Costly Bargaining as a War of Attrition

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Abstract

We study 2-player bargaining over a unit pie, where each player needs to pay a fixed cost in the beginning of every period t, if he wants to stay in the game in period $t+1$ in case a deal has not been reached by the end of t. Whether a player pays this cost is his private information. If only one player stops paying ("drops out") then the other player receives the entire pie. When the frictions are small, every symmetric (not-necessarily-stationary) equilibrium becomes a war of attrition. Specifically, for every $\epsilon > 0$ there exists a cutoff for the aforementioned cost, $c_\epsilon > 0$, and a cutoff for the players' discount factor, $\delta_\epsilon < 1$, such that if the cost is at most c_ϵ and the discount factor is at least δ_ϵ, then in every period in every symmetric equilibrium each player pays the cost with probability at least $1-\epsilon$, the proposer demands for himself a pie-share no smaller than $1-\epsilon$, and the offer is rejected. Consequently, the value of every subgame is at most ϵ. By contrast, with asymmetric strategies every Pareto efficient payoff vector can be approximated in equilibrium, provided that the cost is sufficiently small and that the discount factor is sufficiently close to one.

Key Words: Bargaining; Periodic Costs; War of Attrition.

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1 Introduction

Bargaining is a costly activity. One has to prepare for negotiations, and preparations are costly; sitting at the negotiation table is costly if it is done with the aid of a hired agent, such as a lawyer; and since during the bargaining process the bargainer may be unavailable to pursue other activities, there is an additional cost—the opportunity cost of time. Often these costs have a "periodic nature": one has to prepare for *each round* of bargaining, payments to hired agents are often made on a period-by-period-basis, and the opportunity cost of time is defined per period. In this paper we investigate the implications of adding a periodic-cost parameter to an otherwise standard bargaining game.

Our game is as follows. Two players need to divide a pie. In the beginning of every period there is an initial stage in which each player chooses whether or not to take a costly action ("invest"); the cost of this action is $c > 0$. This choice is unobservable to his opponent. Once these choices are made, bargaining takes place in the second stage of the period, in the following fashion: a proposer is selected at random by Nature, with each player being equally likely to be selected, and then the proposer offers a split of the pie. If the offer is accepted by the responder then it is implemented and the game ends; otherwise, the game moves one period forward, but a player who did not take the costly action drops out of the game. If only one player moves to the next period, he receives the entire pie. If both players move to the next period, the above story repeats itself.

Two central assumptions in our game are that (1) a player who stops investing drops out and receives a null payoff, and (2) once a player drops out, his opponent, if not dropped out himself, receives the entire pie. The interpretation of the cost parameter c that suits assumption (1) best is that of a preparation cost: unless one is prepared for negotiations, one has to leave the table; alternatively, one can "stay at the table," but without any ability to make persuasive arguments or effective moves. Assumption (2) means that in bargaining between a prepared agent and a non-prepared agent, the former can force whatever terms of trade he wishes.

Though the easiest interpretation of the parameter c is that of a preparation cost, it should be noted that in reality the different types of cost that were mentioned in the opening paragraph may be inseparable. For example, if you employ a lawyer in your firm whom you instruct to prepare for negotiations in the next period, then you will bear the costs of preparation today, and may miss some business opportunities in the next period, if these opportunities necessitate the lawyer's availability and readiness. In this scenario the cost is paid for preparations, it is paid for a hired agent, and the opportunity cost of time also plays a role.

In our game, the temptation to try to remain the "last man standing" is hard to resist when the investment cost is small and the discount factor, δ, is close to one. As a result, when the investment cost is small and the discount factor is close to one, the symmetric equilibria of our game are "war of attrition equilibria": each player invests with a high probability in the beginning of every period, demands the entire pie (or almost the entire pie) when he is called by Nature to be the proposer, and rejects the opponent's offer when he is called to be the responder. Consequently, as $(\delta, c) \to (1, 0)$, the payoffs that can be obtained in a symmetric equilibrium converge to zero.

Efficiency is restored in our game if the investment decisions are made public. With publicly observable investments, our game has a unique symmetric subgame perfect equilibrium, which is also its unique stationary subgame perfect equilibrium.[1] Binmore (1987) proved that the random-proposer version of Rubinstein's (1982) game has a unique subgame perfect equilibrium; the equilibrium of the public-investment version of our game is essentially Binmore's equilibrium.

Efficiency can also be restored in our game via asymmetric play. Specifically, we

[1]With observable investments our game is a game of complete information, hence the solution concept for it is subgame perfect equilibrium. In the lion's share of the paper, where investment is non-observable, the solution concept is perfect Bayesian equilibrium.

construct a simple stationary strategy that can approximate any efficient payoff vector in equilibrium, provided that (δ, c) is sufficiently close to $(1, 0)$. The construction is as follows. Suppose that the target payoff vector is (u_1, u_2), with $u_2 \geq u_1$. Take $x \in (\frac{1}{2}, 1)$, arbitrarily close to u_2.[2] We let each player invest with certainty in the beginning of each period, and always demand x for himself whenever he is called by Nature to be the proposer. We let player 2 play aggressively, and always reject the opponent's offer (on the equilibrium path) while player 1 follows a compromising strategy, under which he accepts the opponent's (on-path) offer. Thus, with probability one player 2's proposal is implemented, and he ends up with a pie slice of size x; consequently, when (δ, c) is close to $(1, 0)$ his payoff is approximately x and that of player 1 is approximately $1 - x$.[3] For $u_1 \geq u_2$, the analogous construction, under which player 1 is aggressive and player 2 is compromising, delivers the desired result. In this way, any point in the Pareto frontier can be approximated.

The above construction can be utilized to construct an approximately efficient equilibrium which, while not being exactly symmetric, is *ex ante symmetric*—it is symmetric from the ex ante point of view, but not conditional on every history. This is achieved by viewing Nature's move in the first period as a lottery that determines who will play the aggressive strategy and who will play the compromising strategy, starting from the second period onwards.

The rest of the paper is organized as follows. In Subsection 1.1 we review relevant literature. In Section 2 we formally describe our model. Section 3 is dedicated to the war of attrition. In Sections 4 and 5 we show how this war of attrition can be avoided. In Section 4 we show how it is avoided when investment is public and in Section 5 we show how it is avoided under asymmetric play. In Section 6 we conclude.

[2] It is possible to take $x = u_2$, unless $u_2 \in \{\frac{1}{2}, 1\}$.

[3] $x > \frac{1}{2}$ is necessary for making player 2 willing to reject player 1's offer and trigger another bargaining period. $x < 1$, on the other hand, stems from player 1's incentive constraint: $x = 1$ would mean that player 1 receives a zero share of the pie, in which case investment is suboptimal, no matter how small c is.

1.1 Related literature

Modeling costs of delay in extensive form bargaining games as fixed costs goes back to Rubinstein (1982). Though his model is stated in terms of abstract preferences, fixed costs are one of the two leading applications of time preferences in his model (the other being discounting). In Rubinstein's game with fixed costs the cost is unavoidable: in case of a failure to reach an agreement, each player pays the cost of moving to a new bargaining period. A slightly different role of fixed costs was considered by Perry (1986). In Perry's model, the cost is associated with making an offer. Hence, if player i makes an offer which is rejected by j—in which case play moves to the next period and the roles alternate—only i pays a cost but j does not. A follow-up paper on Perry's work by Cramton (1991) combines (similar to our specification) both fixed costs and discounting in the players' utility functions. In Cramton's model, like in Rubinstein's, a player pays the fixed cost in every period, no matter whether he was the proposer or responder in that period. Contrary to these models, our periodic cost must be paid in advance. More importantly, whether or not a player pays the cost is his private information.

Anderlini and Felli (2001) study a bargaining model with periodic participation costs. Their model is different from ours in several respects. The basic bargaining game is the alternating offers game, each player needs to pay a periodic cost in the beginning of every period if he wants to participate, but not paying the cost only means that the player is suspended from the game for one period; in the next period, he comes back to the game. Under this structure, there is no war of attrition and no competition to remain the "last man standing." Another important difference is that paying (or not paying) the cost is publicly observable in Anderlini and Felli's model.

Another group of related papers consists of those that study bargaining models in which the players compete for a favorable position in the bargaining process. For example, Board and Zwiebel (2012) study a bargaining game in which two players compete, in every period, for the right to make a proposal regarding the split of a pie. The competition for the proposer's position is through a first-price auction, the players have budget constraints, and the horizon is finite. Ali (2015) studies a similar game, in a framework that allows for both finite and infinite horizon, in which n players compete in every period for the right to make a proposal through an all-pay or first-price auction. Yildirim (2007, 2010) studies a model in which the players compete for the proposer's position not through an auction, but by exerting effort that influences the selection of the proposer through a contest success function. An earlier related paper is by Evans (1997), who studies a coalitional bargaining game in which, at each stage, the players compete for the right to make a proposal. In all these papers the players compete for being the proposer, which is a favorable position in the extensive form. In our game, the costly investments guarantee another kind of favorable position—the ability to continue bargaining in case it is necessary.

2 Model

Let G be the following game. Two players, 1 and 2, need to divide a pie of size one. They share a common discount factor, $\delta \in (0,1)$. In the first period of the game, $t = 1$, both players are *active*. In every period t in which both players are active, play is as follows. First, each player privately decides whether or not to take a costly action, *invest*. If a player invests he pays a cost, $c > 0$, upfront. Not investing has no cost. The investment decisions are not publicly observable. After these decisions have been made, Nature selects a *proposer* and a *responder* with equal probabilities. The proposer offers a split of the pie, (x_1, x_2), where x_i is i's proposed share and $x_1 + x_2 = 1$.[4] If the responder accepts the offer, the game ends. If he rejects, the game moves to period $t+1$, but a player who did not invest in the beginning of period t drops out of the game and does not reach period $t+1$. If neither player invested

[4] We will often write an offer as $(x, 1-x)$, with the understanding that the proposer asks x for himself.

in the beginning of t then the game ends and nobody receives anything. If only one player invested then he is the sole active player at $t+1$, and he obtains the entire pie then. If both invested then both reach $t+1$, and both are active at $t+1$.

The payoff of player i from an agreement on (x_1, x_2) in the first period is x_i if he did not invest and $x_i - c$ if he did. His payoff from an agreement on (x_1, x_2) at $t+1$ (for $t \geq 1$), evaluated in the beginning of the game, is:

$$\delta^t x_i - \sum_{i=0}^{\tau-1} \delta^i c,$$

where $\tau \in \{t, t+1\}$. The value $\tau = t$ corresponds to the case where he did not invest in the last period, and $\tau = t+1$ corresponds to the case where he did invest in that period. The utility from perpetual disagreement is $-\sum_{i=0}^{\infty} \delta^i c$.

Let $t+1$ be a period in which both players are active. A *history leading to* $t+1$ is a list $((i_1, x_1), \cdots, (i_t, x_t))$, meaning that in period $k \in \{1, \cdots, t\}$ the proposer was player i_k, he demanded x_k for himself, and the opponent rejected his offer. The set of those histories is denoted H_{t+1}. The initial history is $\emptyset$, and we set $H_1 \equiv \{\emptyset\}$. The set of histories is $H \equiv \cup_t H_t$.

A *strategy* for i is a triplet of functions, $\sigma^i = (I^i, f^i, g^i)$. The function $I^i \colon H \to [0,1]$ prescribes the investment probability in the beginning of every period in which both players are active, as a function of the history leading to this period. The function $f^i \colon H \times \{0,1\} \to [0,1]$ assigns an offer as a function of the history and the player's investment decision; a player uses this component of the strategy whenever he is called by Nature to be the proposer. The function $g^i \colon H \times \{0,1\} \times [0,1] \to \{Accept, Reject\}$ assigns a response to the opponent's offer as a function of the history, the player's investment decision, and the opponent's offer; a player uses this component of the strategy whenever he is called by Nature to be the responder. A pair of strategies is denoted by $\sigma = (\sigma^1, \sigma^2)$.

A *system of beliefs* for player i is a function $\mu^i \colon H \to [0,1]$. Given a history leading to $t+1$, h_{t+1}, the number $\mu^i(h_{t+1})$ is the probability that i attaches to the event that j invested in the beginning of $t+1$. A pair of belief systems is denoted by $\mu = (\mu^1, \mu^2)$.

The pair (σ, μ) is a *perfect Bayesian equilibrium* (PBE) if:

1. For every i and every history after which both players are active, each component of σ^i assigns an optimal action for i, given σ^j and the beliefs μ^i.

2. The beliefs μ obey Bayes' rule whenever possible.

Throughout the paper, "equilibrium" means PBE.[5]

Given a strategy σ and a history $h \in H$, let $Pr_\sigma(h)$ be the probability of h under σ. Given a history h, let $\sigma^i(h) = (I^i(h), f^i(h), g^i(h))$ be the periodic strategy that σ^i induces in the specific period that follows h. We say that an equilibrium (σ, μ) is *symmetric* if:

$$Pr_\sigma(h) > 0 \Rightarrow \sigma^1(h) = \sigma^2(h).$$

That is, symmetry means that in every period the players utilize the same periodic strategy, provided that no deviation occurred in the history that preceded that period. This definition has a strong aspect and a weak aspect. Its weak aspect is that it only applies on the path. Its strong aspect is that the requirement $\sigma^1(h) = \sigma^2(h)$ is imposed also for (on-path) histories h along which the players behaved differently. It is similar to, but weaker than, the notion of "strong symmetry" (Abreu 1986), that requires the players to behave identically, independent of history.

Consider a period of play in a symmetric equilibrium, in which both investment and non-investment occur on the path. This period is *pooling* if the proposer's offer does not reveal whether he invested, otherwise it is *separating*. A symmetric equilibrium is *pooling* (*separating*) if, under this equilibrium, every period is pooling (separating).

[5]In Section 4 we consider a complete information version of our game. There (and only there) the solution concept is subgame perfect equilibrium.

An equilibrium (σ, μ) is *stationary* if σ is history-independent.

Another terminology we will use is "strong" versus "weak." A player who invested is called *strong* (in the period he invested), otherwise he is called *weak*.

Given a history h let h' be identical to h except that each i_k is replaced by $3 - i_k$. Namely, h' is the same as h except that the roles of the players are interchanged. An equilibrium (σ, μ) is *ex ante symmetric* if $Pr_\sigma(h) = Pr_\sigma(h')$ for all h.

3 Symmetric play implies a war of attrition

In this section we show that under symmetric play, and when (δ, c) is close to $(1, 0)$, our game is, effectively, a war of attrition. Specifically, we call an equilibrium the *war of attrition equilibrium* if it is a pooling and stationary equilibrium in which a player mixes in his investment decision and the proposer demands the entire pie, no matter whether he invested or not.[6] The following result provides a necessary and sufficient condition for its existence.

Proposition 1. *A war of attrition equilibrium exists if and only if:*

$$1 - \delta^2 \leq \frac{4c}{\delta} < 1. \tag{1}$$

Proof. In such an equilibrium each player is indifferent between investing and not. The expected utility from not investing is $\frac{1-p}{2}$ and that of investing is $-c + \frac{1}{2}\{p\delta V + (1-p)\} + \frac{1}{2}\delta V$, where V is the equilibrium's continuation value and p is the investment probability. Indifference implies that they are equal, hence $V = \frac{2c}{\delta(1+p)}$. This V also equals the expected utility from not investing, hence that the following equation needs to hold: $\delta p^2 + 4c - \delta = 0$. Its solution, $p = \sqrt{1 - \frac{4c}{\delta}}$, is in $(0, 1)$ if and only if $4c < \delta$. Also, a weak proposer should find it optimal to follow the equilibrium and demand

[6] In several places in the paper (for example, in the abstract) we use the term "war of attrition" informally. Here, we use it to denote a specific equilibrium. We hope that this "double usage" does not cause any confusion.

the entire pie, rather than offering the responder δ, which is an offer the responder "cannot refuse." Therefore $1-p \geq 1-\delta$, or $\delta \geq p$. With the aforementioned p this simplifies to $\frac{4c}{\delta} \geq 1-\delta^2$. Therefore, (1) is necessary.

Conversely, suppose that (1) holds. Consider the following symmetric stationary profile. On the path, behavior is as follows: a player invests with probability $p = \sqrt{1-\frac{4c}{\delta}}$, demands the entire pie if he is the proposer, and rejects the opponent's offer if he invested; a responder who did not invest accepts the opponent's offer. Off the path, behavior is as follows: any offer different from zero is interpreted as a signal that the opponent did not invest, and is therefore accepted by a strong responder if and only if it gives the responder at least δ. In the beginning of every subgame that follows a deviation, the deviation is ignored, and play is as in the first period.

We argue that this is an equilibrium. If a strong proposer demands $x < 1$, his payoff is $(1-p)x + p\delta V < (1-p) + p\delta V$, where V is the value; note that the RHS of this strict inequality is the payoff from following the strategy. Hence, demanding the entire pie is optimal. Now consider a proposer who did not invest. Since a demand $x \in (1-\delta, 1)$ is accepted if and only if the opponent is weak and a demand $x < 1-\delta$ is accepted for sure, the only candidates for a best-response are the demands $1-\delta$ and 1. The inequality $1-\delta \leq 1-p$ guarantees the optimality of the latter. It is easy to see that the prescribed responses are optimal. $\square$

When c is small, every pooling period in a general (i.e., not-necessarily-stationary) symmetric equilibriun looks approximately like in the WOA equilibrium. Interestingly, this result is independent of δ.

Proposition 2. *For every $\epsilon > 0$ there exists a $\tilde{c}_\epsilon > 0$, such that if $c < \tilde{c}_\epsilon$ then the following holds in every pooling period of every symmetric equilibrium:*

- *If the common investment probability is strictly smaller than one, then the common offer, $(d, 1-d)$, satisfies $d \geq 1-\epsilon$.*

- *The common investment probability, p, satisfies $p \geq 1-\epsilon$.*

In Proposition 2's proof, we make use of the following lemmas.

Lemma 1. *Suppose that $c < \frac{\delta}{2}$. Let p be the investment probability in some period in a symmetric equilibrium. Then $p > 0$.*

Proof. Assume by contradiction that $p = 0$. Then each player's payoff is $\frac{1}{2}$. Therefore, each player has a profitable deviation: investing with certainty, demanding the entire pie, and refusing to anything short of the entire pie; this behavior gives the payoff $-c + \frac{1}{2} + \frac{\delta}{2} > \frac{1}{2}$. □

Lemma 2. *Consider a period in a symmetric equilibrium, in the beginning of which each player invests with probability p. Then each player can secure a payoff of no less than:*

$$-c + \frac{(1-p)(1+\delta)}{2}. \tag{2}$$

Proof. Consider the following behavior: a player invests with certainty in the beginning of the period (instead of with probability p), demands the entire pie if he is called to be the proposer, and refuses to anything short of the entire pie if he is called to be the responder. Clearly, this behavior bring a payoff no smaller than $-c + \frac{1-p}{2} + \frac{(1-p)\delta}{2} = -c + \frac{(1-p)(1+\delta)}{2}$. □

Proof of Proposition 2: Consider a pooling period in a symmetric equilibrium. Let p be the investment probability and let d be the common demand. Consider $p < 1$.[7] By Lemma 1 $p > 0$, hence a player is indifferent between investing and not investing in that period. Also, note that a strong responder rejects $1 - d$; otherwise, the offer $1 - d$ is accepted with certainty, which is impossible because (by Lemma 1) $p > 0$.[8] The expected utility from not investing is $\frac{1}{2}(1-p)d + \frac{1}{2}(1-d)$ and that of investing

[7] Note that if $p = 1$, then there is disagreement in this period. Had there been agreement, then clearly investment would have been suboptimal.

[8] Had the common offer been accepted for sure, clearly there would be no reason to invest.

is $-c+\frac{1}{2}\{p\delta V+(1-p)d\}+\frac{1}{2}\delta V$, where V is the continuation value. Indifference implies that they are equal, hence:

$$V=\frac{2c+1-d}{\delta(1+p)}. \tag{3}$$

The fact that a strong responder rejects $1-d$ implies that $\delta V \geq 1-d$. Substituting the expression for V gives $\frac{2c+1-d}{1+p} \geq 1-d$, hence:

$$2c \geq p(1-d). \tag{4}$$

Now, fix a mapping that assigns for each c a pooling period in a symmetric equilibrium of the game in which $p<1$, when the investment cost is c. Let $(p(c), d(c))$ be the investment probability and common demand in this period. By (4), $p(c)(1-d(c)) \rightarrow 0$ as $c \rightarrow 0$.

We argue that $p(c) \nrightarrow 0$ as $c \rightarrow 0$. Otherwise, Lemma 2 implies that a player can secure a payoff of approximately $\frac{1+\delta}{2}$, which is impossible in a symmetric equilibrium. Therefore, $d(c) \rightarrow 1$ as $c \rightarrow 0$. This proves the first part of the proposition.

Now, for a fixed c, a player's expected utility is given by $\frac{1}{2}(1-p)d+\frac{1}{2}(1-d)$.[9] This payoff has to be at least as large as the one described in Lemma 2, $-c+\frac{(1-p)(1+\delta)}{2}$. That is, $\frac{1}{2}(1-p)d+\frac{1}{2}(1-d) \geq -c+\frac{(1-p)(1+\delta)}{2}$, or $c+\frac{1}{2}(1-d) \geq \frac{(1-p)(1+\delta-d)}{2}$. Taking $c \rightarrow 0$ we see that the limit value of p, call it p^*, must satisfy $0 \geq \frac{\delta(1-p^*)}{2}$; therefore, $p^*=1$. This proves the second part of the proposition. $\square$

The next results concern separating periods.

Lemma 3. *The following holds in every separating period in a symmetric equilibrium: if both players invest in the beginning of the period, then there is disagreement in this period.*

Proof. See the appendix. $\square$

[9]Recall that this is the utility from not investing.

Lemma 4. *There exists a $c^* > 0$ such that the following holds in every separating period in a symmetric equilibrium, provided that $c < c^*$: the strong responder accepts the offer of the weak proposer.*

Proof. Consider such a separating period. Let $(s, 1-s)$ and $(w, 1-w)$ be the offers that are made by the strong and weak proposer, where $s \neq w$. By Lemma 3, a strong responder rejects the s-offer. Assume by contradiction that he rejects also the w-offer. Therefore the strong responder rejects both offers. This implies that $w = 1$. Let p denote the investment probability in that period. The indifference condition between investing and not investing is:

$$\frac{p(1-s)}{2} + \frac{1-p}{2} = -c + \frac{1}{2}[p\delta V + (1-p)s] + \frac{1}{2}\delta V, \tag{5}$$

where V is the continuation value. Therefore $\frac{p(1-s)}{2} + c = (\frac{1+p}{2})\delta V + \frac{1-p}{2}(s-1)$, or $p(1-s) + 2c = (1+p)\delta V + (p-1)(1-s)$. Therefore,

$$(1-s) + 2c = (1+p)\delta V.$$

Since a strong responder rejects the s-offer, it must be that $(1-s) \leq \delta V$. Therefore,

$$2c \geq p\delta V. \tag{6}$$

Now, fix a mapping that assigns for each c a separating period in a symmetric equilibrium of the game when the investment cost is c, in which the strong responder rejects the weak proposer's offer (and therefore rejects both offers). In particular, the indifference condition (5) and its above implications apply. Let $(p(c), V(c))$ be the corresponding investment probability and continuation value in this period. Inequality (6) implies $p(c)V(c) \to 0$ as $c \to 0$.

If $p(c) \nrightarrow 0$ as $c \to 0$ then $V(c) \to 0$. In this case, the periodic value, which, in particular, is given by the RHS of (5), converges to $\frac{(1-p^*)s^*}{2}$, where (p^*, s^*) are the limit values of p and s. However, by Lemma 2 a player can secure a payoff of approximately $\frac{(1-p^*)(1+\delta)}{2}$. If, on the other hand, $p(c) \to 0$ as $c \to 0$, then the behavior from

Lemma 2 secures a payoff of approximately $\frac{1+\delta}{2}$, which is impossible in a symmetric equilibrium. Therefore, there is a $c^* > 0$ such that the following is true in every separating period in every symmetric equilibrium, provided that $c < c^*$: the strong responder accepts the weak proposer's offer. $\square$

Combining the above results, we are ready to turn to our first main result.

Theorem 1. *Let $\epsilon > 0$. Then there exist a $c_\epsilon > 0$ and a $\delta_\epsilon < 1$ such that if $c < c_\epsilon$ and $\delta \in (\delta_\epsilon, 1)$ then in every period t in every symmetric equilibrium the following hold:*

1. *Each player invests with probability at least $1 - \epsilon$;*

2. *If the common investment probability is strictly smaller than one, then a strong proposer demands for himself at least $1 - \epsilon$; and*

3. *The utility from starting the period-t subgame is at most ϵ.*

Proof. Parts *1* and *2* were proved in Proposition 2 for the case of a pooling period. Consider then a separating period. Let $(s, 1-s)$ and $(w, 1-w)$ be the offers made by the strong and weak proposers, respectively. By Lemmas 3 and 4 we may assume that the strong responder rejects the s-offer and accepts the w-offer. Since he accepts the later, $1 - w \geq \delta$. By subgame perfection the inequality cannot be strict, hence $w = 1 - \delta$. Indifference between investing and not investing in the beginning of the period implies:

$$\frac{1-\delta}{2} + \frac{1}{2}[p(1-s) + (1-p)\delta] = -c + \frac{1}{2}[p\delta V + (1-p)s] + \frac{1}{2}[p\delta V + (1-p)\delta], \quad (7)$$

where V is the continuation value. Rearranging this equation gives:

$$1 - \delta + p(1-s) = -2c + 2p\delta V + (1-p)s,$$

or $1 - \delta + p = -2c + 2p\delta V + s$.

Since the strong responder rejects the s-offer, $\delta V \geq 1 - s$. Therefore, the above equation implies $1 - \delta + p \geq -2c + s + 2p(1 - s) = -2c + s + 2p - 2ps$. Therefore:

$$1 - \delta + ps \geq -2c + p + (1 - p)s.$$

Taking $(\delta, c) \to (1, 0)$ and denoting by p^* and s^* the corresponding limit-values of p and s, we get:

$$p^* s^* \geq s^* + (1 - s^*)p^* \geq p^*.$$

Since $p^* > 0$ it follows that $s^* = 1$, and therefore $p^* = 1$. This completes the proof of parts *1* and *2* of the theorem for the case of a separating period.

As for part *3*, consider an arbitrary period t in a symmetric equilibrium. Let V_t be the utility from playing the period-t subgame. Suppose first that t is a separating period. The value V_t equals, in particular, the LHS of (7). Since $s^* = p^* = 1$, the value converges to zero as $(\delta, c) \to (1, 0)$.

Next, consider a pooling period. Let p_t be the common investment probability in the beginning of the period-t subgame. It is enough to prove that $V_t \leq \epsilon$ under the assumption $p_t < 1$. The reason is that if t is a period with $p_t = 1$ then there is disagreement in period t.[10] Therefore, the value of starting the subgame of period t, if not zero, is given by $\delta^k V_{t+k}$, where $t + k$ is the first period after t in which the investment probability is smaller than one.

Consider then a period in which the investment probability is in $(0, 1)$ and the common offer is d. As we saw in the proof of Proposition 2, when these quantities are written explicitly as functions of c, we have that $(p(c), d(c)) \to (1, 1)$ as $c \to 0$. Since V_t equals, in particular, the utility from not investing, which is $\frac{1}{2}(1 - p)d + \frac{1}{2}(1 - d)$,

[10] It is easy to see that if $p_t = 1$ and there is agreement in t, then it is profitable to deviate and not invest in the beginning of t, because no matter who will be the proposer, his proposal is supposed to be accepted with certainty.

it follows that V_t converges to zero as $c \to 0$. $\square$

Theorem 1 states that every symmetric equilibrium must have certain properties, but it does not say anything about equilibrium existence. The simplest equilibrium with these properties is the WOA equilibrium, for the existence of which necessary and sufficient conditions were given in Proposition 1. This equilibrium is pooling, which is intuitive: the non-investing player conceals his weakness by making the same offer as the strong player. We end this section by showing that within the class of stationary symmetric equilibria, this pooling is inevitable.

Proposition 3. *There exists a constant $\bar{c} > 0$ such that for every $c < \bar{c}$ there exist a $\delta(c) < 1$ such that if $\delta \in (\delta(c), 1)$ then there does not exist a stationary separating equilibrium.*

Proof. Take $\bar{c} = c^*$, where c^* is the constant from Lemma 4. Consider a stationary separating equilibrium. By Lemmas 3 and 4 the strong responder accepts $1 - w$ and rejects $1 - s$. The combination of stationarity and the fact that he rejects $1 - s$ implies that $s = 1$. As we already noted in the proof of Theorem 1, acceptance of $1 - w$ implies that $w = 1 - \delta$. The indifference condition between investing and not investing is:

$$\frac{1-\delta}{2} + \frac{\delta(1-p)}{2} = -c + \frac{1}{2}[p\delta V + (1-p)] + \frac{1}{2}[p\delta V + (1-p)\delta],$$

where $V = \frac{1-p\delta}{2}$ is the equilibrium value and p is the investment probability. This equation simplifies to $1 - \delta + 2c = 1 - p + 2p\delta V$, or $p - \delta + 2c = p\delta(1 - p\delta)$. At $\delta = 1$ the solution is $p = \sqrt{1 - 2c}$. Therefore, if $\delta \in (\sqrt{1 - 2c}, 1)$ is sufficiently close to one, then $\delta > p$.

We argue that this is impossible. The reason is that the weak proposer's payoff is $1 - \delta$. Since he can secure the payoff $1 - p$ by demanding the entire pie, it must be that $1 - \delta \geq 1 - p$, or $p \geq \delta$. $\square$

4 Complete information

In what follows we show that the war of attrition can be avoided under observable investment.

With observable investment our game becomes a game of complete information. Another version of our game which is effectively a game of complete information is the limit-version, where $c = 0$. The reason is that with zero cost investing weakly dominates not-investing, hence it can be assumed that a player always invests. This costless-investment game is, in fact, the random-proposer bargaining game that has been studied by Binmore (1987), namely the "symmetrized" Rubinstein (1982) game. Binmore demonstrated that this game has a unique subgame perfect equilibrium (SPE). This equilibrium is stationary and symmetric, it involves immediate agreement, and each player's equilibrium expected payoff is $\frac{1}{2}$.

Thus, equilibrium payoffs are discontinuous: when (δ, c) converges to $(1, 0)$ the attainable payoffs in the symmetric equilibria of our game converge to zero, but with $(\delta, c) = (\delta, 0)$, for any value of $\delta \in (0, 1)$, the game has a unique equilibrium, and the payoff of this unique equilibrium is $\frac{1}{2}$. The reason for this discontinuity is that in the limit the incomplete information disappears.

To emphasize the importance of the informational aspect of our game, we study a complete information version of it. In this version—hereafter the *complete information G*—the players' investment choices are publicly observable, hence the proposer can condition his offer, and the responder can condition his response, on the pair of investment choices. The solution concept for this game is SPE.

The complete information G has an equilibrium akin to that of Binmore's game. In it, each player invests with certainty in every period, and the remainder of the strategy (the proposals and accept/reject policy) is like in Binmore's game. In particular, the equilibrium is stationary, there is immediate agreement, the equilibrium's proposal makes the responder indifferent between accepting and rejecting it, and each player's expected payoff is $\frac{1}{2}$. We call this equilibrium the *Binmore equilibrium.*

We do not have a characterization of all the SPEs of the complete information G. We show, however, under suitable assumptions on δ and c, that (1) the Binmore equilibrium is the unique symmetric SPE of the complete information G, and (2) it is also its unique stationary SPE.

Proposition 4. *Suppose that $c < \frac{\delta}{2}$. Then the complete information G has a unique symmetric SPE. It is the Binmore equilibrium.*

Proposition 5. *There exist a discount factor $\hat{\delta} < 1$ and a cost $\hat{c} > 0$, such that the following holds: if $\delta \in (\hat{\delta}, 1)$ and $c < \hat{c}$ then the complete information G has a unique stationary SPE. It is the Binmore equilibrium.*

Had investment been non-observable the Binmore equilibrium would collapse, as each player would be better off deviating from the putative equilibrium and saving the cost. This accounts for the difference between the two propositions above and the results from the previous section. We relegate the proofs of the propositions to the appendix.

5 Asymmetry

In what follows we show that the war of attrition can be avoided under asymmetric play.

Consider the following asymmetric strategy. Fix an $x \in (\frac{1}{2}, 1)$. A player always invests with certainty, and demands x whenever he is called to make an offer. Player 1 accepts player 2's offer if and only if it gives him at least $1 - x$.[11] Player 2 accepts an offer if and only if it gives him at least δ. The prescribed rejections are supported by the following beliefs: whenever a player sees an unexpected offer that the strategy instructs him to reject, he adopts the belief that the proposer did not invest, hence rejecting the offer is optimal, as it guarantees the entire pie in the next period. If a

[11]In equilibrium a player "cannot refuse" an offer that gives him more than δ. Since we are interested in equilibrium for large enough δ's (and small enough c's), we may assume that $1 - x \leq \delta$.

player does not invest (which is a deviation) he accepts any offer of the opponent. If player 1 did not invest and he is selected to be the proposer, he demands $1-\delta$ for himself. If player 2 did not invest and he is selected to be the proposer, he demands x for himself. Denote this strategy by $\sigma(x,2)$. Similarly, let $\sigma(x,1)$ denote the analogous strategy, where player 1 is the aggressive player who always rejects the x-offer and player 2 always accepts it.

Proposition 6. *Fix $x \in (\frac{1}{2},1)$ and $i \in \{1,2\}$. There exist $\delta(x) \in (0,1)$ and $c(x) > 0$ such that the following holds: $\sigma(x,i)$ is sustainable in a stationary equilibrium, provided that $\delta \in (\delta(x),1)$ and $c < c(x)$.*

Proof. Fix an $x \in (\frac{1}{2},1)$. Without loss of generality, we suppose that $i = 2$; namely, player 2 is the aggressive one. We consider $\delta = 1$; the same arguments establish equilibrium existence for δ sufficiently close to one.

Let v_i be player i's value from the abovementioned strategies. For player 1 we have $-c + \frac{1}{2}(1-x) + \frac{1}{2}v_1 = v_1 \Rightarrow v_1 = 1-x-2c$. In particular, $1-x > v_1$, so accepting player 2's offer is better than rejecting it and triggering the next period.

The following condition guarantees that investing is better than not investing: $\frac{1-x}{2} \leq v_1$, or:

$$2c \leq \frac{1-x}{2}. \tag{8}$$

Given that player 1 invested, demanding the equilibrium demand is optimal for him, since $(1-\delta) \leq \delta v_1$ clearly holds for $\delta = 1$. Given that he did not invest, demanding $1-\delta$ is optimal for him, as player 2 will reject any greedier offer.

For player 2 the value satisfies $\frac{1}{2}x + \frac{1}{2}v_2 - c = v_2$, or $v_2 = x - 2c$. The following condition guarantees that investing is optimal: $\frac{1}{2}x + \frac{1}{2}(1-x) = \frac{1}{2} \leq v_2$, or:

$$2c \leq x - \frac{1}{2}. \tag{9}$$

Since player 2's strategy prescribes him a rejection of $1-x$, it needs to be the case that $1-x \leq v_2$, or $1-x \leq x-2c$. This condition is satisfied if c is sufficiently small since $x > \frac{1}{2}$.

Given that player 2 invested, demanding the equilibrium demand is optimal for him, since $x \geq \max\{\delta v_2, 1-\delta\}$ clearly holds for $\delta = 1$. It is also easy to see that given that he did not invest, demanding x is optimal for him.

Combining (8) and (9) we get:

$$2c \leq \min\{\frac{1-x}{2}, x - \frac{1}{2}\}.^{12}$$

Clearly, the above requirement holds for all sufficiently small c's. □

The equilibrium of Proposition 6 is different from that of Rubinstein's/Binmore's game in that that a responder is *not* indifferent between accepting and rejecting the equilibrium offer. To see this, consider first player 1. Since $1 - x > v_1$, accepting the offer is strictly better for him than rejecting it. Note that if player 2 cuts down his offer to some $y \in (v_1, 1-x)$ it would be rejected by player 1, due to player 1's belief that player 2 did not invest; this belief makes it optimal for player 1 to reject this "above-value offer" and take the entire pie in the next period. For a similar reason, $v_2 > 1 - x$.

The following is an immediate consequence of Proposition 6.

Theorem 2. *For every Pareto efficient payoff vector u and every $\epsilon > 0$ there exist $\delta(u, \epsilon) \in (0, 1)$ and $c(u, \epsilon) > 0$ such that the following holds provided that $\delta \in (\delta(u, \epsilon), 1)$ and $c < c(u, \epsilon)$: G has a stationary equilibrium whose payoff vector is ϵ-close to u.*

The equilibria of Proposition 6 can be utilized to construct an equilibrium which approximates efficiency when (δ, c) is close to $(1, 0)$, and which, while not being exactly symmetric, is *ex ante symmetric*. The equilibrium's construction utilizes Nature's first move in the game: the first-period role assignment (proposer versus responder)

[12] For a general δ the corresponding condition is $2c \leq \min\{1 - x + \frac{x}{2}(2-\delta) - \frac{1}{2}(2-\delta)^2, x + \frac{\delta}{2} - 1\}$.

determines which of two continuation equilibria will be played from the second period onwards: $\sigma(x,1)$ or $\sigma(x,2)$.

Corollary 1. *There exist $\delta^{**} < 1$ and $c^{**} > 0$ such that the following holds provided that $\delta \in (\delta^{**}, 1)$ and $c < c^{**}$: G has an ex ante symmetric equilibrium. A player's ex ante expected utility in this equilibrium is:*

$$-c + \frac{\delta}{2} \cdot \frac{1-4c}{2-\delta}.$$

In particular, this payoff converges to $\frac{1}{2}$ as $(\delta, c) \to (1, 0)$.

Proof. Take $x \in (\frac{1}{2}, 1)$ and consider the following profile. In the first period both players invest with certainty. Denote by i the player who is selected by Nature to be the first period's proposer. After i has been selected, play is according to $\sigma(x, j)$. In particular, when i makes his offer it is rejected by j and the game moves to the next period, where the continuation play is as specified in the equilibrium of Proposition 6.

It follows from Proposition 6 that conditional on being at the post-investment stage of the first period, continuation play forms a equilibrium. Note that playing the above strategy is, in effect, starting to play the equilibrium of Proposition 6 in the second period of the game, with each player being equally likely to be in the weak/strong role. The value of this is $\delta\frac{1}{2}(v_1 + v_2)$, where $(v_1 + v_2)$ is the sum of values of the equilibrium of Proposition 6. It is not hard to check that $(v_1 + v_2) = \frac{1-4c}{2-\delta}$. Since the players also invest in the first period, the overall value from the above strategy is $-c + \frac{\delta}{2} \cdot \frac{1-4c}{2-\delta}$, which converges to $\frac{1}{2}$ as $(\delta, c) \to (1, 0)$. It remains to show that the utility from deviating and not investing in the first period is strictly below $\frac{1}{2}$ when (δ, c) is sufficiently close to $(1, 0)$.

Consider then a player who does not invest in the first period. Conditional on being the responder, he accepts the offer $1 - x$ (which, on the equilibrium's path, he is supposed to reject). Conditional on making an offer he can obtain no more than

$1-\delta$. Hence, the overall payoff from the deviation is bounded by $\frac{1}{2}(1-x)+\frac{1}{2}(1-\delta)$, which converges to $\frac{1}{2}(1-x)<\frac{1}{2}$. $\square$

The structure of the equilibrium in Corollary 1 is similar to that of the *randomized dictatorship* mechanism. Under randomized dictatorship, a fair coin flip determines which player will be the "dictator" and obtain his maximum possible payoff, while the other player will receive nothing.[13] The equilibrium in Corollary 1 approaches randomized dictatorship as $x \to 1$. In this equilibrium, the player who enjoys the privilege of making the first rejection signals that continuation play will be an asymmetric equilibrium that favors him. Thus, one can think of Corollary 1 as saying that it is important to establish, early on, a reputation for toughness.

6 Conclusion

We have studied a bargaining model in which a periodic cost parameter plays a central role. Bargaining may involve various types of inter-connected costs, one of which—the one that suits our model most naturally—is the cost of preparations. Costly investments that are made prior to the beginning of negotiations to improve one's bargaining position are ubiquitous. People prepare for negotiations in various ways, and preparations are almost always costly.[14] In our model, one has to pay the cost in the beginning of every period t, if one wants to stay in the game in $t+1$ in case a deal has not been reached by the end of t. We showed that under

[13] This idea/mechanism has been studied extensively in the context of cooperative bargaining. See, for example, Sobel (1981), Myerson (1984), Anbarcı(1998), and Rachmilevitch (2016).

[14] Thompson (2013) labels pre-negotiation preparations as "the magic bullet" and she argues (in Thompson, 2009) that about 80 per-cent of a negotiator's effort should be spent in preparations, which involve activities such as detailed analysis of all issues under consideration, prioritization of issues, fact-finding, perspective taking, identifying all the alternative course of actions, taking into account all contingencies, cooking up an opening proposal, setting aspirations etc. These preparations take time (hence should be done in advance), money, and energy.

symmetric play the bargaining game becomes a war of attrition in which the entire surplus is wasted, but with asymmetric strategies every efficient payoff vector can be approximated in equilibrium, provided that the frictions (cost, discounting) are sufficiently small.

The idea that a player may need to pay some cost every period in order to stay in the game seems to be applicable to other settings and problems, not only bargaining. Investigating it in other contexts remains a task for future research.

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Appendix

Proof of Lemma 3: Assume by contradiction that if both players invest, then there is agreement. Let p denote the common investment probability. Since the period is separating, $p \in (0, 1)$.

Let s and w denote the pie-shares demanded by a weak (non-investing) and strong (investing) proposer. Since a player can secure the payoff $(1 - p) > 0$ by demanding the entire pie, $w > 0$.[15] We argue that the strong responder rejects the w-offer. Otherwise, (i.e., if he accepts it), then the w-offer is accepted for sure—namely, by both types of responders. However, the s-offer is also accepted by both types: it is clearly accepted by the weak responder, and per our assumption it is accepted by the strong responder. Thus, each offer is accepted with certainty, which is clearly impossible in equilibrium (a proposer will choose the offer that maximizes his share of the pie). Therefore, the strong responder rejects the w-offer.

So, the weak proposer's payoff is $(1-p)w$. Since the weak proposer can guarantee the payoff $(1-p)$, it follows that $w = 1$. Incentive compatibility for the weak proposer implies $(1-p) \geq s$ (since, per our assumption, the strong responder agrees if the proposer asks for s, the s-offer is accepted for sure if it is made). Incentive compatibility for the strong proposer implies $s \geq (1-p) + p\psi$, where ψ is the payoff he receives by making the weak proposal to a strong responder. Therefore, $(1-p) \geq s \geq (1-p) + p\psi$. Since $p \in (0,1)$, it is enough to prove that $\psi > 0$ in order to establish a contradiction. This is indeed the case because the player can, for example, avoid investing in the beginning of next-period's subgame and then demand $1-\delta$ if he is selected by Nature to be the proposer. Therefore, $\psi \geq \frac{\delta(1-\delta)}{2}$. The lemma is therefore proved: if both players invest, there is disagreement. $\square$

Proof of Proposition 4: Consider the strategy under which each player invests with certainty in the beginning of every period, the proposer always proposes $(1-\delta(-c+\frac{1}{2}), \delta(-c+\frac{1}{2}))$, and the responder always accepts anything that gives him at least as in this offer, and nothing less. It is easy to verify (by applying the one-shot deviation principle) that $c < \frac{\delta}{2}$ guarantees that this is a SPE; it is obviously symmetric and stationary. We will now prove that this is the unique symmetric SPE.

Since $\frac{\delta}{2} > c$, it follows that in every SPE every player invests with a strictly

[15] If a player demands the entire pie and the opponent is weak, the opponent agrees to this proposal. The probability of the opponent being weak is $(1-p)$.

positive probability in the beginning of every period (the argument given in the proof of Lemma 1 is also valid here). Let V_t denote the equilibrium's value in the subgame that starts in period t (the value may be time-dependent because we do no impose stationarity; it cannot, however, depend on a player's identity, because of the equilibrium's symmetry). We argue that $V_t < \frac{1}{2}$. Otherwise, if $V_t \geq \frac{1}{2}$, then the total value for both players is $2V_t = 1$, which is inconsistent with the fact that each player invests with a strictly positive probability.

Since $V_t < \frac{1}{2}$ for every t, it follows that there is an agreement in the beginning of every subgame. To see this, assume by contradiction that there is disagreement on the path, in some period t. This means that the responder invested in the beginning of the period, and the proposer also invested.[16] Then the proposer's payoff is $\delta V_{t+1} < \frac{1}{2}$. We argue that he has a profitable deviation: if he offers the responder $\delta V_{t+1} + \epsilon$ for an arbitrarily small ϵ, the responder must accept (because of subgame perfection) and for a sufficiently small ϵ the proposer will obtain more than half the pie. Therefore, as argued, there is agreement in every subgame.

To complete the proof, it is enough to show that each player invests with certainty in the beginning of every period. Assume, then, by contradiction, that there is a period t in which each player invests with probability $p \in (0, 1)$. It is easy to check that the utility from not investing in the beginning of t is $\frac{1}{2}[p(1-\delta)+(1-p)] = \frac{1-p\delta}{2}$. To compute the utility from investing, we invoke the just-proved fact, that on the equilibrium path there is agreement in every period. In particular, in the specific period that we consider, t, a proposer-who-invested proposes to the responder δV_{t+1}, and the latter accepts. Therefore, the utility from investing is:

$$-c + \frac{1}{2}[p\delta V_{t+1} + (1-p)\delta] + \frac{1}{2}[p(1-\delta V_{t+1}) + (1-p)] = -c + \frac{1}{2} + \frac{(1-p)\delta}{2}.$$

Indifference between investing and not implies $-c + \frac{1}{2} + \frac{(1-p)\delta}{2} = \frac{1-p\delta}{2}$, and so

[16] Had the proposer not invested, he would have offered δ to the responder, which would be accepted.

$\frac{\delta}{2} = c$—in contradiction to $\frac{\delta}{2} > c$. $\square$

Proof of Proposition 5: Fix an arbitrary stationary SPE. We will prove that it must be the Binmore equilibrium (provided that (δ, c) is sufficiently close to $(1, 0)$). Let p_i denote player i's investment probability in this equilibrium. By the argument from Lemma 3, $p_i > 0$ for both i.

Case 1: $p_i \in (0, 1)$ for some i. Wlog, suppose that $i = 1$. Namely, player 1 is indifferent between investing and not. His utility from not investing is $\frac{1}{2}[p_2(1-\delta) + (1-p_2)] = \frac{1-p_2\delta}{2}$. To calculate his utility from investing, we consider two possibilities separately.

Case 1.1: When both players invest, there is disagreement. In this case player 1's utility from investing is $-c + \frac{1}{2}[p_2\delta V_1 + (1-p_2)\delta] + \frac{1}{2}[p_2\delta V_1 + (1-p_2)] = -c + p_2\delta V_1 + (\frac{1-p_2}{2})(1+\delta)$, where V_1 is player 1's equilibrium's value.[17] This expression is equal to V_1, which, in turn, is equal to $\frac{1-p_2\delta}{2}$. Therefore:

$$-c + p_2\delta(\frac{1-p_2\delta}{2}) + (\frac{1-p_2}{2})(1+\delta) = \frac{1-p_2\delta}{2}.$$

Considering $c = 0$ and simplifying this equation yields:

$$\delta^2 p_2^2 + p_2(1-\delta) - \delta = 0,$$

whose solution is $p_2 = \frac{-(1-\delta)+\sqrt{1-2\delta+\delta^2+4\delta^3}}{2}$. Therefore, as $\delta \to 1$ we have that $p_2 \to 1$ and therefore $V_1 \to 0$.

Note that p_2 is also in $(0, 1)$ and therefore—by the same arguments as above–it follows that $V_2 \to 0$. This, however, is impossible: when the proposer i proposes V_j the responder will accept, which means that i's equilibrium's value is bounded from below by a number which is approximately $\frac{1}{2}(1 - V_i) \sim \frac{1}{2}$.

Case 1.2: The investing players disagree. In this case, subgame perfection implies

[17] The value may depend on the player's identity (because we do not impose symmetry) but not on the subgame (because we do assume stationarity).

that the investing proposer i proposes δV_j to the investing responder, and i's utility from investing is therefore given by $-c+\frac{1}{2}[p_j\delta V_i+(1-p_j)\delta]+\frac{1}{2}[p_j(1-\delta V_i)+(1-p_j)] = -c+\frac{p_j}{2}+(\frac{1-p_j}{2})(1+\delta)$. For $i=1$, equating this utility to the utility of not investing, $\frac{1-p_2\delta}{2}$, yields $\delta = 2c$, which is impossible when (δ, c) is sufficiently close to $(1,0)$.

We conclude that Case 1 is impossible.

Case 2: $p_1 = p_2 = 1$. In this case there must be immediate agreement in equilibrium. Let $(y-c, 1-y-c)$ be the equilibrium's expected utilities. Player 1 offers player 2 $\delta(1-y-c)$ and player 2 offers player 1 $\delta(y-c)$, since in equilibrium each player is indifferent between accepting his opponent's offer and rejecting it. Let us look at player 2's indifference condition:

$$\delta(1-y-c) = \delta\{-c+\frac{1}{2}\delta(1-y-c)+\frac{1}{2}[1-\delta(y-c)]\}.$$

Simplifying this gives $y = \frac{1}{2}$. Therefore, the equilibrium is Binmore's equilibrium. $\square$

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