellini (1999, 2000) and Persson (2002) show that parties target the most contestable districts (i.e., those most likely to swing from one party to the other). Such incentives do not exist in PR systems. Strömberg (2008) stresses the joint importance of the con-

testability and the decisiveness of a district in the national assembly – i.e., the likelihood that it modifies the identity of the candidate or party winning the national election. Our model differs from Strömberg (2008)’s in several dimensions. The key novelty is that we allow for heterogeneity and targeting at the sub-district level. Strömberg instead focuses on targeting and heterogeneity at the U.S. state level. This difference allows us to identify the relative sensitivity effect and explore how it affects the allocation under MAJ vs. PR systems. As we illustrate numerically in Section 6.2, allowing for sub-district heterogeneity and targeting also influences our assessment of the winners and losers of potential reforms of the U.S. electoral system.

Grossman and Helpman (2005) highlight the importance of bargaining between party leaders (who care about national welfare) and legislators (who care about the welfare of their constituents).⁵ They argue that legislators have a national constituency in PR systems, which aligns their incentives with those of party leaders. In contrast, in MAJ systems, legislators’ constituencies are geographically limited, hence the tension with party leaders. Legislator bargaining power immediately leads to more geographically targeted policies than under PR systems. Our approach is complementary. While our model abstracts from the tension between party leaders and legislators, we show how the party leaders’ preferred allocation of resources varies with the electoral system. Due to our relative sensitivity effect, there are situations in which party leaders have stronger incentives to target policies under PR systems than under MAJ systems.

Rogowski and Kayser (2002) point to the seats-votes elasticity as a key factor influencing the targeting of government interventions. When elasticity is higher, parties have stronger incentives to target groups that can deliver many votes at the margin. Given that MAJ systems have a higher seats-votes elasticity than do PR systems (Taagepera and Shugart 1989), there should be more targeting under MAJ systems. Our results refine this prediction. We microfound the electoral sensitivity of localities and show how it differentially affects government interventions under MAJ and PR systems.

⁵Party discipline gives more power to the former, and tends to be higher in parliamentary than in presidential systems (see e.g., Tsebelis 1995; Carey 2007; Dewan and Spirling 2011), a dimension that is not explicitly considered in our model. Yet even in presidential systems, the importance of the party label means that the party’s platform as a whole may end up being more important than that of individual politicians (see, e.g., Snyder and Ting 2002; Krasa and Polborn 2018).

At first sight, some of our results might seem reminiscent of Dixit and Londregan (1998) who study the allocation of governmental resources in federal systems. They model two levels of government, the federal and the state, that move sequentially, and find that state governments target the most sensitive localities while the federal government targets the states with high average electoral sensitivity. Ultimately, governmental resources flow towards highly sensitive localities that have highly sensitive neighbors. The authors thus find a positive effect of the electoral sensitivity of a locality’s neighbors. We find the exact opposite: localities with highly sensitive neighbors receive lower levels of governmental resources. As previously mentioned, recent empirical evidence supports our theoretical prediction of a relative sensitivity effect (Stashko 2020 and Naddeo 2020).

There is a large empirical literature comparing MAJ and PR systems, which can be divided into two strands.⁶ The first assesses the extent to which government interventions are targeted in each system. Cross-country comparisons show that PR systems are associated with lower levels of “targeted” spending and higher levels of “universal” spending (Persson and Tabellini 1999, 2000; Milesi-Ferretti et al. 2002; Blume et al. 2009; Funk and Gathmann 2013). Aidt et al. (2006) instead study the *changes* from MAJ to PR rules that took place in 10 European countries between 1830 and 1938. They find that such reforms led to a *decrease* in spending classified as universal.

The second strand focuses on trade barriers in MAJ and PR systems, interpreting them as targeted transfers. The empirical evidence is mixed. Using cross-country regressions, a number of studies find that MAJ countries are more protectionist (Evans 2009; Hatfield and Hauk 2014; Rickard 2012), while others find more protectionism in PR countries (Mansfield and Busch 1995; Rogowski and Kayser 2002; Chang et al. 2008; Betz 2017). The difference seems to originate from the type of trade barriers considered: Non-tariff barriers tend to be used more often in PR systems, while tariffs seem to be employed more heavily in MAJ systems.

Our relative electoral sensitivity effect sheds new light on this mixed empirical evidence. To this regard, the theoretical literature provides at least one other reason why PR systems

⁶Gagliarducci et al. (2011), who use Italian data, and Stratmann and Baur (2002), who use German data, do not fit this nomenclature as they focus on the behavior of individual politicians rather than the behavior of the parties controlling the government budget.

may lead to more targeting of government interventions: there are usually more parties in PR systems. While there is reason to believe that our results would be robust to an increase in the number of parties (see Seror and Verdier 2017), some models (e.g., Cox 1990, Myerson 1993, and Lizzeri and Persico 2005) suggest that such an increase is associated with a narrower targeting of government interventions. Using Indian data, Chhibber and Nooruddin (2004) find that the provision of public goods decreases when the number of parties increases, and the inverse is true for the provision of club goods. Similar results emerge in multi-country panel analyses such as that of Park and Jensen (2007), who focus on agricultural subsidies in OECD countries, or Castanheira et al. (2012), who assess tax reforms in EU countries.

3 The Economy

Consider a country with a continuum of individuals of total mass 1. The population is partitioned into *localities* $l \in \{1, 2, \dots, L\}$ of size n_l , *s.t.* $\sum_l n_l = 1$. Each locality belongs to an *electoral district* $d \in \{1, 2, \dots, D\}$. If $L > D$, our setup allows governmental resources to be targeted at a finer level than the electoral district (see the discussion in the Introduction and Appendix A). If instead $L = D$, then localities correspond to districts. Section 7 extends the setup to settings with socio-demographic groups that can span multiple localities.

An elected government allocates a total budget y among the different localities. We denote by q_l the amount of government intervention *per capita* in locality l . The vector of interventions is denoted by $\mathbf{q} = \{q_1, \dots, q_L\}$. We cover a variety of local government interventions that range from pure public goods to pure transfers. The central difference between them is the extent to which provision costs vary with population size. Specifically, with pure public goods, costs must be independent of the number of individuals who benefit. With pure transfers, costs must instead be directly proportional to the number of individuals who benefit. To capture intermediate situations, we let the cost of providing q_l to the n_l individuals in locality l be $n_l^\alpha q_l$, with $\alpha \in [0, 1]$. The government's aggregate

budget constraint is thus:

$$\sum_l n_l^\alpha q_l \leq y. \quad (1)$$

When $\alpha = 1$, the government intervention is a pure transfer, and the budget constraint becomes: $\sum_l n_l q_l \leq y$. When $\alpha = 0$, q_l is a pure local public good, and the budget constraint becomes $\sum_l q_l \leq y$.

Individuals of locality l have preferences $u_l(\mathbf{q})$ for the government intervention, with $\partial u_l(\mathbf{q})/\partial q_l > 0 > \partial^2 u_l(\mathbf{q})/\partial q_l^2$ – the function is strictly increasing and concave. In most of the paper, we assume that $u_l(\mathbf{q}) = u(q_l)$, meaning that government interventions do not produce spillover across localities. This simplifies the analysis but is easily generalized (see Section 7).

3.1 The Optimal Allocation

Before introducing a model of electoral competition, we establish the politics-free benchmark. As argued in Becker (1958), any division of the budget across localities is Pareto efficient. However, all allocations are not equivalent when using a Benthamite welfare function. A utilitarian social planner would:

$$\max_{\mathbf{q}} \mathcal{W}(\mathbf{q}) = \sum_l n_l u_l(\mathbf{q}), \quad s.t. \quad \sum_l n_l^\alpha q_l = y. \quad (2)$$

According to this criterion, the social welfare (SW) optimum must satisfy the standard Samuelsonian conditions:

$$\frac{\partial u_l(\mathbf{q})}{\partial q_l} = \lambda^{\text{SW}} n_l^{\alpha-1} \quad \forall l, \quad (3)$$

where λ^{SW} is the Lagrange multiplier associated with the budget constraint.

It is important to note that, except for the limit case of pure transfers (i.e., as long as $\alpha < 1$), the socially optimal allocation responds positively to population size in the locality. This implies that the social optimum tolerates “vertical inequalities.” In other words, inhabitants of larger localities ought to benefit from higher levels of government intervention. Electoral competition may, however, generate incentives that produce a different pattern, which can create undesirable inequality.

3.2 A Measure of Inequality

To assess inequality in government allocation, we build on Atkinson (1970, 1983)'s approach for measuring the social cost of politically motivated distortions. Following Atkinson, assume CRRA preferences, with $\rho(> 0)$ denoting individual risk aversion:

$$u_l(\mathbf{q}) = \begin{cases} \ln(q_l) & \text{if } \rho = 1 \\ \frac{q_l^{1-\rho}}{1-\rho} & \text{if } \rho \neq 1. \end{cases} \quad (4)$$

Under these preferences, maximizing the social welfare function (2) implies that a locality l should receive a share $\sigma_l^{SW} \equiv q_l^{SW}/y = n_l^{(1-\alpha)/\rho} / \sum_{l'} n_{l'}^{(1-\alpha)/\rho}$ of the budget y . Denoting by $\widetilde{W}(y)$ the indirect utility function of a budget y under the social optimum, we have:

$$\widetilde{W}(y) = \begin{cases} \ln(y) + \sum_l n_l \ln \left(\frac{n_l^{(1-\alpha)}}{\sum_{l'} n_{l'}^{(1-\alpha)}} \right) & \text{if } \rho = 1; \\ \frac{1}{1-\rho} \sum_l n_l [\sigma_l^{SW} y]^{1-\rho} & \text{if } \rho \neq 1. \end{cases}$$

Next, contrast that level of welfare with that which results from some *actual* allocation \mathbf{q} . Denote that level by $\mathcal{W}(\mathbf{q})$. Generically, the budget actually needed to reach a welfare level $\mathcal{W}(\mathbf{q})$ can be reduced by re-optimizing the allocation \mathbf{q} . This allows us to define y^E as the smallest possible budget needed to reach $\mathcal{W}(\mathbf{q})$:

$$y^E(\mathbf{q}) = \widetilde{W}^{-1}(\mathcal{W}(\mathbf{q})).$$

Following Atkinson's approach, we use the comparison between y^E and y to measure inequality in government interventions:⁷

$$A(\mathbf{q}) \equiv 1 - \frac{y^E(\mathbf{q})}{y}. \quad (5)$$

This proposed measure of inequality of governmental allocation captures the social cost of politically motivated distortions. That is, the fraction of the budget that could be saved by correcting political distortions, without reducing welfare. All deviations from the optimal

⁷In particular, we have $A_\rho^P \equiv 1 - \frac{y^P}{y} = \begin{cases} 1 - \frac{1}{y} \Pi_l \left(\frac{q_l \sum_k n_k^{1-\alpha}}{n_l^{1-\alpha}} \right)^{n_l} & \text{if } \rho = 1; \\ 1 - \left[\frac{\sum_l n_l (q_l/y)^{1-\rho}}{\sum_l n_l (\sigma_l^{SW})^{1-\rho}} \right]^{\frac{1}{1-\rho}} & \text{if } \rho \neq 1. \end{cases}$

allocation are penalized: those that introduce inequalities for purely political reasons, as well as deviations that fail to differentiate between localities with different population sizes as prescribed by the social optimal in (3).⁸ At the extremes, $A(\mathbf{q})$ is 0 when the allocation is fully efficient, and 1 when it is pure waste.

4 The Politics

4.1 A Model of Electoral Competition

We consider an election with two parties, A and B , that compete for seats in the national assembly.⁹ In the body of the paper, we focus on the case in which parties *maximize their expected number of seats*. This setup is not only more tractable, but also appears empirically more relevant – see discussion in Appendix B. Nonetheless, we show in the same Appendix that our main results hold when we assume that parties maximize their probability of winning a majority of seats.

Central to our analysis is the contrast between *proportional representation* (PR henceforth) and *majoritarian* (or *single-member district*) systems (MAJ henceforth). In most of what follows, we work with the single-district PR system, such that seats are attributed in proportion to the *fraction of national votes* garnered by each party. Our model of the PR system is thus a stylized version of that used in Israel or in the Netherlands. Other countries (e.g., Belgium and Brazil) apply proportional representation at the district level. Such “district-PR systems” produce the same outcomes as the PR version of the Electoral College that we analyze in Section 6. Our modeling of the MAJ system assumes single member districts, to be won by simple majority. This is a stylized version of electoral systems such as that of UK parliamentary elections. In Section 6, we also extend the analysis to other versions of the majoritarian system, in particular the US Electoral College.

⁸Other sources of differentiation could arguably be socially desirable, such as differences in intensity of preference.

⁹As discussed in Section 2, the results in Seror and Verdier (2017) suggest that, in a probabilistic voting model such as ours, under some assumptions about the distributions of idiosyncratic and state shocks, an increase in the number of parties should not substantially change the parties’ targeting strategies.

To maximize their expected seat share (or probability of winning in Appendix B), both parties simultaneously make a binding budget proposal, \mathbf{q}^A and \mathbf{q}^B , that details the allocation of resources across localities. These proposals must satisfy the government budget constraint (1).

Beyond their population size, localities can be heterogeneous along various dimensions. They may differ in turnout rates, and/or the distribution of voter preferences. They may also belong to different electoral districts. Other dimensions of heterogeneity, such as information about electoral promises or partisanship, can easily be accommodated.¹⁰

While n_l is the total population in locality l that will benefit from the public good, $t_l n_l$ is the number of active voters.¹¹ Consider voter i in locality l : she compares the potential utility derived from the allocation proposed by A as opposed to B . Each party $P \in \{A, B\}$ proposes a local public good consumption q_l^P . The associated utility differential between the two parties is thus $\Delta u_l(\mathbf{q}) := u_l(\mathbf{q}^A) - u_l(\mathbf{q}^B)$.

Citizens also care about dimensions other than public goods. In line with the traditional probabilistic voting literature (e.g., Lindbeck and Weibull 1987, Dixit and Londregan 1996, Persson and Tabellini 2000, or Strömberg 2004, 2008), we assume that each voter is characterized by two, independently distributed, ideological preference “shocks.” The former is idiosyncratic and is meant to capture the diversity of ideological positions in each locality. We label individual i ’s ideology $\nu_{i,l}$. It is distributed according to some well-behaved, full support, and strictly continuous cumulative distribution function $\Phi_l(\nu)$:

$$\Phi_l(-\infty) = 0; \quad \Phi_l(\infty) = 1; \quad \frac{\partial \Phi_l(\nu)}{\partial \nu} := \phi_l(\nu) > 0, \quad \forall \nu \in \mathbb{R}.$$

The second ideological shock proxies valence or popularity shocks that are common to all

¹⁰In a previous version of the paper, we incorporated information, as done in Strömberg (2004), and found that it entered our results in the same way as turnout. We also considered the coexistence of swing and partisan voters, as in Dixit and Londregan (1996), finding that the fraction of partisan voters in a locality influences the allocation of governmental resources in the opposite way as does swingness under both the PR and MAJ systems.

¹¹We could also endogenize turnout, as in Lindbeck and Weibull (1987). In such an extended model, voters all face a positive cost of voting. For some of them, this cost is prohibitively high, and they abstain. For others, only voters with a sufficiently large utility differential between the two parties cast a ballot. We can show that the equilibrium allocations in this case would be the same as in the baseline model (proof available upon request).

voters in a district. We let this district shock δ_d follow some other cumulative distribution function $\Gamma_d(\cdot)$, with similar properties:

$$\Gamma_d(-\infty) = 0; \quad \Gamma_d(\infty) = 1; \quad \frac{\partial \Gamma_d(\delta)}{\partial \delta} := \gamma_d(\delta) > 0, \quad \forall \delta \in \mathbb{R}.$$

Note that we can relax the full support assumption, although at the cost of additional assumptions to guarantee interior solutions. This allows us to work under the textbook version of the probabilistic voting model with uniform distributions as in Section 5 (for more detail, see Appendix C).

Combining these three components, individual i votes for party A if and only if:

$$\nu_{i,l} + \delta_d \leq \Delta u_l(\mathbf{q}), \tag{6}$$

which implies that, for some given popularity shock δ_d , A 's vote share in locality l is $\Phi_l(\Delta u_l(\mathbf{q}) - \delta_d)$, and her district vote share is:

$$\pi_d(\mathbf{q}; \delta_d) = \sum_{l \in d} \frac{t_l n_l}{T_d} \Phi_l(\Delta u_l(\mathbf{q}) - \delta_d), \tag{7}$$

which is simply the average of the locality vote shares, weighted by their share of total district votes. $T_d := \sum_{k \in d} t_k n_k$ denotes the total number of votes in the district.

In the next two subsections, we establish the general properties of a pure strategy equilibrium, under the PR and MAJ systems. We will thus be working under the assumption that such an equilibrium exists. Existence conditions have long been established (see, for instance, Lindbeck and Weibull 1987). In essence, a sufficient condition is that district vote shares are quasi-concave in each q_l , which implicitly requires that the distribution functions $\Phi(\nu)$ and $\Gamma(\delta)$ are not too convex.

4.2 Equilibrium Allocation under Proportional Representation

Recall that, under both systems, each party's objective is to maximize its expected number of seats. The difference in objective function stems from how votes translate into seats under the different systems. Under the PR system, maximizing the expected share of seats in the national assembly is equivalent to maximizing the country-wide expected vote count. This translates into the following objective function for party A :

$$\max_{\mathbf{q}^A | \sum_l n_l^\alpha q_l = y} \pi_{PR}(\mathbf{q}) = \mathbb{E}_\delta \left[\sum_l t_l n_l \Phi_l(\Delta u_l(\mathbf{q}) - \delta_{d(l)}) \right], \quad (8)$$

where $d(l)$ is the district that contains locality l . Whenever an interior, pure strategy, equilibrium exists, it is implicitly defined by the following first order conditions:

$$u'_l(\mathbf{q}) t_l n_l \mathbb{E}_\delta [\phi_l(\Delta u_l(\mathbf{q}_0) - \delta_d)] = n_l^\alpha \lambda^{PR}, \quad \forall l, \quad (9)$$

where λ^{PR} is the Lagrange multiplier of the budget constraint under PR. Following the same steps for party B leads to identical FOCs, which implies that $\mathbf{q}^A = \mathbf{q}^B$, and hence $\Delta u_l(\mathbf{q}_0) = 0$, in equilibrium (see e.g., Lindbeck and Weibull 1987).

Let $\bar{\phi}_l = \int_{\delta_d} \phi_l(-\delta_d) d\Gamma_d(\delta_d)$ be the *expected density of swing voters* in locality l when the allocation is symmetric. We define the *electoral sensitivity of locality l* as:

$$s_l := \bar{\phi}_l t_l n_l. \quad (10)$$

A locality is electorally more sensitive if it has a higher proportion of swing voters (a high $\bar{\phi}_l$), and/or a larger number of active voters. With this notation, (9) simplifies into:

$$u'_l(\mathbf{q}^{PR}) s_l = n_l^\alpha \lambda^{PR} \Leftrightarrow u'_l(\mathbf{q}^{PR}) = \frac{\lambda^{PR}}{s_l n_l^{-\alpha}}. \quad (11)$$

Due to the concavity of the utility function, the FOCs imply that *ceteris paribus* localities with higher electoral sensitivity benefit from more government interventions in equilibrium:

Proposition 1 *In the PR system, $q_l \geq q_l$ if and only if $s_l n_l^{-\alpha} \geq s_l n_l^{-\alpha}$.*

That is, the electorally more sensitive localities receive a larger share of the budget. When government interventions have a public good dimension, i.e., $\alpha < 1$, these are the localities that combine a large number of active voters $t_l n_l$ and more swing voters (a high $\bar{\phi}_l$). When the government intervention is a pure transfer, i.e., $\alpha = 1$, population size does not play a role but the other factors influencing electoral sensitivity do still matter. Importantly, in both cases, aggregate popularity shocks δ_d are not explicitly present in (11).

4.3 Equilibrium Allocation under the Majoritarian System

Under the MAJ system, the same objective of maximizing expected seat share amounts to maximizing the number of districts won. Specifically, each seat is associated to a specific district, and winning a district requires winning a plurality of its votes, i.e., $\pi_d(\cdot) \geq \frac{1}{2}$. This introduces a stronger geographic constraint on where votes originate than under the PR system.

From (7), A 's vote share in district d , $\pi_d(\cdot)$, is monotonically and strictly decreasing in δ_d . Hence, A wins d whenever δ_d is sufficiently small, and loses it whenever it is sufficiently large. Define $D_d(\mathbf{q})$ as the unique cutoff value of δ_d that separates district loss from district win given an allocation \mathbf{q} . That is, for each district, $D_d(\mathbf{q})$ is implicitly defined by:

$$\pi_d(\mathbf{q}; D_d(\mathbf{q})) = \sum_{l \in d} \frac{t_l n_l}{T_d} \Phi_l(\Delta u_l(\mathbf{q}) - D_d(\mathbf{q})) = \frac{1}{2}. \quad (12)$$

It follows that A 's probability of winning district d is the CDF of δ_d at this cutoff:

$$p_d(\mathbf{q}) = \Pr_{\delta_d} \left[\pi_d(\mathbf{q}; \delta_d) \geq \frac{1}{2} \right] = \Gamma_d(D_d(\mathbf{q})), \quad (13)$$

and the objective function of the party has become:

$$\max_{\mathbf{q}^A | \sum_l n_l^A q_l = y} \pi_{MAJ}(\mathbf{q}) = \sum_d \Gamma_d(D_d(\mathbf{q})). \quad (14)$$

In comparison with the PR system, the MAJ system thus modifies first order conditions

as:

$$\gamma_d(D_d(\mathbf{q})) \frac{dD_d(\mathbf{q})}{dq_l} = n_l^\alpha \lambda^{MAJ} \quad \forall l, \quad (15)$$

where $\gamma_d(\cdot) := d\Gamma_d(D_d(\mathbf{q}))/d\delta_d$, and λ^{MAJ} is the Lagrange multiplier of the budget constraint under MAJ. Using the implicit function theorem:

$$\frac{dD_d(\mathbf{q})}{dq_l} = -\frac{\partial\pi_d/\partial q_l}{\partial\pi_d/\partial\delta_d} = \frac{\frac{t_l n_l}{T_d} \phi_l(\Delta u_l(\mathbf{q}) - D_d(\mathbf{q}))}{\sum_{j \in d} \frac{t_j n_j}{T_d} \phi_j(\Delta u_j(\mathbf{q}) - D_d(\mathbf{q}))} u'_l(\mathbf{q}^A).$$

Again, the symmetry of the first order conditions for party B implies symmetry in the allocations in equilibrium: $\mathbf{q}^A = \mathbf{q}^B$. Notice that $D_d(\mathbf{q})$ takes on the same value for all symmetric allocations ($\mathbf{q}^A = \mathbf{q}^B$). We denote this value by $\hat{\delta}_d$.

Let $\hat{\phi}_l := \phi_l(-\hat{\delta}_d)$ denote the *swingness* of locality l . In turn, $\hat{s}_l := \hat{\phi}_l t_l n_l$ is the voters' *electoral sensitivity* in locality l , and $\hat{\gamma}_d := \gamma_d(\hat{\delta}_d)$ the *contestability of district d* . In technical terms, $\hat{\gamma}_d$ is the density of the distribution at $\hat{\delta}_d$. In more intuitive terms, it captures the probability that parties end up close to a tie in that district—see the discussion of Proposition 2 below. Note that as in Lindbeck and Weibull (1987), the average bias of the voters in a district may affect its contestability. For instance, if the distributions of district shocks are single-peaked and translations of each other, districts that are very biased towards one candidate or the other will have low contestability.

The first order conditions (15) at the equilibrium tell us that:

$$\hat{\gamma}_d \underbrace{\frac{\hat{s}_l}{\sum_{j \in d} \hat{s}_j}}_{\hat{s}_d} u'_l(\mathbf{q}^A) = n_l^\alpha \lambda^{MAJ}, \quad (16)$$

where \hat{s}_d is the aggregate electoral sensitivity of district d . From the strict concavity of the utility function, it follows directly that:

Proposition 2 *In the MAJ system, $q_l \geq q_{l'}$ if and only if $\hat{\gamma}_{d(l)} \frac{\hat{s}_l n_l^{-\alpha}}{\hat{s}_{d(l)}} \geq \hat{\gamma}_{d(l')} \frac{\hat{s}_{l'} n_{l'}^{-\alpha}}{\hat{s}_{d(l')}}$.*

That is, for a given population size, the localities that receive the largest share of the budget are those in the most *contestable* districts (high values of $\hat{\gamma}_d$), and the ones with

the highest *relative electoral sensitivity* \hat{s}_l/\hat{s}_d .

In the MAJ system, an increase in support for A in locality l affects the winner of the district if and only if it is *pivotal*, i.e., it moves A 's vote share from below to above 50%. For such an increase in A 's support, there is a range of realizations of δ_d , such that the change is pivotal. The more likely δ_d is to fall in that pivotal range, the better the locality is treated.

What determines the likelihood that δ_d falls in this pivotal range? There are two factors. The first is the width of the pivotal range, which is determined by the *relative sensitivity* \hat{s}_l/\hat{s}_d . The higher \hat{s}_l , the more responsive voters in locality l are to an increase in the utility of government allocation. Interpreted loosely, a given increase in q_l buys more votes when electoral sensitivity is higher. This implies a change in the winning party for a wider range of district shocks. However, the electoral sensitivity also influences the voters' responsiveness to the popularity shock δ_d . A higher aggregate electoral sensitivity in the district \hat{s}_d makes the aggregate vote share more unstable, which *reduces* the width of the pivotal range.

The second factor is the *district contestability* $\hat{\gamma}_d$. While relative sensitivity captures the width of the pivotal range, the density of the distribution of the district shock captures the likelihood that the shock takes any of the values in the pivotal range, i.e., its height.

The novel result here is identifying the importance of the relative sensitivity for the allocation in MAJ systems. It means that the resources allocated to a locality also depend on the characteristics of the other localities in the same district, its neighbors. For instance, a locality with high turnout and/or high swingness neighbors should receive fewer resources than a similar locality with low turnout and/or swingness neighbor.¹²

Proposition 2 also highlights that the governmental resources allocated to a locality may be decreasing in its population size. In particular, this is the case when $\hat{s}_l/\hat{s}_d > 1 - \alpha$ (which is always satisfied for pure transfers, and never satisfied for pure public goods). Parties may prefer to target small localities because, for a given relative sensitivity, and

¹²Since swingness is measured at the pivotal value of the state shock, we can easily generate cases in which a locality has low relative sensitivity because it differs from its neighbors in terms of median ideology. This could, for instance, happen when the distribution of $\nu_{i,l}$ is single-peaked at the locality median.

hence likelihood of being pivotal, it is cheaper to buy votes in a less populated locality. Note that this is exactly the opposite of the comparative static under the PR system and what the utilitarian social optimum benchmark prescribes.

An important case to consider is the situation in which there is only one locality per district. Then, $\hat{s}_d = \hat{s}_l$, which implies that the relative sensitivity is equal to one for all localities. Differences in allocations are then exclusively driven by differences in contestability across districts. This setup, which abstracts from the possibility of targeting at the sub-district level, is that typically considered in the literature (see, e.g., Milesi-Ferretti et al. 2002, Persson and Tabellini 2000, and Strömberg 2004, 2008). Whenever targeting capacity (which we assume to be at the locality level but that could also be at a finer level, such as ethnic or age subgroups, or a city block) and electoral districts are not a perfect overlap, differences in relative sensitivities also matter.

5 Comparing the Systems

In this section, we compare government interventions under MAJ and PR systems. Throughout, we focus on $\alpha = 0$ (pure public good), since it captures the essence of the results for all $\alpha < 1$. For simplicity and in line with Persson and Tabellini (1999, 2000), we assume that the individual preference and district-level shocks are uniformly distributed:

$$\nu_{i,l} \sim U \left[\frac{-1}{2\phi_l}, \frac{1}{2\phi_l} \right] \quad \text{and} \quad \delta_d \sim U \left[\beta_d - \frac{1}{2\gamma_d}, \beta_d + \frac{1}{2\gamma_d} \right].$$

Appendix C shows that with a couple of assumptions ensuring interior solutions, the findings from the previous section still apply. Notice that, with uniform distributions, $\hat{s}_l = s_l = n_l t_l \phi_l$ and $\hat{\gamma}_d = \gamma_d$, which facilitates comparisons.

5.1 Winners and Losers

Propositions 1 and 2 above tell us that, in PR systems, a locality that is electorally more sensitive is systematically better treated than one with lower sensitivity. In MAJ systems, the electoral sensitivity of each locality is instead only assessed in comparison to that of

the other localities in its district. Specifically, it is the *relative electoral sensitivity* that matters. Moreover, as already emphasized in the literature, MAJ systems introduce the possible distortion that localities belonging to more contestable districts (high γ_d) receive a disproportionately large share of the resources.

A key implication of this comparison is that whether a locality wins from a PR-to-MAJ reform depends on the characteristics of neighboring localities in the district. We illustrate this with an example: Table 1 considers the utility function $u(q_l) = 2q_l^{1/2}$ (the closed-form solutions for the equilibrium allocations are in Appendix D). Four localities grouped in two districts are sufficient. To isolate the relative electoral sensitivity effect, we first assume that these two districts have the same contestability: $\gamma_A/\gamma_B = 1$.

Consider localities 2 and 3 in Table 1. They have the same electoral sensitivity s_l , but belong to two different districts. As shown in Proposition 1, they must receive the same allocation under the PR system: 11.8% of the total budget in Table 1, column q_l^{PR} .

District	Locality	Sensitivity (s_l)	q_l^{PR}	q_l^{MAJ} ($\gamma_A/\gamma_B = 1$)	q_l^{MAJ} ($\gamma_A/\gamma_B = 6$)
A	1	1	2.9%	9.7%	19.4%
A	2	2	11.8%	38.7%	77.7%
B	3	2	11.8%	7.1%	0.4%
B	4	5	73.5%	44.5%	2.5%

Table 1: Equilibrium allocations under PR and MAJ systems
 $u(q_l) = 2q_l^{1/2}$, $\alpha = 0$

The outcome is noticeably different under the MAJ system (see Table 1, column $q_l^{MAJ}(\gamma_A/\gamma_B = 1)$). Specifically, the allocation is skewed towards locality 2, which ends up receiving about 5.5 times more resources than locality 3, simply because they are surrounded by *other* localities with different characteristics. Locality 2 is the most sensitive in district A, whereas locality 3 is the least sensitive in district B. Following the adage that “in the land of the blind, the one-eyed man is king,” in the MAJ system, more governmental resources flow to locality 2 than to locality 3. By the same token, notice that locality 1 voters, who have a lower sensitivity than those in locality 3, end up receiving more, only because they live in a less sensitive district: their neighbors are respectively localities 2 and 4, and $s_2 \ll s_4$.

Next, we illustrate the effect of contestability. In the last column of Table 1, we raise γ_A/γ_B to 6, maintaining all other parameters constant. This substantially increases the governmental resources for all localities in district A under the MAJ system, from 48.4% to 97% of the total. By contrast, there is no effect on the allocation under the PR system.

5.2 Inequality

Which of these systems generates more inequalities in government interventions? To assess this, we rely on the Atkinson measure of inequality developed in Section 3.2. We shall say that the PR system *Atkinson-dominates* the MAJ system when $A(\mathbf{q}^{PR}) < A(\mathbf{q}^{MAJ})$ and vice versa. The Atkinson measure of inequality increases as political forces distort the allocation further away from the social optimum. The nature of these political forces differ between the MAJ and PR systems.

To illustrate these difference, it is useful to return to the above example. First, imagine that our four localities all have identical turnout and swingness (s_l/n_l constant, $\forall l$). In this case, electoral sensitivity only varies because of different population sizes, and PR necessarily produces the socially optimal allocation ($A^{PR} \equiv 0$), whereas $A^{MAJ} = 0.14$ when $\gamma_A/\gamma_B = 1$ and 0.71 when $\gamma_A/\gamma_B = 6$. Second, consider the opposite scenario: suppose the four localities are identical in terms of population size, $n_l = 1/4$, $\forall l$. In this case, the MAJ system leads to less distortion than the PR system as long as the distortions introduced by γ_A/γ_B are not too large: for $\gamma_A/\gamma_B = 1$, $A^{PR} = 0.26 > A^{MAJ} = 0.13$, and for $\gamma_A/\gamma_B = 6$, $A^{PR} = 0.26 < A^{MAJ} = 0.41$.

Though performing fully general comparisons is complex, Proposition 3 (proof in Appendix E) shows that in the case of log utility or of one locality per district, the comparison between the two systems boils down to comparing the spread in contestabilities to that in electoral sensitivities at the district level, as long as the latter are well apportioned:

Proposition 3 *The PR system Atkinson-dominates the MAJ system if $\gamma_d/\sum_{d'=1}^D \gamma_{d'}$ is a mean preserving-spread of $s_d/\sum_{d'=1}^D s_{d'}$ (and conversely) when either:*

- (1) $\rho \neq 1$, there is one locality per district, and $n_d = 1/D, \forall d$, or
- (2) $\rho = 1$, and $n_d = 1/D, \forall d$.

This difference between the MAJ and PR systems is useful in interpreting findings in the empirical literature. For instance, Strömberg (2008) assumes that the allocation of presidential candidate visits is decided at the U.S. state (and not county) level. This implicitly assumes one locality per district. He finds that replacing the Electoral College with the National Popular Vote (essentially a switch from a MAJ to a PR system as discussed in Section 6.1) would lead to a decrease in cross-state inequalities. This is exactly what our model predicts. That is, for the elections he analyses, the cross-state spread of electoral contestabilities in the U.S. are substantially larger than the cross-state differences in electoral sensitivities. That said, our model suggests that the effect of such a reform could be different if we relaxed the hypothesis that there is only one locality per district, i.e., if the allocation could be made at the sub-district level. In what follows, we investigate this through illustrative examples, and then, in Section 6, through figures calibrated on U.S. data.

5.3 The Importance of Sub-District Targeting

Let us go back to the example in Table 1 (with $\gamma_A/\gamma_B = 6$) and contrast the allocation of resources under a given electoral system when resources can be targeted at the locality level (q_i^{PR} and q_i^{MAJ}) or when we impose that the policymaker cannot discriminate within the district (q_i^{PR-d} and q_i^{MAJ-d}). As shown in Table 2, allowing for locality-level targeting leads to substantial within-district inequality in the allocations of resources.

What are the effects of such changes in terms of Atkinson inequality? This measure being normative, it depends on the distribution of the population across localities. We assume the distribution displayed in column n_l , which implies that in district A sensitivity essentially matches population size. In district B, it is the opposite. This could, for instance, reflect an ideologically very heterogeneous city in locality 3, and a very homogeneous suburban area in locality 4.

Comparing n_4 and q_4^{PR} , we observe that the concentration of spending on locality 4 under the PR system is highly distortionary. This is confirmed by a high Atkinson index: $A_l^{PR} = 0.42$. The allocation under the MAJ system is also distortionary, but slightly less so: $A_l^{MAJ} = 0.38$, implying that the MAJ system (mildly) dominates the PR system.

District	Locality	s_l	n_l	q_l^{PR}	q_l^{MAJ}	q_l^{PR-d}	q_l^{MAJ-d}
A	1	1	17%	2.9%	19.4%	7.8%	48.6%
A	2	2	33%	11.8%	77.7%	7.8%	48.6%
B	3	2	33%	11.8%	1.2%	42.2%	1.4%
B	4	5	17%	73.5%	2.5%	42.2%	1.4%
Atkinson index:				0.42	0.38	0.22	0.40

Table 2: Equilibrium allocations with locality *vs.* district targeting

More importantly, the opposite—and therefore potentially misleading—conclusion emerges if we restrict targeting to be at the district level only: A^{PR-d} drops to 0.22, whereas A^{MAJ-d} increases to 0.40. In other words, overlooking the possibility of within-district discrimination gives the impression that PR would decrease inefficiencies, while the opposite would happen when sub-district targeting is feasible.

What is the intuition? The difference in sensitivities within district B creates within-district inequality under both systems. The difference is in the shares of resources that flow to each district. In the MAJ system, due to low contestability and the higher aggregate sensitivity, district B receives fewer resources, and the inequalities in allocation within district B affect a small share of the overall budget. In contrast, in the PR system the higher the within-district inequality in sensitivity, the more resources are shifted from *district A* to *locality 4*. Hence, the inequalities in allocation within district B affect a larger share of the overall budget.

This example hides two other effects of sub-district targeting. First, the total allocation of a district may change substantially with and without sub-district targeting. Second, the magnitudes of the district gains and losses of a reform, and even the sign of the effect, may change. To illustrate this, we introduce a second example. Table 3 considers the same utility function as in the examples above, but with 3 districts (A, B, and C) each composed of two localities (1 and 2 in A, 3 and 4 in B, and 5 and 6 in C). The districts have different contestability ($\gamma_A = 0.2$, $\gamma_B = 1$, and $\gamma_C = 1.5$), and the localities differ in aggregate sensitivity, s_d .

Table 3 shows that, both under the MAJ and PR systems, districts A and C receive more resources when the government is restricted to providing the same allocation to all localities

District	s_l		q_d^{PR}	q_d^{MAJ}	q_d^{PR-d}	q_d^{MAJ-d}
A	1	1	15.1 %	1%	16.7 %	1.2%
B	0.2	1.8	24.7%	41.7%	16.6 %	30.4%
C	2	2	60.2%	57.3%	66.7 %	68.4%

Table 3: Equilibrium allocations with locality *vs.* district targeting
 $u(q_l) = 2q_l^{0.5}$, $\alpha = 0$

of a same district. The opposite is true for district B. This change is driven by the large inequality in sensitivities between the two localities in district B. Table 3 also shows that, independently of the level of targeting, district A would gain from a MAJ-to-PR reform, and district B would lose. Yet, the magnitudes of these changes depend on the level of targeting. The situation is even more sensitive for district C: it would be a winner of the reform under locality level targeting (+3 percentage points), but a loser under district level targeting (-1.7 percentage points). All these changes are again the consequence of the heterogeneity of localities in district B. The presence of only one highly sensitive locality in B attracts more resources to the district only under locality-level targeting.

6 Reforms: the U.S. Presidential Electoral System

This section extends the analysis to other versions of the MAJ and PR systems in order to study possible reforms of the U.S. presidential electoral system. The election of the U.S. President is organized through the Electoral College. General elections are organized at the state level, and each state is awarded a number of electors equal to its number of representatives in Congress (senators and house representatives). In all states but Maine and Nebraska, electors are allocated to candidates in a “winner-takes-all” fashion.

One oft-proposed reform is to replace the Electoral College by the direct election of the President under the National Popular Vote (NPV henceforth) rule. We find that such a reform would yield allocations identical to those under PR. We then study a broader set of reforms, including a proportional representation version of the Electoral College (PR-EC), whereby the allocation of electors is made proportional to vote shares in each state, in line with Maine and Nebraska today. We next calibrate our theoretical results to the U.S. counties and states. We find that the relative sensitivity effect is the main

driver of several states' eventual gains or losses, and that the PR-EC system produces welfare-superior outcomes. The whole exercise is closely related to Strömberg (2008).

6.1 Theory

The current form of the U.S. Electoral College can be represented as a weighted majoritarian system, in which each state is represented by a district d , and its share of electors represented by ω_d .¹³ The problem for candidate A becomes (see Appendix D):

$$\max_{\mathbf{q}^A | \sum_l n_l^\alpha q_l = y} \pi^{EC}(\mathbf{q}) = \sum_d \omega_d \Gamma_d [D_d(\mathbf{q})] = \frac{1}{2} + \sum_d \gamma_d \omega_d \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{s_d} - \beta_d \right],$$

where swing (resp. safe) states are associated with high (resp. low) values of γ_d .

It is straightforward to see that the Electoral College tilts the allocation of government resources towards districts with a higher ω_d . Indeed, the FOCs are:

$$\frac{\partial u_l(\mathbf{q}^A)}{\partial q_l^A} = \frac{\lambda^{EC}}{\omega_{d(l)}} \frac{s_{d(l)}/\gamma_{d(l)}}{s_l/n_l^\alpha}, \quad \forall l, \quad (17)$$

where the only difference with the MAJ system is the presence of the weight $\omega_{d(l)}$.

A reform of the Electoral College (EC) to replace it with the National Popular Vote (NPV) rule would instead amount to merging all existing districts into a single one. Interestingly, this reform would result in the same government allocation as in the PR system analyzed in Section 4.1.

As a result, we can rely on the comparison between (11) and (17) to anticipate how the NPV reform would affect government interventions. The effect varies with the nature of within- and between-state heterogeneity. If between-state differences in aggregate sensitivity (s_d) are severe, while differences in contestability (γ_d) are mild, switching to the NPV would actually amplify distortions (it is easy to build examples in which EC Atkinson-dominates NPV). In contrast, with small between-state differences in aggregate sensitivities s_d , and large differences in contestabilities γ_d , the NPV reform should reduce

¹³We assume that all states allocate their electors through winner-takes-all.

distortions, and be socially superior. Sub-district targeting and within-state inequality in electoral sensitivity also play an important role, which we discuss further in the following subsection.

Another possible reform is to switch to a proportional representation system within each state (“districts” in the model). Each state retains its total number of electors; it is only the parties’ shares within a state that stops being winner-takes-all. The objective of a candidate in such a system, labeled (PR-EC), turns out to be a simple reweighting of the objective for the single-district PR analyzed thus far. It only needs to be modified for the fact that each district receives some pre-determined fraction ω_d of the seats, with $\sum_d \omega_d = 1$ (see Appendix C for the developments):¹⁴

$$\max_{\mathbf{q}} \pi^{PR-EC}(\mathbf{q}) = \frac{1}{2} + \sum_d \omega_d \sum_{l \in d} \frac{s_l}{T_d} [\Delta u_l(\mathbf{q}) - \mathbb{E}[\delta_d]],$$

where T_d is the total number of active voters in the district. Defining the average turnout rate in a district as $t_d := \sum_{l \in d} t_l \frac{n_l}{n_d}$, with $n_d := \sum_{l \in d} n_l$, we obtain $T_d = t_d n_d$. Taking FOCs and letting λ^{PR-EC} denote the multiplier on the budget constraint, we have:

$$\frac{\partial u_l(\mathbf{q}^A)}{\partial q_l^A} = \left[\frac{\omega_d}{n_d t_d} \ s_l \right]^{-1} n_l^\alpha \lambda^{PR-EC} \ \forall l. \quad (18)$$

Compare this with (11), the FOC under single district PR: we see that $\frac{\omega_d}{n_d t_d}$ was *de facto* equal to 1. This means that the allocation under PR (or the NPV) is as if each district received a share of seats equal to its realized number of votes. Under a well-apportioned PR-EC instead, $\omega_d = n_d$. That is, a pre-determined seat share equal to the district’s share of the population. In other words, high-turnout states will tend to receive less under PR-EC than under the NPV rule.

An important feature of the U.S. Electoral College is, however, that less populated states are over-represented compared to more populated states: ω_d/n_d is weakly decreasing in n_d . Both under the current version of the Electoral College and its PR variant, the effect of this malapportionment is that smaller states get favored in equilibrium. Yet, as seen in (18), reforming the EC system to make it proportional still implies that a state’s allocation

¹⁴The objective function under PR-EC would actually be the same in district-PR systems such as that of Belgium or Brazil.

would no longer be determined by its aggregate sensitivity or its contestability.

Beyond these specific reforms, our model allows us to assess a broader set of electoral systems and identify that:

Proposition 4 *Starting from the Electoral College system, there exists a combination of redistricting and reapportionment that implements the social optimum.*

The underlying rationale for this result is that (i) an arbitrary reweighing of the Electoral College can always tilt the allocation towards any district, and (ii) a division into more districts increases the number of weights available. Hence, there must exist some districting that offers a sufficient number of instruments (district weights) to reach the social optimum. In contrast, the National Popular Vote restricts the number of districts to a single district. In practice, however, achieving such a reform would run into first-order obstacles. First, a modification of both the districts and the distribution of electors in the Electoral College would require a constitutional change. Second, the optimal weights may substantially differ from “one (wo)man, one vote.” Last but not least, the optimal weights are likely to change over time (when the electoral sensitivity of localities change).

6.2 Numerical Simulations

We now apply our results to U.S. presidential election data.¹⁵ Our goal is to assess numerically the implications of possible reforms of the U.S. Electoral College system. In doing so, we emphasize the insights that sub-district targeting brings to the question.

Data

The first step is to match the structure of our model to the U.S. political and administrative structure. In the Electoral College system, U.S. states are what we call districts in our model, and the counties are the localities. Our dataset includes ten presidential elections,

¹⁵We are grateful to JJ Naddeo for generously sharing his dataset with us.

from 1980 to 2016. Due to data availability constraints, we limit the assessment to 48 states ($|D| = 48$), which include 3,106 counties ($|L| = 3106$).¹⁶

We need proxies for the key variables of the model: the county turnout rate (t_l), the county swingness (ϕ_l), and the county population (n_l). With these variables, we can compute the county level sensitivity ($s_l = t_l n_l \phi_l$). Aggregating them at the state level produces t_d , n_d , ϕ_d , and s_d . At the state level, we also need the number of electoral votes (ω_d) and a proxy for contestability (γ_d).

For the county population, we use the decennial census information from 1980-2010 extracted from IPUMS-NHGIS,¹⁷ interpolated for the between-census years. We supplement this data using the American Community Survey for all years post 2010. To obtain the county turnout rate, we divide the number of votes cast by the total county population. Election outcome data was extracted from Congressional Quarterly (CQ) Press Voting and Elections Collection website.¹⁸

For the county swingness, we follow the literature (e.g., Ansolabehere and Snyder 2006) and use the standard deviation in the democratic vote share in previous elections (i.e., between 1980 and the election under consideration). In particular, the democratic vote share is the number of democratic votes divided by the sum of democratic and republican votes for the county in a given year.

For the state contestability, we follow two different and complementary approaches. First, we use the common definition of a swing state in the literature (e.g., Berry et al. 2010), where the contestability of state d in election e is 1 minus the victory margin: $\gamma_{d,e} = (1 - VM_{d,e})$, with $VM_{d,e} = |rep_share_{d,e} - dem_share_{d,e}|$. The second measure relies on the work and data of Strömberg (2008) on electoral visits during the 2000 and 2004 presidential elections in the U.S. The estimation of this measure of contestability involves three steps. First, we impute the contestabilities $g_{d,e}$ such that our model matches Strömberg's predicted number of electoral visits in state d in election e (using district level

¹⁶AK and HI are not included due to irregularities in the data. In addition, two counties in VA and one in NM are missing information for some years.

¹⁷Integrated Public Use Microdata Series National Historical Information System.

¹⁸Accessed here: <https://library.cqpress.com/elections/index.php>.

targeting as in Strömberg 2008). Second, to extrapolate the measure of contestability for the other years, we apply a Newton-Raphson method to estimate the value of p in:

$$\min_p \sum_{d \in D} \sum_{e \in \{2000, 2004\}} (g_{d,e} - (\gamma_{d,e})^p)^2.$$

so as to match the 2000 and 2004 values of $\gamma_{d,e}^S$. We estimate $\hat{p} = 0.0047$. Third, we compute $\gamma_{d,e}^{Str}$, the contestability of state d in election e as $(\gamma_{d,e})^{\hat{p}}$ normalized such that the mean for a given election matches the mean of $\gamma_{d,e}$.

Figure 4 in Appendix G plots the two measures of contestability for the 2016 election. It shows that the Strömberg-like measure produces a more skewed distribution: 10 states emerge as vastly more contestable than all others. This is not surprising given that Strömberg's work focuses on behavior during an electoral campaign, which is likely to magnify the incentives of candidates to focus their effort on a limited number of key states. Arguably, such incentives are milder for government interventions.

Table 4 below provides basic descriptive statistics for the key state-level (d subscript) and county-level (l subscript) variables. We observe variation both across counties and across states in all these parameters. The variations are particularly important for the absolute and relative sensitivity: all these variables have a coefficient of variation above 1. In contrast, contestability variables have coefficients of variation below 1. Note that the substantial variations in the sensitivity variables seem to be largely driven by variations in population across counties and states.

Table 4 also includes the R^2 of regressions of each variable on state-year fixed effects. We see that there is substantial within-state variation in the variables of interest. Only 12% of the variation in county-level sensitivity (s_l) is explained by state-year fixed effects.

Predicted Allocations

For CRRA utility, Appendix D provides the formulae for the allocations under the winner-takes-all version of the Electoral College (q_l^{EC}), the proportional version of the Electoral College (q_l^{PR-EC}), and the National Popular Vote (q_l^{NPV}). With these, we then compute

Table 4: DESCRIPTIVE STATISTICS

Statistics	Mean	Median	Std. Dev	Min	Max	N	R^2 on FE
ϕ_l	0.073	0.067	0.027	0.019	0.222	9314	0.334
t_l	0.43	0.431	0.076	0.119	0.896	9314	0.377
n_l (*)	100	26	321	0	10121	9314	0.119
s_l (*)	3	1	10	0	357	9314	0.116
s_l/s_d	0.015	0.005	0.04	0	0.713	9314	0.206
s_d (*)	190	123	206	17	1209	144	1.000
γ_d	0.83	0.841	0.111	0.486	0.999	144	1.000
γ_d^{Str}	0.83	0.719	0.412	0.248	2.54	144	1.000
ω_d	11	8.5	9.706	3	55	144	1.000

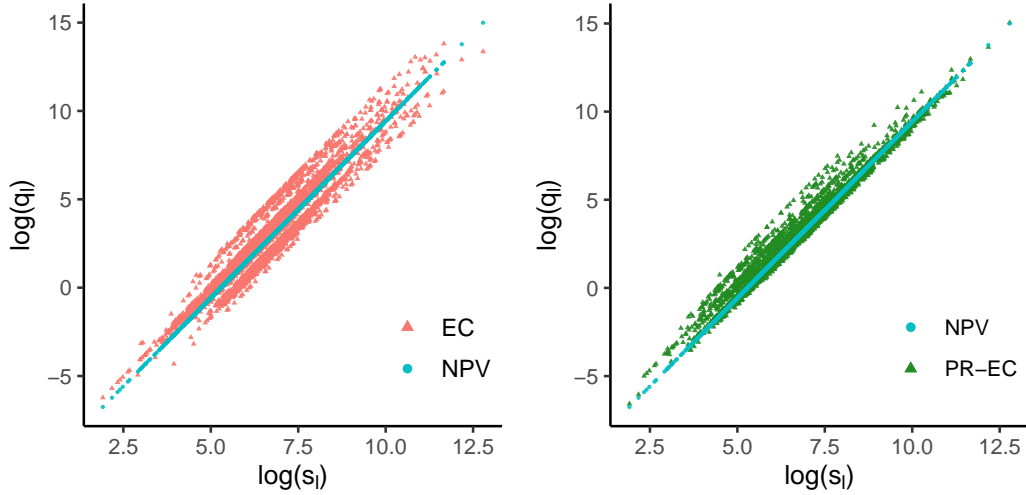
Notes: Averages for years 2008-2016. (*) in thousands.

the predicted allocations for a coefficient of relative risk aversion, ρ , equal to 0.5. These are always proportional to the total budget y , which we set to 10 million.

Figure 1 plots the (log) predicted allocations for various cases: for each county in the sample, it compares the outcome under the National Popular Vote (NPV, the dots on the Figure) with (i) the Electoral College (EC), and (ii) the Electoral College with PR (PR-EC), as a function of the county's log sensitivity. The figure highlights two important features of the allocations under each system.

First, for all electoral systems, the relationship is log-linear in the county's own sensitivity s_l , which drives the bulk of the variations in allocations. To illustrate the magnitude of these variations, consider the example of Cook County and Du Page County, both in Illinois. In 2016, mostly due to population differences, Cook County's electoral sensitivity was 4.24 times higher than that of Du Page County. This difference implies an allocation that is 18 times larger for Cook county than that for Du Page county. While the main driver of a county's sensitivity is its population size, there is also substantial heterogeneity among counties that are similar in population. For instance, Whiteside County and Jackson County, also in Illinois, both had a population of about 57,000 in 2016 but, due to higher turnout rates and swingness, Whiteside County is 1.7 times electorally more sensitive than Jackson County. As a result, our simulations predict that Whiteside County's allocation should be 2.8 times larger than that of Jackson County.

Second, under EC and PR-EC, counties that share a same sensitivity s_l but are in different



Notes: Year 2016. Strömberg-like measure of contestability.

Figure 1: COUNTY ALLOCATIONS AS A FUNCTION OF THEIR ELECTORAL SENSITIVITY

states will typically be treated differently. This explains why the sensitivity of a county does not fully determine its allocation under these systems. Under EC, a county's allocation increases in its state's contestability (γ_d) and number of electors (ω_d), and decreases in its aggregate sensitivity (s_d). For instance, consider Washington County in Missouri and Washington County in Maine. They have almost identical sensitivity. Yet, Washington County in Missouri should receive only about 12% of the amount that Washington County in Maine receives. This is so because Missouri is, in aggregate, 3.5 times more sensitive than Maine ($s_{MO}/s_{ME} = 3.53$), while the two states have similar values of $\gamma_d^{Str} \times \omega_d$ (slightly favoring Missouri).

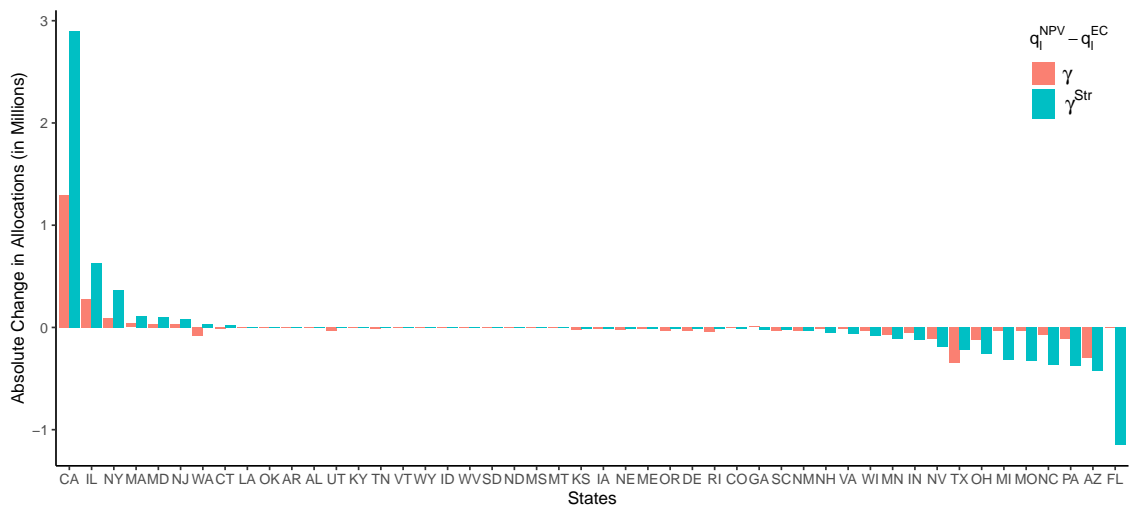
To quantify the effects of each variable on the predicted county allocation q_i^{EC} , we regress the latter on the county's own sensitivity (s_i – the horizontal axis on the figure), and the different state-level characteristics that determine the allocation under EC. We find that s_i explains 85-93% of the total variance of the predicted q_i^{EC} , when we use respectively $\gamma_{d,e}^{Str}$ or $\gamma_{d,e}$. The residual variance decomposes as follows for $\gamma_{d,e}$: 50% is explained by state-level average sensitivity (s_d), 34% by the electoral votes (ω_d), and 16% by the state's contestability ($\gamma_{d,e}$). With $\gamma_{d,e}^{Str}$, these percentages are respectively 23, 17, and 60.

Under PR-EC, only the overall turnout matters among the state characteristics. Neither the state's contestability nor its aggregate swingness matter. The second panel of Figure 1 shows that this reduces variations in allocations across counties with similar sensitivity.

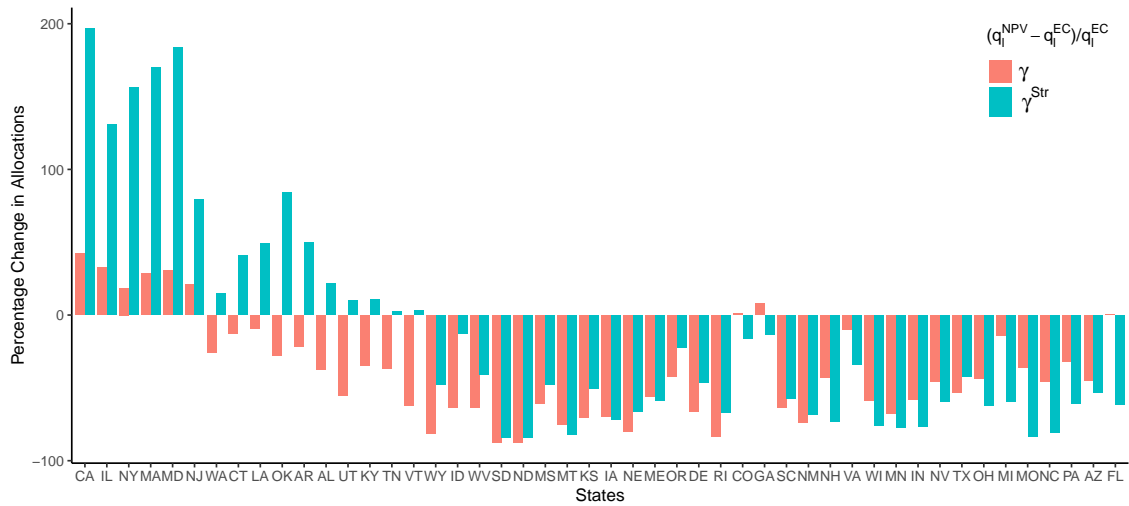
Hence, the allocation under PR-EC more closely resembles that under NPV.

Winners and Losers of the Reform

A reform of the Electoral College (EC) to instead rely on the National Popular Vote (NPV) would generate winners and losers across the country. Counties in a given state win more (or lose less) when the state has (i) a high aggregate sensitivity, s_d , (ii) a small number of electoral votes, ω_d , and (iii) a low contestability, γ_d or γ_d^{Str} .



(a) Absolute gain.



(b) Percentage gain.

Figure 2: WINNERS AND LOSERS OF A REFORM FROM EC TO NPV.

Figure 2 displays the states' gains and losses (in absolute value (panel a) and in percentage

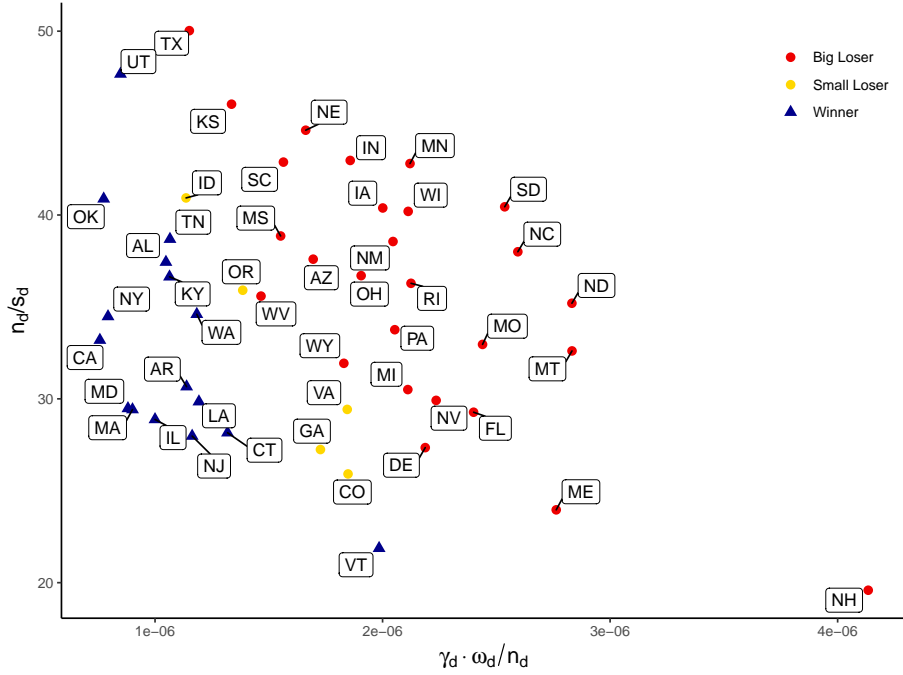
terms (panel b)) due to such a reform for the two different measures of contestability. Several interesting patterns emerge. First, we see that a vast majority of states lose from the reform in favor of a few. The common wisdom is that the identity of winners and losers depends on two factors: state contestability (γ_d) and state malapportionment (ω_d/n_d). Indeed, many of the biggest losers (FL, PA, AZ, NC, and MI) are battleground states that have among the highest contestability, while many of the biggest winners are under-represented states with low contestability (CA, IL, NY and MA).

Notice that the importance of contestability is magnified under the Strömberg-like measure, which is more skewed. The case of Florida is striking: the magnitude of its loss is fundamentally different under the two measures of contestability. Similarly, Figure 2b also highlights that some states (AR, LA, OK, KY, AL, TN, CT, UT, and WA) are winners of the reform only under the Strömberg-like measure of contestability.

This analysis is, however, only partial in that it overlooks the role of the aggregate sensitivity of the state s_d . To this regard, Figure 3 separates out a state's aggregate sensitivity from its other characteristics, highlighting its central relevance for several states. Consider, for instance, Illinois and Texas. They both have relatively low values of $\gamma_d \times \omega_d/n_d$. Yet, Illinois is among the biggest winners of the reform, and Texas among the biggest losers. This is because Texas has a relatively low aggregate sensitivity for its population, due to low turnout rates (lowest in the country) and low swingness (sixth lowest in the country). The relative sensitivity effect is also central to the diverging outcomes between Michigan and Wisconsin, and between Utah and New Jersey.

Interestingly, we see in Figure 2b that the largest winners in absolute value are also for the most part the largest winners in percentage terms. Yet, while some large states are still among the largest losers in percentage terms, they are joined by some of the small states (such as MT, ND, RI, and SD) who are also among the largest losers in proportional terms. This is because small states are over-represented in the EC.

We can also explore the winners and losers of a reform towards PR-EC. Figure 5 in Appendix G shows that the winners and losers are essentially the same as for a reform towards the NPV. The main difference is that, with PR-EC, the states with a low turnout gain more (or lose less) than with NPV. For instance, California and Texas have lower



Notes: Big Loser / Small Loser / Winner if percentage gain $\in (-\infty, -0.5] / (-0.5, 0] / [0, \infty)$. Average for 2008-2016. Strömberg-like contestability.

Figure 3: DECOMPOSITION OF STATE'S CHARACTERISTICS

than average turnout, whereas Florida has higher than average turnout.

Inequality

We can compare electoral systems based on how much inequality in the allocation of resources they generate. Using a Gini coefficient of inequality across individuals tells us that both potential reforms of the Electoral College (i.e., towards NPV or PR-EC) slightly increase inequality for the 2008-2016 elections. Table 5 shows the coefficients for 2016.

	$EC (\gamma^{Str})$	$EC (\gamma)$	NPV	PR-EC
<i>Gini</i>	0.842	0.875	0.909	0.912
<i>Atkinson</i>	0.316	0.089	0.072	0.071

Table 5: INEQUALITY MEASURES 2016

Yet, as discussed in Section 3.2, all inequality is not necessarily socially undesirable. To this regard, our Atkinson-like measure of inequality assesses the social cost of politically motivated inequality. We find that for the 2008-2016 elections, both potential reforms of

the Electoral College decrease inequality. As Table 5 shows, this is particularly pronounced when we use the Strömberg-like measure of contestability. Moreover, it appears that PR-EC produces the lowest Atkinson index. Combined with the higher political feasibility of the EC to PR-EC reform –since it does not require a constitutional amendment– this finding suggests a different path for any reform of the Electoral College.

State-Level *vs.* County-Level Allocations

The estimated gains and losses of a reform would differ in magnitude if one ignored the possibility of differentiating county allocations within a state. This can be seen in Figure 6 in Appendix G.

For instance, for the case of county targeting, our model predicts that Illinois and California are big winners. If we instead impose targeting at the district-level, these states' gains are substantially reduced. Conversely, New Jersey and Massachusetts would gain more. Among the losers, Arizona and Texas lose more with county-level targeting, while Florida and New Hampshire lose less. A key factor explaining these differences is within-state heterogeneity. Illinois and California are, for instance, composed of counties with considerably different electoral sensitivities. In general, highly sensitive counties are bound to gain more under county-level targeting. This is particularly true when other counties in the state are low sensitivity counties. A good example is Cook County in Illinois. It gains about 300 times more with county-level targeting than with state-level targeting (for the Strömberg-like measure of contestability). In contrast, the least sensitive county in Illinois, Scott County, gains almost nothing under county-level targeting but quite substantially under state-level targeting.

7 Extensions

7.1 Beyond Geographically Targeted Interventions

We have thus far worked under the assumption that government interventions are targeted at the locality level, and that there were no externalities across localities or districts. The Introduction and Appendix A instead emphasize that many government interventions are restricted by law to uniformly benefit all citizens of a same group, independently of their location. Think, for instance, of the right to public education, Medicare, or unemployment benefits. Yet, favoring one or the other intervention benefits a different socio-demographic group. In what follows, we investigate how our results extend to such interventions.

Consider a population composed of G socio-demographic groups. Group g represents a fraction $n_g > 0$ of the national population, and a mass $n_{g,l} \geq 0$ of the inhabitants of locality l : $\sum_g n_{g,l} = n_l$, and $\sum_g n_g = \sum_l n_l = 1$. To differentiate the notation from our base case of geographically targeted interventions, let us denote by b_g (e.g., for “benefit”) a group-targeted government intervention. Independently of where she lives, each individual belonging to group g derives a utility $v_g(b_g)$ from it, and the cost of delivering this intervention is $n_g^\beta b_g$ nationally. Apart from the notation and the geographically dispersed allocation of b_g , this is the same setup as before.

To avoid overburdening this section, we relegate the developments for the general model to Appendix H. That said, it should be straightforward that the logic of government intervention remains unchanged under PR: each party maximizes its nation-wide expected vote count, and whether votes come from groups or from localities, the first order conditions share the same features. With uniform distributions, for instance, a group receives more when its nationwide sensitivity $s_g = \sum_l n_{g,l} t_{g,l} \phi_{g,l}$ is larger.

In contrast, interesting differences appear under the MAJ system. Recall that the objective is to win as many districts as possible. From an electoral perspective, a group is more valuable when it helps swing more districts. The first order condition becomes:

$$\sum_d \frac{\gamma_d s_{g,d}}{\gamma s_d} v'(b_g^*) = n_g^\beta \lambda^{MAJ}, \quad (19)$$

with: $\gamma = \sum_d \gamma_d$, $s_{g,d} = \sum_{l \in d} n_{g,l} t_{g,l} \phi_{g,l}$ and $s_d = \sum_{g'} s_{g',d}$. As with locality-targeted

interventions, a group g receives more when it has a higher *relative* sensitivity. Here, a compelling difference is that parties take into account the cross-district *weighted average* of these relative sensitivities, the weights being given by the districts' contestabilities γ_d .

Hence, although the expressions are more complex than in the main model, the contrasts between MAJ and PR remain similar. Consider a group with average sensitivity. Under MAJ, it will receive a higher-than-average level of b_g if its members live in high-contestability districts mainly inhabited by low-sensitivity groups. By the same token, it will benefit from a lower-than-average level of b_g if they live in low-contestability districts mainly inhabited by high-sensitivity groups. In fact, the only exception is:

Proposition 5 *When all districts share a same group composition ($\frac{s_{g,d}}{s_d} = \frac{s_{g,d'}}{s_{d'}}$, $\forall g, d, d'$), the equilibrium allocation is identical under PR and MAJ, for any district contestabilities.*

The proof is immediate: the weights add up to 1 in (19). The important lesson is that differences between the allocations under MAJ and PR systems arise only due to heterogeneity in the districts' socio-demographic compositions. In practice, it is typical for citizens to be geographically concentrated. For instance, in the U.S. the elderly represent a substantially larger share of the total population in some states compared to others. In 2018, the share of older adults (65+) in the population was 20.6% in Maine, and 20.5% in Florida, compared to 11.8% in Alaska and 11.1% in Utah (see U.S. Census Bureau (2018)). These cross-state variations hide even larger variations across counties: e.g., the share of older adults was above 50% in Sumter County but less than 10% in other counties in Florida (see Figure 7 in Mather, Jacobsen, and Pollard 2015). Moreover, various socio-demographic groups appear to have different electoral sensitivities. For instance, older adults have a much higher turnout rate than other age classes. In 2016, the turnout rate was 71% among the 65+ age class compared to 46% among the 18-29 year-old group (see Voting in America 2017).

Under MAJ systems, parties exploit these variations in order to target the districts that are (i) more contestable (high γ_d), and (ii) have a lower overall electoral sensitivity (low s_d). These incentives can either pull in the same or in opposite directions. For instance, Florida is a highly contestable state. Given the high concentration of older adults, parties have incentives to design and maintain generous programs benefiting this age group (e.g.,

Medicare and Social Security).¹⁹ Yet, Florida also has higher-than-average district-level electoral sensitivity (see Figure 3 in Appendix G). The latter produces an opposite force that *reduces* the benefits flowing to that group. Instead, under PR only the electoral sensitivity of a socio-demographic group matters. The higher-than-average electoral sensitivity of older adults should thus advantage them anyway.

7.2 Endogenous Choice: Targeted versus Universal Spending

The previous section analyzed targeting at the socio-demographic group level. A natural follow-up question is how the electoral system modifies the incentive to continue targeting specific localities or groups, or rather to focus on pure public goods that benefit the population more broadly. The literature (see Section 2) argues that politicians have stronger incentives to use geographically targeted instruments under majoritarian than proportional representation systems. In assessing this question, Persson et al. (2000) and Lizzeri and Persico (2001) give politicians a choice between two types of policy instruments: targeted transfers and a global public good. They find that politicians provide less public goods under MAJ systems. In what follows, we revisit this issue in light of our findings.

Following Persson et al. (2000) and Persson and Tabellini (2000), we assume that individuals in locality l have quasi-linear preferences in a transfer q_l (corresponding to $\alpha = 1$ in the previous setup) and a global public good that benefits the entire population:

$$w_l(\mathbf{q}, G) = q_l + u(G),$$

with $u(\cdot)$ strictly increasing and strictly concave in G . The budget is exogenously given as y so that the budget constraint becomes $\sum_l n_l q_l + G = y$.

As detailed in Appendix I, generically under this specification, at most one locality receives a transfer. In the unique equilibrium under PR, this is the locality with the highest s_l/n_l .

¹⁹Various other high-contestability states similarly have a higher-than-average share of older adults, e.g., Pennsylvania, Michigan, Arizona, and South Carolina.

If some transfers are given, then:

$$u'(G) = \max_l \frac{s_l/n_l}{\sum_j s_j}. \quad (20)$$

Under the MAJ system, the equivalent condition is:

$$u'(G) = \max_l \frac{\gamma_{d(l)} \frac{s_l/n_l}{s_d}}{\sum_{d \in D} \gamma_d}. \quad (21)$$

By comparing (20) and (21), we can identify which system leads to the largest provision of the global public good. For simplification purposes, assume localities correspond to electoral districts, $L = D$, and that districts are well apportioned: $n_l = 1/D$ for all l .

Two effects pull in opposite directions. On the one hand, there is the effect of district contestability in the MAJ system, as identified by Persson and Tabellini (2000) and Lizzeri and Persico (2001). Heterogeneous contestabilities increase transfers and decrease the provision of the national public good in the MAJ system versus the PR system. If localities have equal $s_l = \bar{s}$ then there is no transfer under the PR system and heterogeneous district contestabilities make transfers more attractive under the MAJ system. On the other hand, there is the relative electoral sensitivity effect. With one locality per district, $\frac{s_l}{s_d} = 1$, meaning a reduction in variance compared to (20). For $\gamma_d = \gamma, \forall d$, $G^{PR} \leq G^{MAJ}$ in equilibrium – with a strict inequality when $G^{MAJ} > 0$ – as soon as the highest level of electoral sensitivity is larger than the average. In other words, *any* heterogeneity in electoral sensitivities leads to a higher provision of the global public good under MAJ compared to PR systems, a reversal of the standard result in the literature.

The following proposition summarizes the comparison between the two systems:

Proposition 6 *If targetability is at the district level ($L = D$) and districts are well-apportioned ($n_l = \frac{1}{D}$ for all l) then $\max_d \frac{s_d}{\sum_{d' \in D} s_{d'}} > (<) \max_d \frac{\gamma_d}{\sum_{d' \in D} \gamma_{d'}}$ implies $G^{MAJ} \geq (<) G^{PR}$.*

8 Conclusion

This paper compares inequalities in government interventions under different electoral systems. We uncover a novel *relative electoral sensitivity effect* that modifies our understanding of majoritarian systems. Under both the MAJ and PR systems, parties favor localities that are highly electorally sensitive (densely populated, high turnout, very swingable). But, under MAJ systems the aggregate sensitivity of localities in a district decreases the probability that the district may be swung from one party to the other, thereby reducing its appeal. Hence, under MAJ systems, *ceteris paribus*, parties favor localities with a high relative sensitivity.

An important implication of the relative sensitivity effect is that it can *reverse* the common finding in the literature that inequalities in government intervention are higher in MAJ than in PR systems. We show that it can induce parties to target a larger fraction of the population or provide more public goods under MAJ systems.

Our approach also yields novel insights into on-going debates over the U.S. Electoral College, and whether the president should be elected under the National Popular Vote (NPV) or under proportional representation within the electoral colleges (PR-EC). Strömberg (2008) shows that highly contestable and over-represented states benefit from the Electoral College system compared to these other systems. Using numerical simulations, we show that the aggregate sensitivity of the district also matters in distinguishing winners and losers from a reform of the system. Allowing for county-level instead of state-level targeting also matters

The relative sensitivity effect has also implications for the existing empirical literature on distributive politics (see, e.g., the literature reviews in Berry et al. 2010 and Golden and Min 2013). In particular, the allocation of a given locality should systematically depend on the characteristics of its neighbors in the electoral district. This implies that there is a risk of omitted variable bias in studies of the allocations of governmental resources at the sub-district level in MAJ systems.

Finally, an advantage of our framework is that it is both easy to compare with standard

textbook models of probabilistic voting, and tractable enough to explore additional issues. Future research could, for instance, explore the role of the relative sensitivity on important questions such as the gerrymandering process or the effect of redistricting on the allocation of governmental resources. In both cases, the composition of the electoral districts, which directly affects the relative electoral sensitivity, is at the heart of the problem. As political parties increasingly tap into big data to design their campaigns and policies, they are able to target increasingly narrower group of voters. This trend means that our findings will only become more relevant.

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Appendices

A Targeting at the Sub-District Level

Central to our analysis is the fact that electoral districts differ from the actual level at which politicians can target their interventions. More specifically, we work under the assumption that at least some government interventions are targetable at a smaller level than the electoral district.

Several cases fit this assumption. First, government interventions such as public infrastructure and amenities feature *by definition* a local component. Take, for example, roads, bridges, and airports, or public schools, hospitals, and parks. Each requires choosing a very specific location. Other interventions such as disaster relief and military bases similarly share this geographic feature, and empirical evidence shows that the allocation of these expenditures is affected by political factors. Kriner and Reeves (2015), for instance, find that the Obama administration favored both swing and core states in their decisions about natural disaster declarations and military base closures.²⁰ Picci and Golden (2007) find that infrastructure expenditures decided by the Italian central government and allocated to provinces are affected by political factors. Meanwhile, Mahadevan (2019) finds that public electricity provision in India is manipulated by incumbent politicians. In particular, “shortly after a state-level election, there is an increase in electricity consumption, as measured by satellite nighttime lights data, for regions that voted for the winning party. [...] These same regions, however, have discontinuously lower levels of billed consumption, as reported by the electricity provider” (p.2). She also finds that “politicians target consumer categories that have inelastic demands, and groups with greater access to electricity-using infrastructure” (p.4).

Other interventions naturally targeted at a very local level include campaign efforts. In the U.S., for example, TV ads are targeted at the level of the demographic market area (DMA). While these DMAs typically include multiple counties, there are many of them in any given state (see, e.g., the discussion in Spenkuch and Toniatti 2016). Furthermore, there is evidence that political factors are, unsurprisingly, key determinants of TV ad targeting (see, e.g., Strömberg 2008 and Gordon and Hartmann 2016). The same is true of campaign visits (Strömberg 2008; Incerti 2015).

Government interventions are also targetable at the local, sub-district, level because some spending must be channelled through local government units that do not perfectly overlap with electoral

²⁰Other studies find evidence of political manipulation in the allocation of disaster relief funds. There is evidence of political targeting at the state level (see, e.g., Garrett and Sobel 2004) and the county level (see, e.g., Strömberg 2004 and Healy and Malhotra 2009).

districts. For instance, in the case of the US, a substantial fraction of federal and state government expenditures flow through counties and other local governmental units (e.g., municipalities, school districts, and special districts), which are usually smaller than the relevant electoral districts. To this regard, Aidt and Shvets (2012) (p. 14) find that “roughly one quarter of all state spending net of administrative costs is channeled through local government units.” They add that “Gosling (1985) argues that state legislators predominately use spending that goes through local government units to ‘bring home the pork’.”

The fact that government expenditures must flow through sub-district governmental units does not, however, imply that politicians and parties always have the ability to engineer an allocation of funding to districts tailored to their objectives. As Martin (2018) explains on page 479, in many countries, especially advanced democracies, “most government spending is allocated according to criteria approved by a legislature but implemented by the bureaucracy.” In some cases, these criteria are so strict and oriented toward the uniform treatment of electoral units that they even prevent variations that seem justified from a welfare standpoint. Boone et al. (2014) show, for instance, that the 2009 American Recovery and Reinvestment Act did not appear to preferentially target politically sensitive districts, but neither did it flow more to the regions worst hit by the economic crisis.

Yet, mostly due to the non-uniform distribution of demographic characteristics across countries (e.g., retirees, the unemployed, or school-age children are more concentrated in some areas compared to others), there are still many ways for the politicians who design the formulas to target specific groups while impacting different regions in very asymmetric ways.²¹ As stated by Smith (2006) in his book *Formula Funding of Public Services*, “it usually requires only modest levels of ingenuity for a payer to be able to secure almost any desired distributional outcome, while nominally adhering to the use of formulae.” Indeed, there is a large empirical literature that highlights the importance of political factors in explaining variations in the allocations of governmental resources (both across geographic areas and socio-demographic groups). Recent research finds such manipulations in various countries; see, e.g., Gagliarducci et al. (2011) for Italy, Brollo and Nannicini (2012) for Brazil, Albouy (2013) and Berry and Fowler (2016) for the US, Asher and Novosad (2017) for India, and Hodler and Raschky (2014) for 126 countries.

Moreover, some papers in this literature look directly at sub-district targeting. Here too, strong empirical evidence shows that formulaic constraints do not prevent political factors from influencing the allocation of resources at the sub-district level.²² For instance, Kriner and Reeves (2015) finds

²¹To this regard, Vladeck (1999)’s discussion of the distributive effects of Medicare is particularly insightful.

²²There is also evidence that incumbents can manipulate the allocation of resources within the frame-

that, for the period 1984-2008, the allocation of US federal spending to counties was reliably affected by political factors. Martin (2003) and Berry et al. (2010) provide similar evidence (for different political factors). Again in the U.S., Ansolabehere et al. (2002) examine the geographic distribution of funds by states to counties. Using the court-mandated redistricting from the mid-1960s, they find that the number of legislative seats per person of a county affects positively the per person transfer from the state to that county. Various other papers find congruent evidence (see, e.g., Ansolabehere and Snyder 2006, and Cascio and Washington 2014). Clearly, this is not a U.S.-specific phenomenon. There is evidence of similar targeting in other countries and electoral systems, such as Germany (Kauder et al. 2016), Italy (Carozzi and Repetto 2016), and the Nordic countries (Tavits 2009; Fiva and Halse 2016). Evidence of targeting at the individual level has also been established, see, e.g., Hsieh et al. (2011) for individual-level retaliation against political opponents, and the entire patronage literature (e.g., Fafchamps and Labonne 2017 and the references reviewed in Golden and Min 2013).

There is also evidence that transcends this typology of why geographical targetability is possible. To this regard, De Luca et al. (2018) “rely on nighttime light intensity to capture a broad range of preferential policies targeted towards the political leaders’ ethnic homelands.” They consider thousands of ethnographic regions, not congruent with electoral districts, from more than 140 countries, finding that ethnographic regions have 7%-10% more intense nighttime light and roughly 3% higher regional GDP when a member of their ethnic group is the country’s political leader than in other times. Such studies show that the ability of leaders to target their own specific ethnic group is a global phenomenon.

Last but not least, several papers directly test the importance of our relative sensitive effect for the allocation of governmental resources in the U.S. Stashko (2020) focuses on U.S. state governments and legislative elections. She finds that our relative electoral sensitivity effect influences (both statistically and economically) significantly the allocations of state expenditures to counties. That is, the amount received by a county depends not only on the electoral sensitivity of that county, but also on the electoral sensitivity of the other counties in the same district. Naddeo (2020) focuses on the allocation of FEMA funds to counties by state government, demonstrating that the relative sensitivity of a county influences the allocation of funds.

work of existing formulas. As explained in Smith (2006) (p. 129), “Ward and John (1999) and John and Ward (2000) find that under a Conservative government the English local government grant system appeared systematically to favour marginal constituencies, and so-called ‘flagship’ Conservative authorities (but not Conservative local governments in general). They argue that the formulae mechanisms are sufficiently opaque to make such manipulation feasible.” Similarly, he mentions that “Gibson (1998) examines the case of inner London education services, and concludes that ‘there was pure (that is, unadulterated) political manipulation of the Education [funding formula] by the Conservative government in the 1990s’ in order to favour inner London authorities.”

B The Objective of Parties

In the main text, we work under the assumption that parties maximize their expected seat share. However, a number of political economy models assume instead that parties maximize their probability of obtaining a majority of seats in MAJ systems and their expected vote share in PR systems (see, *e.g.*, Lizzeri and Persico (2001), Strömberg (2008)). The main motivation for using system-specific utility functions is to capture the extra payoff that the party winning a majority of seats obtains under MAJ as compared to PR systems. As discussed in Snyder (1989), modeling MAJ systems in this way highlights the pivotability of a seat/district in the national assembly. However, just because a party has a one-seat majority in the legislative assembly does not automatically mean it can pass all the legislation it wants (a case in point is the current situation in the U.S. Senate). Passing legislation is typically much easier when the party has a comfortable super-majority. Hence, even in MAJ systems, parties benefit from earning extra seats beyond a simple majority, an advantage we try to capture with our objective function. Finally, there is empirical evidence in support of our assumption that parties maximize their number of seats in the national assembly (see Jacobson 1985 and Incerti 2015).

To alleviate potential concerns about this assumption, in what follows we study the case of parties that maximize their probability of winning a majority of seats in the national assembly for our model with uniform shocks, both for the PR and the MAJ systems. Under the PR and MAJ systems respectively, the parties' objective functions (28) and (30) become:

$$\text{In PR: } \max_{\mathbf{q}} \frac{1}{2} + \Pr \left[\sum_l \frac{s_l}{T} (\Delta u_l(\mathbf{q}) - \beta_{d(l)}) \geq 0 \right], \quad (22)$$

$$\text{In MAJ: } \max_{\mathbf{q}} \Pr \left[\sum_d \mathbf{1}_d \geq \frac{D}{2} \right], \quad (23)$$

where $\mathbf{1}_d$ takes value 1 if $\pi_d(\mathbf{q}; \delta_d) \geq 1/2$, and 0 otherwise.

The objective function (22) under PR systems is just a monotone transformation of the original objective function (28). For this reason, it produces the same first order conditions, and therefore the same equilibrium allocations as in Section 4.

The differences are more consequential under MAJ systems, where obtaining a majority at the district level is no longer the objective itself. Here, winning a given district only matters insofar as it helps to reach the threshold of 50% of *all* districts. As explained in Lindbeck and Weibull (1987) and Strömberg (2008), this problem is technically intractable with a small number of districts. However, we can focus on its approximate solution, which exploits Lyapunov's central limit

theorem.

Let:

$$\mu(\mathbf{q}) := \sum_d p_d(\mathbf{q}) = \frac{D}{2} + \sum_d \gamma_d \times \left[\frac{\sum_{l \in d} s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]$$

be A 's expected seat share, and define:

$$\sigma_E^2(\mathbf{q}) := \sum_d p_d(\mathbf{q}) [1 - p_d(\mathbf{q})].$$

Since the individuals $p_d(\mathbf{q})$ are statistically independent from one another, the CLT of Liapunov tells us that:

$$\frac{\sum_d \mathbb{1}_d - \mu(\mathbf{q})}{\sigma_E(\mathbf{q})},$$

is asymptotically distributed as a standard normal.

The probability that A wins a majority of the seats given policy platforms \mathbf{q} is therefore:

$$\pi_A(\mathbf{q}) = \Pr \left(\frac{\sum_d \mathbb{1}_d - \mu(\mathbf{q})}{\sigma_E(\mathbf{q})} \geq \frac{D/2 - \mu(\mathbf{q})}{\sigma_E(\mathbf{q})} \right)$$

Using the asymptotic distribution in this, the probability that A wins is:

$$\pi_A(\mathbf{q}) \approx 1 - \Phi[S(\mathbf{q})],$$

where $S(\mathbf{q}) = \frac{D/2 - \mu(\mathbf{q})}{\sigma_E(\mathbf{q})}$ and $\Phi[\cdot]$ is the standard normal cumulative density function.

Note that:

$$\sigma_E^2(\mathbf{q}) = \frac{D}{4} - \sum_{d \in C} \gamma_d^2 \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]^2 \quad (24)$$

which implies:

$$S(\mathbf{q}) = \frac{-\sum_d \gamma_d \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]}{\left(\frac{D}{4} - \sum_d \gamma_d^2 \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]^2 \right)^{1/2}} \quad (25)$$

When parties maximize their approximate probability of winning, the problem of party A becomes:

$$\begin{aligned} \max_{\mathbf{q}^A} \quad & 1 - \Phi \left[\frac{-\sum_d \gamma_d \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]}{\left(\frac{D}{4} - \sum_d \gamma_d^2 \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]^2 \right)^{1/2}} \right] \\ \text{s.t.} \quad & \sum_l n_l^\alpha q_l = y, \end{aligned}$$

which leads to the first order conditions:

$$\begin{aligned} n_l^\alpha \lambda^A &= -\phi(S(\mathbf{q})) S(\mathbf{q}) \times \left[\frac{-\frac{\partial \mu(\mathbf{q})}{\partial q_l}}{\frac{D}{2} - \mu(\mathbf{q})} - \frac{\frac{\partial \sigma_E^2(\mathbf{q})}{\partial q_l}}{\sigma_E^2(\mathbf{q})} \right] \\ &= -\phi(S(\mathbf{q})) S(\mathbf{q}) \times \left[\frac{\gamma_{d(l)} \frac{s_l}{\sum_{j \in d(l)} s_j} u'(q_l)}{\sum_d \gamma_d \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]} - \frac{\gamma_{d(l)}^2 \frac{s_l}{\sum_{j \in d(l)} s_j} u'(q_l) \times \left[p_{d(l)}(\mathbf{q}) - \frac{1}{2} \right]}{\sigma_E^2(\mathbf{q})} \right] \end{aligned}$$

As explained by Strömberg (2008), the first term captures the incentive of the candidate to influence the expected number of electoral votes won, the mean of the distribution, while the second term arises from the incentive to influence the variance in the number of electoral votes.

As before, in equilibrium, $\mathbf{q}_A = \mathbf{q}_B$, which allows us to simplify the FOC into:

$$\frac{\lambda^A \times \sum_d \gamma_d \beta_d}{\phi(S(\mathbf{q})) S(\mathbf{q})} = \gamma_{d(l)} \frac{s_l n_l^{-\alpha}}{\sum_{j \in d(l)} s_j} u'(q_l) \left[1 + \frac{\gamma_{d(l)} \beta_{d(l)} \times \sum_d \gamma_d \beta_d}{\sigma_E^2(\mathbf{q})} \right]$$

where the left-hand side of the equation is independent of l . We can thus label it as λ' and we find that the equilibrium allocation must satisfy:

$$\lambda' = \gamma_{d(l)} \frac{s_l n_l^{-\alpha}}{\sum_{j \in d(l)} s_j} u'(q_l) \left[1 + \frac{\sum_d \gamma_d \beta_d}{\sigma_E^2(\mathbf{q})} \gamma_{d(l)} \beta_{d(l)} \right], \quad (26)$$

which directly compares to (15), the FOC under MAJ systems. We see that the two are identical except for the second term inside the square bracket. This implies that both the relative electoral sensitivity of localities and the contestability of districts are still key in explaining government interventions.

The second term in the square bracket has a natural interpretation. The fraction denotes the average, national, bias in favor of B : If positive, B is more likely to win than A , and vice versa. Let us assume it is positive for the sake of discussion. In this case, the localities benefiting from more government interventions are those belonging to districts that are more contestable *and* also biased toward B ($\gamma_{d(l)} \beta_{d(l)}$ large). This is the same “pivotability effect” as that identified in Lindbeck and Weibull (1987, pp288-289): “[District d] is more likely to be a pivot [district] the

stronger is [its] bias in favour of the more popular party, since the exclusion of such [a district] from the electorate leaves the remaining electorate as little biased as possible, and hence also as likely as possible to produce a tie.”

C Uniform Shocks

This section assumes that the voter’s preference shocks are uniformly distributed:

$$\nu_{i,l} \sim U \left[\frac{-1}{2\phi_l}, \frac{1}{2\phi_l} \right] \quad \text{and} \quad \delta_d \sim U \left[\beta_d - \frac{1}{2\gamma_d}, \beta_d + \frac{1}{2\gamma_d} \right].$$

Whenever taking uniform distributions, we also assume that there are voters to be swung in all localities, that is:

Assumption 1 (Interior) For all \mathbf{q} and δ_d , $\tilde{\nu}_l(\mathbf{q}, \delta) \equiv \Delta u_l(\mathbf{q}) - \delta_d \in \left(-\frac{1}{2\phi_l}, \frac{1}{2\phi_l} \right)$ in all localities for all \mathbf{q} and δ .

With this assumption, locality-level vote shares for A are:

$$\pi_l(\mathbf{q}; \delta_d) = \frac{1}{2} + \phi_l (\Delta u_l(\mathbf{q}) - \delta_d). \quad (27)$$

Objective in PR

In the uniform case, and letting $s_l = n_l t_l \phi_l$ be the *electoral sensitivity* of locality l , Party A’s objective function can be written as:²³

$$\begin{aligned} \max_{\mathbf{q} \mid \sum_l n_l^\alpha q_l \leq y} \pi_{PR}(\mathbf{q}) &:= \mathbb{E}_\delta \left[\sum_l t_l n_l \left[\frac{1}{2} + \phi_l (\Delta u_l(\mathbf{q}) - \delta_{d(l)}) \right] \right] \\ &= \frac{T}{2} + \sum_l s_l (\Delta u_l(\mathbf{q}) - \mathbb{E}[\delta_{d(l)}]), \\ &= \frac{T}{2} + \sum_l s_l (\Delta u_l(\mathbf{q}) - \beta_{d(l)}). \end{aligned} \quad (28)$$

This implies the characterization of the equilibrium allocation in (11) applies.

²³For the last equality, note that $\sum_d \beta_d \sum_{l \in d} s_l$ can be rewritten as $\sum_l s_l \beta_{d(l)}$.

Objective in MAJ

Under MAJ, seats are proportional to the number of districts won by each party. We are interested in p_d from equation (13), the probability that A wins at least 50% of the votes in district d :

$$p_d(\mathbf{q}) = \Pr \left[\delta_d \leq \frac{\sum_{l \in d} s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} \right].$$

To avoid corner solutions and ensure that payoffs are differentiable everywhere, throughout the paper we assume that this probability is non-degenerate for any allocation. In other words, we assume that all districts are *contestable* (districts that are not contestable would receive an allocation equal to 0):

Assumption 2 (Contestability) $p_d(\mathbf{q}) \in (0, 1), \forall d, \mathbf{q}$.

Under Assumption 2, the probability p_d is always strictly between 0 and 1, and can be directly derived from the CDF of a uniform distribution:

$$\begin{aligned} p_d(\mathbf{q}) = F_{\delta_d} \left[\frac{\sum_{l \in d} s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} \right] &= \gamma_d \times \left[\frac{\sum_{l \in d} s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} + \frac{1}{2\gamma_d} - \beta_d \right] \\ &= \frac{1}{2} + \gamma_d \times \left[\frac{\sum_{l \in d} s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]. \end{aligned} \quad (29)$$

Aggregating these probabilities across districts yields A 's expected seat share:

$$\pi_{MAJ}(\mathbf{q}) = \frac{1}{2} + \frac{\sum_d \gamma_d \left[\sum_{l \in d} \frac{s_l \Delta u_l(\mathbf{q})}{\sum_{j \in d} s_j} - \beta_d \right]}{D}. \quad (30)$$

It follows that the optimal allocation is characterized by (16) where we use our constant values of γ_d and s_l .

Discussing Assumptions 1 and 2

First consider Assumption 1. Let $\bar{\Delta} = u(y) - u(0)$ be the largest possible utility difference coming from the allocation of public goods. There are always some swing voters in l if

$$-\bar{\Delta} - \beta_d - \frac{1}{2\gamma_d} > -\frac{1}{2\phi_l} \quad \& \quad \bar{\Delta} - \beta_d + \frac{1}{2\gamma_d} < \frac{1}{2\phi_l}.$$

Notice that the first (second) inequality is more likely to bind if β_d is positive (negative). Assumption 1 requires the variance in the individual preference to be large enough compared to the bias. Indeed, the assumption is satisfied if:

$$|\beta_d| < -\bar{\Delta} - \frac{1}{2\gamma_d} + \frac{1}{2\phi_l}.$$

Next, consider Assumption 2. Contestable districts are such that both candidates always have a strictly positive chance of winning a majority of the votes in that district: $p_d^A(\mathbf{q}) \in]0, 1[\forall \mathbf{q}$. Therefore, a district is contestable if and only if:

$$\frac{\sum_{l \in d} s_l \Delta u_l(\mathbf{q})}{s_d} \in [\beta_d - \frac{1}{2\gamma_d}, \beta_d + \frac{1}{2\gamma_d}].$$

Let $\bar{\Delta U}_d = \max_{\mathbf{q}^A | \sum_l q_l^A = y} \sum_{l \in d} \frac{s_l}{s_d} (u_l(\mathbf{q}^A) - u_l(\mathbf{0}))$ be the largest possible utility gain in the district coming from the allocation of public goods. The district is contestable if

$$-\bar{\Delta U}_d \geq \beta_d - \frac{1}{2\gamma_d} \quad \& \quad \bar{\Delta U}_d \leq \beta_d + \frac{1}{2\gamma_d}.$$

Notice that the first (second) inequality is more likely to bind if β_d is positive (negative). Hence, the assumption is satisfied iff: $\bar{\Delta U}_d + |\beta_d| \leq \frac{1}{2\gamma_d}$. That is, to be contestable, the variance of the district shock must be large enough compared to the bias.

Following Persson and Tabellini (1999) and Galasso and Nunnari (2018), we could also consider some *non-contestable* districts. Non-contestable districts are such that, for any allocation, one of the parties has a zero probability of winning – that is, $p_d(\mathbf{q}) = 0$ or $p_d(\mathbf{q}) = 1 \forall \mathbf{q}$. By definition, non-contestable districts cannot be swung, and therefore parties would not spend any of their budget on localities belonging to such districts.

Equilibrium Existence and Uniqueness

The set of feasible allocations $Q = \{\mathbf{q} \mid \sum_l n_l^\alpha q_l \leq y\}$ is compact and convex. Let's define the expected plurality shares a la Banks and Duggan (1999): $P_l^A(\mathbf{q}) = 2s_l (\Delta u_l(\mathbf{q}) - \mathbb{E}[\delta_{d(l)}]) - n_l t_l$ and $P_l^B(\mathbf{q}) = n_l t_l - 2s_l (\Delta u_l(\mathbf{q}) - \mathbb{E}[\delta_{d(l)}])$. Since $P_l^A(\mathbf{q})$ and $P_l^B(\mathbf{q})$ are jointly continuous in \mathbf{q} , $P_l^j(\mathbf{q})$ is strictly concave in \mathbf{q}^j for $j \in \{A, B\}$ and $P_l^A(\mathbf{q}) + P_l^B(\mathbf{q})$ is constant for all \mathbf{q} then Theorems 2 and 3 of Banks and Duggan (1999) guarantee existence and uniqueness of the equilibrium. The argument for existence and uniqueness of the equilibrium is the same for PR systems.

D Allocations under CRRA Utility Function

This appendix provides the explicit solutions of the planner's optimum and the electoral competition's equilibrium under each electoral system assuming uniform shocks and a particular utility function: the Constant Relative Risk Aversion (CRRA) utility functions:

$$u(q_l) = \begin{cases} \frac{q_l^{1-\rho}}{1-\rho}, & \text{if } \rho \neq 1 \\ \log q_l, & \text{if } \rho = 1. \end{cases}$$

For simplicity, we focus on the case of pure local public goods ($\alpha = 0$) so that the budget constraint is: $\sum_l q_l \leq y$. CRRA utility implies that the budget shares of each locality are independent of the budget size y .

(3) tells us that the socially optimal allocation of public goods is:

$$q_l^* = y \frac{n_l^{\frac{1}{\rho}}}{\sum_k n_k^{\frac{1}{\rho}}} \quad (31)$$

Under PR systems, the characterization of the optimum in (11) and some straightforward manipulations produces the following allocation of public goods:

$$q_l^{PR} = y \frac{(s_l)^{1/\rho}}{\sum_{k=1}^L (s_k)^{1/\rho}}, \quad (32)$$

where $s_l = n_l t_l \phi_l$ is the *electoral sensitivity* of locality l .

The characterization of the optimum for MAJ systems in (16) yield:

$$q_l^{MAJ} = y \frac{(\gamma_{d(l)} s_l / \mathbf{s}_{d(l)})^{1/\rho}}{\sum_{k=1}^L (\gamma_{d(k)} s_k / \mathbf{s}_{d(k)})^{1/\rho}}, \quad (33)$$

where $\mathbf{s}_d = \sum_{k \in d} s_k$.

In the case of the U.S. electoral college [see (17)], the allocation becomes

$$q_l^{EC} = y \left(\frac{(\omega_{d(l)} \gamma_{d(l)} s_l / \mathbf{s}_{d(l)})^{1/\rho}}{\sum_{k=1}^L (\omega_{d(k)} \gamma_{d(k)} s_k / \mathbf{s}_{d(k)})^{1/\rho}} \right), \quad (34)$$

where d are U.S. states, l are counties and ω_d is the number of electors that state d has. While introducing proportional representation within states in the U.S., keeping the same electoral college weights (18) would give:

$$q_l^{PR-EC} = y \left(\frac{(\omega_{d(l)} s_l / n_{d(l)} t_{d(l)})^{1/\rho}}{\sum_{k=1}^L (\omega_{d(k)} s_k / n_{d(k)} t_{d(k)})^{1/\rho}} \right). \quad (35)$$

Finally, if politicians could not discriminate across counties but had to allocate the same amount to all localities within a district (a thought experiment that we carry out in Section 6), we would get

$$q_l^{PR-district} = y \left(\frac{\left(\frac{s_d}{L_d} \right)^{1/\rho}}{\sum_{d'} \left(\frac{s_{d'}}{L_{d'}} \right)^{1/\rho} L_{d'}} \right) \quad \& \quad q_l^{EC-district} = y \left(\frac{\left(\frac{\omega_d \gamma_d}{L_d} \right)^{1/\rho}}{\sum_{d'} \left(\frac{\omega_{d'} \gamma_{d'}}{L_{d'}} \right)^{1/\rho} L_{d'}} \right), \quad (36)$$

where $L_d = \sum_{l \in d} 1$.

E Proof of Proposition 3

We normalize $y = 1$ without loss of generality since, with CRRA utility functions, the equilibrium budget shares are budget invariant.

The first step of the proof consists of proving the following claim: PR systems Atkinson-dominate MAJ systems if and only if:

$$\frac{\sum_l n_l (s_l)^{\frac{1-\rho}{\rho}}}{\left(\sum_k (s_k)^{\frac{1}{\rho}} \right)^{1-\rho}} \leq \frac{\sum_l n_l \left(\frac{\gamma_{d(l)} s_l}{\sum_{k \in d(l)} s_k} \right)^{\frac{1-\rho}{\rho}}}{\left(\sum_k \left(\frac{\gamma_{d(k)} s_k}{\sum_{j \in d(k)} s_j} \right)^{\frac{1}{\rho}} \right)^{1-\rho}} \quad \text{for } \rho \geq 1, \quad (37)$$

and:

$$\sum_d n_d \log \left[\frac{s_d}{\sum_{d'} s_{d'}} \right] < \sum_d n_d \log \left[\frac{\gamma_d}{\sum_{d'} \gamma_{d'}} \right] \text{ for } \rho = 1. \quad (38)$$

To prove this claim, note that $A(\mathbf{q}^{PR}) < A(\mathbf{q}^{MAJ})$ iff $y^E(\mathbf{q}^{PR}) > y_\rho^E(\mathbf{q}^{MAJ})$ where:

$$y^E(\mathbf{q}) = \begin{cases} \prod_l (q_l/n_l)^{n_l} & \text{if } \rho = 1; \\ \left[\frac{\sum_l n_l (q_l)^{1-\rho}}{\left(\sum_j n_j^{\frac{1}{\rho}} \right)^\rho} \right]^{\frac{1}{1-\rho}} & \text{if } \rho \neq 1. \end{cases} \quad (39)$$

Consider first the logarithmic case ($\rho = 1$). Plugging the values \mathbf{q}^{PR} and \mathbf{q}^{MAJ} from Appendix D into y^E/y tells us that $A(\mathbf{q}^{PR}) < A(\mathbf{q}^{MAJ})$ iff:

$$\begin{aligned} \prod_l \left(\frac{t_l \phi_l}{\sum_{k=1}^L s_k} \right)^{n_l} &> \prod_l \left(\frac{t_l \phi_l \gamma_{d(l)} / \mathbf{s}_{d(l)}}{\sum_{k=1}^L s_k \gamma_{d(k)} / \mathbf{s}_{d(k)}} \right)^{n_l} \\ \prod_l \left(\frac{1}{\sum_{k=1}^L s_k} \right)^{n_l} &> \prod_l \left(\frac{\gamma_{d(l)} / \mathbf{s}_{d(l)}}{\sum_{k=1}^L s_k \gamma_{d(k)} / \mathbf{s}_{d(k)}} \right)^{n_l}. \end{aligned} \quad (40)$$

Note that the denominator on the RHS of (40) is equivalent to $\sum_d \frac{\gamma_d}{s_d} \sum_{k \in d} s_k = \sum_d \gamma_d$. Similarly, we can re-write the denominator on the LHS of (40) as $\sum_d \mathbf{s}_d$.

Substituting for these into (40), we get

$$\prod_l \left(\frac{1}{\sum_d \mathbf{s}_d} \right)^{n_l} > \prod_l \left(\frac{\gamma_{d(l)} / \mathbf{s}_{d(l)}}{\sum_d \gamma_d} \right)^{n_l}.$$

Taking logarithms, and noting that $\sum n_l = 1$, yields:

$$\begin{aligned} -\log \left[\sum_{d'} s_{d'} \right] &> \sum_l n_l \log \left[\frac{\gamma_{d(l)}}{s_{d(l)}} \right] - \log \left[\sum_{d'} \gamma_{d'} \right] \text{ or} \\ -\log \left[\sum_{d'} s_{d'} \right] &> \sum_d \mathbf{n}_d \log \gamma_d - \sum_d \mathbf{n}_d \log \mathbf{s}_d - \log \left[\sum_{d'} \gamma_{d'} \right] \text{ or} \\ \sum_d \mathbf{n}_d \log \left[\frac{s_d}{\sum_{d'} s_{d'}} \right] &> \sum_d \mathbf{n}_d \log \left[\frac{\gamma_d}{\sum_{d'} \gamma_{d'}} \right], \end{aligned} \quad (41)$$

where $\mathbf{n}_d = \sum_{l \in d} n_l$.

This proves the claim of our first step for the logarithmic case.

Similarly for $\rho \neq 1$, we substitute the equilibrium values of the allocation under each electoral

system into y^E/y and multiply by $\left(\sum_j n_j^{\frac{1}{\rho}}\right)^{\rho/(1-\rho)}$. This tells us that $A(\mathbf{q}^{PR}) < A(\mathbf{q}^{MAJ})$ iff:

$$\left[\sum_l n_l \left(\frac{(s_l)^{1/\rho}}{\sum_{k=1}^L (s_k)^{1/\rho}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} > \left[\sum_l n_l \left(\frac{(s_l \gamma_{d(l)})^{1/\rho}}{\sum_{k=1}^L (s_k \gamma_{d(k)})^{1/\rho}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \quad (42)$$

which finishes proving the claim of our first step.

Now consider the case where $\rho = 1$. When all districts are well apportioned ($n_d = 1/D \forall d$), inequality (41) becomes

$$\frac{1}{D} \sum_d \log \left[\frac{s_d}{\sum_{d'} s_{d'}} \right] > \frac{1}{D} \sum_d \log \left[\frac{\gamma_d}{\sum_{d'} \gamma_{d'}} \right]$$

Atkinson (1983) shows the strict concavity of the log implies that this inequality holds if $\frac{\gamma_d}{\sum_{d'} \gamma_{d'}}$ is a mean preserving spread of $\frac{s_d}{\sum_{d'} s_{d'}}$ (and vice versa).

Next, consider the case $\rho \neq 1$ and $L = D$. With one locality per district, all relative sensitivities are 1 ($s_l/s_{d(l)} s_l = 1$). Simplifying for $n_l = 1/L$, inequality (42) becomes:

$$\left[\sum_l \left(\frac{(s_l)^{1/\rho}}{\sum_{k=1}^L (s_k)^{1/\rho}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} > \left[\sum_l \left(\frac{(\gamma_{d(l)})^{1/\rho}}{\sum_d (\gamma_d)^{1/\rho}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$

or

$$\frac{1}{1-\rho} \sum_l \left(\frac{(s_l)^{1/\rho}}{\sum_{k=1}^L (s_k)^{1/\rho}} \right)^{1-\rho} > \frac{1}{1-\rho} \sum_l \left(\frac{(\gamma_{d(l)})^{1/\rho}}{\sum_d (\gamma_d)^{1/\rho}} \right)^{1-\rho}. \quad (43)$$

Again, the strict concavity of the CRRA function implies (Atkinson (1983)) that this inequality holds if $\frac{\gamma_d}{\sum_{d'} \gamma_{d'}}$ is a mean preserving spread of $\frac{s_d}{\sum_{d'} s_{d'}}$ (and vice versa).

F Reforms: The U.S. Presidential Electoral System

Proof of Proposition 4

The proof is straightforward. Let us consider what happens when we divide districts into finer and finer partitions of localities with similar characteristics (so that $\frac{s_{d(l)}}{n_{d(l)}} = \frac{s_l}{n_l}, \forall l$).²⁴ This ensures

²⁴To hold everything else constant, the distribution of district-specific shocks δ_d is the same as in the original, larger, district. At the limit $D = L$, however, the distribution of these shocks becomes irrelevant,

that the FOC under the Electoral College becomes:

$$\frac{\partial u_l(\mathbf{q}^{\text{College}})}{\partial q_l^A} \propto \frac{n_{d(l)}/\gamma_{d(l)}}{\omega_{d(l)}} n_l^{\alpha-1}.$$

Then, reapportioning the Electoral College so that $\omega_{d(l)} = n_{d(l)}/\gamma_{d(l)}$ ensures that the resulting allocation of public goods converges towards the social welfare optimum. In the limit, i.e., redistricting up to the point where $D \rightarrow L$, ensures that the first condition can be attained.

Proportional Allocation of Electors

In this section, we study the properties of a PR version of the Electoral College. This transforms the EC system into one of proportional representation with districts. The results thus also apply to district-PR systems as in Belgium or Brazil.

We start with the vote share of A in a district d :

$$\pi_d(\mathbf{q}; \delta_d) = \frac{1}{2} + \sum_{l \in d} \frac{t_l n_l}{n_d} \phi_l [\Delta u_l(\mathbf{q}) - \delta_d],$$

where $n_d := \sum_{k \in d} t_k n_k$ is the expected number of votes in district d .

When there are D local elections under PR and each district receives a fraction ω_d of the Electors, the seat share of party A in the Electoral College becomes: $\sum_d \omega_d \pi_d(\mathbf{q}; \delta_d)$. Taking expectations, the objective function of party A is then:

$$\pi^{PR-EC}(\mathbf{q}) = \frac{1}{2} + \sum_d \frac{\omega_d}{n_d} \sum_{l \in d} s_l (\Delta u_l(\mathbf{q}) - \beta_{d(l)}).$$

To compare, maximizing the probability of winning under the Popular Vote would produce the same allocation of public goods if $\omega_d = n_d$ in the Electoral College under PR.

If instead $\omega_d = \sum_{k \in d} n_k$, i.e., if the Electoral College were well apportioned, then lower turnout and information rates in a given district would not translate into a lower provision of public goods in that district. This brings the allocation closer to the social optimum.

since they can be corrected by reweighting the Electoral College.

G Numerical Simulations

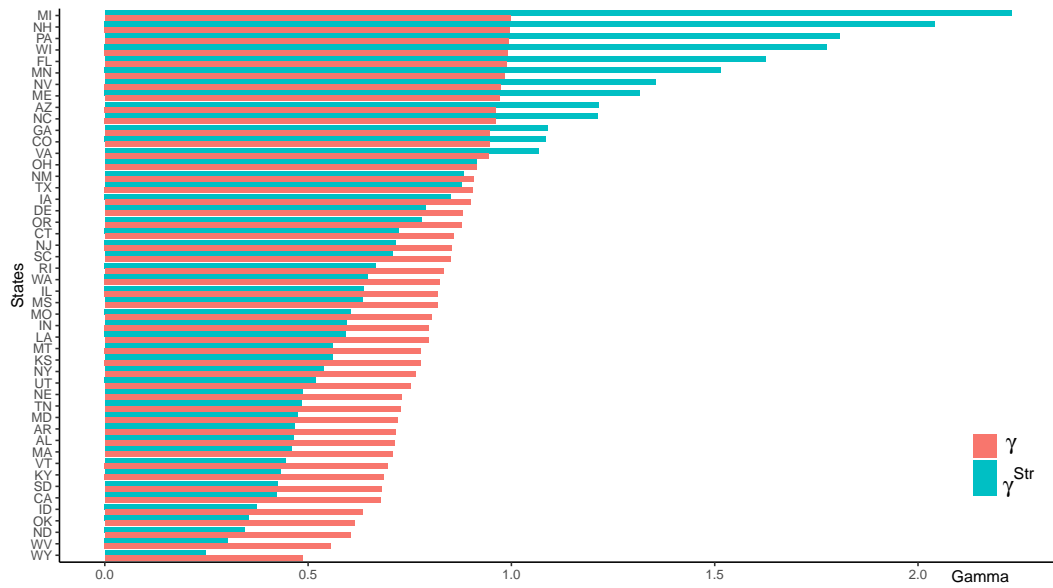
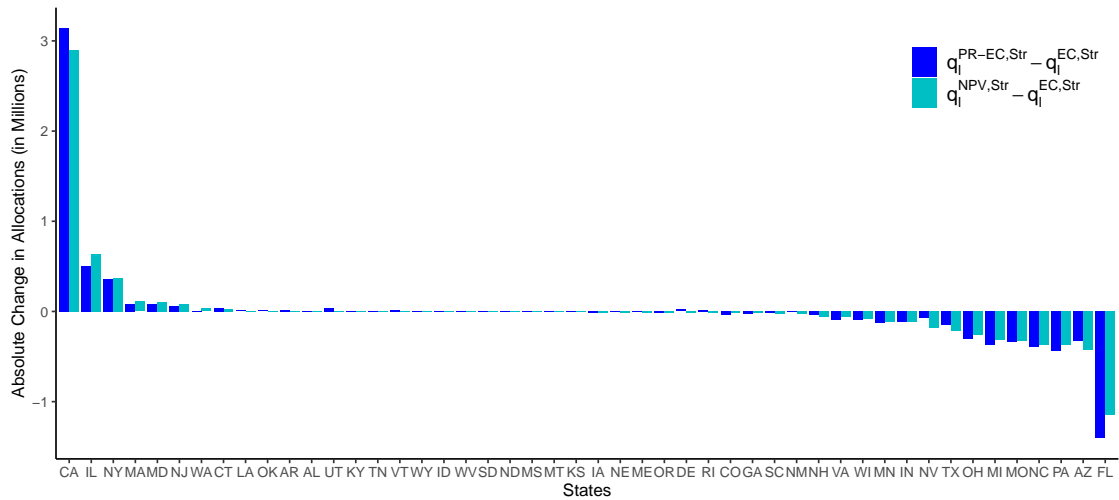
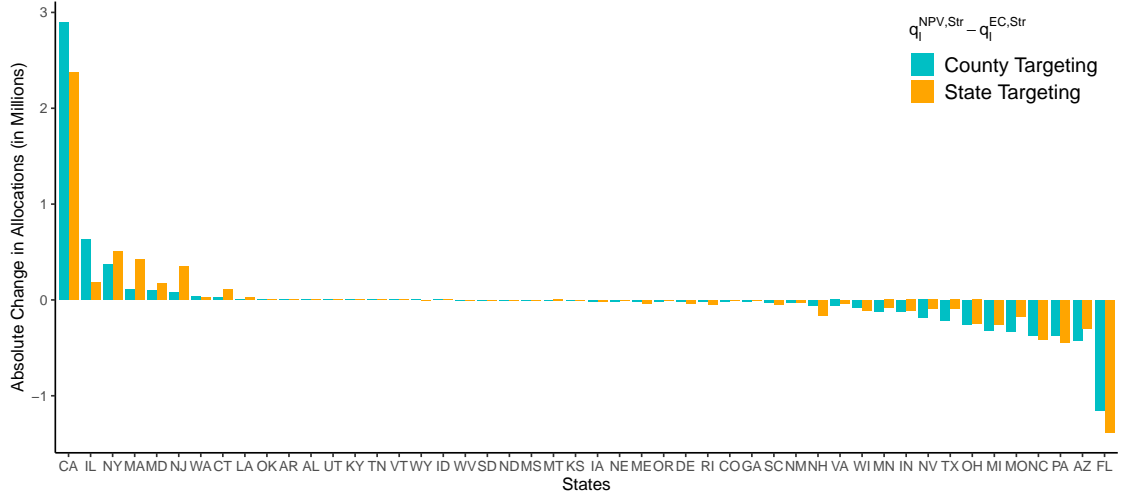


Figure 4: COMPARING THE TWO MEASURES OF CONTESTABILITY FOR 2016



Notes: Average for 2008-2016.

Figure 5: WINNERS AND LOSERS OF A REFORM FROM EC TO NPV OR PR-EC



Notes: Average for 2008-2016.

Figure 6: WINNERS AND LOSERS OF A REFORM FOR COUNTY AND STATE TARGETING

H Beyond Geographically Targeted Interventions

Like in the base model, individual i votes for party A if and only if:

$$\nu_{i,g,l} + \delta_d \leq \Delta v_g(\mathbf{b}), \quad (44)$$

where $\nu_{i,g,l} \sim \Phi_{g,l}(\cdot)$ which can be both locality and group-specific. For tractability reasons, we maintain the assumption that $\delta_d \sim \Gamma_d(\cdot)$ is district-specific and common to all groups and localities in the district. Extending on the notation of the main model, each party P proposes an allocation $\mathbf{b}^P := \{b_1^P, \dots, b_G^P\}$, and voters compare the value of these two proposals $\Delta v_g(\mathbf{b}) := v_g(\mathbf{b}^A) - v_g(\mathbf{b}^B)$.

It follows that A 's vote share in locality l is $\sum_g t_{g,l} n_{g,l} \Phi_{g,l}(\Delta v_g(\mathbf{b}) - \delta_d) / \sum_h t_{h,l} n_{h,l}$, and its district vote share is:

$$\pi_d(\mathbf{b}; \delta_d) = \sum_{l \in d} \sum_g \frac{t_{g,l} n_{g,l} \Phi_{g,l}(\Delta v_g(\mathbf{b}) - \delta_d)}{T_d},$$

with $T_d := \sum_{k \in d} \sum_h t_{h,k} n_{h,k}$ denoting the total number of actual votes in the district. The marginal effect of increasing b_g on a given district's vote share is:

$$\frac{d\pi_d(\mathbf{b}; \delta_d)}{db_g} = \sum_{l \in d} \frac{t_{g,l} n_{g,l}}{T_d} \phi_{g,l}[\Delta v_g(\mathbf{b}) - \delta_d] \times v'_g(b_g^A)$$

The marginal effect of an increase in δ_d is instead:

$$\frac{d\pi_d(\mathbf{b}, \mathbf{q}; \delta_d)}{d\delta_d} = -\frac{1}{T_d} \sum_{l \in d} \sum_g t_{g,l} n_{g,l} \phi_{g,l} [\Delta v_g(\mathbf{b}) - \delta_d]$$

FOCs under PR

Under PR, parties maximise their total expected vote share or vote count:

$$\frac{1}{T} \sum_g \sum_l t_{g,l} n_{g,l} \Phi_{g,l}(\Delta v_g(\mathbf{b}) - \delta_d) \text{ or } \sum_g \sum_l t_{g,l} n_{g,l} \Phi_{g,l}(\Delta v_g(\mathbf{b}) - \delta_d),$$

and it is immediate to check that the FOCs are:

$$\sum_l t_{g,l} n_{g,l} \bar{\phi}_{g,l} v'(b^{PR}) = n_g^\beta \lambda^{PR}.$$

FOCs under MAJ

Under MAJ, the objective function becomes:

$$\max_{\mathbf{b}^A | \sum_g n_g^\beta b_g = y} \sum_d \Gamma_d(D_d(\mathbf{b})),$$

where $D_d(\mathbf{b})$ is the unique cutoff value of δ_d that separates district loss from district win given an allocation \mathbf{b} . It is implicitly defined as the solution to:

$$\pi_d(\mathbf{b}; D_d(\mathbf{b})) \equiv \sum_{l \in d} \sum_g \frac{t_{g,l} n_{g,l}}{T_d} \Phi_{g,l}(\Delta v_g(\mathbf{b}) - D_d(\mathbf{b})) = \frac{1}{2}. \quad (45)$$

The first order conditions thus become:

$$\sum_d \gamma_d(D_d(\mathbf{b})) \frac{dD_d(\mathbf{b})}{db_g^A} = n_g^\beta \lambda^{MAJ} \quad \forall g. \quad (46)$$

where $\gamma_d := d\Gamma_d(D_d(\mathbf{b}))/d\delta_d$. Recall that $D_d(\mathbf{b})$ is implicitly defined in (45). Using the implicit function theorem, we see that:

$$\frac{dD_d(\mathbf{b})}{db_g^A} = -\frac{\partial \pi_d / \partial b_g}{\partial \pi_d / \partial \delta_d} = \frac{\frac{1}{T_d} \sum_{l \in d} t_{g,l} n_{g,l} \phi_{g,l} [\Delta v_g(\mathbf{b}) - D_d(\mathbf{b})] v'(b_g^A)}{\frac{1}{T_d} \sum_{j \in d} \sum_{g'} t_{g',j} n_{g',j} \phi_{g',j} [\Delta v_{g'}(\mathbf{b}) - D_d(\mathbf{b})]}.$$

We know that, when a pure strategy equilibrium exists, it is such that $\mathbf{b}^A = \mathbf{b}^B = \mathbf{b}^*$, and the cutoff value $D_d(\mathbf{b})$ is then a parameter $\hat{\delta}_d$ independent of \mathbf{b}^* . It follows that:

$$\begin{aligned} \frac{dD_d(\mathbf{b}^*)}{db_g} &= -\frac{\partial \pi_d / \partial b_g}{\partial \pi_d / \partial \hat{\delta}_d} = \frac{\sum_{l \in d} t_g n_{g,l} \hat{\phi}_{g,l} [-\hat{\delta}_d] v'(b_g^*)}{\sum_{j \in d} \sum_h t_{h,j} n_{h,j} \hat{\phi}_{h,j} [-\hat{\delta}_d]} \\ &= \frac{t_g \sum_{l \in d} n_{g,l} \hat{\phi}_{g,l}}{\underbrace{\sum_{j \in d} \sum_h t_{h,j} n_{h,j} \hat{\phi}_{h,j}}_{\hat{s}_d}} v'(b_g^*) = \frac{\overbrace{t_g \sum_{l \in d} n_{g,l} \hat{\phi}_{g,l}}^{\hat{s}_{g,d}}}{\hat{s}_d} v'(b_g^*) \end{aligned}$$

It follows that the first order conditions (46) can be expressed as:

$$\sum_d \hat{\gamma}_d \frac{\hat{s}_{g,d}}{\hat{s}_d} v'(b_g^*) = n_g^\beta \lambda^{MAJ}$$

or, after rescaling λ^{MAJ} :

$$\sum_d \frac{\hat{\gamma}_d \hat{s}_{g,d}}{\hat{\gamma} \hat{s}_d} v'(b_g^*) = n_g^\beta \lambda^{MAJ}, \text{ with } \hat{\gamma} := \sum_d \hat{\gamma}_d.$$

I Targeted versus Universal Spending

Equilibrium Allocations

In PR systems, the first order conditions are:

$$\begin{cases} \sum_l s_l u'(G) = \lambda^{PR}, \\ \frac{s_l}{n_l} = \lambda^{PR} \quad \forall l \text{ with } q_l > 0 \end{cases} \quad (47)$$

where λ^{PR} is the Lagrange multiplier of the budget constraint under PR.

In a symmetric equilibrium, only individuals in the locality with the highest s_l/n_l receives a transfer. The following inequality holds:

$$u'(G) \leq \max_l \frac{s_l}{n_l} \frac{1}{\sum_{k=1}^L s_k}, \quad (48)$$

with equality if some positive transfer is given.

In MAJ systems, the first order conditions become:

$$\begin{cases} \sum_{d \in D} \gamma_d u'(G) = \lambda^{MAJ}, \\ \frac{\gamma_{d(l)}}{n_l} \frac{s_l}{\sum_{k \in d(l)} s_k} = \lambda^{MAJ} \quad \forall l \quad \text{with } q_l > 0 \end{cases} \quad (49)$$

where λ^{MAJ} is the Lagrange multiplier of the budget constraint under MAJ.

In a symmetric equilibrium, only individuals in the locality with the highest $\frac{\gamma_{d(l)}}{n_l} \frac{s_l}{\sum_{k \in d(l)} s_k}$ receive a transfer. It follows from (49) that

$$\max_l \frac{\gamma_{d(l)}}{\sum_{d \in D} \gamma_d} \frac{1}{n_l} \frac{s_l}{\sum_{k \in d(l)} s_k} \geq u'(G), \quad (50)$$

with equality if some transfers arise in equilibrium.

Proof of Proposition 6

If $L = D$ and $dn_l = \frac{1}{D} \forall l$ then (20) and (21) tell us that

$$\begin{cases} u'(G^{PR}) \leq \max_l \frac{s_l}{\sum_{k=1}^L s_k} D, \\ u'(G^{MAJ}) \leq \max_l \frac{\gamma_l}{\sum_k \gamma_k} D \end{cases} \quad (51)$$

where the respective equalities hold whenever $G < y$.

It follows directly that if $\max_l \frac{s_l}{\sum_k s_k} > (<) \max_l \frac{\gamma_l}{\sum_k \gamma_k}$ then $G^{PR} \leq (>) G^{MAJ}$.

Table 6: CHARACTERISTICS OF THE STATES FOR 2008-2016

State	n_d	ϕ_d	s_d	t_d	γ^{str}	γ	ω_d	q^{NPV}	q^{EC}	q_{Str}^{EC}	q^{PR-EC}
WY	569	0.068	18	0.422	0.345	0.577	3.0	1	4	1	6
VT	625	0.096	29	0.472	0.413	0.651	3.0	3	7	3	17
ND	704	0.060	20	0.443	0.649	0.771	3.0	1	7	5	5
SD	827	0.054	20	0.434	0.692	0.803	3.0	1	5	4	3
DE	916	0.075	34	0.450	0.670	0.813	3.0	17	51	32	53
MT	1007	0.059	31	0.464	0.941	0.871	3.0	2	7	9	4
RI	1056	0.063	29	0.420	0.561	0.756	4.0	8	51	26	40
NH	1327	0.096	68	0.526	1.376	0.947	4.0	22	38	79	42
ME	1330	0.075	56	0.526	0.919	0.880	4.0	9	21	23	18
ID	1604	0.052	39	0.390	0.453	0.682	4.0	5	14	6	12
WV	1843	0.077	53	0.368	0.541	0.717	5.0	2	6	4	8
NE	1852	0.049	42	0.422	0.613	0.785	5.0	6	31	18	15
NM	2064	0.065	54	0.368	0.846	0.883	5.0	14	52	44	35
NV	2773	0.093	93	0.360	1.086	0.926	5.7	130	240	318	250
UT	2853	0.059	60	0.322	0.431	0.656	5.7	27	61	25	63
KS	2868	0.040	62	0.401	0.638	0.801	6.0	10	33	20	16
AR	2939	0.083	96	0.359	0.557	0.756	6.0	9	12	6	17
MS	2973	0.038	77	0.419	0.769	0.856	6.0	6	15	11	8
IA	3075	0.046	76	0.490	0.974	0.914	6.3	5	15	17	5
CT	3573	0.079	127	0.442	0.673	0.818	7.0	92	106	65	108
OK	3805	0.052	93	0.366	0.420	0.655	7.0	11	15	6	17
OR	3920	0.052	109	0.450	0.776	0.861	7.0	46	80	60	44
KY	4371	0.052	120	0.412	0.580	0.764	8.0	13	19	11	15
LA	4572	0.059	153	0.427	0.653	0.811	8.3	24	27	16	26
SC	4735	0.043	110	0.412	0.854	0.885	8.7	17	47	41	20
AL	4800	0.044	128	0.429	0.558	0.757	9.0	15	24	13	17
CO	5208	0.069	201	0.475	1.070	0.933	9.0	99	98	118	78
MN	5382	0.031	126	0.521	1.146	0.933	10.0	34	107	151	26
WI	5711	0.038	142	0.511	1.210	0.927	10.0	27	64	110	19
MD	5859	0.058	199	0.447	0.515	0.732	10.0	160	122	56	139
MO	6012	0.050	183	0.458	1.371	0.902	10.3	67	104	394	56
TN	6448	0.050	167	0.382	0.622	0.789	11.0	26	41	25	31
IN	6532	0.042	152	0.402	1.097	0.894	11.0	37	89	158	39
AZ	6594	0.069	175	0.351	1.049	0.928	10.7	372	672	795	471
MA	6625	0.070	225	0.461	0.526	0.737	11.3	175	136	65	145
WA	6917	0.061	200	0.434	0.700	0.832	11.7	253	340	220	230
VA	8143	0.053	278	0.460	1.155	0.947	13.0	123	136	186	88
NJ	8810	0.079	315	0.423	0.715	0.839	14.3	185	152	103	163
NC	9738	0.042	257	0.454	1.668	0.979	15.0	87	160	457	59
GA	9904	0.058	364	0.395	1.092	0.938	15.7	127	118	148	122
MI	9932	0.059	326	0.475	1.297	0.911	16.3	213	248	528	152
OH	11566	0.044	315	0.471	1.180	0.946	18.7	156	277	416	109
PA	12721	0.037	377	0.459	1.293	0.944	20.3	244	357	619	177
IL	12819	0.067	444	0.412	0.632	0.798	20.3	1116	840	483	987
NY	19476	0.073	566	0.375	0.521	0.736	29.7	606	511	237	598
FL	19495	0.064	666	0.442	1.647	0.983	28.3	722	718	1873	466
TX	26112	0.040	524	0.313	0.8269	0.876	36.7	303	647	522	367
CA	37925	0.083	1144	0.345	0.521	0.731	55.0	4372	3074	1473	4614