

Dynamic Demand for New and Used Durable Goods without Physical Depreciation: The Case of Japanese Video Games*

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Abstract

For information/digital products, the used goods market has been viewed as a threat by producers. However, it is not clear if this view is justified because the used goods market also provides owners with an opportunity to sell their products. To investigate the impact of the used goods market on new goods sales, we collect a unique data set from the Japanese video game market. Based on the data, we develop and estimate a new dynamic structural model of consumers' buying and selling decisions. The estimation results show that (i) the consumption value of owners depreciates much faster than that of potential buyers, and (ii) consumers are forward-looking but they discount the future more heavily than what the interest rate suggests. Using the estimates, we quantify the impact of eliminating the used game market on publishers' profits and consumer welfare. We find that this policy would increase publishers' profit by 2%, but reduce the consumer surplus by 19% if publishers do not adjust their prices. However, if they adjust prices optimally, it would increase the profit by 78%, and also increase the consumer surplus by 141% due to lower new prices. Overall, our results suggest that the elimination of the used good market improves the social welfare.

Keywords: Information/Digital Products, Durable Goods, Used Goods Market, Demand Estimation, Dynamic Programming, Transaction Costs, Satiation, Discount Factor, Bayesian Estimation.

1 Introduction

The existence of used goods markets has been viewed as a serious problem by producers in information/digital product categories such as books, CDs/DVDs, and video games. They argue that the competition from used goods significantly lowers their profits and reduce the incentive to develop new products. For instance, book publishers and authors expressed their annoyance to Amazon over used books sold on its websites (Tedeschi 2004). Video game publishers in Japan attempted to kill off used video game retailing by suing used video game retailers (Hirayama 2006). Their main argument is that products like books and video games physically depreciate negligibly, but owners' consumption values can decline very quickly due to satiation. As a result, unlike products that physically depreciate more considerably (such as cars), producers of information/digital products may face competition from used goods that appear to be almost identical to new goods soon after the release of a new product.

However, their argument focuses only on one aspect of used goods markets (substitution effect), and ignores the possibility that costs of buying and selling used goods could reduce the substitutability between new and used goods. Moreover, the existence of used goods markets provides consumers with a selling opportunity. If consumers are forward-looking and account for the future resale value when making a buying decision, the effective price consumers pay for a product will be lower than the actual price (resale effect). This feature implies that the existence of used goods markets could increase the sales of new goods. Thus, whether the existence of used goods markets hurts or benefits new-good producers is an empirical question, and the answer depends on which effect, substitution or resale effect, dominates.

The question of whether used goods markets help or hurt new-good producers is not new and has been investigated in the automobile market (e.g., Esteban and Shum 2007, Chen et al. 2013, Schiraldi 2011), and in the housing market (e.g., Tanaka 2013). A general finding among these papers is that the elimination of used goods markets helps new-good producers. For example, Chen et al. (2013) find that opening the used good market lowers firms' profits by 35%. However, it is not straightforward whether these findings will extend to markets such as books and video games, where the sales of new goods is highly concentrated

around the release period, and declines quickly afterwards. If the incentive for consumers to buy new goods in the release period is largely driven by the future resale opportunity, then the elimination of the used goods market could significantly reduce the initial sales of new copies, and new-good producers might not be able to recover its initial loss in subsequent periods even if cannibalization from used goods is eliminated.

This paper contributes to the existing literature in two important dimensions. First, we assemble a new data set from the Japanese video game market, which includes weekly aggregate level data for 20 video game titles released in Japan between 2004 and 2008. The novel aspect of this data set is that, in addition to the sales and prices of new and used goods, which are main variables studied in previous works, it includes three new important variables: (i) resale values of used goods,¹ (ii) quantities of used goods retailers purchased from consumers, and (iii) aggregate inventory level of used goods at retailers. This novel data set allows us to empirically capture important distinctions between video games and other typical durable goods such as cars and houses studied in previous works. One common feature assumed in most of the previous research (both theoretical and empirical) is that durability is measured as the quality deterioration rate, and it is common across buyers and sellers. This assumption will likely be violated in information/digital product categories such as CDs/DVDs and video games because for product owners, consumption values deteriorate mainly due to satiation (satiation-based deterioration); but for potential buyers, consumption values may deteriorate due to freshness of a product (freshness-based deterioration). Our new data on used game trading activities (weekly used-copy quantities demanded and supplied by consumers and associated weekly prices and resale values) help identify these two forms of consumption value deterioration without the need to make an assumption on the used-good market clearing condition.

Second, based on the data set and institutional details about the video game market, we develop and estimate a new structural model of consumers' buying and selling decisions of durable goods which do not exhibit physical depreciation. To our knowledge, this is the first dynamic model of forward-looking consumers that incorporates all of the following features: (i) new and used goods buying decisions, (ii) used

¹We define resale value as the amount consumers receive when they sell their used video games to retailers. In Japan, retailers usually set a take-it-or-leave-it resale value for each game, and consumers sell their games at that resale value. Negotiation is uncommon.

goods selling decision, (iii) consumer expectations about future prices of new and used goods, resale values of used goods, and inventory levels of used goods, (iv) costs of buying and selling used goods, (v) the impact of used goods availability on buying decisions, and (vi) deterioration of both owners' and potential buyers' consumption values. In our model, the expected discounted value of future payoffs from buying a product is determined by a dynamic consumer selling decision problem, which depends on the deterioration rate of owners' consumption values and future resale values. This modeling approach allows us to study the role of consumer expectation about future used-copy prices and resale values on current buying decisions. In particular, the used-copy inventory level, which affects the transition probability of resale values, plays a role of an exclusion restriction in the consumer selling decision problem to identify the discount factor.

We estimate the proposed discrete choice dynamic programming model using the Bayesian algorithm by Ishihara and Ching (2016). To account for the potential endogeneity of prices, we apply the pseudo-policy function approach by Ching (2010b). The preference parameter estimates suggest that consumer heterogeneity in price sensitivity and costs of buying and selling used copies plays an important role in explaining the observed sales paths and substitution patterns. The demand patterns are well-explained by three types of consumers: (1) consumers who purchase new copies and become the main supplier of used copies in the used goods market (about 1% of the population), (2) consumers who purchase new and used copies, but are less likely to sell (about 25%), and (3) consumers who purchase new copies and sell, but their buying and selling probabilities are much lower than those for the first type of consumers, which explain the majority of non-purchasers (about 75%). The substitutability between new and used copies in the Japanese video game market highly depends on the second type of consumers. When the used-copy inventory is relatively low, new and used copies provide relatively similar utility for them as the price differential is canceled out by a high search cost for finding a used copy. Thus a small change in prices of new/used copies can make them switch between new and used copies. However, as the inventory of used copies is accumulated and the used-copy price decreases, these consumers will be more attracted to used copies. We compute the elasticities and examine how different types of consumers switch between new and used copies

over time.

Using the estimated model, we conduct counterfactual experiments and quantify the impact of eliminating the used video game market on new-copy sales, profits, consumer surplus, and social surplus. The video game market has recently tried to move towards using digital download as an alternative distribution channel. A complete switch to digital download would essentially shut down the used video game market. Therefore, the results of this experiment could shed some light on this strategy. We first conduct the experiment by holding the prices of new copies at the observed level. On average, the elimination of the used video game market reduces consumers' willingness-to-pay for new copies of video games in the earlier part of the product lifecycle – this is mainly because the resale effect dominates the substitution effect. However, in the later periods, the substitution effect dominates as some of the consumers who used to purchase a used copy switch to new copies. We find that the profit improves by 2%, but the consumer welfare decreases by 19%. We then compute the optimal prices of new copies when there is no used game market, and quantify the change in profits, consumer surplus, and social welfare. We find that the optimal flat prices are on average 52.2% lower than the observed prices, and the elimination of the used video game market could increase the profit by 78% and the consumer surplus by 141% due to the lower new prices. Overall, our results suggest that the elimination of the used game market could improve the social welfare in the Japanese video game market.

The rest of the paper is organized as follows. Section 2 reviews the previous literature. Section 3 describes the Japanese video game data used in this paper and presents some empirical regularities that have not been documented in the previous literature. Section 4 describes the dynamic discrete choice model of consumer buying and selling decisions. Section 5 explains the estimation strategy and identification. In Section 6, we discuss the parameter estimates and the counterfactual experiment results. Section 7 concludes.

2 Literature Review

There is a large body of theoretical literature in economics and marketing that analyzes the interaction between new and used durable goods. Theoretical studies in economics are mainly concerned with durability

choice, pricing, the role of market frictions, etc. for durable goods monopolists (e.g., Swan 1970, Rust 1986, Bulow 1986, Anderson and Ginsburgh 1994, Waldman 1996, Hendel and Lizzeri 1999, Johnson 2011). In marketing, several papers examine a variety of marketing practices in new and used durable goods markets, including leasing contracts (e.g., Desai and Purohit 1998, Desai and Purohit 1999), channel coordination (Desai et al. 2004, Shulman and Coughlan 2007), trade-ins (Rao et al. 2009), and retail versus P2P used goods markets (Yin et al. 2010). These studies provide important theoretical implications for our research question. For example, in the seminar work, Swan (1970) shows that the existence of used goods markets do not limit profits of a monopoly producer (Swan's Independence Result). Rust (1986) develops an equilibrium model that relaxes the assumption of exogenous scrappage value of used goods, and finds that under certain conditions it is optimal for a monopoly producer to kill off used goods markets by setting zero durability.

Our empirical setting is the closest to Johnson (2011) who develops a model where used goods trading is driven by changing consumer valuation (rather than quality deterioration and consumers with heterogeneous sensitivity to quality). He considers a situation where consumer valuations of a product decline due to consumption, and finds that when the marginal cost of production is small, it is optimal for a monopoly producer to shut down used goods markets and social welfare improves. One important distinction between our model and his model is that he assumes a constant arrival rate of new consumers every period (e.g., new students for textbook purchase every semester), while our model is best described as an optimal stopping problem where the initial set of consumers make buying and selling decisions over time.

Empirical studies on the impact of used goods markets on new-good producers' profits and social welfare are limited partly due to data availability and mostly focused on cars and houses (e.g., Purohit 1992, Esteban and Shum 2007, Engers et al. 2009, Tanaka 2013, Chen et al. 2013, Schiraldi 2011). A notable exception is Shiller (2013) who also investigates the interaction between new and used good demand, but in the U.S. video game industry. A general finding among these papers is that, as we will find in our application, the elimination of used goods markets helps new-good producers. However, these papers simplify the demand-side model due to either their focus on complex supply-side dynamics or limited data availability. In contrast,

we take advantage of our novel data set and estimate a dynamic demand model that has rich features. First, due to limited data availability, these papers do not observe consumers decision on selling used goods and associated resale values. As a result, they impose a market clearing of perfectly competitive used goods markets in every period, so that the quantity sold by consumers equals the observed sales of used goods, and the resale value equals the observed price of used copies. However, our data show that this assumption is far from a good approximation for the Japanese video game market – we consistently observe excess supply of used goods and used game retailers earn positive profits. Thus, our dynamic structural demand model will not impose a market-clearing condition. Instead, we will take advantage of the observed aggregate inventory of used copies and allow the cost of buying a used copy to depend on it (to capture the idea that a high inventory level may reduce the search cost of finding a used copy at retailers). Also, separately observing buying and selling behaviors, we can identify the two forms of consumption value deterioration, assuming that there is no physical depreciation.

Another important difference between our study and the previous studies is that we estimate consumers' discount factor, instead of calibrating it according to the interest rate. As we will describe, our model implies that inventory level of used copies provide exclusion restrictions that help identify the discount factor (Magnac and Thesmar 2002, Fang and Wang 2015). The intuition is that used-copy inventory level does not enter consumers' current utility function for selling decisions, but affect consumers' expected future payoffs from not selling today via the transition probability of future resale values. As a result, the observed correlation between quantity supplied of used copies and the inventory level can help recover the discount factor. Our identification strategy is similar to Chevalier and Goolsbee (2009), who study whether students are forward-looking in their textbook purchase decision. However, since they do not observe when and whether students sell their textbooks, they assume that all of them will sell it at the end of the semester, and the textbook resale value affects all students' utility for buying a new textbook if they are forward-looking. While this assumption may be reasonable in the textbook market, the timing of selling used goods is endogenous in general. Our proposed model endogenizes the timing via a dynamic consumer selling decision

problem.

Finally, our research contributes to structural works on video game markets (e.g., Nair 2007, Dubè et al. 2010, Liu 2010, Lee 2013), and demand models for durable goods in general (e.g., Melnikov 2013, Song and Chintagunta 2003, Gordon 2009, Goettler and Gordon 2011, Carranza 2010, Gowrisankaran and Rysman 2012). This paper extends the literature by incorporating a rich features into a model of consumers' buying and selling decisions for new and used goods, and examines the potential impact of the used good market on the demand for new games, firms' profits, consumer welfare, and social welfare.

3 Data

3.1 Japanese video game industry

Since mid-80s, the Japanese video game market has grown rapidly. The size of the industry in 2009 has reached \$5.5 billion on a revenue basis (including sales of hardware, software, other equipments). This is about three times larger than the theatrical movie revenue in Japan, and it has become one of the most important sectors in the Japanese entertainment industry. The existence of the used good market has been a serious issue for video game publishers since 90s. In 2009, the sales of used video games (software) alone amounts to \$1.0 billion on a revenue basis. One reason for the large used video game market in Japan could be that video game renting by third-party companies is prohibited by law in Japan.² Another reason argued by Hirayama (2006) is the flat-pricing strategy commonly adopted by video game publishers - the price of new games is maintained at the initial level at least one year after the release.³ This may provide an opportunity for used goods market to grow and capture the segment of consumers who do not mind buying used goods. However, it can also be argued that the existence of the used market has induced publishers to adopt the flat-pricing strategy. Liang (1999) uses a theoretical model to show that when used goods markets are present, durable goods monopolists may be able to credibly commit to a high price (avoiding the Coase

²In principle, video game publishers can run the rental business for their own video games. However, only one publisher attempted to operate it in the history and did not succeed and exited.

³Note that in Japan, resale price maintenance is illegal for video games although it is legal for books, magazines, newspapers and music.

Conjecture).⁴ While investigating the optimality of the flat-pricing strategy is interesting, we will leave this topic for future research. Instead, we will take the flat pricing strategy as given, and focus on understanding consumers' dynamic buying and selling decision problem.

3.2 Japanese video game data

We have collected a data set of 20 video games that were released in Japan between 2004 and 2008.⁵ The data come from several sources. For each video game, weekly aggregate sales of new copies and its manufacturer suggested retail price (MSRP) are obtained from the weekly top 30 ranking published in Weekly Famitsu Magazine, a major weekly video game magazine in Japan published by Enterbrain, Inc. The average number of weeks observed across games is 9 weeks. In Japan, the sales of new copies sharply declines after the release week (see Figure 1). In our data set, the median percentage of new game copies sold in the release week (relative to the total annual sales for the first year⁶) is 54%, and the median percentage of new game copies sold within the first month (4 weeks) after release is 82%. Thus, the sales of new copies is highly concentrated within the first month in Japan. In addition to the data from the primary market, we collected weekly aggregate trading volumes (both buying and selling) and the associated weekly average retail prices and resale values in the used market by game title. These are collected from the Annual Video Game Industry Report published by Media Create Co., Ltd. The average number of weeks observed across games is 33 weeks. According to an annual industry report by Enterbrain, Inc., about 80% of used video game trading occur at retailers during our sample period. Thus we do not consider the possibility such as online auctions for buying and selling used copies.

We also collected video game characteristics from Weekly Famitsu Magazine, including average critic and user rating, story-based game dummy, and multi-player game dummy. During our sample period (2004-

⁴The idea is similar to Ching (2010a) and Frank and Salkever (1992), who argue that endogenous market segmentation can allow brand-name firms to sustain a high price when facing entry of generic products in the prescription drug market.

⁵These 20 games in the data capture a small portion of the total number of video games released in Japan during our sample (about 1,000 video games). We had to use a small portion mainly because of the limited data availability of the used video game trading activities. The Annual Industry Report by Media Create publishes the used video game trading data for only 10 games per year. We checked that these 10 games are not necessarily the most popular games of the year. Moreover, increasing the number of games in the sample significantly increases the computational burden of estimating the dynamic programming model we propose in this paper.

⁶We separately collected the total annual sales from the Annual Famitsu Game Hakusho (white paper).

2008), online game features were limited and most of our games are not social games that are played by a large number of users online. Finally, the potential size of market for a video game is measured by the installed base of the platform in which the video game was released. The platforms of the 20 games include three consoles (PlayStation 2, PlayStation 3, Nintendo GameCube). We collected the weekly sales of all three consoles above from their release week to calculate the cumulative sales.

Table 1 shows the summary statistics. The average price of used copies across games and time is about two-thirds of the price of a new copy. The average retailer markup for used copies is large: 1,687.2 in JPY or 73.0%. This number is in contrast to the average retailer markup for new copies, which is around 10% or JPY 760 (Tachibana 2006), and provides strong incentives for retailers to trade used copies. The average relative size of the used game market to the new game market, defined as the ratio of cumulative sales of used copies to that of new copies at the end of used-copy sales sample period, is 0.46 with a maximum of 0.63 and a minimum of 0.35.⁷ Our data show that we have variation in the ratio across games, and it helps identify the difference in the utility function between new- and used-copy purchase after controlling for observed factors such as prices.

3.3 Some empirical regularities

In this section, we will discuss some new empirical regularities along three dimensions: (i) the quantities of used goods demanded and supplied over time, (ii) the inventory level of used goods over time, and (iii) the price and the resale value of used goods.

Figure 2 plots the average quantities of used copies demanded and supplied as well as the average inventory level of used copies over 15 weeks. The inventory level of used copies in week t for a game is defined as the difference between the cumulative quantity of used copies supplied by consumers up to week $t - 1$ and the cumulative quantity of used copies demanded by consumers up to week $t - 1$.

First, both quantities of used copies demanded and supplied sharply increase in the first few weeks after

⁷For new-copy sales, we assume zero sales after a game drops out of the top 30 ranking. This assumption should not affect the number as the sales of new copies is highly concentrated in the first few weeks, so its cumulative number will not increase much in subsequent weeks.

the opening of the used game market (second week after release), reach their peaks, and gradually decrease afterwards. The initial increase is probably because it takes a few weeks for owners of a game to become satiated with their games. As the quantity of used copies supplied by owners increases, the sales of used copies also follows.

Second, on average, the inventory level of used copies carried by retailers grows in the first 15 weeks. About half of the games in our data set exhibit a decline after some point during the sample period. It is clear that in the Japanese video game market, the used market does not clear in every period. As mentioned earlier, unlike previous studies which assume the used goods market clears in each period, we will make use of this excess supply information when estimating our dynamic model. Also note that although retailers accumulate used-copy inventory, their buy-early-sell-late strategy still allows them to make positive profits due to a high markup.

Third, as shown in Figure 3, both the average price and resale value of used goods gradually decrease over time, and the resale value decreases slightly faster. This suggests that both potential buyers' and owners' consumption values depreciate over time, and their deterioration rates could be different. We will incorporate these features into our model.

4 Model

In this section, we present our dynamic discrete choice model of consumer buying and selling decisions for durable goods that do not depreciate physically. To make our presentation more concrete, we will describe our model in terms video games, as this is the market which we will study. We assume that consumers make buying and selling decisions separately for each game.⁸ Let i index consumers, g index games, and t index time. To capture consumer heterogeneity, we allow discrete consumer types. At the beginning of the initial period $t = 1$ (i.e., the period in which the new game is released), no consumers own game g and used games

⁸We do not explicitly model the choice among different games because our focus is to study the choice between new and used copies of the same game title. We control for the impact of the availability of other games on the purchase decision of the focal game by including the cumulative number of other newly introduced games since game g 's release. Note that Nair (2007) finds evidence that the substitutability between two different video games is very low in the US market, and consequently, he also does not model the choice among different games.

are not available yet. Thus, consumers' decision problem is to decide whether to buy a new good or not to buy at all in $t = 1$. In period $t > 1$, consumers who have not bought the game up to $t - 1$ observe the prices of new and used copies, the resale value and inventory level of used copies at retailers, and decide whether to buy a new or used good, or not to buy anything. Let $j = 0, 1, 2$ denote no purchase option, new good purchase, and used good purchase, respectively. If consumers have already bought game g prior to time t and have not sold it yet, then they observe the resale value and inventory level and decide whether or not to sell the game in period t . Let $k = 0, 1$ denote keeping and selling options, respectively. If consumers sell their game, they exit the market. Since video games will eventually become outdated, we assume a terminal period $t = T$ after which consumers can neither buy nor sell. For consumers who own the game and did not sell at the terminal period, we allow them to continue to enjoy the game for additional T' periods (with appropriate satiation-based deterioration).

To capture the institutional details of our empirical application, we make the following assumptions when developing a model: (A) the price of new copies is constant over time, which is motivated by the industry practice in Japan; (B) products do not physically depreciate over time, and thus consumption values from a new copy are identical to those from a used copy;⁹ (C) the decision to buy a used copy may be influenced by factors other than consumption values and prices (such as the availability of used copies at retailers, psychological cost for using pre-owned goods, etc.). Our assumption here is that once consumers overcome this psychological cost at the time of making a buying decision, then consumption values they receive in subsequent periods is not affected by it; (D) products do not generate network externality.¹⁰

The state space of the consumer decision model consists of the following variables: (1) price of new and used goods (p_1, p_2); (2) resale value (r); (3) inventory level of used copies at retailers (Y), which controls for the impact of the availability of used copies on consumer buying decisions; (4) time since release (t), which characterizes the single-period consumption value to potential buyers; (5) time since purchase (τ), which

⁹This assumption excludes a possible situation where retailers adjust the price of used copies based on their physical conditions.

¹⁰Most of the games in our sample are offline games, because of our sample period (2004-2008). Thus, we do not model the network externality.

affects the single-period consumption value to owners; (6) unobserved demand shocks for used copies (ξ_2); (7) unobserved supply shocks for used copies (ξ_s); (8) cumulative number of newly introduced games on the focal game's console since the focal game release (C). As we will describe later, (1), (6), and (8) appear only in the consumer buying decision problem, (5) and (7) appear only in the consumer selling decision problem, and (2), (3) and (4) appear in both consumer buying and selling decision problems. We assume that consumers make their buying and selling decisions to maximize their total discounted expected utility for each game.

We will first describe the single-period utility functions for buying and selling decisions, and then move to the description of the value functions.

4.1 Single-period utility functions

In each period, consumers derive a consumption value from owning game g . Let $v^g(t, \tau)$ be a consumer's single-period consumption value of owning game g at time t if he has owned game g for τ periods prior to time t . Note that if a consumer buys game g at time t , he will receive $v^g(t, 0)$ in that period; if he/she keeps it at time $t + 1$, he will receive $v^g(t + 1, 1)$. Later in this section, we will describe how we allow the two forms of deterioration, freshness-based and satiation-based, to affect the consumption value over time.

Suppose that a consumer has not bought game g up to time $t > 1$. Consumer i 's single-period utility for buying decisions at time t is given by:

$$w_{ijt}^g = \begin{cases} v^g(t, 0) - \alpha_i p_{1t}^g + \rho D_t^g + \epsilon_{i1t}^g & \text{if buying a new copy } (j = 1) \\ v^g(t, 0) - \alpha_i p_{2t}^g - l_Y(Y_t^g; \lambda_i) + \rho D_t^g + \xi_{2t}^g + \epsilon_{i2t}^g & \text{if buying a used copy } (j = 2) \\ l_C(C_t^g; \pi) + \epsilon_{i0t}^g & \text{if no purchase } (j = 0), \end{cases} \quad (1)$$

where p_{1t}^g (p_{2t}^g) is the price of new (used) copies of game g at time t ; ξ_{2t}^g is the unobserved demand shock to used copies;¹¹ α_i is the price-sensitivity. As justified by the stylized facts discussed earlier, we assume that the price of new copies is constant over time, i.e., $p_{1t}^g = p_1^g$ for all t , in our application to the Japanese video game market. We assume that ξ_{2t}^g is *i.i.d.* across time and game, and is normally distributed with zero mean and the standard deviation σ_{ξ_2} ; Y_t^g is the inventory level of used copies for game g at retailers at the

¹¹We do not include the unobserved demand shock to new copies because the price of new copies is constant over time.

beginning of period t . $l_Y(Y_t^g; \lambda_i)$ is the one-time cost that consumers incur when buying a used good (search costs, psychological costs for pre-owned games, etc.), and λ_i is a vector of parameters. In our empirical specification, we specify

$$l_Y(Y_t^g; \lambda_i) = \lambda_{0i} + \frac{1}{\lambda_1 + \lambda_2 Y_t^g} \quad (2)$$

to capture the ideas that (i) consumers might be heterogeneous in psychological costs for pre-owned games (λ_{0i}) and (ii) search costs may depend on the availability of used copies Y_t^g , and the effect is measured by (λ_1, λ_2) . The heterogeneity in λ_{0i} is motivated by a consumer survey conducted by Enterbrain, Inc., which shows that about 15% of consumers never intend to purchase a used copy.¹² This potentially suggests that there is a segment of consumers who have high psychological costs of buying a used copy. This reduced-form specification implies that when no used copies are available at the beginning of a period (i.e., $Y_t^g = 0$),¹³ the cost is $\lambda_{0i} + \frac{1}{\lambda_1}$. As the availability of used copies increases to infinity, the cost approaches λ_{0i} (if $\lambda_2 > 0$). Thus, if λ_2 is positive, the cost decreases as Y_t^g increases; D_t^g is a vector of seasonal dummies and ρ captures the seasonal effects.¹⁴ C_t^g is the cumulative number of games introduced on the console for game g at time t since the introduction of game g (including the games released in the same week as game g), and $l_C(C_t^g; \pi)$ captures the competitive effect from other newly introduced games. In our application, it is specified as

$$l_C(C_t^g; \pi) = \pi_1 \ln(C_t^g + 1). \quad (3)$$

We assume that idiosyncratic errors, ϵ_{ijt}^g , are *i.i.d.* across consumers and time, but allow it to be correlated across options j . We model the correlation in a nested logit framework. Let $\epsilon_{ijt}^g = \zeta_{iht}^g + (1 - \eta_b)v_{ijt}^g$ where ζ_{iht}^g and v_{ijt}^g are extreme value distributed, h indexes nest and takes two possible values: $h = 1$ groups the buying options (i.e., buying a new or used copy), and $h = 0$ is the no purchase option. Thus, the consumer buying decision problem here is equivalent to a two-stage decision making where consumers first decide whether or not to buy, and if buying, then consumers choose a new or used copy. In this setup,

¹²Famitsu Game Hakusho 2006, page 217.

¹³Note that $Y_t^g = 0$ does not mean that there are no used copies available for purchase in period t because some owners would sell their copies to the market during the period.

¹⁴We include this variable only to control for the seasonal variation in sales and do not intend to study its impact on consumers' dynamic decision making. Thus, it is not included as a state variable.

the parameter $\eta_b \in [0, 1)$ measures the within-nest correlation. If $\eta_b = 0$, then our model reduces to a multinomial logit model.

Next, consider consumers' selling decisions. Suppose that a consumer has bought game g and kept it for τ periods. Consumer i 's single-period utility for selling decisions at time t is given by

$$w_{ikt}^g(\tau) = \begin{cases} \alpha_i r_t^g - \mu_i + \xi_{st}^g + e_{i1t}^g & \text{if selling to a retailer } (k = 1) \\ v^g(t, \tau) + e_{i0t}^g & \text{if keeping the game } (k = 0) \end{cases} \quad (4)$$

where r_t^g is the resale value of game g at time t ; μ_i captures any additional cost of selling (cost to go to a retailer and sell in person, endowment effects, etc.) and is allowed to depend on consumer type. This is again motivated by the same consumer survey by Enterbrain, Inc. which indicates that not all consumers sell their games; ξ_{st}^g is an *i.i.d.* unobserved shock to owners for selling decisions at time t . We assume it is normally distributed with zero mean and the standard deviation, σ_{ξ_s} ; e_{ikt}^g is an idiosyncratic error, and we assume it is *i.i.d.* extreme value distributed across consumers and time, with zero mean and the scale parameter η_s .

For the single-period consumption value, $v^g(t, \tau)$, we will assume the following evolution over time. In the release period, we set $v^g(1, 0) = \gamma^g$, where γ^g is a game-specific constant. To capture the deterioration of potential buyers' consumption values due to the aging of a game (freshness-based deterioration), we allow $v^g(t, 0)$ to decay as a function of t . Specifically, we model the deterioration rate as: $v^g(t + 1, 0) = (1 - \varphi(t))v^g(t, 0)$, where $\varphi(t)$ is a function of time trend. In our empirical application, we specify it as

$$\varphi(t) = \frac{\exp(\phi_1 \mathbb{I}(t = 1) + \phi_2 \mathbb{I}(t > 1))}{1 + \exp(\phi_1 \mathbb{I}(t = 1) + \phi_2 \mathbb{I}(t > 1))}, \quad (5)$$

where $\mathbb{I}(\cdot)$ is an indicator function. Note that we treat the deterioration from the first period to the second period differently from the rest of the periods. This is motivated by the observations that the sales of new copies from the release week to the second week usually suffers from a largest decline in the video game market. Next, we capture the deterioration of owners' consumption values due to satiation by modeling the deterioration rate as a function of product characteristics and the duration of ownership: $v^g(t + 1, \tau + 1) = (1 - \kappa(X_{g\tau}))v^g(t, \tau)$, where $\kappa(X_{g\tau})$ is a function of observed product characteristics and

the duration of ownership. In the application, we use the following functional form:

$$\kappa(X_{g\tau}) = \frac{\exp(X'_{g\tau}\delta)}{1 + \exp(X'_{g\tau}\delta)}, \quad (6)$$

where $X_{g\tau}$ includes observed product characteristics of game g (dummies for story-based games and multi-player games, and average critic and user ratings) and the duration of ownership (τ).

Finally, we emphasize that our proposed model allows for consumer heterogeneity in price sensitivity (α_i), and the costs of buying (λ_{0i}) and selling (μ_i) a used copy. In addition, game owners are heterogeneous with respect to (t, τ) : owners' consumption value depends on when they bought the game and how long they have kept it. In general, other preference parameters such as initial consumption value, freshness- and satiation-based deterioration rates, etc. could be heterogeneous. We have experimented several other specifications and found that after controlling for consumer heterogeneity in $(\alpha_i, \lambda_{0i}, \mu_i)$, heterogeneity in other preference parameters does not play a significant role in explaining the observed sales pattern.

4.2 Value functions

Since the dynamic consumer selling decision problem is nested within the dynamic consumer buying decision problem through the expected future payoff, we start off by describing the dynamic consumer selling decision problem, and then describe the dynamic buying decision problem. To simplify the notation, we will drop g superscript. Let β be the discount factor common across consumers.

Let $s_{t,\tau} = (r_t, Y_t, \xi_{st}, t, \tau)$ be the vector of state variables relevant to the selling decision problem. Note that other state variables $(p_{1t}, p_{2t}, C_t, \xi_{dt})$ will not enter here. The inventory level, Y_t , is included since it could affect the distribution of the future resale value. Let $W_i(s_{t,\tau})$ be the integrated value function (or Emax function) of the selling decision problem for consumer i , and $W_{ik}(s_{t,\tau})$ be the corresponding alternative-specific value function for action k . The Bellman equation can be written recursively as:

$$\begin{aligned} W_i(s_{t,\tau}) &= E_e \max_{k \in \{0,1\}} \{W_{ik}(s_{t,\tau}) + e_{ikt}\}, \\ &= \eta_s \ln \left\{ \sum_{k \in \{0,1\}} \exp \left(\frac{W_{ik}(s_{t,\tau})}{\eta_s} \right) \right\}, \end{aligned} \quad (7)$$

where the second equality follows from the assumption that e is extreme value distributed with the scaling parameter η_s , and

$$W_{ik}(s_{t,\tau}) = \begin{cases} \alpha_i r_t - \mu_i + \xi_{st} & \text{if selling } (k = 1), \\ v(t, \tau) + \beta E[W_i(s_{t+1,\tau+1})|s_{t,\tau}] & \text{if keeping } (k = 0). \end{cases} \quad (8)$$

The expectation in $E[W_i(s_{t+1,\tau+1})|s_{t,\tau}]$ is taken with respect to the future resale value (r_{t+1}), inventory level (Y_{t+1}), and unobserved shock for selling decision (ξ_{st+1}).

The probability of selling the game by consumer i at $s_{t,\tau}$ is given by

$$\Pr(k = 1|s_{t,\tau}; i) = \frac{\exp(W_{i1}(s_{t,\tau}))}{\sum_{k'=0}^1 \exp(W_{ik'}(s_{t,\tau}))}. \quad (9)$$

Next, consider the dynamic consumer buying decision problem. Let $b_t = (p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{2t}, t)$ be the vector of state variables relevant to the buying decision problem. Let $V_i(b_t)$ be the integrated value function for consumer i who has not bought the game prior to time t , and $V_{ij}(b_t)$ be the corresponding alternative-specific value functions of action j . The Bellman equation is given by

$$\begin{aligned} V_i(b_t) &= E_\epsilon \max_{j \in \{0,1,2\}} \{V_{ij}(b_t) + \epsilon_{ijt}\} \\ &= \ln \left\{ \exp(V_{i0}(b_t)) + \left[\sum_{j \in \{1,2\}} \exp\left(\frac{V_{ij}(b_t)}{1 - \eta_b}\right) \right]^{1 - \eta_b} \right\} \end{aligned} \quad (10)$$

where

$$V_{ij}(b_t) = \begin{cases} v(t, 0) - \alpha_i p_{1t} + \beta E[W_i(s_{t+1,\tau=1})|s_{t,\tau=0}] & \text{new copy } (j = 1), \\ v(t, 0) - \alpha_i p_{2t} - l_Y(Y_t; \lambda_i) + \xi_{2t} + \beta E[W_i(s_{t+1,\tau=1})|s_{t,\tau=0}] & \text{used copy } (j = 2), \\ l_C(C_t; \pi) + \beta E[V_i(b_{t+1})|b_t] & \text{no purchase } (j = 0). \end{cases} \quad (11)$$

The expectation in $E[V_i(b_{t+1})|b_t]$ is taken with respect to the future prices of new and used copies (p_{1t+1}, p_{2t+1}), resale value (r_{t+1}), inventory level (Y_{t+1}), cumulative number of competing games (C_{t+1}), and unobserved shocks for buying decisions (ξ_{2t+1}). It should be highlighted that the value function of the selling problem is embedded into the value function of the buying problem.¹⁵

The choice probability for option j by consumer i at b_t is given by

$$\Pr(j|b_t; i) = \Pr(h = 1|b_t; i) \cdot \Pr(j|h = 1, b_t; i), \quad (12)$$

¹⁵Note that the Bellman equations stated here applies to $t > 1$. The expected future payoffs component needs to be slightly modified for $t = 1$ because $p_{2,t=1}$ and $r_{2,t=1}$ do not exist. We will discuss how to specify consumers' expectation about these state variables in the next section.

where

$$\Pr(h = 1|b_t; i) = \frac{\left[\sum_{j'=1}^2 \exp\left(\frac{V_{ij'}}{1-\eta_b}\right) \right]^{1-\eta_b}}{\exp(V_{i0}) + \left[\sum_{j'=1}^2 \exp\left(\frac{V_{ij'}}{1-\eta_b}\right) \right]^{1-\eta_b}}, \quad (13)$$

$$\Pr(j|h = 1, b_t; i) = \frac{\exp\left(\frac{V_{ij}}{1-\eta_b}\right)}{\sum_{j'=1}^2 \exp\left(\frac{V_{ij'}}{1-\eta_b}\right)}. \quad (14)$$

Given a finite time horizon, the value functions for both buying and selling decisions can be computed by backward induction from the terminal period, T . We assume that after the terminal period, consumers can neither buy or sell, but can continue enjoying the game if they have bought by the terminal period. We thus assume that consumers who own the game at the end of $t = T$ derive a terminal value equal to the present discounted value of future consumption values, taking satiation-based deterioration into account. In the empirical application, we approximate it by the present discounted value of consumption values for another 100 periods beyond the terminal period, and we set $T = 75$.

4.3 Aggregate sales

Let ψ_l be the population proportion of type- l consumers and $\sum_{l=1}^L \psi_l = 1$. In order to derive the aggregate demand for new and used copies, and aggregate volume of used copies sold to retailers by owners, we need to derive the evolution of the size of each consumer type. Let M_{lt}^d be the size of type- l consumers who have not bought the video game. It evolves according to

$$M_{lt+1}^d = M_{lt}^d \left(1 - \sum_{j=1}^2 \Pr(j|b_t; l) \right) + N_{lt+1}, \quad (15)$$

where N_{lt+1} is the size of new type- l consumers who enter the market at time $t + 1$. We assume that the proportion of new type- l consumers follows the population proportion, ψ_l .¹⁶

Next, let $M_{lt}^s(\tau)$ be the size of type- l consumers who have bought and owned the game for τ periods at time t . It evolves according to

$$M_{lt+1}^s(\tau) = \begin{cases} M_{lt}^d \sum_{j=1}^2 \Pr(j|b_t; l) & \text{if } \tau = 1, \\ M_{lt}^s(\tau - 1) \cdot \Pr(k = 0|s_{t,\tau-1}; l) & \text{if } 1 < \tau \leq t. \end{cases} \quad (16)$$

¹⁶Therefore, we can use the total installed base of the corresponding game console in the release week to calibrate $N_{l,t=1}$. For $t > 1$, N_{lt} can be calibrated based on the weekly sales of the corresponding game console.

The aggregate observed demand at state b_t is then

$$Q_j^d(b_t) = \sum_{l=1}^L M_{lt}^d \Pr(j|b_t; l) + \varepsilon_{jt} \quad (17)$$

where $j = 1$ is new copies and $j = 2$ is used copies, and ε_{jt} represents a measurement error. The aggregate observed quantity supplied to retailers by consumers at state $s_t = s_{t,\tau} \setminus \{\tau\}$ is given by

$$Q^s(s_t) = \sum_{l=1}^L \sum_{\tau=1}^{t-1} M_{lt}^s(\tau) \Pr(k = 1|s_{t,\tau}; l) + \varepsilon_{st}, \quad (18)$$

where ε_{st} represents a measurement error.

5 Estimation Strategy

We estimate the consumer preference parameters using a Bayesian MCMC algorithm for non-stationary finite-horizon models proposed by Ishihara and Ching (2016). We combine the algorithm with the pseudo-policy function approach in Ching (2010b) to control for the potential endogeneity issue (Ishihara 2011). This section briefly explains the Bayesian algorithm, and discusses the empirical specifications of the consumer expectation processes and the pseudo-policy functions. We conclude this section by discussing the identification strategy for the consumer preference parameters.

5.1 Bayesian Dynamic Programming Algorithm

We apply the Bayesian algorithm proposed by Ishihara and Ching (2016), who extend Imai, Jain, and Ching (2009) to non-stationary finite-horizon dynamic programming models. The main idea behind their algorithm for continuous state variables is to compute pseudo-value functions at one randomly drawn state in each iteration and store them. The set of past pseudo-value functions used in approximating the expected future payoffs will then be evaluated at different state points. Thus, one can simply adjust the weight given to each of the past pseudo-value function by the transition density from the current state to the state at which the past pseudo-value function is evaluated.

Similar to Yang et al. (2003) and Musalem et al. (2009), who proposes a Bayesian approach to the estimation of static demand and supply models, Ishihara and Ching (2016) augment unobserved aggregate

demand and supply shocks (ξ_{2t} and ξ_{st}) based on the joint-likelihood of the demand-side model and the pseudo-policy functions. Compared to the GMM approach used in the previous literature (Berry et al. 1995, Gowrisankaran and Rysman 2012), this approach does not require using the contraction mapping procedure to recover the mean utility level from observed market shares. Also, since we use the Bayesian data augmentation technique, unlike the simulated maximum likelihood method, this approach does not need to integrate out the unobserved shocks during the estimation.¹⁷

5.2 Consumer expectation processes

We start with the state transition for consumers' selling decision problem. The state vector is $s_{t,\tau} = (r_t, Y_t, \xi_{st}, t, \tau)$. The time since release (t) and duration of ownership (τ) evolve deterministically and increase by one every period. The unobserved shock to selling (ξ_{st}) is *i.i.d.* normal with zero mean and variance $\sigma_{\xi_s}^2$. We model the process of used-copy resale value r_t to be a function of its lagged value, the lagged used-copy inventory level, and game characteristics. Finally, we model the process of used-copy inventory level to be a function of its lagged value and game characteristics.

The state vector of the buying decision problem is $b_t = (p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{2t}, t)$. The price of new copies (p_{1t}) is constant over time; the time since release (t) evolves deterministically and increases by one every period; ξ_{2t} is *i.i.d.* normal with zero mean and variance $\sigma_{\xi_2}^2$; and the processes for r_t and Y_t are as described above for the selling decision problem. We model the price of used copies (p_{2t}) to be a function of its lagged value, the lagged used-copy inventory level, and game characteristics. Finally, we assume that the process of cumulative number of newly released games on the focal game's console is console-specific, and model it to be a function of its lagged values. Thus we have three processes for each of the consoles in our data (PlayStation 2, Nintendo GameCube, and PlayStation 3).

Note that since used copies are not available at time $t = 1$, there are no lagged values for p_{2t} and r_{2t} at time $t = 2$. We there assume that the initial price and resale value of used copies (i.e., at time $t = 2$) to be

¹⁷Readers who are interested in the details of the algorithm should refer to Ishihara (2011) and Ishihara and Ching (2016), who also provide Monte Carlo evidence for the Bayesian algorithm.

a function of the price of new copies.¹⁸ Also, it is important to note that $Y_t = 0$ for $t = 1, 2$ by construction.

We estimate the parameters of these consumer expectation processes together with the consumer preference parameters using the proposed Bayesian algorithm. For these parameters, we first run a Bayesian linear regression for each consumer expectation process equation with diffuse priors, and obtain the distribution of the consumer expectation process parameters. In the structural estimation, we use this distribution as both the prior and the proposal distribution and take draws from the posterior distribution using the Metropolis-Hastings algorithm. A similar approach is used in Osborne (2014). It is important to note that when using the Metropolis-Hastings algorithm, we need to account for the fact that value functions depend on consumer expectation process parameters. Thus, value functions need to be approximated using the proposed Bayesian algorithm every time we draw consumer expectation process parameters.

5.3 Pseudo-policy function approach

In the proposed model, we face a possible endogeneity issue of used-copy prices and resale values. That is, retailers choose used-copy prices and resale values by observing demand and supply shocks (ξ_{2t} and ξ_{st}) that are not observed by researchers. To address this issue, we adopt the pseudo-policy function (PPF) approach proposed by Ching (2010b). This approach is similar to other limited information approaches (e.g., Villas-Boas and Winer 1999, Ebbes et al. 2005, Park and Gupta 2009, Petrin and Train 2010, Jiang et al. 2009), but differs in that it approximates the pricing policy as a function of observed and unobserved state variables of the underlying equilibrium model. This approach has the potential of generating a more flexible joint distribution of (p, q) .¹⁹ In our model, the state space of the underlying equilibrium model should include unobserved shocks (ξ_{2t}, ξ_{st}), consumption values ($v(t, \tau)$), inventory level (Y_t), cumulative number of newly introduced games (C_t), the size of potential buyers (M_{it}^d), and the size of owners for each each duration of ownership ($M_{it}^s(\tau)$). Furthermore, we include seasonal dummies for Golden Week (D_{1t}) and Christmas (D_{2t}).²⁰ After experimenting with several functional forms, we decided to use the following

¹⁸We find that none of the game characteristics have a significant impact on the initial price and resale value of used copies, and thus do not include them in the process.

¹⁹This approach can also be applied to control for the potential endogeneity of advertising/detailing (e.g., Ching and Ishihara 2010; 2012), and individual retailers' pricing (e.g., Gu and Yang 2011).

²⁰Golden Week in Japan refers to a week in late April and early May that involves multiple public holidays.

specification for the price of used goods (for $t \geq 2$):

$$\ln p_{2t} = \omega_{p1} + \omega_{p2} \frac{1}{L} \sum_{l=1}^L M_{lt}^d + \omega_{p3} Y_t + \omega_{p4} C_t^{other} + \omega_{p5} D_{1t} + \omega_{p6} D_{2t} + \omega_{p7} \xi_{2t} + \omega_{p8} \xi_{st} + \nu_{pt}, \quad (19)$$

where C_t^{other} is the cumulative number of newly introduced games across all consoles and handhelds except for the focal game's console, and ν_{pt} is the prediction error. Also, the pseudo-policy function for the resale value is specified as (for $t \geq 2$):

$$\ln r_t = \omega_{r1} + \omega_{r2} \frac{1}{L} \sum_{l=1}^L M_{lt}^d + \omega_{r3} Y_t + \omega_{r4} C_t^{other} + \omega_{r5} D_{1t} + \omega_{r6} D_{2t} + \omega_{r7} \xi_{2t} + \omega_{r8} \xi_{st} + \nu_{rt}, \quad (20)$$

where ν_{rt} is the prediction error.

Note that C_t^{other} , which is the cumulative number of newly introduced games across all consoles and handhelds except for the focal game's console since the release of the focal game, could play the role of an instrument. This is because publishers usually pre-announce game release dates several months in advance and copies of games are manufactured before the release week. Thus, the pre-announced release date is rarely postponed based on the aggregate shocks in the release week, and we expect C_t^{other} to be uncorrelated with ξ_{2t} and ξ_{st} . Moreover, we expect that the demand for a game depends on other games available on the same console (i.e., the C_t^g variable in Equation 1), but not on games available on other consoles and handhelds, as those games are likely to be irrelevant for consumers' purchase decisions for the focal game.²¹ However, C_t^{other} could affect p_{2t} and r_t via limited shelf spaces at retailers. When new games are released on a console (say, console A), the demand for used copies of console A's games will drop. This creates pressure on the shelf space at used goods retailers, and causes the price and resale value of used games on console A to drop. But the pressure on the shelf space in general could also cause the price and resale value of used games on other consoles to drop as well. As a result, we expect that C_t^{other} will have a negative impact on the price and resale value of used copies for the focal game. Inclusion of C_t^{other} , which is exogenous at time t in our model, should help reduce the reliance on functional form restrictions for identification.

Assuming the normal distributions for measurement errors in the sales of new and used copies as well as the quantities sold by consumers to retailers, and prediction errors in the pseudo-pricing policy functions,

²¹This may be violated if consumers own and actively use multiple consoles and handhelds.

we derive the joint likelihood of the demand-side model and the pseudo-policy functions, which will be used to compute the acceptance probabilities in the Metropolis-Hastings algorithm. Appendix A.2 describes the construction of the likelihood function.

Remarks

Note that if consumers observe all the state variables and understand how the equilibrium prices are generated, then one would gain efficiency by using the pseudo-policy functions to form the consumers' future price expectations as well. However, we decided not to take this approach for the following reasons. First of all, it is unclear if consumers are aware of all the state variables of the equilibrium model, especially, the size of potential buyers (i.e., M_{it}^d) and the sizes of owners (i.e., $\{M_{it}^s(\tau)\}_{\tau=1}^{t-1}$), which are "hidden" and not available in any public domain. It seems more plausible that consumers use a simpler Markov process to forecast future prices and resale value based on what are observed in the public domain, which include current prices, resale value and game characteristics, as these variables are much more salient. In particular, Hendel and Nevo (2006) argue that a simple Markov process might be a reasonable assumption about consumers' memory and formation of expectations. Février and Wilner (2016) provide evidence to support such an approach. Besides, if one uses the pseudo-policy functions to form the consumer price expectation, one needs to specify the state space for the dynamic consumer buying and selling decision model to include $(M_{it}^d, \{v(t, \tau), M_{it}^s(\tau)\}_{\tau=1}^{t-1})$. Such a modification will dramatically increase the number of continuous state variables compared to our current approach, and thus make the model infeasible to estimate.

5.4 Identification

In this section, we provide an informal discussion for the identification of our proposed model. To facilitate our discussion, we first describe the identification when there is no heterogeneity in the costs of buying and selling a used copy (i.e., $\lambda_{0l} = \lambda_0$, $\mu_l = \mu$, and $\alpha_l = \alpha$ for all l). We then discuss what data variation helps us identify the heterogeneity.

We first consider the price sensitivity (α), the impact of used-copy inventory on the costs for buying a used copy ($\lambda_0, \lambda_1, \lambda_2$), and the competitive effect (π_1) in $l_C(C_t^g; \pi)$. These are identified by variation in the

sales of new and used copies, and the variation in used-copy prices and resale values, used-copy inventory, and the number of competing games, respectively. The within-group correlation (η_b) is identified by the extent to which the conditional market share of new (or used) copies is correlated with the unconditional market share of new (or used) copies (Berry 1994). The scaling parameter for selling decision (η_s) is identified because the price sensitivity parameter is common in both buying and selling decisions, and the scaling parameter of the idiosyncratic errors for buying decision (the error associated with the first-stage decision of the nested logit model) is normalized to one.

The game-specific constant (γ^g) is identified by the level of sales over time for each game. The freshness-based deterioration rate (φ 's), which are common across games, is identified by the average declining rate of sales of new games across games over time. Given γ^g , φ 's and α , the parameters that determine the satiation-based deterioration rate (δ 's) and the cost of selling (μ) are identified by the variation in the volume of video games sold by consumers to retailers over time across games.

The identification of the discount factor (β) hinges crucially on exclusion restrictions, by which we mean the existence of at least one variable which affects the future payoffs, but does not affect the current utility of buying used and new games (Chevalier and Goolsbee 2009, Fang and Wang 2015).²² In our model, the used-copy inventory (Y_t^g) plays exactly this role in the selling decision problem because it does not enter the current period utility and consumers only use its current value to predict its value next period, which gives them the expected return of selling the game next period. Therefore, the extent to which the volume of used copies sold is affected by the used-copy inventory provides us with information about how consumers discount the future.

So far we have assumed that there is no consumer heterogeneity. Now we consider how the heterogeneity parameters ($\alpha_l, \lambda_{0l}, \mu_l$) and the proportion of each consumer type (ψ_l) are identified. To simplify the argument, assume that there are two types of consumers and the model has only three periods. Furthermore, it will be useful to ignore the idiosyncratic errors (ϵ, e) in the utility functions. In period 1, consumers have

²²Chung et al. (2014), Lee (2013), and Ching and Ishihara (2015) also explore such exclusion restrictions to identify the discount factor in different applications. Magnac and Thesmar (2002) discuss another exclusion restriction that is closely related.

two options: buy a new copy, or delay purchase. In period 2, consumers can choose to buy a used copy. Note further that the price of new copies is constant over time, and the used-copy price is always lower than the new-copy price. If consumers are homogeneous and buy the game, then we should see either (i) they all buy a new copy in period 1, or (ii) they all buy a used copy in period 2. Only a model with heterogeneous consumers can generate sales of new copies (in period 1) and used copies (in period 2). So the relative sales of new and used copies provides us with some information about the extent of consumer heterogeneity. In particular, the reason why a consumer chooses to buy a used copy in period 2 is because his/her expected utility of buying a used copy is higher than the utility of buying a new copy in period 1.

Let type-1 consumers be those who buy a new copy in period 1 and type-2 consumers be those who buy a used copy in period 2. If $\alpha_1 = \alpha_2$, then it must be that $\lambda_1 > \lambda_2$. That is, type-1 consumers have a high cost of buying a used copy and thus buy a new copy in period 1. Now if $\alpha_1 \neq \alpha_2$, then we cannot determine the order of λ_1 and λ_2 .²³ However, we know one of the types has to buy in period 1 (and we assumed it is type 1 without loss of generality). Thus, the cross-sectional variation in the sales of new copies in period 1 across games identifies the proportions of type 1 and type 2 consumers (i.e., ψ_l for $l = 1, 2$). In period 2, type-1 consumers are the potential suppliers of used copies. Thus the cross-sectional variation in the volume of used copies supplied and the resale value of used copies in period 2 across games allows us to identify α_1 and μ_1 . Type-2 consumers are the potential purchasers of used copies in period 2. The cross-sectional variation in the sales of used copies and used-copy price across games identifies α_2 and λ_2 (note that type-2 will not buy a new copy in period 2 because the price of new copies is constant over time and type-2 did not buy a new copy in period 1). In period 3, type-2 consumers are the potential suppliers of used copies. Once again, the cross-sectional variation in the volume of used copies supplied and the resale value of used copies in period 3 across games identifies α_2 and μ_2 (over-identification restriction on α_2). Finally, given λ_2 , (α_l, μ_l) for $l = 1, 2$, λ_1 has to satisfy a condition such that type-1 consumers buy a new copy in period 1. Note that since type-1 consumers do not buy a used copy, we can only identify the lower bound of λ_1

²³Note that even if $\lambda_1 = \lambda_2$, it is not necessarily the case that $\alpha_1 < \alpha_2$ in our setting because consumers with a higher α also value the resale value more, so they may buy a new copy in period 1.

(i.e., λ_1 can be infinity so that type 1 consumers do not buy a used copy). In the proposed empirical model, idiosyncratic errors could make some of type-1 consumers buy a used copy in period 2.

Our identification arguments for consumer heterogeneity only make use of three periods of data. But certainly, the data variation beyond these three periods also helps identify the heterogeneity parameters and potentially more consumer types. Finally, note that the extreme value error terms (or any idiosyncratic error terms with continuous distribution) make the demand and used-copy supply become a smooth function of parameters. So even if we add them back to the model, the main idea of our identification argument still applies. Under this environment, both types of consumers will buy new copies in period 1, and used copies in period 2. But the consumers who buy new (used) copies are still mainly type 1 (2). Therefore, the cross-sectional data variation in the first three periods (and the cross-sectional and time-series variation in the subsequent periods) still provides us with many over-identification restrictions to identify the heterogeneity parameters.

6 Results

6.1 Parameter Estimates

We allow for three types of consumers who differ in their price sensitivity and costs of buying and selling at used goods retailers (i.e., α_l , λ_{0l} and μ_l). The parameter estimates for the demand model, pseudo-policy functions, and consumer expectation processes are reported in Tables 2, 3, and 4, respectively. We report the posterior mean and standard deviation (s.d.). Unless otherwise noted, we use diffuse priors on the parameters.

We start with the demand estimates in Table 2. All of the parameters show the expected signs. The estimated discount factor is 0.859. Recall that the unit of periods in our application is a week. Our estimate is much lower than the discount factor calibrated from the weekly interest rate ($\simeq 0.999$), which is typically done in the literature (note that most dynamic models do not have exclusion restrictions to help identify the discount factor). However, our result is consistent with the previous studies in experimental/behavioral economics (see Frederick et al. 2002 for a survey on this literature), and three recent empirical studies (Yao

et al. 2012, Lee 2013, Chung et al. 2014) which also find that the discount factor is lower than the interest rate. Price-sensitivity parameters (α_l) are positive because it enters the utility function as a negative term. Its magnitude is small because prices are in JPY (1 JPY \simeq 0.01 US dollar). We find that type-1 consumers (about 1% of the population) have the lowest price sensitivity, and type-3 consumers (about 74%) have the highest price sensitivity. Type-3 consumers have the lowest probability of purchasing a game among the three segments, which helps explain the overall low probability of purchase (relative to the number of console owners) for each game.

As for the cost of buying a used copy ($\lambda_{0l}, \lambda_1, \lambda_2$), recall that we use the following functional form: $l_Y(Y_t^g; \lambda_l) = \lambda_{0l} + \frac{1}{\lambda_1 + \lambda_2 Y_t^g}$. We find λ_2 is positive, indicating that the cost of buying a used copy diminishes as the inventory level rises. This is intuitive because as the availability of used copies increases, consumers' search costs may decrease. We quantify the range of the cost of buying a used copy in monetary terms by dividing l_Y by the price coefficient. As we will show later, we find that λ_{0l} 's for type-1 and type-3 consumers are so large that these consumers never purchase a used copy, even if the used-copy inventory level is high. Thus, we examine the range for type-2 consumers: it falls in [175, 4,419] in JPY.²⁴ The average price difference between new and used copies in the first few weeks is around JPY 1,500. Thus, when the used-copy inventory is very low in the first weeks, type-2 consumers will also purchase new copies. As the used-copy inventory increases, their demand will shift to used copies. We also find a significant difference in the selling costs between these three types. Type-1 and type-3 consumers have a relatively low cost of selling (relative to their price sensitivity).

We find that both the Golden Week and Christmas season have a positive impact on sales, but the magnitude is larger for the Christmas season. The parameter for the competitive effect from other games on the same console (π_1) is positive, suggesting that the increasing number of new game introduction may make it less attractive for consumers to buy the focal game.

Parameters for the freshness-based deterioration rate include two parameters (ϕ_1 for the first week, and ϕ_2 from the second week on). The estimated deterioration rate from the first to the second week (captured

²⁴The minimum number represents the cost when $Y_t^g \rightarrow \infty$, and the maximum represents the cost when $Y_t^g = 0$.

via ϕ_1) is about 50%, and that from the second on (captured via ϕ_2) is 0.3%. These numbers are consistent with the observed pattern of the sales of new copies, which declines quickly during the first few weeks after release.

For the deterioration rate of owners' consumption values due to satiation, we include the following product characteristics in $X_{g\tau}$: an intercept, story-based game dummy, multi-player game dummy, average critic and user rating, and duration of ownership. A positive coefficient of a variable implies that the variable will increase the deterioration rate. Our estimates suggest that story-based games and games with a higher user rating exhibit a lower deterioration rate. Depending on product characteristics, the weekly deterioration rate for owners at $\tau = 1$ ranges from 22% to 49%. Finally, the positive coefficient for the duration of ownership suggests that the per-period deterioration rates become larger as consumers keep the game longer, but the effect is not significant.

The parameters for pseudo-policy functions are reported in Table 3. In particular, we find that unobserved demand shocks for used-copy purchase (ω_{p7}, ω_{r7}) have a small and non-significant impact on both used-copy price and resale value, but unobserved shocks to selling (ω_{p8}, ω_{r8}) have a negative and significant impact on both price and resale value of used copies. Note that the unobserved shocks to selling affect the resale value negatively and the resale value affects the selling utility positively. Therefore, we expect that the price coefficient might be biased downwards if we did not control for the endogeneity problem.

Finally, the estimates for consumer expectation processes are reported in Table 4.

6.2 Goodness-of-fit

Our estimated dynamic model provides a good fit to the data. To show the goodness-of-fit, we simulate the sales of new and used copies as well as the volume sold to retailers by consumers by drawing 1,000 sets of parameters (including unobserved shocks) from the posterior distribution, computing the predicted quantities, and averaging them over 1,000 draws. Figures 4-6 show the fit of (1) new-copy sales, (2) used-copy sales, (3) volume of used copies supplied to retailers by consumers, respectively, for all 20 games. In general, our dynamic model is able to explain the data very well with three types of consumers. Similarly,

we also simulate the predicted price and resale value of used copies by the pseudo-pricing policy functions. Figures 7-8 show the fit of (1) used-copy price and (2) resale value, respectively, for all 20 games. These results show that the pseudo-pricing policy functions specified in Equations (19) and (20) are also able to capture the data trend quite well.

6.3 Roles of Heterogeneity in Transaction Costs

Having recovered three latent classes of consumers, we investigate how the heterogeneity in transaction costs generates different buying and selling decisions across consumer types. We use our simulation results to compute the average proportion of (1) new-copy sales, (2) used-copy sales, (3) volume of used copies supplied by owners, by consumer type. The results are reported in Table 5. The following observed patterns characterize the three consumer segments. First, type-1 consumers (about 1% of the population) are relatively price-insensitive and purchase new copies in the earlier weeks, but because of their high cost of buying a used copy, they do not purchase used copies. They are also the main supplier of used copies, especially in the earlier weeks. Second, type-2 consumers (about 25%) initially purchase new copies because the cost of buying a used copy is high in the first few weeks due to a low used-copy inventory level. As the inventory is accumulated, these consumers shift to used copies. This tendency is further strengthened by the decreasing price of used copies. As a result, the proportion of new-copy sales by type-2 consumers decreases quickly after week 3. As for selling behavior, type-2 consumers hardly supply used copies because of their high cost of selling a used copy. Finally, type-3 consumers (about 74%) have a small probability of buying a game because of their high price sensitivity and high costs of buying and selling used copies. However, because of their large segment size, they generate a significant proportion of new-copy sales. Since they have the highest price sensitivity and a lower cost of selling than type-2 consumers, some of them supply used copies. The proportion of used-copy supply by type-3 consumers increases over time. This is mainly because most of the type-1 consumers' new-copy purchase happens in the first few weeks, and thus, as time goes, the size of type-1 game owners decreases.

6.4 Elasticities and Switching Behavior

This section discusses elasticities of demand and supply and consumer switching behavior. The cross-price elasticity of demand between used and new copies will inform us whether they are close substitutes from consumers' viewpoint (e.g. Ghose et al. 2006). Moreover, our dynamic model is able to quantify (i) the inventory elasticities of demand, and (ii) the price elasticities of used-copy supply, i.e., volume sold to retailers by consumers. These two types of elasticities have not been examined in the literature because data on used-copy inventory and volume of used copies supplied were not available. Our results for these two elasticities may shed some light on why retailers accumulate used-copy inventory for future sales instead of procuring just enough for their current period sales.

Table 6 shows seven types of elasticities (labeled by E.1-E.4) and switching behavior (S.1 and S.2). E.1.1 and E.1.2 investigate the percentage change in the sales of new and used copies in week t in response to a 1% change in new-copy price in week t , while E.2.1 and E.2.2 show the percentage change in the sales of new and used copies in response to a 1% change in used-copy price. Our primary focus here is on E.1.2 and E.2.1, which show the cross-price elasticity of demand for new and used copies, respectively.

Our estimates show that the cross-price elasticities are very high in the first two week, but decrease quickly afterwards. For example, under E.1.2, a 1% increase in new-copy price in week 2 increases the demand for used copies by 35%. This high number is partly driven by a very small base sales of used copies in week 2. Under S.1, we compute the proportion of consumers who switch from new to used copies as a result of a 1% increase in new-copy price. We find that out of those who switch away from new copies (i.e., 2.9% of original new-copy purchasers), only 0.6% of consumers switch to used copies and the rest of them switch to no purchase. Thus, the majority of consumers do not switch to used copies in week 2. However, in week 3, we see that 53% of consumers switch to used copies. Those are type-2 consumers, and the high percentage mainly comes from an increase in the used-copy inventory, which results in a lower cost of buying a used copy for type-2 consumers. As times goes on, the cross-price elasticities under E.1.2 become small. This is because once the used-copy inventory is accumulated, most of type-2 consumers only demand used

copies. Thus, the majority of new-copy sales in later weeks is driven by type-1 and type-3 consumers. As a result, an increase in new-copy price will not make them switch to used copies.

Under E.2.1, we investigate the cross-price elasticities when the used-copy price increases by 1%. In week 2, the elasticity is very small (0.011). This is once again due to a very small base sales of used copies, relative to a very large base sales of new copies. However, out of those who switch away from used copies, 83% of them switch to new copies. In week 3, the elasticity increases to 1.67. This is because (1) the base sales of used copies become larger, and (2) the cost of buying a used copy is still relatively high because of a low used-copy inventory. As a result, type-2 consumers are likely to switch to new copies. However, in later weeks, the used-copy inventory is accumulated and the price of used copies decreases. As a result, the majority of type-2 consumers purchase used copies, and a used-copy price increase hardly induces them to switch to new copies. This pattern can also be seen under S.2, where the proportion of consumers who switch to new copies decreases quickly over time.

In summary, the substitution between new and used copies is mainly determined by type-2 consumers. Initially, new copies are more attractive for them because the high cost of buying a used copy outweighs the price differential between new and used copies. As the used-copy inventory increases in the first few weeks, the cost decreases and new and used copies become equally attractive for them, resulting in the high cross-price elasticities. As the inventory is further accumulated, used copies become more attractive for them, reducing the cross-price elasticities.

Next, E.3.1 and E.3.2 of Table 6 show the inventory elasticities of demand, i.e., the percentage change in the sales of new and used copies due to a 1% change in the inventory of used copies. We find that in week 3 where not many used copies are available, a 1% increase in the inventory has a large effect on the demand for new and used copies. This suggests that the availability of used copies could be playing an important role in increasing the sales of used copies by reducing the cost for buying a used copy. Finally, E.4 examines the elasticity of used-copy volume supplied to retailers by consumers. Note that due to a high selling cost, type-2 consumers hardly sell their games. Moreover, most of type-1 and type-3 consumers'

new-copy purchases occur in the earlier weeks. Thus, as more of these consumers sell, the proportion of these owners shrinks, which causes the elasticity of supply of used-copy to decrease over time. This pattern also suggests one possible reason why retailers accumulate used-copy inventory in the earlier stage of product lifecycle – it becomes harder for them to procure used copies in the later part of product lifecycle as game owners become more inelastic with respect to the resale value.

6.5 Elimination of the Used Game Market

Video game publishers often claim that the existence of the used game market lowers the sales of new games. The claim is often based on the conjecture that if there were no used game market, most of the used-copy buyers would switch to a new copy. Our cross-price elasticity analysis in the previous section indicates that this concern might be valid because type-2 consumers might purchase new copies if used copies are not available. However, if the used game market is shut down, it is possible that the demand for new copies may drop because the total expected discounted value from buying a new copy could be lowered due to the lack of selling opportunities. As a result, we expect type-1 consumers to demand less new copies. Also, type-2 consumers might reduce demand for new copies because some of the used-copy supply is generated by them. To find out whether it is worthwhile for video game publishers to pursue the strategy to shut down the used game market, we will use our estimated model to conduct a counterfactual experiment.

We conduct the experiment under two scenarios. In the first scenario, we assume that video game publishers keep the currently observed prices of new copies even after the elimination of the used good market. Thus the profit change is purely due to the change in demand for new copies. In the second scenario, we compute the optimal flat-prices of new copies in the absence of the used good market. We maintain the flat-pricing scheme because it has been an industry practice in Japan, and there might be some obstacles for the industry to shift to the price-skimming scheme. In both scenarios, the supply-side is modeled as a monopoly publisher who sets the price of new copies prior to the release of the focal game, and the marginal cost is JPY 1,000 (Tachibana 2006).²⁵

²⁵We do not consider an interaction between the publisher and retailers who sell new copies because we do not observe the whole-sale price.

Similar to the goodness-of-fit, we simulate 1,000 sets of parameters from the posterior distribution, compute the predicted quantities, and average them over the 1,000 draws. We compute the statistics on the percentage and absolute changes in video game publishers' profits, consumer surplus, and social surplus, all aggregated across all 20 games. For consumer surplus, we follow McFadden (1981) and Small and Rosen (1981) and compute the *ex ante* consumer surplus for each consumer type r using the following closed form formula.

$$E[CS_r] = \frac{1}{\alpha_r} E \left[\max_{\epsilon} V_{rj}(b_1) + \epsilon_{ij1} \right] = \frac{1}{\alpha_r} \ln \left[\sum_{j=0}^1 \exp(V_{rj}(b_1)) \right],$$

where $V_{rj}(b_1)$ is the alternative-specific value function at time $t = 1$, and $b_1 = (p_1, C_1, t = 1)$. We use the observed market size M_t^d and the proportion of each type ψ_l to construct the market-level consumer surplus for each game, and then aggregate them across games to get the aggregate consumer surplus. For publishers' profits (producer surplus), we also compute the *ex ante* present discounted value of future profits for a game as:

$$E[PS] = E \left[\sum_{t=1}^T \beta_f^{t-1} \pi \right] = E \left[\sum_{t=1}^T \beta_f^{t-1} (p_1 - c) \sum_{l=1}^L M_{lt}^d \Pr(j = 1 | b_t; l) \right],$$

where c is the marginal cost (JPY 1,000), and β_f is the publishers' discount factor. We assume that β_f is calibrated based on the weekly interest rate ($\beta_f = 0.999$). To compute the expected present discounted value over future demand shocks, we simulate a sequence of demand shocks and integrate them out using the Monte Carlo integration.

Table 7 summarizes the results. We first discuss the scenario where we maintain the price of new copies at the currently observed level. The aggregate profit change across all games is 2%, or USD 20m. About a half of games experience a decrease in profit, with the minimum of -27%, and another half experience an increase in profit, with the maximum of 205%. This finding suggests that while the resale effect exists, substitution effects may dominate resale effects for most of the games. The elimination of used goods markets does reduce the consumer surplus for almost all games. Type-1 consumers no longer benefits from the selling opportunity, so they either purchase new copies or switch to no purchase. Either way, the overall utility decreases significantly (41% or USD 368). However, type-2 and type-3 consumers are hardly affected.

Their consumer surplus decreases but the absolute change is relatively small (USD 0.82 and USD 2.32 for type-2 and type-3 consumers, respectively). Some of type-2 consumers who used to purchase used copies switch to new copies in the early period, and thus, they do not get hurt so much. Since they are not the supplier of used copies, the reduction in their consumer surplus is not very large. Type-3 consumers have a low probability of purchasing a game, and thus less affected by the elimination. However, some of them are the supplier of used copies, and thus the lost opportunity for future resale slightly lowers their consumer surplus. Overall, the social surplus is also negative for most of the games and the aggregate social surplus is negative because of the large decrease in the consumer surplus.

Next, we consider a situation where video game publishers adjust the price of new copies optimally after the used game market is shut down. We compute the optimal flat-prices and then examine changes in price, profits, consumer surplus, and social surplus. We find that the optimal flat-prices are on average 52.2% lower than the observed prices. This large decline is mainly to induce price-sensitive type-2 and type-3 consumers to purchase new copies. It also helps attract type-1 consumers, but the proportion of type-2 and type-3 consumers (99% in total) is much larger than that of type-1 consumers (1%). The aggregate producer surplus increases by 77.5% due to the elimination of the used game market. Also, the aggregate consumer surplus increases significantly due to the lower new prices. Type-1 consumers are still hurt by the elimination of the used game market, but the consumer surplus significantly improves for type-2 and type-3 consumers. Overall, the aggregate social surplus across all games improves by 104.9%.

In Table 8, we report the welfare changes, satiation-based deterioration rate (first week of ownership), and observed size of the used good market on a per-game basis under the optimal flat-pricing scenario. The observed size of the used goods market is defined by the ratio of cumulative sales of new copies to that of used copies (see Section 3). We compute the correlation of each of the welfare changes and the satiation-based deterioration rate with the observed size of the used goods market. The correlations suggest that in general, the larger the observed size of the used goods market, (1) the larger the profit changes and social welfare changes, and (2) the smaller the consumer welfare changes. These are in line with our expectation that

for games with a larger size of the used goods market, eliminating the used goods market helps publishers but hurts consumers. However, we find a negative correlation between the satiation-based deterioration rate and the observed size of the used goods market. One might expect that the higher the satiation-based deterioration rate, the quicker consumers sell games to the used good market, resulting in a larger used goods market. However, for games with the higher satiation-based deterioration rate, it is possible that resale values will be lower as more consumers will want to sell. Furthermore, the higher satiation-based deterioration might lower consumers' willingness-to-pay for used copies. Together, it is not obvious if the correlation between the satiation-based deterioration rate and the observed size of the used goods market should be positive. Also, partly due to the small sample of games (20 games), none of the correlations are significantly different from.

These counterfactual experiment results provide an important implication for recent pricing practices for digital distribution of video games. One can view our counterfactual experiment to be close to having publishers completely switch to the digital downloading format, which will essentially shut down the used game market. Thus, the results of this counterfactual experiment should be able to shed some light on the pricing and profitability of this marketing strategy. In general, our results suggest that if video game publishers are able to lower the game price significantly, then shifting to the digital downloading format could improve both profits and consumer surplus, leading to an improvement in social welfare.

7 Conclusion

Based on our newly collected data set from the Japanese new and used video game markets, we develop a new empirical framework for studying consumers dynamic buying and selling decisions. Our framework, together with this unique data set, allows us to estimate the discount factor, deterioration rates of potential buyers' and owners' consumption values separately, and costs for buying and selling used goods separately. We find that our estimated model is able to explain dynamic patterns of buying and selling decisions very well.

Our estimation results suggest that consumers in the Japanese video game market are forward-looking,

and there are three important segments of consumers that characterize the substitution pattern between new and used games. We also compute two new elasticities, inventory elasticity of used-copy demand and elasticity of used-copy supply, which have not been examined before. Our counterfactual experiment suggests that the resale effect plays an important role in generating new-copy sales: without adjusting the price of a new copy optimally, video game publishers could lose profits by shutting down the used game market. If publishers can adjust the price optimally, the elimination of the used game market is beneficial to both publishers and consumers, leading to an improvement in social welfare.

We develop our dynamic discrete choice model based on institutional details of the Japanese video game market. For example, we assume that the price of new copies is constant over time. Also, our model is not suitable for applications where digital products can physically depreciate over time or network externality exists. Readers who are applying our approach should carefully adjust the model based on their specific empirical applications.

This paper does not explicitly model the supply-side competition between video game publishers and used game retailers. While this is a limitation, the current approach is suitable for our research purpose for two reasons. First, it avoids potential biases due to the mis-specification of the supply-side competitive structure. Second, since our interest is to examine the impact of eliminating the used game market on video game publishers' profits and social welfare relative to the current situation, our counterfactual scenarios do not require us to model the competition between new game publishers and used game retailers. However, if we combine our demand-side model with a supply-side model, we will be able to address some additional research questions. For example, our estimated elasticities suggest that if the opening of the used game market were delayed by several weeks, it could avoid the competition from used copies, and yet maintain the future selling opportunity for consumers. This remedy was actually proposed during the used video game lawsuit in Japan, but was not adopted. To examine the impact of this policy, we need to model the supply-side competition between new game publishers and used game retailers.

In relation to this, one should also be careful when interpreting the results of our counterfactual exercises.

Although we find that the average profits of the games we examined could increase after eliminating the used game market, we do not know if the game publishers are setting their prices optimally in the current situation. To investigate this, we would also need a model of used good retailers decisions, so that we can predict how the used-copy prices will react to any changes of the new-copy prices. Finally, the accumulation of aggregate inventory of used copies over time is another puzzling fact that requires more research. We plan to study these important research topics in the future.

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Table 1: Summary statistics

	Average	S.D.	Min	Max
Price of new copies (in JPY)	7,613.1	629.1	7,140	9,240
Price of used copies (in JPY)	4,515.3	1,087.8	2,219	7,433
Resale value of used copies (in JPY)	2,828.1	1,182.7	1,036	6,547
Markup for used copies (in JPY)	1,687.2	382.2	435	2,786
Markup for used copies (%)	73.0%	36.5%	7.3%	210.7%
Weekly sales of new copies	100,650.4	259,022.3	2,772	2,236,881
Weekly sales of used copies	7,184.6	6,478.8	458	62,734
Ratio of cumulative sales of used to new copies ^a	0.462	0.091	0.350	0.627
Weekly quantity sold by consumers	8,121.4	8,436.8	1,012	55,830
Weekly inventory of used copies	31,022.5	28,347.7	0	129,462
Market size (installed base)	14,866,067.6	6,097,167.2	746,971	20,822,775
Weekly # new game introduction	7.01	4.02	0	17
Dummy for story-based games	0.700	0.470	0	1
Dummy for multi-player games	0.450	0.510	0	1
Critic rating (in 10-point scale)	8.99	0.656	7.75	10
User rating ^b	56.4	9.20	41.6	67.4

Note: USD 1 \approx JPY 100

a. computed by setting sales of new copies to zero for those weeks in which it is below top 30 ranking

b. user rating is a standardized score against a set of video games released in the same year (by Enterbrain, Inc.)

Table 2: Demand estimates

	mean	s.d.		mean	s.d.
preference parameters			segment proportion		
discount factor (β)	0.859**	0.015	type 1 (ψ_1)	0.012**	0.001
<i>price sensitivity</i>			type 2 (ψ_2)	0.247**	0.055
type 1 (α_1)	1.12e-04**	2.35e-05	type 3 (ψ_3)	0.742**	0.055
type 2 (α_2)	3.69e-04**	3.25e-05	deterioration rates		
type 3 (α_3)	5.75e-04**	1.71e-05	<i>potential buyers</i>		
<i>cost for buying a used copy</i>			1 st -week (ϕ_1)	0.012	0.133
intercept: type 1 (λ_{01})	47.3**	7.37	from 2 nd -week on (ϕ_2)	-5.58**	1.23
intercept: type 2 (λ_{02})	0.065	0.053	<i>game owners</i>		
intercept: type 3 (λ_{03})	27.0**	4.75	intercept (δ_1)	1.56	0.973
inventory-related parameters			dummy for story-based games (δ_2)	-0.520 ⁺	0.311
constant (λ_1)	0.638 ⁺	0.314	dummy for multi-player games (δ_3)	-0.145	0.231
coefficient for inventory level (λ_2)	0.004**	3.54e-04	critic rating (δ_4)	0.012	0.107
<i>cost for selling a used copy</i>			user rating (δ_5)	-0.036**	0.007
type 1 (μ_1)	0.178	0.155	ownership duration (logged) (δ_6)	0.939 [*]	0.409
type 2 (μ_2)	16.3**	3.18	parameters for error terms		
type 3 (μ_3)	6.71**	0.353	within-group correlation for buying decision (η_b)	0.905**	0.027
<i>seasonal dummies</i>			scaling parameter for selling decision (η_s)	1.42**	0.131
golden week (early May) (ρ_1)	0.207**	0.076	<i>s.d. (measurement error)</i>		
christmas (late Dec) (ρ_2)	0.397**	0.085	sales new (ϵ_1)	83519.0**	13652.6
<i>outside option</i>			sales used (ϵ_2)	2689.4**	256.4
same-console competitive effect (τ_1)	0.206**	0.031	volume sold by consumers (ϵ_3)	1899.3**	159.0
marginal log-likelihood		-23558.3	<i>s.d. (unobserved aggregate shock)</i>		
			sales used (ξ_2)	0.288**	0.038
			volume sold by consumers (ξ_3)	0.382**	0.022

Notes: ⁺, ^{*}, and ^{**} indicate that zero is not contained in the 10%, 5%, and 1% credible interval, respectively. 20 game-specific initial consumption values (ψ^g) are estimated but not reported here.

Table 3: Estimates for pseudo-pricing policy functions

	<u>price of used copies</u>		<u>resale value of used copies</u>	
	mean	s.d.	mean	s.d.
pseudo-pricing policy function parameters				
intercept (ω_1)	8.93**	0.027	8.65**	0.047
avg. size of potential buyers (ω_2)	-4.99e-08**	4.32e-09	-5.57e-08**	7.75e-09
inventory of used goods (ω_3)	-1.09e-06**	3.41e-07	-2.09e-06**	6.08e-07
cumulative # new games on other consoles (ω_4)	-0.003**	1.31e-04	-0.005**	2.31e-04
seasonal dummy for golden week (ω_5)	0.015	0.020	0.060 ⁺	0.033
seasonal dummy for christmas (ω_6)	1.64e-04	0.023	0.054	0.040
unobserved shock to buying used copy (ω_7)	0.004	0.012	0.009	0.015
unobserved shock to selling (ω_8)	-0.494**	0.028	-0.881**	0.048
s.d. (v_t)	0.062**	0.004	0.074**	0.010

Notes: ⁺, ^{*}, and ^{**} indicate that zero is not contained in the 10%, 5%, and 1% credible interval, respectively.

Table 4: Estimates for consumer expectation processes: price of used games, resale value, inventory level, and cumulative # competing games on the same console

variable	price of used games				resale value of used games				inventory level	
	t=2		t>2		t=2		t>2		t>2	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
lagged value	0.796**	0.006	0.958**	0.005	0.666**	0.009	0.928**	0.005	0.966**	0.006
lagged inventory			-0.002**	2.04e-04			-0.002**	2.07e-04		
dummy for story-based games									1951.0**	375.6
critic rating			29.9**	5.01			25.2**	4.48	592.8**	124.8
user rating			-1.63**	0.603			-1.09 ⁺	0.601	-82.3**	20.0
s.d.	187.6**	33.2	123.0**	3.43	299.1**	52.1	125.1**	3.48	3618.6**	98.4

Notes: ⁺, *, and ** indicate that zero is not contained in the 10%, 5%, and 1% credible interval, respectively. For price and resale value of used games, the lagged value for t=2 is the price of new games. For inventory level, the lagged value and the lagged inventory are identical.

variable	cumulative # of newly introduced games					
	PlayStation 2		Nintendo GameCube		PlayStation 3	
	mean	s.d.	mean	s.d.	mean	s.d.
intercept	3.58**	0.225	0.375	0.334	0.475**	0.158
lagged value	1.00**	0.003	1.02**	0.054	1.01**	0.011
s.d.	2.80**	0.084	0.766**	0.124	0.806**	0.061

Notes: ⁺, *, and ** indicate that zero is not contained in the 10%, 5%, and 1% credible interval, respectively.

Table 5: Proportion of Predicted Quantities by Consumer Segment

weeks in release	new-copy demand			used-copy demand			used-copy supply to retailers		
	type 1	type 2	type 3	type 1	type 2	type 3	type 1	type 2	type 3
1	0.215	0.380	0.406						
2	0.227	0.367	0.406	0.000	1.000	0.000	0.691	0.001	0.308
3	0.292	0.139	0.569	0.000	1.000	0.000	0.676	0.001	0.322
4	0.319	0.035	0.647	0.000	1.000	0.000	0.651	0.002	0.348
5	0.306	0.017	0.677	0.000	1.000	0.000	0.638	0.002	0.361
6	0.302	0.010	0.687	0.000	1.000	0.000	0.618	0.002	0.380
7	0.294	0.008	0.698	0.000	1.000	0.000	0.604	0.002	0.394
8	0.283	0.010	0.707	0.000	1.000	0.000	0.585	0.003	0.413
9	0.284	0.009	0.708	0.000	1.000	0.000	0.553	0.003	0.444
10	0.284	0.007	0.708	0.000	1.000	0.000	0.528	0.004	0.469
11	0.281	0.008	0.711	0.000	1.000	0.000	0.507	0.004	0.489
12	0.279	0.008	0.713	0.000	1.000	0.000	0.478	0.005	0.518
13	0.276	0.007	0.718	0.000	1.000	0.000	0.455	0.005	0.540
14	0.274	0.006	0.720	0.000	1.000	0.000	0.428	0.006	0.567
15	0.273	0.005	0.722	0.000	1.000	0.000	0.403	0.006	0.590

Table 6: Elasticities

type of elasticities	E.1.1	E.1.2	S.1	E.2.1	E.2.2	S.2	E.3.1	E.3.2	E.4
	elasticities of demand w.r.t. new-copy price		proportion of switchers	elasticities of demand w.r.t. used-copy price		proportion of switchers	elasticities of demand w.r.t. used-copy inventory		elasticities of supply w.r.t. resale value
weeks in release	new-copy demand	used-copy demand	from new to used copies	new-copy demand	used-copy demand	from used to new copies	new-copy demand	used-copy demand	used-copy supply to retailers
1	-2.71								
2	-2.91	35.1	0.006	0.011	-22.6	0.831			0.905
3	-4.94	7.53	0.525	1.67	-7.96	0.484	-0.134	0.716	0.873
4	-3.67	1.35	0.228	0.528	-3.29	0.184	-0.020	0.112	0.870
5	-3.48	0.581	0.142	0.265	-2.58	0.095	-0.005	0.046	0.847
6	-3.37	0.412	0.118	0.136	-2.37	0.050	-0.004	0.050	0.823
7	-3.39	0.307	0.099	0.112	-2.23	0.040	-0.004	0.046	0.789
8	-3.44	0.346	0.115	0.126	-2.23	0.042	-0.004	0.039	0.772
9	-3.43	0.303	0.106	0.114	-2.16	0.038	-0.002	0.033	0.786
10	-3.42	0.259	0.097	0.098	-2.09	0.033	-0.002	0.028	0.763
11	-3.43	0.283	0.105	0.102	-2.03	0.034	-0.002	0.027	0.734
12	-3.44	0.282	0.104	0.095	-1.99	0.032	-0.003	0.029	0.736
13	-3.44	0.229	0.089	0.083	-1.89	0.027	-0.003	0.030	0.713
14	-3.44	0.220	0.084	0.077	-1.84	0.025	-0.003	0.032	0.709
15	-3.42	0.156	0.067	0.059	-1.74	0.020	-0.002	0.020	0.697
Average	-3.46	3.38	0.135	0.248	-4.07	0.138	-0.014	0.093	0.787

Table 7: Welfare changes due to elimination of used game market: aggregate across all games

	percentage change	dollar change (in USD)
<u>under observed flat-pricing</u>		
producer surplus	2.14%	20.6 m
consumer surplus (per person)	-18.8%	-5.42
type 1	-41.2%	-368.40
type 2	-1.40%	-0.82
type 3	-21.0%	-2.32
consumer surplus (aggregate)	-18.8%	-60.4 m
social surplus	-3.06%	-39.8 m
<u>under optimal flat-pricing</u>		
new-copy price ^a	-52.2%	-39.6
producer surplus	77.5%	748.5 m
consumer surplus (per person)	141.0%	40.74
type 1	-35.8%	-320.57
type 2	98.4%	57.59
type 3	369.4%	40.69
consumer surplus (aggregate)	141.0%	603.7 m
social surplus	104.0%	1,352.2 m

Note: USD 1 = JPY 100 is used for currency conversion.

a. per-game change from observed price.

Table 8: Game-level welfare changes (proportion), satiation-based deterioration rate, and observed size of the used goods market

Game title	producer surplus	consumer surplus	social surplus	satiation-based deterioration rate ^a	size of used goods market
Dragon Quest 5	0.959	3.145	1.440	0.342	0.242
Samurai Warriors	0.427	2.311	0.900	0.269	0.265
Onimusha 3	0.694	1.138	0.837	0.222	0.349
Dragon Ball Z2	1.617	0.929	1.389	0.465	0.299
Grand Theft Auto: Vice City	0.792	1.114	0.896	0.260	0.388
Dragon Quest 8	0.085	1.585	0.412	0.235	0.176
Dynasty Warriors 4	1.370	1.498	1.409	0.229	0.232
Metal Gear Solid 3: Snake Eater	0.721	1.974	1.070	0.215	0.295
Dragon Ball Z3	2.000	0.911	1.645	0.450	0.265
Gran Turismo 4	0.357	0.947	0.561	0.374	0.145
Resident Evil 4	11.719	1.391	10.238	0.228	0.326
Final Fantasy 12	0.146	3.067	0.757	0.413	0.263
Kingdom Hearts II	0.400	2.699	1.031	0.226	0.284
Winning Eleven 10	1.242	0.871	1.122	0.360	0.258
Yakuza	0.594	1.245	0.803	0.217	0.775
Gundam Seed: OMNI vs. ZAFT	0.915	0.616	0.826	0.493	0.465
Yakuza 2	1.339	1.586	1.418	0.226	0.651
Dynasty Warriors: Gundam	1.309	0.258	1.117	0.371	0.505
Winning Eleven 2008	9.414	1.308	8.271	0.419	0.387
Persona 4	0.868	0.548	0.791	0.300	0.308
Correlation with size of used goods markets	0.047	-0.258	0.031	-0.151	1

a. satiation-based deterioration rate for the first week of ownership.

Figure 1: Average quantities demanded for new video games

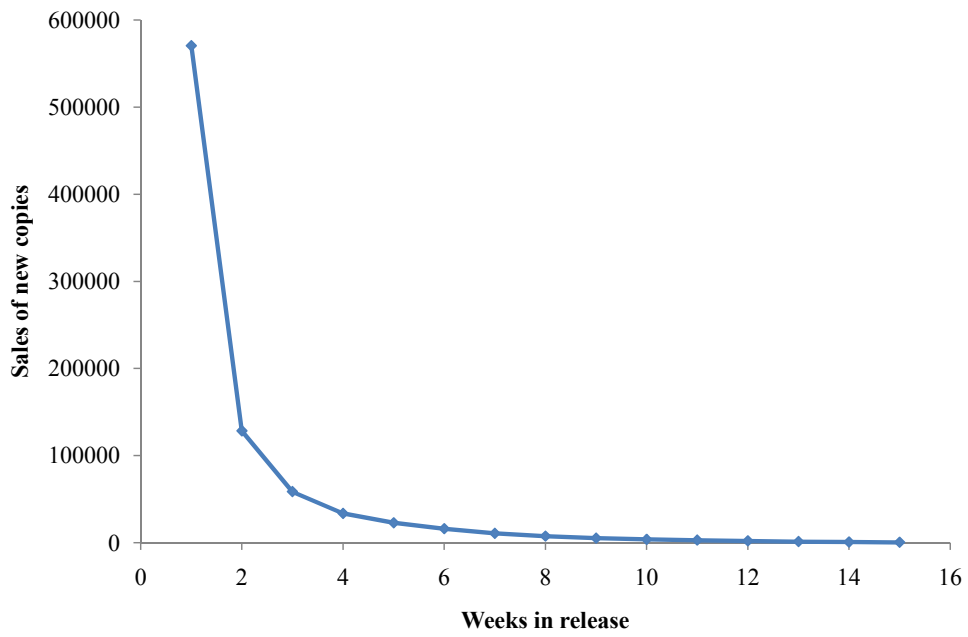


Figure 2: Average quantities demanded and supplied and inventory level for used video games

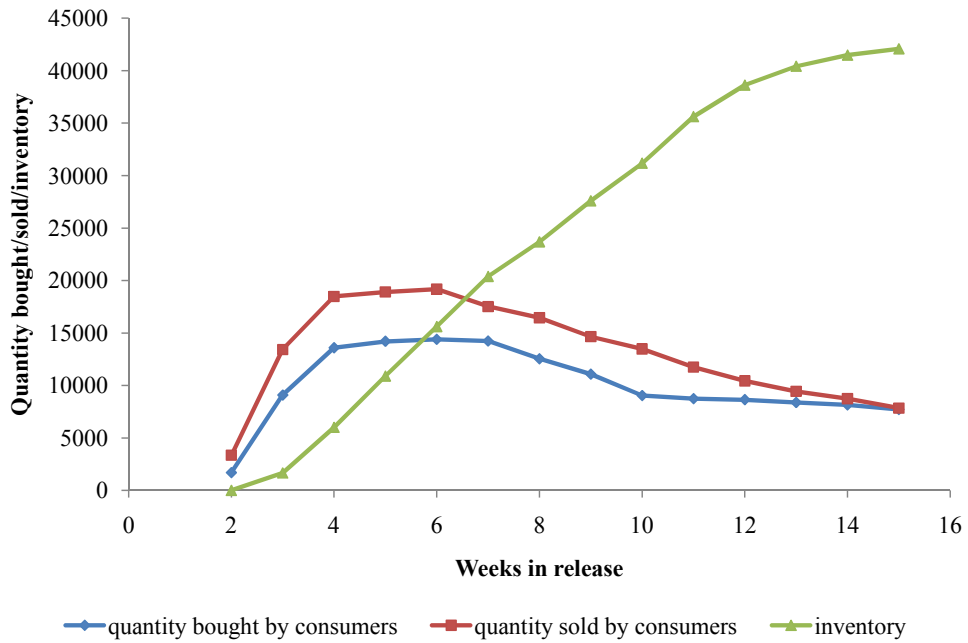


Figure 3: Average price and resale value of used video games

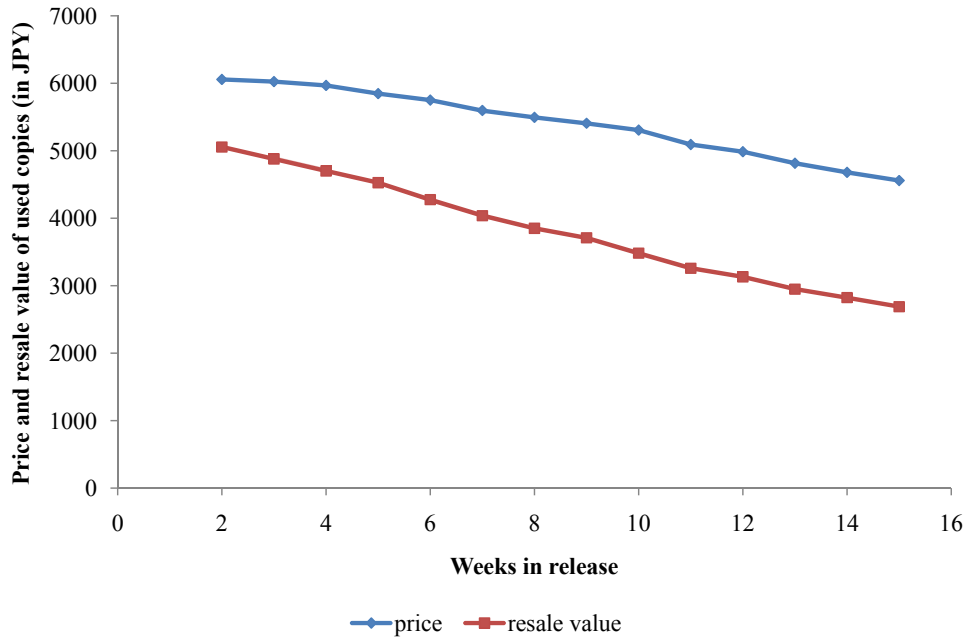


Figure 4: Observed versus Predicted Sales of New Copies

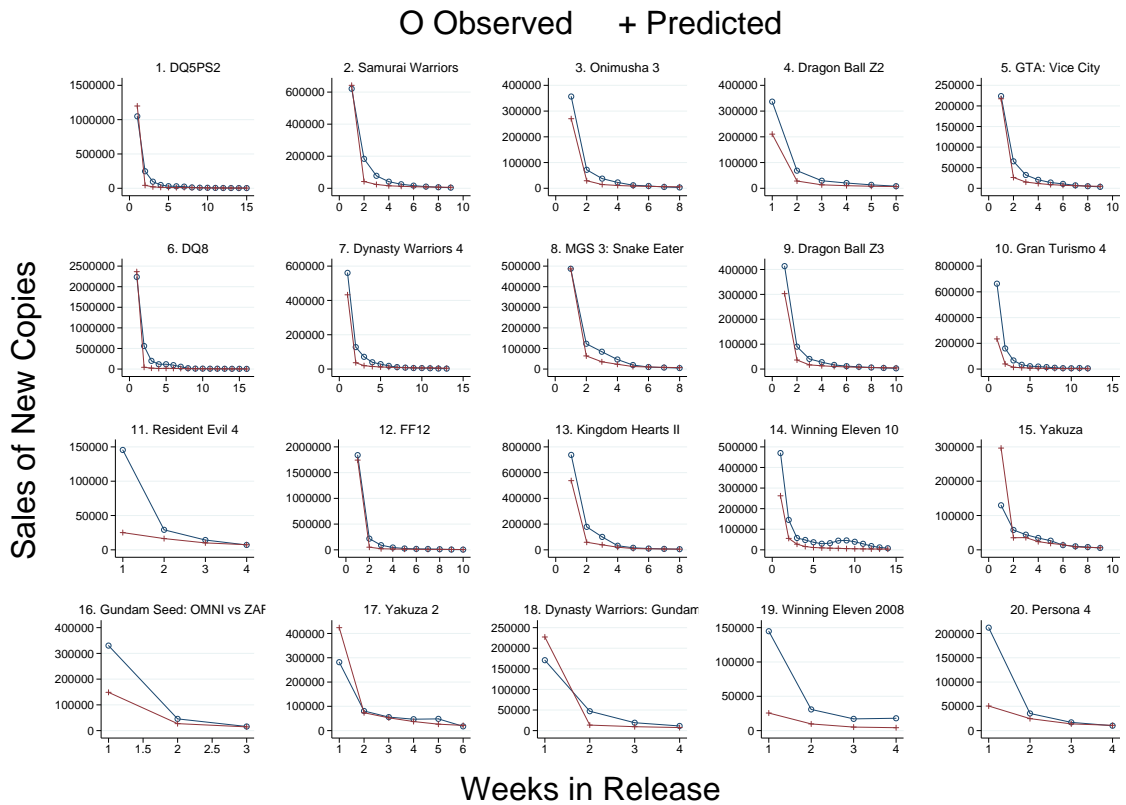


Figure 5: Observed versus Predicted Sales of Used Copies

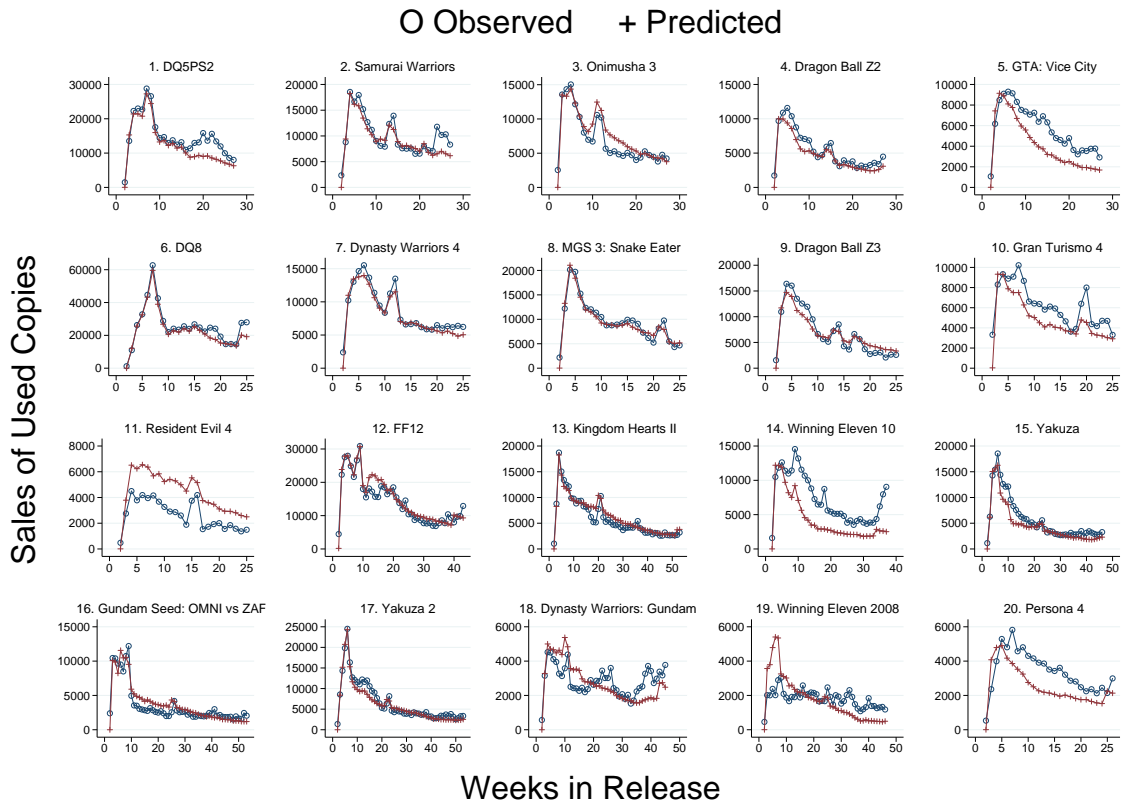


Figure 6: Observed versus Predicted Volume Sold to Retailers by Consumers

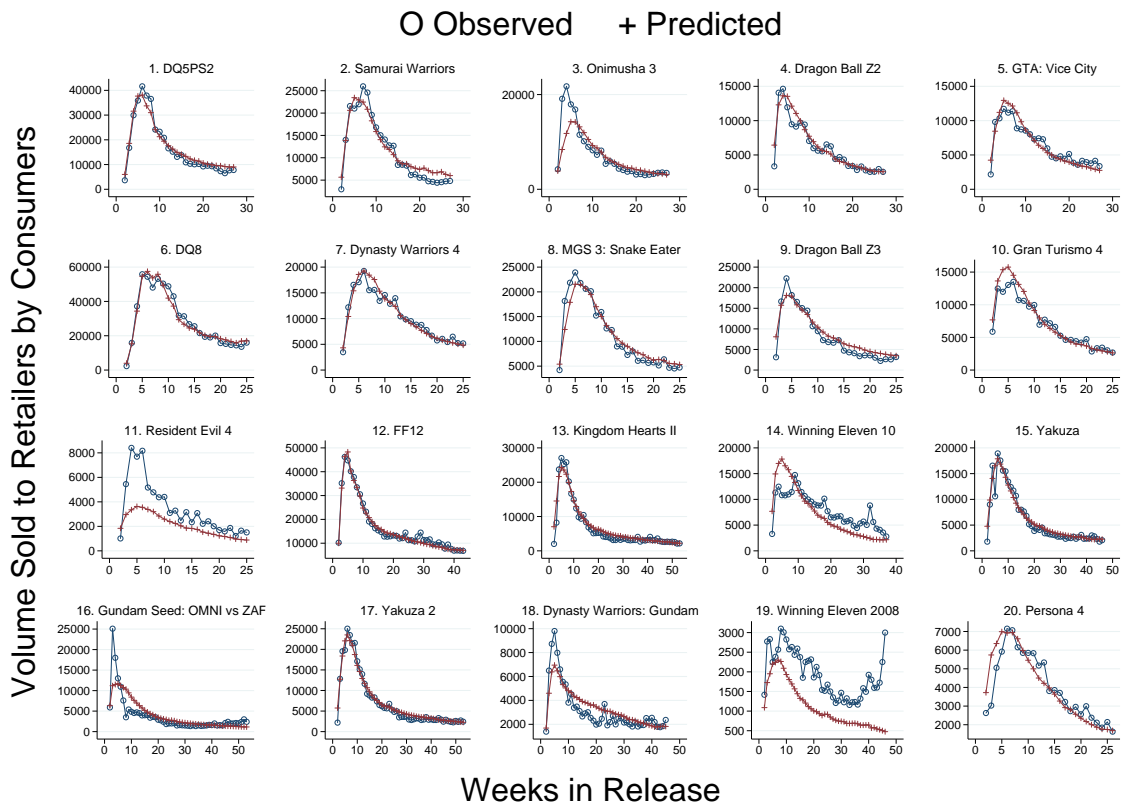


Figure 7: Observed versus Predicted Prices of Used Copies

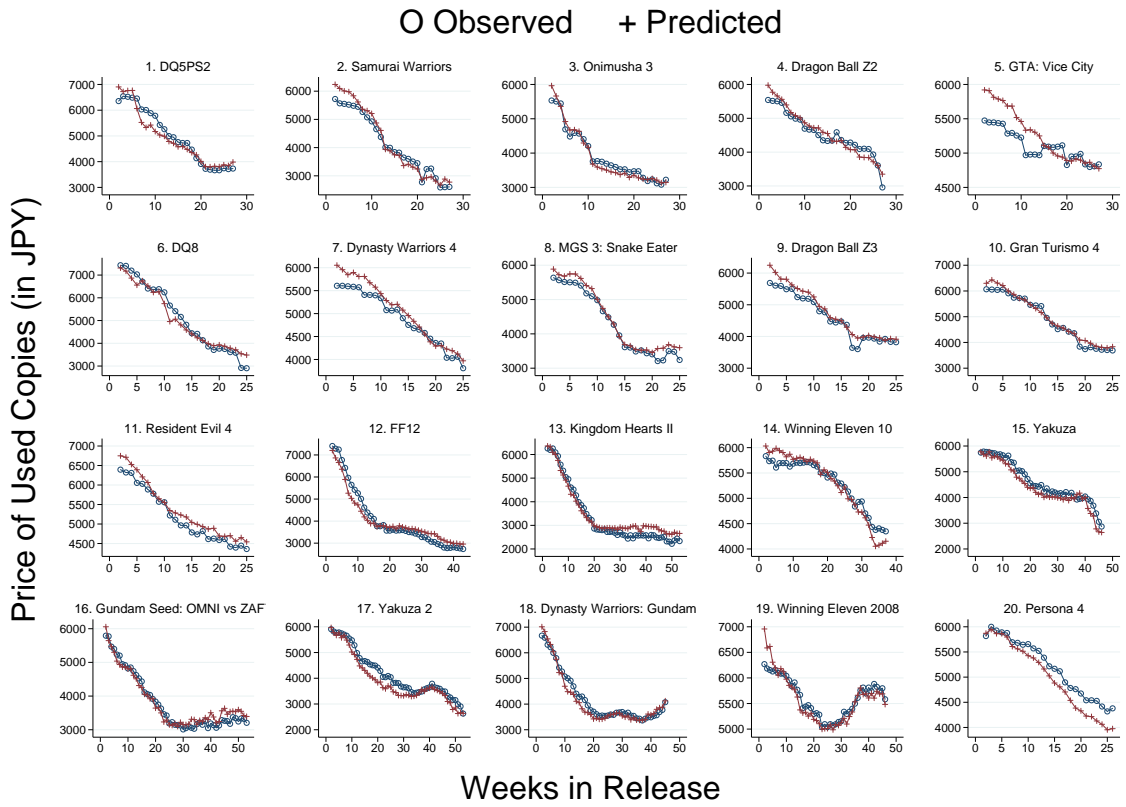
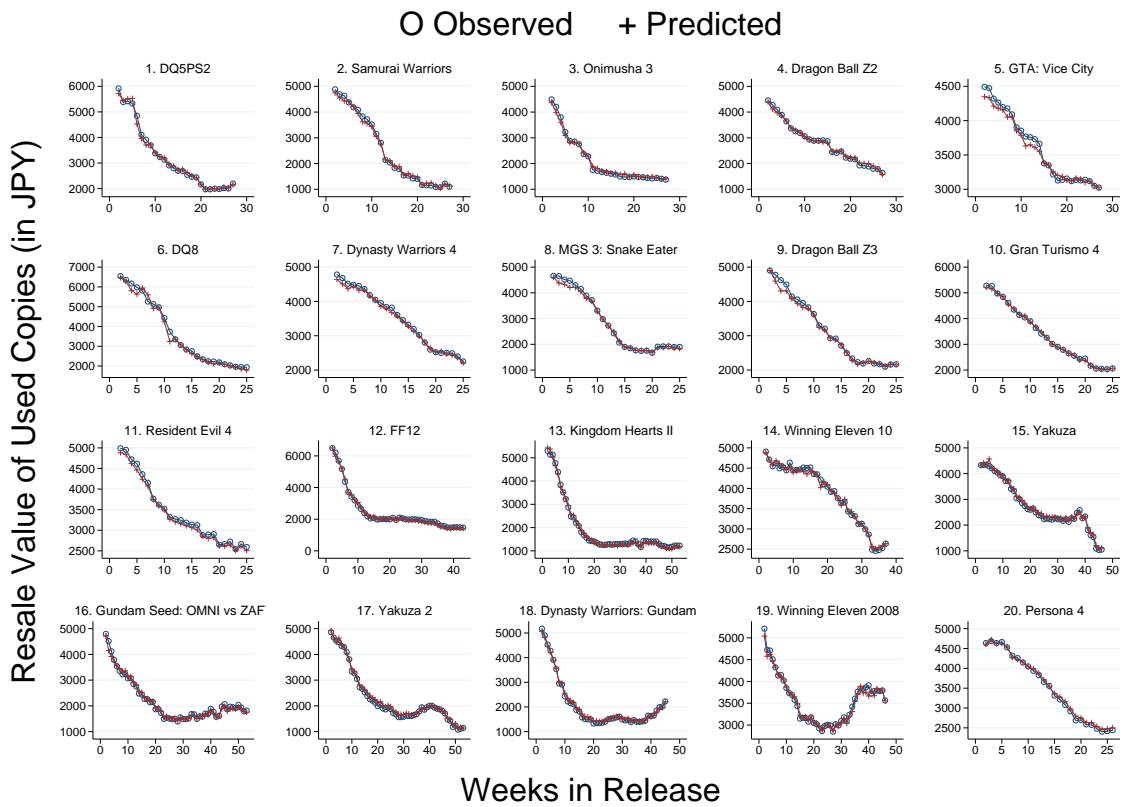


Figure 8: Observed versus Predicted Resale Value of Used Copies



A Appendix

A.1 The procedure for the estimation algorithm

This appendix discusses the details for the estimation algorithm described in Section 5. For more details about the general algorithm, see Ishihara and Ching (2016).

In the following exposition, we drop the subscript for type (l) and the superscript for game (g). Let θ_d and θ_s be the vectors of demand-side parameters (including consumer expectation process parameters) and pseudo-policy function parameters, respectively. In the context of the present model, the output of the algorithm in outer-loop iteration m is,

$$H^m = \{\theta_d^n, \theta_s^n, \theta_d^{*n}, \{\tilde{V}^n(b_t^n; \theta_d^{*n})\}_{t=2}^T, \{\tilde{W}^n(s_{t,\tau}^n; \theta_d^{*n}) \forall \tau\}_{t=2}^T\}_{n=m-N}^{m-1},$$

where \tilde{V}^n and \tilde{W}^n are consumer's pseudo-value functions for buying and selling decisions in iteration n , respectively; $b_t^n = (p_1, \tilde{p}_{2t}^n, \tilde{r}_t^n, \tilde{Y}_t^n, \tilde{C}_t^n, \tilde{\xi}_2^n, t)$ and $s_{t,\tau}^n = (\tilde{r}_t^n, \tilde{Y}_t^n, \tilde{\xi}_s^n, t, \tau)$ are vectors of state variables for buying and selling decisions in iteration n , respectively; N is the number of past pseudo-value functions used for approximating the expected value functions; θ_d^n and θ_s^n are the accepted parameter vectors of the demand-side model and the pseudo-policy functions in iteration n , respectively; θ_d^{*n} is the candidate parameter vector for the demand-side model in iteration n ; $(\tilde{p}_{2t}^n, \tilde{r}_t^n, \tilde{Y}_t^n, \tilde{C}_t^n)$ is a vector of random draw from a uniform distribution with the range defined by the lower- and upper-bound of their observed values; $(\tilde{\xi}_2^n, \tilde{\xi}_s^n)$ are drawn from the normal distributions specified in the model.

A.1.1 Pseudo-value function setup

A consumer's pseudo-value functions for selling decision at time t with duration of ownership τ in iteration m are defined as follows:

$$\begin{aligned} \tilde{W}^m(s_{t,\tau}^m; \theta_d^{*m}) &= E_e \max_{k \in \{0,1\}} \{\tilde{W}_k^m(s_{t,\tau}^m; \theta_d^{*m}) + e_{ikt}\} \\ &= \eta_s \ln \left\{ \sum_{k \in \{0,1\}} \exp \left(\frac{\tilde{W}_k^m(s_{t,\tau}^m; \theta_d^{*m})}{\eta_s} \right) \right\}, \end{aligned} \quad (21)$$

where the second equality follows from the assumption that e_{ikt} is type 1 extreme value distributed; \tilde{W}_k^m 's are consumer's pseudo alternative-specific value functions in iteration m , which are given by

$$\tilde{W}_k^m(s_{t,\tau}^m; \theta_d^{*m}) = \begin{cases} \alpha \tilde{r}_t^m - \mu_i + \tilde{\xi}_s^m & \text{if selling,} \\ v(t, \tau) + \beta \hat{E}^m[W(s_{t+1, \tau+1}; \theta_d^{*m}) | s_{t,\tau}^m] & \text{if keeping.} \end{cases} \quad (22)$$

The *pseudo*-expected value function for selling decision, $\hat{E}^m[W_l(\cdot; \theta_d^{*m}) | \cdot]$, is defined as the weighted average of the past pseudo-value functions for selling decision in period $t + 1$:

$$\begin{aligned} & \hat{E}^m[W(s_{t+1, \tau+1}; \theta_d^{*m}) | s_{t,\tau}^m] \\ &= \sum_{n=m-N}^{m-1} \tilde{W}^s(s_{t+1, \tau+1}^n; \theta_d^{*n}) \frac{K_h(\theta_d^{*m} - \theta_d^{*n}) f_s(\tilde{r}_{t+1}^n, \tilde{Y}_{t+1}^n | s_{t,\tau}^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) f_s(\tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q | s_{t,\tau}^m)}, \end{aligned} \quad (23)$$

where $K_h(\cdot)$ is a Gaussian kernel with bandwidth h , and $f_s(\cdot | \cdot)$ is the transition density recovered in the first-stage estimation. The relevant state variables for the transition density are (r, Y) . Other variables in $s_{t,\tau}$ are either *i.i.d.* (ξ_s) or deterministic (t, τ) . Note that the kernel captures the idea that one assigns higher weights to the past pseudo-value functions which are evaluated at parameter vectors that are closer to θ_d^{*m} . Also, we integrate r_{t+1} and Y_{t+1} out by the weighted average, where weights are given by the transition probabilities. In contrast, we do not need to weigh ξ_{st+1} because they are drawn from the distribution specified in the model.

Consumers' pseudo-value functions for buying decision at time t in iteration m are defined as follows:

$$\begin{aligned} \tilde{V}^m(b_t^m; \theta_d^{*m}) &= E_\epsilon \max_{j \in \{0,1,2\}} \{ \tilde{V}_j^m(b_t^m; \theta_d^{*m}) + \epsilon_{ijt} \}, \\ &= \ln \left\{ \exp(\tilde{V}_0^m(b_t^m; \theta_d^{*m})) + \left[\sum_{j \in \{1,2\}} \exp\left(\frac{\tilde{V}_j^m(b_t^m; \theta_d^{*m})}{1 - \eta_b}\right) \right]^{1 - \eta_b} \right\} \end{aligned} \quad (24)$$

where \tilde{V}_j^m 's are consumer's pseudo alternative-specific value functions in iteration m , which are given by

$$\begin{aligned} & \tilde{V}_j^m(b_t^m; \theta_d^{*m}) \\ &= \begin{cases} v(t, 0) - \alpha p_1 + \beta \hat{E}^m[W(s_{t+1, \tau=1}; \theta_d^{*m}) | s_{t, \tau=0}^m] & \text{new copy,} \\ v(t, 0) - \alpha \tilde{p}_{2t}^m - l_Y(\tilde{Y}_t^m; \lambda_i) + \tilde{\xi}_2^m + \beta \hat{E}^m[W(s_{t+1, \tau=1}; \theta_d^{*m}) | s_{t, \tau=0}^m] & \text{used copy,} \\ l_C(\tilde{C}_t^m; \pi) + \beta \hat{E}^m[V(b_{t+1}; \theta_d^{*m}) | b_t^m] & \text{no purchase.} \end{cases} \end{aligned} \quad (25)$$

The *pseudo*-expected future value function for buying decision, $\hat{E}^m[V(\cdot; \theta_d^{*m}) | \cdot]$, is defined as the weighted

average of the past pseudo-value functions for buying decision in period $t + 1$:

$$\hat{E}^m[V(b_{t+1}; \theta_d^{*m}) | b_t^m] = \sum_{n=m-N}^{m-1} \tilde{V}(b_{t+1}^n; \theta_d^{*n}) \frac{K_h(\theta_d^{*m} - \theta_d^{*n}) f_b(\tilde{p}_{2t+1}^n, \tilde{r}_{t+1}^n, \tilde{Y}_{t+1}^n, \tilde{C}_{t+1}^n | b_t^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) f_b(\tilde{p}_{2t+1}^q, \tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q, \tilde{C}_{t+1}^q | b_t^m)}. \quad (26)$$

Note again that this weighted average integrates out p_{2t+1} , r_{t+1} , Y_{t+1} , C_{t+1} , and ξ_{2t+1} .

A.1.2 Step-by-step procedure

Each MCMC iteration in the proposed algorithm consists of five blocks:

1. Draw $\sigma_\xi^m = (\text{sigma}_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions conditional on $\xi_t^{g,m-1} = (\xi_{2t}^{g,m-1}, \xi_{st}^{g,m-1})$ for all observed t and g .
2. Draw $\xi_t^{g,m}$ for all observed t and g conditional on the data, σ_ξ^m , θ_d^{m-1} and θ_s^{m-1} using random-walk Metropolis-Hastings. In the Metropolis-Hastings algorithm, the joint-likelihood of the quantity demanded, quantity supplied, used price, and resale value will be used to compute the acceptance probability.
3. Draw θ_d^m conditional on the data, $\{\xi_t^m\}$ and θ_s^{m-1} using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, the joint-likelihood of all the observed data will be used. Note that θ_d^m affects the likelihood of used price and resale value through the pseudo-policy functions.
4. Draw θ_s^m conditional on the data, $\{\xi_t^m\}$ and θ_d^m using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, only the likelihood of the pseudo-policy functions will be used because θ_s^m does not enter the demand-side model.
5. Compute the pseudo-value functions for buying and selling decision problems. Starting from the terminal period, we sequentially compute the pseudo-value functions backwards at only one randomly drawn state point in each period. We store them and update H^m to H^{m+1} .

In deriving the posterior distribution of parameters, we use an inverted gamma prior on σ_ξ , and a diffuse prior on θ_s . For θ_d , we use a diffuse prior on parameters other than consumer expectation process

parameters. For consumer expectation process parameters, we first run a Bayesian linear regression with diffuse priors, and use the posterior distribution of parameters from the Bayesian linear regression as priors in structural estimation. Finally, note that the likelihood used in this algorithm is *pseudo*-likelihood as it is a function of *pseudo* alternative-specific value functions. Below, we provide a step-by-step procedure for the five blocks described above.

1. Suppose that we are at iteration m . We start with

$$H^m = \{\theta_d^n, \theta_s^n, \theta_d^{*n}, \{\tilde{V}^n(b_t^n; \theta_d^{*n})\}_{t=2}^T, \{\tilde{W}^n(s_{t,\tau}^n; \theta_d^{*n})\}_{t=2}^T\}_{n=m-N}^{m-1},$$

where N is the number of past iterations used for expected value function approximations.

2. *Block 1*: Draw $\sigma_\xi^m = (\sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions (inverted gamma) conditional on $\xi_t^{g,m-1} = (\xi_{2t}^{g,m-1}, \xi_{st}^{g,m-1})$ for all observed t and g .

3. *Block 2*: For each observed t and g , draw $\xi_t^{g,m}$ from its posterior distribution conditional on $\sigma_{\xi_1}^m, \theta_d^{m-1}, \theta_s^{m-1}, \{\xi_k^{g,m}\}_{k=2}^{t-1}$, and $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$ where T^g is the length of sample periods for game g . Here, we will draw $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$, separately, and from $t = 2$. Below, we will describe how to draw $\xi_{2t}^{g,m}$, but a similar procedure can be applied for drawing $\xi_{st}^{g,m}$ with appropriate modifications.

(a) Draw $\xi_{2t}^{g,*m}$ (candidate parameter value).

(b) We compute the pseudo-joint likelihood at $\xi_{2t}^{g,*m}$ conditional on $\{\xi_k^{g,m}\}_{k=1}^{t-1}, \xi_{st}^{g,m-1}, \{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}, \theta_d^{m-1}$ and θ_s^{m-1} . Note that conditional on $\sigma_{\xi_2}^m$, the pseudo-joint likelihood prior to time t does not depend on $\xi_{2t}^{g,*m}$. But $\xi_{2t}^{g,*m}$ would affect the potential size of buyers and owners in the future, so the pseudo-joint likelihood after time t would depend on $\xi_{2t}^{g,*m}$. Thus, we need to compute the pseudo-joint likelihood at time t and later. To compute the pseudo-joint likelihood, we need to obtain the pseudo-alternative specific value functions for both buying and selling decisions at the observed state vectors denoted by $(\mathbf{b}_t, \mathbf{s}_{t,\tau})$ from time t to T^g : $\tilde{V}_j^m(\mathbf{b}_t; \theta_d^{m-1})$ and $\{\tilde{W}_k^m(\mathbf{s}_{t,\tau}; \theta_d^{m-1})\}_{\tau=1}^{t-1}$. First, to obtain $\tilde{V}_j^m(\mathbf{b}_t; \theta_d^{m-1})$, we need to calculate both

$\hat{E}^m[V(b_{t+1}; \theta_d^{m-1}) | \mathbf{b}_t]$ (pseudo-expected value function when consumers choose no purchase option) and $\hat{E}^m[W(s_{t+1, \tau=1}; \theta_d^{m-1}) | \mathbf{s}_{t, \tau=0}]$ (pseudo-expected value function when consumers choose to buy a new or used copy), which are computed as the weighted average of past-pseudo value functions evaluated at time $t + 1$:

- i. For $\hat{E}^m[V(b_{t+1}; \theta_d^{m-1}) | \mathbf{b}_t]$, we take the weighted average of $\{\tilde{V}^n(b_{t+1}^n; \theta_d^{*n})\}_{n=m-N}^{m-1}$ as in Equation (26).
- ii. For $\hat{E}^m[W(s_{t+1, \tau=1}; \theta_d^{m-1}) | \mathbf{s}_{t, \tau=0}]$, we take the weighted average of $\{\tilde{W}^n(s_{t+1, \tau=1}^n; \theta_d^{*n})\}_{n=m-N}^{m-1}$ as in Equation (23). Note that if consumers buy at time t , they will have owned the game for one period when they reach $t + 1$. Thus, the set of past pseudo-value functions used here only include those evaluated at $\tau = 1$.

Next, to obtain $\{\tilde{W}_k^m(\mathbf{s}_{t, \tau}; \theta_d^{m-1})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m[W(s_{t+1, \tau+1}; \theta_d^{m-1}) | \mathbf{s}_{t, \tau}]\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}^n(s_{t+1, \tau+1}^n; \theta_d^{*n})\}_{n=m-N}^{m-1}$ as in Equation (23).

- (c) Similarly, we compute the pseudo-joint likelihood at $\xi_{2t}^{g, m-1}$ conditional on $\{\xi_k^{g, m}\}_{k=2}^{t-1}$, $\xi_{st}^{g, m-1}$, $\{\xi_k^{g, m-1}\}_{k=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1} .
- (d) Based on the pseudo-joint likelihoods at $\xi_{2t}^{g, *m}$ and $\xi_{2t}^{g, m-1}$, we compute the acceptance probability for $\xi_{2t}^{g, *m}$ and decide whether to accept (i.e., set $\xi_{2t}^{g, m} = \xi_{2t}^{g, *m}$) or reject (i.e., set $\xi_{2t}^{g, m} = \xi_{2t}^{g, m-1}$).

Using a similar procedure, draw $\xi_{st}^{g, m}$. Note that drawing $\xi_{st}^{g, m}$ does not require us to compute \tilde{V}_j^m because conditional on $\sigma_{\xi_s}^m$, $\xi_{st}^{g, *m}$ does not influence the likelihood function for buying decisions.

4. *Block 3*: Use the Metropolis-Hastings algorithm to draw θ_d^m conditional on $\{\xi_t^m\}$ and θ_s^{m-1} .

- (a) Draw θ_d^{*m} (candidate parameter vector).
- (b) We compute the pseudo-joint likelihood at θ_d^{*m} conditional on $\{\xi_t^m\}$ and θ_s^{m-1} based on the pseudo-alternative specific value functions for both buying and selling decisions at θ_d^{*m} : $\tilde{V}_j^m(\mathbf{b}_t; \theta_d^{*m})$ and $\{\tilde{W}_k^m(\mathbf{s}_{t, \tau}; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all observed t and g . To obtain $\tilde{V}_j^m(\mathbf{b}_t; \theta_d^{*m})$, we need to calculate

both $\hat{E}^m[V(b_{t+1}; \theta_d^{*m})|\mathbf{b}_t]$ and $\hat{E}^m[W(s_{t+1, \tau=1}; \theta_d^{*m})|\mathbf{s}_{t, \tau=0}]$, which are computed as the weighted average of past-pseudo value functions evaluated at time $t + 1$:

- i. For $\hat{E}^m[V(b_{t+1}; \theta_d^{*m})|\mathbf{b}_t]$, we take the weighted average of $\{\tilde{V}^n(b_{t+1}^n; \theta_d^{*n})\}_{n=m-N}^{m-1}$ as in Equation (26).
- ii. For $\hat{E}^m[W(s_{t+1, \tau=1}; \theta_d^{*m})|\mathbf{s}_{t, \tau=0}]$, we take the weighted average of $\{\tilde{W}^n(s_{t+1, \tau=1}^n; \theta_d^{*n})\}_{n=m-N}^{m-1}$ as in Equation (23). Again, note that if consumers buy at t , they will have owned the game for one period when they reach $t + 1$. Thus, the set of past pseudo-value functions used here are all evaluated at $\tau = 1$.

To obtain $\{\tilde{W}_k^m(\mathbf{s}_{t, \tau}; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we only need to calculate $\{\hat{E}^m[W(s_{t+1, \tau+1}; \theta_d^{*m})|\mathbf{s}_{t, \tau}]\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}^n(s_{t+1, \tau+1}^n; \theta_d^{*n})\}_{n=m-N}^{m-1}$ as in Equation (23).

- (c) Similarly, we compute the pseudo-joint likelihood at θ_d^{m-1} conditional on $\{\xi_t^m\}$ and θ_s^{m-1} .
- (d) Based on the pseudo-joint likelihoods at θ_d^{*m} and θ_d^{m-1} , we compute the acceptance probability for θ_d^{*m} and decide whether to accept (i.e., set $\theta_d^m = \theta_d^{*m}$) or reject (i.e., set $\theta_d^m = \theta_d^{m-1}$).

5. *Block 4*: Use the Metropolis-Hastings algorithm to draw θ_s^m conditional on $\{\xi_t^m\}$ and θ_d^m .

- (a) Draw θ_s^{*m} (candidate parameter vector).
- (b) We compute the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} conditional on $\{\xi_t^m\}$ and θ_d^m . Note that the pseudo-alternative specific value functions do not depend on θ_s^{*m} , but are required to compute the pseudo-likelihood at θ_s^{*m} since they influence the evolution of equilibrium state variables, which enter pseudo-policy functions. However, they have already been computed in step 4(b) (if θ_d^{*m} has been accepted) or 4(c) (if θ_d^{*m} has been rejected).
- (c) To form the acceptance probability of θ_s^{*m} , we need the pseudo-likelihood for pseudo-policy functions at θ_s^{m-1} conditional on $\{\xi_t^m\}$ and θ_d^m as well. Note that this value has been computed in step 4 and needs not be re-computed here.

- (d) Based on the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} and θ_s^{m-1} , we compute the acceptance probability for θ_s^{*m} and decide whether to accept (i.e., set $\theta_s^m = \theta_s^{*m}$) or reject (i.e., set $\theta_s^m = \theta_s^{m-1}$).

6. *Block 5*: Compute the pseudo-value functions for buying and selling decision problems.

- (a) For each $t = 2, \dots, T$, make a draw of used-copy price (\tilde{p}_{2t}^m), resale value (\tilde{r}_t^m), inventory level (\tilde{Y}_t^m), and cumulative number of newly introduced games (\tilde{C}_t^m) from uniform distributions with appropriate upper- and lower-bound (e.g., upper- and lower-bound of observed values).

- (b) Make a draw of $\tilde{\xi}_2^m$ and $\tilde{\xi}_s^m$ from the corresponding distribution based on $\sigma_{\xi_2}^m$ and $\sigma_{\xi_s}^m$.

- (c) Start from the terminal period T .

- i. Compute the value functions $\tilde{V}^m(b_T^m; \theta_d^{*m})$ and $\{\tilde{W}^m(s_{T,\tau}^m; \theta_d^{*m})\}_{\tau=1}^{T-1}$. Note that at time T , there is no need to compute the pseudo-expected value function. Thus, the value functions computed at time T are not pseudo-value functions.

- ii. Store $\tilde{V}^m(b_T^m; \theta_d^{*m})$ and $\{\tilde{W}^m(s_{T,\tau}^m; \theta_d^{*m})\}_{\tau=1}^{T-1}$.

- (d) For $t = T - 1, \dots, 2$, compute the pseudo-value function $\tilde{V}^m(b_t^m; \theta_d^{*m})$ and $\{\tilde{W}^m(s_{t,\tau}^m; \theta_d^{*m})\}_{\tau=1}^{t-1}$.

- i. To compute $\tilde{V}^m(b_t^m; \theta_d^{*m})$, we need to calculate $\hat{E}^m[V(b_{t+1}; \theta_d^{*m})|b_t^m]$ and $\hat{E}^m[W(s_{t+1,\tau=1}; \theta_d^{*m})|s_{t,\tau=0}^m]$ based on Equations (26) and (23), respectively.

- ii. To compute $\{\tilde{W}^m(s_{t,\tau}^m; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m[W(s_{t+1,\tau+1}; \theta_d^{*m})|s_{t,\tau}^m]\}_{\tau=1}^{t-1}$ based on Equation (23).

- iii. Store $\tilde{V}^m(b_t^m; \theta_d^{*m})$ and $\{\tilde{W}^m(s_{t,\tau}^m; \theta_d^{*m})\}_{\tau=1}^{t-1}$.

7. Go to iteration $m + 1$.

In our application in Section 6, we set $N = 100$ (# past pseudo-value functions used for the approximation of expected value functions) and $h = 0.01$ (kernel bandwidth).

A.2 The likelihood function

Assuming that the prediction errors, ν_{pt} and ν_{rt} , in Equations (19) and (20) are normally distributed, we obtain the conditional likelihood of observing (p_{2t}^g, r_t^g) ,

$$f_s(p_{2t}^g, r_t^g | \{M_{lt}^{s,g}(\tau)\}_{\tau=1}^{t-1}, \xi_{2t}^g, \xi_{st}^g, Y_t^g, C_t^{g,other}; \theta_s) \quad (27)$$

where θ_s is the parameter vector of pseudo-policy functions. Note that $M_{lt}^{s,g}(\tau)$ (size of type- l owners) are a function of $X_g, p_1^g, \{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}, \{C_m^g\}_{m=1}^{t-1}, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^{t-1}, M_1^{d,g}$ (initial size of potential buyers), and $\{N_m^g\}_{m=2}^t$ (potential buyers who arrive at time m). Thus, we can rewrite f_s as

$$f_s(p_{2t}^g, r_t^g | \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^{g,other}, Z_t^g; \theta_s). \quad (28)$$

where $Z_t^g = \{X_g, p_1^g, \{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}, \{C_m^g\}_{m=1}^{t-1}, M_1^{d,g}, \{N_m^g\}_{m=2}^t\}$ is a vector of observed variables.

Assume further that the measurement errors, $\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{st}$, in Equations (17) and (18) are normally distributed. Then, the conditional likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g)$ is written as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | M_t^{d,g}, v^g(t, 0), \{M_{lt}^{s,g}(\tau), v^g(t, \tau)\}_{\tau=1}^{t-1}, \xi_{2t}^g, \xi_{st}^g, p_1^g, p_{2t}^g, r_t^g, Y_t^g, C_t^g; \theta_d), \quad (29)$$

where θ_d is the vector of demand-side parameters. Similar to f_s , f_d can be rewritten as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, Y_t^g, C_t^g, Z_t^g; \theta_d). \quad (30)$$

The joint likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g)$ is the product of f_s and f_d :

$$\begin{aligned} l(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g | \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^g, C_t^{g,other}, Z_t^g; \theta_d, \theta_s) = \\ f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, Y_t^g, C_t^g, Z_t^g; \theta_d) \times \\ f_s(p_{2t}^g, r_t^g | \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^{g,other}, Z_t^g; \theta_s). \end{aligned} \quad (31)$$

The likelihood of observing $\mathbf{D} = \{\{Q_{1t}^g\}_{t=1}^{T^g}, \{Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g\}_{t=2}^{T^g}\}_{g=1}^G$ is

$$\begin{aligned} L(\mathbf{D} | \xi, C, C^{other}, Y, Z; \theta_d, \theta_s) = \\ \prod_{g=1}^G \left[f_d(Q_{11}^g | C_1^g, Z_1^g; \theta_d) \prod_{t=2}^{T^g} l(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g | \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, C_t^g, C_t^{g,other}, Z_t^g; \theta_d, \theta_s) \right] \end{aligned} \quad (32)$$

where G is the total number of games, T^g is the length of observations for game g , $Y = \{\{Y_t^g\}_{t=2}^{T^g}\}_{g=1}^G$, $C = \{\{C_t^g\}_{t=1}^{T^g}\}_{g=1}^G$, $C^{other} = \{\{C_t^{g,other}\}_{t=2}^{T^g}\}_{g=1}^G$, and $Z = \{\{Z_t^g\}_{t=1}^{T^g}\}_{g=1}^G$.

Note that $\{\xi_{2t}^g, \xi_{st}^g\}_{t=2}^{T^g}$ are unobservable to the econometricians. In the proposed Bayesian framework, these variables are augmented from the corresponding distributions to form the likelihood $L(\mathbf{D}|\cdot)$.