Brand Reallocation and Market Concentration

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***** PRELIMINARY*****

Abstract

This paper connects the distribution of products across firms to aggregate brand innovation and market concentration. To do so, we employ a dataset of the universe of USPTO trademarks to document key facts regarding the lifecycles of products: the trademark market is dynamic with a lot of exit and entry and trademark reallocation across firms; Small firms are more likely to create new products, while big firms are more likely to poach existing products from other firms; Product reallocation is associated with an increase in the profit margin of buying firms. We introduce an endogenous growth model with product creation, product transfer, and variable markups that embeds these empirical findings among others from the market for trademarks. Using the estimated model, we quantify the net impact of product reallocation on the growth of real consumption. We find that the reallocation of brands increases market concentration, but policies that reduce reallocation also induce a decline in brand creation and efficiency.

Key Words: Markups, Trademarks, Firm Dynamics, Reallocation

JEL Code: D22, D43, L11, L13, L22

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1 Introduction

Large firms are multi-product in nature. Further, the product scope of firms explains a large share of sales variation (Hottman et al., 2016), and thus the market power of large firms. However, the market for products is quite dynamic; products are constantly created, destroyed, and transferred across firms. As the creation of new varieties and the distribution of markups both play a central role in economic growth, it is crucial to understand the dynamics of the distribution of brands and the implications on efficiency.

This paper connects the role of multi-product firms to the role of product creation and transfer across firms. We do so by integrating a dataset on the universe of trademarks in the US Patent and Trademark Office (USPTO) to a quantitative endogenous growth model of product creation and exchange. We open by documenting several facts regarding the lifecycle of brands employing the trademark data, with a focus on the ownership of products across firms. We then build a model with product creation and transaction that embeds these facts. Using the calibrated model, we quantify how transactions of brands affect the distribution of firm-level sales and the consequences of brand exchange on the efficiency of the decentralized equilibrium.

We open with five facts about the distribution of trademarks across firms and its reallocation:

Fact 1 *The firm size distribution in trademarks is i) heavily skewed (99th/50th ratio is 50), and ii) the market experiences lots of entry and exit; approximately 12% of the active firms each year are entrants, while 10% of active firms will exit next period.*

Fact 2 *The market for trademarks i) contains a lot of trademark reallocation across firms, ii) trademark reallocation is a larger share of fast-growing firms, iii) the largest firms are built the most by reallocation.*

Fact 3 *There is rising dynamism in the trademark market over time.*

Fact 4 *When a firm purchases a brand, the firm's sales and costs both go up with an increase in the markup.*

Fact 5 *When a firm sells a brand, there is a weak decline in markup.*

We connect these facts to a model of endogenous growth with product creation. Firms are born small. During their lifecycles, they can grow the number of brands through creating new brands or buying brands. They decline in size through brand destruction or brand sale. Expanding product scope creates value to the firm through two margins: (1) they increase the total sales of the firm holding markups unchanged; (2) it enables firms to charge a higher markup on all products due to the reduced competition from their own products.

The transactions of product ownership are organized through directed search. A firm that looks to buy new products posts vacancies with a constant cost, which captures either the fee to intermediaries or the regulatory burdens from antitrust bodies. Buyers specify the price to be paid when matched and selling firms direct search to the vacancies. With standard assumptions on the matching process, both sellers and buyers tradeoff the speed of trading against the terms of trade. The transaction market clears when the total sales in the market equalize the total purchase. The cost of posting vacancies is the focus of our comparative statics. By increasing this parameter, we ask: *How does tightening regulation of brand transactions affect the efficiency of decentralized equilibrium?*

The existence of variable markups across firms in the decentralized equilibrium creates two wedges. First, firms that are more productive or have large product scope charge higher markups and thus produce too little compared to the efficient allocation. We refer to this as the static distortion; Second, firms do not produce to an efficient scale after they create new products and thus under-innovate. We refer to this as the dynamic distortion. The reallocation of products interacts with these distortions and presents a tension on the tradeoff between optimal allocation and market concentration.

This paper then connects the model results to the quantitative nature of product markets and product transactions. We connect the model to the main facts to back out the importance of the market for brand transactions. First, we match the firm size distribution, entry and exit rates, and sources of firm growth and decline. We use these moments to back out the costs of transaction, registration, and the rate of destruction in the economy.

With the underlying parameters, we focus on policy counterfactuals that shift the cost of transferring brands. These counterfactuals can be thought of as similar to antitrust policy (strict inducement against buying brands) or taxes on intellectual property reassignment. We find that even though larger firms induce higher markups, the overall efficiency gain from both the increase in overall creation and optimal allocation of trademarks outweighs the efficiency loss.

To discuss an illustrative example of the role of trademarks with multi-product firms, we focus on the expansion of a classic consumer goods product firm, Procter & Gamble. As of 2010, Procter & Gamble (P&G) holds 1700 trademarks, which represent a host of well-known brands. Many of these brands were not created by P&G. 40% of P&G brands have come initially from other firms and were transferred over. This includes famous brands such as *Pantene*, *NyQuil*, and *Crest*.[1]

P&G provides an example of potentially efficient expansion, as they are known as a company that can build and market brands. However, this connects a rising concern on the role of concentration in markups. As firms get better and engage in acquisitions of smaller brands and firms, this both provides an efficiency gain through an optimal allocation of brands and an efficiency loss through the relationship between concentration and markups. This is a key question within this paper.

The paper is structured as follows. The rest of this section reviews the literature. Section 2 introduces the USPTO Trademark Dataset and merged Compustat data. Section 3 documents the key facts that frame our investigation. Section 4 introduces a novel model of product creation and transfer with variable firm productivity and variable markups. Section 5 builds a bridge from the empirical facts to the model to investigate the role of shutting down specific channels of product exchange. We use this to study policy counterfactuals. Section 6 concludes.

Related Literature

This paper builds on and contributes to several literatures: the study of endogenous growth and firm dynamics, the study of multi-product firms and their implications, and the study of firm market power.

The introduction of new products is a central element of economic growth. This is a bedrock component of much of modern endogenous growth theory (Romer, 1990; Grossman and Helpman, 1991a). Product creation has also been noted as a key empirical component of both economic growth and gains from trade (Bils and Klenow, 2001; Broda and Weinstein, 2006). Further, the ability of individuals to exchange products allows for products to expand into new markets and

[1]We note in Appendix A the example of P&G. Figure A1 shows some of the many brands associated with P&G. Figure A2 shows the stock of brands P&G holds, which saw a large expansion in the late 1990s through the 2000s.

may spur upstream innovation (Eaton and Kortum, 1996). The quantitative model in this paper is based on the endogenous product creation model developed by Grossman and Helpman (1991b) and Atkeson and Burstein (2010). We contribute to this literature by documenting the size and direction of the reallocation of products among firms and provide a framework to understand the effects of this reallocation on productivity in an environment with variable markups.

Intellectual property transfer plays an important role in the distribution of technologies and products across firms. Our paper is thus related to Akcigit et al. (2016), who study the effect of patent transfers on productivity growth, where the gains from trade in patent transfers come from matching firms to technologies. Shi and Hopenhayn (2017) study how the appropriability of innovation, e.g. the ability to license or sell intellectual property induces upstream incentives. This is related to Abrams et al. (2019), who illustrate the contentious role of middlemen and intellectual property transfers. We focus this paper on the demand side by documenting facts of trademark transfers; this allows us to turn to variable markups depending on consumer demand and firm market power, adding intellectual property transfer to a framework developed in Peters (2020). This extension enables us to discuss the potential welfare loss of transfers due to the shifting dispersion of markups.

This paper connects the literature on economic growth and intellectual property to the study of multi-product firms. Hottman et al. (2016) study multi-product firms and find the scope of products explains a large share of sales variations across firms, while Berger et al. (2019) apply this reasoning to study the role of market power in the labor market. In this paper, the sources of market power come from oligopolistic competition across firms, which follows Atkeson and Burstein (2008).

Argente et al. (2018, 2020) document there is substantial reallocation of products on the creation and destruction margins. Argente et al. (2021) and Einav et al. (2021) document that the expansion of product sales is largely due to expansion of the customer base. Our discussion is inspired by these theoretical and empirical explorations of large firms. We contribute by studying the incentives of large firms in an environment where the customer base can be reallocated via the transfers of brands.

Building a brand is about building consumer relationships. Academics have noted that advertisement overcomes information frictions (Stigler, 1961), and provides incentives for firms to maintain or build their reputation (Nelson, 1970, 1974). Other papers have suggested that branding is socially wasteful because it inspires a zero-sum spirit and increases barriers to entry (Galbraith, 1958). Bronnenberg et al. (2012) show that because of brand inertia, individuals may stay attached to brands over long time horizons, implying leverage for increasing prices based on consumer goodwill. This paper connects these ideas to the role of brand transactions and explores the multi-product nature of firms.

This paper applies insights from search theory to study the market for trademark exchange. Some previous work has stressed the importance of reallocation and labor market frictions in driving economic growth. For instance, Lentz and Mortensen, 2008 apply a random search framework to uncover the importance of entry, exit, and reallocation in how labor markets interact with firm dynamics. This current paper considers the frictions in the market for intellectual property, applying competitive search theory as developed in Menzio and Shi (2011). More recent work such as Kaas and Kircher (2015) and Schaal (2017) has brought competitive search theory to firm dynamics. Characterizing the incentives of sellers and buyers in a frictional market of brand transfer is complicated, as the incentives depend on the equilibrium distribution of productivity and brand scope. The framework of competitive search enables us to characterize the trading decisions in a tractable manner. The constrained efficiency property also allows us to focus on how the transfer of brands interacts with other distortions in an endogenous growth framework.

A discussion of the reallocation of trademarks naturally connects to a rich empirical literature on firm dynamics. Many researchers have noted a declining reallocation in the economy. For example, the reallocation rate of jobs has been decreasing, and the entry and exit rate of firms has been decreasing (Decker et al., 2014, Davis and Haltiwanger, 2014, and Decker et al., 2020). Our reallocation measure follows the work of Davis and Haltiwanger (1992) and Davis et al. (1996). We show that even during recent periods of diminishing dynamism we also experience a rise in the reallocation of brands. A lot of this reallocation is due to exchanges from small firms to large firms. This connects to work on rising concentration and markups have been studied extensively, both empirically (Barkai, 2020; De Loecker et al., 2020; Traina, 2018) and theoretically (Edmond et al., 2018; Peters, 2020; Akcigit and Ates, 2021).

Lastly, we extend a burgeoning literature on the role of trademarks in marketing and strategy to a macroeconomic context. Graham et al. (2013) provide a general overview of the dataset and provide insights about the uses of trademarks. Schautschick and Greenhalgh (2016), who document the importance of trademarks to firms, review other literature that confirms the growing recognition of the importance of trademarks. Dinlersoz et al. (2018) document the newly available USPTO bulk dataset on trademarks and document facts about trademarks over a firm's life cycle. Heath and Mace (2019) focus the role of trademarks in the strategic interaction of firms. Castaldi (2019) discusses the potential of this rich dataset in providing empirical analogs of a host of subjects in management research.

2 Data

Our project uses two main datasets, the most central of which is the US Patent and Trademark Office (USPTO) Trademark data. We connect this data to CRSP Compustat data on firm sales, firm costs, and industry, which admit studying the dynamics of trademarks across firms and industries, firm response to trademark events, and the holdings of trademarks at each firm.

2.1 USPTO Trademark Data

USPTO Trademark data provides a unique and comprehensive insight into brand-building. Trademarks are a central and dynamic arena of the economy: firms register for trademarks whenever they want their brand protected.

To register for a trademark, a firm must undergo the following process. First, an individual who applies must pay a fee that ranges from $225-$400. Within three months of filing, an examining attorney checks for compliance, and if the application is approved, it "publishes for opposition." After this, there is a 30-day period during which third parties affected by the trademark registration can step forward to file an "Opposition Proceeding" to stop the registration. This process is again evaluated by an examiner. If each aspect passes, the trademark will get filed in "due course."

With a registered trademark in hand, the owner now has *exclusive* rights to use the mark within the sphere of activity designated in the process. For the most part, trademark law also allows the owner to prevent unauthorized use even outside the domain of their products and services. Underlying this law is the principle that consumer confusion should be minimized. If consumer confusion is possible, the trademark owner has a case. However, one can still petition to cancel a trademark and end the exclusive rights of the owner. The petition to cancel often comes from competing firms that think the intellectual property is too broad. Cancellations are a significant share of overall trademark activity. In addition to registration and cancellation, firms exchange a large share of trademarks, which delivers the rights to brand and sell the product.

Trademarks have a long history. Henry III passed the first legislative act concerning trademarks in 1266. In the USPTO dataset, the first registered trademark was granted to Averill Chemical Paint Company in 1870. Since then, there has been massive growth in trademarks through the 20th and 21st century.

The USPTO Trademark data consists of more than 5.3 million unique trademark registrations since 1870. Using a fuzzy match, we identify over 1.3 million unique owners from 1870 to the present. Table 1 provides summary statistics for the dataset. Overall, one million unique firms in our dataset have produced at least one trademark in the past. Lots of firms are active, but the median firm has only two trademarks.

Table 1: Summary statistics on Trademarks from USPTO

	Overall
# unique firms	1.35M
# unique registrations	5.36M
# unique transactions by bundle	915076
# unique transactions by ID	4.46M
# unique cancels	2.12M
99th percentile firm size	83
75th percentile firm size	5
Median firm size	1
Mean firm size	5

Note: Firm size is defined as the number of trademarks within a firm

One striking feature of the data noted in Table 1 is the number of cancellations and transactions. This activity indicates that the market for trademarks is highly contested and dynamic. Cancels either require that other firms are concerned about the territory – many cancellations suggest a competitive market for accruing goodwill, or that a firm is not using its trademark. The contested aspect of the trademark market has been noted in prior literature as an important component of firm dynamics (Fosfuri and Giarratana, 2009).

Figure 1 illustrates the time trend of trademarks and real GDP, showing a natural trend in trademark growth that coincides with output growth. Figure 1a shows how these two growth patterns coincide. Figure 1b normalizes trademark registrations by GDP. We note in particular trademarks are more pro-cyclical than GDP historically (e.g. larger declines in the Great Depression and World War II), and that recently trademark growth has outpaced output growth.

2.2 Compustat/CRSP Data

We link trademarks to Compustat/CRSP data to evaluate how company balance sheet information changes with trademark transactions and registrations. There is no unique firm ID that bridges these two datasets, and, thus, we proceed by string name-matching.

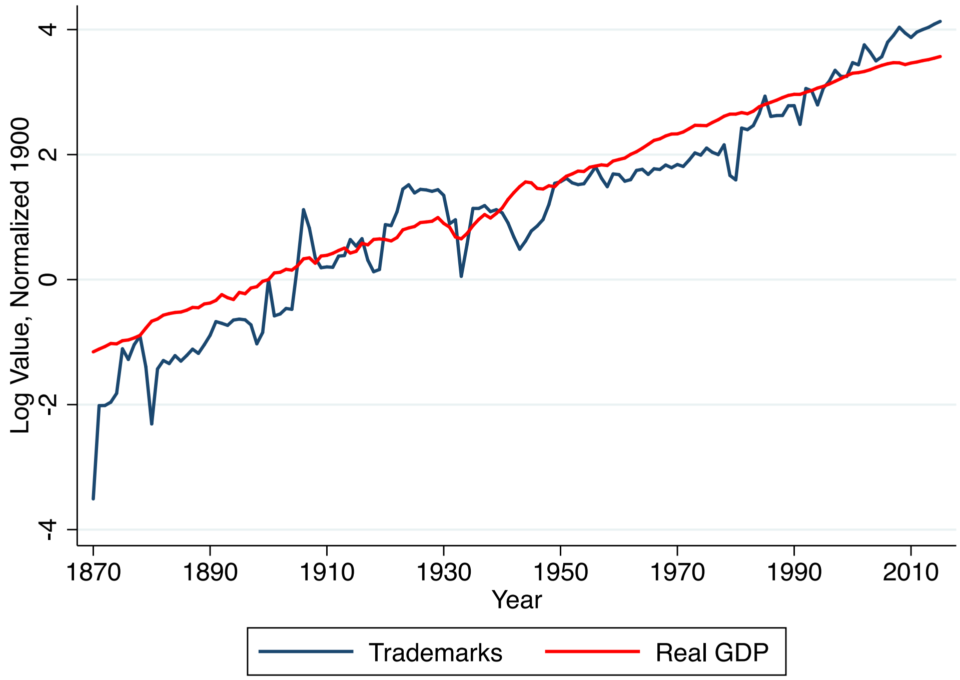
After trimming the data for punctuation and spaces, we perform an exact match on company names. We then supplement this algorithm with a fuzzy match. Following Autor et al. (2016), we build in company location information with an exact match on year.

The merge links 40% of Compustat firms (70% weighted by observations) to firms with at least one trademark in the USPTO data. Table 2 provides summary statistics on the data and merge. The matched trademarking firms are larger on average than the average Compustat firm across many dimensions, including total assets, capital, and sales. They also are over-represented in manufacturing and services. This is to be expected given the public nature of the firms in Compustat, and we use this to analyze the difference between small and large firms over time.

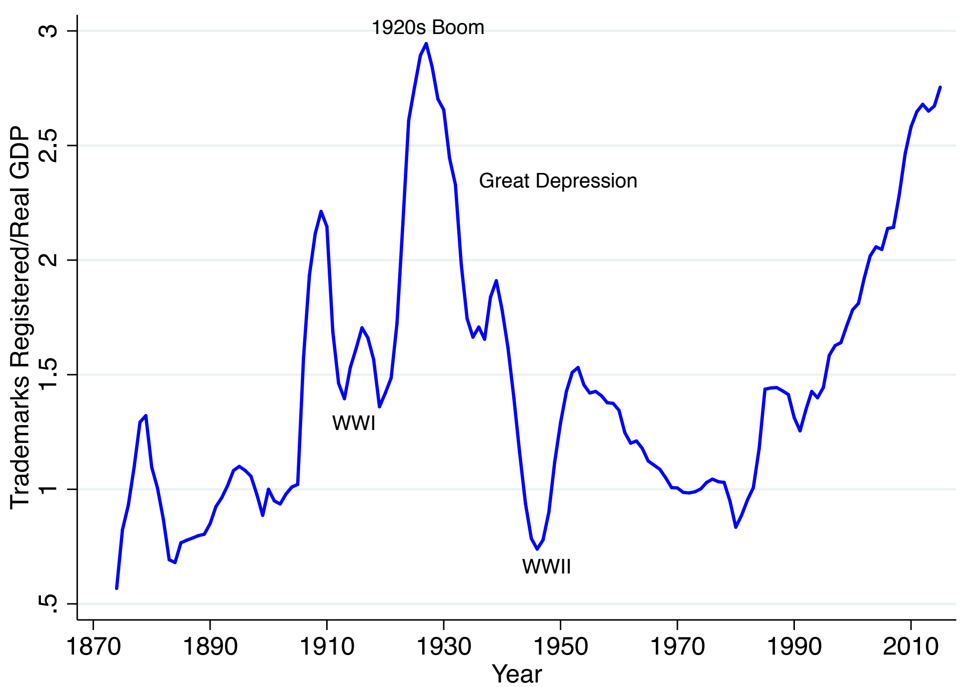
3 Empirical Facts

This section discusses the main empirical facts that frame our investigation. We focus primarily on how firms grow and decline through trademarks. We address the five facts discussed in the introduction. This includes the overall dynamism of the market, the role of reallocation in

Figure 1: Trademarks over Time



(a) Log Trademarks and Real GDP, Normalized to 0 in 1900



(b) The Ratio of Trademark Registrations to Real GDP, Normalized to 1 in 1900

Table 2: Summary statistics on Trademarks from USPTO

	Unmatched mean	Matched mean	Difference
total assets	964.71	2865.96	1901.25***
capital	458.25	1059.66	601.41***
net invest	6.84	11.34	4.50**
real sales	102.02	316.45	214.44***
agriculture	0.02	0.00	-0.02***
mining	0.15	0.03	-0.12***
construction	0.02	0.01	-0.01***
manufacturing	0.38	0.57	0.18***
transportation	0.10	0.06	-0.04***
wholesale	0.05	0.04	-0.01***
retail	0.08	0.08	-0.00***
services	0.19	0.22	0.02***
N	99888	371979	471867

determining the firm size distribution, trends in reallocation and the market for trademarks, and the marginal effect of a trademark transaction.

Firm Dynamics and Trademarks

The trademark market is active, as many firms enter (register or buy their first trademark), exit (cancel or trade their last trademark). Once firms enter, they create and transact brands throughout their life-cycle. However, the distribution is skewed – few firms hold many brands. We first address this fact in detail.

> **Fact 1** *The firm size distribution in trademarks is i) heavily skewed (99th/50th ratio is 50), and ii) the market experiences lots of entry and exit; approximately 10% of the active firms each year are entrants, while 10% will exit next period.*

Table 3 documents the firm size distribution and entry and exit rates. The top 5% of firms hold over half the trademarks, while the top 1% holds almost 30% of trademarks. Further, the entry and exit rates are similar and high. There is lots of registration each year.

Figure 2 maps out the firm size distribution in the stock of trademarks. Due to the distribution being highly skewed, we take the log of a firm's trademark holdings benchmarking the year at 2010,

Table 3: Firm Dynamics in Trademarks

	Overall
Entry Rate	12%
Exit Rate	10%
Registration Rate	8.8%
Poach Rate	3.5%
Top 5% Trademark Share	51%
Top 1% Trademark Share	30%
Top 0.1% Trademark Share	22%
99th/50th percentile ratio	49

Note: Firm size is defined as the number of trademarks within a firm

using cumulative firm growth until this point. One can see both the mass of small firms (about 50% of firms hold only 1 trademark) and the fat right tail.

We now turn to the dynamics of trademark reallocation–the creation/destruction margin and exchange margin–to understand the drivers of the firm size distribution and pricing power at the firm.

Trademark Reallocation

When it comes to trademark exchange, we start by exploring some long-running features of the market before turning to the time trend. Fact 2 focuses on the persistent aspects of the market.

> **Fact 2** *The market for trademarks i) contains a lot of trademark reallocation across firms, ii) trademark reallocation is a larger share of fast-growing firms, iii) the largest firms are built the most by reallocation.*

To explore Fact 2, we plot the distribution of firm growth in trademarks, weighted by firm size in the previous period. Figure 3 plots the weighted histogram. Many firms experience zero growth, but over half of firms experience either positive or negative growth.

Figure 2: Firm Size Distribution

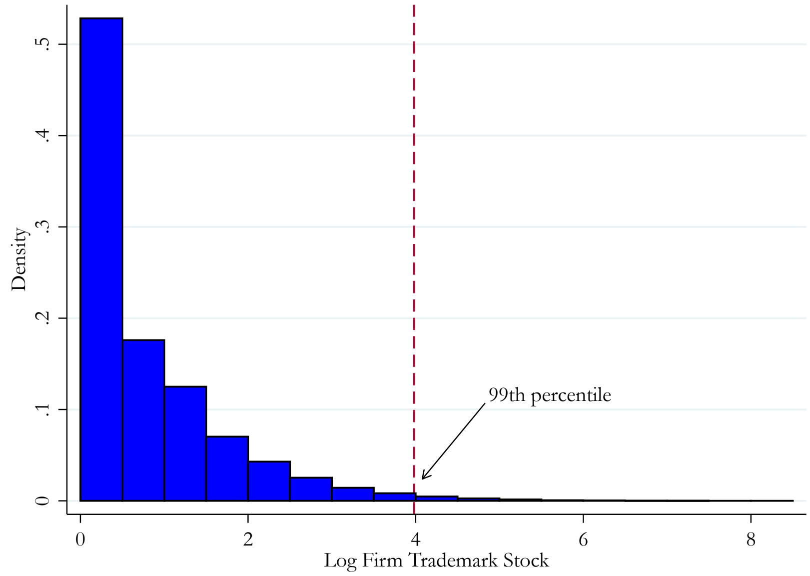


Figure 3: Growth Rates of Firms

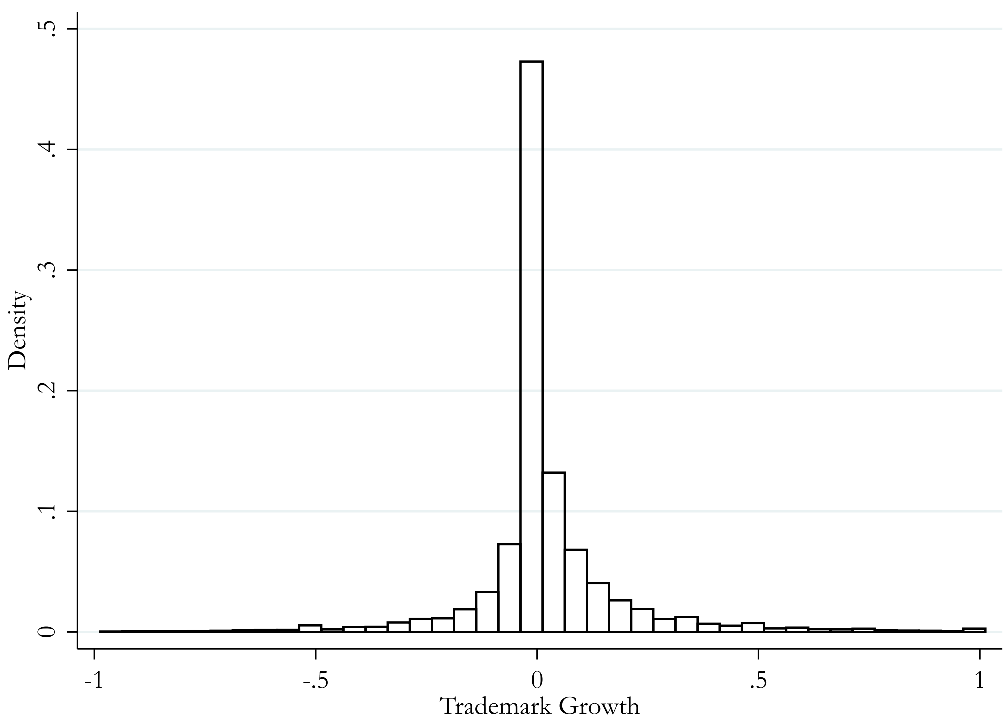
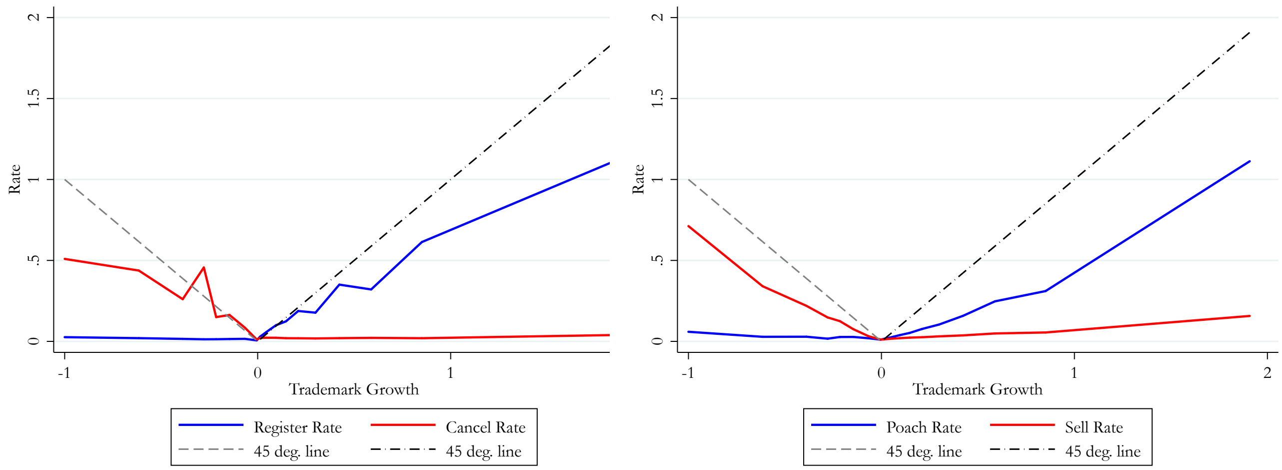


Figure 4: The Contribution of Registration and Transaction to Firm Growth



(a) Registered

(b) Transacted

When firms do grow, they grow from both trademark registration and exchange. Figure 4 illustrates the role of registrations (Figure 4a) and exchanges (Figure 4b) in driving firm growth.

We observe the importance of poaching in driving firm growth in particular when focusing on the contribution of registrations and exchanges to firm size by firm percentile. Figure 5 illustrates this striking fact. Among top 1% firms, about half of their trademarks come from poaching, whereas for small firms less than 5% of their stock comes from poaching. Given these facts about the persistent nature of the market for trademarks, we turn now to the overall time trend in reallocation.

Time Trend in Trademark Market Dynamism

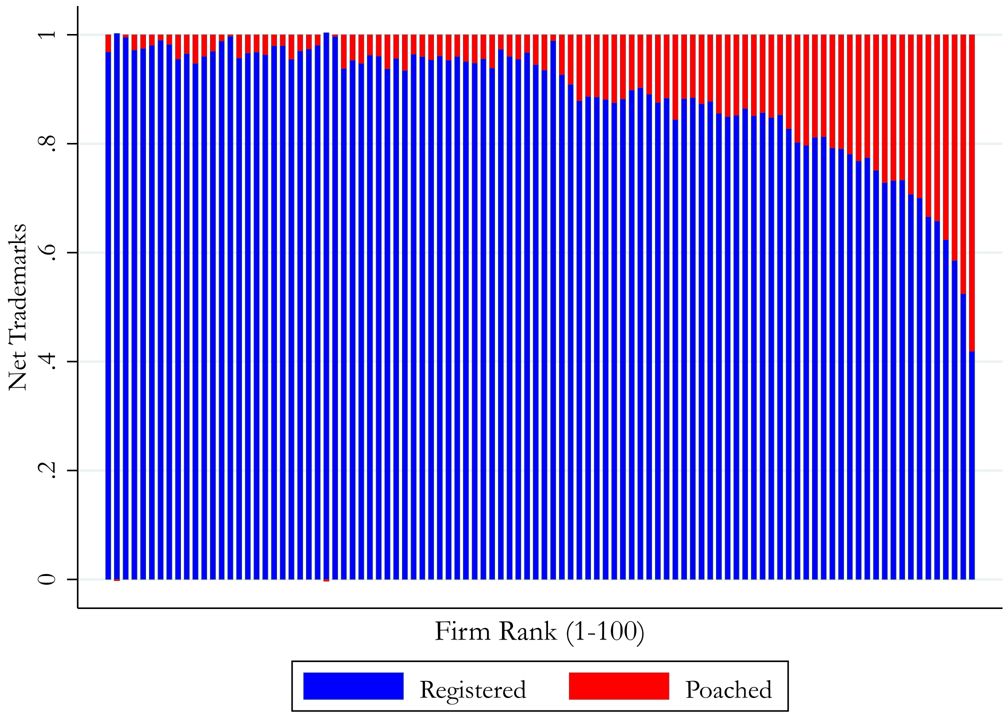
Fact 3 *There has been rising dynamism in the trademark market over time with trademarks.*

To explore Fact 3, we apply the reallocation measure developed by Davis et al. (1996) to measure the dynamism in trademark ownership. The positive growth rate is defined as the total number of inflow of trademarks (including new registration and purchase) divided by the lagged stock of trademark:

$$g_{j,t}^{+} = \frac{\text{Registration}_t + \text{Purchase}_t}{\text{Average Number of Marks}_{j,t-1,t}}.$$

Correspondingly, we define the negative growth rate as the total number of outflow of trademarks

Figure 5: Contribution of Poaching and Registration to Firm Size



(including cancellation and selling) divided by the lagged stock of trademarks:

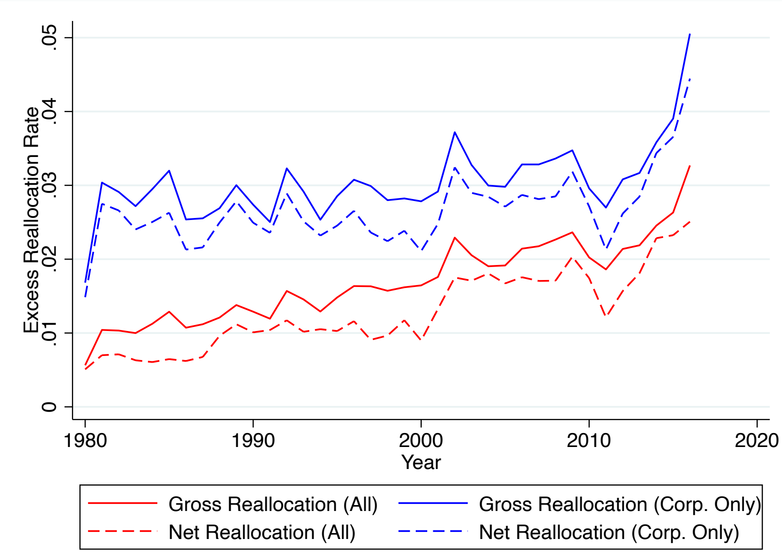
$$g^{-}_{j,t} = \frac{\text{Cancellation}_t + \text{Sell}_t}{\text{Average Number of Marks}_{j,t-1,t}}.$$

Our baseline definition of reallocation rate is the sum of positive growth rate and negative growth rate. We then weight the firm-level reallocation rate by the firm's share of trademarks within the focal year. Using this measure, we ask what is the probability a representative trademark experiences a change of ownership in year t:

$$R_t = g^{+}_{j,t} + g^{-}_{j,t}.$$

Figure 6 illustrates the trend since the 1960s in reallocation among trademarks. We note that the overall reallocation in trademarks has risen over time. This is of interest for two reasons. First, the rising reallocation confirms that dynamism in the trademark market remains high even though previous work has documented falling reallocation in other areas of the economy (Gourio et al., 2014; Decker et al., 2018). Second, the overall reallocation rate of trademarks is consistent with what we have previously noted: as the discussion of markups heats up, so does the trademark market.

Figure 6: Excess Reallocation Rate



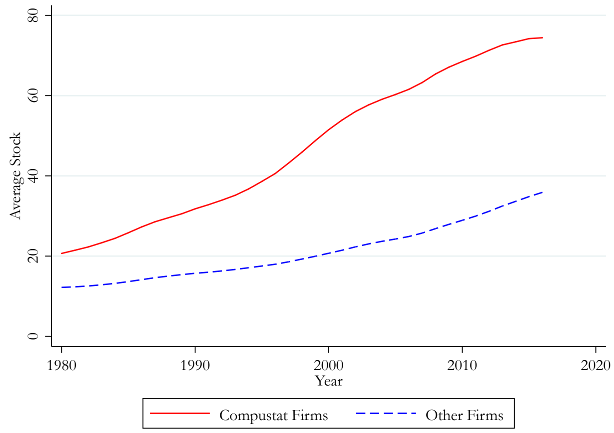
Rising reallocation could be the result of two main forces. First, firms could be registering trademarks at higher rates, as brand entry could be growing. Second, trademark transactions across firms could be rising. The effect on overall market power for both of these forces would depend on whether incumbents are growing or collecting new brands versus entrants. We link our data to CRSP Compustat data to evaluate the concentration across and within firms.

Large and Small Firms – Figure 7 plots the stock of trademarks by firms in Compustat (which we treat as "large" firms) and not in Compustat (which we treat as "small firms"). Figure 7a illustrates that the ratio of trademarks in large firms versus small firms is growing over time. We normalize the initial stocks to be equal in Figure 7b to illustrate the more significant growth in large firms' stocks of trademarks over time.

To better understand the forces behind the growth of large and small firms, we split the inflows of trademarks into registrations, transactions, and cancellations in Figure 8. We plot the relationship between these flows and firm stocks separately. While registrations remain persistently the most significant addition to small firms, large firms are more frequently poaching trademarks.

There are two main results in this section. First, the movement of trademarks in aggregate

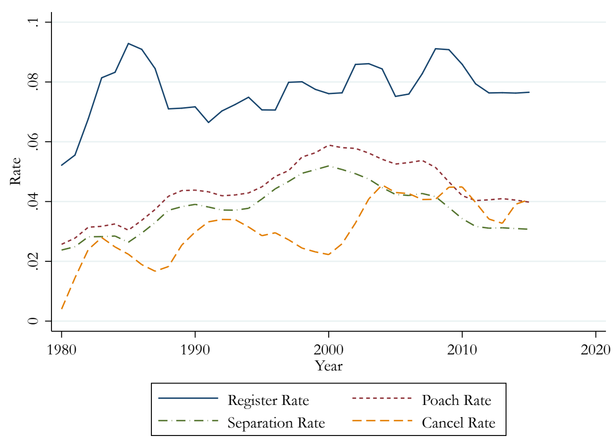
Figure 7: Stock per Firm



(a) No Normalization (b) Normalize to 1980

Note: Flow Rate=Flow/Total Stock; 3-year moving-average

Figure 8: Trademark Inflow from Registrations and Transactions



(a) Small Firm (Non-Compustat) (b) Large Firm (Compustat)

Note: Flow Rate=Flow/Total Stock; 3-year moving-average

indicates that unlike other markets (i.e., firm, labor), trademark dynamism is rising. Given that economists have noted the declining dynamism in other markets, this should be of interest to those thinking about the sources and consequences of falling dynamism on these other margins. Second, a lot of the action is different among large and small firms. Large firms tend to purchase brands, while small firms tend to create brands. Understanding the role of trademark transactions at the firm level can help us further understand how this dynamic plays out in terms of the firm market power.

Trademark Events

Fact 4 *When a firm purchases a brand, the firm's sales and costs both go up with an increase in the markup.*

Fact 5 *When a firm sells a brand, there is a weak decline in markup.*

In the previous section, we illustrated trends in the distribution and dynamism of trademarks. Now we turn our attention to the role of trademarks at the firm level. This section uses event study to understand how sales and costs respond to trademark transactions. A firm could take two sides in an event, as a buyer or as a seller. We evaluate both sides for firms that we managed to find in both the USPTO trademark dataset and the Compustat dataset. Our baseline specification is detailed in Equation (1):

$$\log \frac{\text{sales}_{it}}{\text{variable cost}_{it}} = \sum_{\tau=-5}^{\tau=5} \beta_\tau \text{Transaction}_{i,t+\tau} + \Gamma' \mathbf{X}_{i,t} + \xi_i + \phi_t + \epsilon_{i,t}. \tag{1}$$

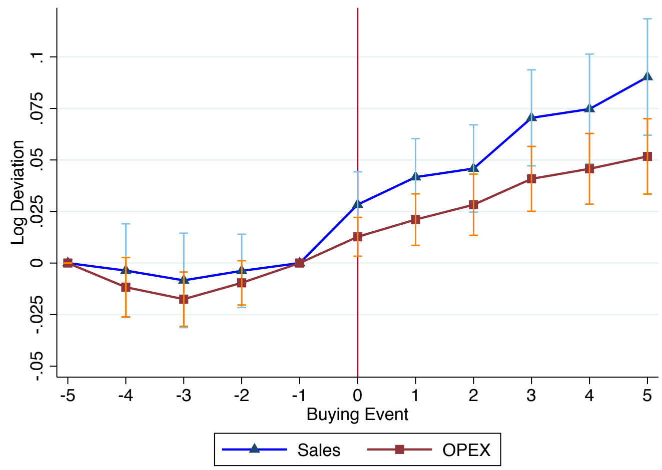
The outcome variable is the profit margin of a firm, which is defined as the quarterly sales divided by the quarterly variable cost. We use two measures of variable costs, the Cost of goods sold (*COGS*) and the operating cost (*OPEX*). In equation (1), transaction$_{i,t+\tau}$ is an indicator for the quarters that is τ quarters away from a transaction event. We control for other firm-level variables indicated by $\mathbf{X}_{i,t}$: *capital stock*; *assets*; *current ratio*; *long-term debt ratio*; and *lagged profit margin*, as well as a firm fixed effect and a time fixed effect.

By estimating equation (1), we ask how the profit margin of a firm compares to its average profit margin, during the quarters around the transaction events. Following the approach of De Loecker et al. (2020), the estimates β_τ in Equation (1) have a structural interpretation as the effect on mark-up of a firm. The insight of De Loecker et al. (2020) is that the markup of a firm can be written as the product of a with ξ_i, the output elasticity with respect to variable cost, and the profit margin measured by variable cost. This structural equation implies that the log of a firm's markup is additive in elasticity and the profit margin.

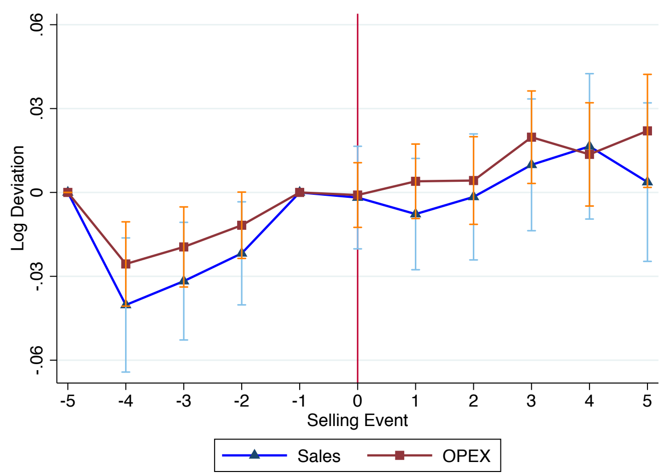
$$\log \text{markup}_{it} = \log \xi_i + \log \frac{\text{sales}_{it}}{\text{variable cost}_{it}}$$

As we are interested in the change of a firm's markup around the transaction event, the firm-specific output elasticity is irrelevant.

Figure 9: IRF to Buying Events



(a) Sales and OPEX, Buying Event



(b) Sales and OPEX, Selling Event

We use log sales and look at the first event of trademarking. Figure 10 plots two separate regressions on one graph with different outcome variables of interest: costs and sales. We plot each coefficient with the clustered standard error.

The results are consistent with the hypothesis that accumulating brands lead to growth in markups (Figure 10a) and selling brands may lead to a decline (Figure 10b). For firms buying a trademark, there is a striking trend-break of sales and costs to the event. Both are relatively flat prior to the event. Once a trademark transaction happens, sales and costs increase significantly – by almost 10% after 5 quarters. Further, sales increase more than costs. In just a regression of the markup on an event, we find an initial and significant increase in the markup by 1.5%, eventually settling to 2.8% markup 5 quarters after the event. This is not true with a selling event, where costs rise slightly more than sales over time.

The results in Figure 10 provide evidence that after adding additional brands, firms may increase their market power over time. Combining this with the rising rate of transfer from small to large firms can help connect the importance of brand dynamism with the aggregate distribution of markups across firms. Further, the change in markups will be a key outcome of our model, which we turn to next.

4 Model

We introduce in this section a model of product creation and transfer where firms pursue different strategies for growth. Growth occurs through brand creation and reallocation. Brands expand as firms choose to create them, and are reallocated as firms find a buyer of their brand.

4.1 Environment

The economy is composed of a representative household that supplies L units of labor to the economy at each instant. The household receives income from their labor and profits in the corporate sector. At each instant t, the total measure of available product is N_t. The utility of a household is given by a CES aggregator across all products, with substitution elasticity σ:[2]

$$C_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

[2]The baseline environment can also be extended to a setting where the household makes consumption-saving choices. Given our focus on steady state equilibrium, it does not change our quantitative analysis.

The household takes as given prices p_{it}, wage w_t, and aggregate profit Π_t, and chooses consumption c_{it} to maximize its lifetime utility. We define $E_t = w_t L + \Pi_t$ as the total expenditure of the household.

There is a unit measure of firms at any instant of time. Firms can own a positive measure of products. They produce all their products using a linear technology in labor, with firm productivity $z = exp(\frac{x_{jt}y_j}{\sigma-1})$. The productivity has a permanent component y_j, which is drawn from a distribution $G_0(y)$ upon entry. The time-varying component y_{jt} follows a Ornstein-Uhlenbeck process:

$$d\ln x_j = -\theta \ln x_j dt + \sigma dB_t,$$

where B_t is a standard Brownian process. Each firm enters the market with an infinitesimal product. The firm can increase its brand holdings by (1)*creation* or (2) *purchase*. To create a flow of ι brands requires $R(\iota, n)$ units of labor, where $R(\iota, n)$ is increasing in (ι, n) and is jointly convex. We lay out the details of the brand transaction in the next paragraph. Meanwhile, existing brands become obsolete with rate δ^I. Each existing firm exits with an exogenous rate δ^J, upon which it is immediately replaced by an entrant firm.

The market for brands is organized through directed search. Firms can be either a buyer or seller in this market. To buy a trademark, a firm hires k_s units of labor in order to post a vacancy. In the advertisement, the buying firm specifies the price to be paid, denoted by τ. The rest of the firms in the economy observe all advertisements in the market, and decide which post they want to search for. We denote the buyer-seller ratio for individual post as θ, the contact rate for a seller is $p(\theta)$ and the contact rate of a seller is $q(\theta) = \frac{p(\theta)}{\theta}$. We make the standard assumptions on the matching technology: (1) The contact rate for a seller is increasing and concave in θ; (2) The contact rate for a buyer is decreasing and convex in θ.

4.2 Characterization

Households' Decision.- The household's full problem is:

$$\max_{c_{it}} \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

s.t.

$$\int_0^{N_t} c_{it} p_{it} di \leq w_t L + \Pi_t.$$

The household's optimal consumption choice constitutes the demand curve for products $c_j = \left(\frac{p_{jt}}{P_t}\right)^{-\sigma} C_t$. The price index is defined by $P_t = \left(\int_0^{N_t} p_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$.

Product Market Equilibrium.- We first take as given the joint distribution of number of products and productivity $G(n,z)$, and characterize the equilibrium price and consumption arise from this equilibrium. Consider a firm with state (n,z), its profit maximization problem is:

$$\pi(z,n;G) = \max_{p_i} \int_{i \in n_j} (p_i - \frac{w_t}{z^{1/(\sigma-1)}}) c_i \, di$$

s.t.

$$c_i = \left(\frac{p_{it}}{P_t}\right)^{-\sigma} \frac{E_t}{P_t}$$

The optimal pricing strategy of a firm is to set a constant markup on top of its unit labor cost $\frac{w_t}{z^{1/(\sigma-1)}}$. Under Bertrand competition, the optimal markup of a firm is increasing in its total market share

$$\mu = \frac{\sigma + (1-\sigma)s}{\sigma + (1-\sigma)s - 1},$$

where

$$s = \frac{nz\mu^{1-\sigma}}{\int_{n,z} n'z'\mu'^{1-\sigma} dG(n,z)}$$

The market share of a firm depends on both the focal firm's productivity and markup, as well as the joint distribution of markup and (n,z) across firms. The following lemma states the conditions for the pricing equilibrium.

Lemma 1 *Given any distribution $G(n,z)$, the pricing equilibrium is characterized by $\mu(n,z)$ and $s(n,z)$ such that:*

$$\mu(n,z) = \frac{\sigma + (1-\sigma)s(n,z)}{\sigma + (1-\sigma)s(n,z) - 1},$$

$$s(n,z) = \frac{\mu(z,n)^{1-\sigma} zn}{\int_{z',n'} \mu(z',n')^{1-\sigma} z'n' dG(z',n')}.$$

We note that the denominator of the market share equation is related to the aggregate price index P_t. We thus write:

$$s(n,z) = \mu(z,n)^{1-\sigma} z n P_t^{\sigma-1}$$

With the calculated market share and markup, and two aggregates (C_t, P_t), the total profit of the firm is the total expenditure adjusted by the Lerner index. We thus reach the following static profit of a firm with state (n,z):

$$\pi_t(n,z) = \frac{s(n,z;P_t)}{\sigma + (1-\sigma)s(n,z;P_t)} P_t C_t \tag{2}$$

Competitive Search Equilibrium.- As the cost of posting a vacancy is constant $k_s w_t$, a firm that looks to purchase would be indifferent in posting any price that is traded with positive probability. Denote ω_t the equilibrium expected price for one brand. The indifference condition for a buyer ensures that:

$$\omega_t = \frac{k_s}{q(\theta)} + \tau.$$

Dynamics Decision.- We are now ready to write out the Bellman equation for a firm with current productivity z and n number of products. We continue by directly writing the equations as if the market is in steady state.

$$\begin{aligned}(\rho+\delta)V(n,x,y) = \max_{\iota,b,s,\tau,\theta} \quad & \underbrace{\pi(nxy) - k_p n}_{\text{Flow Payoff}} \\ & + V_n\left(\iota + b - p(\theta)s - \delta_I n\right) - \underbrace{R(\iota,n)}_{\text{Innov. Cost}} - \underbrace{\omega b^+}_{\text{Buying Cost}} + \underbrace{p(\theta)\tau s}_{\text{Selling Proceed}} \\ & - \theta x V_x + \frac{\sigma^2}{2} V_{xx}.\end{aligned} \tag{3}$$

s.t.

$$\omega = \frac{k}{q(\theta)} + \tau$$

The choices of a firm are all based on the marginal value of a brand $v = V_n$. The optimal innovation decision equalize the marginal cost of innovation to the marginal value: $R_e(\iota^*, n) = v$. As $R_{ee} > 0$, there is a unique level of innovation given (z,n). Denote this solution as $\iota(v,n)$. Because the expected cost of purchasing a brand is constant, a buying firm would immediately adjust to its target level of brands. This target is given by $V_n(n^*, z) = \omega$. Denote this boundary as $n_b(z;\omega)$. Similarly, a firm would only start to sell if the marginal value of a brand falls below the lowest possible selling price τ_{min}. Denote this boundary as $n_s(z;\omega)$.

We denote the corresponding creation flow as $e(n,z)$ and the cost as $R(n,z)$. A firm starts to sell if $\frac{\partial V}{\partial n} < \omega - k$. In this case it will not buy, thus the value function follows:

$$(\rho+\delta)V^s(n,z) = \pi(n,z;\Phi) - R(n,z) + \lambda(z,n)\tau(z,n)n + V_n\bigg(e - \lambda(n,z)n - \chi n\bigg) + V_{zt} \tag{4}$$

A firm starts to buy if $n < n_b(z)$, for any $n \leq n_b(z)$:

$$V^b(n,z) = V^b(n_b(z),z) - \omega(n_b(z) - n)$$

Aggregation.- As the evolution of distribution is relatively standard given the value function, we detail the forward equation in the appendix. The market for trademarks has to clear: the total sales from all the sub-markets have to equal the total purchase. The total sales is aggregated across all existing firms, while the purchase can be made by both the entrants and the incumbents that are below their adjustment target. Formally, this market clearing condition requires

$$\underbrace{\int n\, p(n,z,y) dG(n,z,y)}_{\text{Sales}} = \underbrace{\int [n_b(z,y) - n]^+ d\tilde{G}(n,z,y)}_{\text{Incumbent Purchase}} + \underbrace{\delta(1 - G(z_b))n_b(z,y)}_{\text{Entrant Purchase}} \tag{5}$$

Labor in the economy is used for either production, innovation, or setting up transactions. The market clearing for production labor yields the last aggregation condition needed for characterization of the equilibrium:

$$C_t = \mathcal{M}_t(L - L_s - L_I), \tag{6}$$

where:

$$\mathcal{M}_t = \frac{\bigg(\int nz\mu(z,n)^{1-\sigma} dG(z,n)\bigg)^{-\sigma/(1-\sigma)}}{\int nz\mu(z,n)^{-\sigma} dG(z,n)}$$

After describing the decisions and evolution of brand ownership, we are now ready to write down the system of equations that characterize the equilibrium allocation and prices.

Definition 2 *A stationary equilibrium is characterized by* $\{v(n,x,y), g(n,x,y)\}$ *and* $\{\omega, P, C\}$ *such that:*

1. $v(n,x,y)$ *solves the HJB equation given* $\{\omega, P, C\}$*;*
2. $g(n,x,y)$ *solves the KF equation given* $v(n,x,y)$*;*
3. *Market clears for brands;*
4. *Price index follows its definition;*
5. *Goods Market Clearing*

$$C = \mathcal{M}(L - L_\iota - L_s)$$

5 Quantitative Analysis

(*Preliminary*)

To quantify the model, we assume the innovation cost takes the form $R(\iota, n) = \frac{r_0}{r_1+1} n \left(\frac{\iota}{n}\right)^{r_1+1}$ and the contact rate for a seller is Cobb-Douglas in the buyer-seller ratio $p(\theta) = \theta^{\alpha}$. We normalize the matching efficiency to be 1 as an increase in the matching efficiency is equivalent to a reduction in the cost of posting a vacancy. We set the discount rate to be 0.02. The obsolete rate δ_I is set at 0.02 to match the cancellation rate of expanding firms. The exit rate of firms is set at the observed exit rate in the trademark dataset: $\delta_J = 0.1$. We take the productivity estimates from the firm dynamics literature to calibrate the productivity process. We assume $\theta = -0.1$, which leads to a yearly autocorrelation 0.9 at the firm productivity and set the dispersion of the shock to be $\nu = 0.03$, which is common in the literature.

We are left with two sets of parameters to estimate: the parameters on adjustment cost (innovation cost and matching function) and the parameter on productivity. We use several moments to discipline these parameters. The shape of innovation cost (r_0, r_1) governs the gross creation rate and how the creation rate varies by the size of firms. We thus use the slope of an auxiliary regression as our target for these parameters. Similarly, the search cost parameters (k_s, θ) govern the gross selling rate and its variations. We thus use the estimates of a regression on sales rate on log firm size as our target. Finally, we estimate the holding cost k_p and the initial firm productivity distribution to match the mean firm size and its dispersion.

Table 4: Estimation

Name	Symbol	Value	Target	Data	Model
Innovation Cost - Const.	r_0	50.0	mean reg. rate	.085	.064
Innovation Cost - Curv.	r_1	5.50	slop of reg. rate	-.012	-0.08
Vacancy Cost	k_s	0.62	mean sell rate	0.029	0.031
Matching Elasticity	γ	0.82	slope sell rate	0.002	0.004
Fixed Brand Cost	k_I	0.51	mean firm size	4.59	9.27
Initial Productivity	$[y_l, y_h]$	[0.01, 60]$\sim$[0.9,0.1]	dispersion of firm size	-	-

Aux. regression: $rate = \beta_0^r + \beta_1^r \ln L.Stock$

Table 5: Calibration

Name	Symbol	Target	Value
Substitution Elasticity	σ	*HRW* median	4.1
Discount Rate	ρ	Yearly Calibration	0.08
Exit Rate	δ_J		0.10
Obsolete Rate	δ_I		0.02
Persistence	$1-\theta$	Serial Correlation of Firm Productivity	.1
Variance	σ_z	Dispersion of Firm Shocks	0.03

Model Fit

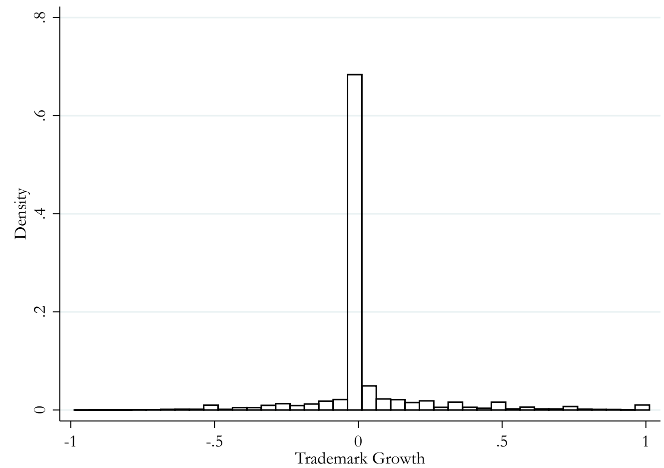
To evaluate the quantitative performance of our model, we first benchmark the simulated data from the model to corresponding objects in the data. As an out-of-sample test, we replicate the flow rates by the net growth rates from the model. Although we set the correlation of sell/registration rates and firm size as our targets, we do not set the buying rates and cancellation rates as targets.

Welfare Analysis

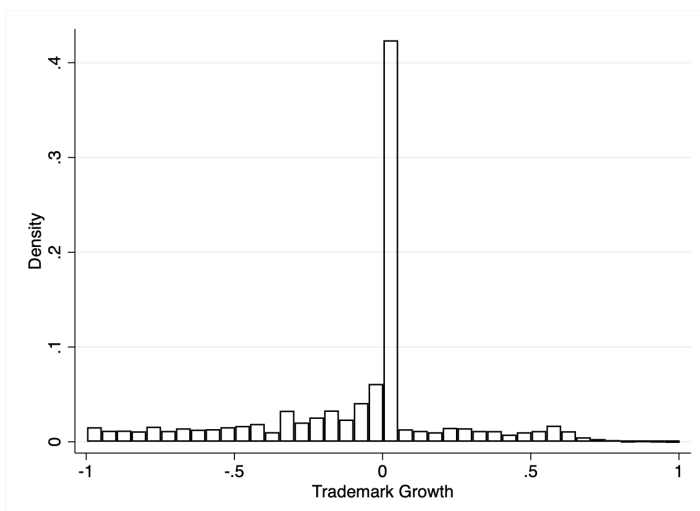
An increase in search cost is isomorphic to two topical policy discussions. First, a stringent antitrust policy would induce the probability of a successful trademark transaction between firms to decline. Second, taxing or subsidizing the price of trademarks would induce a similar decline in the returns to reallocation. We explore this through shifting the cost of transacting a trademark.

In the following table, we compare the productivity in the baseline economy to a counterfactual

Figure 10: Growth Rate Distribution: Data and Model



(a) Growth Rate Distribution - Data



(b) Growth Rate Distribution - Model

setting where the cost of posting vacancies increase by 10%. We interpret this counterfactual either as a change in the intermediation technology in trademark transactions, or as a stronger anti-trust policy that incurs higher cost on transactions.

Table 6: Counterfactual: Change in Search Cost

Total %Δ $\mathcal{M}$	Creation	Productivity Reallocation	Markup Dispersion
-1.00	-3.70	-0.40	3.10

6 Conclusion

Market power in brand innovation has potential positive and negative forces. To study the role of market power in a realistic model of product innovation and exchange, we apply new data on trademarks to cover the brands that firms hold. This paper argues that trademarks are the best empirical analog for brands and connects facts in trademark to a model of endogenous growth with production creation, destruction, and transfer.

After illustrating key facts related to the dynamism of the trademark market and the importance of large firms, we turn to a model of multi-product firms with pricing power and different efficiency. We use this model to study market power in brand innovation. In particular, we focus on a natural tension in brand innovation that emerges from the fact that efficient firms should have more brands, but are able to achieve market power in the process.

Our quantitative model matches the distribution of firms, entry and exit rates, the registration rate of firms and transfer rates. We use this matched model to study a relevant policy counterfactual: how does restricting brand exchange impact consumer welfare? We find that the two tensions rely importantly on how substitutable goods are to consumers and the underlying distribution of firm efficiency. Overall, the efficiency gains outweigh the potential gains from pricing power when firms market substitutable goods.

This project is a first step in unifying two key mechanisms, pricing power and efficient brand allocation, in product-variety driven growth. In doing so, we connect the framework to rich data on brands that enable a detailed study of frictional markets in brand transactions and sales. The results shed light on the importance of policymakers to understand the various forces at play when managing antitrust policies, and have awareness of the efficiency gains from product consolidation.

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Appendix

A Example of Brand-Building: Procter & Gamble

Figure A1 illustrates how many firms that rely specifically on their brand relationships are held by P&G.

Figure A1: Example of P&G Brands

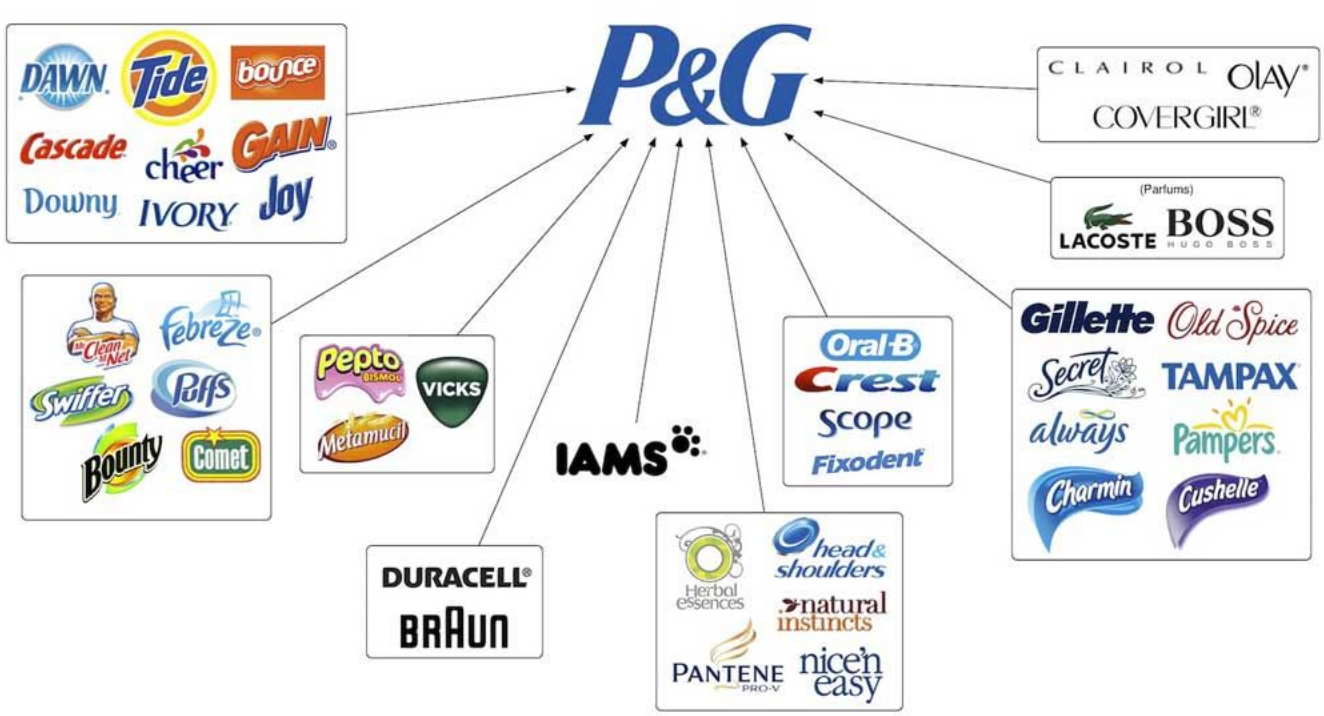


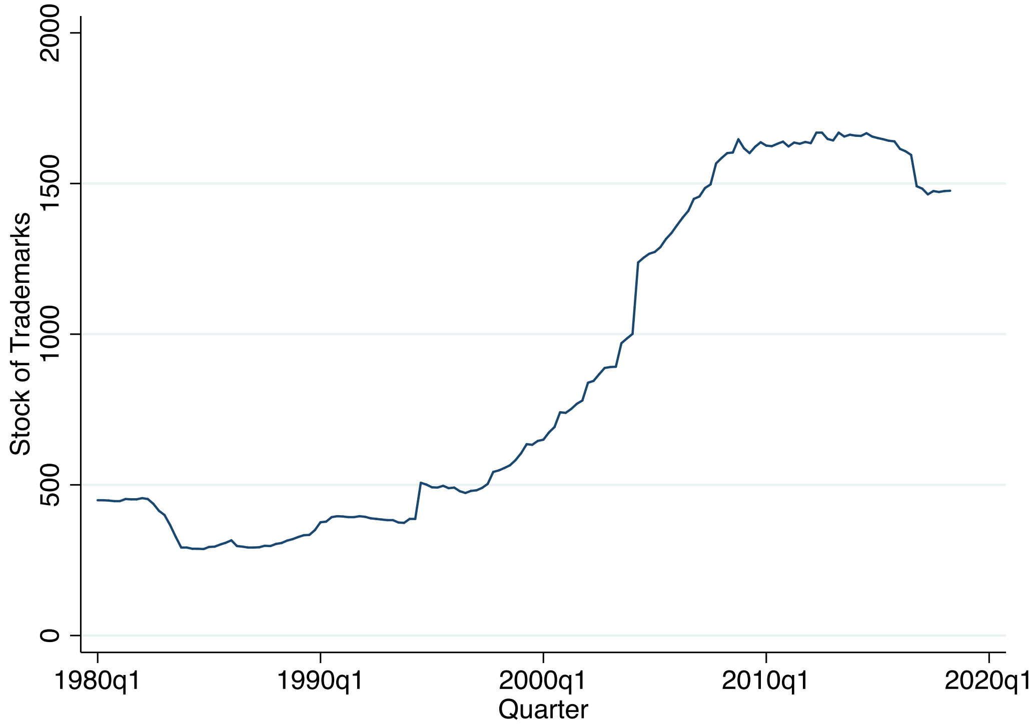
Figure A2 shows how P&G's trademark holdings have grown over time. Much of this trademark increase has come through poaching trademarks from other firms or purchasing other firms.

B Evolution of Productivity and Brand Distribution

In this section, we describe the evolution of state variables in the model. Denote $g(n,x,y)$ the density of firms with product scope n, transitory productivity x and permanent productivity y. In the steady state, this density must balance the inflows and outflows into (n,x,y):

$$\begin{aligned}\delta \mathbb{I}_{x=0,n=n_b(y)} g_0(y) = &- \partial_n \Big(\iota(n,x,y) - p(n,x,y)n - \delta^I n \Big) g(n,x,y) \\ &- \delta^J g(n,x,y) \\ &- \partial_x \theta g_x(n,x,y) + \frac{1}{2} \partial_{xx} \nu^2 g(n,x,y).\end{aligned}$$

Figure A2: Tracing the brands of P&G over time



The left-hand side of the equation is the inflow due to new firms. It is only positive when $x = 0$. The total inflow of new firms is δ and $g_0(y)$ fraction of them will become a type $(n_b(y), 0, y)$ firm. The density drifts due to the innovation choice of firms, with the net growth $(\iota - pn - \delta n)$. Meanwhile, the productivity also change due to the transitory shocks. At each instant, δ_J fraction of firms exit the economy.