Electoral Accountability and Responsive Democracy*

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Abstract

We consider elections with adverse selection (hidden preferences) and moral hazard (hidden actions), in which neither voters nor politicians can commit to future choices. We show that some below average politicians may randomize between "taking it easy" (choosing policies near their ideal points) and "going for broke" (mimicking above average politicians by choosing high policies). When politicians are highly office motivated, they respond by choosing high policies to signal they are above average. Normative implications depend on the nature of policy: if voter preferences are increasing, then elections deliver positive outcomes; but if voter preferences are single-peaked, then politicians overshoot in the first period by choosing policies above the voters' ideal. Regardless, all politician types are re-elected with probability converging to one; thus, electoral incentives shift to sanctioning, rather than selection, as office motivation becomes large.

1 Introduction

Representative democracy, by definition, entails the delegation of power by society to elected officials. A main concern for representative democracy is whether elected politicians are responsive to voter preferences and produce desirable policy outcomes for citizens. Political thinkers since Madison, if not earlier, have viewed elections as an effective mechanism for achieving responsiveness.[1] The goal of

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[1] *The Federalist* 57, in particular, offers a discussion of the role of "frequent elections" in the selection of politicians and the control of politicians while in office.

this paper is to study formally the incentives provided by democratic elections and the implied linkage between voter preferences and policy outcomes. In doing so, we must move beyond the basic Downsian model of static elections, the stalwart of formal work on electoral competition, to explicitly incorporate a temporal dimension within the analysis. An active and growing literature on electoral accountability, starting with the seminal work of Barro (1973) and Ferejohn (1986), has undertaken this line of inquiry, with the goal of improving our understanding of the operation of real-world political systems and the conditions under which democracies succeed or fail. This, in turn, has the potential to facilitate the design of political institutions that produce socially desirable policy outcomes.

Nevertheless, our understanding of the fundamental interplay between disciplining incentives provided by the possibility of future re-election, on the one hand, and the temptation to engage in opportunistic behavior in the present, on the other, remains incomplete. With few exceptions, such as Fearon (1999), Ashworth (2005), Acemoglu et al. (2013), and Besley (2006), the literature on electoral accountability has paid relatively little attention to situation where the preferences *and* actions of politicians are unobserved by voters. Such settings combine salient aspects of real-world elections, but they are analytically challenging, and as a consequence, research has been conducted under special modeling assumptions about the type space, the action space, or the information generated by policy choices.[2] In particular, models with continuous policy choices assume that policies generate signals with variance that is high relative to the rewards of office, and they restrict attention to equilibria in pure strategies. Banks and Sundaram (1993, 1998) and Duggan (2017) allow for general type spaces and information, but those articles analyze infinite-horizon models and must address difficulties that arise in the fully dynamic setting.

We present a two-period model of elections that allows us to study the dynamic incentives facing politicians and the policy choices emerging from those incentives. In contrast to much previous work, we eschew functional form assumptions and work in an environment similar to that of Banks and Sundaram (1998) and Duggan (2017). An advantage of this approach is that our analysis sheds greater light on the structure underlying different results. Importantly, we can also allow for an arbitrary degree of office motivation; this is not possible when only pure strategy equilibria are considered, and rewards of office cannot be too large relative to the variance of the signaling technology. We prove existence of equilibrium under general conditions, and we provide a characterization of equilibrium behavior in the

[2]For example, Ashworth (2005) assumes symmetric learning, so that politicians do not observe their own abilities; Acemoglu et al. (2013) and Besley (2006) assume there are just two politician types, and the latter assumes two possible policies. Ashworth (2005) and Acemoglu et al. (2013) assume normally distributed signals, and the latter add the assumption of quadratic utility.

model. We impose sufficient structure (satisfied in special cases of interest) such that in the first period, a politician can have at most two optimal policy choices, "taking it easy" and "going for broke," and voters follow a straightforward retrospective rule: re-elect the incumbent if and only if the observed policy outcome exceeds a cutoff level. The first-period office holder's choice must take account of the cutoff used by voters, and the updating of voter beliefs (and thus the voters' cutoff) depends on choices of the first period office holder via Bayes' rule; thus, electoral equilibria must solve a non-trivial fixed point problem.

As politicians become more office motivated, the re-election cutoff used by voters becomes arbitrarily demanding, and politicians respond to re-election incentives: above average politicians (i.e., politician types that are better in expectation for voters than the challenger) choose arbitrarily high policies; some low politician types choose policies that are above but close to their ideal points; and there is a unique "marginal type" that may mix between taking it easy and going for broke. Despite the increasing cutoff used by voters, politicians who go for broke are re-elected with probability close to one, demonstrating a strong form of incumbency advantage for politicians above the marginal type. Imposing additional restrictions on the politicians' payoff functions (still admitting the quadratic-normal special case), the marginal type is in fact the lowest politician type, and as politicians become more office motivated, this politician goes for broke with probability approaching one. In particular, *all* politician types go for broke and are re-elected with probability converging to one, and the incentive effects of elections shift to sanctioning (inducing politicians to choose high policy), rather than selection (screening higher types as office holders in the second period).

Our analysis implies that office motivation is a limited instrument to promote responsiveness, for several reasons. First, below average politicians may be discouraged when office motivation is high, as signaling becomes costly, and adopt policies that are close to their ideal policies with positive probability. Second, we assume voter preferences are strictly increasing over the ideal policies of the politician types, but we allow for the possibility that voter preferences are single-peaked, so that arbitrarily high policy choices are damaging to voters. In that case, excessive responsiveness may be damaging for voters. For example, if politicians choose economic stimulus policies, then it may be that voter preferences are initially increasing, but that overstimulation of the economy is harmful. In this case, politicians above the marginal type still respond to electoral incentives by choosing arbitrarily high policy, but the signaling technology has negative welfare effects; in terms of the growth example, politicians have an incentive to overstimulate the economy in the first period, in order to signal to voters that they will do a better than average job of managing the economy in the second period. Third, with very high office motivation, selection ceases to operate.

Normative implications of our results depend on the nature of policy. If policies are a public good, so that voters' preferences are increasing, then our responsiveness result has positive welfare implications for the effectiveness of elections: all above average politician types (and under stronger assumptions, all types) choose arbitrarily high policies, increasing the ex ante expected payoff of voters as office motivation becomes large. If policies are ideological, however, then the welfare implications of our analysis are qualified: when voter preferences are single-peaked, politician types who go for broke will overshoot in the first period, choosing policies above the voters' ideal. For example, if politicians choose economic stimulus policies, then it may be that voter preferences are initially increasing, but that overstimulation of the economy is harmful; or if politicians choose aggressive trade policies, then voter preferences may initially increase as politicians obtain advantageous terms, but the imposition of tariffs may generate costs that offset the benefits of the policy. In these cases, the signaling technology has negative effects, as politicians respond to office incentives by taking actions that, while causing harm to voters, signal that their type is above average.

In this light, electoral accountability may be seen in the best circumstances as one among several complementary formal and informal mechanisms for achieving responsiveness.[3] Ideally, for instance, political parties may be useful in candidate selection, and checks and balances may be useful in curbing excessive signaling. Our work highlights the delicate nature of the electoral accountability mechanism and its sensitivity to politicians' preferences and to the ways the public is informed about the consequences of policy decisions. Changes in the information environment, in particular, will have implications in terms of the effects of office motivation. We believe our results may be useful for further political economy work exploring these issues, and we conclude our analysis by discussing numerous applications and extensions of the accountability model to symmetric learning, populism, political cycles, harsh dictators and endogenous revolt, and electoral uncertainty.

In relation to the literature, at an intuitive level, our results bridge the work of Fearon (1999), who focuses on pure strategy equilibria in which politicians become more compliant as office motivation increases, and the work of Besley and Smart (2007), featuring mixed strategy behavior by the bad type of politician and pure strategy behavior by the good type. We find that electoral incentives shift from selection to sanctioning when politicians are highly motivated. The same feature of moral hazard trumping adverse selection is present in, e.g., Besley and Smart (2007), and it is consistent with recent empirical literature, e.g., Aruoba et al. (forthcoming), who find under some circumstances a strong disciplining effect but

[3] The idea that elections are insufficient to ensure good results for voters goes back at least to classical public choice contributions like Brennan and Buchanan (1980).

only a weak selection effect of elections. The normative conclusions of our analysis with ideological policies shares offers lessons similar to those of the "pandering" results of Canes-Wrone et al. (2001) and Maskin and Tirole (2004), but our result differs in subtle ways. Overshooting is not the result of the voter's incorrect beliefs or inability to learn, but rather the result of the voter inability to commit. The voter rewards politicians who signal more effectively, because they are more likely to be of the high type and will provide better policies next period, even if signaling is costly to the voter in the first period. This result has the flavor of the model of populism of Acemoglu et al. (2013) and of earlier work on signaling in models of the political business cycle as in Rogoff and Sibert (1988) and Rogoff (1990).

In Section 2, we present the two-period electoral accountability model, and discuss its relation with models which are widely used. In Section 3, we define the concept of electoral equilibrium, a refinement of perfect Bayesian equilibrium on which our analysis is focussed. In Section 4, we develop a simple two-type version of the model to introduce themes and results from the general analysis. In Section 5, we prove existence and present our results on responsive democracy as politicians become office motivated. In Section 6, we discuss applications and extensions; in Section 7, we gather final remarks; and we collect proofs of results in the Appendix.

2 Electoral accountability model

We analyze a two-period model of elections involving a representative voter, an incumbent politician, and a challenger. Prior to the game, nature chooses the types of the incumbent and challenger from the finite set $T = \{1,\dots,n\}$, with $n \geqslant 2$. These types are private information—in particular, they are unobserved by the voter—and are drawn identically and independently.[4] We let $p_j > 0$ denote the prior probability that a politician is type j. In period 1, the incumbent makes a policy choice $x_1 \geqslant 0$, which is unobserved by the voter, and a policy outcome $y_1 = x_1 + \varepsilon_1$ is stochastically determined, where ε_1 is an unobserved noise term that is distributed according to a distribution F with density f. In contrast to x_1 and ε_1, the outcome y_1 is observed by the voter. Then, after updating beliefs about the incumbent's type, the voter chooses between the incumbent and the challenger. In period 2, the winner of the election makes a policy choice $x_2 \geqslant 0$, a policy outcome $y_2 = x_2 + \varepsilon_2$ is determined, where ε_2 is again distributed according to F, and the game ends. Figure 1 illustrates the timeline of events in the model.

Given policy choice x and outcome y in either period, each player obtains a

[4]The analysis easily extends to allow the distribution of the challenger's type to differ from the incumbent's; we assume identical distributions only to reduce notation.

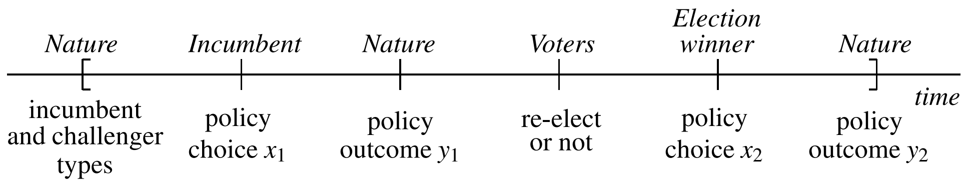

Figure 1: Timeline

payoff of $u(y)$ if not in office, while an office holder of type j receives a payoff of $w_j(x)+\beta$, where $\beta \geqslant 0$ represents the benefits of holding office.[5] Total payoffs for the voter and politicians are the sum of per-period payoffs. We assume that for all $j=1,\dots,n$, the type j politician's utility function w_j is twice differentiable and has unique maximizer $\hat{x}_j$, so that we admit the class of *power utility* functions such that $w_j(x) = -|x-\hat{x}_j|^r$ for $r>1$, including *quadratic utility,* with $w_j(x) = -(x-\hat{x}_j)^2$. As well, we admit the class of *exponential utility* functions, in which $w_j(x) = -e^{r(x-\hat{x}_j)} + r(x-\hat{x}_j)+1$ with $r>0$. In these cases, utility is strictly concave with ideal point $\hat{x}_j$. We assume that the incumbent's marginal utilities are ordered by type: for all $j<n$, we have

(A1) $w'_j(x) < w'_{j+1}(x)$ for all x, $w'_1(0) \geqslant 0$, and $w'_n(x) < 0$ for large enough x.

Here, beyond the politician's preferences as a citizen, w_j reflects her interests as a policy maker, e.g., the cost of effort or a preference for rents captured from voters. Note that (A1) implies that the ideal policies are ordered according to type, i.e.,

$$0 \leqslant \hat{x}_1 < \hat{x}_2 < \cdots < \hat{x}_n,$$

and that in the case of power or exponential utility, (A1) is automatically satisfied by indexing types in the order of their ideal points.

We assume that the density f of the noise term ε is differentiable and strictly positive on the real line, and we impose the standard monotone likelihood ratio property (MLRP), i.e., for all $x>x'$ and all $y>y'$, we have

(A2) $$\frac{f(y-x)}{f(y-x')} > \frac{f(y'-x)}{f(y'-x')},$$

where we use the fact that the likelihood of outcome $y=x+\varepsilon$ given policy choice x is $f(y-x)$. This implies that greater policy outcomes induce the voter to update

[5] It is simple to allow out-of-office payoffs to vary across types, as long as all types are interested in reelection; see (A5) below.

favorably her beliefs about the policy adopted by the incumbent in the first period. As is well-known, the MLRP implies that the density function is unimodal, and that both the density and the distribution functions are strictly log-concave.[6] Moreover, we assume that for all $x > x'$,

$$\text{(A3)} \qquad \lim_{y\to-\infty} \frac{f(y-x)}{f(y-x')} \;=\; \lim_{y\to+\infty} \frac{f(y-x')}{f(y-x)} \;=\; 0,$$

so that arbitrarily extreme signals become arbitrarily informative. In particular, we capture the benchmark case, used by Ashworth et al. (2018) and others, that $f(\cdot)$ is a mean-zero, normal density, which implies that conditional on the policy choice x, the outcome is normally distributed with mean x. We also admit the logistic density used by Fearon (1999).

Finally, we allow the voter's utility function u to be single-peaked or strictly increasing, with the only assumption being that the voter's expected utility is increasing in the range of politician ideal points. Specifically, defining the voter's expected utility from policy choice x by $\mathbb{E}[u(y)|x] = \int u(y)f(y-x)dy$, we assume that for all $x, x' \in [\hat{x}_1, \hat{x}_n]$ with $x < x'$, we have

$$\text{(A4)} \qquad \mathbb{E}[u(y)|x] \;<\; \mathbb{E}[u(y)|x'].$$

Obviously, given the MLRP, (A4) is satisfied if voter utility is strictly increasing, but we allow for the possibility that u is single-peaked with an ideal policy weakly greater than $\hat{x}_n$. Note that we can assume politicians share the voter's preferences over policy outcomes by specifying cost parameters $\theta_j > 0$ that are decreasing in type, i.e., $\theta_j > \theta_{j+1}$ for all $j < n$, assuming f is twice differentiable, and setting

$$w_j(x) \;=\; \mathbb{E}[u(y)|x] - \theta_j c(x)$$

for some twice differentiable cost function $c(\cdot)$ with $c(x)' > 0$ for all $x > 0$ and $c'(0) = 0$, as long as the resulting function w_j has a unique maximizer.

3 Electoral equilibrium

As in the citizen-candidate model, we assume that neither the incumbent nor the challenger can make binding promises before an election. We also assume that the voter cannot commit her vote, so that voting as well as policy making must be time consistent. Thus, our analysis focusses on perfect Bayesian equilibria of the electoral accountability model, under an additional refinement to preclude implausible

[6]See Bagnoli and Bergstrom (2005) for an in-depth analysis of log concavity and related conditions.

behavior on the part of the voter and politicians. Letting $X = \mathbb{R}_+$ and $Y = \mathbb{R}$ denote the spaces of policy choices and outcomes, respectively, a *strategy for the type j incumbent* is a pair (π_j^1, π_j^2), where

$$\pi_j^1 \in \triangle(X) \quad \text{and} \quad \pi_j^2 \colon X \times Y \to \triangle(X),$$

specifying mixtures over policy choices in period 1 and policy choices in period 2 for each possible previous policy choice and observed outcome.[7] A *strategy for the type j challenger* is a mapping

$$\gamma_j \colon Y \to \triangle(X),$$

specifying mixtures over policy choices in period 2 for each policy type and observed outcome. A *strategy for the voter* is a mapping

$$\rho \colon Y \to [0,1],$$

where $\rho(y)$ is the probability of a vote for the incumbent given outcome y. A *belief system for the voter* is a probability distribution $\mu(\cdot|y_1)$ on $T \times X$ as a function of the observed outcome.

A strategy profile $\sigma = ((\pi_j, \gamma_j)_{j \in T}, \rho)$ is *sequentially rational* given belief system μ if neither the incumbent nor the challenger can gain by deviating from the proposed strategies at any decision node, and if the voter votes for the candidate that makes her best off in expectation following all possible realizations of y_1. Beliefs μ are *consistent* with the strategy profile σ if for every y_1, the distribution $\mu(j, x|y_1)$ is derived from $(\pi_j^1)_{j \in T}$ via Bayes' rule. Note that the assumption that the output density has full support ensures that beliefs are always obtained from Bayes' rule and there is no need to consider off-equilibrium beliefs. A *perfect Bayesian equilibrium* is a pair (σ, μ) such that the strategy profile σ is sequentially rational given the beliefs μ, and μ is consistent with σ.

Sequential rationality implies that challengers will choose their ideal policies since there are no further elections, so that γ_j assigns probability one to $\hat{x}_j$ for all y_1. This implies that the expected payoff of electing the challenger for the voter is

$$V^C = \sum_k p_k \mathbb{E}[u(y)|\hat{x}_k].$$

Similarly, sequential rationality implies that $\pi_j^2(\cdot|x_1, y_1)$ assigns probability one to $\hat{x}_j$ for all x_1 and all y_1, so henceforth we assume politicians choose their ideal

[7] The notation $\triangle(\cdot)$ indicates the set of Borel probability measures over a given Borel measurable subset of Euclidean space. Measurability of strategies or subsets of policies will be assumed implicitly, as needed, without further mention.

policies in the second period, and we simplify notation by dropping the superscript from π_j^1 for the mixture over policies used by the type j politician in the first period. It follows that the expected payoff to the voter from re-electing the incumbent is

$$V^I(y_1) \;=\; \sum_k \mu_T(k|y_1)\mathbb{E}[u(y)|\hat{x}_k],$$

where $\mu_T(j|y_1)$ is the marginal distribution of the incumbent's type given policy outcome y_1. Thus, the incumbent is re-elected if $V^I(y_1) > V^C$ and only if $V^I(y_1) \geqslant V^C$.

An *electoral equilibrium* is any perfect Bayesian equilibrium that is *monotonic,* in the following sense: the voter follows a simple retrospective rule given by $\bar{y} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that she re-elects the incumbent if $y_1 > \bar{y}$ and only if $y_1 \geqslant \bar{y}$. Monotonicity includes the possibility that the voter always reelects, or never reelects, and it follows naturally from the interpretation of outcomes as signals of politicians' effort in the first period. In Proposition 1, below, we provide weak conditions on politician payoffs under which every perfect Bayesian equilibrium is an electoral equilibrium, in which case the natural restriction to monotonic equilibria is without loss of generality.

Observe that electoral equilibria are characterized by three conditions. First, updating of voter beliefs respects Bayes' rule, after observing outcome y_1. In particular, when the policy mixtures π_j are discrete, we can write

$$\mu_T(j|y_1) \quad = \quad \frac{p_j \sum_x f(y_1 - x)\pi_j(x)}{\sum_k p_k \sum_x f(y_1 - x)\pi_k(x)}.$$

Since the outcome density is positive, every outcome is on the path of play, so Bayes' rule pins down the voter's beliefs. We henceforth summarize an electoral equilibrium by the strategy profile σ, leaving beliefs implicit. Second, the threshold $\bar{y}$ must be such that, anticipating that politicians choose their ideal policies in the second period, the expected utility of re-electing the incumbent after observing y_1, given the voters' belief system, is greater than $\sum_k p_k\mathbb{E}[u(y)|\hat{x}_k]$ if $y_1 > \bar{y}$, and is greater than or equal to $\sum_k p_k\mathbb{E}[u(y)|\hat{x}_k]$ if and only if $y_1 \geqslant \bar{y}$. Since $\mu_T(j|y_1)$ is continuous in y_1, by the previous condition, it follows that *if* $\bar{y}$ is finite, then it must satisfy the indifference condition $V^I(\bar{y}) = V^C$ for the voter, or equivalently,

$$\sum_k \mu_T(k|\bar{y})\mathbb{E}[u(y)|\hat{x}_k] \quad = \quad \sum_k p_k\mathbb{E}[u(y)|\hat{x}_k]. \tag{1}$$

Third, since the incumbent is re-elected if $y_1 > \bar{y}$ and only if $y_1 \geqslant \bar{y}$, the type j incumbent's policy strategy π_j places probability one on maximizers of

$$w_j(x) + (1 - F(\bar{y} - x))[w_j(\hat{x}_j) + \beta] + F(\bar{y} - x)V^C, \tag{2}$$

so that the politician mixes over optimal actions in the first period.

To facilitate the analysis, we henceforth assume that all incumbent types are in principle interested in re-election, and that no type is willing to work arbitrarily hard to win, i.e., for all types j,

$$\text{(A5)} \qquad w_j(\hat{x}_j) + \beta \; > \; V^C \; > \; \lim_{x\to\infty} w_j(x) + \beta.$$

Note that the first part of (A5) is satisfied if β is sufficiently large,[8] and the second part holds if w_j is concave. By (A5), it is never optimal for a politician to choose a policy below her ideal policy in response to a retrospective voting rule, and the set of possible optimal choices is bounded above, so there is at least one solution to the incumbent's problem in the first period. Denoting such a solution by x_j^*, the necessary first order condition for a solution of the incumbent's maximization problem (2) is:[9]

$$w'_j(x_j^*) \quad = \quad -f(\bar{y} - x_j^*)[w_j(\hat{x}_j) + \beta - V^C]. \tag{3}$$

Thus, the marginal disutility in the current period from increasing the policy choice is just offset by the marginal utility in the second period, owing to the politician's increased chance of re-election, and regardless of the cutoff $\bar{y}$, the incumbent optimally exerts a positive amount of effort, i.e., chooses $x_j^* > \hat{x}_j$, in the first term of office.

A sufficient condition for every perfect Bayesian equilibrium to be monotonic is that the value of the ideal policy $w_j(\hat{x}_j)$ is constant in type, a condition that holds for all power and exponential payoff functions for the politicians. In fact, the next proposition shows that less is needed: it is enough that the ideal payoffs do not change too much relative to the difference in marginal costs of effort of the politicians.[10,11]

[8] The inequality $w_j(\hat{x}_j) + \beta > V^C$ is very reasonable, and it would be satisfied in a version of the model in which challengers entered at some cost.

[9] More precisely, the first order condition is $w'_j(x_j^*) \leqslant -f(\bar{y} - x_j^*)[w_j(\hat{x}_j) + \beta - V^C]$, with equality if $x_j^* > 0$. In fact, by (A1), we have $w'_j(0) \geqslant 0$ for all types, and by the assumption that $f(\cdot)$ is everywhere positive, with (A5), the right-hand side of (3) is negative; thus, we cannot have $x_j^* = 0$.

[10] In a non-monotonic equilibrium, for instance, it could be that a low type who is not very interested in re-election adopts a policy close to her ideal policy, while a high type who is extremely interested in re-election signals her type by adopting even lower policies because the voter expects her to do so; thus, lower policy outcomes are a signal of higher type. We consider such behavior implausible as it involves signaling the willingness to adopt high policies next period by adopting low policies in the current period.

[11] Note that Condition A6 of Banks and Sundaram (1998) and (C2) of Duggan (2017) each imply that optimal values are weakly increasing in type; they use this to establish existence of a monotonic equilibrium, and their characterization results are restricted to this class. In contrast, we do not restrict optimal values for our existence result. The necessity result of our Proposition 1 addresses a shortcoming by precluding non-monotonic equilibria.

Proposition 1. *Assume (A1)–(A5). If in addition, for all $j < n$ and for all $x > \hat{x}_{j+1}$,*

$$w_j'(x) \quad < \quad w_{j+1}'(x) \times \frac{w_j(\hat{x}_j) + \beta - V^C}{w_{j+1}(\hat{x}_{j+1}) + \beta - V^C}, \tag{4}$$

then in every perfect Bayesian equilibrium there is a cutoff $\overline{y} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that the voter votes to re-elect if $y_1 > \overline{y}$ and only if $y_1 \geqslant \overline{y}$. That is, every perfect Bayesian equilibrium is an electoral equilibrium.

4 Two-type model

For the special case of two types, we can calculate electoral equilibria explicitly, and we can demonstrate the necessity of mixing when politicians are sufficiently office motivated. The voter's cutoff is simply the solution to the equation $\mu_T(2|y) = p_2$, so that conditional on the cutoff $y^*(\pi_1, \pi_2)$, the probability that the incumbent is the high type is just equal to the prior probability. If policy strategies are pure, then we let x_1 and x_2 be the policies chosen by the two types, so that $y^*(x_1, x_2)$ solves the equation

$$p_2 \quad = \quad \frac{p_2 f(y - x_2)}{p_1 f(y - x_1) + p_2 f(y - x_2)},$$

or after manipulating, the likelihood of y is the same given the policy choices of the politician types, i.e., $f(y - x_1) = f(y - x_2)$. Adding the assumption that the density $f(\cdot)$ is symmetric around zero, the cutoff is simply the midpoint of the politicians' choices, i.e.,

$$\overline{y} \quad = \quad y^*(x_1, x_2) \quad = \quad \frac{x_1 + x_2}{2}.$$

We can depict an electoral equilibrium graphically by reformulating the incumbent's problem in terms of optimization subject to an inequality constraint. Define a new objective function

$$W_j(x, r) \quad = \quad w_j(x) + r[w_j(\hat{x}_j) + \beta - V^C],$$

which is the expected utility if the politician chooses policy x and is re-elected with probability r, minus a constant term corresponding to the current enjoyment of office. Note that W_j is well-behaved: in particular, if w_j is concave, then W_j is concave in (x, r), and it is quasi-linear in r. Of course, given x, the re-election probability is in fact pinned down as $1 - F(\overline{y} - x)$. Defining the constraint function

$$g(x, r) \quad = \quad 1 - F(\overline{y} - x) - r,$$

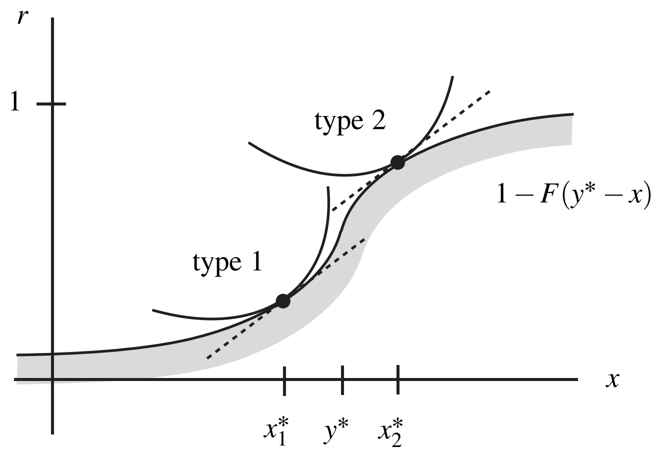

Figure 2: Electoral equilibrium in pure strategies with two types

we can then formulate the politician's optimization problem as

$$\begin{array}{c}\max_{(x,r)} W_j(x,r) \\ \text{s.t. } g(x,r) \geqslant 0,\end{array}$$

where the constraint is written as an inequality for heuristic purposes. As clearly illustrated by Figure 2, the politician's constraint set is non-convex, leading to the possibility of multiple optima and discontinuity with respect to parameters.

In the figure, we draw the indifference curves of the type 1 and type 2 politicians through their optimal policies, x_1^* and x_2^*, given the constraint set determined by the cutoff y^*. This is reflected in the tangency condition at each optimal policy. Moreover, the voter's indifference condition implies that the likelihood of outcome y^* is equal given either optimal policy, and this implies that the two tangent lines have equal slopes. Indeed, using the first order condition for office holders of types 1 and 2, we have

$$\frac{w_1'(x_1^*)}{w_1(\hat{x}_1)+\beta-V^C} \;=\; -f(y^*-x_1^*) \;=\; -f(y^*-x_2^*) \;=\; \frac{w_2'(x_2^*)}{w_2(\hat{x}_2)+\beta-V^C},$$

as claimed.

As an example, consider the quadratic-normal case with two types, e.g., $w_j(x) = -(x-\hat{x}_j)^2$ for $j = 1,2$, and f equal to the standard normal density. Assume that $\beta > V^C$, and that the voter is risk neutral, i.e., $u(y) = y$ for all y. From the necessary first order condition (3), we deduce that an equilibrium in pure policies must satisfy:[12]

$$x_j^* \;=\; x_j(\beta) \;=\; \hat{x}_j + \left(\frac{\beta - V^C}{2}\right) f\left(\frac{\hat{x}_2 - \hat{x}_1}{2}\right) \tag{5}$$

for $j = 1, 2$, and

$$y^* \;=\; y(\beta) \;=\; \frac{\hat{x}_1 + \hat{x}_2}{2} + \left(\frac{\beta - V^C}{2}\right) f\left(\frac{\hat{x}_2 - \hat{x}_1}{2}\right).$$

In terms of Figure 2, in an equilibrium in pure strategies, as β increases, the indifference curves of the two types move in parallel to the right; the equilibrium cutoff y^* increases so that the curve $1 - F(y^* - x)$ moves to the right in parallel as well.

It can be checked that if the incumbent is not too office motivated, i.e.,

$$\beta \;<\; V^C + \frac{2 - (\hat{x}_2 - \hat{x}_1)}{f\left(\frac{\hat{x}_2 - \hat{x}_1}{2}\right)},$$

then the objective function (2) of the type j politician is quasi-concave, so that the optimal policy choices, $x_1^* = x_1(\beta)$ and $x_2^* = x_2(\beta)$, and the voter's cutoff $y^* = y(\beta)$ determine a pure strategy equilibrium. However, if office benefit is too high, i.e.,

$$\beta \;>\; V^C + \frac{2}{-f'\left(\frac{\hat{x}^2 - \hat{x}_1}{2}\right)}, \tag{6}$$

then the second order condition of the problem of type 1 fails at $x_1(\beta)$. In fact, for high enough office motivation, the solution $x_1(\beta)$ of the type 1 politician's first order condition, from (5), becomes a local *minimum*, and thus a pure strategy equilibrium fails to exist. The politician of type 1 is better off by either deviating toward $x_2(\beta)$ and being rewarded with a higher probability of re-election, or deviating toward $\hat{x}_1$ and exploiting the time in office. Either deviation is incompatible with the cutoff $y(\beta)$.

For a numerical illustration, let the two politician types have ideal policies $\hat{x}_1 = 1$, $\hat{x}_2 = 2$, and set $\beta - V^C = 20$. We can construct an equilibrium recursively, beginning with the observation that inequality (6) holds, so equilibria will necessarily involve mixing. First, we obtain the unique cutoff such that the type 1 politician has two distinct optimal policies; in this example, given cutoff $y^* \approx 4.21$, the type 1 politician has optimal policy choices $x_{*,1} \approx 1$ and $x_1^* \approx 4.51$. Solving the first-order condition of the type 2 politician, we obtain $x_2^* \approx 4.98$. Finally, we back out

[12]The first order condition gives $x_j^* = \hat{x}_j + \left(\frac{\beta - V^C}{2}\right) f\left(\frac{x_2^* - x_1^*}{2}\right)$ for $j = 1, 2$. Subtracting the expression for type 1 from that for type 2, we deduce $x_2^* - x_1^* = \hat{x}_2 - \hat{x}_1$, which we use in the argument of $f(\cdot)$.

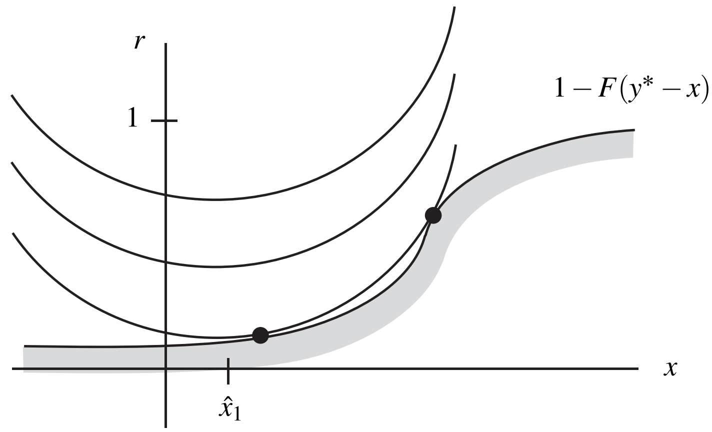

Figure 3: Type 1 politician's optimization problem with high office motivation

the mixing probabilities for the type 1 politician as determined by the voter's indifference condition to obtain $\pi_1(x_{*,1}) \approx 0.21$ and $\pi_1(x_1^*) \approx 0.79$. The probabilities of reelection for type 1 are approximately 0 and 0.62 depending on whether the politician adopts $x_{*,1}$ or x_1^*, and the probability of reelection for type 2 is 0.78.

The problem of the type 1 politician is depicted in Figure 3. Intuitively, as the voter becomes more demanding, the type 1 politician is tempted to quit the "rat race" with the type 2 politician and adopt a policy close to her ideal point, rather than choose a higher, more costly policy that generates a higher probability of reelection. The mixed strategy adopted by the type 1 politician is such that the cutoff adopted by the voter makes the type 1 politician indifferent between taking it easy and going for broke. Between the two equilibrium policy choices of the type 1 politician, there is a unique local minimum; a condition for this is given by (A6) in the next section.

Several features of the above construction are noteworthy. First, beyond the obvious fact that an electoral equilibrium exists, the voter's cutoff is demanding, in the sense that it substantially exceeds the ideal policies of the politicians. Second, the type 2 politician has a unique optimal policy choice, while the type 1 politician has two optimal policies and mixes with positive probability on each. Third, both politician types are responsive, as their greater optimal policy choices are also significantly above the ideal policy choices, while the type 1 politician's lower optimal policy is close to the ideal policy. Fourth, the greater policy choice x_1^* of the type 1 politician and the policy choice x_2^* of the type 2 politician are relatively close to each other. Last, the type 1 politician chooses the greater policy x_1^* with probability close to one, and conditional on policies x_1^* and x_2^*, the probability that the incumbent is reelected is high.

The above observations extend beyond the two-type model and the functional forms assumed here—to the model with general utilities and noise density, and with multiple politician types—when politicians are highly office motivated. Specialized to the quadratic-normal model, our results are summarized in the following proposition.

Proposition 2. *In the two-type model with quadratic politician payoffs, standard normal density, and risk neutral voter, there is an electoral equilibrium, and every perfect Bayesian equilibrium is an electoral equilibrium. Let the office benefit* β *become arbitrarily large. Then for every selection of electoral equilibria* σ*:*

(i) the voter's cutoff y^* *grows without bound, i.e.,* $y^* \to \infty$*;*

(ii) the type 1 politician mixes with positive probability on exactly two policies, $x_{*,1}$ *and* x_1^**, and the type 2 politician puts probability one on a policy* x_2^* *such that* $x_{*,1} < x_1^* < x_2^*$*;*

(iii) the lowest policy of the type 1 politician converges to type 1's ideal policy, the highest policies of both types increase without bound, i.e., $x_{*,1} \to \hat{x}_1$ *and* $x_1^*, x_2^* \to \infty$*;*

(iv) the highest optimal policies of the two types become arbitrarily close, i.e., $x_2^* - x_1^* \to 0$*;*

(v) the probability of reelection converges to one for both types, and in particular, the probability that the type 1 politician chooses the highest optimal policy converges to one, i.e., $1 - F(y^* - x_j^*) \to 1$ *for* $j = 1,2$ *and* $\pi_1(x_1^*) \to 1$.

Theorems 1–4, below, show that the properties stated in Proposition 2 are general features of electoral equilibria as politicians become highly office motivated. In particular, as politicians become more office motivated, the equilibrium cutoff y^* increases without bound; the greater optimal policy choice of the type 1 politician and the unique optimal choice of the type 2 politician become arbitrarily high; the type 1 politician mixes between its two optimal policies, with the probability of going for broke converging to one; the greater optimal policies of the politician types become close to each other; and the probability of re-election converges to one for both types. Our characterization is stated for to electoral equilibria, but by Proposition 1, if ideal payoffs of politicians do not vary too much across types, then in every perfect Bayesian equilibrium, politicians are responsive to high office incentives, and the sanctioning effect of elections leads to high policy choices in the first period, but this comes at the cost of selection, as elections fail to screen out the worst types.

5 Responsive democracy

A useful and intuitive property of electoral equilibria highlighted in Proposition 2 is that each politician type has at most two optimal policy choices. Lemma 1, in the Appendix, shows that this property extends to a class of politician payoffs and noise distributions including the quadratic-normal model. To formalize general conditions under which the result holds, define the functions $h = f'/f$ and $ARA_j = -w''_j/w'_j$ which are, respectively, the derivative of the log of the density f and the coefficient of absolute risk aversion of the type j politician. Next, we state a simple condition that is sufficient for the desired property, but one that is substantially stronger than needed: assume that for all j and all finite $\overline{y}$, we have

(A6) $ARA_j(x) - h(\overline{y} - x)$ is strictly concave on $x > \hat{x}_j$.

The key to this condition is the fact that if a policy choice $x > \hat{x}_j$ satisfies the necessary first and second order conditions for a local minimizer of the type j politician's objective function (2), then $ARA_j(x) + h(x) \geqslant 0$. Moreover, x satisfies the sufficient first and second order conditions for a strict local minimizer if the inequality holds strictly. By (A6), it follows that there cannot be a local maximizer between two local minimizers, and this leads to Lemma 1.[13,14] Although (A6) is a technical condition, it is permissive: if the politician payoffs w_j belongs to the power utility class, then $ARA_j(x) = -\frac{r-1}{x-\hat{x}_j}$, and if the noise density f is normal, then h is actually linear, so (A6) is fulfilled. Likewise, if w_j is exponential, then we have $ARA_j(x) = -1/(1 - e^{r(\hat{x}_j - x)})$, and again the condition is satisfied.

The usefulness of (A6) is that for an arbitrary cutoff $\overline{y}$, each type of incumbent has at most two optimal policies as a function of the cutoff. The greater solution to the incumbent's optimization problem, $x^*_j(\overline{y})$, corresponds to "going for broke," while the lesser solution, which is denoted $x_{*,j}(\overline{y})$, corresponds to "taking it easy." When these two policy choices coincide, the politician has a unique optimal policy; a gap between the two policy choices reflects the possibility that the increase in effort involved in going for broke is just offset by the increase in probability of being re-elected.

Our first main result establishes existence of electoral equilibrium, along with a partial characterization. Importantly, electoral equilibria must solve a fixed point problem: optimal policy choices of politician types depend on the cutoff used by the voter, and the cutoff used by the voter depends, via Bayes rule, on the policy

[13] In fact, our analysis does not rely crucially on the possibility of at most two optimal policies; our existence and characterization results would hold with any finite bound on the number of optimal policies, so (A6) can be correspondingly weakened.

[14] In the Appendix, we define a weaker condition (A6′) to this effect, and we prove all of our results using it, rather than the simpler condition stated above.

choices of politician types. The problem presents some unusual complications, but we provide a fixed point argument that addresses them in the Appendix.

Theorem 1. *Assume (A1)–(A6). Then there is an electoral equilibrium, and every electoral equilibrium is given by mixed policy strategies $\pi^*_1, \ldots, \pi^*_n$ and a finite cutoff y^* such that:*

(i) each type j politician mixes over policies using π^*_j, which places positive probability on at most two policies, say x^*_j and $x_{*,j}$, where $\hat{x}_j < x_{*,j} \leqslant x^*_j$,

(ii) the supports of policy strategies are strictly ordered by type, i.e., for all $j < n$, we have $x^*_j < x_{*,j+1}$,

(iii) the voter re-elects the incumbent if and only if $y \geqslant y^*$, and the cutoff satisfies $x_{*,1} + \hat{z} < y^* < x^*_n + \hat{z}$ where $\hat{z}$ is the mode of the outcome density.

To prove the theorem, we first establish that politicians' best responses to the voter's strategy are necessarily strictly monotonic in type. That is, politician strategies are given by an n-tuple $(\pi_1, \ldots, \pi_n)$ with supports that are ordered by type. An interesting technical aspect of the existence proof is that it is resistant to the application of standard fixed point theorems, such as Kakutani's theorem or Glicksberg's theorem. This is because the domain of ordered n tuples of strategies is not convex.[15] We deal with this issue by using a particular representation of a politician's strategy: rather than view it as a probability measure over policies, we view it as an ordered triple (x_j, z_j, r_j), where x_j represents the lowest policy choice, z_j represents the highest choice, and r_j is the probability that x_j is chosen (so that z_j is chosen with probability $1 - r_j$). We specify a profile of politician strategies as an ordered $3n$-tuple $((x_1, z_1, r_1), \ldots, (x_n, z_n, r_n))$ such that $z_j \leqslant x_{j+1}$ for all $j < n$, and along with politician strategies, we specify a cutoff for the voter, y. This gives us convexity of the domain, but the cost is that we lose the property that best response sets are convex.

To illustrate, in Figure 4, we use one axis to represent the lowest policy choice x_j, one for the highest choice z_j, and one for the probability r_j. Fixing a cutoff y for the voter, we depict a politician type j with lowest optimal policy $x_{*,j}(y)$ and highest optimal policy $x^*_j(y)$, so that the set of best responses is the union of bold lines in the cube. Here, if both policy choices are optimal, then any mixture between them is also optimal (the vertical portion of the best response set); but if $r_j = 1$, for example, then the higher policy z_j is chosen with probability zero, so any choice $z_j \geqslant x_{*,j}(y)$ is optimal (the portion of the best response set on the top

[15] See the discussion following Theorem 2 of Duggan (2017) for an explanation of this problem and how it affect the existence proof of Banks and Sundaram (1998).

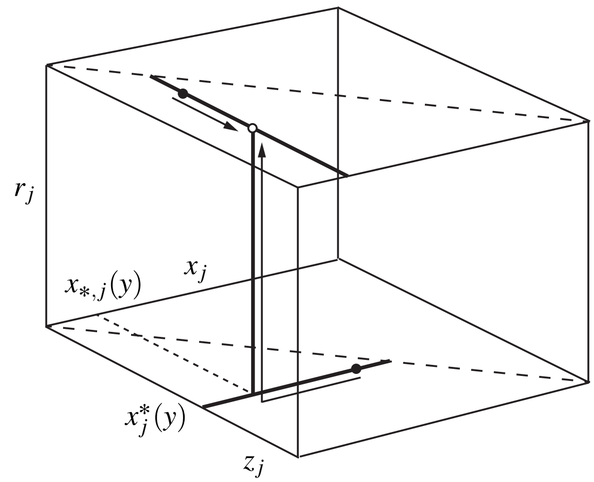

Figure 4: Contractibility of best responses

face). Nevertheless, the best response set is contractible, in the sense that it can be continuously deformed to a single element, which is $(x_{*,j}(y), x_j^*(y), 1)$ in the figure. This allows us to use the Eilenberg-Montgomery fixed point theorem (see McLennan (2018), Theorem 14.1.5) to deduce existence of a fixed point of the best response correspondence, which yields the desired electoral equilibrium.

We now consider the possibility of responsive democracy, meaning that incumbents choose high levels of policy, despite the short run temptation to choose their ideal policies. Under general conditions, we find that as office benefit grows large: the voter becomes arbitrarily demanding, in the sense that the equilibrium cutoff y^* diverges to infinity; that the policy choices of all politician types become close to their ideal policy or arbitrarily high; and that, in fact, all above average types choose arbitrarily high policies. Under additional restrictions on the curvature of politician payoffs, we find that the probability of re-election, conditional on the incumbent going for broke, converges to one. Finally, we isolate conditions such that the type 1 politician mixes with positive probability between taking it easy and going for broke, and such that the probability of shirking by the type 1 politician in fact goes to zero; in this case, and the probability of re-election goes to one for all politician types, demonstrating a form of incumbency advantage. We make use of a standard Inada-type condition: for all j,

(A7) $$\lim_{x\to\infty} w_j'(x) = -\infty.$$

Intuitively, this assumption requires that the marginal cost of effort to increase without bound as effort increases; it is satisfied in the quadratic and exponential cases and many other cases of interest. Henceforth, let $G = \{j : \mathbb{E}[u(y)|\hat{x}_j] > V^C\}$ denote the set of *above average* types, which are such that the expected utility from their ideal policy exceeds the expected utility from a challenger. Let $\ell = \min G$ be the smallest above average type, and note that $\ell \geqslant 2$.

Before proceeding to the formal analysis, we comment on the welfare implications of our responsiveness results. Under the interpretation of policy as a public good, we would assume that voter preferences are monotonic in policy outcomes, so that u is strictly increasing. Given the short time horizon (and limited ability of the voter to sanction politicians), and given the divergence in preferences between the voter and politicians, the prospects for well-functioning democratic elections may seem dim. Nevertheless, when β is large, so that politicians are substantially office-motivated, we obtain a potentially positive welfare result. Letting $\lim_{y\to\infty} u(y) = \overline{u}$, our analysis implies that when politicians are highly office motivated, the voter's ex ante expected payoff in the first period is bounded below by

$$\sum_{j\notin G} p_j \mathbb{E}[u(y)|\hat{x}_j] \quad + \quad \sum_{j\in G} p_j \overline{u}.$$

Of course, if the voter's utility function is not bounded above, then an immediate implication, since type n is above average and $p_n > 0$, is that the voter's expected utility from politicians' choices in the first period increases without bound as office benefit becomes large.

However, our analysis admits the possibility that voter utility u is single-peaked, so that high policy choices are actually damaging to the voter. In this case, the previous lower bound still holds, but it may be unrestrictive. If u is single-peaked and not bounded below as policy outcomes increase, then $\overline{u} = -\infty$, and the ex ante expected payoff of the voter decreases without bound, as each politician type chooses high policies at great cost in order to signal to voters that it is above average. At work is the fact that current policy outcomes are used only to update beliefs about the incumbent's type; utility from current policy (whether high or low) is "sunk," and is not used to reward or punish politicians.

The next theorem states our initial result on responsive democracy: as politicians become highly office motivated, the voter becomes demanding, so that y^* diverges to infinity, by requiring arbitrarily high policy outcomes to re-elect the incumbent; and all above average politician types go for broke, choosing arbitrarily high policies in pursuit of this increasing bar for re-election. An implication, because there is at least one above average type, is that policy choices are responsive to electoral incentives with positive probability. Note that condition (A7) is used only for parts (iii) and (iv) of the result.

Theorem 2. *Assume (A1)–(A7). Let the office benefit β become arbitrarily large. Then for every selection of electoral equilibria σ, the voter's cutoff diverges to infinity; the policy choices of each politician type either accumulate at their ideal policy or increase without bound; and the policy choices of all above average types increase without bound:*

(i) $y^* \to \infty$;

(ii) for all j, either $x_{*,j} = x_j^* \to \hat{x}_j$, or $x_{*,j} = x_j^* \to \infty$, or both $x_{*,j} \to \hat{x}_j$ and $x_j^* \to \infty$;

(iii) $x_{\ell-1}^* \to \infty$ and $\max\{\text{supp}(\pi_{k-1}^*)\} \to \infty$;

(iv) for all $j \geqslant \ell$, we have $x_{*,j} = x_j^* \to \infty$.

Theorem 2 implies that there is a "marginal type" $m \leqslant \ell - 1$ such that the policy choices of all types above the marginal type go to infinity, and the policy choices of all types below the marginal type converge to their ideal policies. For the marginal type itself, the highest optimal policy also increases without bound, while the lowest optimal policy (if different from the highest) converges to the marginal type's ideal point. Note that for a type $j < m$ below the marginal type, we have $x_j^* \to \hat{x}_j$, and since the voter becomes arbitrarily demanding, i.e., $y^* \to \infty$, we have $F(y^* - x_j^*) = 1$. Thus, the probability of reelection of every type below the marginal type converges to zero, and elections effectively screen out all politicians below the marginal type.

Corollary 1. *Under the assumptions of Theorem 2, for office benefit β sufficiently large and for every electoral equilibrium σ, there is a marginal politician type (which may depend on σ), denoted $m \leqslant \ell - 1$, such that:*

(i) for all $j < m$, $x_j^* = x_{*,j} \to \hat{x}_j$ and $1 - F(y^* - x_j^*) \to 0$;

(ii) for all $j > m$, $x_j^* = x_{*,j} \to \infty$;

(iii) $x_m^* \to \infty$ and if $x_m^* \neq x_{*,m}$, then $x_{*,m} \to \hat{x}_m$.

Theorem 2 does not identify which type is the marginal type, nor does it inform us about the probability that the incumbent politician is re-elected. It leaves open, for example, the possibility that the re-election probability of all above average politician types, which go for broke, is bounded strictly below one; in this case, elections would be effective in screening out undesirable types, but they would also screen out desirable types too often. To sharpen our characterization of the equilibrium behavior of politicians and to understand the electoral prospects of incumbents for large office benefit, we slightly strengthen assumption (A7) to require that absolute risk aversion go to zero for high policy choices: for all j,

$$\text{(A8)} \qquad w_j \text{ is concave and } \lim_{x\to\infty} ARA_j(x) = 0.$$

Note that (A8) is satisfied if (A7) holds and the second derivative of w_j is bounded below, i.e., $\lim_{x\to\infty} w_j''(x) > -\infty$; in turn, this is clearly satisfied for quadratic utility and all power utility functions.[16]

The next theorem shows that under the latter assumption, the probability of reelection of every type above the marginal type converges to one, and the lowest optimal policy of the type 1 politician converges to that type's ideal point. Thus, all above average types are re-elected with probability close to one, whereas it is optimal for type 1 politicians to shirk in equilibrium, so that elections can, in principle, at least screen out the type 1 incumbent from being re-elected. However, it may be that the type 1 politician is the marginal type—and then there is *no* type below marginal, and it may also optimal for the type 1 politician to go for broke. In this case, it is theoretically possible that *all* politician types go for broke with probability close to one, and that elections become ineffective at screening; we will see shortly that this possibility is realized under conditions that generalize the quadratic-normal model.

Theorem 3. *Assume (A1)–(A8). Let the office benefit β become arbitrarily large. Then for every selection of electoral equilibria σ, the least equilibrium policy $x_{*,1}$ of the type 1 politician converges to that type's ideal policy; the difference between the highest equilibrium policy and the voter cutoff goes to infinity for every type whose highest equilibrium policy goes to infinity; and in consequence the probability of reelection conditional on adopting that policy converges to one:*

(i) $x_{*,1} \to \hat{x}_1$;

(ii) if $x_j^* \to \infty$, *then both* $x_j^* - y^* \to \infty$ *and* $1 - F(y^* - x_j^*) \to 1$.

Our last result imposes additional structure to pin down the marginal type as type 1, and to characterize the response of type 1 politicians to equilibrium incentives. Recall that (A1) requires that marginal disutility of effort is ranked according to type, so that $\frac{w_{j+1}'(x)}{w_j'(x)} < 1$ for large enough x. In the case of power utility, this ratio

[16] If instead $w_j''(x)$ goes to $-\infty$, then by L'Hôpital's rule, the assumption is satisfied if (A7) holds and $w_j'''(x)$ is bounded, and so on for higher order derivatives. Assumption (A8) fails in the exponential case, in which the limit is equal to one.

actually goes to one as x increases. We use this restriction, in addition to concavity, to extend the findings of Proposition 2 from the quadratic-normal case to more general functional forms: for all $j < n$,

$$\text{(A9)} \qquad \lim_{x\to\infty} \frac{w'_{j+1}(x)}{w'_j(x)} \;=\; 1.$$

With our earlier assumptions, (A9) allows us to identify the marginal type as type 1. As office benefit becomes large, the type 1 politician has two optimal policy choices: take it easy by choosing policies arbitrarily close to her ideal point, or go for broke by choosing arbitrarily high policies. The type 1 politician shirks with positive probability, but that probability goes to zero, and the probability that the type 1 politician is reelected goes to one as office benefit increases. Moreover, the high policy choices of all politician types become arbitrarily close to each other when politicians are sufficiently office motivated.

Theorem 4. *Assume (A1)–(A9). Let the office benefit β become arbitrarily large. Then for every selection of electoral equilibria σ, the type 1 politician mixes between taking it easy and going for broke, and the probability of shirking goes to zero; for all politician types, the probability of reelection goes to one; and the greatest optimal policies of the types become arbitrarily close:*

(i) $x_{*,1} \to \hat{x}_1$, $x_1^* \to \infty$, *and* $\pi_1(x_1^*) \to 1$*;*

(ii) *for all* $j = 1,\ldots,n$, $1 - F(y^* - x_j^*) \to 1$*;*

(iii) $x_n^* - x_1^* \to 0$*;*

(iv) $0 < \pi_1(x_1^*) < 1$.

Thus, under (A1)–(A9), a form of incumbency advantage arises as office benefit becomes large, as all politician types go for broke and are re-elected with probability close to one. As a consequence, the positive selection effects established in Theorem 2 and Corollary 1 fail to kick in, because there are no types below marginal to screen out; all types $j \geqslant 2$ go for broke and, by Theorem 3, are reelected with probability converging to one. At most the marginal type 1 may be screened, but that possibility is precluded in Theorem 4. We conclude that elections lead to responsive policy choices when politicians are highly office motivated, but that the effect of electoral incentives shifts away from selection to sanctioning: in the limit, all types choose approximately the same high policy in the first period, and all types are reelected with probability close to one.

6 Applications and extensions

6.1 Competence and symmetric learning

The literature on political careers assumes that politicians have innate talent that augments their choice of effort; see, e.g., Persson and Tabellini (2000, Section 4.5), Ashworth (2005), and Ashworth and Bueno de Mesquita (2008). Here, an incumbent has competence $\gamma_j \in \{\gamma_1,\ldots,\gamma_n\}$, the politician chooses effort $e \geqslant 0$, and then a policy outcome $y = e+\gamma_j+\varepsilon$ is observed by the voter, where ε is a normally distributed shock. The politician's payoff is $-c(e)$, where $c(\cdot)$ is a twice differentiable cost of effort, with $c(e)' > 0$ and $c(e)'' > 0$ for $e > 0$, and $c'(0) = 0$. The voter's utility from y is either linearly increasing or quadratic in the outcome y. We can replicate this structure in the electoral accountability model by specifying policy payoffs $w_j(x) = -c(x-\gamma_j)$. These models are equivalent, upon identifying e with $x-\gamma_j$. Specifically, the politician's payoff from e in the competence model is equal to her payoff from $x-\gamma_j$ in ours, and the distribution of y conditional on e in the first coincides with the distribution conditional on $x-\gamma_j$ in ours, so that voter updating is preserved. Thus, in an equilibrium of the electoral accountability model, the choice of x with realized outcome $y = x+\varepsilon$ is equivalent to the choice of $e = x-\gamma_j$ with realized outcome $y = e+\gamma_j+\varepsilon$, as required.

The preceding discussion provides a specialization of the model that is consistent with an interpretation of policy choices as the sum of effort and competence, but we did not touch on politician information. In the electoral accountability model, we assume the incumbent's type is private information, so that γ_j is observed by the politician (but not the voter) prior to her policy choice in the first period. With this information structure, our results apply to the model of competence directly: under the maintained assumptions of the paper, an electoral equilibrium exists, and as office motivation increases, the voter becomes arbitrarily demanding, and all above average types choose arbitrarily extreme policies in an attempt to retain office. Notably, neither the usual assumption of linear or quadratic voter utility, nor a normally distributed shock, are needed. In the quadratic-normal special case, however, we can say that all types $j > 1$ choose arbitrarily extreme policies, and the lowest type mixes, with probability close to one on extreme policies; moreover, the probability that the incumbent is reelected goes to one, so that elections are unable to screen competent politicians.

In contrast to our assumption of privately informed politicians, much of the literature assumes symmetric learning, i.e., the incumbent does not observe her type, but rather shares the priors of the voter. To begin, we let $\hat{x}$ be the politician's ideal policy choice, and we reformulate (A6) in terms of the ex ante distribution, $\tilde{f}(z) = \sum_j p_j f(z)$. Specifically, we assume $ARA_j(x) - \tilde{f}'(\overline{y}-x)/\tilde{f}(\overline{y}-x)$ is strictly concave on $x > \hat{x}$. In this setting, given a cutoff $\bar{y}$ for the voter, the incumbent chooses x to solve

$$\max_{x \geqslant 0} w_j(x) + \sum_j p_j(1 - F(\bar{y} - x - \gamma_j))[w_j(\hat{x}_j) + \beta - V^C],$$

and Lemma 1 implies that the incumbent has at most two optimal policies. Moreover, given a mixed policy π, the voter's cutoff satisfies the indifference condition

$$\sum_k \mu_T(k|\bar{y})\mathbb{E}[u(y)|\hat{x}_k] \quad = \quad \sum_k p_k \mathbb{E}[u(y)|\hat{x}_k],$$

where μ_T is derived from Bayes' rule. Setting $\tilde{\pi}_1 = \pi$, and letting $\tilde{\pi}_j$ denote the result of "shifting" π to the right by $\gamma_j - \gamma_1$, we can express posterior beliefs conditional on outcome $\bar{y}$ as

$$\mu_T(j|\bar{y}) \quad = \quad \frac{p_j \int f(\bar{y} - x)\tilde{\pi}_j(dx)}{\sum_k p_k \int f(\bar{y} - x)\tilde{\pi}_k(dx)}.$$

Because these induced mixtures are ordered by type, Lemma 3 implies that there is a unique cutoff satisfying the voter's indifference condition.

Because the "gaps" between the induced mixtures $\tilde{\pi}_j$ and $\tilde{\pi}_{j+1}$ are determined exogenously as $\gamma_{j+1} - \gamma_j$, the analysis of the symmetric learning model is somewhat simpler than the model with asymmetric information. Our arguments imply that, again, an electoral equilibrium exists, and as office motivation increases, the voter becomes arbitrarily demanding. The incumbent mixes between going for broke and taking it easy, with the probability of extreme policies going to one, and the probability of reelection goes to one: again, elections lose the ability to discriminate between competent and incompetent politicians.

6.2 Populism

A recent literature has contemplated the possibility that electoral incentives may actually operate in a perverse manner, as the pursuit of re-election induces politicians to take actions that actually reduce the welfare of voters. In recent work on pandering, Acemoglu et al. (2013) consider a two-period model of elections with adverse selection and moral hazard. A representative voter has quadratic utility and ideal policy $\hat{z}_m$ at zero, and there are two politician types: an honest type with quadratic utility and ideal point $\hat{z}_h = 0$ equal to the median, and a type that accepts bribes with quadratic utility and (effective) ideal point $\hat{z}_b > 0$. The authors assume that conditional on policy choice z, the policy outcome is $y = z + \varepsilon$, where ε is distributed normally with mean zero and variance σ, and they assume the variance is sufficiently high (relative to office benefit) to permit an analysis of pure strategy equilibria. The authors show that although the politicians are conservative, the honest politician type chooses a liberal policy $z < 0$, which is bad for both the politician and the voter. When office benefit is sufficiently high, in fact they show that if there is an equilibrium in pure strategies, then both politician types choose liberal policies, in order to signal that they are not extreme.

The above model is obtained as a special case of the model of electoral accountability after an inessential transformation: locate the dishonest type to the left of the honest type, and translate ideal points so that $\hat{z}_b = 0$ and $\hat{z}_h = \hat{z}_m = 1$. Once this is done, since the voter's utility is increasing on the range of politician ideal points, we can apply the analysis of the accountability model. In particular, electoral equilibria exist regardless of office benefit, and the honest type chooses a conservative policy $z > 0$, and when office benefit is sufficiently high, the policy choice of the honest type becomes extreme, and the dishonest type mixes between policies close to her ideal point and arbitrarily extreme, conservative policies; moreover, the probability of the extreme choice goes to one, and both politician types are reelected with probability converging to one. Importantly, by Theorem 4, it is not possible to set the variance σ of the shock in advance, and then to derive extremism results for high office benefit—unless mixed strategies are accounted for.

Our analysis allows us to generalize the results of Acemoglu et al. (2013) significantly. We can assume any finite number of politician types with effective ideal points $0 \leqslant \hat{z}_1 < \cdots < \hat{z}_n = 1 = \hat{z}_m$ to the left of the voter, with type n representing an honest politician, who will choose the voter's ideal point in the second period, and types $1, \ldots, n-1$ representing politicians whose honest is increasing in type. The first-period incumbent chooses policy z, and the outcome is $y = z + \varepsilon$, but rather than assuming the shock is normally distributed, we can allow it to be distributed according to a density f satisfying (A2) and (A3). Generalizing quadratic utility, we can assume simply that the voter's utility is single-peaked, and that politician preferences satisfy (A1), (A5), and, along with the density f, (A6). By Theorem 1, an electoral equilibrium exists, and the honest type chooses policy to the right of the voter's ideal point. Adding (A7), it follows that as office motivation increases, the voter becomes arbitrarily demanding, and all above average types choose arbitrarily extreme policies to signal to the voter that they are not captured by interest groups. Thus, the incentive to engage in populism and the political inefficiency it entails, identified by Acemoglu et al. (2013), is a robust property of elections and not contingent on details of the model.

6.3 Political cycles

The electoral accountability model can be specialized to political business cycles, in the spirit of Persson and Tabellini (1990). Those authors consider a two-period model, in which the first-period incumbent chooses a level of inflation. Employment in the model is determined by the competence of the politician and inflation in an expectations-augmented Phillips equation. The payoff of the politician is the same as voters, with the exception of an additive benefit of holding office. At the time of election, voters observe employment but not inflation. Since politicians are interested in reelection, low-competence politicians may mimic high competence ones via surprise inflation. Persson and Tabellini study pooling and separating pure strategy equilibria.

We propose a model of political cycles that is equally natural and can be analyzed using the electoral accountability model. First, we allow for any finite set of types, $\{\theta_1,\ldots,\theta_n\}$, where $0<\theta_1<\cdots<\theta_n$, with higher types representing more competent politicians. In contrast to the above authors, we do not assume that the first-period incumbent controls inflation completely; rather, the incumbent chooses the level $x_1 \geqslant 0$ of a monetary instrument that influences inflation. This choice is not observed by voters, but it determines an economic variable (analogous to inflation) $y_1 = x_1+\varepsilon_1$ that is observed, where ε_1 is drawn from a density f satisfying (A2) and (A3). Let $y_1^e = \mathbb{E}[y_1]$ be the level of the economic variable expected by voters in the first period, and assume that employment is given by $z_1 = y_1 - y_1^e$. Voter utility in period t is quadratic in y_1 and linear in z_1, plus the constant $\sigma/2$, i.e.,

$$u(y_1) \quad = \quad -\frac{y_1^2}{2}+y_1-y_1^e+\frac{\sigma}{2}.$$

Using the fact that ε has mean zero, we have

$$\mathbb{E}[u(y)|x_1] \quad = \quad -\frac{x_1^2}{2}+x_1-y_1^e,$$

and thus the voters' payoff as a function of policy has the same form as assumed in Persson and Tabellini (1990). We define payoffs in the second period just as in the first.

We define the utility of the type j politician from policy x as

$$w_j(x) \quad = \quad -x^2/2+x-\frac{1}{\theta_j}c(x),$$

where $c(\cdot)$ is a twice differentiable cost function satisfying $c'(x)>0$ and $c''(x)>0$ for all $x>0$ and $c'(0)=0$. This means that politicians incur an extra cost in generating inflation, perhaps due to personal financial interests or commitments to interested parties, and importantly, the cost incurred is higher for less competent politician types. In particular, politicians have ideal points $0 < \hat{x}_1 < \hat{x}_2 < \cdots < \hat{x}_n < 1$, and voter preferences are increasing in this range. Thus, voters prefer to reelect more competent politician types, and the incumbent in the first period has the incentive to signal that they are a higher type.

This model of political cycles with adverse selection and moral hazard can be analyzed as a special case of the electoral accountability model, upon noting that the expected inflation term x_t^e in period $t = 1,2$ does not affect equilibrium. Expected inflation is pinned down by the politicians' equilibrium strategies, and it enters voter utility as a constant term, so it does not affect the voters' re-election decision even though it is important for welfare analysis. Therefore, adding (A6), Theorem 1 implies that that electoral equilibria exist, that each politician type mixes over at most two policy choices, and that all policy choices are strictly positive: incumbents have an incentive to inflate the economy to influence voter perceptions and increase their probability of re-election. An implication is that expected inflation is positive in equilibrium, i.e., $y^e > 0$. It follows that the voters' expected payoff in the first period is

$$\sum_j p_j \sum_x \mathbb{E}[u(y)|x]\pi_j(x) \quad = \quad -\frac{1}{2}\sum_j p_j \sum_x \mathbb{E}[y^2|x]\pi_j(x) \; < \; 0.$$

Moreover, as office benefit β increases, expected inflation becomes arbitrarily high, and voter welfare decreases without bound, as politicians seek to manipulate the economy to increase their chances of winning election.

6.4 Harsh dictators and endogenous revolt

The focus of the electoral accountability approach is democratic elections, but the model can be employed to elucidate the choices of any politician in the face of removal from her position, by democratic or other means. Assume a leader can be one of two types $j \in \{1,2\}$, and chooses an unobserved level of effort $x \geqslant 0$ to oppress the populace; in the analysis, the type 2 leader is interpreted as tough, and the type 1 as soft. Following the choice of x, a shock ε is drawn according to the density f satisfying (A2) and (A3), and the result, $y = x + \varepsilon$, is an observed level of oppression. The populace, which is modeled as a unitary player, then chooses whether to revolt or not, and the probability of a successful revolt is $\rho_j \in (0,1)$. Assume that $\rho_1 > \rho_2$, so that a revolt against the tough leader is less likely to be successful than one against a soft leader. After the outcome of the revolt is determined, the game ends.

Payoffs are as follows. A type j leader who chooses effort x receives a payoff $\tilde{w}_j(x)$, where $\tilde{w}_j$ is twice differentiable, has unique maximizer $\hat{x}_j$, and with the density f, satisfies (A6). If a successful revolt occurs, then the leader receives an additional payoff $-\pi$, and otherwise she receives $\tilde{\beta}$, where $\pi, \tilde{\beta} > 0$. Here, we interpret π as the level of punishment inflicted on the outgoing leader. If the populace attempts a revolt, then its payoff is R if it is successful, and its payoff is zero otherwise; and it does not attempt a revolt, then it receives a peaceful payoff P. We assume $\rho_1 R > P > \rho_2 R > 0$ to give the populace a non-trivial decision consistent with the interpretation of the model, and we impose a strengthening of (A1) as follows:

(A1$'$) $\dfrac{w_1'(x)}{\rho_1} < \dfrac{w_2'(x)}{\rho_2}$ for all x, $w_1'(0) \geqslant 0$, and $w_2'(x) < 0$ for large enough x.

To support equilibria analogous to the analysis of the paper, the division by ρ_j is needed to ensure that the tough leader is willing to exert greater oppressive effort than the soft leader. We do not specify the payoff of the populace from the outcome y, as it is a sunk cost; the realization of y only affects the decision of the populace through the updating of beliefs.

We analyze equilibria in which the tough leader type exerts higher oppressive effort, and the decision of the populace is determined by a cutoff $\overline{y}$ such that the populace revolts if and only if $y < \overline{y}$. In such an equilibrium, the cutoff is defined by the following indifference condition:

$$[\mu(1|\overline{y})\rho_1 + \mu(2|\overline{y})\rho_2]R \quad = \quad P,$$

where the left-hand side is the payoff to the populace from revolt, conditional on $\overline{y}$, and the right-hand side is the payoff from not attempting the revolt. Equivalently,

$$\mu(2|\overline{y}) \quad = \quad \frac{P - R\rho_1}{R(\rho_2 - \rho_1)}. \tag{7}$$

Given the cutoff, the type j leader solves

$$\max_{x \geqslant 0} \tilde{w}_j(x) + F(\overline{y} - x)(\rho_j(-\pi) + (1 - \rho_j)\tilde{\beta}) + (1 - F(\overline{y} - x))\tilde{\beta},$$

which is equivalent, up to a positive affine transformation, to

$$\max_{x \geqslant 0} \frac{\tilde{w}_j(x)}{\rho_j} + F(\overline{y} - x)(\pi + \tilde{\beta}). \tag{8}$$

We can map the equilibrium conditions (7) and (8) of the leader model into the equilibrium conditions of the accountability model. The voter's indifference condition in the two-type model of electoral accountability is

$$\mu(2|\bar{y}) \quad = \quad \frac{V^C - \mathbb{E}[u(y)|\hat{x}_1]}{\mathbb{E}[u(y)|\hat{x}_2] - \mathbb{E}[u(y)|\hat{x}_1]},$$

so we can replicate (7) by specifying u such that

$$\frac{V^C - \mathbb{E}[u(y)|\hat{x}_1]}{\mathbb{E}[u(y)|\hat{x}_2] - \mathbb{E}[u(y)|\hat{x}_1]} \quad = \quad \frac{P - R\rho_1}{R(\rho_2 - \rho_1)}$$

In the accountability model, the type j incumbent solves

$$\max_{x \geqslant 0} w_j(x) + F(\bar{y} - x)(w_j(\hat{x}_j) + \beta - V^C),$$

and we can replicate (8) by specifying policy utility $w_j(x) = \frac{\tilde{w}_j(x)}{\rho_j}$, and setting office benefit equal to $\beta = \pi + \tilde{\beta}$. Moreover, we can add a constant to policy utility so that $\frac{\tilde{w}_j(\hat{x}_j)}{\rho_j} = V^C$. Formulated this way, we have

$$w_j(x) + F(\bar{y} - x)(w_j(\hat{x}_j) + \beta - V^C) \quad = \quad \frac{\tilde{w}_j(x)}{\rho_j} + F(\bar{y} - x)(\pi + \tilde{\beta}),$$

so that a strategy profile is an equilibrium in the leader model if and only if it is an equilibrium of the accountability model.

Adding (A5), it is immediate that in the leader model, there exist equilibria such that the tough leader exerts higher oppressive effort than the soft leader, and that the decision of the populace to revolt is given by a cutoff: if the realized level of oppression is below the cutoff, then the leader is more likely to be soft, and the populace attempts a revolt; and otherwise, no revolt is attempted. Adding (A7), as the incentive to hold power $\pi + \tilde{\beta}$ increases, our results imply that the tough leader exerts arbitrarily high oppressive effort to remain in power, and the soft leader as well puts positive probability on arbitrarily high oppression. At the same time, ever greater oppression outcomes are needed to dissuade the populace from revolt. This is so even if the difference in vulnerability of the two leader types, $\rho_1 - \rho_2$, is small, as the leaders engage in a battle of beliefs to avoid revolt and possible removal. This is particularly applicable, given that the incentives to hold power hinge on the difference between staying in power, $\tilde{\beta}$, and the punishment from removal, $-\pi$. The empirical literature in international relations has demonstrated the frequency of violent removal (including death) of autocratic leaders, in which case π is naturally expected to be large. Adding (A8), our results also imply that the probability of attempted revolt is small when the incentive to retain power is large.

6.5 Electoral uncertainty

A feature of the electoral accountability model is that, conditional on the realized policy outcome, the decision of the voter is predictable by the first-period incumbent. The literature on candidate competition has incorporated "probabilistic voting," which reflects components of voters' decisions that are unobserved by candidates, as in Lindbeck and Weibull (1993) and Banks and Duggan (2005). We can extend the accountability model in this direction, and despite additional technicalities due to the increased complexity of voter behavior, we can derive the results on existence and extremism as office motivation increases.

Assume that following the incumbent's policy choice in the first period, the voter's payoff from the unknown challenger is subject to a random shock γ, distributed according to the density g, such that the voter's expected payoff from electing the challenger is $V^C + \gamma$. If policy choice strategies are ordered by type, then the voter's optimal strategy is a retrospective rule such that, conditional on the shock γ, the cutoff $\overline{y}(\gamma)$ satisfies the following indifference condition:

$$\sum_{j=1}^{n} \mu_T(j|y)\mathbb{E}[u(y)|\hat{x}_j] \quad = \quad V^C + \gamma.$$

Such a retrospective voting strategy $\overline{y}(\cdot)$ is continuous and increasing in γ, and the probability that the incumbent is not re-elected following policy choice x is then

$$\Phi(x|\overline{y}(\cdot)) \quad = \quad \int F(\overline{y}(\gamma) - x)g(\gamma)d\gamma,$$

reflecting the politician's uncertainty about the valence of the challenger and the corresponding cutoff used by the voter. The type j incumbent's payoff function becomes

$$w_j(x) + (1 - \Phi(x|\overline{y}(\cdot)))[w_j(\hat{x}_j) + \beta - V^C],$$

with necessary first order condition

$$w'_j(x) \quad = \quad \phi(x|\overline{y}(\cdot))[w_j(\hat{x}_j) + \beta - V^C],$$

where $\phi(x|\overline{y}(\cdot))$ is the derivative of $\Phi(x|\overline{y}(\cdot))$ with respect to x.

For the analysis to carry over to the model with electoral uncertainty, we modify (A6) by defining $\tilde{h}(x|\overline{y}(\cdot)) = \phi'(x|\overline{y}(\cdot))/\phi(x|\overline{y}(\cdot))$ and requiring that for all j and all continuous, increasing $\overline{y}(\cdot)$, the function $ARA_j(x) - \tilde{h}(x|\overline{y}(\cdot))$ is strictly concave on $x > \hat{x}_j$. Then Lemma 1, in the Appendix, carries over, and each politician type has at most two optimal policy choices in the first period. The proof of Lemma 2 goes through, using the objective function $W_j(x, 1 - \Phi(x|\overline{y}(\cdot))$, so that given $\overline{y}(\cdot)$, optimal policies are ordered by type. Most of Lemma 3 goes through unchanged, but now a realization γ must be selected, and the unique solution to

$$\sum_k \mu_T(k|\overline{y})\mathbb{E}[u(y)|\hat{x}_k] \quad = \quad V^C + \gamma \tag{9}$$

is also a function of the shock. The bound on the cutoff does not go through: for example, if γ is sufficiently large that $\mathbb{E}[u(\hat{x}_n)] < V^C + \gamma$, then the voter cannot be indifferent between the incumbent and challenger. But if

$$\mathbb{E}[u(y)|\hat{x}_1)] - V^C \; < \; \gamma \; < \; \mathbb{E}[u(y)|\hat{x}_n] - V^C, \tag{10}$$

then there is a finite solution $y^*(\pi_1, \ldots, \pi_n, \gamma)$ that is unique and is a continuous, strictly increasing function of the shock: if the voter has a greater preference for the challenger, then a higher realized outcome, signaling that the incumbent's type is higher, is needed to maintain voter indifference. Because the behavior of voters following shocks greater than $\mathbb{E}[u(y)|\hat{x}_n] - V^C$ or less than $\mathbb{E}[u(y)|\hat{x}_1)] - V^C$ is independent of the realized policy outcome, we assume without loss of generality that the density g has support on the interval $(\underline{\gamma}, \overline{\gamma})$ defined by $\overline{\gamma} = \mathbb{E}[u(y)|\hat{x}_n] - V^C$ and $\underline{\gamma} = \mathbb{E}[u(y)|\hat{x}_n] - V^C$.

The analysis of equilibrium existence must now confront several technical challenges. First, the current proof of Theorem 1 uses a bound on possible equilibrium cutoffs used by the voter, established in Lemma 3. This result no longer holds: for realizations close to $\overline{\gamma}$ or $\underline{\gamma}$, the voter's cutoff diverges to positive or negative infinity, respectively. But for a given γ, we can still deduce a bound on possible equilibrium cutoffs $\overline{y}(\gamma)$. Second, the current fixed point argument benefits from the fact that possible cutoffs belong to a compact interval, but now the argument involves a set of cutoff functions, $\overline{y}(\cdot)$, which must be topologized carefully. Nevertheless, a similar closed graph argument can be used to verify the conditions of the Eilenberg-Montgomery fixed point theorem, again yielding an electoral equilibrium.

As in Theorem 2, we can consider the responsiveness of policy choices as office motivation increases. Now, part (i) of Theorem 2 must be stated for all realizations of the shock: as β increases, if we consider any realization $\gamma \in (\underline{\gamma}, \overline{\gamma})$, then the equilibrium cutoff $\overline{y}(\gamma)$ used by the voter goes to infinity. That is, once again, voters become arbitrarily demanding in equilibrium. The argument for part (ii) is more nuanced, but we can again show that the optimal policy choices of each politician type either converge to her ideal point or diverge to infinity, as office motivation grows large.[17] Parts (iii) and (iv) of the theorem carry over as stated,

[17] In part (ii) of the current version of Theorem 2, we show more: if a politician does not mix between one policy close to her ideal point and one that becomes extreme, then she has a unique optimal policy choice.

with the implication that all above average types choose arbitrarily extreme policies in order to increase their chances of reelection.

Thus, the principle results carry over to the framework with electoral uncertainty, indicating that the incentives highlighted in the analysis are an inherent aspect of elections, rather than artifacts of modeling assumptions. They do not arise in the earlier literature on elections with probabilistic voting, which assumes the ability to commit on the part of politicians. In the absence of such commitment, the analysis points to the importance of private information, which can generate strong signaling incentives with significant normative implications, depending on the nature of the signaling technology and the welfare properties of effort exerted by the incumbent.

7 Concluding remarks

The ability of elections to provide incentives for politicians to deliver good policies for voters, via the expectation of re-election or career promotion, has been the object of much attention, both in academic work and policy debate.[18] The two-period model of elections provides a canonical setting for analysis of the interplay between short-term opportunistic incentives and long-term re-election incentives in determining politicians' behavior, and thus it is potentially useful in framing the analysis of electoral accountability. In this paper, we consider an environment with realistically sparse information, in which voters are imperfectly informed about both the preferences and the actions of politicians. We assume that politicians and voters cannot commit to future actions, opening the scope for opportunistic behavior and creating potential difficulties for the success of democratic electoral mechanisms. In contrast to much previous work, we assume an arbitrary number of politician types and general preferences and signaling technology, permitting an arbitrary degree of office motivation, and shedding greater light on the structure underlying different results.

We establish existence of an electoral equilibrium, and we show that in all such equilibria, voters use a simple cutoff rule to determine electoral outcomes, and politicians mix over at most two policy choices,"taking it easy" and "going for broke," with a unique marginal type that may mix between the two. We then show the possibility of responsive democracy: as politicians become highly office motivated, voters become more demanding; and policy choices of above average politicians in the first period increase without bound. Responsiveness is not an

[18]Achen and Bartels (2016), for instance, offer a version of Fearon's (1999)model in an appendix. They focus exclusively on pure strategy equilibria and first order conditions, which we show are not sufficient.

unalloyed blessing, as politicians in the first period may go beyond the ideal policies of the voter, but the harm imposed by extreme policy choices is sunk; thus, in equilibrium, the role played by the observed policy outcome purely informational, and higher policy outcomes are evidence that the incumbent is the high type. Under additional assumptions, incentive effects of elections shift from selection to sanctioning, as all politician types go for broke and are re-elected with probability approaching one, implying a strong form of incumbency advantage.

Beyond the specific issues addressed in this paper, the accountability model, with the machinery we have developed for it, invites application to the study of a broad range of questions. We illustrated a number of such avenues in the previous section, namely, applications and extensions to symmetric learning, populism, political cycles, harsh dictators and endogenous revolt, and electoral uncertainty. Another topic that deserves special mention is the changing information environment and its impact in the way that politicians communicate both intentions and results to the electorate. In this vein, for instance, Kartik and Van Weelden (2018) examine the consequences of good and bad signaling technology in a simplified environment with a myopic voter and binary policy space. A related topic is the effect of cognitive limitations on accountability; for example, Matějka and Tabellini (2016) study the effect of rational inattention on electoral competition, and closer to our focus, Lockwood (2017) considers the implications of confirmation bias for political agency. The accountability framework would seem to be a promising setting for the analysis of these problems, and a host of others characterized by sparse information, signaling via policy choices, and limited scope for commitment.

A Proofs of results

In the Appendix, we replace (A6) with the following weakening: assume that for all j, all finite $\bar{y}$, and all $x, \tilde{x}, z$ with $\hat{x}_j < x < \tilde{x} < z$,

$$\text{(A6}'\text{)} \qquad \text{if } ARA_j(x) \geqslant h(\bar{y}-x) \text{ and } ARA_j(z) \geqslant h(\bar{y}-z), \\ \text{then } ARA_j(\tilde{x}) > h(\bar{y}-\tilde{x}).$$

We first show that we can limit the need for mixing to at most two policy choices for each type.[19]

Lemma 1. *Assume (A1), (A5), and (A6′). For every cutoff $\bar{y} \in Y$ and every type j, there are at most two optimal policies, i.e., two maximizers of the objective function*

[19]The possibility of multiple optimizers has a counterpart in static models of elections with probabilistic voting, where log concavity is used to ensure existence of a unique optimal policy for each candidate (cf. Roemer (1997) and Bernhardt et al. (2009)).

(2). For every type j, the greatest and least optimal policies for type j, $x_j^*(\cdot)$ *and* $x_{*,j}(\cdot)$ *are, respectively, upper and lower semi-continuous functions of* $\overline{y}$*.*

Proof of Lemma 1: Suppose toward a contradiction that there are three distinct local maximizers of the type j politician's objective function, say x', x'', and x''' with $x' < x'' < x'''$. Thus, there are local minimizers z' and z'' such that $x' < z' < x'' < z'' < x'''$. With (A5), inspection of the first order condition (3) at $x = z', z''$ reveals that $w_j'(z') < 0$ and $w_j'(z'') < 0$, so we can write the first order condition at z' and z'' as

$$w_j(\hat{x}_j) + \beta - V^C \;=\; -\frac{w_j'(z')}{f(\overline{y}-z')} \;=\; -\frac{w_j'(z'')}{f(\overline{y}-z'')}.$$

By the necessary second order condition for a local minimizer, the second derivative at z' satisfies

$$0 \;\leqslant\; w_j''(z') - f'(\overline{y}-z')[w_j(\hat{x}_j) + \beta - V^C] \;=\; w_j''(z') - f'(\overline{y}-z')\left[-\frac{w_j'(z')}{f(\overline{y}-z')}\right],$$

or equivalently,

$$\frac{w_j''(z')}{w_j'(z')} \;\leqslant\; -\frac{f'(\overline{y}-z')}{f(\overline{y}-z')}.$$

Similarly, we have

$$\frac{w_j''(z'')}{w_j'(z'')} \;\leqslant\; -\frac{f'(\overline{y}-z'')}{f(\overline{y}-z'')}.$$

Since x'' is a local maximizer, the first order condition holds at x'', and the second derivative at x'' is non-positive, but then we have

$$\frac{w_j''(x'')}{w_j'(x'')} \;\geqslant\; -\frac{f'(y-x'')}{f(y-x'')},$$

contradicting (A6$'$). We conclude that the objective function has at most two local maximizers, and therefore there are at most two optimal policies for type j.

From previous arguments and (A1), optimal policies for type j are bounded below by $\hat{x}_j \geqslant 0$ and above by $\overline{x}_j > \hat{x}_j$ such that $w_j(\overline{x}_j) + \beta = V^C$. A standard application of Berge's theorem of the maximum (see e.g. Border (1985), Theorem 12.1) implies that the correspondence of optimal best responses is nonempty-valued and is upper hemi-continuous in $\overline{y}$. Since the correspondence of optimal best-responses includes at most two policies for each cutoff, upper hemi-continuity of the best response correspondence is equivalent to the greatest and least optimal policies for type j, $x^*_j(\cdot)$ and $x_{*,j}(\cdot)$ being, respectively, upper and lower semi-continuous functions of $\overline{y}$. $\square$

The next lemma establishes that, given an arbitrary value $\overline{y}$ of the cutoff, the maximizers of (2) are ordered by type—a property that is key for the subsequent analysis.

Lemma 2. *Assume (A1) and (A5). For every cutoff $\overline{y}$, maximizers of (2) are strictly ordered by type, i.e., for all $j < n$, if x is optimal for type j and z is optimal for type $j+1$, then $x < z$.*

Proof of Lemma 2: Consider $j > k$ and $x > z$. Suppose first that $w_j(\hat{x}_j) \geqslant w_k(\hat{x}_k)$. Recall that, for every cutoff $\overline{y}$, the objective function (2) for j can be written as $W_j(x, 1-F(\overline{y}-x)) = w_j(x) + (1-F(\overline{y}-x))[w_j(\hat{x}_j) + \beta - V^C]$. We claim that

$$\begin{aligned} & W_j(x,1-F(\overline{y}-x)) - W_j(z,1-F(\overline{y}-z)) \\ & \quad > \quad W_k(x,1-F(\overline{y}-x)) - W_k(z,1-F(\overline{y}-z)), \end{aligned}$$

or equivalently,

$$\begin{aligned} & w_j(x) - w_j(z) + (F(\overline{y}-z) - F(\overline{y}-x))[w_j(\hat{x}_j) + \beta - V^C] \\ & \quad > \quad w_k(x) - w_k(z) + (F(\overline{y}-z) - F(\overline{y}-x))[w_k(\hat{x}_k) + \beta - V^C]. \end{aligned}$$

Since $x > z$, we have $F(\overline{y}-z) - F(\overline{y}-x) > 0$. Using $w_j(\hat{x}_j) \geqslant w_k(\hat{x}_k)$ and (A5),

$$\begin{aligned} & (F(\overline{y}-z) - F(\overline{y}-x))[w_j(\hat{x}_j) + \beta - V^C] \\ & \quad > \quad (F(\overline{y}-z) - F(\overline{y}-x))[w_k(\hat{x}_k) + \beta - V^C]. \end{aligned}$$

In addition, continuous differentiability of w_j and (A1) imply $w_j(x) - w_j(z) > w_k(x) - w_k(z)$, as required. The ordering of maximizers of $W_j(x, 1-F(\overline{y}-x))$ then follows from standard supermodularity arguments.[20]

Now suppose that $w_j(\hat{x}_j) < w_k(\hat{x}_k)$. Using (A5), we can renormalize the objective function as

$$\widetilde{W}_j(x, 1-F(\overline{y}-x)) \quad = \quad \frac{w_j(x)}{w_j(\hat{x}_j) + \beta - V^C} + (1-F(\overline{y}-x)).$$

[20] Supermodularity has been of course used in a variety of contexts by Milgrom and Shannon (1994) and Athey (2002) and others in order to obtain ordered best-responses.

We claim that

$$\begin{aligned}&\widetilde{W}_j(x, 1-F(\overline{y}-x)) - \widetilde{W}_j(z, 1-F(\overline{y}-z)) \\ &> \quad \widetilde{W}_k(x, 1-F(\overline{y}-x)) - \widetilde{W}_k(z, 1-F(\overline{y}-z)),\end{aligned}$$

or equivalently,

$$\frac{w_j(x) - w_j(z)}{w_j(\hat{x}_j) + \beta - V^C} \quad > \quad \frac{w_k(x) - w_k(z)}{w_k(\hat{x}_k) + \beta - V^C}.$$

This follows from continuous differentiability of w_j, (A1), and $w_k(\hat{x}_k) + \beta - V^C > w_j(\hat{x}_j) + \beta - V^C > 0$. Again, the ordering of maximizers of $\widetilde{W}_j(x, 1-F(\overline{y}-x))$ follows from standard supermodularity arguments. □

The ordering of optimal policies is very useful in combination with the fact that given arbitrary policy choices $x_1 < x_2 < \cdots < x_n$ of the politician types in the first period, there is a unique outcome, which we denote $y^*(x_1, \ldots, x_n)$, such that conditional on realizing this value, the voter is indifferent between re-electing the incumbent and electing a challenger. Moreover, as shown below, this extends to the case of mixed policy strategies $\pi_1, \ldots, \pi_n$ with supports that are strictly ordered by type, i.e., for all $j < n$,[21]

$$[x \in \text{supp}(\pi_j) \text{ and } z \in \text{supp}(\pi_{j+1})] \quad \Rightarrow \quad x < z.$$

That is, given such mixed policy strategies, there is a unique solution to the voter's indifference condition in (1), and we let $y^*(\pi_1, \ldots, \pi_n)$ denote the solution to the voter's indifference condition as a function of policy choices. In addition to uniqueness, the next lemma establishes that the cutoff is continuous in policy strategies and lies between the choices of the type 1 and type n politicians, shifted by the mode of the density of $f(\cdot)$, which we denote by $\hat{z}$.

Lemma 3. *Assume (A2) and (A3). For all mixed policy strategies $\pi_1, \ldots, \pi_n$, with supports that are bounded and strictly ordered by type, and for all belief systems μ derived via Bayes rule, there is a unique solution to the voter's indifference condition (1), and the solution $y^*(\pi_1, \ldots, \pi_n)$ is continuous as a function of mixed incumbent strategies. Moreover, this solution lies between the extreme policy choices shifted by the mode of the outcome density, i.e.,*

$$\min\big(\text{supp}(\pi_1)\big) + \hat{z} \; < \; y^*(\pi_1, \ldots, \pi_n) \; < \; \max\big(\text{supp}(\pi_n)\big) + \hat{z}.$$

[21] In Lemma 3, we assume supports of mixed policies are bounded, so they are in fact compact.

Proof of Lemma 3: For existence of a solution to the indifference condition, fix $\pi_1,\ldots,\pi_n$ with supports that are strictly ordered by type, and note that the left-hand side of (1) is continuous in $\bar{y}$. For each type j, let $x_j = \min\big(\text{supp}(\pi_j)\big)$ be the lowest policy in the support of the type j politician's policy mixture, and let $x^j = \max\big(\text{supp}(\pi_j)\big)$ be the highest policy in the support. Since f is single-peaked, by (A2), it follows that for all cutoffs $\bar{y} > x^n$, all types $j < k$, and all policies $x \in \text{supp}(\pi_j)$ and $x' \in \text{supp}(\pi_k)$, we have

$$f(\bar{y}-x) \;\leqslant\; f(\bar{y}-x^j) \;\leqslant\; f(\bar{y}-x_k) \;\leqslant\; f(\bar{y}-x').$$

The last inequality above implies $f(\bar{y}-x_n) \leqslant \int f(\bar{y}-x)\pi_n(dx)$. Given $j<n$, the first inequality above implies $\int f(\bar{y}-x)\pi_j(dx) < f(\bar{y}-x^j)$. We conclude that

$$\mu_T(j|\bar{y}) \;=\; \frac{p_j \int f(\bar{y}-x)\pi_j(dx)}{\sum_k p_k \int f(\bar{y}-x)\pi_k(dx)} \;\leqslant\; \frac{p_j}{p_n}\frac{f(\bar{y}-x^j)}{f(\bar{y}-x_n)}$$

for all $j<n$. By (A3), it follows that $f(\bar{y}-x^j)/f(\bar{y}-x_n) \to 0$ as $\bar{y}\to\infty$, and thus

$$\lim_{\bar{y}\to\infty} \mu_T(j|\bar{y}) \;=\; 0,$$

which implies that $\mu_T(n|\bar{y}) \to 1$ as $\bar{y}\to\infty$. In words, when the policies of the politicians are ordered by type, high realizations of the outcome become arbitrarily strong evidence that the incumbent is the best possible type as the voter's cutoff increases. Similarly, $\mu_T(1|\bar{y})$ goes to one as $\bar{y}$ decreases without bound. Thus, the left-hand side of (1) approaches $\mathbb{E}[u(y)|\hat{x}_n]$ when the cutoff is large, and it approaches $\mathbb{E}[u(y)|\hat{x}_1]$ when the cutoff is small, and existence of a solution follows from the intermediate value theorem.

To show uniqueness, we claim that the left-hand side of (1) is strictly increasing in $\bar{y}$. Since higher types choose better policies for the voter in the second period, to prove the claim it is enough to show that $\mu_T(\cdot|\bar{y})$ exhibits first order stochastic dominance over $\mu_T(\cdot|\bar{y}')$ for $\bar{y} > \bar{y}'$; we claim the slightly stronger condition that for each $1 \leqslant j \leqslant n$, the inequality $\bar{y} > \bar{y}'$ implies

$$\sum_{k\geqslant j} \mu_T(j|\bar{y}) \;>\; \sum_{k\geqslant j} \mu_T(j|\bar{y}').$$

This is the case if

$$\frac{\sum_{k=j}^n p_k \int f(\bar{y}-x)\pi_k(dx)}{\sum_{m=1}^n p_m \int f(\bar{y}-x')\pi_m(dx')} \;>\; \frac{\sum_{k=j}^n p_k \int f(\bar{y}'-x)\pi_k(dx)}{\sum_{m=1}^n p_m \int f(\bar{y}'-x')\pi_m(dx')},$$

or equivalently, cancelling terms appearing on both sides of the inequality,

$$\sum_{m=1}^{j-1}\sum_{k=j}^{n} p_m p_k \left(\int f(\overline{y}'-x')\pi_m(dx')\right)\left(\int f(\overline{y}-x)\pi_k(dx)\right)$$
$$> \sum_{m=1}^{j-1}\sum_{k=j}^{n} p_m p_k \left(\int f(\overline{y}-x')\pi_m(dx')\right)\left(\int f(\overline{y}'-x)\pi_k(dx)\right).$$

Writing $\pi_k \otimes \pi_m$ for the product measure on $\mathbb{R}^2$ generated by π_k and π_m, this becomes

$$\sum_{m=1}^{j-1}\sum_{k=j}^{n} p_m p_k \int f(\overline{y}'-x')f(\overline{y}-x)(\pi_k \otimes \pi_m)(d(x,x'))$$
$$> \sum_{m=1}^{j-1}\sum_{k=j}^{n} p_m p_k \int f(\overline{y}-x')f(\overline{y}'-x)(\pi_k \otimes \pi_m)(d(x,x'))$$

Since supports are strictly ordered by type, it follows that for all types $m < k$ and all policies $x \in \text{supp}(\pi_k)$ and $x' \in \text{supp}(\pi_m)$, we have $x > x'$. Since $\overline{y} > \overline{y}'$, it follows from (A2) that

$$f(\overline{y}'-x')f(\overline{y}-x) > f(\overline{y}-x')f(\overline{y}'-x),$$

and the desired inequality follows.[22] Standard continuity arguments imply that $y^*(\pi_1,\ldots,\pi_n)$ is continuous as a function of mixed policy strategies with discrete supports.

To obtain the upper bound on the cutoff, consider any $\overline{y} \geqslant x^n + \hat{z}$. Recall that the posterior probability that the politician is type j, conditional on observing $\overline{y}$, is

$$\mu_T(j|\overline{y}) \quad = \quad \frac{p_j \int f(\overline{y}-x)\pi_j(dx)}{\sum_k p_k \int f(\overline{y}-x)\pi_k(dx)}.$$

Note that for all types $j < k$ and all policies $\tilde{x}_j \in \text{supp}(\pi_j)$ and $\tilde{x}_k \in \text{supp}(\pi_k)$, we have $\hat{z} \leqslant \overline{y} - \tilde{x}_k < \overline{y} - \tilde{x}_j$. Since $f(\cdot)$ is single-peaked by (A2), we see that for all $\tilde{x}_1,\ldots,\tilde{x}_n$ such that each $\tilde{x}_k$ is in the support of π_k, we have

$$f(\overline{y}-\tilde{x}_1) \ < \ f(\overline{y}-\tilde{x}_2) \ < \ \cdots \ < \ f(\overline{y}-\tilde{x}_n).$$

Therefore, the coefficients on prior beliefs are ordered by type, i.e.,

$$\frac{\int f(\overline{y}-x)\pi_1(dx)}{\sum_k p_k \int f(\overline{y}-x)\pi_k(dx)} \ < \ \cdots < \ \frac{\int f(\overline{y}-x)\pi_n(dx)}{\sum_k p_k \int f(\overline{y}-x)\pi_k(dx)},$$

[22]Banks and Sundaram (1998) develop a similar argument in a related problem (Lemma A.6).

and we conclude that the posterior distribution $\mu_T(\cdot|\overline{y})$ first order stochastically dominates the prior, contradicting the indifference condition. An analogous argument leads to a contradiction for the case $\overline{y} \leqslant x_1 + \hat{z}$. □

Proof of Proposition 1: Fix a perfect Bayesian equilibrium, and let ρ be the strategy of the voter, let $(\pi_j)_{j\in T}$ be the probability measures representing the strategies pursued by the politician types in the first period, and let μ be the belief system of the voter. We claim first that the supports of $(\pi_j)_{j\in T}$ are ordered by type. To see this, note that the probability of re-election given a choice of policy x is given by $\int_y \rho(y) f(y-x)\,dy$. Hence, by a positive affine transformation, we can rewrite the objective of a politician of type j as

$$\tilde{W}_j(x) \quad = \quad \frac{w_j(x)}{w_j(\hat{x}_j)+\beta-V^C} + \int_y \rho(y) f(y-x)\,dy.$$

As in Lemma 2, consider types $j<k$ and policy choices $z<x$. Then

$$\tilde{W}_j(x)-\tilde{W}_j(z) \quad > \quad \tilde{W}_k(x)-\tilde{W}_j(z)$$

holds if and only if

$$w_j(x)-w_j(z) \quad > \quad (w_k(x)-w_k(z)) \times \left(\frac{w_j(\hat{x}_j)+\beta-V^C}{w_k(\hat{x}_k)+\beta-V^C}\right),$$

and this follows from (4) and continuous differentiability of w_j. Then standard supermodularity arguments imply that the supports of the mixtures π_j adopted by $j=1,\dots,n$ are strictly ordered by type. By (A5), the optimal policy choices of each politician type are bounded, and thus the supports of each π_j are compact. Thus, Lemma 3 yields a unique cutoff $\overline{y}$ such that, conditional on observing $\overline{y}$, the voter is indifferent between the incumbent and the challenger. Then the argument, in the proof of Lemma 3, that the left-hand side of (1) is strictly increasing in $\overline{y}$ delivers the desired result. □

Proof of Theorem 1: In proving the theorem, we must address three technical subtleties. The first is that when supports of mixed policy choices are only weakly ordered, the left-hand side of (1) is only weakly increasing, so that the equality has a closed, convex (not necessarily singleton) set of solutions. In fact, if all politician types choose the same policy with probability one, then updating does not occur and incumbents are always re-elected, so that the voter's cutoff is negatively infinite. As policy choices of politician types converge to the same policy, this means that the cutoff either jumps discontinuously (from a bounded, finite level) to, or diverges to, negative infinity. We circumvent this problem by deriving a positive lower bound on the distance between optimal policy choices of the different types. To this end, we first observe that by (A5), equilibrium policy choices for the type j politician are bounded above by some $\overline{x}_j > \hat{x}_j$ such that $V^C > w_j(\overline{x}) + \beta$, and we set $\overline{x} = \max\{\overline{x}_j \mid j \in T\}$ to obtain a bound across types.

Next, given any cutoff $\overline{y}$ and any type j politician, we observe that there are at most two optimal policies, by Lemma 1, and each satisfies the first order condition (3). Note that $f(\overline{y}-x) \to 0$ uniformly on $[0,\overline{x}]$ as $|\overline{y}| \to \infty$ (otherwise (A3) would be violated) and from the first order condition, this implies that the optimal policies of the type j politician converge to the ideal policy, i.e., $x^*_j(\overline{y}) \to \hat{x}_j$ and $x_{*,j}(\overline{y}) \to \hat{x}_j$. Thus, we can choose a sufficiently large interval $[y_L, y_H]$ and $\varepsilon' > 0$ such that for all $\overline{y}$ outside the interval, optimal policies differ across types by at least ε', i.e., for all $j < n$, we have $|x_{*,j+1}(\overline{y}) - x^*_j(\overline{y})| > \varepsilon'$. By upper semi-continuity of $x^*_j(\cdot)$ and lower semi-continuity of $x_{*,j+1}(\cdot)$, the function $|x_{*,j+1}(\overline{y}) - x^*_j(\overline{y})|$ is lower semi-continuous and therefore attains its minimum on the (nonempty, compact) interval $[y_L, y_H]$. Since, from Lemmas 1 and 2, $x_{*,j+1}(\overline{y}) > x^*_j(\overline{y})$ for all $\overline{y}$, this minimum is positive. Thus, there exists $\varepsilon'' > 0$ such that for all $\overline{y} \in [y_L, y_H]$, optimal policies differ by at least ε''. Finally, we set $\varepsilon = \min\{\varepsilon', \varepsilon''\}$ to establish the desired lower bound.

We are interested in the profiles $(\pi_1, \ldots, \pi_n)$ such that for all politician types j, π_j places positive probability on at most two alternatives, and the supports of mixed policy strategies are strictly ordered by type and separated by a distance of at least ε, i.e., for all $j < n$ and all policies x_j with $\pi_j(x_j) > 0$ and x_{j+1} with $\pi_{j+1}(x_{j+1}) > 0$, we have $x_j + \varepsilon \leqslant x_{j+1}$. It is convenient to represent such a profile by a $3n$-tuple (x,z,r), where $x = (x_1, \ldots, x_n) \in [0,\overline{x}]^n$, $z = (z_1, \ldots, z_n) \in [0,\overline{x}]^n$, and $r = (r_1, \ldots, r_n) \in [0,1]^n$. In addition, we require that for all j, we have $x_j \leqslant z_j$, and that for all $j < n$, we have $z_j + \varepsilon \leqslant x_{j+1}$. We then associate (x,z,r) with the profile of mixed policy strategies such that the type j politician places probability r_j on x_j and the remaining probability $1 - r_j$ on z_j. Letting D^ε consist of all such $3n$-tuples (x,z,r), we see that D^ε is nonempty, convex, and compact. Using this representation, we can define (abusing notation slightly) the induced cutoff $y^*(x,z,r)$, which is continuous as a function of its arguments.

The second difficulty is that the set Y of policy outcomes is not compact, so that the voter's cutoff is, in principle, unbounded. To circumvent this problem, we note that by continuity of the function $y^*(\cdot)$ the image $y^*(D^\varepsilon)$ is compact, and we can let $\overline{Y}$ be a closed interval containing this image. The existence proof then proceeds with an application of a fixed point theorem that relaxes Kakutani's conditions. We define the correspondence $\Phi\colon D^\varepsilon \times \overline{Y} \rightrightarrows D^\varepsilon \times \overline{Y}$ so that for each $(x,z,r,\overline{y})$, the value of Φ consists of $(3n+1)$-tuples $(\tilde{x}, \tilde{z}, \tilde{r}, \tilde{y})$ such that for every politician type j, the mixed policy strategy represented by $(\tilde{x}_j, \tilde{z}_j, \tilde{r}_j)$ is optimal given $\overline{y}$, and $\tilde{y}$ is the unique cutoff induced by the indifference condition:

$$\Phi(x,z,r,\bar{y}) \quad = \quad \left\{ (\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in D^{\varepsilon} \times \overline{Y} \;\middle|\; \begin{array}{c} \text{for all } j,\, \tilde{x}_j \leqslant \tilde{z}_j, \\ \tilde{r}_j > 0 \Rightarrow \tilde{x}_j \in \{x_{*,j}(\bar{y}), x_j^*(\bar{y})\}, \\ \tilde{r}_j < 1 \Rightarrow \tilde{z}_j \in \{x_{*,j}(\bar{y}), x_j^*(\bar{y})\}, \\ \text{and } \tilde{y} = y^*(x,z,r) \end{array} \right\}.$$

Of note, we require that the first policy coordinate $\tilde{x}_j$ is less than or equal to the second, $\tilde{z}_j$, and we require that these are optimal when chosen with positive probability.

To deduce the existence of a fixed point of Φ, we first verify that the correspondence is upper hemi-continuous with closed values, i.e., it has closed graph. This property is not immediately obvious, because optimal policies are not unique, and the functions $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ are not continuous. It is important that we allow for the possibility that $\tilde{x}_j = \tilde{z}_j$, in which case both policies are equal to either the least optimal policy $x_{*,j}(\bar{y})$ or to the greatest optimal policy $x_j^*(\bar{y})$. Of course, these policies can coincide as well. Let $\{(x^m,z^m,r^m,\bar{y}^m)\}$ be any sequence converging to $(x,z,r,\bar{y})$ in $D^{\varepsilon} \times \overline{Y}$, and consider a corresponding sequence $\{(\tilde{x}^m,\tilde{z}^m,\tilde{r}^m,\tilde{y}^m)\}$ such that $(\tilde{x}^m,\tilde{z}^m,\tilde{r}^m,\tilde{y}^m)$ belongs to $\Phi(x^m,z^m,r^m,\bar{y}^m)$ for all m and $(\tilde{x}^m,\tilde{z}^m,\tilde{r}^m,\tilde{y}^m) \rightarrow (\tilde{x},\tilde{z},\tilde{r},\tilde{y})$. We must show that $(\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in \Phi(x,z,r,\bar{y})$. Since limits preserve weak inequalities, it is immediate that for all j, we have $\tilde{x}_j \leqslant \tilde{z}_j$, and continuity of $y^*(\cdot)$ implies $\tilde{y} = y^*(x,z,r)$. It remains to establish optimality of policies adopted with positive probability, and we consider x_j, as the argument for z_j is analogous. To this end, suppose $\tilde{r}_j > 0$, so that for sufficiently high m, we also have $\tilde{r}_j^m > 0$, implying $\tilde{x}_j^m \in \{x_{*,j}(\bar{y}^m), x_j^*(\bar{y}^m)\}$. Since the best response correspondence is upper hemi-continuous (Lemma 1), $\tilde{x}_j$ is an optimal policy for the type j politician given cutoff $\bar{y}$. If $\tilde{x}_j \notin \{x_{*,j}(\bar{y}), x_j^*(\bar{y})\}$, then this implies the politician has at least three optimal policies, contradicting Lemma 1. Thus, $\tilde{x}_j$ is either the least or greatest optimal policy given $\bar{y}$, as desired.

This formulation yields a correspondence that is defined on a convex and compact domain and that is upper hemi-continuous and has nonempty, closed values. The typical application of Kakutani's fixed point theorem also proceeds by verifying convex values of the correspondence, and this leads to the third difficulty: Φ does not have this property. In particular, this property fails if $(x,z,r,\bar{y})$ is such that $x_j^*(\bar{y}) \neq x_{*,j}(\bar{y})$ for some j. Nevertheless, the values of the correspondence are contractible, and this is sufficient for existence of a fixed point. A subset $C \subseteq \Re^d$ of Euclidean space is *contractible* if there is an element $\bar{c} \in C$ and a continuous mapping $h\colon C \times [0,1] \rightarrow C$ such that for all $c \in C$, $h(c,0) = c$ and $h(c,1) = \bar{c}$. That is, the set can be continuously deformed to a single element. Convex sets are contractible, but convexity is not necessary for contractibility. It is straightforward to see that $\Phi(x,z,r,\bar{y})$ is contractible to the element $(\hat{x},\hat{z},\hat{r},\hat{y})$ such that: for all j,

- $\hat{x}_j = x_{*,j}(\overline{y})$,
- $\hat{z}_j = x^*_j(\overline{y})$,
- $\hat{r}_j = 1$,

where of course $\hat{y} = y^*(x,z,r)$ is fixed by construction. To reduce notation, we provide an informal description of the mapping h. Considering any $(\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in \Phi(x,z,r,\overline{y})$, we break the unit interval into three components. For $t \in [0,.3]$, if $\tilde{r}_j = 0$, then we continuously adjust $\tilde{x}_j$ to $x_{*,j}(\overline{y})$; and if $\tilde{r}_j > 0$, then we keep $\tilde{x}_j$ fixed at $x_{*,j}(\overline{y})$. For $t \in [.3,.7]$, we continuously increase each $\tilde{r}_j$ up to one. And for $t \in [.7,1]$, we continuously adjust $\tilde{z}_j$ to $x^*_j(\overline{y})$ for each j; note that this last step is material only if $\tilde{r}_j = 1$, in which case the first two steps leave $\tilde{x}_j$ and $\tilde{r}_j$ unchanged. See Figure 4, above, for an illustration. This completes the construction, and we conclude that the values of Φ are contractible.

The correspondence Φ is upper hemi-continuous with nonempty, closed, contractible values, and the domain $D^\varepsilon \times \overline{Y}$ is nonempty, compact, and convex. Therefore, the Eilenberg-Montgomery fixed point theorem (see McLennan (2018), Theorem 14.1.5) implies that Φ has a fixed point,[23] (x^*,z^*,r^*,y^*), which yields an electoral equilibrium. Finally, the characterization results in (i)–(iii) follow directly from Lemma 1 and Lemma 3. □

Proof of Theorem 2: Consider an electoral equilibrium as β becomes large. By Theorem 1, each politician type j mixes between two policies, x^*_j and $x_{*,j}$, and the voter uses a finite cutoff y^*. Suppose there is a subsequence such that y^* is bounded above, say $y^* \leqslant \overline{y}$. By Theorem 1, the equilibrium cutoff lies in the compact set $[\hat{x}_1 + \hat{z}, \overline{y}]$. Then the first order condition for the type 1 politician in (3) implies that $x_{*,1} \to \infty$, and in particular, we have $x_{*,1} > \overline{y} - \hat{z}$ for large enough β, but this contradicts $x_{*,1} + \hat{z} \leqslant y^* \leqslant \overline{y}$. We conclude that y^* diverges to infinity, which proves (i).

To prove (ii), suppose toward a contradiction that there is a type j, an $\varepsilon > 0$, and a subsequence of office benefit levels such that $\hat{x}_j + \varepsilon \leqslant x^*_j \leqslant \frac{1}{\varepsilon}$. Going to a subsequence, we can assume $x^*_j \to \tilde{x}_j$ such that $\hat{x}_j < \tilde{x}_j < \infty$. Then for sufficiently large β, we have $\hat{x}_j < x^*_j$. For these parameters, the payoff to the type j politician from choosing $\hat{x}_j$ instead of x^*_j is non-positive, and thus we note that

$$(F(y^* - \hat{x}_j) - F(y^* - x^*_j))[w_j(\hat{x}_j) + \beta - V^C] \quad \geqslant \quad w_j(\hat{x}_j) - w_j(x^*_j).$$

[23] The Eilenberg-Montgomery fixed point theorem holds for a domain that is a nonempty compact absolute retract. Every compact, convex set is an absolute retract (McLennan, 2018), so this assumption is satisfied automatically.

That is, the current gains from choosing the ideal policy are offset by future losses. Since $x_j^* \to \tilde{x}_j$, we can fix $\eta \in (0,1)$, and for high enough office benefit, we have $x_j^* > \tilde{x}_j - \eta$, and thus

$$\frac{F(y^* - x_j^*) - F(y^* - \tilde{x}_j - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)} \geqslant \frac{F(y^* - \tilde{x}_j - \eta) - F(y^* - \tilde{x}_j - 1)}{F(y^* - \hat{x}_j) - F(y^* - \tilde{x}_j - \eta)}.$$

Since $y^* \to \infty$, the limit of the right-hand side above as β becomes large is indeterminate, and by L'Hôpital's rule, this limit is equal to

$$\lim \frac{f(y^* - \tilde{x}_j - \eta) - f(y^* - \tilde{x}_j - 1)}{f(y^* - \hat{x}_j) - f(y^* - \tilde{x}_j - \eta)} = \lim \frac{f(y^* - \tilde{x}_j - 1)\left(\frac{f(y^* - \tilde{x}_j - \eta)}{f(y^* - \tilde{x}_j - 1)} - 1\right)}{f(y^* - \tilde{x}_j - \eta)\left(\frac{f(y^* - \hat{x}_j)}{f(y^* - \tilde{x}_j - \eta)} - 1\right)} = \infty,$$

where we use (A2) and (A3). Then, however, the future gain from choosing $\tilde{x}_j + 1$ instead of x_j^* strictly exceeds current losses, i.e.,

$$(F(y^* - x_j^*) - F(y^* - \tilde{x}_j - 1))[w_j(\hat{x}_j) + \beta - V^C] > w_j(x_j^*) - w_j(\tilde{x}_j + 1), \quad (11)$$

for high enough β. To be specific, let

$$\begin{aligned} A &= w_j(\hat{x}_j) + \beta - V^C, \\ B &= w_j(\hat{x}_j) - w_j(x_j^*), \text{ and} \\ C &= w_j(x_j^*) - w_j(\tilde{x}_j + 1), \end{aligned}$$

where A is evaluated at sufficiently large β. Note that since $\hat{x}_j < \tilde{x}_j < \infty$, we have $\lim B > 0$ and $\lim C < \infty$. We have noted that $(F(y^* - \hat{x}_j) - F(y^* - x_j^*))A \geqslant B$ for sufficiently large β, and we have shown that as β becomes large, we have

$$\frac{F(y^* - x_j^*) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)} > \frac{C}{B}.$$

Combining these facts, we have

$$(F(y^* - \hat{x}_j) - F(y^* - x_j^*))A\left(\frac{F(y^* - x_j^*) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)}\right) > B\left(\frac{C}{B}\right),$$

which yields (11) for large β. This gives the type j politician a profitable deviation from x_j^*, a contradiction. A similar argument holds for $x_{*,j}$. It follows that for all j, all $\varepsilon > 0$, and sufficiently large β, we have $\{x_{*,j}, x_j^*\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon) \cup (\frac{1}{\varepsilon}, \infty)$, To establish that $\{x_{*,j}, x_j^*\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon)$ for all $\varepsilon > 0$, and sufficiently large β, implies $x_{*,j} = x_j^*$ for sufficiently large β, suppose otherwise. Then there must be a sequence of equilibria such for all $\varepsilon > 0$, for sufficiently large β, there exists j such that $\{x_{*,j}, x_j^*\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon)$ and $x_{*,j} \neq x_j^*$. Using part (i), we can find a subsequence of equilibria for increasing values of the office benefit such that along that subsequence the voter cutoff is strictly increasing in β. For each equilibrium along the sequence, there must be a local minimizer located in between $x_{*,j}$ and x_j^*. Note that a local minimizer must satisfy the necessary second order condition:

$$\frac{w_j''(x)}{w_j'(x)} \leqslant -\frac{f'(y-x)}{f(y-x)}$$

for $y = y^*$. Let $\tilde{X}(y) \subseteq X$ denote the set of policies satisfying the inequality above for a given voter cutoff. By (A6$'$), $\tilde{X}(y)$ is convex for any y. From the necessary second order condition for a maximizer, we get

$$\frac{w_j''(x_{*,j})}{w_j'(x_{*,j})} \geqslant -\frac{f'(y^*-x_{*,j})}{f(y^*-x_{*,j})} \quad \text{and} \quad \frac{w_j''(x_j^*)}{w_j'(x_j^*)} \geqslant -\frac{f'(y^*-x_j^*)}{f(y^*-x_j^*)}.$$

Since there must be a minimizer in the interval $[x_{*,j}, x_j^*]$, these two inequalities imply $\varnothing \neq \tilde{X}(y^*) \subseteq [x_{*,j}, x_j^*]$. Now fix one value of β, say β', and let x' denote the minimizer in between $x'_{*,j}$ and $x_j'^*$, so that $x' > \hat{x}_j$ and $x' \in \tilde{X}(y'^*)$. Since f is log-concave by (A2) and y^* is increasing along the sequence, it follows that for for each $\beta > \beta'$, we have $x' \in \tilde{X}(y^*)$. But then for each $\beta > \beta'$, we have $x_{*,j} \leqslant x' \leqslant x_j^*$. Choosing $\varepsilon \in (0, x' - \hat{x}_j)$, we conclude that for all $\beta > \beta'$, the inequality $x_j^* > \hat{x}_j + \varepsilon$ holds, a contradiction. A similar argument establishes that $\{x_{*,j}, x_j^*\} \subset (\frac{1}{\varepsilon}, \infty)$ implies $x_{*,j} = x_j^*$ for sufficiently large β.

To prove (iii), suppose that $x_{\ell-1}^*$ does not diverge to infinity. By (ii), there is a subsequence such that $x_{\ell-1}^* \to \hat{x}_{\ell-1}$. Now fix politician type $j \leqslant \ell - 1$, and note that since equilibrium policy choices are ordered by type, we have $x_j^* \to \hat{x}_j$. Using the expression for Bayes rule, the posterior probability of type $j \leqslant \ell - 1$ conditional on observing y^* satisfies

$$\mu_T(j|y^*) \quad = \quad \frac{p_j \sum_x f(y^*-x)\pi_j(x)}{\sum_k p_k \sum_x f(y^*-x)\pi_k(x)} \;\leqslant\; \frac{p_j f(y^*-x_j^*)}{\sum_{k\geqslant\ell} p_k \sum_x f(y^*-x)\pi_k(x)},$$

where the inequality uses (A3), which implies $f(y^*-x_{*,j}) \leqslant f(y^*-x_j^*)$ as $y^* \to \infty$. Note that

$$\sum_{k\geqslant\ell} p_k \sum_x f(y^*-x)\pi_k(x) \quad = \quad \sum_{k\geqslant\ell} p_k[f(y^*-x_k^*)\pi_k(x_k^*) + f(y^*-x_{*,k})\pi_k(x_{*,k})].$$

Dividing by $f(y^* - x_j^*)$, we obtain the expression

$$\sum_{k \geqslant \ell} p_k \left[\frac{f(y^* - x_k^*)}{f(y^* - x_j^*)} \pi_k(x_k^*) + \frac{f(y^* - x_{*,k})}{f(y^* - x_j^*)} \pi_k(x_{*,k}) \right].$$

For each $k \geqslant \ell$, if $x_k^* \to \hat{x}_k$, then (A2) and (A3) imply that $\frac{f(y^* - x_k^*)}{f(y^* - x_j^*)} \to \infty$. By (ii), the remaining case is $x_k^* \to \infty$. Note that in this case, (A8) implies $w_k'(x_k^*) \to -\infty$, and thus the first order condition in (3) implies that $f(y^* - x_k^*)\beta \to \infty$. The first order condition for type j implies $f(y^* - x_j^*)\beta \to 0$, and we infer that $\frac{f(y^* - x_k^*)}{f(y^* - x_j^*)} \to \infty$. Similarly, $\frac{f(y^* - x_{*,k})}{f(y^* - x_j^*)} \to \infty$ for all $k \geqslant \ell$. Thus, we have

$$\mu_T(j|y^*) \;\leqslant\; \frac{p_j}{\sum_{k \geqslant \ell} p_k \sum_x \frac{f(y^* - x)}{f(y^* - x_j^*)} \pi_k(x)} \;\to\; 0.$$

We conclude that the voter's posterior beliefs conditional on y^* place a probability arbitrarily close to one on above average types $j \geqslant \ell$, contradicting the indifference condition in (1). Therefore, we have $x_{\ell-1}^* \to \infty$.

To complete the proof of (iii), suppose toward a contradiction that for some subsequence, the support of $\pi_{\ell-1}^*$ is bounded. It follows that for sufficiently large β, we have $\pi_j^*(x_{*,\ell-1}) = 1$. But then

$$\mu_T(\ell - 1|y^*) \quad = \quad \frac{p_{\ell-1} f(y^* - x^{*,\ell-1})}{\sum_{k \geqslant \ell} p_k \sum_x f(y^* - x)\pi_k(x)},$$

and the above arguments imply that $\mu_T(\ell - 1|y^*) \to 0$, and again $\mu_T(j|y^*) \to 0$ for all $j < \ell - 1$. But then the voter's posterior beliefs place probability close to one on above average types, again contradicting the voter's indifference condition.

Finally, since policy choices are ordered by type, it follows directly from (iii) that $x_{*,j} \to \infty$ for all $j \geqslant \ell$, proving (iv), as required. □

Proof of Theorem 3: For part (i), note that, from part (ii) of Theorem 2, either $x_{*,1} \to \hat{x}_1$ or $x_{*,1} = x_1^* \to \infty$. Suppose toward a contradiction that $x_{*,1}$ increases without bound. From the necessary first and second order conditions of the type 1 politician's problem, we have

$$\frac{w_1''(x_{*,1})}{w_1'(x_{*,1})} \;\geqslant\; -\frac{f'(y^* - x_{*,1})}{f(y^* - x_{*,1})}.$$

From (A8), the left-hand side of the above inequality converges to zero from above as $x_{*,1} \to \infty$. Since (A2) implies f is log-concave, it follows that $-f'(z)/f(z)$ is strictly increasing in z, and moreover $-f'(z)/f(z) > 0$ if and only if $z > \hat{z}$. From Lemma 3, we have $y^* - x_{*,1} > \hat{z}$, where $\hat{z}$ is the mode of the outcome density, so that the right-hand side of the inequality above is strictly positive. We conclude that $y^* - x_{*,1}$ must converge to $\hat{z}$ from above as $x_{*,1} \to \infty$, so the probability of reelection of the lowest type must converge to $1 - F(\hat{z})$. Now consider the indifference curve through $\hat{x}_1$, given by (x, r) pairs satisfying

$$w_1(x) + r[w_1(\hat{x}_1) + \beta - V^C] \quad = \quad w_1(\hat{x}_1),$$

or equivalently, satisfying $r = r_1(x)$, where

$$r_1(x) \quad \equiv \quad \frac{w_1(\hat{x}_1) - w_1(x)}{w_1(\hat{x}_1) + \beta - V^C}.$$

Let $\tilde{x} > \hat{x}_1$ be defined by $r_1(\tilde{x}) = 1 - F(y^* - x_{*,1})$, so that

$$\tilde{x} \quad = \quad w_1^{-1}\big(F(y^* - x_{*,1})w_1(\hat{x}_1) - (1 - F(y^* - x_{*,1}))(\beta - V^C)\big),$$

where w_1 is invertible on $(\hat{x}_1, \infty)$ by concavity and uniqueness of the ideal policy $\hat{x}_1$.

We claim that $\tilde{x} < x_{*,1}$ for large β, so that the type 1 politician would be better off by adopting the ideal policy and being reelected with probability zero than by adopting the policy $x_{*,1}$ and being reelected with probability $1 - F(y^* - x_{*,1})$. Since the politician's indifference curves are vertically parallel and convex, it is enough to check that $r_1'(\tilde{x}) < f(y^* - x_{*,1})$ for large β, or equivalently,

$$-\frac{w_1'\big(w_1^{-1}\big(F(y^* - x_{*,1})w_1(\hat{x}_1) - (1 - F(y^* - x_{*,1}))(\beta - V^C)\big)\big)}{w_1(\hat{x}_1) + \beta - V^C} \quad < \quad f(y^* - x_{*,1}).$$

By concavity of w_1, it follows that w_1^{-1} and w_1' are weakly decreasing, so the left-hand side above is weakly decreasing in $y^* - x_{*,1}$. Since $y^* - x_{*,1} \geqslant \hat{z}$, we have

$$\begin{aligned} & w_1'\big(w_1^{-1}\big(F(y^* - x_{*,1})w_1(\hat{x}_1) - (1 - F(y^* - x_{*,1}))(\beta - V^C)\big)\big) \\ & \quad \leqslant \quad w_1'\big(w_1^{-1}\big(F(\hat{z})w_1(\hat{x}_1) - (1 - F(\hat{z}))(\beta - V^C)\big)\big). \end{aligned}$$

Thus, the desired inequality holds for large enough β if

$$\lim_{\beta \to \infty} -\frac{w_1'\left(w_1^{-1}\left(F(\hat{z})w_1(\hat{x}_1) - (1 - F(\hat{z}))(\beta - V^C)\right)\right)}{w_1(\hat{x}_1) + \beta - V^C} \quad < \quad f(\hat{z}),$$

which holds if

$$\lim_{z \to \infty} \frac{w_1''(z)}{w_1'(z)} \quad < \quad f(\hat{z}),$$

which holds by (A8). This contradicts optimality of $x_{*,1}$ and establishes part (i).

For part (ii), assume x_j^* increases without bound. We claim that either $y^* - x_j^* \to -\infty$ or $y^* - x_j^* \to \infty$. To prove this, suppose toward a contradiction that there is a subsequence of electoral equilibria such that $y^* - x_j^* \to K$ with K finite. Along this subsequence, the probability of reelection conditional on adopting x_j^* converges to $1 - F(K)$. As in the previous argument, consider the indifference curve through $\hat{x}_j$, given by

$$w_j(x) + r[w_j(\hat{x}_j) + \beta - V^C] \quad = \quad w_j(\hat{x}_j),$$

or equivalently, satisfying $r = r_j(x)$, where

$$r_j(x) \quad \equiv \quad \frac{w_j(\hat{x}_j) - w_j(x)}{w_j(\hat{x}_j) + \beta - V^C}.$$

Let $\tilde{x}$ be defined by $r_j(\tilde{x}) = 1 - F(y^* - x_j^*)$, so that

$$\tilde{x} \quad = \quad w_j^{-1}\big(F(y^* - x_j^*)w_n(\hat{x}_j) - (1 - F(y^* - x_j^*))(\beta - V^C)\big).$$

We claim that $\tilde{x} < x_j^*$ for large β, so that the politician of type j would be better off by adopting the ideal policy and being reelected with probability zero than by adopting the policy x_j^* and being reelected with probability $1 - F(y^* - x_j^*)$. Since the politician's indifference curves are vertically parallel and convex, it is enough to check that $r_j'(\tilde{x}) < f(y^* - x_j^*)$ for large β or equivalently

$$-\frac{w_j'\big(w_j^{-1}\big(F(y^* - x_j^*)w_j(\hat{x}_j) - (1 - F(y^* - x_j^*))(\beta - V^C)\big)\big)}{w_j(\hat{x}_j) + \beta - V^C} \quad < \quad f(y^* - x_j^*).$$

Again, the left-hand side is weakly decreasing in $y^* - x_j^*$, and thus given any $\varepsilon > 0$, the above inequality holds for large enough β if

$$\lim_{\beta\to\infty} -\frac{w_j'\left(w_1^{-1}\left(F(K-\varepsilon)w_j(\hat{x}_j) - (1 - F(K-\varepsilon))(\beta - V^C)\right)\right)}{w_j(\hat{x}_j) + \beta - V^C} \; < \; f(K),$$

which holds if

$$\lim_{z\to\infty} \frac{w_j''(z)}{w_j'(z)} \quad < \quad f(K),$$

which holds by (A8). To complete the proof of part (ii), it remains to show that $y^* - x_j^*$ cannot diverge to infinity if x_j^* goes to infinity as β grows arbitrarily large.

To see this, note that the first and second order condition for x_j^* to be maximum imply

$$\frac{w_j''(x_j^*)}{w_j'(x_j^*)} \geqslant -\frac{f'(y^*-x_j^*)}{f(y^*-x_j^*)},$$

which cannot hold for large enough β since the left-hand side converges to zero by (A9), but the right hand side is increasing since f is log-concave by (A2), and is positive for $y^*-x_j^*>\hat{z}$. □

Proof of Theorem 4: Part (ii) follows from part (i) and Theorem 3. Thus, we focus on parts (i), (iii), and (iv). Note that $x_{*,1}\to\hat{x}_1$ follows from Theorem 3. Now, by Corollary 1, there is a marginal type m such that $x_{*,j}=x_j^*\to\hat{x}_j$ for all $j<m$, $x_{*,j}=x_j^*\to\infty$ for all $j>m$, and $x_m^*\to\infty$. Let

$$\begin{aligned}
A_j &= p_j\frac{f(y^*-x_{*,j})}{f(y^*-x_m^*)}, \text{ for all } j<m,\\
B &= p_m\pi_m(x_{*,m})\frac{f(y^*-x_{*,m})}{f(y^*-x_m^*)}\\
C &= p_m\pi_m(x_m^*)\\
D_j &= p_j\frac{f(y^*-x_j^*)}{f(y^*-x_m^*)}, \text{ for all } j>m,
\end{aligned}$$

and define

$$A = \sum_{j:j<m} A_j \quad\text{and}\quad D = \sum_{j:j>m} D_j.$$

Then the indifference condition for the voter conditional on y^* can be written as

$$\frac{\sum_{j:j<m}A_j\mathbb{E}[u(y)|\hat{x}_j]+(B+C)\mathbb{E}[u(y)|\hat{x}_m]+\sum_{j:j>m}D_j\mathbb{E}[u(y)|\hat{x}_j]}{A+B+C+D} = V^C. \quad (12)$$

By the first order conditions for the type $j\leqslant m$ and type m politicians, we have

$$\frac{f(y^*-x_{*,j})}{f(y^*-x_m^*)} = \frac{w_j'(x_{*,j})}{w_m'(x_m^*)} \to 0, \quad (13)$$

and thus $A,B\to 0$.

We claim that for all for all $j=m,\dots,n-1$, we have

$$\lim\frac{f(y^*-x_j^*)}{f(y^*-x_m^*)} = 1.$$

Indeed, for β sufficiently large, we have $y^* < x_m^* \leqslant x_j^*$, and then single-peakedness of f implies that the above limit is less than or equal to one. For the opposite inequality, the first order condition for the type m and type j politician imply

$$\frac{f(y^* - x_j^*)}{f(y^* - x_m^*)} \;=\; \frac{w_j'(x_j^*)}{w_m'(x_m^*)} \;\geqslant\; \frac{w_j'(x_m^*)}{w_m'(x_m^*)},$$

where we use the facts that w_j is concave and that $x_j^* \geqslant x_m^*$. Since $x_m^* \to \infty$, (A9) implies that the limit of the right-hand side of the preceding inequality equals one, and the claim follows.

Note that the left-hand side of (12) can be written as an expectation with respect to a probability distribution $q = (q_1, \ldots, q_n)$, namely, $\sum_{j=1}^n q_j \mathbb{E}[u(y)|\hat{x}_j]$, where

$$\begin{aligned} q_j &= \frac{A_j}{A+B+C+D}, \text{ for all } j < m, \\ q_m &= \frac{B+C}{A+B+C+D}, \\ q_j &= \frac{D_j}{A+B+C+D}, \text{ for all } j > m. \end{aligned}$$

Going to a subsequence along which these terms converge, we can assume that $q \to \tilde{q}$ as β becomes large. By (13), we have $\tilde{q}_j = 0$ for all $j < m$. Moreover, (13) implies that $B \to 0$, and thus we have

$$\tilde{q}_m \;=\; \lim \frac{p_m \pi_m(x_m^*)}{A+B+C+D}.$$

From our claim, it follows that for all $j = m+1, \ldots, n-1$, we have

$$\frac{\tilde{q}_{j+1}/p_{j+1}}{\tilde{q}_j/p_j} \;=\; \lim \frac{D_{j+1}/p_{j+1}}{D_j/p_j} \;=\; 1.$$

Moreover, using $B \to 0$ and setting $j = m$ in our claim, we have

$$\frac{\tilde{q}_{m+1}/p_{m+1}}{\tilde{q}_m/p_m} \;=\; \lim \frac{D_{m+1}/p_{m+1}}{(B+C)/p_m} \;=\; \frac{1}{\lim \pi_m(x_m^*)}$$

if $\lim \pi_m(x_m^*) > 0$, and $\tilde{q}_m = 0$ otherwise. Therefore, we have shown that for all $j = m+1, \ldots, n$, we have $\frac{\tilde{q}_{j+1}}{p_{j+1}} \geqslant \frac{\tilde{q}_j}{p_j} \geqslant \frac{\tilde{q}_m}{p_m}$. If either $m > 1$ or both $m = 1$ and $\lim \pi_m(x_m^*) < 1$, then the limiting distribution $\tilde{q}$ stochastically dominates the prior, and we have

$$\lim \sum_{j=1}^n q_j \mathbb{E}[u(y)|\hat{x}_j] \;=\; \sum_{j=1}^n \tilde{q}_j \mathbb{E}[u(y)|\hat{x}_j] \;>\; V^C,$$

contradicting the voter's indifference condition for large enough β. This establishes part (i).

To prove part (iii), suppose toward a contradiction that $x_n^* - x_1^* \not\to 0$. Going to a subsequence, we can assume $x_n^* - x_1^* \to \Delta > 0$. From the first order condition for the type 1 and type n politician, we have

$$\frac{w_n'(x_n^*)}{w_1'(x_1^*)} \quad = \quad \frac{f(y-x_n^*)}{f(y-x_1^*)} \ \to \ 0,$$

where the limit follows from $\Delta > 0$ and (A4). But by $x_1^* \to \infty$ and (A9), we have

$$\liminf \frac{w_n'(x_n^*)}{w_1'(x_1^*)} \quad \geqslant \quad \lim \frac{w_n'(x_1^*)}{w_1'(x_1^*)} \quad = \quad 1,$$

where the first inequality uses concavity of w_n and $\hat{x}_n < x_1^* < x_n^*$. This contradiction establishes (iii).

To prove part (iv), we must argue that $0 < \pi_1(x_1^*) < 1$ for β sufficiently large. First, note that $\pi_m(x_m^*) \to 1$, so that $\pi_m(x_m^*) > 0$ holds for sufficiently large β. Suppose toward a contradiction that $\pi_1(x_1^*) = 1$ along some subsequence. By Lemma 3, we must have $x_1^* + \hat{z} < y^*$, or equivalently, $x_1^* - y^* < -\hat{z}$, but Theorem 3 implies that $x_1^* - y^* \to \infty$ a contradiction. □

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