Estimating Matching Efficiency with Variable Search Effort*

Andreas Hornstein Marianna Kudlyak
FRB Richmond FRB San Francisco

First Draft: May 21, 2015

This Draft: September 2, 2016

Preliminary and Incomplete
Please do not quote

Abstract

We augment the standard matching function by allowing for heterogeneity of job seeker input and introducing a simple model of endogenous search effort. We then decompose changes in the average employment transition rate into changes in inputs and in aggregate matching efficiency. First, the matching function elasticity with respect to vacancies (α) and the groups' search effort elasticities are not separately identified, i.e., data on group-specific transition rates are consistent with low α and pro-cyclical search effort as well as with high α and counter-cyclical effort. Second, matching efficiency is identified up to a positive scalar. Third, data provide evidence for variable search effort; and the decline in the matching efficiency in the Great Recession was exceptional, even after controlling for variable search effort. Fourth, a model with group-specific variable search effort and only one exogenous shock – aggregate matching efficiency (as opposed to a number of group-specific exogenous shocks) – captures well both co-movement and relative movement in group-specific transition rates. If the variable search effort is shut, as is typical in the literature, the data then require

*Email: andreas.hornstein@rich.frb.org, marianna.kudlyak@sf.frb.org. The authors thank Yongsung Chang and Mark Watson for insightful comments, Eran Yashiv for a useful discussion, and the participants at the 2016 NBER SI Macro Perspectives group, UC Santa Cruz, UT Austin, UC San Diego, U.S. Census, the Fall 2015 Midwest Macro Meetings, University of Rochester, the 2015 Kiel-FRB Richmond conference at CREI Barcelona, the 2015 FRB Richmond-UVA Jamboree at UVA, the 20th ENSAI Economics day "On Labor Markets Instability" in Paris, the 2015 Macro Montreal "New Developments in Business Cycle Analysis". The views expressed here are those of the authors and do not reflect those of the FRB Richmond, FRB San Francisco or the Federal Reserve System.

large group-specific shocks to capture transition rates. Finally, in contrast to the standard approach, the matching function with heterogeneity and variable search effort requires larger aggregate matching efficiency for larger α.

Key Words: Matching efficiency. Search effort. Matching elasticity. Aggregate matching function. **JEL Codes**: E24, J63, J64.

1 Introduction

In Diamond-Mortensen-Pissarides matching models of the labor market, the matching function plays a role similar to the aggregate production function in macroeconomic models of the goods market.[1] The matching function is a reduced form representation of how in a frictional labor market the combination of workers who are looking for employment and vacant positions that need to be filled—the inputs to the matching function—results in new matched workers and positions—the output of the matching function. Changes in the number of new matches that cannot be accounted for by changes in inputs need to be attributed to the residual - matching efficiency. How much variation in the number of new matches can be attributed to inputs versus to the matching efficiency? The question has gained a renewed interest recently: while prior to 2009 variation in the aggregate number of vacancies and unemployed accounted for most of the variation in aggregate number of new matches, it did less so afterwards.

Measurement of the matching efficiency depends on the measured inputs, similarly to the measurement of total factor productivity in the growth accounting literature. In the standard matching function, inputs are homogeneous and there is no utilization variation of inputs. We augment the standard matching function by allowing for heterogeneity of job seeker input and endogenous search effectiveness of these inputs. Through the lens of the matching function, heterogeneity of job seeker input is reflected in search effectiveness, which is a multiplicative shifter of the job seeker's rate of finding a job. We allow the search effectiveness of a group of job seekers to vary endogenously as well as exogenously. We refer to the endogenous component as group-specific search effort and introduce a simple model of search effort, in which search effort is a function of the aggregate transition rate and therefore time-varying.

Given the augmented matching function, we perform an accounting exercise for variation in the average employment transition rate in the economy using variation in the labor market tightness, search effort, and matching efficiency. While an emerging literature recognizes the importance of heterogeneity in search effectiveness, all existing approaches attribute the heterogeneity entirely to the exogenous variation in search effectiveness among different groups of job seekers.[2] As we show below, allowing for endogenous search effort in the aggregate

[1] Petrongolo and Pissarides (2001) in their survey of the matching function provide an in-depth description of the role the matching function plays in macroeconomics.

[2] For example, Şahin Song, Topa, and Violante (2012), Veracierto (2011), Elsby, Michaels, and Ratner (2016), Kroft, Lange, Notowidigdo and Katz (2016), Hall and Schulhofer-Wohl (2015), and Davis, Faberman,

matching function accounting has important implications for identification of parameters of the aggregate matching function and, consequently, for estimating the contribution of variation in matching efficiency to the variation of aggregate employment transition rate.

Motivated by the standard search and matching model (e.g., Pissarides, 2000), we model search effort of an observable type of job seekers as a constant elasticity function of the aggregate employment transition rate, which is itself determined by the matching function. We then derive a set of identification results. First, from the data on employment transition rates, the groups' search effort elasticities with respect to the aggregate transition rate and the matching function elasticity (α) are not separately identified. That is, the data are consistent with low role of vacancies and pro-cyclical search effort as well as well as high role of vacancies and counter-cyclical search effort. Second, the aggregate matching efficiency is identified up to a positive scalar, $1/\alpha$. Third, the matching elasticity from the standard matching function, which ignores variable search effort, is identified, but is not equal to the underlying 'true' matching elasticity.

Our model of endogenous search effort defines the observed groups transition rates and the unobserved aggregate transition rate as nonlinear functions of identified parameters, unobserved stochastic state, and measured series of the group transition rates, groups shares in the job seeker pool and vacancies. We use an extended Kalman filter to estimate the parameters and infer the unobserved state. In our benchmark model with variable search effort, we estimate a flexible random walk process for the aggregate matching efficiency and constant group-specific exogenous effects. We then compare the results allowing for group-specific exogenous effects to also be time-varying.

We apply our estimation procedure to three alternative groupings of the pool of non-employed individuals: unemployed job seekers by duration of unemployment, unemployed job seekers by reason of unemployment, and non-employed job seekers by labor force status (unemployed and out of the labor force) and gender.[3] The decompositions are motivated by large and persistent differences in employment transition rates across groups within each decomposition and large compositional changes of the search pool by these groupings. In the matching function framework, we attribute differences in transition rates across job seekers to the differences in search effectiveness. We take job seeker's membership in a group at the beginning of a period as given, and are agnostic of whether the differences

and Haltiwanger (2013).

[3]In the estimation, we focus on the non-employed job seekers. However, the framework can be easily extended to take into account employed job seekers.

in transition rates reflect some ex ante heterogeneity or are a result of being/becoming a member of a particular group.[4] As a robustness exercise, we additionally allow for time-varying unobserved heterogeneity within the observed groups.

First, we find that data provide evidence for group-specific variable search effort and rejects the hypothesis under which our model is isomorphic to a model with no variable search effort. Second, the decline of the aggregate matching efficiency in the Great Recession was exceptional, even after controlling for variable search effort. This conclusion is robust to alternative sample periods, measures of vacancies or allowing structural breaks in the parameters.

Furthermore, we find that a model with group-specific variable search effort and only one exogenous shock – aggregate matching efficiency (as opposed to a number of type-specific exogenous shocks) is sufficient to capture both co-movement and relative changes in group-specific transition rates. Little improvement is gained from adding small group-specific exogenous shocks. If we instead shut down the group-specific variable search effort, as is typical in the literature, and let exogenous group-specific shocks capture the heterogeneity, the data then require large group-specific shocks, which often move in the counter-intuitive direction, to capture both co-movement and relative movements in the group-specific transition rates.

Finally, the contribution of the aggregate matching efficiency to the variation of the average employment transition rate depends on the elasticity of the matching function. Under the standard matching function approach which ignores heterogeneity and endogenous search effort, larger elasticity with respect to vacancies, α, implies smaller contribution of the aggregate matching efficiency. In the matching function with endogenous search effort the opposite holds: larger α requires larger matching efficiency shocks. Specifically, when α is low, the role of vacancies in the production of new hires is low. The role of search effort in the decomposition of the average transition rate is high; and the model calls for pro-cyclical search effort to drive movements in the employment transition rate. Small contribution from the aggregate matching efficiency is then needed. When α is high, the role of vacancies in the production of new hires is high. The model calls for counter-cyclical search effort to amplify fluctuations in the job seeker input. The role of search effort in the decomposition of the average transition rate is low and it is countercyclical, and more volatile matching efficiency compensates for that. While the group transition rates data are consistent with both pro-/counter-cyclical search effort, we find the counter-cyclical effort for a wide range

[4] For a similar groupings in matching function accounting exercises see Şahin, Song, Topa, and Violante (2014), Barnichon and Figura (2015), and Hall and Schulhofer-Wohl (2015).

of α, $(1/3, 1)$.

Is the labor market better characterized by low α, pro-cyclical search effort and thus low volatility of the aggregate matching efficiency needed to explain volatility of the average employment transition rate or is it better characterized by high α, counter-cyclical search effort and thus high volatility of the aggregate matching efficiency? As we show, the matching elasticity obtained from the standard matching function approach is not equal the data-generating α under variable search effort, thus we need to look elsewhere. One approach is to bring additional data to bear on the question of the cyclicality of search effort. While literature on the cyclicality of search effort is scarce, some evidence emerges. Specifically, using the Current Population Survey data, Shimer (2004) finds that the number of search methods used by the unemployed increases during the 2001 recession. Using the Current Population Survey and the data from the Annual Time Use Survey (ATUS), Mukoyama, Patterson and Sahin (2014) conclude that the time spent on search is countercyclical. Faberman and Kudlyak (2014) find that the number of applications sent by a job seeker per week on an online job board is significantly higher in metropolitan areas with more slack labor markets. Using ATUS, DeLoach and Kurt (2013), however, argue for a-cyclical search effort, and Gomme and Lkhagvasuren (2015) argue for pro-cyclical effort of an individual worker and counter-cyclical average search effort.

If the evidence tilts towards countercyclical search effort, it implies that the matching function elasticity is closer to 1 than to 0. That is, vacancies play an important role in the production of new hires in the matching function framework and the countercyclical search effort exacerbates changes in the job seeker input. Consequently, large changes in the aggregate matching efficiency are required to rationalize changes in the observed average transition rate in the matching accounting framework. The evidence for endogenous and counter-cyclical search effort poses a hurdle for the standard search and matching model, both in terms of its technical capacity to accommodate counter-cyclical search effort and in terms of its ability to generate fluctuations of the employment transition rate of empirical magnitudes.

To our knowledge, ours is the first study that allows for endogenous search effort in the matching function accounting framework. A number of papers have studied how 'mismeasurement' of the aggregate matching function might affect estimates of matching efficiency. Sahin, Song, Topa, and Violante (2014) show how the potential misallocation of unemployed workers across disaggregated labor markets affects measured matching efficiency in the reduced form aggregate matching function. They derive correction factors for the effects of misallocation of searchers across markets and find that observed misallocations do not generate large movements in these correction factors. Veracierto (2011) broadens the measure of the worker search input to include OLF participants. Even though employment transition rates from OLF are significantly smaller than from unemployment, total transitions from OLF are significant. Veracierto therefore includes OLF non-employed as an input to aggregate worker search effort, and assumes that their search effort is reflected in their employment transition rate relative to the unemployed. Implicitly this fixes the search effort of the unemployed at one. Barnichon and Figura (2015), Hall and Schulhofer-Wohl (2015) and Sedlacek (2014) follow up on Veracierto (2011) allowing for different matching efficiencies across groups of employed and non-employed workers. For each group they estimate an efficiency parameter that combines matching efficiency and search intensity. In their framework aggregate matching efficiency is a weighted average of the group-specific matching efficiencies which are taken as exogenous. Based on the evidence of declining search effort with unemployment duration, Davis (2011) proposes correction factor for search effort that depends on the average duration of unemployment, and constructs the effective input of workers to the matching function as the product of total unemployment and the correction factor. Kroft, Lange, Notowidigdo, and Katz (2016) generalize this approach and provide a more detailed disaggregation of the unemployed by duration, but again their approach implicitly fixes search effort for the group with the highest employment transition rate. On the theory side, the cyclicality of search effort is potentially relevant for potential amplification of the volatility of vacancies and unemployment in search and matching models (for example, Costain and Reiter (2008) or Gomme and Lkhagvasuren (2015)).

The remainder of the paper is structured as follows. Section 2 describes a model of matching function with heterogeneity and endogenous search effort. Section 3 derives identification results and estimation procedure. Section 4 describes the data and basic facts. Section 5 presents our main estimation results for identified aggregate matching efficiency with group-specific variable search effort, compares the estimates from this model to the estimates from the models with variable search effort that also allow for time-varying type-specific exogenous effects, and compares the results to the estimates from alternative approaches to modelling job seeker heterogeneity, which do not allow for variable search effort. Section 6 presents the decomposition of the average employment transition rate conditional on pro- and countercyclical search effort. Section 7 concludes.

2 Matching with heterogeneous search effectiveness

The aggregate search and matching function in macro-labor models describes the 'production' of hires as a function of the stocks of job seekers and vacancies, and an exogenous shift term denoting the aggregate efficiency of the matching process. The standard approach for the search and matching function assumes that the inputs are homogenous and that search effort does not vary endogenously with the state of the labor market. We augment the standard search and matching function by allowing for time-varying heterogeneity across observed groups of job seekers. In particular, the search effectiveness of a group of job seekers may vary for exogenous reasons or it may change in response to the aggregate matching rate, that is, the rate per unit of search effort at which employment opportunities arise. We refer to the endogenous component as group-specific search effort and introduce a simple model in which search effort is a function of the aggregate matching rate.

2.1 A model of heterogeneous search effectiveness

In this section we describe a simple extension of the aggregate matching function approach that allows for heterogeneity in search effectiveness of job seekers.

Consider an economy with a finite number of search types, $i \in I$.[5] Time is continuous. At any point in time u_i job seekers of type i engage in search and the types differ in their search effectiveness, ρ_i, to be described in more detail below. Total effective search input, $u^* \equiv \sum_i \rho_i u_i$, and vacancies, v, are inputs to a Cobb-Douglas matching function that generates hires h

$$h_t = \exp(\kappa_t)\, v_t^{\alpha} (u_t^*)^{1-\alpha}, \tag{1}$$

for a given aggregate matching efficiency, κ, and matching elasticity, $0 < \alpha < 1$. In the standard matching function the search types are homogeneous, $\rho_i = 1$, and aggregate search effort is simply the sum of all job seekers, $u \equiv \sum_i u_i$.

The aggregate matching rate per search unit, $\lambda \equiv h/u^*$, is

$$\lambda_t = \exp(\kappa_t)\, \theta_t^{\alpha} \bar{\rho}_t^{-\alpha}, \tag{2}$$

where $\theta \equiv v/u$ is the standard aggregate labor market tightness, and $\bar{\rho} \equiv \sum_i \omega_i \rho_i$ is average search effectiveness, with $\omega_i = u_i/u$ being the share of type i in the search pool. The transi-

[5] Throughout the paper, we use terms "type" and "group" interchangeably. In Section 4, we provide the exact definitions of job seeker groupings used in the empirical analysis.

tion rate to employment for a type i searcher is then the product of the type-specific search effectiveness and the per search unit matching rate, $\lambda_i = \rho_i \lambda$, and the average aggregate transition rate, $\bar{\lambda} \equiv \sum_i \omega_i \lambda_i$, is

$$\bar{\lambda}_t = \exp(\kappa_t)\, \bar{\rho}_t^{1-\alpha} \theta_t^{\alpha}. \tag{3}$$

For the analysis of the average transition rate the standard matching function approach ignores changes in average search effectiveness, either due to changes in the composition of the search pool or due to variations in effort and sets $\bar{\rho} = 1$. In other words, with time-varying heterogeneity in search effectiveness, the standard approach conflates changes in matching efficiency with changes in composition and search effort.

We assume that type-specific search effectiveness is a constant elasticity function of the aggregate matching rate

$$\ln \rho_{it} = z_{it} + \eta_i \ln \lambda_t, \tag{4}$$

where z_{it} reflects exogenous type-specific matching efficiency. Our approach is motivated by the basic search and matching model, e.g. Pissarides (2000) or Gomme and Lkhagvasuren (2015), for which the employment transition rate of searchers is the product of search effort and the aggregate matching rate per unit of search effort, and searchers face an increasing and convex cost from search effort. For this model search effort increases if the expected gains from search increase, in particular, if the aggregate matching rate increases. In our set-up the search elasticity η_i reflects the endogenous response of search effort to changes in the aggregate matching rate, assuming that more effort makes a searcher more effective. We will call search effort pro-cyclical if $\eta_i > 0$ and countercyclical if $\eta_i < 0$. We do not restrict search effort to be pro-cyclical as implied by the basic search and matching model, but we impose a lower bound on the search elasticity, $\eta_i \geq -1$, such that a type's employment transition rate is always pro-cyclical,

$$\ln \lambda_{it} = z_{it} + (1 + \eta_i) \ln \lambda_t \text{ for } i \in I. \tag{5}$$

Our model of heterogenous search effectiveness thus defines observed employment transition rates of groups as a function of the unobserved aggregate matching rate and exogenous group-specific effects, that may be fixed or time-varying. Combining equations (2) and (5) then yields the aggregate matching rate as a non-linear function of the search pool composition, aggregate matching efficiency, and exogenous type-specific effects

$$\ln \lambda_t = \kappa_t + \alpha \ln \theta_t - \alpha \ln \sum_i \omega_{it} \exp\left(z_{it} + \eta_i \ln \lambda_t\right). \tag{6}$$

In equations (5) and (6), $y_t = \left\{\theta_t, (\lambda_{it})_{i=1}^I, (\omega_{it})_{i=1}^I\right\}$ is observable, $x_t = \left\{\kappa_t, (z_{it})_{i=1}^I\right\}$ describes the unobserved state, and $\left\{\alpha, (\eta_i)_{i=1}^I\right\}$ are parameters. Our goal is to estimate the parameters and infer the unobserved state conditional on observable variables.

2.2 Identification

We first show that conditional on using only observations on group-specific transition rates, the parameters of our model are not uniquely identified. In particular, the sign of the search effort elasticity, that is, the cyclicality of search effort, is not identified.

Proposition 1 *Conditional on observations $\{y_t\}$ the matching elasticity and search effort elasticities are identified only up to the restriction*

$$\frac{\alpha}{1-\alpha}\left(1+\eta_i\right) = \phi_i \geq 0 \text{ for } i \in I. \tag{7}$$

and the aggregate matching efficiency and matching rate are identified up to the restriction

$$\hat{\kappa}_t \equiv \frac{\kappa_t}{\alpha} \text{ and } \ln \hat{\lambda}_t \equiv \frac{1-\alpha}{\alpha} \ln \lambda_t. \tag{8}$$

Using the definition of ϕ_i together with the transformation of variables $\hat{\kappa}$ and $\hat{\lambda}$, we can rewrite our model for employment transition rates (5) and (6) as follows

$$\ln \lambda_{it} = z_{it} + \phi_i \ln \widehat{\lambda}_t, \tag{9}$$

$$\ln \widehat{\lambda}_t = \widehat{\kappa}_t + \ln \theta_t - \ln \sum_i \omega_{it} \exp\left(z_{it} + \phi_i \ln \widehat{\lambda}_t\right). \tag{10}$$

From these equations it is obvious that given observable y and a solution $\left(z, \hat{\kappa}, \hat{\lambda}, \phi\right)$, any other solution $(z, \kappa, \lambda, \alpha, \eta)$ that satisfies the constraints (7) and (8) is observationally equivalent. In other words, working with observations on group-specific transition rates only, the observations are consistent with either pro- or counter-cyclical search effort. In particular, if we choose a matching elasticity sufficiently close to zero (one) then search effort will be pro-cyclical, $\eta_i > 0$ (counter-cyclical, $\eta_i < 0$) for all types. Finally note that if the identified transition elasticities are the same for all types, $\phi_i = \phi \; \forall i \in I$, then by setting $\eta = 0$ and $\alpha = 1/(1+\phi)$, our model is equivalent to one without endogenous search effort. Alternatively, if the identified transition elasticities differ across types we can argue for the presence of endogenous search effort, even if the observations do not uniquely determine the cyclicality of that search effort. It is an empirical question whether the identified transition elasticities differ across types and we address this issue in the estimation section.

We get a better understanding of the model's inability to identify the cyclicality of search effort if we ignore heterogeneity and consider the case of a representative searcher. We then obtain a closed form solution of the aggregate matching rate, λ, and the employment transition rate, λ_1, in terms of the underlying parameters, matching elasticity and search effort elasticity,

$$\begin{aligned}
\ln \lambda &= \frac{\alpha}{1+\alpha\eta_1}\left(\ln \theta + \hat{\kappa}\right) \text{ and} \\
\ln \lambda_1 &= \underbrace{\alpha \frac{1+\eta_1}{1+\alpha\eta_1}}_{\equiv \hat{\alpha}}\left(\ln \theta + \hat{\kappa}\right)
\end{aligned}$$

where $\hat{\alpha}$ is the effective matching elasticity which relates observed transition rates to market tightness.[6] Let us start with a matching function elasticity α and constant search effort, $\eta = 0$. Then a 1 percentage point increase of market tightness results in an α ppt increase of the matching rate λ and the transition rate λ_1. Suppose that search effort is pro-cyclical, $\eta_1 > 0$, then the higher matching rate leads to an increase in search effort, ρ_1, and a decline in effective tightness, θ/ρ_1, equation (2). Thus the matching rate increases less than α ppts. The direct effect of increased search effort on the transition rate, however, more than compensates for the decline in the matching rate, and the transition rate increases by more than α ppts. The converse holds when search effort is counter-cyclical, $\eta_1 < 0$. In this case, a 1 ppt increase of market tightness results in a more than α ppt increase in the matching rate because search effort declines, and the effective market tightness increases by more than 1 ppt. Again, the direct effect of reduced search effort more than compensates for the increase in the matching rate, and the transition rate increases by less than α ppts. Thus the effective matching elasticity is increasing in the matching elasticity and the search effort elasticity. Any given effective matching elasticity $\hat{\alpha}$ can then be accounted for by the same matching elasticity, $\alpha = \hat{\alpha}$, and a-cyclical search effort, $\eta_1 = 0$, or by a larger (smaller) matching

[6] Here we ignore exogenous type-specific effects since there is only one type.

elasticity, $\alpha > \hat{\alpha}$ ($\alpha < \hat{\alpha}$) with a counter-cyclical (pro-cyclical) search effort, $\eta_1 < 0$ ($\eta_1 > 0$).

Even though the matching elasticity and search effort elasticities are not separately identified, for a log-linear approximation of the average employment transition rate as a function of market tightness, the coefficient on market tightness is uniquely determined.

Proposition 2 *The log-linear approximation of the average employment transition rate at a point* $\left(\hat{\lambda}_0, \hat{\kappa}_0, z_0, \omega_0\right)$ *is*

$$\Delta \ln \bar{\lambda} = \bar{\alpha} \left(\Delta \ln \theta + \Delta \hat{\kappa}\right) + (1 - \bar{\alpha}) \Delta \bar{z} + \Delta \bar{\varepsilon}, \text{ with} \tag{11}$$

$$\bar{\alpha} = \frac{\bar{\phi}}{1 + \bar{\phi}} \tag{12}$$

and $\bar{\phi}$, $\Delta \bar{z}$, *and* $\Delta \bar{\varepsilon}$ *are weighted averages of type-specific elasticities, efficiencies and measurement errors.*

The expression for the log-linear approximation of the average employment transition rate is analogous to the standard homogeneous search model, but with the reduced form matching elasticity $\bar{\alpha}$ being a function of the weighted average of the identified transition elasticities ϕ_i. Therefore the 'reduced form' matching function elasticity $\bar{\alpha}$ is identified.

Finally, we note that the overall level of the aggregate matching efficiency and the type-specific effects is not identified. Using equations (9) and (10) it is straightforward to show the following proposition.

Proposition 3 *The level of the state* $x_t = \left\{\hat{\kappa}_t, (z_{it})_{i=1}^{I}\right\}$ *is identified up to an additive shift term.*

When we estimate our model we therefore normalize one of the states.

3 Estimation

Our model for the evolution of the type-specific employment transition rates has a straightforward state-space representation, albeit with a non-linear measurement equation. Conditional on the parameters we are using an extended Kalman-filter approach to infer the state of the system from observations on the type transition rates. We obtain parameter estimates by maximizing the likelihood function.

Given the unobserved state $x_t = (\hat{\kappa}_t, \{z_{it}\}_i)$, equation (9) for the measured type-transition rate λ_i^m with measurement noise ε_i,

$$\ln \lambda_{it}^m = z_{it} + \phi_i \ln \hat{\lambda}_t + \varepsilon_{it} \text{ for } i = 1, .., I \tag{13}$$

with $\varepsilon_{it} \sim N(0, \Sigma_\varepsilon)$, and equation (10) for the identified matching rate define the measurement equations of the state-space model. We use three different specifications for the evolution of the unobserved state x_t. The baseline specification has aggregate matching efficiency only, and the two other alternatives use either time-varying type-specific efficiencies only or aggregate and TVTS efficiencies jointly.

Model 1. Time-varying aggregate matching efficiency and fixed type-specific matching efficiencies,

$$\begin{aligned} \widehat{\kappa}_t &= \widehat{\kappa}_{t-1} + \zeta_t, \ \zeta_t \sim N\left(0, \sigma_\zeta^2\right) \\ z_{it} &= c_i \ \forall i \in I. \end{aligned}$$

We use a random walk to capture any trend in the aggregate matching efficiency and also any potentially substantial drop of the matching efficiency after 2007. We allow for permanent differences across types through fixed effects, and time-varying differences across types are captured through differences in the identified transition elasticities. In view of proposition 3 we normalize $c_1 = 0$.

Model 2. Time-varying type-specific matching efficiencies and fixed aggregate matching efficiency,

$$\begin{aligned} \widehat{\kappa}_t &= 0 \\ z_{it} &= z_{it-1} + \xi_{it}, \ \xi_{it} \sim N\left(0, \sigma_{\xi_i}^2\right) \ \forall i \in I \end{aligned}$$

Similar to Model 1, we allow for flexible random walk processes for type-specific matching efficiencies. We normalize $\widehat{\kappa} = 0$.

Model 3. Time-varying aggregate matching efficiency and time-varying type-specific matching efficiencies,

$$\begin{aligned}\widehat{\kappa}_t &= \widehat{\kappa}_{t-1} + \zeta_t,\ \zeta_t \sim N\left(0, \sigma_\zeta^2\right), \\ z_{it} - c_i &= \gamma_i \left(z_{it-1} - c_i\right) + \xi_{it},\ \xi_{it} \sim N\left(0, \sigma_{\xi_i}^2\right)\ \forall i \in I.\end{aligned}$$

In Model 3, we specify the aggregate matching efficiency as a random walk and type-specific matching efficiencies as stationary $AR(1)$-processes. We normalize $c_1 = 0$. Anticipating our estimation results for Model 2, we see substantial co-movement of type-specific efficiencies at low frequencies, which suggests a common trend. We identify this common trend with the aggregate matching efficiency. Furthermore, with this specification the stochastic processes for $\widehat{\kappa}_t$ and $\{z_{it}\}$ are separately identified to a first order approximation. Separate identification of the states is potentially an issue for Model 3 since at any point in time we have I observations on type employment transition rates, but $I+1$ unobserved states. For a linear Kalman-filter the state is identified, if the state-space system is 'observable.' Formally, the n-dimensional state of a system is said to be observable, if the last n past observations are sufficient to determine the entire history of states for the last n periods. If the measurement equations for our model were linear, we could easily verify that the observability condition is satisfied for at least one type-specific efficiency being stationary. Since our measurement equations are non-linear we are using an extended Kalman-filter and no general conditions for identification are available. In our estimation we check that the observability condition is satisfied for the extended Kalman-filter along the inferred path of the state.

4 Data and basic facts

We now construct alternative measures of the search pool using data from the Current Population Survey (CPS) and characterize these measures from the perspective of our matching framework with heterogeneity and variable search effort.

4.1 Definition of job seeker groups

In the matching function framework, we attribute differences in transition rates across job seekers to differences in search effectiveness. Therefore, we want to decompose the pool of job seekers into observable groups that are characterized by large and persistent differences in transition rates to employment. Specifically, we consider three alternative characterizations of the pool of job seekers using the CPS.

The first two characterizations define the search pool narrowly as the unemployed, that is, those reporting to be actively engaged in search. We consider two decompositions of the unemployed, by duration and by reason of unemployment. The first classification groups the unemployed into those that report unemployment of less than 5 weeks, 5-26 weeks, or more than 26 weeks. The second classification consists of the four groups that report being unemployed because they are on temporary layoff, on permanent layoff, have quit a job, or have previously been out of the labor force.

The third characterization of the search pool includes all non-employed, that is the unemployed and those that are out of the labor force (OLF). Although those that are OLF do not report to be actively engaged in search, in any month a substantial number of them do make the transition to employment. Thus a clear cut distinction between those that are unemployed and those that are OLF may not be appropriate for a matching framework that expressly allows for differences in search effectiveness.[7] We decompose this broader definition of the search pool into four groups of non-employed job seekers characterized by their labor market status (unemployed or OLF) and gender (male or female).

For our characterizations of the search pool we take a job seeker's membership in a group at the beginning of a period as given.[8] We are agnostic of whether the groups within each decomposition reflect some ex-ante (inherent) heterogeneity among job seekers in terms of their transition rates, or whether different transition rates are a result of being/becoming a member of a particular group.

4.2 Data sources and construction of the series

For each definition of the search pool we construct the employment transition rates and the search pool shares of the different groups of job seekers.

We construct the employment transition rates for the different groups of non-employed job seekers using the micro data from the Current Population Survey (CPS) basic monthly files, from January 1976 to December 2015. We follow Madrian and Lefgren (1999) and Shimer (2012) and match individuals from month to month using information on race, age and sex besides individual and household's identification number. In the analysis, we weight each individual by the average of the individual's CPS sampling weights from adjacent months.

[7] For these reasons Veracierto (2011), Barnichon and Figura (2015), and Hall and Schulhofer-Wohl (2015) have all used a broader concept of the search pool in the matching function.

[8] The same approach is taken in other recent work that accounts for heterogeneity in matching efficiency, for example, Barnichon and Figura (2015) or Hall and Schulhofer-Wohl (2015).

The transition probability of a group is the fraction of individuals that transition between labor market states in two adjacent months.[9] We transform the month-to-month transition probabilities to monthly continuous time transition rates. For the search pool definitions that cover the unemployed we use the exit probabilities to employment (E) and OLF (I), and assume that job seekers who exit do not return to unemployment in the same month. This defines a relation between the discrete time transition probabilities p and the continuous time employment transition rates λ, which we can solve for the transition rate from unemployment to employment

$$\lambda_{UE} = -\frac{\log(1 - p_{UE} - p_{UI})}{1 + p_{UI}/p_{UE}}.$$

When the exit probability to OLF is small relative to the exit probability to employment then the employment transition rate is approximately $-\log(1 - p_{UE})$. For most of our samples this is not a good approximation. We therefore prefer to use information on exit rates to all states when calculating the employment transition rate. For the search pool definition that covers all non-employed we use the exit probabilities of unemployed (OLF) to employment and OLF (unemployment). Otherwise we proceed the same way to calculate the monthly employment transition rates.

We employ two alternative aggregate vacancy series: (1) the Help Wanted Index (HWI) from the Conference Board, which is available since 1951, and (2) the Job Openings and Labor Turnover Survey (JOLTS) program of the BLS, which is available starting in January 2001.[10]

For the baseline analysis, we use data from the CPS and the HWI covering the period from January 1994 to December 2015. We start the baseline sample in 1994 because of the structural breaks in the search pool shares and transition rates associated with the 1994 CPS redesign. We use the adjustment factors from Polivka and Miller (1998) to adjust the search pool share series prior to 1994. Since no comparable adjustment factors are available for our constructed exit probabilities, we introduce an additive adjustment factor in our measurement equations for the group-specific employment transition rates prior to 1994.

All monthly series are seasonally adjusted using the Watson (1996) implementation of the X-11 procedure. Since the monthly series remain highly volatile, even after this adjustment,

[9] In the analysis, we follow the BLS approach and treat the reported labor force status as a true status. Frazis, Robinson, Evans, and Duff (2005) describes that the main reason for why the BLS does not correct responses for a potential error is a lack of methodology or the data that would guide the correction.

[10] Given the shift in job advertising from print media to web-based means the HWI may not be consistent over time. Barnichon (2010) corrects for structural changes in the HWI series and we use his adjusted series.

we estimate our models on quarterly averages of the seasonally adjusted monthly data.

4.3 Basic facts

Accounting for heterogeneity in the matching function framework matters for the measurement of matching efficiency if there are large and persistent differences in employment transition rates across different groups of job seekers and large and systematic changes in the composition of the search pool over time. We now show that this is indeed the case for our two decompositions of the pool of unemployed. In particular, in recessions the search pool composition shifts towards those with relatively low employment transition rates, that is, the average search effectiveness declines. Not accounting for this change in average effectiveness is likely to bias ones estimate of the aggregate matching efficiency downward. For our broader definition of the search pool which includes unemployed and OLF this kind of cyclical bias is not as pronounced.

First, consider the decomposition of the search pool of unemployed by duration of unemployment. Large differences in transition rates among the unemployed by duration are immediately apparent: short-term unemployed are three times as likely to transition to employment than long-term unemployed (Figure 1, Panel A.1). Furthermore, the differences in the transition rates among these groups persist over time, keeping the ranking of transition rates unchanged. But even though there is substantial comovement among the transition rates the relative transition rates do change (Figure 1, Panel A.2). For example, in the 2007-09 recession the transition rates of the medium- and long-term unemployed decline more than those of the short-term unemployed. We will attribute changes in relative transition rates to time-varying differences in search effectiveness. Finally, the composition of the pool of the unemployed by duration changes systematically and substantially over time: the share of long-term unemployed increases in recessions, e.g., following the 2007-09 recession the share of long-term unemployed more than doubles (Figure 1, Panel A.3).

The alternative decomposition of the search pool of unemployed by reason has the same features as the decomposition by duration (Figure 1.B). Relative to those who give a temporary layoff as the reason for unemployment, the unemployed on permanent layoff tend to be half as likely to find employment, they suffer a relatively larger decline of transition rates in recessions, and they make up a larger share of the search pool in recessions.[11]

[11] The sharp jump of the transition rate in 1994 for those who report being on temporary layoff likely reflects the structural break associated with the CPS redesign.

For the broader definition of the search pool that includes all of the non-employed working-age population the pattern that emerges for average search effectiveness is more ambiguous (Figure 1.C). The unemployed are relatively more effective at search than those that are OLF, their employment transition rates are about five to ten times higher than the OLF transition rates. But the transition rates of the unemployed are also much more cyclical such that the relative transition rates of those that are OLF are increasing in recessions. At the same time, the search pool share of those that are OLF tends to decline in recessions. The cyclical bias of composition changes for this search pool definition is therefore not obvious.

5 Matching efficiency with variable search effort

In this section, we present estimates of the identified transition elasticities (ϕ_i) and the identified aggregate matching efficiency ($\widehat{\kappa}$). We turn to the interpretation of these results conditional on the aggregate matching elasticity α in the next section. We start with estimates of the baseline model with type-specific variable search effort and time-varying aggregate matching efficiency only (Model 1) for the search pool decomposition by unemployment duration. For this case we find evidence for variable search effort and an unprecedented decline of the identified matching efficiency following the Great Recession. We then study the role of composition effects in a restricted version of the baseline model that effectively eliminates variable search effort, but allows for type-fixed effects. Relative to a framework with homogeneous search, the composition effects associated with the type-fixed effects reduce the volatility in the identified aggregate matching efficiency, but substantial volatility remains. The results are robust with respect to unobserved heterogeneity in search effort and the use of the HWI index for vacancies. We then compare the results from this baseline model with the two alternative models that allow for time-varying type-specific matching efficiencies (Models 2 and 3) and find that relative to these alternatives the baseline model with variable search effort captures changes in relative transition rates well. Furthermore, the identified aggregate matching efficiency in the baseline model appears to capture a common trend in the time-varying type-specific matching efficiencies in Model 2. Finally, we show that estimates of the baseline model for our two alternative search pool definitions, unemployment by reason and labor force status, yield qualitatively similar paths for the identified matching efficiency. In particular, we find comparable declines of the identified aggregate matching efficiency following the Great Recession.

For ease of exposition, in this section we will frequently drop the qualifier 'identified' for the transition elasticities and aggregate matching elasticity when no confusion can arise.

5.1 Aggregate matching efficiency with variable search effort

The baseline model with endogenous search effort and aggregate matching efficiency captures well the comovement of type-specific employment transition rates and changes in relative transition rates.

5.1.1 Transition elasticities and matching efficiency in a baseline model

We begin with the baseline model with aggregate matching efficiency only, Model 1 of Section 3, when heterogeneity in the search pool of unemployed workers is defined by duration of unemployment. For different sub-samples and specifications, Table 1 displays the parameter estimates of the model and Figure 2 displays the smoothed posterior of the identified aggregate matching efficiency.

Identified transition elasticities The baseline specification covers the years 1994-2015 and uses the HWI for the vacancy measure, Table 1, Column $(1a)$. The type-specific fixed effects, c_i, decline with unemployment duration, capturing the persistently lower employment transition rates of the unemployed with longer durations.[12][13] The transition elasticities, ϕ_i, are monotonically increasing with the duration of unemployment, that is, the employment transition rates of those unemployed with longer durations are more cyclically sensitive than the transition rates of those with shorter durations. The estimates of the type-specific transition elasticities are sufficiently precise to reject the hypothesis that they are the same. As noted in the discussion of Proposition 1, if the identified transition elasticities are the same for all types, then the model is consistent with constant search effort for a particular choice of the aggregate matching elasticity. Since the transition elasticities are significantly different from each other we reject the hypothesis of no variable search effort.

Identified aggregate matching efficiency For the baseline specification the estimate of the identified aggregate matching efficiency declined dramatically following the Great Recession and it has only partially recovered over the last years; the solid black line $(1a)$ in Figure 2. Since the aggregate matching efficiency is identified only up to a positive scalar,

[12]The fixed effect for short duration unemployment is normalized at one.

[13]For the Kalman filter we take the initial prior of the identified aggregate matching efficiency as given, and this prior becomes a parameter of the likelihood function reported in the first row of Table 1.

namely the matching elasticity, we will defer discussion of the quantitative magnitude of changes in the matching efficiency to the next section. The behavior of the identified matching efficiency is, however, informative about the magnitude of movements in the aggregate matching efficiency during the Great Recession relative to the overall sample period.

To gain more perspective on the magnitude of the decline in the matching efficiency, we re-estimate the model for the extended sample period 1976-2015. As noted in section 4.2 we allow for a structural break in the employment transition rates prior to 1994 to account for the 1994 CPS data revision. For the years after 1994 the estimated aggregate matching efficiency for the extended sample and the baseline sample follow each other closely, lines (1a) and (2a) in Figure 2. In particular the matching efficiency declines dramatically in the Great Recession, and that decline remains exceptional even for the extended sample period. The estimated transition elasticities are somewhat lower in absolute value for the extended sample period, but the differences between them remain significant and the cyclical sensitivity continues to increase with the duration of unemployment, columns (1a) and (2a) in Table 1.

5.1.2 Robustness

Constant search effort We evaluate the contribution of endogenous search effort by estimating a restricted version of the model where the transition elasticities are the same for all types. As we have argued in the discussion of Proposition 1 in Section 2.2, imposing this restriction makes the model equivalent to one with constant search effort for a particular value of the matching elasticity. The model can then be reinterpreted as having fixed type-specific search efficiencies, similar to Barnichon and Figura (2016).

The results of estimating the restricted model are presented in Table 1, column $(1b)$, and the aggregate matching efficiency is the black o-line $(1b)$ in Figure 2. The restricted model has no time-varying heterogeneity, only type-specific fixed effects, and the restricted transition elasticity is an average of the unrestricted type-specific transition elasticities. The estimated aggregate matching efficiency of the restricted model is adjusted to compensate for the model's lack of ability to accommodate relative movements in type-specific transition rates. Specifically, to compensate for the decline in relative transition rates of long duration unemployment after 2007 the restricted model requires a somewhat higher aggregate matching efficiency than the unrestricted model with type-specific time-varying search effort, Figure 2, lines $(1a)$ and $(1b)$. In other words, allowing for variable search effort makes the estimates matching efficiency somewhat more volatile relative to a constant search effort restriction.

For the estimated restricted model search effort is constant, that is, the implicit effort elasticity is zero, if the matching elasticity is $\alpha = 0.38$. For this matching elasticity we construct the aggregate matching efficiency using the standard assumption of a homogeneous search pool, that is, we use the expression for the average transition rate, equation (3), with a constant match quality. The scaled matching efficiency from this homogeneous case, $\overline{\kappa}/\alpha$, is comparable to our identified matching efficiencies, and it is displayed as line ($1c$) in Figure 2. As we have discussed in previously, Section 4.3, for the decomposition of the unemployed search pool by duration of unemployment the composition of the search pool shifts towards groups with relatively lower employment transition rates when unemployment is high and the average transition rate is low, that is, average search quality declines. Moving from lines ($1c$) to ($1b$) demonstrates how accounting for compositional changes alone reduces the volatility of the estimated aggregate matching efficiency.

Unobserved heterogeneity If one believes that changes in the duration distribution of unemployment mainly reflect unobserved heterogeneity, then the composition of our search pool groups is systematically changing over time.[14] The time-varying employment transition rates of a group may therefore reflect not only the actual variation of the transition rate of the job seekers in the group, but also the unobserved compositional shifts within the group. Such time-varying unobserved heterogeneity within our search pool groups might then be reflected in the time-varying endogenous as well as exogenous components of group-specific matching efficiencies.

If unobserved heterogeneity is quantitatively important, the identified transition elasticities, ϕ_i, should vary systematically with the aggregate state of the economy. To check for this possibility, we allow for a structural break in the identified transition elasticities that depends on the level of the unemployment rate. In particular, the identified transition elasticities are allowed to be different for periods when the unemployment rate exceeds the average of the minimum and maximum unemployment rate. For this exercise we use the extended sample period starting in 1974 and to allow for the possibility of low frequency changes in the unemployment rate, we define separate minima and maxima for the sub-samples 1974-1987, 1987-1997, 1997-2005, and 2005-2012. Column ($2b$) of Table 1 shows the estimates from this specification. The estimated breaks for the transition elasticities are quite small.

[14] For recent contributions see Hornstein (2012), Ahn and Hamilton (2015), and Alvarez, Borovickova, and Shimer (2016).

Furthermore, allowing for these breaks does not much affect the estimates for the aggregate matching efficiency, the dashed-x red line ($2b$) in Figure 2. We could also allow for structural breaks in the group-specific fixed effects, but do not pursue this option now since in the following we will estimate the model with aggregate and time-varying type-specific matching efficiencies and the latter will reflect possible time-varying within-group heterogeneity.

JOLTS vacancies Finally, we consider how an alternative measure for vacancies, namely the vacancy posting series from JOLTS, affects our estimate of matching efficiency. JOLTS is available only from 2001 so we re-estimate our model with the HWI vacancy measure for the shorter sample period, line ($3a$) in Figure 2, and also estimate our model with the JOLTS vacancy measure, line ($3b$) in Figure 2. The decline in matching efficiency starting in 2008 is comparable for the two vacancy measures, but the JOLTS based measure does replicate the pre-2008 decline that is quite prominent for the HWI measure. The estimates for the identified transition elasticities for the shorter sample using either the HWI or JOLTS vacancy measure are quite close to their respective estimates for the baseline sample, but with larger standard errors, Columns (1a) and (3a) respectively (3b) of Table 1.

5.2 Alternative models of matching efficiency

In the results presented so far, the comovement of the type-specific employment transition rates is captured by the aggregate matching efficiency, and relative changes of type-specific transition rates are captured by differential responses of search effort to the aggregate transition rate while exogenous differences in type-specific efficiencies remain fixed. In this section, we present results for models that also allow for time-varying type-specific matching efficiencies, Models 2 and 3 of Section 3. A comparison across the models is informative regarding the extent to which type-specific variable search effort with a common aggregate shock is sufficient to capture both comovement and relative changes in type-specific transition rates.

In Table 2.A and Figure 3 we display the results from estimating Models 1, 2, and 3 for the extended sample period from 1976-2015 when the search pool of unemployed is differentiated by unemployment duration.[15] The top panel of 3 displays the smoothed posterior for the aggregate matching efficiency of Models 1 and 3, and the lower three panels display the smoothed posteriors for the type-specific matching efficiencies of Models 2 and 3.

[15] The results for Model 1 represent specification (2a) in Table 1 and Figure 2. The Appendix contains a complete listing of the results for Models 2 and 3 similar to Table 1 and Figure 2 for Model 1.

For the model with time-varying type-specific matching efficiencies and fixed aggregate matching efficiency, Model 2, we find that the transition elasticities are more similar across groups than in the baseline Model 1 with fixed type-specific effects, Table 2.A, Columns (1) and (2). This means that Model 2 captures changes in relative type-specific transition rates not so much through differences in the types' responsiveness to the aggregate transition rate, but through differential changes in type-specific matching efficiencies. For example, the matching efficiency of the long-duration unemployment group is relatively more volatile and displays a larger decline in the post-2008 period than the matching efficiencies for the other two groups, lower three panels of Figure 3. Nevertheless, there is a substantial comovement between the type-specific matching efficiencies of the three groups, which suggests the presence of a common component.

Comparing the model with time-varying aggregate and type-specific matching efficiencies, Model 3, to the baseline Model 1, we find only small differences for the estimated transition elasticities and the estimated time paths for aggregate matching efficiency, Columns (1) and (3) of Table 2.A, and the top panel of Figure 3. Relatively small movements in type-specific matching efficiencies achieve some marginal improvement over the model with an aggregate matching efficiency only, but otherwise differences in search effort elasticities across types are enough to capture changes in relative transition rates.

Comparing the estimates across the three alternative models of aggregate and type-specific matching efficiencies with variable search effort we would argue that the variation of the type-specific transition rates is well captured by movements in aggregate matching efficiency together with differential endogenous type-specific responses to movements in the aggregate transition rate. Through the lens of the aggregate matching function with heterogeneity and endogenous search effort it means that there is little exogenous movement in the type-specific transition rates beyond what is captured by the type-specific search effort and an aggregate exogenous shock.

5.3 Alternative search pool definition

We now estimate models of heterogeneous employment transition rates with variable search effort for our two alternative definitions of the search pool, unemployed by reason of unemployment and non-employed by labor force status and gender. Estimated transition elasticities are displayed in Table 2, panels B and C, for all three model specifications. and smoothed posteriors of the aggregate matching efficiencies for Model 1 are displayed in Figure 4.[16] For both definitions of the search pool we continue to find evidence in favor of different transition elasticities across types, that is, variable search effort. We also find declines of the aggregate matching efficiency in the years following the Great Recession that are comparable to what we find for the baseline search pool of unemployed by duration.

Comparing the decomposition of unemployment by reason with the previous decomposition by duration, we find that the characteristics of those who claim to have been laid off temporarily are similar to those who report a short unemployment duration: their employment transition rates are higher and less cyclically sensitive than for the other groups. For Model 1 with aggregate matching elasticity only, this is reflected in their transition elasticities being lower than for the other groups, Table 2.B, Column (1). Similar to the baseline search pool definition, the differences between estimated transition elasticities of different groups are less pronounced for Model 2 with type-specific matching efficiencies only, Table 2.B, Column (1).

The second alternative definition of the search pool includes all non-employed and differentiates between those that are actively engaged in search and those that are not. For this broader definition of the search pool, the groups with the higher employment transition rates tend to be more cyclically sensitive. This is the opposite of what we see for the two previous search pool definitions which cover only the unemployed. The differences in cyclical sensitivity of the transition rates are so large across groups that they show up as differences in estimated transition elasticities for all specifications of aggregate and type-specific efficiencies, Table 2.C.

The qualitative features of the estimated aggregate matching efficiency are very similar for the different search pool definitions, Figure 4. All of them are characterized by a decline of aggregate matching efficiency in the years following the Great Recession that is exceptional relative to the full sample. Compared with the baseline search pool of unemployed by duration, the two alternative search pool definitions indicate more volatility of the aggregate matching efficiency.

[16] The Appendix contains a complete listing of results for the two alternative search pool definitions similar to the case of the unemployment search pool by duration.

6 Aggregate matching efficiency and the cyclicality of search effort

In this section, we decompose the observed average employment transition rate in the economy into the components due to the aggregate matching efficiency, the aggregate labor market tightness, and the search effectiveness. The first two components are direct effects while the search effectiveness contains an indirect effect of the aggregate matching efficiency and the aggregate labor market tightness via endogenous search effort.

The average employment transition rate is given by

$$\ln \overline{\lambda_t} = \underbrace{\alpha \ln \theta_t}_{\substack{\text{measured} \\ \text{tightness}}} + \underbrace{\kappa_t}_{\substack{\text{agg matching} \\ \text{efficiency}}} + \underbrace{(1-\alpha)\ln \bar{\rho}_t(\cdot)}_{\substack{\text{heterogeneity and} \\ \text{exog+endog search effectiveness}}}. \tag{14}$$

The value of matching function elasticity, α, matters for the decomposition of the average transition rate. In the standard matching function with homogenous search input, larger a implies larger contribution of θ and thus smaller contribution of aggregate matching efficiency, $\overline{\kappa}_t$,

$$\overline{\kappa}_t = \ln \overline{\lambda_t} - \alpha \ln \theta_t. \tag{15}$$

In contrast, with heterogeneous inputs and endogenous effort α not only controls the contribution of θ but also the contribution and sign of the average search effort, $(1-\alpha)\ln \bar{\rho}_t(\cdot)$. The average transition rate in (14) can be rewritten as

$$\ln \overline{\lambda_t} = \alpha\left(\ln \theta_t + \widehat{\kappa_t}\right) + (1-\alpha)\ln \bar{\rho}_t(\cdot),$$

where $\kappa_t = \alpha\widehat{\kappa_t}$ and $\widehat{\kappa_t}$ are independent of α. That is, larger α implies larger movements in $\alpha\widehat{\kappa_t}$.

Without additional information beyond data on type-specific transition rates and the vacancy-unemployment ratio, the model cannot separately identify the importance of vacancies versus job seeker input in the production of hires (i.e., matching function elasticity, α), from the co-movement of the search effort with the aggregate transition rate (i.e., search effort elasticity η_i). Instead, the data allow identifying the elasticity of type-specific transition rates with respect to the aggregate transition rate (ϕ_i), which is a nonlinear function of α

and η_i. Using ϕ_i, we can compute search effort elasticties η_i for a given value of α as

$$\eta_i = \phi_i \frac{1-\alpha}{\alpha} - 1.$$

Without any additional data, the model can accommodate low α with pro-cyclical search effort ($\alpha < \frac{\phi_i}{1+\phi_i}$ and $\eta_i > 0$) as well as high α with counter-cyclical search effort ($\alpha > \frac{\phi_i}{1+\phi_i}$ and $\eta_i < 0$).

We can now construct the decomposition of the average employment transition rate in eq. (14), contingent on the matching function elasticity $\alpha = 0.2$, when search effort is pro-cyclical for all three unemployment groups by duration, and the decomposition contingent on the matching function elasticity $\alpha = 0.5$, when search effort is counter-cyclical for all three unemployment groups by duration. Note that for this decomposition we calculate the aggregate matching efficiency, κ, and not the identified matching efficiency, κ/α. By construction, the larger is α the larger is the contribution of labor market tightness to variations in the average transition rate (the red line). For the standard matching function with homogeneous inputs this means that for a larger α the implied measure of matching efficiency is smaller. However, with heterogeneous inputs to the matching function and endogenous search effort, average search quality is pro-cyclical for low values of α and counter-cyclical for larger values of α (the green line). For low values of α pro-cyclical average search quality then more than compensates for the small impact of market tightness and matching efficiency. Conversely, for high values of α counter-cyclical average search quality requires a larger contribution coming from matching efficiency than is suggested by the large contribution from market tightness. Furthermore, average search effort changes because the composition of the search pool changes, relative search efforts of the different types in the search pool change, and a residual interaction effect, i.e.,

$$\ln \bar{\rho}_t = \underbrace{\ln \sum_i \frac{\overline{u_{it}}}{u_t} \rho_{it}}_{\text{fixed composition}} + \underbrace{\ln \sum_i \frac{u_{it}}{u_t} \overline{\rho_{it}}}_{\text{varying composition}} + \underbrace{\varrho_t}_{\text{residual}}.$$

In Figure 4, Panels A and B, display a sequence of decompositions of the aggregate employment transition rate, $\ln \overline{\lambda_t}$ (the solid black lines), for low, $\alpha = 0.2$, and high, $\alpha = 0.5$, values of the matching efficiency, respectively. Subtracting the contribution of market tightness, the red lines, yields the standard estimate of matching efficiency that assumes a homogeneous search pool, the dashed blue line. Allowing for changes in average search quality due to composition effects, the dashed green line, yields a refinement of the matching efficiency estimate, the dotted blue line. For the reasons given above accounting for the composition changes reduces the estimated volatility of matching efficiency for either value of α. Finally, accounting for changes in search effort and the composition of the search pool, the solid green, yields an estimate of structural matching efficiency, solid blue line. As just explained accounting for changes in search effort and composition effects can make structural matching efficiency less volatile than accounting for composition effects only when search effort is pro-cyclical, Figure 4 Panel A with $\alpha = 0.2$. The converse is true when search effort is counter-cyclical, Figure 4 Panel B with $\alpha = 0.5$.

6.1 Existing empirical evidence on the cyclicality of search effort

We show above that data on type-specific transition rates and the vacancy-unemployment ratio cannot separately identify α from search effort elasticities, η_i. To separately identify α, additional data are required. Such data might come from the evidence on the cyclical behavior of search effort.

While evidence on the cyclicality of search effort is relatively scarce, the existing studies suggest that search effort is countercyclical. Specifically, using the Current Population Survey data, Shimer (2004) finds that the number of search methods used by the unemployed increases during the 2001 recession. Using the Current Population Survey and the data from the Annual Time Use Survey (ATUS), Mukoyama, Patterson and Sahin (2014) conclude that the time spent on search is countercyclical. Faberman and Kudlyak (2014) find that the number of applications sent by a job seeker per week on an online job board is significantly higher in metropolitan areas with more slack labor markets. Using ATUS, DeLoach and Kurt (2013), however, find search appears to be acyclical. Specifically, they argue that workers reduce their search in response to deteriorating labor market conditions, but these effects are offset by the increase in search effort due to declines in household wealth. Gomme and Lkhagvasuren (2015) argue that search effort of an individual worker is pro-cyclical and that the measured counter-cyclical average search effort is due to a composition effect.

Taking as given the evidence that search effort is countercyclical implies that the matching function elasticity is closer to 1 than to 0. That is, vacancies play an important role in the production of new hires in the matching function framework and the countercyclical search effort exacerbates changes in the job seeker input. Consequently, large changes in the aggregate matching efficiency are required to describe changes in the average transition rate.

7 Conclusion

Modelling search effort as a constant elasticity function of the aggregate transition rate, we find a substantial decline of the aggregate matching efficiency after 2007, even after accounting for endogenous search effort. Endogenous search effort accounts well for variation in relative transition rates of different groups of job seekers. The data are consistent with both pro- and countercyclical search effort. Without additional data, we can only make statement about cyclicality conditional on the elasticity of the matching function. We find counter-cyclical effort for a wide range of the elasticity of the matching function with respect to vacancies, $(1/3, 1)$. Counter-cyclical effort dampens transition rate volatility and larger volatility in aggregate matching efficiency is required to compensate for that, in contrast with the standard model that ignores endogenous search effort.

A Endogenous Search Effort

A simple modification of the basic matching model allows for variation of individual search effort that is related to the aggregate employment transition rate, e.g. Pissarides (2000) or recently Gomme and Lkhagvasuren (2015). Let U and W denote the value of being unemployed and employed, respectively. Then, the return on unemployment is

$$rU = b - c(s) + \rho\lambda (W - U),$$

where r is the rate of time discount and b is the flow return from unemployment. Devoting effort to search increases the rate at which the worker becomes employed but it comes at a cost, $c(s)$. Determining the optimal choice of effort is a well-defined problem if the effort cost is an increasing convex function of effort. For simplicity, assume that the cost function is of the constant elasticity variety,

$$c(s) = s_0 s^v \text{ with } \nu > 1.$$

The first order condition yields the optimal search effort as

$$s = \lambda^{1/(\nu-1)} \left[(W - U) / (s_0 \nu)\right]^{1/(\nu-1)}, \tag{16}$$

that is, search effort is a constant elasticity function of the aggregate transition rate.

For the basic matching model, search effort is an increasing function of the aggregate transition rate: as the marginal benefit from search increases, the worker will devote more effort to search, yielding pro-cyclical search effort. We propose to estimate a reduced form expression that relates the search intensity for each type to the aggregate matching rate, and a type-specific persistent component, z_{it}. The elasticity of search effort with respect to the aggregate transition rate is η_i. We do not impose any restrictions on search effort to be pro- or counter-cyclical, but we do impose the restriction that the type transition rate is a non-decreasing function of the aggregate transition rate, $\eta_i \geq -1$.

References

[1] Ahn, Hie Joo , and James Hamilton. 2015. "Heterogeneity and Unemployment Dynamics," Working Paper, University of San Diego.

[2] Alvarez, Fernando, Katarina Borovickova, and Robert Shimer. 2016. Decomposing Duration Dependence in a Stopping Time Model, mimeo.

[3] Barnichon, Regis. 2010. Building a composite help-wanted index. Economics Letters 109, 175-178.

[4] Barnichon, Regis, and Andrew Figura. 2015. Labor Market Heterogeneity and the Aggregate Matching Function. American Economic Journal: Macroeconomics, Vol. 7(4): 222-49.

[5] Baxter, Marianne, and Robert G. King. 1999. Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. Review Of Economics And Statistics. Vol. 81 (4): 575-593.

[6] Christensen, Bent Jesper, Rasmus Lentz, Dale T. Mortensen, George R. Neumann, and Axel Werwatz. 2005. On-the-job Search and the Wage Distribution. Journal of Labor Economics, Vol. 23 (1): 31-58.

[7] Costain, James S., and Reiter, Michael. 2008. Business Cycles, Unemployment Insurance, and the Calibration of Matching Models. Journal of Economic Dynamics and Control, Vol. 32(4): 1120-1155.

[8] Davis, Steven J. 2011. Comment on "Job Search, Emotional Well-Being, and Job Finding in a Period of Mass Unemployment," by Alan Krueger and Andreas Mueller, Brookings Papers on Economic Activity: 58–81.

[9] Davis, Steven J., R. Jason Faberman, and John C. Haltiwanger. 2013. The Establishment-Level Behavior of Vacancies and Hiring. Quarterly Journal of Economics 128 (2): 581-622.

[10] DeLoach, Steve, and Mark R. Kurt. 2013. Discouraging Workers: Estimating the Impact of Macroeconomic Shocks on Search Intensity of the Unemployed. Journal of Labor Research, Vol. 34(4): 433-454.

[11] Elsby, Michael W.I., Ryan Michaels, and David Ratner. 2015. The Beveridge Curve: A Survey. Journal of Economic Literature: Vol. 53 (3): 571-630.

[12] Faberman, Jason R., and Marianna Kudlyak. 2014. The Intensity of Job Search and Search Duration. Richmond FED WP No. 14-12.

[13] Frazis, Harley J., Edwin L. Robinson, Thomas D. Evans, and Martha A. Duff. 2005. Estimating Gross Flows Consistent with Stocks in the CPS. Monthly Labor Review, Vol. 128 (9): 3-9.

[14] Fujita, Shigeru, and Giuseppe Moscarini. 2013. "Recall and Unemployment," NBER Working Paper No. 19640.

[15] Gomme, Paul, and Damba Lkhagvasuren. 2015. Worker Search Effort as an Amplification Mechanism. Journal of Monetray Economics, Vol. 75 (October): 106–122.

[16] Hall, Robert E., and Sam Schulhofer-Wohl. 2015. Measuring Matching Efficiency with Heterogeneous Jobseekers. NBER Working Paper No. 20939.

[17] Hornstein, Andreas. 2012. Accounting for Unemployment: The Long and Short of It. FRB Richmond WP No. 12-07.

[18] Kroft, Kory, Fabian Lange, Matthew J. Notowidigdo, and Lawrence Katz. 2015. Long-Term Unemployment and the Great Recession: The Role of Composition, Duration Dependence, and Non-Participation. Journal of Labor Economics, Vol. 34 (S1): S7-S54.

[19] Krueger, Alan, and Andreas Mueller. 2011. Job Search, Emotional Well-Being, and Job Finding in a Period of Mass Unemployment, Brookings Papers on Economic Activity, Vol. 42(1): 1-81.

[20] Lise, Jeremy. 2013. On-the-job Search and Precautionary Savings. Review of Economic Studies (80): 1086–1113.

[21] Madrian, Brigitte C., and Lars John Lefgren. 1999. A Note on Longitudinally Matching Current Population Survey (CPS) Respondents. NBER Working Paper No. t0247.

[22] Mukoyama, Toshihiko, Christina Patterson, and Aysegul Sahin. 2014. Job Search Behavior over the Business Cycle. Federal Reserve Bank of New York Staff Report No. 689.

[23] Petrongolo, Barbara, and Christopher A. Pissarides. 2001. Looking into the Black Box: A Survey of the Matching Function. Journal of Economic Literature 39, 390-431.

[24] Pissarides, Christopher A.. 2000. Equilibrium Unemployment Theory. The MIT Press.

[25] Polivka, Anne E., and Stephen M. Miller. 1998. The CPS after the Redesign: Refocusing the Economic Lens. In "Labor Statistics Measurement Issues, " John Haltiwanger, Marilyn E. Manser and Robert Topel, editors. University of Chicago Press.

[26] Şahin, Ayşegül, Joseph Song, Giorgio Topa, and Givanni L. Violante. 2014. Mismatch Unemployment. American Economic Review, Vol. 104(11): 3529-64.

[27] Sedlacek, Petr. 2016. The Aggregate Matching Function and Job Search from Employment and Out of the Labor Force. Review of Economic Dynamics, Vol. 21(7): 16-28.

[28] Shimer, Robert. 2004. Search Intensity. Mimeo, downloadable from https://sites.google.com/site/robertshimer/research/workingpapers.

[29] Shimer, Robert. 2012. Reassessing the Ins and Outs of Unemployment. Review of Economic Dynamics 15, 127-148.

[30] Veracierto, Marcelo. 2011. Worker Flows and Matching Efficiency. Federal Reserve Bank of Chicago Economic Perspectives v.35 4th Quarter, 147-169.

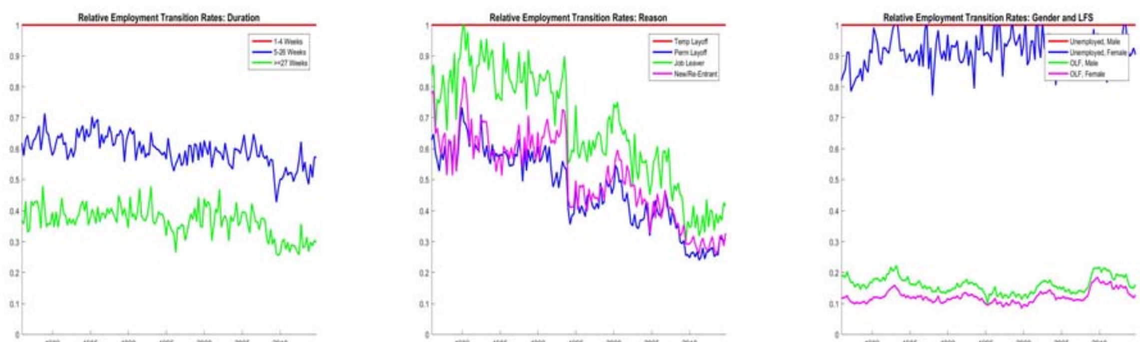
[31] Watson, Mark M. 1996. Comment on "Is Seasonal Adjustment a Linear or Nonlinear Data Filtering Process," Journal of Business and Economic Statistics, Vol. 14, No. 3.

Figure 1: Transition Rates to Employment and Search Pool Composition

A. Duration B. Reason C. LFS and Gender

1. Transition Rates

2. Relative Transition Rates



3. Search Pool Composition

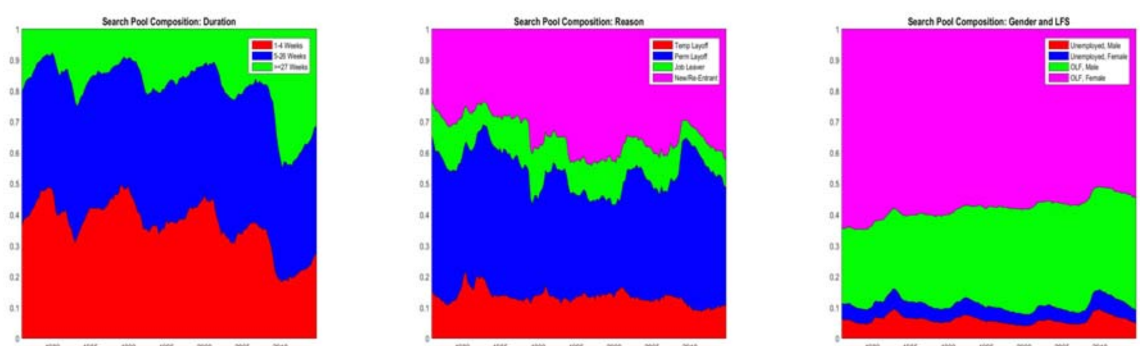
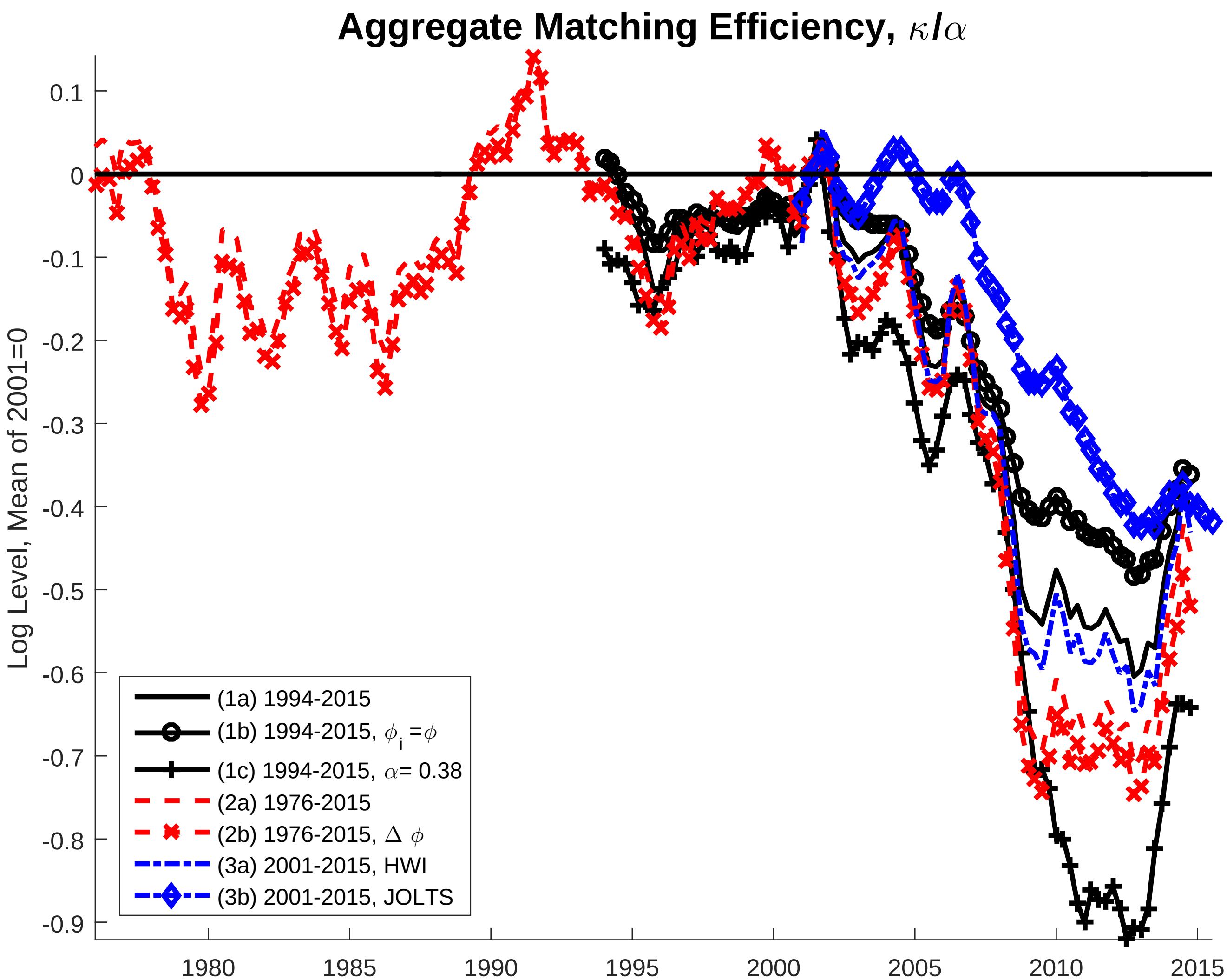
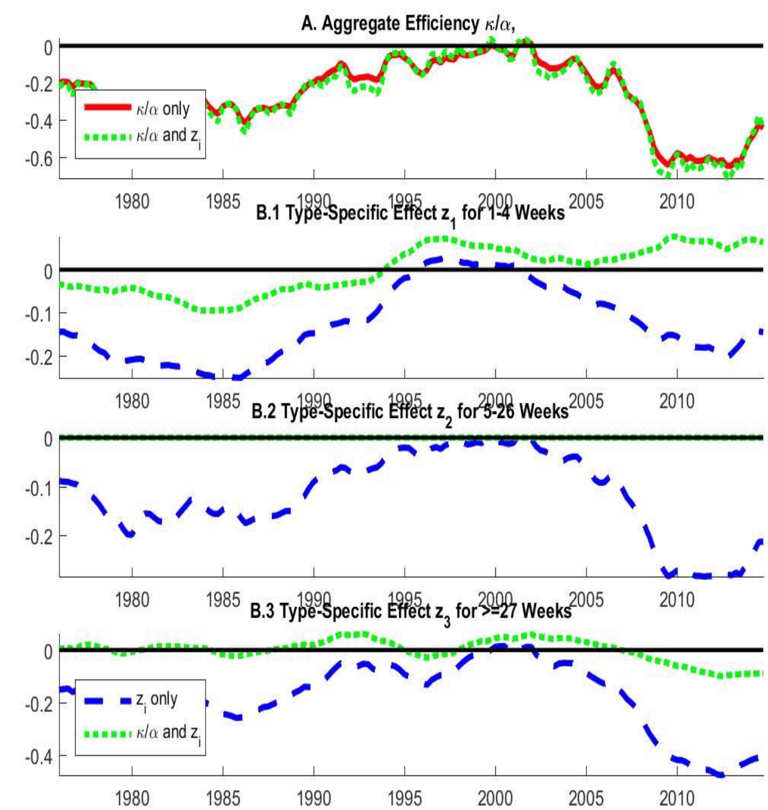


Figure 2: Identified Aggregate Matching Efficiency from the Model with Aggregate Matching Efficiency and Variable Search Effort, Unemployment by Duration



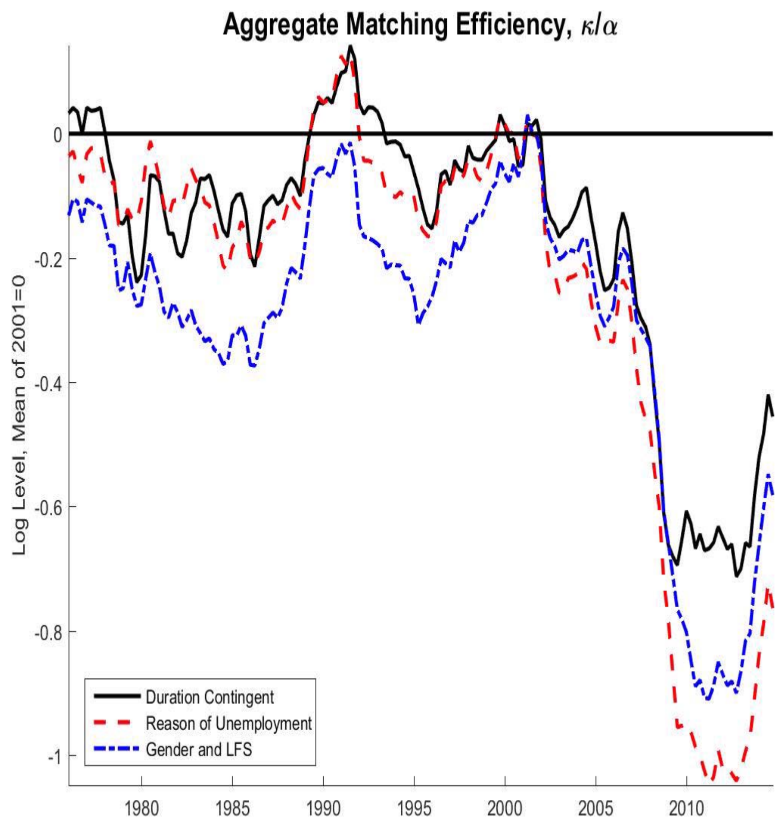
Note: Lines (1a)-(3b) show smoothed posterior estimate of $\widehat{\kappa}_t$ from a model with a random walk in aggregate matching efficiency (Model 1 in the text). Lines (1a), (1b), (2a)-(3b) show the estimates from the respective columns in Table 1, see note to Table 1 for details. Line (1c) shows the constructed $\widehat{\kappa}_t$ for the standard matching function without heterogeneity and $\alpha = 0.38$; see Section 5.3.3 for details. All efficiencies are normalized to zero in 2001Q1.

Figure 3: Aggregate and Type-Specific Matching Efficiencies from Alternative Models of Stochastic State, Unemployment by Duration



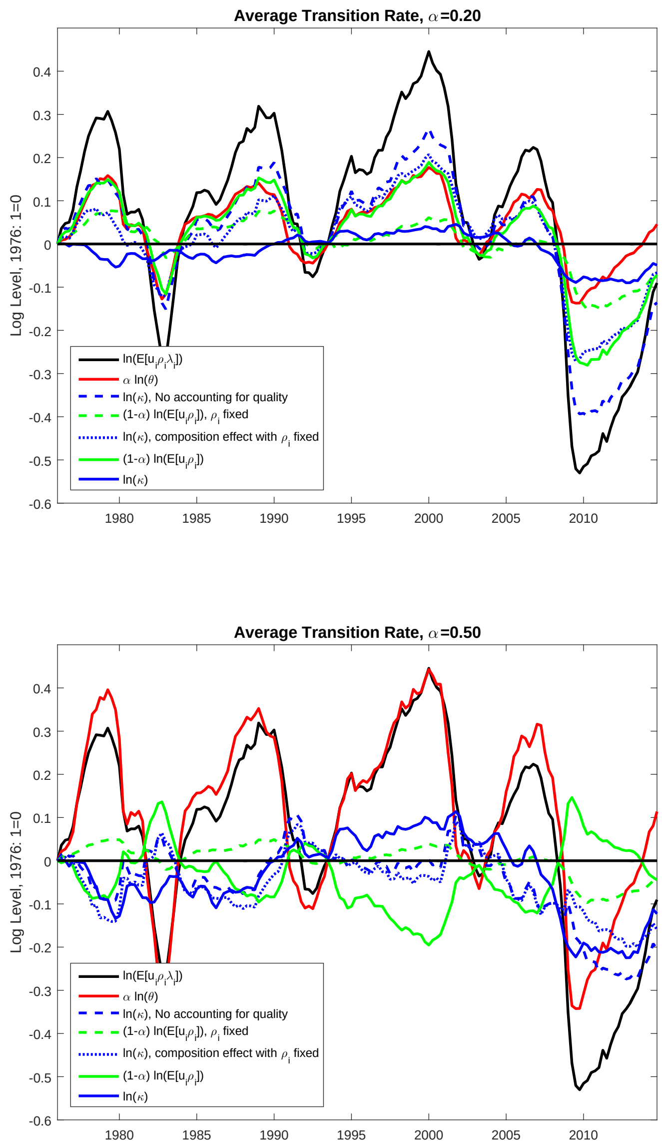
Note: Smoothed posterior estimates of the aggregate and type-specific matching efficiency from Models 1-3 defined in the text. The specification follows column (1a) in Table 1. All efficiencies are normalized to zero in 2001Q1.

Figure 4: Aggregate Matching Efficiencies from Alternative Search Pool Definitions



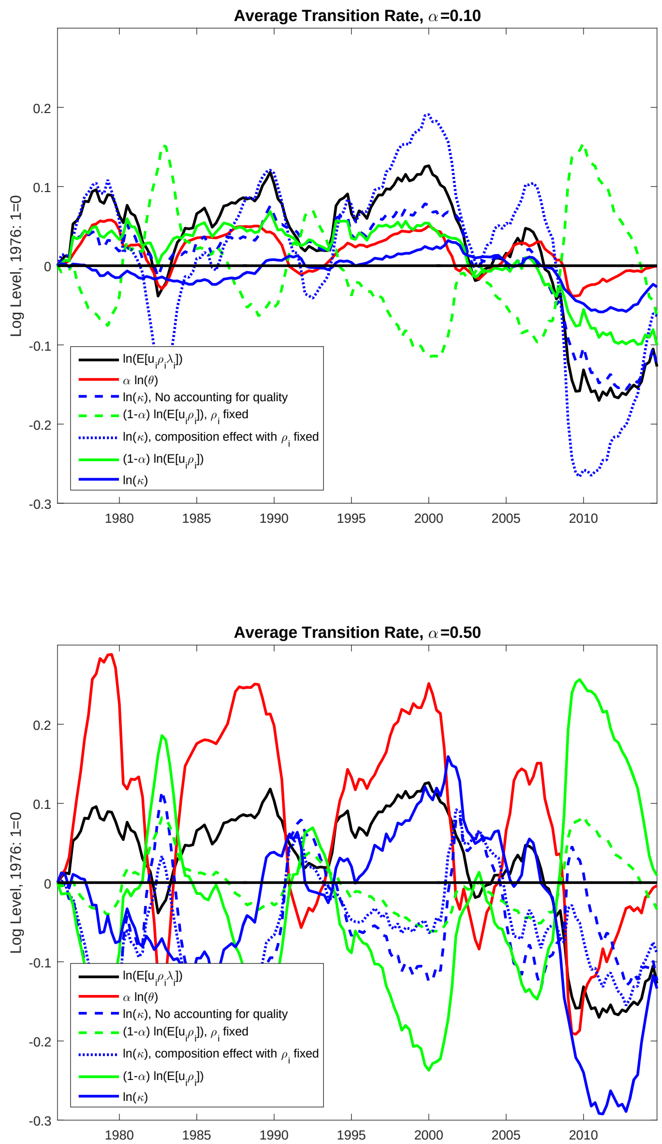
Note: Smoothed posterior estimates of the aggregate matching efficiency from a model with a random walk in aggregate efficiency (Model 1 in the text) for search pool defined as (1) unemployed by duration, (2) unemployed by reason, or (3) LFS by gender. The specification follows column (2a) Table 1. All efficiencies are normalized to zero in 1994Q1.

Figure 5: Decomposition of the Average Transition Rate, Unemployment by Duration



Note:

Figure 6: Decomposition of the Average Transition Rate, LFS by Gender



Note:

Table 1: Estimates from the Model with Aggregate Matching Efficiency and Variable Search Effort, Unemployment by Duration

	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
$E[\kappa\vert 0]$	-2.401	-1.914	-2.548	-2.838	-2.749	-2.881
	(1.416)	(0.809)	(0.912)	(0.218)	(0.938)	(6.912)
c_2	-0.325	-0.572	-0.349	-0.314	-0.216	-0.212
	(0.245)	(0.008)	(0.051)	(0.039)	(0.417)	(0.415)
c_3	-0.587	-1.076	-0.682	-0.451	-0.423	-0.461
	(0.091)	(0.014)	(0.058)	(0.171)	(0.421)	(0.839)
φ_1	0.405	0.611	0.358	0.303	0.355	0.377
	(0.021)	(0.073)	(0.024)	(0.036)	(0.107)	(0.111)
φ_2	0.562	-	0.483	0.429	0.543	0.579
	(0.139)	-	(0.047)	(0.072)	(0.067)	(0.070)
φ_3	0.717	-	0.579	0.628	0.702	0.730
	(0.081)	-	(0.029)	(0.087)	(0.092)	(0.584)
$\sigma_{\varepsilon 1}$	0.048	0.065	0.045	0.045	0.049	0.050
	(0.009)	(0.006)	(0.003)	(0.003)	(0.006)	(0.006)
$\sigma_{\varepsilon 2}$	0.038	0.042	0.036	0.036	0.039	0.038
	(0.006)	(0.004)	(0.003)	(0.003)	(0.005)	(0.005)
$\sigma_{\varepsilon 3}$	0.088	0.110	0.086	0.079	0.085	0.091
	(0.017)	(0.009)	(0.005)	(0.006)	(0.010)	(0.009)
σ_{κ}	0.054	0.036	0.060	0.063	0.069	0.038
	(0.017)	(0.012)	(0.012)	(0.013)	(0.019)	(0.014)
λ_{adj1}	-	-	0.156	0.154	-	-
	-	-	(0.025)	(0.027)	-	-
λ_{adj2}	-	-	0.069	0.066	-	-
	-	-	(0.019)	(0.031)	-	-
λ_{adj3}	-	-	0.055	0.050	-	-
	-	-	(0.031)	(0.050)	-	-
φ_1^{hU}	-	-	-	0.017	-	-
	-	-	-	(0.009)	-	-
φ_2^{hU}	-	-	-	0.015	-	-
	-	-	-	(0.012)	-	-
φ_3^{hU}	-	-	-	-0.025	-	-
	-	-	-	(0.021)	-	-
Log Likehood	346.910	305.460	664.070	675.045	224.958	243.407
No Obs	84	84	156	156	56	59
No Par	10	8	13	16	10	10

Note: Col. (1a) - (3a) display parameter estimates of a model with a random walk for aggregate matching efficiency using the HWI as a measure of vacancies. Col. (1a) displays estimates for the sample period 1994-2015, and col. (1b) imposes the restriction that ϕ_i is the same for all types. Col. (2a) displays estimates for the sample period 1976-2015 with adjustment of the search pool data prior to the 1994 CPS revision based on Polivka and Miller (1998) and a fixed estimated structural change in measured transition rates λ_i, and col. (2b) introduces an unemployment rate contingent structural break in the parameter ϕ_i as explained in the text. Col. (3a) re-estimates the model from col. (1a) for the sub-sample 2001-2015 with HWI for

Table 2: Identified Matching Elasticities

	(1)	(2)	(3)
A. Unemployment by Duration			
1-4 weeks	0.381	0.405	0.368
	(0.037)	(0.036)	(0.174)
5-26 weeks	0.506	0.538	0.488
	(0.037)	(0.075)	(0.294)
27 weeks and above	0.604	0.523	0.521
	(0.038)	(0.108)	(0.138)
B. Unemployment by Reason			
Laid off temporarily	0.116	0.587	0.498
	(0.018)	(0.061)	(0.074)
Laid off permanently	0.563	0.719	0.634
	(0.053)	(0.103)	(0.069)
Left job	0.571	0.668	0.584
	(0.056)	(0.069)	(0.062)
LF entrant	0.516	0.461	0.451
	(0.049)	(0.070)	(0.051)
C. Unemployment and OLF by Reason			
Unemployed, male	0.686	0.731	0.588
	(0.058)	(0.064)	(0.082)
Unemployed, female	0.686	0.588	0.589
	(0.059)	(0.058)	(0.079)
OLF, male	0.261	0.218	0.195
	(0.028)	(0.043)	(0.041)
OLF, female	0.215	0.193	0.189
	(0.022)	(0.042)	(0.033)

Note: The columns display the estimated identified matching elasticities ϕ of a model with (1) a random walk for aggregate matching efficiency only, (2) random walks for type-specific matching efficiencies, and (3) a random walk for aggregate matching efficiency and stationary AR(1)'s for type-specific matching efficiencies. All models are estimated for specification (2a) in Table 1, that is, the 1976-2015 sample period with appropriate adjustment for the 1994 CPS revision. See note to Table 1 for details.