Electoral Systems and Inequalities in Government Interventions

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- **This paper:** focus on electoral systems (**MAJ vs. PR**)

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- **In PR systems**
  - fewer electoral districts
  - each select at least 2 representatives
  - seats assigned in proportion to the vote shares of each party

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      40 cases MAJ $\longrightarrow$ PR, 13 cases PR $\longrightarrow$ MAJ

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- **Results relevant for Electoral College vs. NPV**
  - Whitaker and Neale (2004): "[...] more proposed constitutional amendments have been introduced in Congress regarding electoral college reform than on any other subject."
  - current initiative: National Popular Vote Interstate Compact

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  - PR: no geographical constraint

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- **Numerical simulations** to assess **Electoral College reforms**

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- Voters consume **locality-specific public resources**: $\mathbf{q} = \{q_1, ..., q_L\}$
  - q_l is amount per capita in locality l
- Preferences $u_l(\mathbf{q}) = u(q_l)$
  - $u' > 0 > u''$
  - no spillover across localities; no differences in utility functions

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  - budget constraint: $\sum_l n_l^{\alpha} q_l = y$

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- Social planner maximizes **utilitarian welfare function**:

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- Socially optimal allocation:

$$\frac{\partial u_l(\mathbf{q})}{\partial q_l} = \lambda^{SW} n_l^{\alpha-1}, \ \forall l$$

  - socially optimal q_l increases in n_l → only **vertical inequality**
  - no effect of electoral districts, nor of political characteristics

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- We build upon Atkinson (1970, 1983)
  - assume CRRA utility:

$$u_l(\mathbf{q}) = \begin{cases} \ln(q_l) & \text{if } \rho = 1 \\ \frac{(q_l)^{1-\rho}}{1-\rho} & \text{if } \rho \neq 1 \end{cases}$$

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    - ★ ρ is individual risk aversion
  - define the equivalent budget: $y^E(\mathbf{q}) = \tilde{W}^{-1}(\mathcal{W}(\mathbf{q}))$
    - ★ were $\tilde{W}(y)$ is the indirect social utility function

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- A is a measure of financial cost of political distortions
  - the smaller A, the more efficient the allocation

The Politics

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  - make budget allocation proposals: $\mathbf{q}^A$ and $\mathbf{q}^B$

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A Model of Electoral Competition

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- **Objective**: maximize expected number of seats in national assembly
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- **Objective**: maximize expected number of seats in national assembly
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- **Electoral system**: maps votes into seats
  - *PR*: seats attributed proportionally to fraction of national votes
    - ★ as if one nationwide district
    - ★ extension: PR with districts
  - *MAJ*: seats are proportional to the fraction of districts won
    - ★ one seat per district
    - ★ districts won by FPTP

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  - $s_l = \bar{\phi}_l t_l n_l$ is the **electoral sensitivity** of locality l
    - ★ $\bar{\phi}_l = \int_{\delta_d} \phi_l\left(-\delta_d\right) d\Gamma_d\left(\delta_d\right) \rightarrow$ expected density of swing voters in l
  - λ^{PR} is the Lagrange multiplier of the budget constraint under PR

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- No effect of γ_d

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  - $\hat{\gamma}_d$ is the **contestability** of district d
    - ⋆ intuitively: proba that parties end up close to a tie in d
    - ⋆ $\hat{\delta}_d$ is the value of δ s.t. district is tied when $\mathbf{q}^A = \mathbf{q}^B$

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- For given pop. size, share of budget of locality l increases with
  - contestability of district, $\hat{\gamma}_{d(l)}$
  - **relative electoral sensitivity,** $\frac{\hat{s}_l}{\hat{s}_{d(l)}}$
    - resources allocated to a locality depend on characteristics of neighbors

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  - for given increase in support, there is a range of realizations of δ_d s.t. the change is pivotal
  - the more likely δ_d fall in pivotal range, the better the locality is treated
  - two factors determine the likelihood δ_d falls in pivotal range
    - ★ width and height

Equilibrium under MAJ

Proposition

In MAJ, $q_l > q_{l'}$ *if and only if* $\hat{\gamma}_{d(l)} \frac{\hat{s}_l n_l^{-\alpha}}{\hat{s}_{d(l)}} > \hat{\gamma}_{d(l')} \frac{\hat{s}_{l'} n_{l'}^{-\alpha}}{\hat{s}_{d(l')}}$.

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  - higher $\hat{s}_{d(l)}$ $\rightarrow$ voters in d more responsive to the shock δ_d

    $\rightarrow$ aggregate vote share in d more unstable

    $\rightarrow$ **reduces width of pivotal range**

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- $\hat{s}_l = \hat{s}_{d(l)} \longrightarrow$ all localities have the same relative sensitivity
- Differences in allocations exclusively driven by differences in contestability across district
  - trade-off MAJ vs. PR: contestability vs. sensitivity
  - overlooks role of relative sensitivity

Comparing the Systems

- Comparison of government interventions under MAJ and PR systems
  - PR: electoral sensitivity
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    - ★ district specific shock: $\delta_d \sim U[\beta_d - \frac{1}{2\gamma_d}, \beta_d + \frac{1}{2\gamma_d}]$

      ($\hat{\gamma}_d = \gamma_d =$ **contestability**)

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Comparing the Systems

Winners and Losers

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- Numerical example with 4 localities and 2 districts
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District	Locality	Sensitivity (s_l)	q_l^{PR}	q_l^{MAJ} ($\gamma_A/\gamma_B = 1$)	q_l^{MAJ} ($\gamma_A/\gamma_B = 6$)
A	1	1	2.9%	9.7%	19.4%
A	2	2	11.8%	38.7%	77.7%
B	3	2	11.8%	7.1%	0.4%
B	4	5	73.5%	44.5%	2.5%

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    - ★ PR: social optimum $A\left(\mathbf{q}^{PR}\right) = 0$
    - ★ MAJ: distortions $A\left(\mathbf{q}^{MAJ}\right) = 0.14$ for $\gamma_A/\gamma_B = 1$

      $A\left(\mathbf{q}^{MAJ}\right) = 0.71$ for $\gamma_A/\gamma_B = 6$

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PR Atkinson-dominates MAJ if $\frac{\gamma_d}{\sum_{d'=1}^{D} \gamma_{d'}}$ *is a mean preserving-spread of* $\frac{s_d}{\sum_{d'=1}^{D} s_{d'}}$ *(and conversely) when either*

1. $\rho \neq 1$*, there is one locality per district, and* $n_d = 1/D \; \forall d$*, or*

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- For those specific cases, comparison boils down to comparing
  - spread in contestabilities
  - spread in electoral sensitivities

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  - Stromberg (2008): replacing Electoral College with NPV
    $\rightarrow$ decrease in cross-states inequalities in campaign resources

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- **What if we allow for targeting at sub-district level?**

Comparing the Systems

Importance of Sub-District Targeting

Affects comparison in terms of inequalities

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- Numerical example: same as before (with $\gamma_A/\gamma_B = 6$)
  - new columns with targeting at district level

District	Locality	s_l	n_l	q_l^{PR}	q_l^{MAJ}	q_l^{PR-d}	q_l^{MAJ-d}
A	1	1	17%	2.9%	19.4%	7.8%	48.6%
A	2	2	33%	11.8%	77.7%	7.8%	48.6%
B	3	2	33%	11.8%	1.2%	42.2%	1.4%
B	4	5	17%	73.5%	2.5%	42.2%	1.4%
			Atkinson index:	**0.42**	**0.38**	**0.22**	**0.40**

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  - targeting creates within district inequality under both systems
  - what matters is the share of resources that flow to each district (weight put on new distortions)

Comparing the Systems

Importance of Sub-District Targeting

Affects gains and loses of districts

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- Different numerical example:
  - same utility function
  - 3 districts (A, B, and C)
    - each composed of two localities
    - different contestabilities: $\gamma_A = 0.2$, $\gamma_B = 1$, and $\gamma_C = 1.5$

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C	2	2	60.2%	57.3%	66.7 %	68.4%

- A and C receive more resources with district targeting, B less

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- MAJ-to-PR reform:
  - C wins under locality targeting (+3 p.p.)
  - C loses under district targeting (-1.7 p.p.)

Reforms: the U.S. Presidential Electoral System

- Study possible reforms of the Electoral College

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- **Electoral College**:
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  - National Popular Vote (NPV)
    - ★ equivalent to PR
  - PR version of the Electoral College (PR-EC)
    - ★ allocation of electors proportional to vote shares in each state

Reforms: the U.S. Presidential Electoral System

Theory

- Electoral College in our model

  $\simeq$ MAJ system with district weight ω_d

$$\frac{\partial u_l\left(\mathbf{q}^A\right)}{\partial q_l^A} = \frac{1}{\omega_{d(l)}} \frac{\lambda^{\textbf{College}}}{\Upsilon_{d(l)}} \frac{\sum_{k\in d(l)} s_k}{s_l}, \ \forall l$$

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- Comparison with MAJ:
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  - same role of contestability and relative sensitivity

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- PR version of the Electoral College in our model

$$\frac{\partial u_l\left(\mathbf{q}^A\right)}{\partial q_l^A} = \frac{n_d t_d}{\omega_d}\frac{1}{s_l}\lambda^{PR-EC},\ \forall l$$

  - $t_d := \sum_{l\in d} t_l \frac{n_l}{n_d}$ is the average turnout in d
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    - ⋆ allocation as if each district received a share of seats equal to its realized number of votes
    - ⋆ high-turnout districts tend to receive less under PR-EC than PR
  - still no effect of contestability

Reforms: the U.S. Presidential Electoral System

Numerical Simulations

- Application of results to U.S. presidential election data

Reforms: the U.S. Presidential Electoral System

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- Goal: assess numerically the implications of possible reforms of the U.S. Electoral College

Reforms: the U.S. Presidential Electoral System

Numerical Simulations

- Application of results to U.S. presidential election data
- Goal: assess numerically the implications of possible reforms of the U.S. Electoral College
- Focus on the insights that sub-district targeting brings to the question

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Data

- Match model and US political and administrative structure

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- Our dataset covers 10 presidential elections (1980-2016)

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Data

- Match model and US political and administrative structure
  - states are the districts (48 in our dataset)
  - counties are the localities (3106 in our dataset)
- Our dataset covers 10 presidential elections (1980-2016)
- We need proxies for key variables

Proxies for key variables

- n_l: decennial census information from IPUMS-NHGIS
  - post-2010, supplemented with American Community Survey

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Data

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Reforms: the U.S. Presidential Electoral System

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    - ★ Berry et al. (2010)
  - $\gamma_{d,e}^{Str}$ relies on the work and data from Stromberg (2008)
    - ★ roughly, we fit Stromberg's predictions, find relationship between fitted values and $\gamma_{d,e}$, and then extrapolate for other years

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Data

Table 4: DESCRIPTIVE STATISTICS

Statistics	Mean	Median	Std. Dev	Min	Max	N	R^2 on FE
ϕ_l	0.073	0.067	0.027	0.019	0.222	9314	0.334
t_l	0.43	0.431	0.076	0.119	0.896	9314	0.377
n_l (*)	100	26	321	0	10121	9314	0.119
s_l (*)	3	1	10	0	357	9314	0.116
s_l/s_d	0.015	0.005	0.04	0	0.713	9314	0.206
s_d (*)	190	123	206	17	1209	144	1.000
γ_d	0.83	0.841	0.111	0.486	0.999	144	1.000
γ_d^{Str}	0.83	0.719	0.412	0.248	2.54	144	1.000
ω_d	11	8.5	9.706	3	55	144	1.000

Notes: Averages for years 2008-2016. (*) in thousands.

- Variations both across counties and across states
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Reforms: the U.S. Presidential Electoral System

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- Variations both across counties and across states
  - particularly important for the absolute and relative sensitivity
- R^2 of regressions of each variable on state-year fixed effects
  - substantial within-state variation in the variables of interest

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Predicted Allocations

- We can compute the predicted allocation for
  - CRRA utility ($\rho = 0.5$)
  - uniform shocks
  - total budget of \$10 million

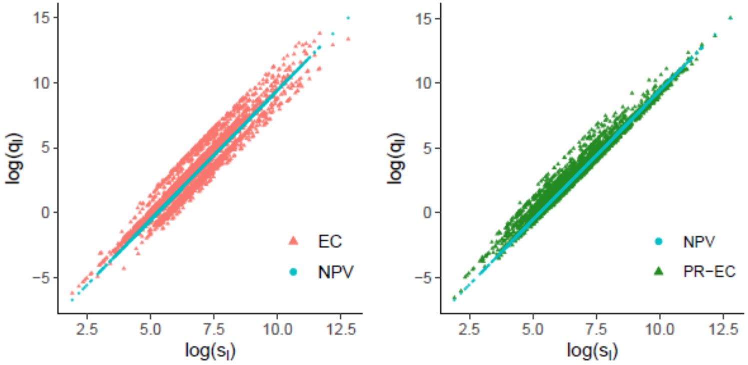
Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Predicted Allocations

- We can compute the predicted allocation for
  - CRRA utility ($\rho = 0.5$)
  - uniform shocks
  - total budget of \$10 million
- Three systems: EC, NPV, and PR-EC

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Predicted Allocations



Notes: Year 2016. Strömberg-like measure of contestability.

Figure 1: County allocations as a function of their electoral sensitivity

- Relationship is log-linear in s_l (drives most of variations in allocations)

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Predicted Allocations

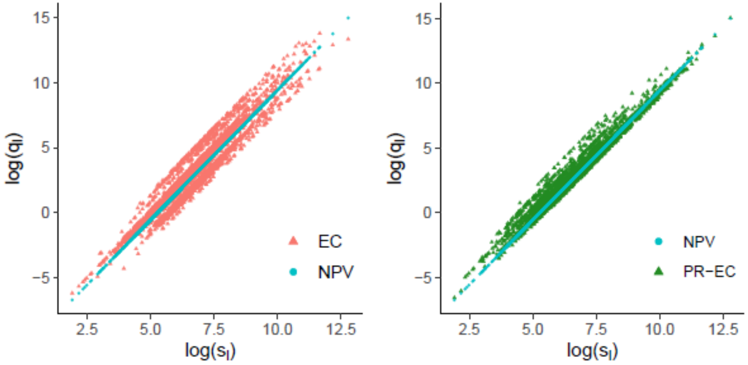


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Figure 1: COUNTY ALLOCATIONS AS A FUNCTION OF THEIR ELECTORAL SENSITIVITY

- Variations not only due to differences in n_l, also t_l and ϕ_l

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Predicted Allocations

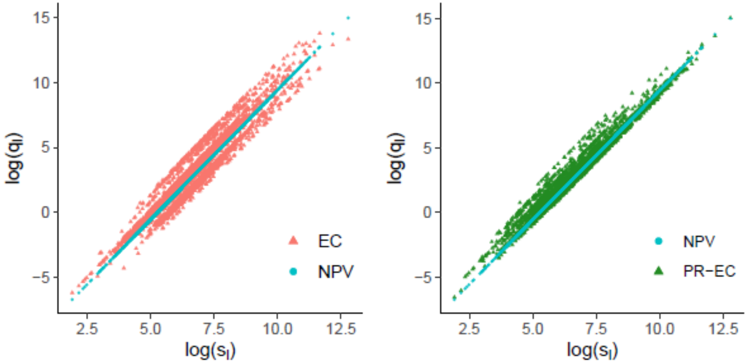

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Figure 1: COUNTY ALLOCATIONS AS A FUNCTION OF THEIR ELECTORAL SENSITIVITY

- EC and PR-EC: counties with same s_l typically be treated differently

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Winners and Losers of the Reform

- A reform of the EC towards NPV would generate winners and losers

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- Counties in a given state win more (or lose less) when the state has
  - a high aggregate sensitivity s_d
  - a small number of electoral votes ω_d
  - a low contestability γ_d or γ_d^{Str}

Reforms: the U.S. Presidential Electoral System

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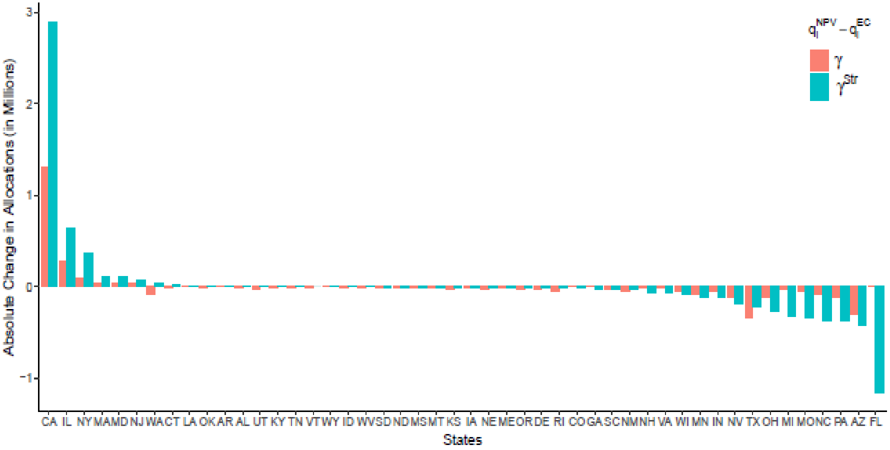
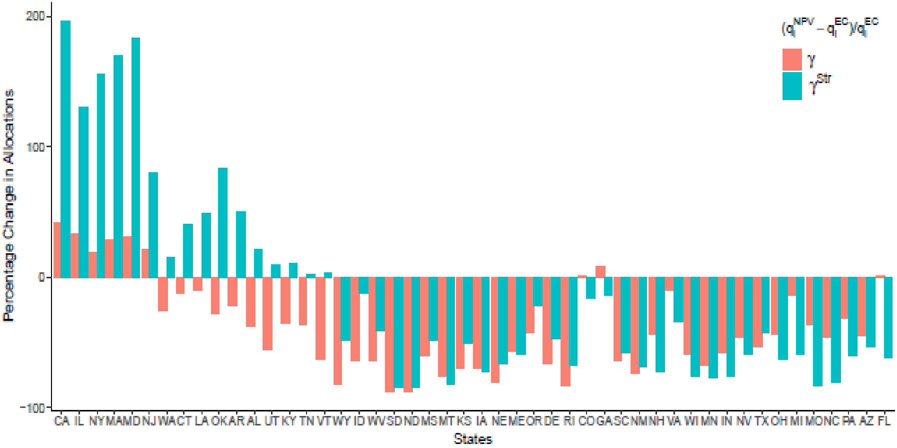


(a) Absolute gain.

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Winners and Losers of the Reform



(b) Percentage gain.

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Winners and Losers of the Reform

Several interesting patterns emerge

1. A majority of states lose from the reform in favor of a few

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2. Common wisdom: winners and losers depends on γ and ω
   - many of biggest losers (FL, PA, AZ, NC, MI) battleground states
   - many of biggest winners have low ω and γ (CA, IL, NY, MA)
   - importance of contestability is magnified under γ^{Str}
     - FL: magnitude of loss is fundamentally different under γ and γ^{Str}
     - some states (AR, LA, OK, KY, AL, TN, CT, UT, WA) win only for γ^{Str}

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Reforms: the U.S. Presidential Electoral System

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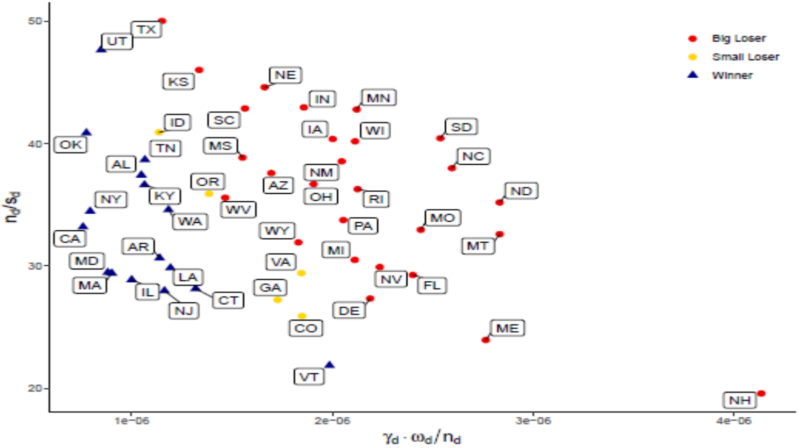
3. Overlooks the role of the aggregate sensitivity of the state
    - new figure to highlight the importance of that component
    - IL vs. TX: similar contestability and malapportionment
    - yet, IL among biggest winners, TX among biggest losers
        - ★ TX has relatively low s_d, due to low t_d and ϕ_d

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Winners and Losers of the Reform



Notes: Big Loser / Small Loser / Winner if percentage gain $\in (-\infty, -0.5]$ / $(-0.5, 0]$ / $[0, \infty)$. Average for 2008-2016. Strömberg-like contestability.

Figure 3: Decomposition of State's Characteristics

Reforms: the U.S. Presidential Electoral System

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Reforms: the U.S. Presidential Electoral System

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2. Common wisdom: winners and losers depends on γ and ω
3. Overlooks the role of the aggregate sensitivity of the state
4. Winners and losers in absolute value vs. percentage terms
    - largest winners in absolute value, also among those in percentage terms
    - largest losers in percentage also small states (MT, ND, RI, SD)
        - ★ over-represented in the EC

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5. Similar results for reform to PR-EC

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2. Common wisdom: winners and losers depends on γ and ω
3. Overlooks the role of the aggregate sensitivity of the state
4. Winners and losers in absolute value vs. percentage terms
5. Similar results for reform to PR-EC
   - but, states with low turnout gain more (or lose less) than with NPV
   - e.g., CA and TX lower than average t_d, FL higher

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Inequality

- Comparison electoral systems based on inequality in allocation

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Inequality

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- Two measures:
  - Gini of inequality across individuals: includes all inequalities
  - Atkinson measure: socially inefficient inequality

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Inequality

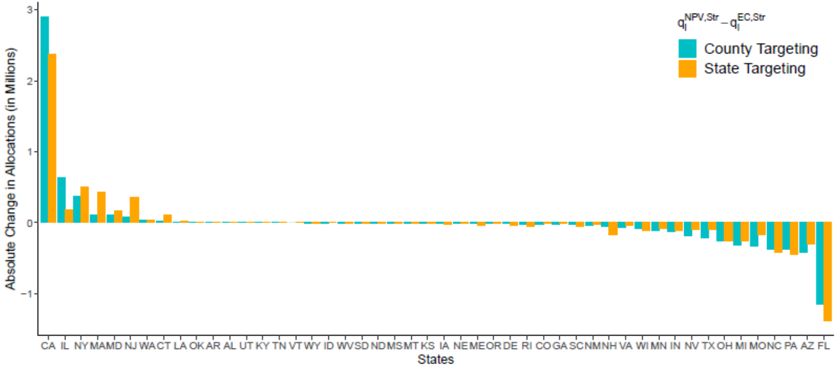
- Comparison electoral systems based on inequality in allocation
- Two measures:
  - Gini of inequality across individuals: includes all inequalities
  - Atkinson measure: socially inefficient inequality
- Results:

	$EC\ (\gamma^{Str})$	$EC\ (\gamma)$	NPV	PR-EC
Gini	0.842	0.875	0.909	0.912
Atkinson	0.316	0.089	0.072	0.071

Table 5: INEQUALITY MEASURES 2016

  - Gini: both reforms slightly **increase** inequality for 2008-2016
  - Atkinson: both reforms slightly **decrease** inequality for 2008-2016

What if no county targeting?



Notes: Average for 2008-2016.

Figure 6: Winners and losers of a reform for County and State Targeting

What if no county targeting?

- IL and CA gain less, while NJ and MA gain more
- AZ and TX lose less, while FL and NH lose more
- Key factor: within-state heterogeneity
  - IL and CA composed of counties with considerably different s_l
  - highly sensitive counties gain more under county-level targeting, especially when other counties in the state are low sensitivity

Extensions

- Beyond Geographically Targeted Interventions
- Endogenous Choices: Targeted vs. Universal Spending

Conclusions

- **Effects of electoral systems on inequality in govt interventions**
  - focus on **PR vs. MAJ**
- **Main novelty:** sub-district targeting and heterogeneity
- **Main result:** *relative electoral sensitivity effect* only in MAJ
  - can reverse common wisdom that inequalities higher in MAJ
- **Implications** for reforms of U.S. Electoral College
  - not only contestability and apportionment of the states
  - also, aggregate sensitivity of the states
  - relevance confirmed by numerical simulations

BONUSES

This Paper

- **New light on mixed empirical evidence** about MAJ vs. PR
  - types of spending: targeted vs. universal
    (Persson&Tabellini 99, 03; Milesi-Ferretti et al. 02; Aidt et al. 06; Blume et al 09; Funk&Gathmann 13)
    - ★ arbitrary (and inconsistent) choices of what is targeted vs. universal
    - ★ unlikely to fit all countries
  - trade policy: free trade vs. trade barriers
    (Mansfield&Busch 95; Rogowski&Kayser 02; Chang et al. 08, 10; Evans 09; Hatfield&Hauk 14; Betz 17)
    - ★ results vary with type of barriers

Different approaches of PR vs. MAJ

1. *Votes* $\longrightarrow$ *Seats*
    - objectives of parties: same under both systems
    - systems affect how to achieve this objective
    - abstracts from pivotability of a district in national assembly
2. *Seats* $\longrightarrow$ *Influence*
    - MAJ: parties maximize proba. of obtaining majority of seats/votes
    - PR: parties maximize number of votes
    - premium for majority in national assembly larger under MAJ

The Politics

A Model of Electoral Competition

- **Probabilistic voting model**

  (Enelow&Hinich 82, Lindbeck&Weibull 87; Dixit&Londregan 95; Persson&Tabellini 01, Stromberg 04,08)

- Turnout varies across localities: t_l

- When voting, individual i in locality l casts ballot for A iff:

$$\Delta u_l(\mathbf{q}) \geq \nu_{i,l} + \delta_d$$

  - $\nu_{i,l}$: individual's ideology, cdf $\Phi_l(\cdot)$
  - δ_d : district-level popularity shock, cdf $\Gamma_d(\cdot)$
  - we can relax full support assumption $\rightarrow$ uniform distributions
    - ★ $\nu_{i,l} \sim U[\frac{-1}{2\phi_l}, \frac{1}{2\phi_l}]$ (ϕ_l = **swingness**)
    - ★ $\delta_d \sim U[\beta_d - \frac{1}{2\gamma_d}, \beta_d + \frac{1}{2\gamma_d}]$ (γ_d = **contestability**, β_d: bias in favor of B)

- Easy to compute vote share of party A
  - in locality l:

$$\Phi_l\left(\Delta u_l(\mathbf{q}) - \delta_{d(l)}\right)$$

    where $d(l)$ is the district to which l belongs

  - in district d:

$$\pi_d(\mathbf{q};\delta_d) = \sum_{l\in d} \frac{t_l n_l}{T_d} \Phi_l(\Delta u_l(\mathbf{q}) - \delta_d)$$

    where $T_d = \sum_{k\in d} t_k n_k$

Equilibrium under PR

- **Under PR**: parties maximize the country-wide expected vote share subject to the aggregate budget constraint

$$\max_{\mathbf{q}^A \mid \sum_l n_l^\alpha q_l = y} \pi_{PR}(\mathbf{q}) = E_\delta \left(\sum_l t_l n_l \Phi_l \left(\Delta u_l(\mathbf{q}) - \delta_{d(l)} \right) \right)$$

- If equilibrium exists, $\mathbf{q}^A = \mathbf{q}^B$, and implicitly defined by:

$$\frac{\partial u_l \left(\mathbf{q}^A \right)}{\partial q_l^A} s_l = n_l^\alpha \lambda^{PR} \ \forall l$$

  - $s_l = \bar{\phi}_l t_l n_l$ is the **electoral sensitivity** of locality l
    - $\bar{\phi}_l = \int_{\delta_d} \phi_l \left(-\delta_d \right) d\Gamma_d \left(\delta_d \right) \rightarrow$ expected density of swing voters in l
  - λ^{PR} is the Lagrange multiplier of the budget constraint under PR

Equilibrium under MAJ

- **Under MAJ**: parties maximize the number of districts won
  - winning a district requires $\pi_d(\cdot) \geq 1/2$
- Remember:

$$\pi_d(\mathbf{q}; \delta_d) = \sum_{l \in d} \frac{t_l n_l}{T_d} \Phi_l(\Delta u_l(\mathbf{q}) - \delta_d)$$

- Thus, A wins district d when δ_d is sufficiently small, and loses when sufficiently large
  - $D_d(\mathbf{q})$ is the unique cutoff value of δ_d that separates district loss from win for a given allocation $(\pi_d(\mathbf{q}; D_d(\mathbf{q})) = 1/2)$
- The probability that A wins district d is:

$$p_d(\mathbf{q}) = \Pr\left(\pi_d(\mathbf{q}; \delta_d) \geq \frac{1}{2}\right) = \Gamma_d(D_d(\mathbf{q}))$$

Equilibrium under MAJ

- Party A's objective function is:

$$\max_{\mathbf{q}^A | \sum_l n_l^\alpha q_l = y} \pi_{MAJ}(\mathbf{q}) = \sum_d \Gamma_d (D_d(\mathbf{q}))$$

- If equilibrium exists, $\mathbf{q}^A = \mathbf{q}^B$, and implicitly defined by:

$$\hat{\gamma}_{d(l)} \frac{\hat{s}_l}{\hat{s}_{d(l)}} u_l' \left(\mathbf{q}^A\right) = n_l^\alpha \lambda^{MAJ} \ \forall l$$

  - $\hat{\gamma}_d$ is the **contestability** of district d
    - ★ intuitively: proba that parties end up close to a tie in d
  - $\hat{\phi}_l = \phi_l \left(-\hat{\delta}_d\right)$ is the **swingness** of locality l
    - ★ $\hat{\delta}_d$ is the value of $D_d(\mathbf{q})$ when $\mathbf{q}^A = \mathbf{q}^B$
  - $\hat{s}_l = t_l n_l \hat{\phi}_l$ is the electoral sensitivity of locality l
  - $\hat{s}_d = \sum_{j \in d} t_j n_j \hat{\phi}_j$ is the aggregate sensitivity in district d

Equilibrium under MAJ

Effect of Population Size

Proposition

In MAJ, $q_l > q_{l'}$ if and only if $\hat{\gamma}_{d(l)} \frac{\hat{s}_l n_l^{-\alpha}}{\hat{s}_{d(l)}} > \hat{\gamma}_{d(l')} \frac{\hat{s}_{l'} n_{l'}^{-\alpha}}{\hat{s}_{d(l')}}$.

- Other result: effect of population size
  - share of budget may be decreasing in n_l
  - requires $\hat{s}_l / \hat{s}_{d(l)} > 1 - \alpha$
    (satisfied for pure transfers, not for pure public goods)
- Intuition: for a given relative sensitivity, it is cheaper to buy votes in a less populated locality

Comparing the Systems

Importance of Sub-District Targeting

- **What if we allow for targeting at sub-state level?**
- Substantial effect on equilibrium allocation
- Affects the comparison in terms of inequalities
- Affects the total allocation to a district
  - winner and loser of reform
  - magnitude of gain or loss

Comparing the Systems
Importance of Sub-District Targeting

Substantial effect on equilibrium allocation

- Numerical example: same as before (with $\gamma_A / \gamma_B = 6$)
  - new columns with targeting at district level

District	Locality	s_l	n_l	q_l^{PR}	q_l^{MAJ}	q_l^{PR-d}	q_l^{MAJ-d}
A	1	1	17%	2.9%	19.4%	7.8%	48.6%
A	2	2	33%	11.8%	77.7%	7.8%	48.6%
B	3	2	33%	11.8%	1.2%	42.2%	1.4%
B	4	5	17%	73.5%	2.5%	42.2%	1.4%
Atkinson index:				**0.42**	**0.38**	**0.22**	**0.40**

- Substantial change in the allocation

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Data

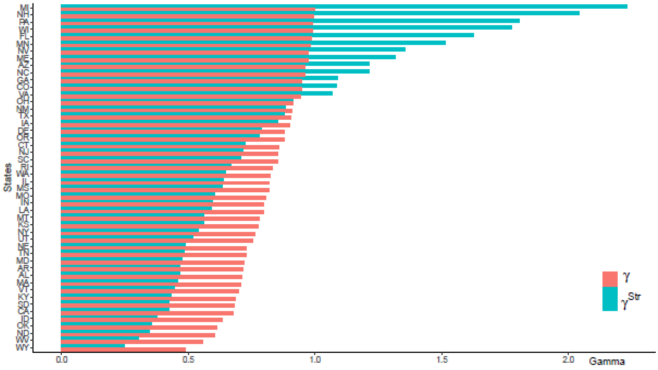


Figure 4: COMPARING THE TWO MEASURES OF CONTESTABILITY FOR 2016

- Stromberg-like measure produces a more skewed distribution

Reforms: the U.S. Presidential Electoral System

Numerical Simulations: Predicted Allocations

- To quantify the effects of the different variables on the predicted county allocation q_l^{EC}, we regress it on s_l, s_d, γ_d, and ω_d
- We find that s_l explains 85%-93% of the total variance of the predicted q_l^{EC} (for γ_d^{Str} or γ_d)
- Residual variance decomposition:
  - for γ_d^{Str}: s_d explains 50%, ω_d 34%, γ_d 16%
  - for γ_d: s_d explains 23%, ω_d 17%, γ_d^{Str} 60%

Extensions

Max proba of winning

- **Our results are robust to alternative objective function**
- Modified version of our model
  - BOTH under PR and MAJ: given shocks $\tilde{\gamma}_d$ and biases β_d, calculate proba that d is pivotal for majority in national assembly
  - **Messy!** Need Lyapunov's CLT to calculate approximate probability that A wins,

$$P(\mathbf{q}) = 1 - \Phi\left[S(\mathbf{q})\right],$$

with $S(\mathbf{q}) = \frac{\frac{D}{2} - \mu(\mathbf{q})}{\sigma_E(\mathbf{q})}$ and $\sigma_E^2(\mathbf{q}) := \sum_d p_d(\mathbf{q})\left[1 - p_d(\mathbf{q})\right]$

  - $S(\mathbf{q}) > 0$ means that A has less than a 50% proba of winning

Extension: Max proba of winning

- For L and D large enough that we can apply Lyapunovs' CLT:

Proposition

In **PR**, $q_l > q_{l'}$ *iff* $s_l n_l > s_{l'} n_{l'}, \forall l, d$ *(unchanged).*

In **MAJ**, *and* $\forall l, d$, $q_l > q_{l'}$ *iff*

$$\frac{\gamma_{d(l)} s_l n_l}{\sum_{k \in d(l)} s_k n_k} \left[1 + \frac{S(\mathbf{q})}{\sigma_E(\mathbf{q})} \gamma_{d(l)}^2 \beta_d \right] \text{ larger than for } l'$$

(unchanged if $S(\mathbf{q}) = 0$*).*

- Spend more on frontrunner leaning districts than if max expected vote share

Extensions

Targeted vs. Universal Spending

- **Literature: incentives to target stronger under MAJ**
- **Relative sensitivity effect works as opposite force**
- Modified model ($\sim$ P&T)
  - transfers instead of local public goods ($k(q_l) = n_l q_l$)
  - national public good: G
  - payoff of i in locality l: $q_l + u(G)$

Extensions

Targeted vs. Universal Spending

- **Traditional contestability effect:**

  if same sensitivity but $\neq$ contestability then $G^{MAJ} < G^{PR}$

  - PR: transfers useless
  - MAJ: neglect less contestable districts, makes transfers attractive

- **New relative sensitivity effect:**

  if same contestability, $\neq$ sensitivity, one locality per district then $G^{MAJ} > G^{PR}$

  - PR: transfers useful to target locality with higher sensitivity
  - MAJ: transfers useless (same relative sensitivity)

Extensions

Size of government

- Modified model
  - preferences: $w_l(\mathbf{q}, \tau) = v(y(1-\tau)) + u_l(\mathbf{q})$
    - ★ proportional tax on income τ
    - ★ local public goods ($\alpha = 0$)
    - ★ $u' > 0 > u''$ and $v' > 0 > v''$
  - all individuals have the same income (no targeting through taxes)
  - all districts have same contestability

Extensions

Size of government

Proposition

If $u''' \leq 0$:
(i) same sensitivities but different relative sensitivities: $\tau^{PR} > \tau^{MAJ}$;
(ii) same relative sensitivities but different sensitivities: $\tau^{PR} < \tau^{MAJ}$.

- Intuition:
  - spread of sensitivities + diminishing marginal utility q

    $\rightarrow$ smaller effect of marginal \$
  - inequality in q can increase or decrease average marginal utility (u''')
- $u''' \leq 0$ is NOT a necessary condition
  - same result holds for log utility