# Merger Review for Markets with Buyer Power* 

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#### Abstract

We analyze the competitive effects of mergers in markets with buyer power. Using mechanism design arguments, we show that without cost synergies, a merger of suppliers is harmful for a buyer regardless of buyer power, although post merger the buyer demands a lower price from the merged entity than from symmetric premerger suppliers. Buyer power can deter harmful mergers and mitigate but never eliminate harm from mergers that occur. With buyer power, a merger of symmetric suppliers increases rivals' incentives to invest in cost reduction and entrants' incentives to enter, and can increase the merging parties' incentives to invest. Buyer power increases the profitability of perfect collusion relative to a merger and makes such collusion harder to detect. Although cost synergies can eliminate merger harm, they can also render an otherwise profitable merger unprofitable. Extensions allow for asymmetric suppliers, coordinated effects, multi-product firms, and cross-market effects.


Keywords: unilateral effects, coordinated effects, cost efficiencies, merger simulation JEL Classification: D44, D82, L41

[^0]
## 1 Introduction

Buyer power features prominently in the antitrust analysis of mergers. That buyer power will prevent merging suppliers from being able to negotiate higher prices is a frequent merger defense. ${ }^{1}$ As one observer put it, buyer power is sometimes embraced by courts as if it had "talismanic power." ${ }^{2}$ Because powerful buyers can withstand upward price pressure from mergers of suppliers, so this appealing argument goes, they are not harmed by such mergers. ${ }^{3}$

In large part because the most commonly used models in merger review, which are based on Cournot or Bertrand competition, do not accommodate powerful buyers, it is often not clear what exactly is meant by buyer power. However, this does not mean that buyer power lacks empirical plausibility. For example, computer manufacturers such as Dell and HP procure components from upstream suppliers using competitive procurements and face-to-face negotiations. Although these buyers might value having the latest generation of a component, they may also be willing to continue to manufacture using a prior generation in the absence of a sufficiently low price for the new generation. As another case in point, oil companies such as Shell, Exxon-Mobil, and BP procure oilfield services for their wells using competitive procurements and negotiations in which they play oilfield services provides off against one another. Likewise, municipalities procuring road improvements, park landscaping, and other city services sometimes cancel procurements in the face of what they view as insufficiently competitive pricing. ${ }^{4}$

The use of competitive procurement processes by the buyers in these examples makes it natural to model buyer power in the context of a procurement. This is the approach we take in this paper. We view the buyer as designing a procurement mechanism in which suppliers participate. The suppliers' costs are their private information, and so the buyer's mechanism is constrained by incentive compatibility and individual rationality for the suppliers. Similar to Bulow and Klemperer's (1996) approach to modeling environments

[^1]with and without bargaining power, in our model a buyer with no buyer power must rely on the outcome of an efficient auction among potential suppliers, with no ability to further negotiate. In contrast, a powerful buyer can demand discriminatory discounts from suppliers and negotiate with the auction winner, thereby implementing the buyer surplus maximizing mechanism, subject to incentive compatibility and individual rationality.

Our approach captures, in a general way, procurement markets, where purchasing can involve combinations of requests for proposals, auctions, and negotiations. It combines this generality with a disciplined, principled analysis of bargaining and bargaining power, without being subject to the pitfall of complete information models that bargaining is always efficient. ${ }^{5}$ Our procurement-based approach allows us to build on insights from the theory of optimal auctions and to exploit results from mechanism design. The framework accounts for market power because each supplier is treated as having a monopoly over her private information. Yet, suppliers are always "oligopolistic" in the sense that when there are more of them, the monopoly power of each individual supplier decreases.

We model a merger without cost synergies as allowing the merged entity to produce at a cost equal to the minimum of the costs of the merging suppliers. Using mechanism design arguments, we show that without cost synergies, a merger of suppliers is harmful for a buyer regardless of buyer power, despite the fact that the buyer demands a lower price from the merged entity than from symmetric pre-merger suppliers. Although buyer power can deter harmful mergers, it can thus never eliminate harm if mergers occur. Absent buyer power, mergers without synergies are neutral for rivals and potential entrants and profitable for the merging suppliers. With buyer power, a merger of symmetric suppliers is beneficial for rivals and increases the expected profits of entrants. Modelling investments as inducing a shift in a supplier's cost distribution according to stochastic dominance and assuming symmetric suppliers, we show that with buyer power, mergers increase the incentives to invest for rivals. Without buyer power, mergers increase the merging suppliers' incentives to invest, but with buyer power, mergers may or may not increase merging suppliers' incentives to invest. We show by example that all suppliers' incentives to invest can increase in the wake of a merger.

A natural view is that mergers and perfect collusion are equivalent. Although this way of thinking is correct without buyer power, it is misleading in its presence. First,

[^2]mergers are public events, while collusion happens under the surface. Powerful buyers can thus be expected to take defensive actions against the increased market power of a merged entity, but not necessarily against a cartel. Therefore, buyer power makes collusion more profitable relative to mergers, assuming the collusion is not detected (or suspected). Moreover, because buyer power induces more aggressive price demands by the buyer, losing bids above these price demands are more likely with buyer power even with competitive bidding, making collusive bidding involving deliberately losing bids harder to detect. Consequently, buyer power makes collusion not only more profitable, but also more difficult to detect.

This contrasts with Carlton and Israel (2011), who argue that powerful buyers may actually be harmed more by a merger than those without buyer power. They base this on the possibility that, in the absence of powerful buyers, suppliers could collude to set monopoly prices, and so a merger would have no effect, but that with powerful buyers, pre-merger pricing would be below monopoly levels and increase as a result of a merger. They conclude that "there is no theoretical necessity that the presence of powerful buyers must always lessen the price effects from a merger." (Carlton and Israel, 2011, p. 132) As we show, the presence of powerful buyers is more likely to invite collusion than the absence of buyer power, and in our procurement setting, it is a theoretical necessity that powerful buyers are less affected by a merger than those without buyer power. Thus, our results are consistent with empirical evidence that collusion in, for example, products such as disk drives and LCD panels, negatively affected seemingly powerful buyers, like Dell, HP, and Microsoft.

While the results described above may strike one as surprising and counterintuitive at first, there is a clear and simple intuition for them. Without buyer power, the allocation is efficient before and after the merger, which explains the neutrality result for rivals. Because a merger eliminates a competing bid for the merged entity, it follows that the merger is profitable for the merging suppliers and harmful for the buyer, whose expected payments increase. In our model of mergers without cost synergies, the merged entity's cost distribution is dominated in terms of the reverse hazard rate by the pre-merger distribution. This means that, like the seller in Myerson's optimal auction, a powerful buyer will discriminate against the merged entity by setting a more aggressive reserve price for the merged entity and by procuring from the merged entity less often than he procured from the two suppliers before the merger. ${ }^{6}$ But this discrimination implies that with buyer power, the merger benefits rivals and potential entrants. Although the powerful buyer can mitigate some of the harm of a merger, a simple revealed preference

[^3]argument implies that he cannot eliminate the harm.
The results summarized above imply that, without cost synergies, mergers are always detrimental to buyers irrespective of their power. This motivates us to extend the analysis to account for such cost efficiencies. We model cost efficiencies as a commonly known percentage decrease in the cost of the merged entity relative to the minimum cost of the two merging suppliers before the merger. The assumption that this percentage is known is based on cost efficiencies being part of a merger review and therefore information that both the buyer and the merging suppliers have. We show that, as expected, cost efficiencies make mergers unambiguously less harmful for the buyer and, if the efficiencies are large enough, can cause the buyer to welcome the merger. However, arguments based on cost synergies do not constitute a slam-dunk defense: The cost synergies required to make a merger acceptable to the buyer may make it unprofitable to the merging suppliers. Indeed, whether the buyer is powerful or not, post-merger profit of the merged entity goes to zero as synergies approach 100 percent. Although cost synergies reduce costs, they also squeeze informational rents and thereby profits. Eventually, the latter effect dominates.

We also analyze the effect of a merger on incentives to merge, enter, and innovate. Related to incentives to merge, we show that the merging suppliers gain from a merger in the absence of buyer power, and may or may not with buyer power. A merger does not provide incentives for entry in the absence of buyer power, but does so with buyer power. Related to innovation, we show that the effect on incentives for cost reducing investment depend on buyer power and differ for the merging and nonmerging suppliers. We show that (i) without buyer power, a merger does not affect incentives for investment by nonmerging suppliers and increases incentives for investment by merging suppliers; and (ii) with buyer power, a merger increases incentives for investment by nonmerging suppliers can increase or decrease incentives for investment by merging suppliers depending on the details of how investment affects the merging suppliers' cost distributions.

Our approach provides a merger simulation tool that captures a procurement-based market. Importantly, it does not require an assumption that all bidders are symmetric, and it is based on an optimal mechanism, which involves optimal price discrimination. It also permits us to develop a measure of coordinated effects by assuming that imperfectly colluding suppliers use a rotation scheme, which can readily be quantified using the merger simulation apparatus.

There is a related literature on merger analysis based on auction models, including Waehrer (1999), Waehrer and Perry (2003), Miller (2014), and Froeb, Mares, and Tschantz (2017). Waehrer (1999) examines mergers in both asymmetric first-price and second-price auction markets. ${ }^{7}$ Waehrer and Perry (2003) focus on open auctions and allow the optimal

[^4]reserve to adjust post merger. Our approach differs in considering optimal procurements with asymmetric bidders and allowing varying buyer power. ${ }^{8}$ Miller (2014) considers a procurement setting in which buyers purchase from suppliers of differentiated products using a variant of a second-price auction, developing a stochastic model that can be calibrated to estimate merger effects. In concurrent work, Froeb, Mares, and Tschantz (2017) consider effects of a merger between bidders in an optimal (ascending) auction when bidders draw their values from a family of power-related distributions, where each bidder's value can be viewed as the maximum of some number of draws from a common distribution. They show that a merger reduces the auctioneer's expected revenue and that, under certain conditions, a merger to monopoly is not profitable for the merging bidders.

Our results can be related to other work on buyer power and its role in merger analysis, including theoretical work such as Dobson and Waterson (1997) and Horn and Wolinsky (1988) and empirical work like Crawford and Yurukoglu (2012), Gowrisankaran, Nevo, and Town (2015), Collard-Wexler, Gowrisankaran, and Lee (2016). ${ }^{9}$ On coordinated effects, see Gayle et al. (2011) for a theoretical approach based on an explicit collusion benchmark and Miller and Weinberg (2016) for an empirical analysis of coordinated effects following the Miller-Coors merger. ${ }^{10}$ In addition, our framework allows one to analyze multi-product firms that produce complementary products. ${ }^{11}$

Other authors have considered the effect of a merger on incentives for investment. Motta and Tarantino (2017) show that in Bertrand oligopoly with differentiated products, absent efficiency gains, a merger lowers total output and as a result lowers total investment in cost reduction. They also consider investment in quality enhancement, but find ambiguous results for that case. In our model, results for quality enhancing investment are also ambiguous. López and Vives (2016) focus on the effect of a symmetric
lated equilibrium bidding strategies. Thomas (1999) examines mergers in asymmetric first-price auctions by deriving equilibrium bidding strategies for the binomial cost distribution.
${ }^{8}$ We model the merged firm as having a cost that is distributed as the minimum of the two cost draws from the merging firms. This is the approach also taken by Salant, Switzer, and Reynolds (1983), Perry and Porter (1985), Waehrer (1999), Dalkir, Logan, and Masson (2000), and Froeb, Tschantz, and Crooke (1999). This type of merger is equivalent to efficient (observable) collusion as discussed by Mailath and Zemsky (1991) and McAfee and McMillan (1992).
${ }^{9}$ See also, Sheffman and Spiller (1992), Chipty and Snyder (1999), Inderst and Wey (2007), Inderst and Shaffer (2007), Dana (2012), and Caprice and Rey (2015).
${ }^{10}$ If colluding firms can make side payments and can allocate which firm is to win efficiently, then collusion and mergers have the same effect in auction markets if and only if both the collusion and the merger is observed by the buyer. However, clandestine collusion has a different effect from merger because of the difference in reaction by the buyer. See DeBrock and Smith (1983), Graham and Marshall (1987), von Ungern-Sternberg (1983), Mailath and Zemsky (1991), McAfee and McMillan (1992), McAfee (1994), and Kumar et al. (2015).
${ }^{11}$ O'Brien and Shaffer (2005) also consider mergers with multi-product suppliers.
increase in cross-ownership in a symmetric Cournot model with R\&D spillovers. ${ }^{12}$ They show that an increase in cross-ownership increases cost-reducing investment for high levels of $R \& D$ spillovers but decreases investment for low levels of $R \& D$ spillovers. ${ }^{13}$ Our model has no R\&D spillovers (investment by one supplier has no effect on other suppliers' costs), but we focus on the change in cross-ownership resulting from a merger, which necessarily involves asymmetries. Although we do not pursue it here, our approach can also be extended to account for cross-ownership among suppliers along the lines of Lu (2012).

The paper is structured as follows. Section 2 defines the setup. Section 3 analyzes a merger in our setup. Section 4 considers extensions. Section 5 concludes.

## 2 Setup

In the baseline setup, we consider one product and one buyer. ${ }^{14}$ In the pre-merger market, there are $n \geq 2$ suppliers, indexed $1, \ldots, n$. Each supplier $i \in\{1, \ldots, n\}$ draws a cost $c_{i}$ independently from continuously differentiable distribution $G$ with support $[\underline{c}, \bar{c}]$ and density $g$ that is positive on the interior of the support. Each supplier is privately informed about her type, and so the suppliers' types are unknown to the buyer. The buyer has value $v>\underline{c}$ for one unit of the product. All of this is common knowledge

We let suppliers 1 and 2 be the merging suppliers. Like Farrell and Shapiro (1990), we model a merger as allowing the merging suppliers to rationalize production by producing using the lower of their two costs. (We discuss the possibility of further cost efficiencies from the merger in Section 3.3.) Thus, given pre-merger costs $\mathbf{c}=\left(c_{1}, \ldots, c_{n}\right)$, in the corresponding post-merger market, the nonmerging suppliers have the same cost as before the merger, and the merged entity has cost $c=\min \left\{c_{1}, c_{2}\right\}$. We denote the distribution for the minimum of the pre-merger costs of suppliers 1 and 2 by $\hat{G}(c) \equiv 1-(1-G(c))^{2}$, with density $\hat{g}$. Thus, although suppliers are symmetric in the pre-merger market, in the post-merger market they are not because the merged entity draws its cost from a different distribution. ${ }^{15}$ (We relax the assumption of pre-merger symmetry in Section 4.1.)

[^5]Buyers, suppliers, and the designer are risk neutral. A buyer's payoff is zero if he does not trade and is equal to his value minus the price he pays if he does trade. Similarly, a supplier's payoff is zero if she does not trade and is equal to the payment she receives minus her cost if she does trade.

The setup's merits include the assumption of private information, which neither presumes nor precludes efficiency and results in an efficiency-profit tradeoff, and of independently distributed private types, which means that the optimal Bayesian mechanism provides a practical benchmark, such as the procurement-plus-bargaining procedure described next. ${ }^{16}$

The buyer uses a procurement procedure to select a supplier and determine a purchase price. We adapt to our procurement setup and extend to asymmetric suppliers the approach used by Bulow and Klemperer (1996), who model a seller without bargaining power as using a second-price auction and a seller with bargaining power as using an ascending auction followed by a take-it-or-leave-it offer by the seller to the auction winner, which is an optimal sales mechanism in their setup. In our procurement context, the case of no bargaining power corresponds to a buyer using a descending clock auction with a reserve (or starting point) equal to the minimum of the buyer's value and the upper bound of support of the cost distribution. The case of bargaining power corresponds to a buyer using an optimal procurement mechanism, which can be implemented as a discriminatory descending clock auction followed by a take-it-or-leave-it offer by the buyer to the auction winner.

The case without bargaining power and the case with bargaining power can be nested within a single procurement process parameterized by $\beta \in\{0,1\}$, where $\beta=0$ corresponds to the case without bargaining (or buyer) power and $\beta=1$ corresponds to the case with bargaining (or buyer) power. ${ }^{17}$ We denote a nonmerged supplier's virtual cost function

[^6]by
$$
\Gamma(c) \equiv c+\frac{G(c)}{g(c)}
$$
and the merged entity's virtual cost function by
\[

$$
\begin{equation*}
\hat{\Gamma}(c) \equiv c+\frac{\hat{G}(c)}{\hat{g}(c)}=c+\frac{G(c)}{g(c)} \frac{2-G(c)}{2(1-G(c))} \tag{1}
\end{equation*}
$$

\]

where the equality uses the definition of $\hat{G}(c) .{ }^{18}$ We impose the standard regularity assumption that $\Gamma$ and $\hat{\Gamma}$ are increasing. A sufficient condition for this is that $G / g$ is nondecreasing. For expositional compactness, it is also useful to define, for $\beta \in\{0,1\}$, the weighted virtual cost functions by

$$
\begin{equation*}
\Gamma_{\beta}(c) \equiv(1-\beta) c+\beta \Gamma(c) \quad \text { and } \quad \hat{\Gamma}_{\beta}(c) \equiv(1-\beta) c+\beta \hat{\Gamma}(c) . \tag{2}
\end{equation*}
$$

Observe that $\Gamma_{\beta}(c)=c+\beta G(c) / g(c)$ and $\hat{\Gamma}_{\beta}(c)=c+\beta \hat{G}(c) / \hat{g}(c)$. By construction, when $\beta=0$, the weighted virtual cost corresponds to the true cost, and when $\beta=1$, to the usual notion of a supplier's virtual cost. Because we allow the possibility that the densities are zero at $\underline{c}$ (and also possibly $\bar{c}$ ), define $\Gamma_{\beta}(\underline{c})=\lim _{c \rightarrow \underline{c}} \Gamma_{\beta}(c)=\underline{c}$, and analogously for $\hat{\Gamma}_{\beta}$. If $\Gamma_{\beta}(\bar{c})$ is finite, then for $x>\Gamma_{\beta}(\bar{c})$, define $\Gamma_{\beta}^{-1}(x) \equiv \bar{c}$, and analogously for $\hat{\Gamma}_{\beta}$.

We assume that, given $\beta \in\{0,1\}$, a buyer uses the following procurement-plusbargaining procedure: First, the buyer conducts a (possibly discriminatory) descending clock auction. The clock price starts at price $\min \{v, \bar{c}\}$. As the clock price decreases, participating suppliers can choose to exit. When a supplier exits, she becomes inactive and remains so. The auction is discriminatory in that activity by supplier $i$ at a clock price of $p$ obligates supplier $i$ to supply the product at the clock price $p$ less a supplier-specific discount of $p-\Gamma_{\beta}^{-1}(p)$ if $i$ is an independent supplier and $p-\hat{\Gamma}_{\beta}^{-1}(p)$ if $i$ is the merged entity, should the buyer choose to trade with that supplier. Observe that for $\beta=0$, the weighted virtual cost functions are the identity functions and, therefore, the discounts are 0 . The clock stops when only one active supplier remains, with ties broken by randomization. Then, denoting the final clock price by $\hat{p}$, the buyer makes a take-it-or-leave-it offer of $\min \left\{\Gamma_{\beta}^{-1}(v), \Gamma_{\beta}^{-1}(\hat{p})\right\}$ to a winning nonmerged supplier or $\min \left\{\hat{\Gamma}_{\beta}^{-1}(v), \hat{\Gamma}_{\beta}^{-1}(\hat{p})\right\}$ to a winning merged entity. A supplier is obligated to accept an offer of $\Gamma_{\beta}^{-1}(\hat{p})$ (or $\hat{\Gamma}_{\beta}^{-1}(\hat{p})$ for the merged entity), but can accept or reject a lower offer. If the buyer's offer is accepted,

[^7]trade occurs at the accepted price. Otherwise there is no trade.
When $\beta=0$, the procurement-plus-bargaining procedure treats all bidders symmetrically and does not require discounts off the clock price (put differently, the discounts are 0 , as noted above). In addition, the take-it-or-leave-it offer is equal to the minimum of $v$, $\bar{c}$, and the final clock price. Thus, when $\beta=0$, the procedure reduces to descending clock auction with reserve equal to $\min \{v, \bar{c}\}$.

However, when $\beta=1$, the merged entity is subject to a larger discount off the clock price relative to the nonmerged suppliers. This is so because $\hat{G}$ is dominated by $G$ in terms of the reverse hazard rate. ${ }^{19}$ To see this, observe that for all $c \in[\underline{c}, \bar{c}]$,

$$
\begin{equation*}
\frac{\hat{g}(c)}{\hat{G}(c)}=\frac{g(c)}{G(c)} \frac{2(1-G(c))}{2-G(c)} \leq \frac{g(c)}{G(c)}, \tag{3}
\end{equation*}
$$

with a strict inequality for $c$ in the interior of the support. Consequently, for all $c \in[\underline{c}, \bar{c}]$, we have $\hat{\Gamma}(c) \geq \Gamma(c)$ and hence, for all $p, \hat{\Gamma}^{-1}(p) \leq \Gamma^{-1}(p)$, with strict inequalities for $c \in(\underline{c}, \bar{c})$ and $p \in(\underline{c}, \hat{\Gamma}(\bar{c}))$. Thus, with buyer power, the buyer behaves more aggressively towards the merged entity than towards nonmerging suppliers, demanding greater discounts from the merged entity in the auction and making a lower take-it-or-leave-it offer to the merged entity when it wins the auction. Dominance in terms of the reverse hazard rate also implies first-order stochastic dominance, that is, (3) implies $G(c) \leq \hat{G}(c)$.

In the essentially unique equilibrium in non-weakly dominated strategies of the procure-ment-plus-bargaining procedure, each supplier remains active until the clock price reaches her weighted virtual cost and then exits, and if she wins the auction, she accepts the buyer's take-it-or-leave-it offer if and only if it is greater than or equal to her cost.

As shown in the proof of following lemma, the procurement-plus-bargaining procedure maximizes social surplus when $\beta=0$, and expected buyer surplus when $\beta=1 .{ }^{20}$

Lemma 1 The equilibrium outcome of the procurement-plus-bargaining procedure corresponds to the allocation and payments in the dominant strategy implementation of the

[^8]optimal mechanism for a designer whose objective is to maximize the expected value of
\[

$$
\begin{equation*}
\beta \text { (buyer surplus) }+(1-\beta) \text { (social surplus), } \tag{4}
\end{equation*}
$$

\]

subject to incentive compatibility and individual rationality.

Proof. See the Appendix.

It follows from standard mechanism design results that expected buyer surplus in the equilibrium of the procurement-plus-bargaining procedure can be written in terms of virtual costs. Specifically, expected buyer surplus in the pre-merger market is

$$
\begin{equation*}
E_{\mathbf{c}}\left[\sum_{i=1}^{n} q_{i}^{\beta}(\mathbf{c})\left(v-\Gamma\left(c_{i}\right)\right)\right], \tag{5}
\end{equation*}
$$

where $\mathbf{q}^{\beta}(\mathbf{c})=\left(q_{1}^{\beta}(\mathbf{c}), . ., q_{n}^{\beta}(\mathbf{c})\right)$ with $q_{i}^{\beta}(\mathbf{c}) \in[0,1]$ and $\sum_{i=1}^{n} q_{i}^{\beta}(\mathbf{c}) \leq 1$ is the incentivecompatible pre-merger allocation rule. Having $q_{i}^{\beta}(\mathbf{c})$ equal to one (zero) means that, upon cost realization $\mathbf{c}=\left(c_{1}, \ldots, c_{n}\right)$, supplier $i$ does (does not) produce. If for all $i$ and all $\mathbf{c}$, $q_{i}^{\beta}(\mathbf{c})$ is nonincreasing in $c_{i}$, then there are transfers that make that mechanism incentive compatible. Expected buyer surplus in the post-merger market is

$$
\begin{equation*}
E_{\mathbf{c}}\left[\hat{q}^{\beta}(\mathbf{c})\left(v-\hat{\Gamma}\left(\min \left\{c_{1}, c_{2}\right\}\right)\right)+\sum_{i=3}^{n} \hat{q}_{i}^{\beta}(\mathbf{c})\left(v-\Gamma\left(c_{i}\right)\right)\right], \tag{6}
\end{equation*}
$$

where $\hat{q}^{\beta}(\mathbf{c})$ is the incentive-compatible post-merger allocation rule for the merged entity and $\hat{q}_{i}^{\beta}(\mathbf{c})$ is the incentive-compatible post-merger allocation rule for nonmerging supplier $i \in\{3, \ldots, n\}$. These satisfy $\hat{q}^{\beta}(\mathbf{c}) \in[0,1], \hat{q}_{i}^{\beta}(\mathbf{c}) \in[0,1]$, and $\hat{q}^{\beta}(\mathbf{c})+\sum_{i=3}^{n} \hat{q}_{i}^{\beta}(\mathbf{c}) \leq 1$, where the interpretation is the same as pre-merger. Thus, expected buyer surplus depends on buyer power only through its affect on the allocation rule.

Denote by $\Delta B S^{\beta}$ the change in expected buyer surplus as a result of a merger, which is (6) minus (5), and by $\Delta S S^{\beta}$ the change in expected social surplus as a result of a merger, which is (6) minus (5) but evaluated at actual costs instead of virtual costs, that is:

$$
\Delta S S^{\beta}:=E_{\mathbf{c}}\left[\sum_{i=1}^{n} q_{i}^{\beta}(\mathbf{c})\left(v-c_{i}\right)\right]-E_{\mathbf{c}}\left[\hat{q}^{\beta}(\mathbf{c})\left(v-\min \left\{c_{1}, c_{2}\right\}\right)+\sum_{i=3}^{n} \hat{q}_{i}^{\beta}(\mathbf{c})\left(v-c_{i}\right)\right] .
$$

We denote by $\Delta P^{\beta}$ the change in the buyer's expected payment as result of a merger, which is (6) minus (5) with $v$ set equal to zero, and by $\Delta Q^{\beta}$ the expected change in the
quantity traded as a result of a merger, that is:

$$
\Delta Q^{\beta}:=E_{\mathbf{c}}\left[\hat{q}^{\beta}(\mathbf{c})+\sum_{i=3}^{n} \hat{q}_{i}^{\beta}(\mathbf{c})-\sum_{i=1}^{n} q_{i}^{\beta}(\mathbf{c})\right] .
$$

## 3 Merger analysis

We begin by analyzing the effects of a merger on prices, quantities, and buyer surplus, assuming no cost efficiencies from the merger or other countervailing effects. Then we consider suppliers' incentives to merge, enter, or invest in cost-reducing innovation, and we consider the possibility of merger-related cost efficiencies.

### 3.1 Merger effects

A merger has two key effects on the supply side of the market. First, the number of suppliers is reduced by one. Second, as noted above, the merged entity has a better cost distribution than any of the individual suppliers, that is, $\hat{G}(c) \geq G(c)$ for all $c \in[\underline{c}, \bar{c}]$. Because of the change in distribution, the merger affects the powerful buyer's optimal mechanism, which means the merger affects the procurement-plus-bargaining procedure for a powerful buyer.

We first discuss the case without buyer power. Because a buyer without buyer power uses an efficient mechanism before and after the merger, the allocation is efficient with and without the merger. The merger only eliminates one of the bidders. The allocation not being affected by the merger means $\hat{q}^{0}(\mathbf{c})=q_{1}^{0}(\mathbf{c})+q_{2}^{0}(\mathbf{c})$ and, for $i \in\{3, \ldots, n\}$, $\hat{q}_{i}^{0}(\mathbf{c})=q_{i}^{0}(\mathbf{c})$. If $c_{1}=\min \{\mathbf{c}\}$, then supplier 1 wins in the pre-merger market and receives payment $\min \left\{v, c_{2}, . ., c_{n}\right\}$, and the merged entity wins in the post-merger market, but receives the weakly larger payment $\min \left\{v, c_{3}, \ldots, c_{n}, \bar{c}\right\} .{ }^{21}$ The only effect of a merger is to increase the buyer's payment in cases in which the pre-merger outcome would have been for one of the merging suppliers to win and the other to determine the price paid.

Thus, although the allocation is not affected by the merger when there is no buyer power, the expected payment by the buyer increases, and so the merger decreases expected buyer surplus.

Proposition 1 In the absence of buyer power, a merger results in the same allocation for any realization of costs (implying that $\Delta Q^{0}=0$ and $\Delta S S^{0}=0$ ), a higher expected payment by the buyer $\left(\Delta P^{0}>0\right)$, and lower expected buyer surplus $\left(\Delta B S^{0}<0\right)$.

[^9]Proof. See the Appendix.

Proposition 1 highlights a contrast with the results of Farrell and Shapiro (1990), who show that under Cournot competition, in the absence of cost synergies, a merger causes the quantity to decrease and the price to increase. In our setup, there can be a price effect without a quantity effect.

Next we analyze the case with buyer power. To do so, we must account for the change in the buyer's procurement procedure as a result of the merger. An immediate implication of the reverse hazard rate dominance noted in (3) is that, for all $c_{1}, c_{2} \in[\underline{c}, \bar{c}]$,

$$
\begin{equation*}
\hat{\Gamma}\left(\min \left\{c_{1}, c_{2}\right\}\right) \geq \min \left\{\Gamma\left(c_{1}\right), \Gamma\left(c_{2}\right)\right\} \tag{7}
\end{equation*}
$$

with a strict inequality when $\min \left\{c_{1}, c_{2}\right\} \geq \underline{c}$. This implies that the merger weakly reduces the expected quantity. The merger does not affect the quantity if $v$ is sufficiently large, that is, if $v \geq \Gamma(\bar{c})$ or, in the case of a merger to monopoly, if $v \geq \hat{\Gamma}(\bar{c})$, and otherwise strictly reduces the expected quantity.

Figure 1 illustrates the mechanics at work. Panel (a) shows the effects of a merger with buyer power and $n=2$, and panel (b) shows the effects of a merger with buyer power and $n \geq 3$. (Both panels assume $v<\Gamma(\bar{c})$.) As shown in panel (a), when $n=2$, the effect of a merger is to reduce the set of types for which trade occurs. As shown in panel (b), when $n \geq 3$, the merger not only reduces the set of types for which trade occurs, but also shifts trade away from the merging entity and towards higher cost nonmerging suppliers. In particular, for types in the lighter shaded region along the diagonal, one of the merging suppliers trades before the merger, but a higher cost nonmerging supplier trades after the merger.

With buyer power, a revealed preference argument implies that the buyer's expected surplus is reduced by the merger: The buyer could in the pre-merger market use the "same" allocation rule as in the post-merger market by procuring from the lower-cost supplier of suppliers 1 and 2 whenever $\hat{q}^{1}(\mathbf{c})=1$, in which case the additional competing bid would increase his pre-merger expected buyer surplus. Adopting the pre-merger mechanism that is optimal for the buyer can only further increase pre-merger expected buyer surplus.

We summarize these results as follows:
Proposition 2 With buyer power, a merger results in a weakly lower expected quantity traded $\left(\Delta Q^{1} \leq 0\right)$ (strict if $n=2$ and $v<\hat{\Gamma}(\bar{c})$ or if $n \geq 3$ and $v<\Gamma(\bar{c})$ ), lower expected buyer surplus $\left(\Delta B S^{1}<0\right)$, and weakly lower expected social surplus $\left(\Delta S S^{1} \leq 0\right)$ (strict if $n \geq 3$ or $v<\hat{\Gamma}(\bar{c})$ ).


Figure 1: Quantity effects of a merger. Shaded areas between the dashed and solid lines are affected by the merger, either becoming an area of no trade (darker shading) or an area with trade shifted away from the merged entity and towards the nonmerging suppliers (lighter shading). Assumes buyer power and $v<\Gamma(\bar{c})$.

Proposition 2 implies that social surplus decreases strictly with a merger in the presence of buyer power if the merger is not a merger to monopoly, i.e. $n \geq 3$, or the buyer's take-it-or-leave-it offer to the merged firm binds some of the time, i.e. if $\hat{\Gamma}^{-1}(v)<\bar{c}$. To appreciate the breadth of the conditions under which $\Delta S S^{1}<0$ is the case, observe that $\hat{\Gamma}^{-1}(v)<\bar{c}$ holds under the fairly weak conditions that $v$ is finite and $g$ is such that $g(\bar{c})(1-G(\bar{c}))=0$, which holds whenever $g(\bar{c})$ is finite. ${ }^{22}$

The decrease in quantity in Proposition 2 is particularly notable because merger review tends to focus on the price effects of a merger, with little consideration to effects on quantity. For example, after a merger to monopoly, the buyer makes a lower take-it-or-leave-it offer, provided $v<\hat{\Gamma}(\bar{c}) .{ }^{23}$ Despite this lower offer, which might be perceived as a reduction in the price paid by the buyer, the buyer is strictly worse off as a result of the merger. Thus, in this example, a focus on price effects leads to the wrong conclusion regarding the effects of the merger on the buyer.

In both our setup with buyer power and that of Farrell and Shapiro (1990), a merger results in a shift in quantity away from the merged entity and towards other suppliers (or towards zero if $n=2$ ). In Farrell and Shapiro (1990), the internalization of externalities by the merging suppliers results in a decrease in their equilibrium quantity, and a cor-

[^10]responding increase in the equilibrium quantities of nonmerging suppliers. The resulting decrease in total quantity implies an increase in price. In contrast, in our setup with buyer power, a merger causes the buyer to protect himself from the increased market power of the merged entity by more aggressively discriminating against and more aggressively bargaining with the merged entity. This response by the buyer results in a decrease in quantity for the merged entity, both due to a greater probability of no trade at all, and a shift towards trade with nonmerging suppliers. But, despite the buyer's more aggressive stance towards the merged entity, the loss of competition causes a decrease in the buyer's expected surplus.

Carlton and Israel (2011) argue that because the threat from a buyer's outside optionfor example, due to vertical integration that allows the buyer to produce in-house - is not diminished by a merger of suppliers, a merger of suppliers may have little effect on prices. In contrast, Proposition 2 shows that even when buyer power is derived from an outside option that leaves the buyer with the same willingness to pay (equal to $v$ ) before the merger and after the merger, the harmful effects of the merger on the buyer are not eliminated.

Propositions 1 and 2 also provide an interesting contrast with a classic result from the auctions literature due to Bulow and Klemperer (1996). This result says that a seller is better off with $n+1$ bidders drawing their types from identical distributions and no bargaining power than having $n$ bidders and maximal bargaining power (in the sense of being able to commit to the optimal reserve). This suggests the following thought experiment in our setting: Would the buyer prefer facing the merged entity and having buyer power to having no merger and no buyer power? Intuition based on Bulow and Klemperer (1996) might suggest that the buyer is better off with no merger and no buyer power. However, this intuition is wrong because, in a nutshell, a merger only eliminates a bid, not a cost draw, which is the key difference relative to the Bulow and Klemperer (1996) thought experiment. In a post-merger market with buyer power, the buyer could use a random reserve drawn from the distribution of the higher of two independent draws from $G$, that is, from $G^{2}$, thereby reintroducing or replicating the bidding game that he would face with no buyer power and no merger. Because the random reserve is not optimal, it follows that the buyer is strictly better off in a post-merger market with buyer power than in a pre-merger market with no buyer power. ${ }^{24}$

As we show below, buyer power mitigates some harmful effects of a merger but exac-

[^11]erbates others. Propositions 1 and 2 imply that $0=\Delta Q^{0} \geq \Delta Q^{1}$ and $0=\Delta S S^{0} \geq \Delta S S^{1}$ with strict inequalities in the cases described in Proposition 2. Thus, quantity and social surplus effects are exacerbated by buyer power. In contrast, buyer power mitigates the effect of a merger on expected buyer surplus, as shown in the proof of the following proposition:

Proposition $30=\Delta Q^{0} \geq \Delta Q^{1}$ and $0=\Delta S S^{0} \geq \Delta S S^{1}$ and $0>\Delta B S^{1}>\Delta B S^{0}$.
Proof. See the Appendix.

The effect on expected buyer surplus described in Proposition 3 implies that, although a merger harms a buyer with buyer power, that harm is less than the harm to a buyer without buyer power.

Another relevant question is how the number of rivals affects the harm from mergers. Competition authorities typically view the presence of large numbers of nonmerging suppliers as a factor that mitigates the harm from a merger. ${ }^{25}$ We now show that without buyer power this is indeed the case in our setup. Furthermore, as $n$ goes to infinity, it is also true for powerful buyers. Under the technical conditions that

$$
\text { (i) } \int_{\underline{c}}^{\bar{c}}\left((1-G(c))^{n-2}-\left(1-G\left(\Gamma^{-1}(\min \{\hat{\Gamma}(c), \bar{c}\})\right)\right)^{n-2}\right) \max \{v-\hat{\Gamma}(c), 0\} d \hat{G}(c) \text {, }
$$

is nondecreasing in $n$ and

$$
\text { (ii) } \quad\left(\hat{G}(c)-\hat{G}\left(\hat{\Gamma}^{-1}(\Gamma(c))\right)\right) \max \{v-\Gamma(c), 0\}
$$

is nonincreasing in $c$, we can also show that merger harm decreases in $n$ for powerful buyers.

Proposition 4 With no buyer power, buyer harm decreases in $n$, i.e., $\Delta B S^{0}$ increases in $n$. With buyer power, buyer harm and harm to social surplus go to zero as $n$ goes to infinity, $\lim _{n \rightarrow \infty} \Delta B S^{1}=0$ and $\lim _{n \rightarrow \infty} \Delta S S^{1}=0$. Moreover, with buyer power, under conditions (i) and (ii), buyer harm monotonically increases in n, i.e., $\Delta B S^{1}$ monotonically increases in $n$.

Proof. See the Appendix.

[^12]Although Proposition 4 leaves open the possibility that buyer surplus could change nonmonotonically with $n$ when buyers are powerful, numerical results show that harm to buyers decreases monotonically in $n$ for a variety of distributions. We illustrate the case of the uniform distribution, which does not satisfy conditions $(i)$ and (ii), in Figure 2.


Figure 2: Change in buyer surplus as a result of a merger as a function of $n$, for varying values of the buyer's value $v$, assuming costs are drawn from the uniform distribution on $[0,1]$.

### 3.2 Merger-related incentives

As we show, the incentives to merge, enter, and innovate are affected by buyer power.

## Incentives to merge

A well-known result for mergers in the Cournot model is that a merger without cost synergies always benefits rival firms, but is not profitable for the merging firms unless it is a merger to monopoly (see Salant, Switzer, and Reynolds (1983) and Perry and Porter (1985)). ${ }^{26}$ As we now show, in our procurement model, things are starkly different.

As already foreshadowed by the preceding analysis, without buyer power, mergers are always profitable for the merging suppliers and neutral for the rivals. As we now show, with buyer power, mergers always benefit rivals but may or may not be profitable for the merging suppliers. In particular, with buyer power and $v$ sufficiently small, a merger to monopoly is not profitable.

We illustrate the possible effects of a merger on the merging suppliers in Figure 3. The figure focuses on effects for a cost vector $\mathbf{c}$ with $c_{1}<c_{2}$. The effects for $c_{1}>c_{2}$ are symmetric.

[^13]As Figure 3(a) shows, in the absence of buyer power, a merger either does not change or increases the joint surplus of the merging suppliers. The increase occurs when, before the merger, one of the merging suppliers wins and is paid the cost of the other merging supplier. Then, after the merger, the merged entity wins but receives a larger payment.

With buyer power, as Figure 3(b) shows, a merger can cause the merging suppliers to trade at a different price, cause them not to trade when they would have before the merger, or have no effect. When the effect of the merger is to cause the merging suppliers to trade at a different price than they would have before the merger, there are three possibilities. The merger can reduce the payment to the merging suppliers from $p \equiv$ $\min \left\{\Gamma_{\beta}^{-1}(v), \min _{j=3}^{n}\left\{c_{j}\right\}\right\}$ to $\hat{p} \equiv \min \left\{\hat{\Gamma}_{\beta}^{-1}(v), \min _{j=3}^{n}\left\{\hat{\Gamma}_{\beta}^{-1}\left(\Gamma_{\beta}\left(c_{j}\right)\right)\right\}\right\}$, it can reduce the payment to the merging suppliers from $c_{2}$ to $\hat{p}$, or it can increase the payment to the merging suppliers from $c_{2}$ to $\hat{p}$. When the effect of the merger is to cause the merging suppliers not to trade when they would have before the merger, then the merging suppliers' surplus falls from either $p-c_{1}$ or $c_{2}-c_{1}$ to zero.
(a) No buyer power

(b) Buyer power


Figure 3: Effect of a merger on the merging suppliers for cost vector $\mathbf{c}$ with $c_{1}<c_{2}$, where $p \equiv \min \left\{\Gamma_{\beta}^{-1}(v), \min _{j=3}^{n}\left\{c_{j}\right\}\right\}$ and $\hat{p} \equiv \min \left\{\hat{\Gamma}_{\beta}^{-1}(v), \min _{j=3}^{n}\left\{\hat{\Gamma}_{\beta}^{-1}\left(\Gamma_{\beta}\left(c_{j}\right)\right)\right\}\right\}$.

In the following proposition, which summarizes our results, a merger being profitable means that the expected post-merger profit of the merged entity exceeds the sum of the expected profits of the merging suppliers before the merger, while a merger benefitting rivals means that the nonmerging suppliers are better off after the merger than before the merger. A merger is called neutral for rivals if it does not affect the profits of nonmerging suppliers.

Proposition 5 With no buyer power, a merger is profitable for the merging suppliers and neutral for nonmerging suppliers. With buyer power, any merger, whether it is profitable or not, benefits the nonmerging suppliers. With buyer power and $v$ sufficiently small,
a merger to monopoly is not profitable if $g(\underline{c})>0$. Likewise, with buyer power and $n$ sufficiently large, a merger is not profitable if $g(\underline{c})>0$. With buyer power and $v$ sufficiently large, a merger to monopoly is profitable.

Proof. See the Appendix.

The intuition for why with buyer power a merger to monopoly is not profitable for $v$ sufficiently small is straightforward. With buyer power, a merger has two effects. It eliminates a competing bid, which is good (for the merging suppliers), and it induces the buyer to discriminate against the merged entity, which is bad for the merged entity. When $v$ is small, the elimination of the competing bid is second-order because, both with and without the merger, the transaction almost surely occurs (if it occurs at all) at the take-it-or-leave-it offer set by the buyer. Because this offer becomes more aggressive after the merger, the negative effect dominates.

Interestingly, essentially the same logic drives the result that in sufficiently competitive environments, mergers in the presence of buyer power are not profitable. The lowest cost draw by any of the nonmerging suppliers, denoted $c_{(1)}$, provides the buyer with an outside option. Rather than $v$, the buyer's maximum willingness to pay to one of the merging suppliers is $c_{(1)}$ before the merger and $\hat{\Gamma}^{-1}\left(c_{(1)}\right)$ after the merger. As $n$ goes to infinity, $c_{(1)} \rightarrow \underline{c}$ almost surely, which is akin to $v \rightarrow \underline{c}$.

Proposition 5 has a number of implications. First, for powerful buyers, collusion may be a greater concern than mergers. To see this, note that collusion between two suppliers has a similar effect to a merger in that it eliminates a competing supplier. However, if collusion is not detected or anticipated by the buyer, it has no effect on the procurement mechanism. Thus, collusion always increases the expected payoff of the colluding suppliers, even when a merger is not profitable.

Second, in the absence of buyer power, when $v>\bar{c}$, perfect collusion between two suppliers requires the second bid to be equal to $\bar{c}$ (to guarantee that it never binds). Such a bid potentially raises suspicions because it is a probability zero event. However, if $\Gamma^{-1}(v)<\bar{c}$, perfect collusion only requires the second bid to be greater than or equal to the reserve $\Gamma^{-1}(v)$. Such cost realizations happen with positive probability and are thus not (or much less) suspicious. Thus, buyer power makes collusion not only more profitable relative to a merger, but also more difficult to detect (relative to the case without buyer power).

Third, the broad insight of Proposition 4 is that an increase in the number of rivals decreases the buyer's harm from the merger. To that, Proposition 5 adds the insight that a larger number of suppliers also makes the merger less profitable for the merging suppliers. Thus, an argument that a merger is profitable and that large numbers of nonmerging
suppliers mitigate harms may be incompatible when there are powerful buyers.
Fourth, with powerful buyers, even a merger to monopoly may not be profitable. ${ }^{27}$ In this sense, powerful buyers are better at deterring mergers than at eliminating harmful merger effects.

## Incentives to enter

When evaluating the likely competitive effects of a merger, competition authorities regularly consider whether the merger (together with any price increases that result from the merger) might induce entry into the market and whether such entry might ameliorate any competitive harms from the merger. ${ }^{28}$

In assessing the likelihood of entry, considerations include, among other things, whether entry is likely to be profitable. ${ }^{29}$ We say that a merger potentially induces entry if the expected profit of an entrant is greater after the merger than before the merger, and we say that a merger does not induce entry if the expected profit of an entrant is no greater after the merger than before the merger.

Because the probability of trade is not affected by a merger when there is no buyer power, the profitability of entry is also not affected by a merger when there is no buyer power. However, when there is buyer power, a merger increases the weighted virtual type of the most competitive of the merging suppliers, so a merger increases the profitability of entry. Thus, we have the following result.

Proposition 6 In the absence of buyer power, a merger is neutral for nonmerging suppliers and so does not induce entry, but with buyer power, a merger increases the expected payoff from entry and so potentially induces entry.

[^14]
## Incentives to innovate

To analyze a merger's effect on incentives to innovate by investing in cost-reducing technologies, we stipulate that, prior to its cost realization, each supplier can make an investment that induces a first-order stochastic dominance shift of its cost distribution. Specifically, letting $G_{I}$ denote a supplier's cost distribution after investment, we assume $G_{I}(c) \geq G(c)$ for all $c \in[\underline{c}, \bar{c}]$. To begin, we assume that investments are not observable by the buyer, which allows us to keep the allocation mechanism fixed as we change investments, and then we consider observable investment. (Of course, in equilibrium the buyer will anticipate equilibrium levels of investment and make the mechanism depend on the equilibrium investments; however, the mechanism would not vary as a supplier considers deviating from a "prescribed" equilibrium investment.)

To account for the effects of buyer power on innovation, we need to account for the effects of buyer power on the weighted virtual cost function and, in particular, on the distribution of the lowest weighted virtual cost. The probability that the lowest weighted virtual cost of $n-1$ suppliers in the pre-merger market is not more than $z$, denoted $L(z)$, is

$$
L(z) \equiv 1-\left(1-G\left(\Gamma_{\beta}^{-1}(z)\right)\right)^{n-1} .
$$

For the post-merger market, the probability that for a nonmerging supplier the lowest of the other post-merger suppliers' weighted virtual costs is not bigger than $z$, denoted by $L^{m}(z)$, is

$$
L^{m}(z) \equiv 1-\left(1-G\left(\Gamma_{\beta}^{-1}(z)\right)\right)^{n-3}\left(1-\hat{G}\left(\hat{\Gamma}_{\beta}^{-1}(z)\right)\right)
$$

In what follows it will be useful to have the following lemma relating $L^{m}(z)$ and $L(z)$.
Lemma 2 For $\beta=0, L^{m}(z)=L(z)$ for $z \geq \underline{c}$; and for $\beta=1, L^{m}(z) \leq L(z)$ for $z \geq \underline{c}$, with a strict inequality for $z \in(\underline{c}, \hat{\Gamma}(\bar{c}))$.

According to Lemma 2, when the buyer has buyer power, a merger induces a first order stochastic dominance shift in the distribution of the lowest competing "bid" faced by a nonmerging supplier.

We can now define a nonmerging supplier's pre-merger and post-merger profits in terms of $L$ and $L^{m}$. Supplier $i$ 's expected pre-merger profit $\pi\left(c_{i}\right)$ is, for $c_{i}<\Gamma_{\beta}^{-1}(v),{ }^{30}$

$$
\pi\left(c_{i}\right)=\int_{\Gamma_{\beta}\left(c_{i}\right)}^{v}\left(\Gamma_{\beta}^{-1}(z)-c_{i}\right) d L(z)+(1-L(v))\left(\Gamma_{\beta}^{-1}(v)-c_{i}\right),
$$

and for $c_{i} \geq \Gamma_{\beta}^{-1}(v), \pi\left(c_{i}\right)=0$. Post merger, a nonmerging supplier $i$ with cost $c_{i}<\Gamma_{\beta}^{-1}(v)$

[^15]has expected profit $\pi^{m}\left(c_{i}\right)$ given by
$$
\pi^{m}\left(c_{i}\right)=\int_{\Gamma_{\beta}\left(c_{i}\right)}^{v}\left(\Gamma_{\beta}^{-1}(z)-c_{i}\right) d L^{m}(z)+\left(1-L^{m}(v)\right)\left(\Gamma_{\beta}^{-1}(v)-c_{i}\right)
$$
and for $c_{i} \geq \Gamma_{\beta}^{-1}(v), \pi^{m}\left(c_{i}\right)=0$.
We say that a merger increases incentives to invest for a nonmerging supplier if the incremental profit to a nonmerging supplier from investment is greater post merger than pre merger:
\[

$$
\begin{equation*}
\int_{\underline{c}}^{\bar{c}} \pi^{m}(c)\left(d G_{I}(c)-d G(c)\right)>\int_{\underline{\underline{c}}}^{\bar{c}} \pi(c)\left(d G_{I}(c)-d G(c)\right) . \tag{8}
\end{equation*}
$$

\]

Using integration by parts, we can write (8) as

$$
\int_{\underline{c}}^{\bar{c}}\left(\pi^{m \prime}(c)-\pi^{\prime}(c)\right)\left(G_{I}(c)-G(c)\right) d c<0
$$

which holds when $\beta=1$ because $\pi^{m \prime}(c)-\pi^{\prime}(c)=L^{m}\left(\Gamma_{\beta}(c)\right)-L\left(\Gamma_{\beta}(c)\right)$, which is negative by Lemma 2 when $\beta=1$. However, (8) holds with equality when $\beta=0$. Thus, with buyer power, a merger increases the incentives of the nonmerging suppliers to invest, but without buyer power a merger is neutral for the nonmerging suppliers' incentives to invest.

Let us now turn to the profits of the merged entity. Let $\hat{\pi}^{m}(c)$ be the expected profit from one of the merged entity's plants when that plant has cost $c .{ }^{31}$ We say that a merger increases incentives to invest for the merging suppliers (at each plant) if (8) holds with $\hat{\pi}^{m}$ replacing $\pi^{m}$, which we can write as

$$
\begin{equation*}
\int_{\underline{c}}^{\bar{c}}\left(\hat{\pi}^{m}(c)-\pi(c)\right)\left(d G_{I}(c)-d G(c)\right)>0 . \tag{9}
\end{equation*}
$$

When $\beta=0,(9)$ is satisfied because then $\pi^{m}(c)-\pi(c)$ is decreasing in $c$ for $c<$

$$
\begin{aligned}
& { }^{31} \text { Focusing on the plant of supplier 1, for } c_{1}<\hat{\Gamma}_{\beta}^{-1}(v), \\
& \qquad \hat{\pi}^{m}\left(c_{1}\right)=\left(1-G\left(c_{1}\right)\right)\left(\int_{\hat{\Gamma}_{\beta}\left(c_{1}\right)}^{v}\left(\hat{\Gamma}_{\beta}^{-1}(z)-c_{1}\right) d \hat{L}^{m}(z)+\left(1-\hat{L}^{m}(v)\right)\left(\hat{\Gamma}_{\beta}^{-1}(v)-c_{1}\right)\right),
\end{aligned}
$$

and otherwise $\hat{\pi}^{m}\left(c_{1}\right)=0$, where $\hat{L}^{m}(z)$ is the probability that the lowest weighted virtual type of the $n-2$ nonmerging suppliers is not greater than $z$, i.e., $\hat{L}^{m}(z) \equiv 1-\left(1-G\left(\Gamma_{\beta}^{-1}(z)\right)\right)^{n-2}$.
$\min \{v, \bar{c}\}$. However, when $\beta=1, \hat{\pi}^{m}(c)-\pi(c)$ is decreasing in $c$ for $c<\hat{\Gamma}_{1}^{-1}(v),{ }^{32}$ increasing in $c$ for $c \in\left(\hat{\Gamma}_{1}^{-1}(v), \Gamma_{1}^{-1}(v)\right)$, and zero for $c \geq \Gamma_{1}^{-1}(v)$. For an illustration of $\hat{\pi}^{m}$ and $\pi$, see Figure 4(a), which provides a case in which the merger is profitable for the merging suppliers. Because $\hat{\pi}^{m}$ and $\pi$ are each larger for a portion of the type space, (9) may or may not hold, depending on the nature of the shift in distribution due to investment. In the illustration provided in Figure 4, panel (b) shows two possible values for $G_{I}$, one such that the merger increases a merging supplier's incentives to invest and another such that the merger decreases a merger supplier's incentives to invest.


Figure 4: Effect of a merger on the incentives to invest for a merging supplier. Panel (a): Pre-merger profit of a merging supplier (solid line) and post-merger profit of the merged entity at one plant (dashed line) as a function of cost. The merger is profitable for the merging suppliers in expectation. Panel (b): Pre-merger cost distribution $G$ and two possible post-investment cost distributions, one such that a merger increases the incentive for a merging supplier to invest (dashed line, $G_{I}(c)=\sqrt{c}$ for $c \in[0,1]$ ) and one such that a merger decreases the incentive for a merging supplier to invest (solid line, $G_{I}(c)=c$ for $c \in[0,1 / 4]$ and $G_{I}(c)=1 / 9\left(1+24 x^{2}-16 x^{3}\right)$ for $\left.c \in(1 / 4,1]\right)$. Both panels assume $\beta=1$, $n=2, v=2.1$, and $G$ uniform on $[0,1]$.

As this suggests, investments that shift probability weight to the low end of the support are more favorable for having a merger that increases incentives to invest than investments that merely shift probability weight towards the middle of the support. In addition, this

$$
\begin{aligned}
&{ }^{32} \text { For } c<\hat{\Gamma}_{1}^{-1}(v) \\
& \begin{aligned}
\hat{\pi}^{m \prime}(c)-\pi^{\prime}(c) & =-\frac{g(c)}{1-G(c)} \hat{\pi}^{m}(c)+(1-G(c))\left(\hat{L}^{m}\left(\hat{\Gamma}_{1}(c)\right)-1\right)-\left(L\left(\Gamma_{1}(c)\right)-1\right) \\
& =-\frac{g(c)}{1-G(c)} \hat{\pi}^{m}(c)+(1-G(c))\left(\left(1-G\left(\Gamma_{1}^{-1}\left(\hat{\Gamma}_{1}(c)\right)\right)\right)^{n-2}-(1-G(c))^{n-2}\right)
\end{aligned}
\end{aligned}
$$

which is negative if $\hat{\Gamma}_{1}(c)>\Gamma_{1}(c)$, which holds for $c>\underline{c}$.
shows that even when a merger is profitable, the merging suppliers' incentives to invest can go either up or down as a result of the merger.

Together with the implications of Lemma 2, we have the following result:
Proposition 7 Without buyer power, a merger is neutral for nonmerging suppliers' incentives to invest and increases the merging suppliers' incentives to invest. With buyer power, a merger increases incentives to invest for nonmerging suppliers, but can increase or decrease incentives to invest for the merging suppliers.

Proposition 7 contrasts with results for investment in complete information models of mergers, where incentives to invest increase or decrease with a supplier's equilibrium quantity, implying that a merger increases incentives to invest for nonmerging suppliers and decreases them for merging suppliers (see, e.g., Motta and Tarantino, 2017). In contrast, in our model, the effect of a merger on a supplier's incentive to invest depends on the interaction between investment and the supplier's profitability. When there is buyer power, a merger is profitable for a nonmerging supplier, and incrementally more so the lower is her cost, so a merger increases a nonmerging supplier's incentive to invest in cost reduction. The same is true for the merging suppliers when there is no buyer power, so with no buyer power a merger increases the merging suppliers' incentives to invest.

With buyer power, a merger is profitable for the merging suppliers for some cost realizations and not for others, as depicted in Figures 3(b) and 4(a). Thus, when there is buyer power, the effect of a merger on the merging suppliers' incentives to invest depends on the details of how the investment affects the cost distribution. For example, a merger creates a disincentive for an investment that shifts probability away from costs such that a supplier never trades and towards costs such that the supplier trades pre-merger but not post-merger. In contrast, a merger enhances the merging suppliers' incentives for investments that shift probability weight away from costs such that the merger is not profitable and towards costs such that it is profitable.

At this point, the analysis is not an equilibrium analysis because we have only looked at individual rewards from investment without accounting for interaction effects arising from other suppliers' investment incentives (and without modeling the costs of investment). This approach has the benefit of generality and detail freeness - we do not have to impose assumptions on how exactly investment affects the cost distribution or on the cost of investment. In this approach, the rewards for innovation also appear to correspond to what antitrust authorities are most likely to be evaluating. However, it leaves unanswered questions such as whether industry investment can increase after a merger occurs.

To illustrate that the answer to this question is affirmative, it is sufficient to consider the following simple example. Suppose that there are two suppliers drawing their costs
from distribution $G(c)=c^{2}$, or from $G_{I}(c)=c$ if they invest, where investment has cost $k$. Consider a simultaneous moves investment game prior to cost realizations, and assume investments are observed by the buyer. Assuming buyer power and $v=10$, pre-merger expected payoffs for the suppliers as a function of investments are shown in Figure 5(a).
(a) Pre-merger expected surplus of suppliers 1 and 2
(b) Post-merger expected surplus of merged entity

2


| Invest in one plant |  |
| :--- | :--- |
| Invest in two plants | $\begin{array}{l}0.5551-k \\ \text { invest }\end{array}$ |
| $0.6117-2 k$ |  |

Figure 5: Payoffs with and without investment, where investment has cost $k$. Assumes $n=2, \beta=1, v=10, G(c)=c^{2}$, and $G_{I}(c)=c$.

For $k>0.0209$, the unique equilibrium of the pre-merger investment game is for neither supplier to invest. In the post-merger market shown in Figure 5(b), there is investment in at least one plant for $k<0.1026$. Thus, for $k \in(0.0209,0.1026)$, there is no investment pre-merger, but there is investment (in one or both plants) post-merger.

### 3.3 Cost efficiencies

Absent cost efficiencies, any merger is detrimental to the buyer regardless of whether or not there is buyer power. Cost efficiencies are thus necessary for mergers not to harm the buyer. To model cost efficiencies, we now assume that the merged entity's cost is reduced by multiplicative factor $1-s$, where $s \in[0,1]$ measures the strength of cost synergies. Thus, the merged entity's cost is $(1-s) \min \left\{c_{1}, c_{2}\right\}$. Because likely cost efficiencies are part of the merger review and therefore information that both the buyer and the merging suppliers share, ${ }^{33}$ we assume that $s$ is commonly known. For simplicity, we assume $\underline{c}=0$. The distribution of costs after the merger for the merged entity with cost synergies $s$ is now, for $c \in[0,(1-s) \bar{c}]$,

$$
\bar{G}(c) \equiv \hat{G}(c /(1-s)),
$$

with density $\bar{g}(c) \equiv \hat{g}(c /(1-s)) /(1-s)$.
Because cost synergies improve the merged entity's distribution, cost synergies increase the set of trades that are beneficial to the buyer. Post merger, the buyer could do exactly

[^16]the same as he does without cost synergies, and, as a result, the buyer is better off post merger with cost synergies if he adjusts his mechanism to account for those synergies.

This raises the question of whether the presence of buyer power might mean that a lower level of cost synergies is required to offset merger harm than without buyer power. However, this is not necessarily the case. For a level of synergies $s$ sufficiently close to zero, the result that $\Delta B S_{s}^{1}>\Delta B S_{s}^{0}$, where $\Delta B S_{s}^{\beta}$ is the analog to $\Delta B S^{\beta}$ for the case with synergies, follows from Proposition 3 and continuity. But when $s=1$, the postmerger buyer has payoff $v$ regardless of buyer power, while the pre-merger buyer has higher expected surplus with buyer power than without, implying that $\Delta B S_{s}^{0}>\Delta B S_{s}^{1}$.

We summarize with the following proposition:
Proposition 8 Post-merger expected buyer surplus increases with cost synergies, i.e., $\Delta B S_{s}^{0}$ and $\Delta B S_{s}^{1}$ are both increasing in s. For s sufficiently close to zero, $\Delta B S_{s}^{1}>\Delta B S_{s}^{0}$, and for $s$ sufficiently close to one, $\Delta B S_{s}^{0}>\Delta B S_{s}^{1}$.

It follows from Proposition 8 that merging suppliers cannot argue that, in general, the cost efficiencies required to eliminate harm from the merger are lower when they face powerful buyers than when they face weak buyers.

## Effect on the merged entity of cost synergies with no buyer power

Next we analyze the effects of cost synergies on the merged entity's expected surplus, and thereby on the incentives to merge. We begin with the case without buyer power.

The optimal take-it-or-leave-it offer for a buyer without buyer power that accounts for cost synergies, denoted $p^{0}(s)$ is

$$
p^{0}(s) \equiv \min \{v,(1-s) \bar{c}\} .
$$

This merely adjusts the powerless buyer's optimal take-it-or-leave-it offer from $\min \{v, \bar{c}\}$ to $\min \{v,(1-s) \bar{c}\}$, recognizing that the highest possible cost draw for the merged entity is now $(1-s) \bar{c}$. This fact has an immediate implication that may not have been fully anticipated. As described in Proposition 9, where we let $\hat{\Pi}(s)$ denote the merged entity's expected surplus with cost synergies $s$, cost synergies are not necessarily beneficial for the merged entity.

Proposition 9 With no buyer power and two suppliers: (i) if $v<\bar{c}$, then the merged entity's expected surplus increases and then decreases as cost synergies increase (there exists $\hat{s} \in(0,1)$ such that $\hat{\Pi}(s)$ increases in $s$ for $s<\hat{s}$ and decreases in $s$ for $s>\hat{s})$; (ii) if $v \geq \bar{c}$, then the merged entity's expected surplus decreases as cost synergies increase
$(\hat{\Pi}(s)$ decreases in $s$ for all $s \in[0,1])$; and (iii) the merged entity's expected surplus is zero when $s=1(\hat{\Pi}(1)=0)$.

Proof. See the Appendix.

Proposition 9 implies that for merging suppliers, cost synergies are, at best, a mixed blessing. While they unambiguously increase buyer surplus (Proposition 8) and therefore make mergers more likely to be approved, they eventually (if $v<\bar{c}$ ) or universally (if $v \geq \bar{c}$ ) decrease post-merger expected surplus for the merging suppliers. The intuition is straightforward. Although synergies decrease production costs, which is good for the merging suppliers, they also reduce information rents and squeeze markups, which is bad for the merging suppliers. As synergies increase, the latter, detrimental effect dominates.

## Effect on the merged entity of cost synergies with buyer power

Letting $\hat{\sigma}(c) \equiv \hat{G}(c) / \hat{g}(c)$ be the reverse hazard rate without cost efficiencies, the price elasticity of supply without cost efficiencies, denoted $\hat{\varepsilon}(p)$, is ${ }^{34}$

$$
\hat{\varepsilon}(c) \equiv \frac{c}{\hat{\sigma}(c)} .
$$

The following lemma describes how the powerful buyer's optimal post-merger take-it-or-leave-it offer to the merged entity $p^{*}(s)$ varies with $s$, provided that the buyer's value $v$ (and not $\bar{c}$ ) determines the buyer's offer.

Lemma 3 If $s$ is sufficiently small that $v<(1-s) \hat{\Gamma}(\bar{c})$, then
(i) if $\hat{\varepsilon}^{\prime}(c)=0, p^{*}(s)$ does not vary with $s$;
(ii) if $\hat{\varepsilon}^{\prime}(c)<0, p^{*}(s)$ decreases in $s$;
(iii) if $\hat{\varepsilon}^{\prime}(c)>0, p^{*}(s)$ increases in $s$.

Proof. See the Appendix.

Lemma 3 is useful in proving Proposition 10 below, but also of independent interest. As mentioned, the focus of merger analysis tends to be on the merger's impact on prices with little consideration to its effect on quantity; however, Lemma 3 implies that such an approach can be misleading. To see this, assume first that the merged entity has an

[^17]isoelastic supply function, that is, that its cost distribution without cost efficiencies is of the form $\hat{G}(c)=c^{x}$ with support $[0,1]$ and $x>0 .{ }^{35}$ In this case, by only noting the absence of price effects, one would incorrectly conclude that cost efficiencies do not benefit the buyer.

Next assume that the merged entity's cost distribution is a convex combination of two isoelastic supply functions $c^{x_{1}}$ and $c^{x_{2}}$, that is, $\hat{G}(c)=a c^{x_{1}}+(1-a) c^{x_{2}}$ with $x_{1}, x_{2}>0$, $a \in(0,1)$ and $x_{1} \neq x_{2}$. In this case, the price elasticity of supply is increasing in price, implying by Lemma 3 that cost efficiencies induce the buyer to make a higher take-it-or-leave-it offer. ${ }^{36}$ Considering only price effects, one would thus conclude that when supply exhibits increasing price elasticity, cost efficiencies exacerbate the merger's harmful effects because they lead to further price increases. Although it is true that with increasing price elasticity of supply, the buyer's take-it-or-leave-it offer increases with cost synergies, the buyer is strictly better off the larger are the cost efficiencies. This is easiest to see when the merged entity is a monopoly. In that case, because the buyer always has the option of setting the same price $p$ that he sets when there are no cost synergies, his expected surplus with cost synergies $s>0$ must be at least $(v-p) \hat{G}(p /(1-s))$, which for $p<\bar{c}$ is strictly larger than his maximal expected surplus without cost synergies, $(v-p) \hat{G}(p)$. The reason why the buyer is better off is the quantity effect, that is, the fact that $\hat{G}(p /(1-s)) \geq \hat{G}(p)$ with strict inequality when $p<\bar{c}$.

Lemma 3 is also instrumental for the analysis of cost synergies on the merged entity's post-merger profit when the buyer is powerful. To complete this analysis, one has to account for the possibility that, in the language of the lemma, $s$ is large in the sense that $v>(1-s) \hat{\Gamma}(\bar{c})$. Whether such an $s$ exists depends on the seemingly technical detail whether $g(\bar{c})(1-G(\bar{c}))$ is positive or not. To see why this matters, recall that

$$
\hat{\Gamma}(c)=c+\frac{G(c)}{g(c)} \frac{2-G(c)}{2(1-G(c))} .
$$

Thus, as mentioned above, $\lim _{c \rightarrow \bar{c}} \hat{\Gamma}(c)=\infty$ if $g(\bar{c})(1-G(\bar{c}))=0$, in which case there is no $s<1$ such that $(1-s) \hat{\Gamma}(\bar{c})<v$. In this case, we are always in the case analyzed in Lemma 3. In contrast, if $g(\bar{c})(1-G(\bar{c}))>0$, then $\lim _{c \rightarrow \bar{c}} \hat{\Gamma}(c)<\infty$, in which case there

[^18]exists a $\hat{s} \in(0,1)$ such that $(1-s) \hat{\Gamma}(\bar{c})<v$ if and only if $s>\hat{s}$. We note, in passing, that if supply exhibits nondecreasing elasticity, that is, if $\hat{\varepsilon}^{\prime}(c) \geq 0$, we have $\lim _{c \rightarrow \bar{c}} \hat{\Gamma}(c)<\infty .{ }^{37}$ In the following proposition, which is illustrated in Figure 6, we assume that the sign of $\hat{\varepsilon}^{\prime}(c)$ does not vary with $c$.

Proposition 10 With buyer power and $n=2$, the buyer's optimal post-merger take-it-or-leave-it offer $p^{*}(s)$ satisfies $p^{*}(1)=0$. Further, if $v \geq \hat{\Gamma}(\bar{c})$, then $p^{*}(s)$ decreases in $s$, and if $v<\hat{\Gamma}(\bar{c})$, then:
(i) if $\hat{\varepsilon}^{\prime}(c)=0, p^{*}(s)$ does not vary with $s$ for $s \in[0, \hat{s})$ and decreases linearly in $s$ for $s \in(\hat{s}, 1] ;$
(ii) if $\hat{\varepsilon}^{\prime}(c)<0, p^{*}(s)$ decreases in $s$;
(iii) if $\hat{\varepsilon}^{\prime}(c)>0, p^{*}(s)$ increases in $s$ for $s \in[0, \hat{s})$ and decreases linearly in $s$ for $s \in(\hat{s}, 1]$. Proof. See the Appendix.
(a) Constant elasticity

(b) Decreasing elasticity

(c) Increasing elasticity


Figure 6: Post-merger reserve as a function of cost synergies $s$. The three panels correspond to three different post-merger supply elasticities in the absence of cost efficiencies. All panels assume $n=2, \beta=1$, and $v=1$ and have distributions defined on $[0,1]$. Panel (a) assumes $G(c)=1-\sqrt{1-c^{2}}$. Panel (b) assumes $G(c)=c$. Panel (c) assumes $G(c)=1-\sqrt{1-\left(e^{c}-1\right) /(e-1)}$.

Although the details of Proposition 10 are more involved than in Proposition 9 for the case without buyer power, the basic intuition from Proposition 9 generalizes to the case with buyer power. Cost synergies squeeze informational rents and thereby profit margins. In the extreme, the surplus of the merged entity goes to zero because the merged entity is left with no private information.

Propositions 9 and 10 have the following corollary:

[^19]Corollary 1 The expected surplus of the merged entity is maximized at a level of cost synergies strictly less than one.

Two tensions thus emerge from this analysis for cost synergies (and buyer power) as a merger defense. First, buyer power requires (weakly) larger synergies for the merger to be profitable. This is immediate because without buyer power, the merger is already profitable without synergies. Thus, merging suppliers who invoke buyer power as an countervailing effect may also have to make a case for why there are cost synergies that are large enough to outweigh the buyer power effects. Second, synergies are a two-edged sword for merging suppliers. Although cost synergies reduce production costs, they also reduce information rents and, eventually, profits, implying that post-merger profits are maximized at a moderate level of synergies.

Much of the above analysis rests on the assumption that there are only two suppliers, so that the merger being considered is a merger to monopoly. With rival suppliers after the merger, business stealing from rivals is an additional effect of cost synergies.

## 4 Extensions

We now extend our analysis. We first allow for asymmetric suppliers before the merger. Then we provide a measure of coordinated effects, and finally we allow for multi-product suppliers.

### 4.1 Asymmetric suppliers

Although we assume above that suppliers are symmetric, our framework easily accommodates asymmetric suppliers pre merger, where each supplier $i \in\{1, \ldots, n\}$ has a cost distribution $G_{i}$ with support $[\underline{c}, \bar{c}]$. In that case, the merged entity's cost distribution is $\hat{G}(c) \equiv 1-\left(1-G_{1}(c)\right)\left(1-G_{2}(c)\right)$. Each pre-merger supplier $i$ has a supplier-specific weighted virtual cost function $\Gamma_{\beta, i}$ defined analogously to the symmetric case.

As we now discuss, all of our results continue to hold in the asymmetric setup as long as either there is no buyer power, and almost all of our results continue to hold if the merging suppliers are symmetric with one another. Furthermore, even with buyer power and full asymmetry, almost all results continue to hold under the virtual dominance condition described below.

To begin, our result that the buyer is harmed by a merger holds generally, regardless of buyer power and for any distributional asymmetries. This can be seen because our proof above does not rely on distributional assumptions. If the buyer were to use the optimal post-merger mechanism in the pre-merger market, applying the virtual cost function for
the merged entity to both supplier 1 and supplier 2, then the buyer would be better off pre-merger versus post-merger because of the additional competition in the pre-merger market. And buyer surplus would be even greater in the pre-merger market if the optimal pre-merger mechanism were used.

Corollary 2 With or without buyer power and regardless of distributional asymmetries, a merger reduces buyer surplus.

Further, all results that assume no buyer power continue to hold with distributional asymmetries. Because a buyer with no buyer power does not differentially discriminate among suppliers, asymmetries among suppliers have no effect. Thus, Propositions 1 and 9 and the parts of the other results that relate to the case of no buyer power, continue to hold, although for Propositions 4 and 5, which consider an increase in the number of suppliers, one would need to specify a specific supplier or set of suppliers to be replicated.

Almost all results that assume buyer power also continue to hold if the merging suppliers are symmetric, that is if $G_{1}=G_{2}$, even if the nonmerging suppliers have cost distributions that are different from the merging suppliers and different from each other. The conditions in Proposition 2 for a strict inequality $0>\Delta Q^{1}$ must be adjusted to say that either $n=2$ and $v<\hat{\Gamma}(\bar{c})$ or $n \geq 3, v<\hat{\Gamma}(\bar{c})$, and $v<\min _{i \in\{3, \ldots, n\}} \Gamma_{i}(\bar{c})$. In addition, the result in Proposition 2 that $\Delta S S^{0} \geq \Delta S S^{1}$ no longer holds in general. And the same caveat as above for Propositions 4 and 5 applies. When $G_{1}=G_{2}$, for all cost realizations of the merging suppliers, $\hat{\Gamma}_{\beta}\left(\min \left\{c_{1}, c_{2}\right\}\right) \geq \min \left\{\Gamma_{\beta, 1}\left(c_{1}\right), \Gamma_{\beta, 2}\left(c_{2}\right)\right\}$, with a strict inequality when $\beta=1$ and $\min \left\{c_{1}, c_{2}\right\}>\underline{c}$. This corresponds to condition (7) for the symmetric setup, which drives our results with buyer power, except the result that $\Delta S S^{0} \geq \Delta S S^{1}$, which relies on the merger creating asymmetries where none existed. When there are pre-merger asymmetries, a merger can reduce or even eliminate asymmetries, which can improve social surplus.

As this suggests, symmetry between the merging suppliers is not necessary. With appropriate adjustments, almost all results continue to hold as long as virtual dominance holds, which says that for all $c_{1}, c_{2} \in[\underline{c}, \bar{c}]$,

$$
\begin{equation*}
\hat{\Gamma}_{\beta}\left(\min \left\{c_{1}, c_{2}\right\}\right) \geq \min \left\{\Gamma_{\beta, 1}\left(c_{1}\right), \Gamma_{\beta, 2}\left(c_{2}\right)\right\} \tag{10}
\end{equation*}
$$

where $\Gamma_{\beta, i}$ is the pre-merger virtual cost function for supplier $i$, with a strict inequality for a positive measure set of costs when $\beta=1$. Virtual dominance is satisfied whenever the merging suppliers are symmetric and in some cases when they are asymmetric. ${ }^{38}$ With

[^20]virtual dominance, Propositions 1-3 hold for asymmetric suppliers with the same adjustment as above for Proposition 2. Propositions 4 and 5 hold with the caveat mentioned above that one must specify how suppliers are to be replicated. In addition, for the last part of Proposition 4, conditions (i) and (ii) must be revised to account for asymmetries, and for the last part of Proposition 5, the required condition is that $g_{1}(\underline{c}) g_{2}(\underline{c})>0$. Propositions 6-10 hold without adjustment.

Finally, even virtual dominance, although it is sufficient for almost all of our results, is not necessary. For example, if $G_{1}(c)=c$ and $G_{2}(c)=c^{2}$, both with support $[0,1]$, then for $c_{1}$ sufficiently small and $c_{2}$ sufficiently large, (10) does not hold, ${ }^{39}$ but numerical calculations show that with buyer power, merger effects are as described in Proposition 2. In particular, with buyer power, the expected quantity traded, expected buyer surplus, and expected social surplus are all reduced by a merger to monopoly as shown in Figure 7.


Figure 7: Effects of a merger to monopoly by asymmetric merging suppliers when there is buyer power. Assumes $\beta=1, G_{1}(c)=c, G_{2}(c)=c^{2}$, and values for $v$ as shown.

### 4.2 Coordinated effects

According to the U.S. Guidelines, the U.S. competition authorities recognize that "A merger also can enhance market power by increasing the risk of coordinated, accommodating, or interdependent behavior among rivals" (U.S. Guidelines, p. 2). Similarly, the EC recognizes that possibility that a merger changes "the nature of competition in such a way that firms that previously were not coordinating their behaviour, are now significantly more likely to coordinate and raise prices or otherwise harm effective competition." (EC

[^21]Guidelines, para. 22(b)) In addition, the U.S. Guidelines suggest a possible relation between buyer power and coordinated effects, ${ }^{40}$ saying that buyer power can mitigate merger effects if, for example, "the conduct or presence of large buyers undermines coordinated effects." (U.S. Guidelines, p. 27) To quantify coordinated effects, the U.S. Guidelines suggest looking at the profitability of coordination, saying "The Agencies regard coordinated interaction as more likely, the more the participants stand to gain from successful coordination." (U.S. Guidelines, p. 26) This motivates us to ask: By how much does the merger increase the profitability of coordination? In turn, this requires us to say how coordination works.

To avoid explicit, frictionless collusion, which would always be profitable, we focus on coordination that does not involve the communication of competitively sensitive information. ${ }^{41}$ Without such communication, the coordinating suppliers are not able to identify which is the lowest cost supplier, and, consequently, efficient collusion is not possible. However, other types of coordination are conceivable and may be profitable. In particular, even in the absence of communication regarding costs, suppliers have been known to coordinate on bid rotation schemes, whereby the suppliers allocate the gains from the suppression of rivalry across time, buyers, and/or products. ${ }^{42}$

To model imperfect collusion, we consider a "rotation" among coordinating suppliers. ${ }^{43}$ Specifically, assume that a randomly designated coordinating supplier bids as she would in the absence of coordination, while the other coordinating suppliers submit deliberately losing bids. In our setup, unlike Cournot and Bertrand setups, the incentives of the noncoordinating suppliers are not affected by presence of coordination by rivals, so we assume noncoordinating suppliers continue to bid as they would in the absence of coordination. In addition, also in contrast to Cournot and Bertrand setups, the coordinated bidding is consistent with equilibrium bidding and so is not detectable ex post (at least not deterministically and not in a one-shot game), so we hold constant the buyer's purchasing procedure.

The tradeoffs involved in this type of coordination are that coordinating suppliers may lose sales to outside suppliers by suppressing potentially winning bids, and may lose

[^22]surplus even when they win if the "wrong" (not lowest cost) supplier wins. But the coordinating suppliers stand to gain higher payments by suppressing all but one of the coordinating suppliers' bids. Thus, on net, coordination can be positive or negative for the coordinating suppliers.

To quantify this, if the concern is coordination between suppliers $1, \ldots, m$, define:

$$
\hat{\pi}=\sum_{i=1}^{m} \frac{1}{m} E[\text { joint surplus of } 1, \ldots, m \text { when } i \text { is designated }]
$$

and

$$
\pi=E[\text { joint surplus of } 1, \ldots, m \text { under competition }],
$$

and then quantify the gain from coordination as $\hat{\pi}-\pi$.
For example, as a benchmark for the incremental profits due to bilateral tacit collusion, consider the incremental profits due to a rotation scheme between suppliers 1 and 3 in the pre-merger market. We model this as those two suppliers alternating across a large number of procurements in submitting noncompetitive bids, leaving their partner in coordination to compete against the other suppliers participating in the procurements. Then we consider a similar scenario under the post-merger configuration of suppliers. For example, if supplier 1 merges with supplier 2, we can consider the incremental profits due to a rotation scheme between the merged entity and supplier 3. If the incremental profits from coordination are positive and significantly larger than they were in the pre-merger market, that suggests that the merger significantly increases the likelihood of coordinated effects.

Figure 8 illustrates that pairwise coordination can be unprofitable for at least some of the individual coordinating suppliers prior to a merger, but significantly positive for all coordinating suppliers following the merger. In the example of Figure 8, suppliers 1 and 2 draw their costs from a different distribution from supplier 3, but the merger of suppliers 1 and 2 creates a merged entity that is symmetric with supplier 3. In the premerger market, the dominant effect of coordination between suppliers 1 and 3 is to shift surplus to supplier 2, who benefits significantly from the suppression of bids by suppliers 1 and 3. As a result, the coordination is not profitable for supplier 3. However, in the post-merger market, where the coordinating suppliers face no other rivals, the dominant effect of coordination between the merged entity and supplier 3 is to shift surplus away from the buyer. As a result, both the merged entity and supplier 3 experience a more than $100 \%$ increase in their expected surplus, while the buyer experiences a $10 \%$ decrease in his expected buyer surplus. Thus, for the merger in this example, incremental harm associated with coordinated effects would likely be a concern.

To examine further how coordinated effects are affected by buyer power, focus on
(a) Pre-merger expected surplus

|  | Competition | Coordination <br> by 1 and 3 | Change |
| :--- | :---: | :---: | ---: |
| Supplier 1 | 0.06 | 0.083 | $34 \%$ |
| Supplier 2 | 0.06 | 0.150 | $139 \%$ |
| Supplier 3 | 0.09 | 0.079 | $-16 \%$ |

(b) Post-merger expected surplus

|  | Competition | Coordination <br> by 1 and 3 | Change |
| :--- | :---: | :---: | :---: |
| Merged entity | 0.13 | 0.27 | $106 \%$ |
| Supplier 3 | 0.13 | 0.27 | $106 \%$ |
| Buyer | 4.53 | 4.06 | $-10 \%$ |

Figure 8: Effect of a merger on the profitability of coordination between, in panel (a), suppliers 1 and 3 , and in panel (b), the merged entity and supplier 3. Assumes $n=3$, $\beta=1$, and $v=5$, with $G_{1}$ and $G_{2}$ uniform on $[0,1]$ and $G_{3}(c)=1-(1-c)^{2}$.
the case of two suppliers. Let the suppliers' costs be $c_{(1)}<c_{(2)}$. Without coordination, their joint payoff is $\max \left\{0, \min \left\{\Gamma_{\beta}^{-1}(v), c_{(2)}\right\}-c_{(1)}\right\}$. If the supplier with the lower cost is selected to bid while other does not, their joint payoff is $\max \left\{0, \Gamma_{\beta}^{-1}(v)-c_{(1)}\right\}$, and if the supplier with the higher cost is selected, their joint payoff is $\max \left\{0, \Gamma_{\beta}^{-1}(v)-c_{(2)}\right\}$. Thus, the change in the suppliers' joint surplus assuming a $50-50$ probability of being selected is:

$$
\begin{aligned}
& \frac{1}{2} \max \left\{0, \Gamma_{\beta}^{-1}(v)-c_{(1)}\right\}+\frac{1}{2} \max \left\{0, \Gamma_{\beta}^{-1}(v)-c_{(2)}\right\} \\
& -\max \left\{0, \min \left\{\Gamma_{\beta}^{-1}(v), c_{(2)}\right\}-c_{(1)}\right\} .
\end{aligned}
$$

If $c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)$, then the change is

$$
\Gamma_{\beta}^{-1}(v)-c_{(2)}-\frac{c_{(2)}-c_{(1)}}{2}
$$

which can be positive or negative - there is a gain from suppressing rivalry but a loss of efficiency. The expression is positive if $c_{(2)}-c_{(1)}$ is not too large. Thus, coordinating suppliers gain from the rotation if both of them have costs below $\Gamma_{\beta}^{-1}(v)$ and the efficiency loss is not too large.

If instead $c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}$, then the change in suppliers' joint surplus is:

$$
-\frac{1}{2}\left(\Gamma_{\beta}^{-1}(v)-c_{(1)}\right),
$$

which is negative. Thus, the rotation reduces supplier surplus when only one of the suppliers has a cost below $\Gamma_{\beta}^{-1}(v)$ because they get no benefit from suppressing rivalry and half of the time they lose a transaction.

As we show in the following proposition, when the buyer has a sufficiently high value and the gain from coordination is positive, that gain decreases with buyer power. Thus, coordinated effects from a merger may be viewed as less likely in the face of powerful,
high-value buyers.
Proposition 11 Assuming two symmetric suppliers, there exists $\hat{v}<\bar{c}$ such that for all $v>\hat{v}$, the gain from coordination $\hat{\pi}-\pi$ is decreasing in buyer power when the gain is positive.

Proof. See the Appendix.

Proposition 11 implies that, at least in some settings, coordinated effects are less of a concern for powerful buyers, which is consistent with the notion in the U.S. Guidelines (p. 27) that "the conduct or presence of large buyers" could undermine coordinated effects. ${ }^{44}$

### 4.3 Multi-product suppliers

The proposed merger of oilfield services firms Halliburton and Baker Hughes posed challenges for competition authorities because of the multi-product nature of the merging firms. ${ }^{45}$ In particular, existing methodologies, which typically focus on individual relevant antitrust markets, have limited ability to account for cost synergies associated with multi-product production, demand side complementarities that cause buyers to prefer a single source for multiple products, or the interaction of these with buyer power. ${ }^{46}$

In order to address cross-market issues such as buyer preferences for one-stop shopping, we consider a setup with two products, $A$ and $B$. The buyer has value zero for product $A$ or $B$ individually, value $v$ for the pair of products $A$ and $B$ if purchased from two different suppliers, and value $V \geq v$ for the pair of products $A$ and $B$ if purchased from the same supplier. Thus, the difference between $V$ and $v$ captures demand-side complementarities, sometimes referred to as the value of "one-stop shopping."

This multi-product extension accommodates the case in which suppliers produce multiple complementary products, as well as the case in which products are location specific, with one product supplied in location $A$ and the other in location $B$, where the buyer

[^23]demands coverage that spans both locations. Further, this extension can be interpreted in terms of vertically related products. For example, product $A$ might be the transportation or marketing of product $B$, both of which are demanded by the buyer. In that case, a merger of a supplier of $A$ with a supplier of $B$ is a vertical merger.

Let $\mathbb{M}$ be the set of multi-product suppliers, $\mathbb{A}$ be the set of suppliers of only $A$, and $\mathbb{B}$ be the set of suppliers of only $B$, with $|\mathbb{M}|+|\mathbb{A}|+|\mathbb{B}|=n$. The cost type of a multi-product supplier is the cost to that supplier of producing both products, whereas the cost type of a single-product supplier is the cost to that supplier of producing only one product. Thus, each supplier has a single-dimensional type. Multi-product suppliers can supply individual products to the buyer for a commonly known proportion of their joint production cost, where we allow for the possibility of cost synergies in production. Specifically, the cost to multi-product supplier $i \in \mathbb{M}$ of supplying just product $A$ is $\gamma^{A} c_{i}$ and of supplying just product $B$ is $\gamma^{B} c_{i}$, where $\gamma^{A}, \gamma^{B}<1$ and where $\gamma^{A}$ and $\gamma^{B}$ are known by the buyer. As a matter of notation, it will be useful to set $\gamma_{i}^{A}=\gamma^{A}$ and $\gamma_{i}^{B}=\gamma^{B}$ for $i \in \mathbb{M}$ and $\gamma_{i}^{A}=\gamma_{i}^{B} \equiv 1$ for $i \in \mathbb{A} \cup \mathbb{B}$. Cost synergies are accommodated by letting $\gamma^{A}+\gamma^{B}>1$, which implies that multi-product suppliers can produce both products at a cost that is less than the sum of the costs of producing each separately.

As in the single-product case, we model the merger of two suppliers of substitute products (both producing $A$, both producing $B$, or both producing $A$ and $B$ ) by assuming that the merged entity has cost type equal to the minimum of the cost types of the merging suppliers. We model a merged entity that combines a supplier of product $A$ with a supplier of product $B$ as drawing its cost type from the distribution for the sum of the cost types of the two merging suppliers. For a merger that combines a single-product supplier with a multi-product supplier, various approaches can be considered, but we do not focus on this case.

Analogous to the single-product case, we assume that market outcomes correspond to the allocation and payments of the optimal mechanism with objective that is a weighted average of buyer surplus and social surplus, with weight $\beta \in\{0,1\}$ on buyer surplus. Thus, when the type vector is $\mathbf{c}$, quantities traded in the pre-merger market (and analogously for the post-merger market) are determined by the maximizer of the set

$$
\{0\} \cup\left\{V-\Gamma_{\beta, i}\left(c_{i}\right)\right\}_{i \in \mathbb{M}} \cup\left\{v-\gamma_{i}^{A} \Gamma_{\beta, i}\left(c_{i}\right)-\gamma_{j}^{B} \Gamma_{\beta, j}\left(c_{j}\right)\right\}_{i \in \mathbb{M U A}, j \in \mathbb{M U B}, i \neq j}
$$

as follows: If there exists $i \in \mathbb{M}$ such that $V-\Gamma_{\beta, i}\left(c_{i}\right)$ is a maximizer, then $q_{i}^{A}(\mathbf{c})=$ $q_{i}^{B}(\mathbf{c})=1$; otherwise, if there exist $i \in \mathbb{M} \cup \mathbb{A}$ and $j \in \mathbb{M} \cup \mathbb{B}$ with $i \neq j$ such that $v-\gamma_{i}^{A} \Gamma_{\beta, i}\left(c_{i}\right)-\gamma_{j}^{B} \Gamma_{\beta, j}\left(c_{j}\right)$ is a maximizer, then $q_{i}^{A}(\mathbf{c})=q_{j}^{B}(\mathbf{c})=1$; otherwise, zero is a
maximizer and there is no trade. Expected buyer surplus is

$$
E_{\mathbf{c}}\left[\begin{array}{c}
\sum_{i \in \mathbb{M}} q_{i}^{A}(\mathbf{c}) q_{i}^{B}(\mathbf{c})\left(V-\min \left\{\bar{c}, \Gamma_{1, i}\left(c_{i}\right)\right\}\right) \\
+\sum_{i \in \mathbb{M U A}, j \in \mathbb{M} \cup \mathbb{B}, i \neq j} q_{i}^{A}(\mathbf{c}) q_{j}^{B}(\mathbf{c})\left(v-\gamma_{i}^{A} \min \left\{\bar{c}, \Gamma_{1, i}\left(c_{i}\right)\right\}-\gamma_{j}^{B} \min \left\{\bar{c}, \Gamma_{1, j}\left(c_{j}\right)\right\}\right)
\end{array}\right] .
$$

The allocation is monotone in the sense that, focusing on a direct implementation of the mechanism, if reporting type $c_{i}$ causes supplier $i$ to supply both $A$ and $B$, then for all $\hat{c}_{i}<c_{i}$, reporting $\hat{c}_{i}$ also causes $i$ to supply both $A$ and $B$; and if reporting type $c_{i}$ causes supplier $i$ to supply only one product, then for all $\hat{c}_{i}<c_{i}$, reporting $\hat{c}_{i}$ causes $i$ to supply either one or both products. If reporting type $c_{i}$ causes supplier $i$ not to trade, then for all $\hat{c}_{i}>c_{i}$, reporting $\hat{c}_{i}$ also causes supplier $i$ not to trade.

In the dominant strategy implementation for the multi-product setup, payments to suppliers are defined by multiple threshold cost types as described in the Appendix.

Lemma 4 There exist threshold cost types that define the dominant-strategy implementation of the optimal direct mechanism with objective (4), subject to incentive compatibility and individual rationality.

Proof. See the Appendix.

Using the characterization of outcomes in the multi-product setup as described above, we can analyze a merger in that setup. We begin by considering the effects of a buyer preference for one-stop shopping.

Consider a merger of two multi-product suppliers. An increase in the value of one-stop shopping, that is, an increase in $V$ for a given $v$, increases set of cost types such that one of the merging suppliers supplies both products to the buyer and the other defines the second-best option for the buyer. Thus, an increase in the value of one-stop shopping exacerbates the harm associated with a merger of two multi-product suppliers.

We summarize with the following proposition:
Proposition 12 An increase in the value of one-stop shopping exacerbates the adverse effects of a merger of multi-product suppliers on buyer surplus $\left(\Delta B S^{0}\right.$ and $\Delta B S^{1}$ decrease in $V$ ).

## Merger of suppliers of complements

Our framework also allows us to consider the merger of supplier 1 producing only product $A$ with supplier 2 producing only product $B$, which is a merger of suppliers of complements. As mentioned, these complementary products could be two inputs used together
by the buyer, or inputs in different geographic locations for a buyer that demands coverage for both locations, or vertically related products, such as a product and its distribution.

To illustrate effects, consider the case of $n=2$. Let $\bar{\Gamma}_{\beta}$ be the weighted virtual cost function for a supplier who draws her cost type from the distribution that is the convolution of $G_{1}$ and $G_{2}$. Even if the virtual cost functions for suppliers 1 and 2 are bounded on $[\underline{c}, \bar{c}], \bar{\Gamma}_{1}$ is necessarily unbounded on $[2 \underline{c}, 2 \bar{c}] .{ }^{47}$

If $v \geq \Gamma_{\beta, 1}(\bar{c})+\Gamma_{\beta, 2}(\bar{c}) \geq 2 \bar{c}$, then in the pre-merger market the buyer always purchases and pays $2 \bar{c}$. After the merger, the buyer purchases if $\bar{\Gamma}_{\beta}\left(c_{1}+c_{2}\right) \leq V$, in which case the buyer pays $\bar{\Gamma}_{\beta}^{-1}(V)$, which is equal to $2 \bar{c}$ when $\beta=0$ and is less than $2 \bar{c}$ when $\beta=1$ because $V<\bar{\Gamma}_{1}(2 \bar{c})=\infty$. Thus, the buyer's quantity and payment are not affected by the merger when $\beta=0$, but he benefits from one-stop shopping if $V>v$.

With buyer power, by revealed preference, the buyer's expected surplus following the merger is greater than if the buyer committed to always purchase from the merged entity at price $2 \bar{c}$, which would be incentive compatible and generate the same surplus as in the case of no merger. Thus, with buyer power, the buyer's expected surplus increases as a result of a merger, and more so the greater is his value for one-stop shopping.

Thus, we have the following result:
Proposition 13 Assuming $v \geq \Gamma_{\beta, 1}(\bar{c})+\Gamma_{\beta, 2}(\bar{c})$, a merger of monopoly suppliers of complementary products increases a buyer's expected surplus if the buyer has buyer power or has a positive value for one-stop shopping, and the effect on expected buyer surplus is increasing in the value of one-stop shopping.

This result is consistent with the usual intuition that the merger of suppliers of complementary products typically produces benefits for the buyer.

## 5 Conclusion

We provide a framework for analyzing markets with buyer power. The framework captures procurement-based price formation and explicitly incorporates buyer power. It allows us to consider incentives for merger, entry, and innovation, and it permits an analysis of merger efficiencies, and a quantification of coordinated effects. An extension to multiproduct suppliers enables us to address cross-market effects.

We show that powerful buyers may deter harmful mergers that buyers without buyer power cannot deter. However, even powerful buyers are harmed by a merger without cost synergies. Because powerful buyers take defensive measures against a merged firm by

[^24]making more aggressive take-it-or-leave-it offers and discriminating against it in the procurement, the benefits of perfect (undetected and unsuspected) collusion among suppliers are larger than the benefits from merging in the presence of buyer power. In this sense, powerful buyers may deter mergers and invite collusion.

Moreover, buyer power does not mitigate all, and exacerbates some, merger concerns. In particular, with symmetric suppliers pre merger, social surplus decreases with the merger under fairly general conditions if the buyer is powerful and is not affected without buyer power. Buyer power also affects the ability of entry and cost efficiencies to remedy merger effects. Entry cannot be relied upon as a remedy without buyer power. Cost synergies can eliminate merger harm, but they can also render an otherwise profitable merger unprofitable. Last but not least, with symmetric suppliers pre merger, a merger increases rivals' incentives to invest in cost reduction if and only if the buyer is powerful. Without buyer power, the merged entity's incentives to invest increase with the merger. Whether its incentives to invest are stronger or weaker post merger when the buyer is powerful depends on the specific details of the setup.

## A Appendix: Proofs

Proof of Lemma 1. Focus on the pre-merger market. Analogous arguments apply to the post-merger market. Standard mechanism design arguments imply that (4) is maximized, subject to incentive compatibility and individual rationality, if trade occurs between the buyer and the supplier with the lowest weighted virtual cost if and only if that weighted virtual cost is less than or equal to $v$. If no supplier has a weighted virtual cost less than or equal to $v$, then there is no trade. Because suppliers remain active up to their weighted virtual costs in the essentially unique equilibrium in non-weakly dominated strategies of the procurement-plus-bargaining procedure, the procurement-plus-bargaining procedure implements this outcome.

Proof of Proposition 1. If $\beta=0$, trade occurs pre-merger if and only if $\min _{i \in\{1, \ldots, n\}} c_{i} \leq v$, which is also the condition for trade post-merger. Thus, the probability of trade is not affected by a merger when $\beta=0$. However, when $\beta=0$, the expected payment by the buyer increases as a result of the merger because the elimination of a supplier increases the expected threshold payments: from a pre-merger payment conditional on trade of $\min \left\{v, 2^{\text {nd }}\left\{c_{1}, \ldots, c_{n}\right\}\right\}$ to a post-merger payment conditional on trade of $\min \left\{v, 2^{\text {nd }}\left\{\min \left\{c_{1}, c_{2}\right\}, c_{3}, \ldots, c_{n}, \bar{c}\right\}\right.$, where the operator $2^{\text {nd }}$ selects the second-lowest element of a set. Because quantity is unchanged but expected payment is greater, expected buyer surplus decreases as a result of a merger.

Proof of Proposition 3. The results that $0=\Delta Q^{0} \geq \Delta Q^{1}$ and $0=\Delta S S^{0}>\Delta S S^{1}$ and that $\Delta B S^{1}$ and $\Delta B S^{0}$ are negative follow from arguments made in the text. We show that $\Delta B S^{1}>\Delta B S^{0}$. To do so, it is useful to consider a range of buyer power $\beta \in[0,1]$. Suppose temporarily that a buyer with buyer power $\beta$ uses virtual cost $\Gamma_{\beta}$ to evaluate all of the pre-merger and post-merger suppliers (in our model the buyer with buyer power uses $\hat{\Gamma}$ to evaluate the merged entity). Let $x \equiv \min \left\{c_{3}, \ldots, c_{n}, \bar{c}\right\}$ and $F$ be the distribution of $x$. For such a buyer, the probability of trade is not affected by the merger, and the payment is affected only when $\min \left\{c_{1}, c_{2}\right\}<\max \left\{c_{1}, c_{2}\right\}<\min \left\{\Gamma_{\beta}^{-1}(v), x\right\}$, in which case the buyer pays $\min \left\{\Gamma_{\beta}^{-1}(v), x\right\}$ instead of $c_{2}$. Thus, the expected change in buyer surplus as a result of a merger is

$$
2 E_{\mathbf{c}}\left[c_{2}-\min \left\{\Gamma_{\beta}^{-1}(v), x\right\} \mid c_{1}<c_{2}<\min \left\{\Gamma_{\beta}^{-1}(v), x\right\}\right] \operatorname{Pr}\left(c_{1}<c_{2}<\min \left\{\Gamma_{\beta}^{-1}(v), x\right\}\right),
$$

which we can write as

$$
\begin{equation*}
2 \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\min \left\{\Gamma_{\beta}^{-1}(v), x\right\}} \int_{\underline{c}}^{c_{2}}\left(c_{2}-\min \left\{\Gamma_{\beta}^{-1}(v), x\right\}\right) d G\left(c_{1}\right) d G\left(c_{2}\right) d F(x) . \tag{11}
\end{equation*}
$$

Differentiating (11) with respect to $\beta$, we get:

$$
-2 \frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \int_{\Gamma_{\beta}^{-1}(v)}^{\bar{c}} \int_{\underline{c}}^{\Gamma_{\beta}^{-1}(v)} \int_{\underline{c}}^{c_{2}} d G\left(c_{1}\right) d G\left(c_{2}\right) d F(x) \geq 0
$$

where the inequality follows because $\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \leq 0$. Thus, for such a buyer, the change is surplus as a result of a merger is weakly greater (closer to zero) when $\beta=1$ than when $\beta=0$. This implies that for a buyer who uses virtual cost $\hat{\Gamma}_{1}$ for the merged entity, $\Delta B S_{1}>\Delta B S_{0}$.

Proof of Proposition 4: We first consider the case with $\beta=0$. Let $L_{n-2}(c) \equiv 1-(1-$ $G(c))^{n-2}$ be the distribution of the lowest cost draw of the $n-2$ rivals of the merging suppliers. Denoting by $c \equiv \min \left\{c_{1}, c_{2}\right\}$ the lowest cost draw of the merging suppliers, the buyer's expected payoff pre-merger is

$$
\begin{aligned}
U_{p r e}^{0} & =\int_{\underline{c}}^{\min \{v, \bar{c}\}}\left(1-L_{n-2}(c)\right)(v-\Gamma(c)) d \hat{G}(c) \\
& +\int_{\underline{c}}^{\min \{v, \bar{c}\}}\left(1-\hat{G}\left(c_{(1)}\right)\right)\left(v-\Gamma\left(c_{(1)}\right)\right) d L_{n-2}\left(c_{(1)}\right) .
\end{aligned}
$$

Post-merger, the buyer's expected payoff is

$$
\begin{aligned}
U_{\text {post }}^{0} & =\int_{\underline{c}}^{\min \{v, \bar{c}\}}\left(1-L_{n-2}(c)\right)(v-\hat{\Gamma}(c)) d \hat{G}(c) \\
& +\int_{\underline{c}}^{\min \{v, \bar{c}\}}\left(1-\hat{G}\left(c_{(1)}\right)\right)\left(v-\Gamma\left(c_{(1)}\right) d L_{n-2}\left(c_{(1)}\right) .\right.
\end{aligned}
$$

Taking the difference, we obtain

$$
\begin{aligned}
\Delta B S^{0} & \equiv U_{\text {post }}^{0}-U_{\text {pre }}^{0} \\
& =\int_{\underline{c}}^{\min \{v, \bar{c}\}}\left(1-L_{n-2}(c)\right)(\Gamma(c)-\hat{\Gamma}(c)) d \hat{G}(c) .
\end{aligned}
$$

Because $\Gamma(c)-\hat{\Gamma}(c)<0$ for any $c>\underline{c}$, we have $\Delta B S^{0}<0$. Because $1-L_{n-2}(c)$ decreases in $n, \Delta B S^{0}$ increases in $n$.

For $\beta=1$, we have pre-merger

$$
\begin{aligned}
U_{\text {pre }}^{1} & =\int_{\underline{c}}^{\bar{c}}\left(1-L_{n-2}(c)\right) \max \{v-\Gamma(c), 0\} d \hat{G}(c) \\
& +\int_{\underline{c}}^{\bar{c}}\left(1-\hat{G}\left(c_{(1)}\right)\right) \max \left\{v-\Gamma\left(c_{(1)}\right), 0\right\} d L_{n-2}\left(c_{(1)}\right),
\end{aligned}
$$

and post-merger

$$
\begin{aligned}
U_{\text {post }}^{1} & =\int_{\underline{c}}^{\bar{c}}\left(1-L_{n-2}\left(\Gamma^{-1}(\hat{\Gamma}(c))\right)\right) \max \{v-\hat{\Gamma}(c), 0\} d \hat{G}(c) \\
& +\int_{\underline{c}}^{\bar{c}}\left(1-\hat{G}\left(\hat{\Gamma}^{-1}\left(\Gamma\left(c_{(1)}\right)\right)\right)\right) \max \left\{v-\Gamma\left(c_{(1)}\right), 0\right\} d L_{n-2}\left(c_{(1)}\right) .
\end{aligned}
$$

Taking the difference, we obtain

$$
\begin{aligned}
\Delta B S^{1} & \equiv U_{\text {post }}^{1}-U_{\text {pre }}^{1} \\
& =\int_{\underline{c}}^{\bar{c}}\left(1-L_{n-2}(c)\right)[\max \{v-\hat{\Gamma}(c), 0\}-\max \{v-\Gamma(c), 0\}] d \hat{G}(c) \\
& +\int_{\underline{c}}^{\bar{c}}\left(L_{n-2}(c)-L_{n-2}\left(\Gamma^{-1}(\min \{\hat{\Gamma}(c), \bar{c}\})\right)\right) \max \{v-\hat{\Gamma}(c), 0\} d \hat{G}(c) \\
& +\int_{\underline{c}}^{\bar{c}}\left(\hat{G}\left(c_{(1)}\right)-\hat{G}\left(\hat{\Gamma}^{-1}\left(\Gamma\left(c_{(1)}\right)\right)\right)\right) \max \left\{v-\Gamma\left(c_{(1)}\right), 0\right\} d L_{n-2}\left(c_{(1)}\right) .
\end{aligned}
$$

The first line is, as is the equivalent expression for $\beta=0$, negative and increasing in $n$ because $1-L_{n-2}(c)$ decreases in $n$.

The second and third line captures differences in the allocation rule pre merger and post merger. The effect of increase in $n$ on the last term can readily be signed if $\left(\hat{G}\left(c_{(1)}\right)-\right.$ $\left.\hat{G}\left(\hat{\Gamma}^{-1}\left(\Gamma\left(c_{(1)}\right)\right)\right)\right) \max \left\{v-\Gamma\left(c_{(1)}\right), 0\right\}$, which is positive, decreases in $c_{(1)}$ (which is condition (ii)) because an increase in $n$ induces a stochastic dominance shift of $L_{n-2}$, implying that the last term increases in $n$. Condition (i) implies that the expression in the second line is nondecreasing in $n$. Thus, under conditions (i) and (ii), $\Delta B S^{1}$ increases in $n$.

Proof of Proposition 5. The result that with buyer power and $v$ sufficiently large, a merger to monopoly is profitable follows from Figure 3(b) because as $v$ goes to infinity, $p$ and $\hat{p}$ go to $\bar{c}$, leaving only the region in which the merging suppliers gain from the merger.

We next show that for $\beta=1$ and $v$ sufficiently close to $\underline{c}$, assuming $g(\underline{c})>0$, a merger to monopoly $(n=2)$ is not profitable. It is useful to begin with the properties of how the buyer's optimal take-it-or-leave-it offers vary with $v$. Let $p(v)$ be such that $\Gamma(p(v))=v$,
and let $\hat{p}(v)$ be such that $\hat{\Gamma}(\hat{p}(v))=v$. For any $v \in(\underline{c}, \Gamma(\bar{c}))$, we have

$$
\hat{p}(v)<p(v),
$$

while for $v=\underline{c}$ we have $\hat{p}(\underline{c})=p(\underline{c})=\underline{c}$. Let $\sigma(c)=G(c) / g(c)$. Observe that $\sigma(\underline{c})=0$,

$$
\sigma^{\prime}(\underline{c})=1-\left.\sigma(c) \frac{g^{\prime}(c)}{g(c)}\right|_{c=\underline{c}}=1,
$$

and

$$
\sigma^{\prime \prime}(\underline{c})=\left.\left(-\sigma^{\prime}(c) \frac{g^{\prime}(c)}{g(c)}-\sigma(c) \frac{g^{\prime \prime}(c) g(c)-\left(g^{\prime}(c)\right)^{2}}{g^{2}(c)}\right)\right|_{c=\underline{c}}=-\frac{g^{\prime}(\underline{c})}{g(\underline{c})} .
$$

By the definition of $\sigma, p(v)$ satisfies

$$
p(v)+\sigma(p(v))=v .
$$

Differentiating we get

$$
p^{\prime}(\underline{c})=\left.\frac{1}{1+\sigma^{\prime}(p(v))}\right|_{v=\underline{c}}=\frac{1}{2}
$$

and differentiating once more we get

$$
p^{\prime \prime}(\underline{c})=-\left.\frac{\sigma^{\prime \prime}(p(v))\left(p^{\prime}(v)\right)^{2}}{1+\sigma^{\prime}(p(v))}\right|_{v=\underline{c}}=-\frac{\sigma^{\prime \prime}(\underline{c})}{(2)^{3}}=\frac{1}{8} \frac{g^{\prime}(\underline{c})}{g(\underline{c})} .
$$

Letting $\hat{\sigma}(c)=\sigma(c) \frac{2-G(c)}{2(1-G(c))}$, we have

$$
\hat{p}^{\prime}(\underline{c})=\left.\frac{1}{1+\hat{\sigma}^{\prime}(p(v))}\right|_{v=\underline{c}}=\frac{1}{2}=p^{\prime}(\underline{c}),
$$

and differentiating once more we have

$$
\hat{p}^{\prime \prime}(\underline{c})=-\left.\frac{\hat{\sigma}^{\prime \prime}(\hat{p}(v))\left(\hat{p}^{\prime}(v)\right)^{2}}{1+\hat{\sigma}^{\prime}(\hat{p}(v))}\right|_{v=\underline{c}}=-\frac{\hat{\sigma}^{\prime \prime}(\underline{c})}{(2)^{3}}=\frac{1}{8}\left(\frac{g^{\prime}(\underline{c})}{g(\underline{c})}-g(\underline{c})\right) .
$$

Using

$$
\hat{\sigma}^{\prime}(\underline{c})=\left.\left(\sigma^{\prime}(c) \frac{2-G(c)}{2(1-G(c))}+\sigma(c) \frac{2 g(c)}{(2(1-G(c)))^{2}}\right)\right|_{c=\underline{c}}=\sigma^{\prime}(\underline{c})=1 .
$$

and

$$
\left[\frac{2-G(c)}{2(1-G(c))}\right]^{\prime}=\frac{2 g(c)}{(2(1-G(c)))^{2}},
$$

we obtain

$$
\begin{aligned}
\hat{\sigma}^{\prime \prime}(\underline{c}) & =\left.\left\{\sigma^{\prime \prime}(c) \frac{2-G(c)}{2(1-G(c))}+2 \sigma^{\prime}(c) \frac{2 g(c)}{(2(1-G(c)))^{2}}+\sigma(c) \frac{d}{d c}\left[\frac{2 g(c)}{(2(1-G(c)))^{2}}\right]\right\}\right|_{c=\underline{c}}, \\
& =\sigma^{\prime \prime}(\underline{c})+g(\underline{c}) .
\end{aligned}
$$

Consequently, we have

$$
\begin{equation*}
\hat{p}^{\prime \prime}(\underline{c})=\frac{1}{8}\left[-\sigma^{\prime \prime}(\underline{c})-g(\underline{c})\right]=\frac{1}{8}\left[\frac{g^{\prime}(\underline{c})-g(\underline{c})^{2}}{g(\underline{c})}\right]=p^{\prime \prime}(\underline{c})-\frac{1}{8} g(\underline{c})<p^{\prime \prime}(\underline{c}) . \tag{12}
\end{equation*}
$$

Given a take-it-or-leave-it offer $p$ before the merger, the joint expected profit of the two merging suppliers $\Pi(p)$ is

$$
\Pi(p)=\int_{\underline{c}}^{p}(p-c) d \hat{G}(c)-2 \int_{\underline{c}}^{p} \int_{c}^{p}(p-y) d G(y) d G(c) .
$$

The first term is the joint profit of the two suppliers if they did not compete against each other and always were always paid the price $p$, producing at the lowest cost. From that we have to subtract the lost profits due to the competing bids, in which the winner is paid $y \leq p$ instead of $p$. Changing the order of integration in the double integral, we obtain

$$
2 \int_{\underline{c}}^{p} \int_{c}^{p}(p-y) d G(y) d G(c)=2 \int_{\underline{c}}^{p}(p-y) G(y) d G(y) \equiv h(p) .
$$

Consequently,

$$
\Pi(p)=\int_{\underline{c}}^{p}(p-c) d \hat{G}(c)-h(p)
$$

Observe that $h^{\prime}(p)=2 \int_{\underline{c}}^{p} G(y) d G(y), h^{\prime \prime}(p)=2 G(p) g(p)$ and $h^{\prime \prime \prime}(p)=2 G(p) g^{\prime 2}$. Evaluated at $p=\underline{c}$, we thus have $h^{\prime}(\underline{c})=0=h^{\prime \prime}(\underline{c})$ and $h^{\prime \prime \prime}(\underline{c})=2 g(\underline{c})^{2}$.

Given a take-it-or-leave-it offer $p$, the post-merger expected profit is

$$
\hat{\Pi}(p)=\int_{\underline{c}}^{p}(p-c) d \hat{G}(c) .
$$

Observe that, evaluated at $p=\underline{c}$, we have $\Pi^{\prime}(\underline{c})=0=\hat{\Pi}^{\prime}(\underline{c})$ and $\Pi^{\prime \prime}(\underline{c})=2 g(\underline{c})=\hat{\Pi}^{\prime \prime}(\underline{c})$. Furthermore, $\Pi^{\prime \prime \prime}(\underline{c})=2 g^{\prime}(\underline{c})-4 g(\underline{c})^{2}$ and $\hat{\Pi}^{\prime \prime \prime}(\underline{c})=2 g^{\prime}(\underline{c})-2 g(\underline{c})^{2}$.

Define next $f(v) \equiv \Pi(p(v))$ and $\hat{f}(v) \equiv \hat{\Pi}(\hat{p}(v))$. We have

$$
\begin{aligned}
f^{\prime}(v) & =\Pi^{\prime}(p(v)) p^{\prime}(v) \\
f^{\prime \prime}(v) & =\Pi^{\prime \prime}(p(v))\left(p^{\prime 2}+\Pi^{\prime}(p(v)) p^{\prime \prime}(v)\right. \\
f^{\prime \prime \prime}(v) & =\Pi^{\prime \prime \prime}(p(v))\left(p^{\prime 3}+3 \Pi^{\prime \prime}(p(v)) p^{\prime}(v) p^{\prime \prime}(v)+\Pi^{\prime}(p(v)) p^{\prime \prime \prime}(v)\right.
\end{aligned}
$$

and analogously for $\hat{f}(v)$ with all $\Pi$ 's and $p$ 's and their derivatives replaced by $\hat{\Pi}$ 's and $\hat{p}$ 's.

Because $\Pi^{\prime}(\underline{c})=0=\hat{\Pi}^{\prime}(\underline{c})$ and $\Pi^{\prime \prime}(\underline{c})=2 g(\underline{c})=\hat{\Pi}^{\prime \prime}(\underline{c})$ and $p^{\prime}(\underline{c})=\hat{p}^{\prime}(\underline{c})=1 / 2$, we have $f^{\prime}(\underline{c})=0=\hat{f}^{\prime}(\underline{c})$ and $f^{\prime \prime}(\underline{c})=g(\underline{c}) / 2=\hat{f}^{\prime \prime}(\underline{c})$. Turning to the third derivatives, we obtain

$$
f^{\prime \prime \prime}(\underline{c})=\frac{1}{4}\left(g^{\prime}(\underline{c})-2 g(\underline{c})^{2}\right)+\frac{3}{8} g^{\prime}(\underline{c})
$$

and

$$
\hat{f}^{\prime \prime \prime}(\underline{c})=\frac{1}{4}\left(g^{\prime}(\underline{c})-g(\underline{c})^{2}\right)+\frac{3}{8} g^{\prime}(\underline{c})-\frac{3}{8} g(\underline{c})^{2},
$$

whence we conclude

$$
f^{\prime \prime \prime}(\underline{c})=\hat{f}^{\prime \prime \prime}(\underline{c})+\frac{1}{8} g(\underline{c})^{2}>\hat{f}^{\prime \prime \prime}(\underline{c}),
$$

where the inequality holds because $g(\underline{c})>0$ by assumption. Therefore, at $v=\underline{c}, \Pi(p(v))$ increases faster in $v$ than $\hat{\Pi}(\hat{p}(v))$. By continuity, $\Pi(p(v))>\hat{\Pi}(\hat{p}(v))$ holds for some $v$ in the neighborhood of but larger than $\underline{c}$. This proves the result that merger to monopoly is not profitable for $v$ sufficiently small.

The last part of the proposition holds by similar logic. We sketch that proof. Denote by $c_{(1)}$ the lowest cost draw by any of the nonmerging suppliers. When $n>2$, rather than $v$ being the buyer's maximum willingness to pay to one of the merging suppliers, it is instead is $c_{(1)}$ pre-merger and $\hat{\Gamma}^{-1}\left(c_{(1)}\right)$ post-merger. As $n$ goes to infinity, $c_{(1)} \rightarrow \underline{c}$ almost surely. The relevant take-it-or-leave-it offers $p$ and $\hat{p}$ in the proof above become $p\left(c_{(1)}\right)=c_{(1)}$ because there is no discrimination pre-merger, where $p^{\prime}\left(c_{(1)}\right)=1$, and $\hat{p}\left(c_{(1)}\right)$, whose derivative at $c_{(1)}=\underline{c}$ is $\left.\hat{p}^{\prime}\left(c_{(1)}\right)\right|_{c_{(1)}=\underline{c}}=1 / 2$. Thus, using the notation above, we have $f^{\prime \prime}(\underline{c})=2 g(\underline{c})>g(\underline{c})=\hat{f}^{\prime \prime}(\underline{c})$.

Proof of Proposition 9. Consider first $(1-s) \bar{c} \leq v$ (a sufficient condition for which is obviously $\bar{c} \leq v$ ). Then there will always be trade post merger and the buyer pays $(1-s) \bar{c}$.

Consequently,

$$
\begin{aligned}
\hat{\Pi}(s) & =\int_{0}^{(1-s) \bar{c}}((1-s) \bar{c}-c) \bar{g}(c) d c \\
& =\int_{0}^{(1-s) \bar{c}}(\bar{c}-c /(1-s)) \hat{g}(c /(1-s)) d c .
\end{aligned}
$$

Making the change of variables with $y \equiv c /(1-s)$, which implies $d c=(1-s) d y$, we obtain

$$
\begin{equation*}
\hat{\Pi}(s)=(1-s) \int_{0}^{\bar{c}}(\bar{c}-y) \hat{g}(y) d y=(1-s) \hat{\Pi}(0) \tag{13}
\end{equation*}
$$

implying, for $(1-s) \bar{c} \leq v$,

$$
\frac{\partial \hat{\Pi}(s)}{\partial s}=-\hat{\Pi}(0)<0 \text { and } \hat{\Pi}(1)=0
$$

Assume next that $\bar{c}>v$, which means there is a $\hat{s} \in(0,1)$ such that $(1-s) \bar{c}<v$ if and only of $s>\hat{s} \equiv 1-v / \bar{c}$. For all $s \leq \hat{s}$, the buyer will offer $v$ whenever there is trade. Consequently, for $s \leq \hat{s}$,

$$
\begin{aligned}
\hat{\Pi}(s) & =\int_{0}^{v}(v-c) \bar{g}(c) d c \\
& =\frac{1}{1-s} \int_{0}^{v}(v-c) \hat{g}(c /(1-s)) d c .
\end{aligned}
$$

Making the change of variables $y \equiv c /(1-s)$ again, we obtain

$$
\begin{equation*}
\hat{\Pi}(s)=\int_{0}^{v /(1-s)}(v-(1-s) y) \hat{g}(y) d y \tag{14}
\end{equation*}
$$

whose derivative is

$$
\frac{\partial \hat{\Pi}(s)}{\partial s}=\int_{0}^{v /(1-s)} y \hat{g}(y) d y>0
$$

For $s>\hat{s}$, the analysis is as in the case with $\bar{c} \leq v$, with the result that $\frac{\partial \hat{\Pi}(s)}{\partial s}<0$.

Proof of Lemma 3. The adjusted virtual cost function for a merged entity with synergies, $\check{\Gamma}(c)$, is $^{48}$

$$
\begin{equation*}
\check{\Gamma}(c) \equiv(1-s) \hat{\Gamma}(c /(1-s)) . \tag{15}
\end{equation*}
$$

[^25]Using the definition of the reverse hazard rate and (15), we have

$$
\check{\Gamma}(c)=(1-s) \hat{\Gamma}(c /(1-s))=c+(1-s) \frac{\hat{G}(c /(1-s))}{\hat{g}(c /(1-s))}=c+(1-s) \hat{\sigma}(c /(1-s)) .
$$

Defining $h(s) \equiv(1-s) \hat{\sigma}(c /(1-s))$, we have $\check{\Gamma}(c)=c+h(s)$ and so

$$
\frac{\partial \check{\Gamma}(c)}{\partial s}=h^{\prime}(s)=-\left(\hat{\sigma}(c /(1-s))+\frac{c}{1-s} \hat{\sigma}^{\prime}(c /(1-s))\right) .
$$

Thus, the sign of $\frac{\partial \check{\Gamma}(c)}{\partial s}$ is the same as the sign of $-\left[\hat{\sigma}(p)+p \hat{\sigma}^{\prime}(p)\right]=-\hat{\varepsilon}^{\prime}(p)$ with $p=$ $c /(1-s)$. An increase (decrease) of the weighted virtual cost function implies a decrease (increase) of the optimal take-it-or-leave-it offer.

Proof of Proposition 10: Assume first that $v<\hat{\Gamma}(\bar{c})$.
Case (iii): Because $\hat{\varepsilon}^{\prime}(c)>0$, we know that $\hat{\Gamma}(\bar{c})<\infty$ and, from Lemma 3, that there is a $\hat{s} \in(0,1)$ such that $p^{*}(s)$ is increasing for $s \leq \hat{s}$. At $s=\hat{s}$, we have $(1-\hat{s}) \hat{\Gamma}(\bar{c})=v$. For $s>\hat{s}$, the optimal take-it-or-leave-it offer is $p^{*}(s)=(1-s) \bar{c}$, which is linear in $s$ and implies $p^{*}(1)=0$.
Case (ii): If $\hat{\varepsilon}^{\prime}(c)<0$ and $\lim _{c \rightarrow \bar{c}} \hat{\Gamma}(c)=\infty$, then, from Lemma 3, we know that $p^{*}(s)$ is such that it decreases with $s$ and satisfies $(1-s) \hat{\Gamma}(p(s) /(1-s))=v$, which implies $p^{*}(1)=0$. If $\lim _{c \rightarrow \bar{c}} \hat{\Gamma}(c)<\infty$, then the upper bound $(1-s) \bar{c}$ may or may not become binding at some value of $s$. Either way, $p^{*}(s)$ is decreasing in $s$ and $p^{*}(1)=0$.
Case (i): If $\hat{\varepsilon}^{\prime}(c)=0$, the same logic as in Case (i) applies except that $p^{*}(s)$ does not vary with $s$ for $s \leq \hat{s}$.

If $v \geq \hat{\Gamma}(\bar{c})$, then the buyer's take-it-or-leave-it offer is $p^{*}(s)=(1-s) \bar{c}$, which is decreasing in $s$ and implies $p^{*}(1)=0$.

Proof of Proposition 11. We consider $v$ sufficiently large that

$$
\begin{align*}
& E\left[c_{(2)} \mid c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right)  \tag{16}\\
> & E\left[\left.\frac{1}{2} c_{(2)} \right\rvert\, c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right),
\end{align*}
$$

which holds strictly for $v \geq \bar{c}$ and so by continuity for all $v>\hat{v}$ for some $\hat{v}<\bar{c}$.
The suppliers' joint surplus with coordination is (here we write $\hat{\pi}$ and $\pi$ with a $\beta$
is

$$
\check{\Gamma}(x) \equiv x+\frac{\bar{G}(x)}{\bar{g}(x)}=x+\hat{s} \frac{\hat{G}(x / \hat{s})}{\hat{g}(x / \hat{s})}=\hat{s}\left(x / \hat{s}+\frac{\hat{G}(x / \hat{s})}{\hat{g}(x / \hat{s})}\right)=\hat{s} \hat{\Gamma}(x / \hat{s}) .
$$

subscript to emphasize the dependence on buyer power):

$$
\begin{aligned}
\hat{\pi}_{\beta}= & \frac{1}{2} E\left[\Gamma_{\beta}^{-1}(v)-c_{(1)} \mid c_{(1)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)\right) \\
& +\frac{1}{2} E\left[\Gamma_{\beta}^{-1}(v)-c_{(2)} \mid c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(2)}<\Gamma_{\beta}^{-1}(v)\right)
\end{aligned}
$$

and without coordination is

$$
\begin{aligned}
\pi_{\beta}= & E\left[c_{(2)}-c_{(1)} \mid c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right) \\
& +E\left[\Gamma_{\beta}^{-1}(v)-c_{(1)} \mid c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right) .
\end{aligned}
$$

Thus, the gain from coordination is

$$
\begin{aligned}
\hat{\pi}_{\beta}-\pi_{\beta}= & \frac{1}{2} E\left[\Gamma_{\beta}^{-1}(v)-c_{(1)} \mid c_{(1)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)\right) \\
& +\frac{1}{2} E\left[\Gamma_{\beta}^{-1}(v)-c_{(2)} \mid c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(2)}<\Gamma_{\beta}^{-1}(v)\right) \\
& -E\left[c_{(2)}-c_{(1)} \mid c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right) \\
& -E\left[\Gamma_{\beta}^{-1}(v)-c_{(1)} \mid c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right) .
\end{aligned}
$$

Rewriting the above expression, we get

$$
\begin{align*}
& \hat{\pi}_{\beta}-\pi_{\beta}= \\
& E\left[\left.\Gamma_{\beta}^{-1}(v)-c_{(2)}-\frac{c_{(2)}-c_{(1)}}{2} \right\rvert\, c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right)  \tag{17}\\
& +E\left[\left.-\frac{1}{2}\left(\Gamma_{\beta}^{-1}(v)-c_{(1)}\right) \right\rvert\, c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right) .
\end{align*}
$$

It follows that if $\hat{\pi}_{\beta}-\pi_{\beta}>0$, then, replacing $c_{(1)}$ with $c_{(2)}$ in one place in the second line of (17),

$$
\begin{aligned}
& E\left[\left.\Gamma_{\beta}^{-1}(v)-c_{(2)}-\frac{c_{(2)}-c_{(1)}}{2} \right\rvert\, c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right) \\
& +E\left[\left.-\frac{1}{2}\left(\Gamma_{\beta}^{-1}(v)-c_{(2)}\right) \right\rvert\, c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right)>0,
\end{aligned}
$$

which we can rewrite as

$$
\begin{aligned}
& \Gamma_{\beta}^{-1}(v) \frac{1}{2} G\left(\Gamma_{\beta}^{-1}(v)\right)\left(2 G\left(\Gamma_{\beta}^{-1}(v)\right)-1\right) \\
& -E\left[\left.\frac{c_{(2)}}{2}-\frac{c_{(1)}}{2} \right\rvert\, c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right) \\
& +E\left[-c_{(2)} \mid c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right] \operatorname{Pr}\left(c_{(1)}<c_{(2)}<\Gamma_{\beta}^{-1}(v)\right) \\
& +E\left[\left.\frac{1}{2} c_{(2)} \right\rvert\, c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right] \operatorname{Pr}\left(c_{(1)}<\Gamma_{\beta}^{-1}(v)<c_{(2)}\right)>0 .
\end{aligned}
$$

The second line is negative and, if (16) holds, then the third and fourth lines, taken together, are negative. Thus, we have the implication that the first line is positive, which implies

$$
\begin{equation*}
2 G\left(\Gamma_{\beta}^{-1}(v)\right)-1>0 . \tag{18}
\end{equation*}
$$

Now, using (17), we can write $\hat{\pi}_{\beta}-\pi_{\beta}$ as

$$
\int_{\underline{c}}^{\Gamma_{\beta}^{-1}(v)} \int_{\underline{c}}^{c_{2}}\left(\Gamma_{\beta}^{-1}(v)-c_{2}-\frac{c_{2}-c_{1}}{2}\right) g\left(c_{2}\right) g\left(c_{1}\right) d c_{1} d c_{2}+\int_{\Gamma_{\beta}^{-1}(v)}^{\bar{c}} \int_{\underline{c}}^{\Gamma_{\beta}^{-1}(v)}\left(-\frac{\Gamma_{\beta}^{-1}(v)-c_{1}}{2}\right) g\left(c_{2}\right) g\left(c_{1}\right) d c_{1} d c_{2} .
$$

Taking the view, for the moment, that $\beta \in[0,1]$ and differentiating with respect to $\beta$, we get

$$
\begin{aligned}
& \frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \int_{\underline{\underline{\alpha}}}^{\Gamma_{\beta}^{-1}(v)}\left(-\frac{\Gamma_{\beta}^{-1}(v)-c_{1}}{2}\right) g\left(\Gamma_{\beta}^{-1}(v)\right) g\left(c_{1}\right) d c_{1}+\int_{\underline{\underline{\beta}}}^{\Gamma_{\beta}^{-1}(v)} \int_{\underline{c}}^{c_{2}} \frac{\Gamma_{\beta}^{-1}(v)}{\partial \beta} g\left(c_{2}\right) g\left(c_{1}\right) d c_{1} d c_{2} \\
& -\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \int_{\underline{\varepsilon}}^{\Gamma_{\beta}^{-1}(v)}\left(-\frac{\Gamma_{\beta}^{-1}(v)-c_{1}}{2}\right) g\left(\Gamma_{\beta}^{-1}(v)\right) g\left(c_{1}\right) d c_{1}+\int_{\Gamma_{\beta}^{-1}(v)}^{\bar{c}} \int_{\underline{\beta}}^{\Gamma_{\beta}^{-1}(v)}-\frac{1}{2} \frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} g\left(c_{2}\right) g\left(c_{1}\right) d c_{1} d c_{2},
\end{aligned}
$$

which we can rewrite as

$$
\begin{aligned}
& \frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \int_{\underline{\underline{c}}}^{\Gamma_{\beta}^{-1}(v)}\left(-\frac{\Gamma_{\beta}^{-1}(v)-c_{1}}{2}\right) g\left(\Gamma_{\beta}^{-1}(v)\right) g\left(c_{1}\right) d c_{1}+\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \int_{\underline{c}}^{\Gamma_{\beta}^{-1}(v)} g\left(c_{2}\right) G\left(c_{2}\right) d c_{2} \\
& -\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \int_{\underline{\underline{~}}}^{\Gamma_{\beta}^{-1}(v)}\left(-\frac{\Gamma_{\beta}^{-1}(v)-c_{1}}{2}\right) g\left(\Gamma_{\beta}^{-1}(v)\right) g\left(c_{1}\right) d c_{1}-\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \frac{1}{2}\left(1-G\left(\Gamma_{\beta}^{-1}(v)\right)\right) G\left(\Gamma_{\beta}^{-1}(v)\right),
\end{aligned}
$$

where two terms cancel, giving us

$$
\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta}\left(\int_{\underline{c}}^{\Gamma_{\beta}^{-1}(v)} g\left(c_{2}\right) G\left(c_{2}\right) d c_{2}-\frac{1}{2}\left(1-G\left(\Gamma_{\beta}^{-1}(v)\right)\right) G\left(\Gamma_{\beta}^{-1}(v)\right)\right) .
$$

Because integration by parts implies that $\int_{\underline{c}}^{\Gamma_{\beta}^{-1}(v)} g\left(c_{2}\right) G\left(c_{2}\right) d c_{2}=\frac{1}{2} G^{2}\left(\Gamma_{\beta}^{-1}(v)\right)$, we can
write this as

$$
\begin{aligned}
& \frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \frac{1}{2} G\left(\Gamma_{\beta}^{-1}(v)\right)\left[G\left(\Gamma_{\beta}^{-1}(v)\right)-\left(1-G\left(\Gamma_{\beta}^{-1}(v)\right)\right)\right] \\
= & -\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta} \frac{1}{2} G\left(\Gamma_{\beta}^{-1}(v)\right)\left(1-2 G\left(\Gamma_{\beta}^{-1}(v)\right)\right),
\end{aligned}
$$

which is negative because $\frac{\partial \Gamma_{\beta}^{-1}(v)}{\partial \beta}<0$ and, by (18), $1-2 G\left(\Gamma_{\beta}^{-1}(v)\right)$ is negative. Thus, $\hat{\pi}_{\beta}-\pi_{\beta}$ is decreasing in $\beta$, which implies that $\hat{\pi}_{1}-\pi_{1}<\hat{\pi}_{0}-\pi_{0}$.

Proof of Lemma 4. Given $\mathbf{c}_{-i}$, possible threshold types for supplier $i$ are $c_{i}^{A B,-}, c_{i}^{A,-}$, $c_{i}^{B,-}, c_{i}^{A B, A}, c_{i}^{A B, B}, c_{i}^{B, A}$, and $c_{i}^{A, B}$, where $c^{X, Y}$ is defined so that for cost types below $c_{i}^{X, Y}$ and above the next lower threshold, supplier $i$ supplies product $X$, and for cost types above $c_{i}^{X, Y}$ and below the next higher threshold, supplier $i$ supplies product $Y$, where "-" denotes the empty set.

For example, if for a given $\mathbf{c}$, supplier $i$ supplies both $A$ and $B$, then

$$
\begin{equation*}
V-\Gamma_{\beta, i}\left(c_{i}\right) \geq \max \left\{0, \max _{\substack{j \in \mathbb{M} \\ j \neq i}}\left(V-\Gamma_{\beta, j}\left(c_{j}\right)\right) \max _{\substack{k \in \mathbb{M} \\ j \in \mathbb{M U A} \\ j \neq k}}\left(v-\gamma_{k}^{A} \Gamma_{\beta, k}\left(c_{k}\right)-\gamma_{j}^{B} \Gamma_{\beta, j}\left(c_{j}\right)\right)\right\} . \tag{19}
\end{equation*}
$$

If the highest element in curly brackets on the right side of (19) does not involve supplier $i$, then there exists a cost type for supplier $i, c_{i}^{A B,-}$, such that if supplier $i$ reports a cost less than $c_{i}^{A B,-}$, supplier $i$ supplies $A$ and $B$, but if supplier $i$ reports a cost greater than $c_{i}^{A B,-}$, supplier $i$ supplies nothing. The cost type $c_{i}^{A B,-}$ is the threshold type for supplier $i$ between supplying both $A$ and $B$ and supplying nothing. However, if the highest element in curly brackets on the right side of (19) involves supplier $i$, then the threshold types must account for that. Fixing the cost types of suppliers other than $i$, as the cost type of supplier $i$ varies, the set of products that supplier $i$ supplies also varies. For example, it may be that for low cost types, supplier $i$ supplies $A$ and $B$, but for intermediate cost types, supplier $i$ supplies only $A$, and for higher cost types, supplier $i$ supplies nothing. In this case, the cost types defining the cutoffs between the regions in type space would be denoted $c_{i}^{A B, A}$ and $c_{i}^{A,-}$. Other threshold types are defined analogously. In all cases, the threshold types for supplier $i$ depend only on the cost types of the other suppliers.

Given the cost vector for the suppliers, the identities of the trading suppliers, and the threshold types for the trading suppliers, payments in the dominant strategy implementation are as shown in Figure 9. As we show, the payments defined in Figure 9 correspond to the dominant-strategy implementation of the optimal mechanism for the multi-product
setup.

| Threshold types | Supplier $i$ 's type | Products $i$ supplies | Payment to supplier $i$ |
| :--- | :--- | :--- | :--- |
| $c_{i}^{A B,-}$ | $c_{i}<c_{i}^{A B,-}$ | $A$ and $B$ | $c_{i}^{A B,-}$ |
| $c_{i}^{A B, A}<c_{i}^{A,-}$ | $c_{i}<c_{i}^{A B, A}$ | $A$ and $B$ | $c_{i}^{A B, A}+\gamma_{i}^{A}\left(c_{i}^{A,-}-c_{i}^{A B, A}\right)$ |
|  | $c_{i} \in\left(c_{i}^{A B, A}, c_{i}^{A,-}\right)$ | $A$ only | $\gamma_{i}^{A} c_{i}^{A,-}$ |
| $c_{i}^{A B, B}<c_{i}^{B,-}$ | $c_{i}<c_{i}^{A B, B}$ | $A$ and $B$ | $c_{i}^{A B, A}+\gamma_{i}^{B}\left(c_{i}^{B,-}-c_{i}^{A B, B}\right)$ |
|  | $c_{i} \in\left(c_{i}^{A B, B}, c_{i}^{B,-}\right)$ | $B$ only | $\gamma_{i}^{B} c_{i}^{B,-}$ |
| $c_{i}^{A B, A}<c_{i}^{A, B}<c_{i}^{B,-}$ | $c_{i}<c_{i}^{A B, A}$ | $A$ and $B$ | $c_{i}^{A B, A}+\gamma_{i}^{A}\left(c_{i}^{A, B}-c_{i}^{A B, A}\right)$ |
|  |  |  | $+\gamma_{i}^{B}\left(c_{i}^{B,-}-c_{i}^{A, B}\right)$ |
|  | $c_{i} \in\left(c_{i}^{A B, A}, c_{i}^{A, B}\right)$ | $A$ only | $\gamma_{i}^{A} c_{i}^{A, B}+\gamma_{i}^{B}\left(c_{i}^{B,-}-c_{i}^{A, B}\right)$ |
|  | $c_{i} \in\left(c_{i}^{A, B}, c_{i}^{B,-}\right)$ | $B$ only | $\gamma_{i}^{B} c_{i}^{B,-}$ |
| $c_{i}^{A B, B}<c_{i}^{B, A}<c_{i}^{A,--}$ | $c_{i}<c_{i}^{A B, B}$ | $A$ and $B$ | $c_{i}^{A B, B}+\gamma_{i}^{B}\left(c_{i}^{B, A}-c_{i}^{A B, B}\right)$ |
|  | $c_{i} \in\left(c_{i}^{A B, B}, c_{i}^{B, A}\right)$ | $B$ only | $+\gamma_{i}^{A}\left(c_{i}^{A,--}-c_{i}^{B, A}\right)$ |
|  | $c_{i} \in\left(c_{i}^{B, A}, c_{i}^{A,-}\right)$ | $A$ only | $\gamma_{i}^{B} c_{i}^{B, A}+\gamma_{i}^{A}\left(c_{i}^{A,-}-c_{i}^{B, A}\right)$ |

Figure 9: Threshold types in the multi-product setup with associated payments

To see that dominant strategy incentive compatibility is satisfied, suppose that suppliers other than $i$ report truthfully. If supplier $i$ does not trade when he reports truthfully, then all threshold types for supplier $i$ are less than $c_{i}$, and so any report that results in supplier $i$ trading gives supplier $i$ a payment that is less than $c_{i}$, and so no deviation is profitable. If supplier $i$ does trade when he reports truthfully, then a downward deviation $r_{i}<c_{i}$ only changes supplier $i$ 's payoff if $r_{i}$ is less than a type threshold that is less than $c_{i}$. For example, if supplier $i$ is a multi-product supplier and if $r_{i}<c_{i}^{A B, A}<c_{i}<c_{i}^{A,-}$, then under truthful reporting supplier $i$ supplies $A$, for a payoff of $\gamma_{i}^{A} c_{i}^{A,-}-\gamma_{i}^{A} c_{i}$, but under report $r_{i}$, supplier $i$ supplies $A$ and $B$, for a payoff of

$$
\begin{aligned}
c_{i}^{A B, A}+\gamma_{i}^{A}\left(c_{i}^{A,-}-c_{i}^{A B, A}\right)-c_{i} & =\left(1-\gamma_{i}^{A}\right)\left(c_{i}^{A B, A}-c_{i}\right)+\gamma_{i}^{A} c_{i}^{A,-}-\gamma_{i}^{A} c_{i} \\
& <\gamma_{i}^{A} c_{i}^{A,-}-\gamma_{i}^{A} c_{i},
\end{aligned}
$$

where the inequality uses $c_{i}^{A B, A}-c_{i}<0$ and $\gamma_{i}^{A}<1$ for a multi-product supplier, and so the deviation is not profitable. A similar analysis shows that no other downward deviation
is profitable.
If supplier $i$ reports $r_{i}>c_{i}$, then his payoff is only affected if $r_{i}$ is greater than a type threshold that is greater than $c_{i}$. For example, consider a multi-product supplier $i$, with $c_{i}<c_{i}^{A B, A}<r_{i}<c_{i}^{A,-}$. Then under truthful reporting, supplier $i$ supplies $A$ and $B$ for a payoff of $c_{i}^{A B, A}+\gamma_{i}^{A}\left(c_{i}^{A,-}-c_{i}^{A B, A}\right)-c_{i}$, but under report $r_{i}$, supplier $i$ supplies only $A$ for a payoff of $\gamma_{i}^{A} c_{i}^{A,-}-\gamma_{i}^{A} c_{i}$, which is less for any $c_{i}<c_{i}^{A B, A}$, and so the deviation is not profitable. Similarly, no other upward deviation is profitable.

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[^1]:    ${ }^{1}$ See, e.g., Steptoe (1993) and Carlton and Israel (2011).
    ${ }^{2}$ As stated by Steptoe (1993, p. 494), "Although the strong-buyer defense may be valid in a variety of circumstances, I believe that the courts have sometimes embraced it as if it had talismanic power that cured all doubts about a merger."
    ${ }^{3}$ According to the U.S. Horizontal Merger Guidelines (hereafter U.S. Guidelines) which guide courts in the United States in how to evaluate the potential anticompetitive effects of a merger, "The Agencies consider the possibility that powerful buyers may constrain the ability of the merging parties to raise prices." (U.S. Guidelines, p. 27) Merger guidelines in other jurisdictions provide a similar treatment of buyer power. The European Commission's Guidelines on the Assessment of Horizontal Mergers (hereafter EC Guidelines) discuss the possibility that "buyer power would act as a countervailing factor to an increase in market power resulting from the merger." (para. 11) The Australian Competition and Consumer Commission's Merger Guidelines view "countervailing power" as a competitive constraint that can limit merger harms. (paras. 1.4, 5.3, 7.48)
    ${ }^{4}$ See Kumar et al. (2015, online appendix).

[^2]:    ${ }^{5}$ For an early criticism of complete information model, see, for example, Samuelson (1985, p. 322), who writes: "In pursuit of a preferred agreement, one party may threaten the other and, for credibility's sake, bind himself to carry out the threat some portion of the time. When he does, efficiency fails. Alternatively, the parties may adopt the standard negotiation bluff, insisting on ultrafavorable (and incompatible) terms of agreement. If agents persist in these demands, a mutually beneficial agreement may be lost. Although proponents of the Coase presumption may regard these actions as irrational, it is no less true that such behavior (e.g., strikes, the carrying out of costly threats) frequently occurs. Moreover, it is unrealistic to suppose that the bargaining setting is one of perfect information."

[^3]:    ${ }^{6}$ See also McAfee and McMillan (1987) on how optimal ascending auctions involve discrimination in favor of weaker bidders.

[^4]:    ${ }^{7}$ Dalkir, Logan, and Masson (2000) examine mergers in asymmetric first-price auctions using simu-

[^5]:    ${ }^{12}$ Nocke and Whinston (2010) and Mermelstein, Nocke, Satterthwaite, and Whinston (2015) provide models of sequential mergers in the Cournot setup.
    ${ }^{13}$ In the model of López and Vives (2016), when spillovers are high, the dominant effect of the decrease in competition associated with cross-ownership is to allow investing firms to better appropriate the benefits of their investments, increasing incentives for investment. However, when spillovers are low, the dominant effect of increased cross-ownership is a reduction in output and corresponding reduction in incentives for cost-reducing investment.
    ${ }^{14}$ The assumption of one buyer is conservative from the perspective of providing the scenario in which buyer power is most likely to enable a buyer to remedy merger harms.
    ${ }^{15}$ If one perceives a merger as an acquisition of one firm by another, then the acquiring firm might view itself as more efficient because it draws its cost from $\hat{G}$ rather than $G$; however, because the distribution of the minimum cost across all firms is unchanged, it seems appropriate to refer to this as

[^6]:    an acquisition/merger without cost efficiencies.
    ${ }^{16}$ The assumption of independently distributed types is made on theoretical grounds and captures the notion that there may be a trade-off between profit and efficiency. With correlated types, no matter how small the degree of correlation, profit-maximization and efficiency are in no conflict, as shown by Crémer and McLean $(1985,1988)$, but this requires mechanisms that involve gambles that do not seem plausible or realistic; see, for example, Kosmopoulou and Williams (1998) and Börgers (2015). At the cost of additional notation, the assumption of private value could easily be relaxed by allowing for interdependent values along similar lines as Myerson (1981).
    ${ }^{17}$ We assume that buyer power itself is not affected by a merger among suppliers, which seems natural if buyer power derives from the size and/or sophistication of the buyer, as suggested by the EC Guidelines(para. 65), or from the ability to vertically integrate upstream or sponsor entry, as suggested by the U.S. Guidelines(p. 27). However, the EC Guidelines also raise the possibility that a merger could reduce buyer power "because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative." (EC Guidelines, para. 67)

[^7]:    ${ }^{18}$ As noted by Bulow and Roberts (1989), the virtual cost function can be interpreted as a supplier's marginal cost, treating the (change in the) probability of trade as the (marginal change in) quantity. To see why this is so, interpret $q=G(p)$ as the quantity supplied by a seller who draws his cost from $G$ at price $p$. The inverse supply function of this seller is then $P(q)=G^{-1}(q)$. Consequently, the cost of of procuring quantity $q$ from this seller is $C(q)=q P(q)$, whose derivative is $C^{\prime}(q)=q P^{\prime}(q)+P(q)$. Substituting $P^{\prime}(q)=1 / g\left(G^{-1}(q)\right)$ and $q=G(p)$ then gives $C^{\prime}(q)=\Gamma\left(G^{-1}(q)\right)$.

[^8]:    ${ }^{19}$ Recall that the reverse hazard rate for a distribution $F(c)$ with density $f(c)$ is $f(c) / F(c)$ and that $F(c)$ dominates $G(c)$ in terms of the reverse hazard rate if $f(c) / F(c) \geq g(c) / G(c)$ for all $c \in[\underline{c}, \bar{c}]$.
    ${ }^{20}$ When $\bar{c}<v$ and $\beta=0$, trade always occurs. Apart from the need to identify the supplier with the lowest cost, which can be done via an efficient auction, such as a second-price auction, the model becomes akin to a complete information model. For example, if there were only one supplier, any take-it-or-leave-it offer $p \in[\bar{c}, v]$ would induce the surplus maximizing allocation. To avoid this indeterminacy, we assume that in such a case the buyer's take-it-or-leave-it offer is $p=\bar{c}$. This can be justified on a number of grounds. With multiple suppliers, Bertrand competition between them would reduce the highest price they can ever get to $\bar{c}$. It is also consistent with the idea that the buyer organizes the procurement, with the difference between $\beta=1$ and $\beta=0$ being only whether he can or cannot commit to a binding price offer that sometimes prevents ex post efficient trade from occurring. Technically, our assumption means that the individual rationality constraint always binds for a supplier with the highest possible cost draw.

[^9]:    ${ }^{21}$ Likewise, if $c_{2}=\min \{\mathbf{c}\}$, then supplier 2 wins in the pre-merger market and receives payment $\min \left\{v, c_{1}, c_{3}, . ., c_{n}\right\}$, and the merged entity wins in the post-merger market, but receives the weakly larger payment $\min \left\{v, c_{3}, \ldots, c_{n}, \bar{c}\right\}$.

[^10]:    ${ }^{22}$ Because $\hat{\Gamma}(c)=c+\frac{G(c)}{g(c)} \frac{2-G(c)}{2(1-G(c))}, g(\bar{c})(1-G(\bar{c}))=0$ implies $\lim _{c \rightarrow \bar{c}} \hat{\Gamma}(c)=\infty$ and hence $v<\hat{\Gamma}(\bar{c})$.
    ${ }^{23}$ In this example, the post-merger take-it-or-leave-it offer $\hat{\Gamma}^{-1}(v)$ is the transaction price. For $v$ sufficiently small, $\Gamma^{-1}(v)$ is a good proxy for the transaction price before the merger, but because it is higher, the transaction is both more likely and more expensive pre-merger. For example, with buyer power, if $G$ is uniform on $[0,1]$ and $v=1$, the buyer makes a lower expected payment post merger. Of course, he also trades a smaller quantity and, on balance, is made worse off by the merger.

[^11]:    ${ }^{24}$ A superficial reading of Bulow and Klemperer's result (and proof) might suggest that the buyer can always protect himself from the harmful effects of mergers by replicating the pre-merger bidding game. But only half of this is true - a buyer with buyer power can always replicate the bidding game without the merger and with no buyer power. However, keeping buyer power fixed, the buyer is always hurt by the merger.

[^12]:    ${ }^{25}$ For models of vertical integration in which harm increases with the competitiveness of the industry, see Riordan (1998) and Loertscher and Reisinger (2014).

[^13]:    ${ }^{26}$ For the Bertrand model, mergers have been shown to be more beneficial for outsiders than for the merging firms; see Deneckere and Davidson (1985).

[^14]:    ${ }^{27}$ The proposition shows that this is, for example, the case when $v$ is sufficiently small and $g(\underline{c})$ is positive. However, the latter is merely a technical assumption, which at the cost of having to take additional derivatives could be relaxed.
    ${ }^{28}$ As stated in the U.S. Guidelines (p. 28): "As part of their full assessment of competitive effects, the Agencies consider entry into the relevant market. The prospect of entry into the relevant market will alleviate concerns about adverse competitive effects only if such entry will deter or counteract any competitive effects of concern so the merger will not substantially harm customers." Related to the likely sufficiency of entry to remedy harms, see the U.S. Guidelines, Section 9.3 and EC Guidelines, Section VI. The EC Guidelines (para. 68) state: "When entering a market is sufficiently easy, a merger is unlikely to pose any significant anti-competitive risk. Therefore, entry analysis constitutes an important element of the overall competitive assessment. For entry to be considered a sufficient competitive constraint on the merging parties, it must be shown to be likely, timely and sufficient to deter or defeat any potential anti-competitive effects of the merger."
    ${ }^{29}$ See the U.S. Guidelines, Section 9.2 and EC Guidelines, Section VI.

[^15]:    ${ }^{30}$ Recall that $\Gamma_{\beta}^{-1}(z)$ is defined to be $\bar{c}$ for $z>\Gamma_{\beta}(\bar{c})$, so for such values of $z, L(z)=1$ and $L^{\prime}(z)=0$.

[^16]:    ${ }^{33}$ The EC Guidelines state that, to be considered, cost efficiencies must be "verifiable." (EC Guidelines, para. 78) The U.S. Guidelines, p. 30 state that "it is incumbent upon the merging firms to substantiate efficiency claims so that the Agencies can verify by reasonable means the likelihood and magnitude of each asserted efficiency...."

[^17]:    ${ }^{34}$ To see this, let $\hat{q}(p) \equiv \hat{G}(p)$ be the quantity supplied given cost distribution $\hat{G}$ and price $p$. The price elasticity of supply is then $\hat{\varepsilon}(p)=\hat{q}^{\prime}(p) p / \hat{q}(p)=\hat{g}(p) p / \hat{G}(p)=p / \hat{\sigma}(p)$.

[^18]:    ${ }^{35}$ To see that this function exhibits a constant price elasticity of supply, notice that it implies $\sigma(p)=$ $p / x$ and thus $\varepsilon(p)=x$. Notice also that if $\hat{G}(c)=c^{x}$ is the cost distribution (not accounting for cost efficiencies) after the merger of two symmetric suppliers, the pre-merger distribution of the two suppliers is $G(c)=1-\sqrt{1-c^{x}}$. The distribution $G(c)=1-\sqrt{1-c^{x}}$ fails to exhibit a monotone virtual cost function $\Gamma^{1}(c)$ for $c$ sufficiently large ( $c$ larger than $2^{\frac{1}{x}}\left(\frac{-x^{2}+\sqrt{2} \sqrt{x^{4}+x^{3}}+x+2}{x^{2}+4 x+4}\right)^{\frac{1}{x}}$ ); however, this non-monotonicity does not pose a problem if the buyer's value $v$ is not too large. For any $x \geq 1, v \leq 1$ is sufficient.
    ${ }^{36}$ The derivative of $\hat{\varepsilon}(p)$ is $\hat{\varepsilon}^{\prime}(p)=\frac{(1-a) a p^{x}+x^{x}-1}{\left(x_{1}-x_{2}\right)^{2}}$, which is positive under the assumptions stated.

[^19]:    ${ }^{37}$ To see this, note that $\hat{\varepsilon}^{\prime}(c) \geq 0$ is equivalent to $\hat{\sigma}(c) \geq c \hat{\sigma}^{\prime}(c)$, which implies that $\hat{\sigma}(\bar{c}) \leq \int_{\underline{c}}^{\bar{c}} \hat{\sigma}(c) / c d c<$ $\infty$.

[^20]:    ${ }^{38}$ Virtual dominance holds, for example, when $G_{1}$ is uniform on $[0,1]$ and $G_{2}(c)=c$ for $c \in[0,1 / 4]$ and $G_{2}(c)=1 / 9\left(1+24 x^{2}-16 x^{3}\right)$ for $c \in(1 / 4,1]$, which is depicted as the solid line in Figure $4(\mathrm{~b})$ and has continuous density and increasing virtual cost.

[^21]:    ${ }^{39}$ In this example, when $\beta=1$ (dropping the $\beta$ subscript on $\left.\Gamma_{\beta, i}\right), \Gamma_{1}\left(c_{1}\right)=2 c_{1}, \Gamma_{2}\left(c_{2}\right)=3 c_{2} / 2$, and $\hat{\Gamma}(c)=c+\frac{c+c^{2}-c^{3}}{1+2 c-3 c^{2}}$. For $c_{1} \in(0,1 / 2), \hat{\Gamma}\left(c_{1}\right)<\Gamma_{1}\left(c_{1}\right)$, and for $c_{2} \in(2 / 3,1], \Gamma_{2}\left(c_{2}\right)>\Gamma_{1}(1 / 2)$. Thus, for $c_{1} \in(0,1 / 2)$ and $c_{2}$ sufficiently large, $\hat{\Gamma}\left(\min \left\{c_{1}, c_{2}\right\}\right)=\hat{\Gamma}\left(c_{1}\right)<\Gamma_{1}\left(c_{1}\right)<\min \left\{\Gamma_{1}\left(c_{1}\right), \Gamma_{2}\left(c_{2}\right)\right\}$, contrary to (10).

[^22]:    ${ }^{40}$ See Grout and Sonderegger (2005) on the implications of buyer power for the ability of suppliers to sustain collusion. See Green, Marshall, and Marx (2015) illustrating how explicit collusion can defeat the buyer power of even a small number of large strategic buyers.
    ${ }^{41}$ As noted in the EC Guidelines, a concern is that a merger may increase the likelihood of coordination among firms that increases prices "even without entering into an agreement or resorting to a concerted practice within the meaning of Article 81 of the Treaty." (EC Guidelines, para. 39)
    ${ }^{42}$ On bid rotation schemes, see Hendricks, McAfee, and Williams (2015) and U.S. Department of Justice (2015).
    ${ }^{43}$ One example of a rotation scheme is the electrical contractors conspiracy, which is sometimes referred to as the "phases of the moon" conspiracy. See Richard A. Smith, "The Incredible Electrical Conspiracy," Fortune, April 1961, 63, 132-80, and May 1961, 63, 161-224.

[^23]:    ${ }^{44}$ The U.S. Guidelines (p. 27) also state: "In some cases, a large buyer may be able to strategically undermine coordinated conduct, at least as it pertains to that buyer's needs, by choosing to put up for bid a few large contracts rather than many smaller ones, and by making its procurement decisions opaque to suppliers."
    ${ }^{45}$ The proposed merger was announced in 2014, but the parties ultimately abandoned attempts to merge in the face of opposition from the DOJ: United States of America v. Halliburton Co. and Baker Hughes, Inc., Complaint, April 6, 2016, Case 1:16-cv-00233-UNA, hereafter "DOJ Complaint."
    ${ }^{46}$ The argument that the DOJ put forward in its opposition to the Halliburton-Baker Hughes transaction focused on effects within individual product markets, ignoring possible synergies and complementarities, but then returned to these issues as overarching concerns that amplified competitive concerns created by the transaction, noting that the merging parties "offer similar types of integrated solutions, bundled services, and other multiple-product and service combinations." (DOJ Complaint, p. 30)

[^24]:    ${ }^{47}$ The density of the convolution is $g_{m}(c)=\int_{\underline{c}}^{\bar{c}} g_{1}(x) g_{2}(c-x) d x$, and so $g_{m}(2 \bar{c})=\int_{\bar{c}}^{\bar{c}} g_{1}(x) g_{2}(2 \bar{c}-x) d x=$ 0 , which follows because $g_{2}(2 \bar{c}-x)$ is zero for all $x \in[\underline{c}, \bar{c})$.

[^25]:    ${ }^{48}$ Let $\hat{s} \equiv 1-s$ and note that the distribution of $\hat{s} \min \left\{c_{1}, c_{2}\right\}$ is $\bar{G}(x) \equiv \operatorname{Pr}\left(\hat{s} \min \left\{c_{i}, c_{j}\right\} \leq x\right)=$ $\hat{G}(x / \hat{s})$ defined on $[\hat{s} \underline{c}, \hat{s} \bar{c}]$ with density $\bar{g}(x)=\frac{1}{\hat{s}} \hat{g}(x / \hat{s})$. The weighted virtual cost associated with $\bar{G}(x)$

