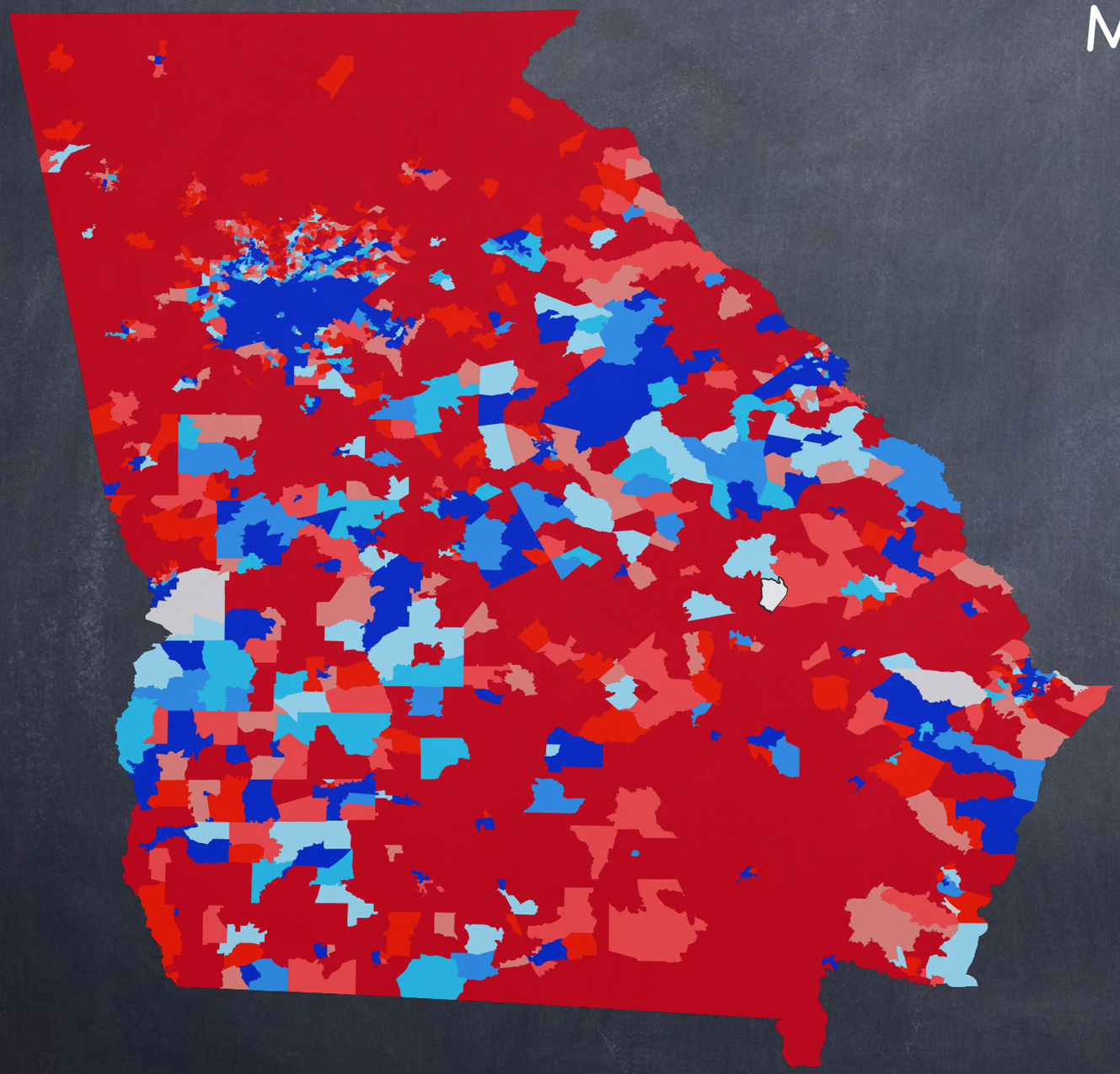
Georgia



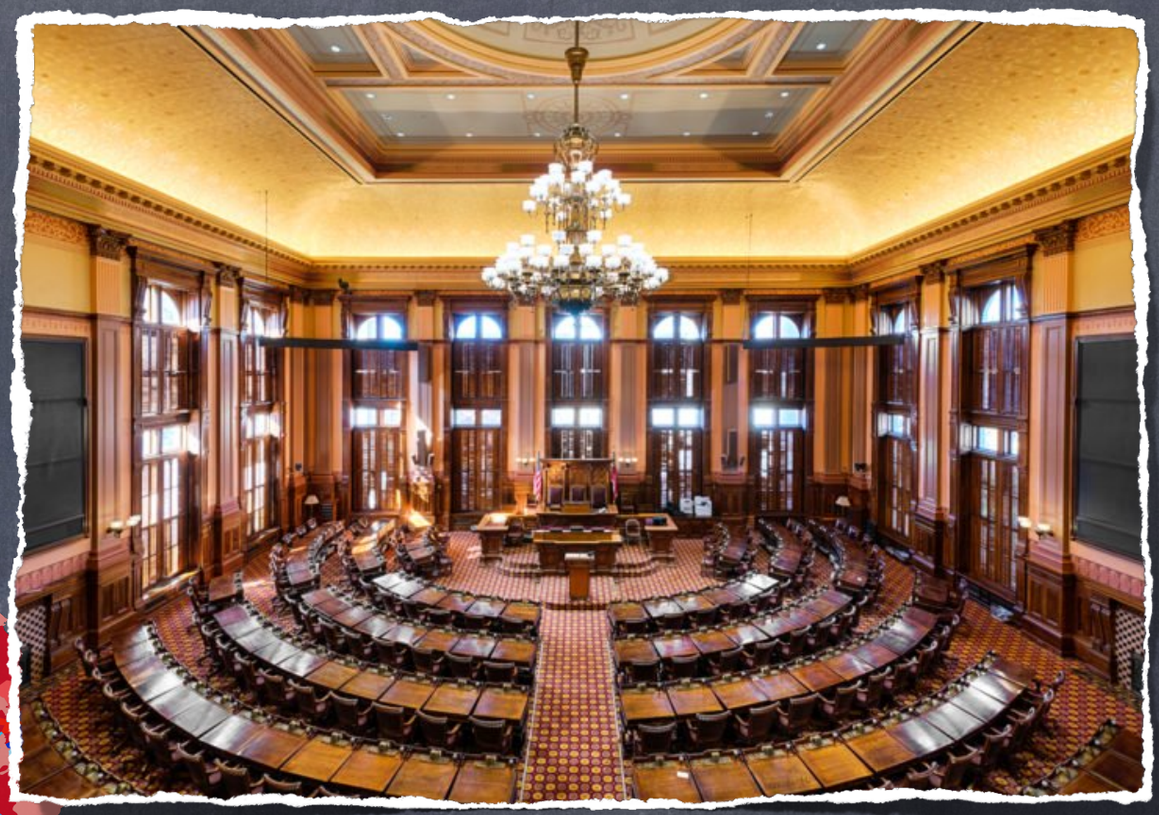
Mean Ideology at a Precinct

McCain
(Republican)

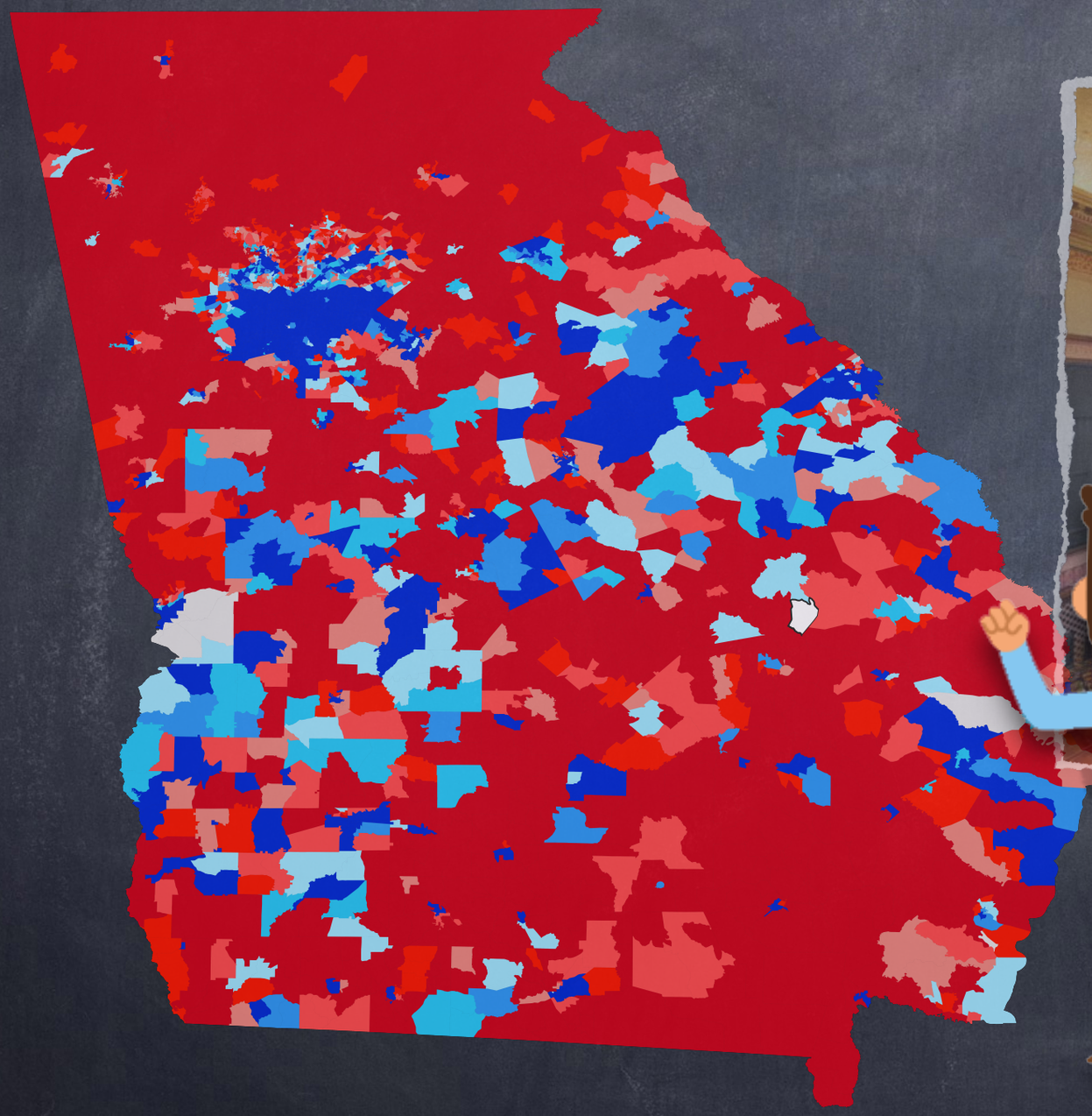
Obama
(Democrat)

Georgia

The House of Representatives



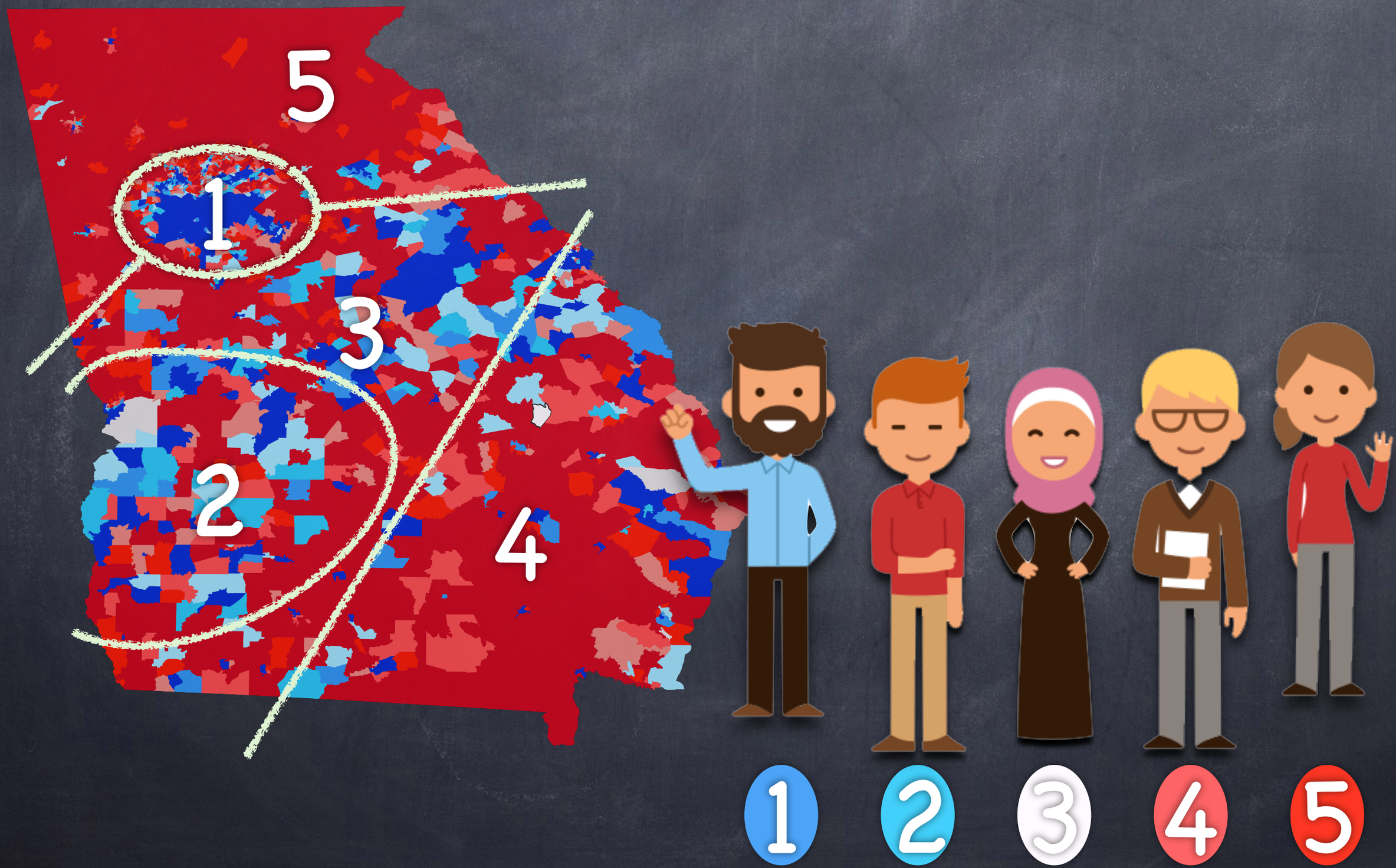
Georgia



The House of Representatives

Representatives

Georgia



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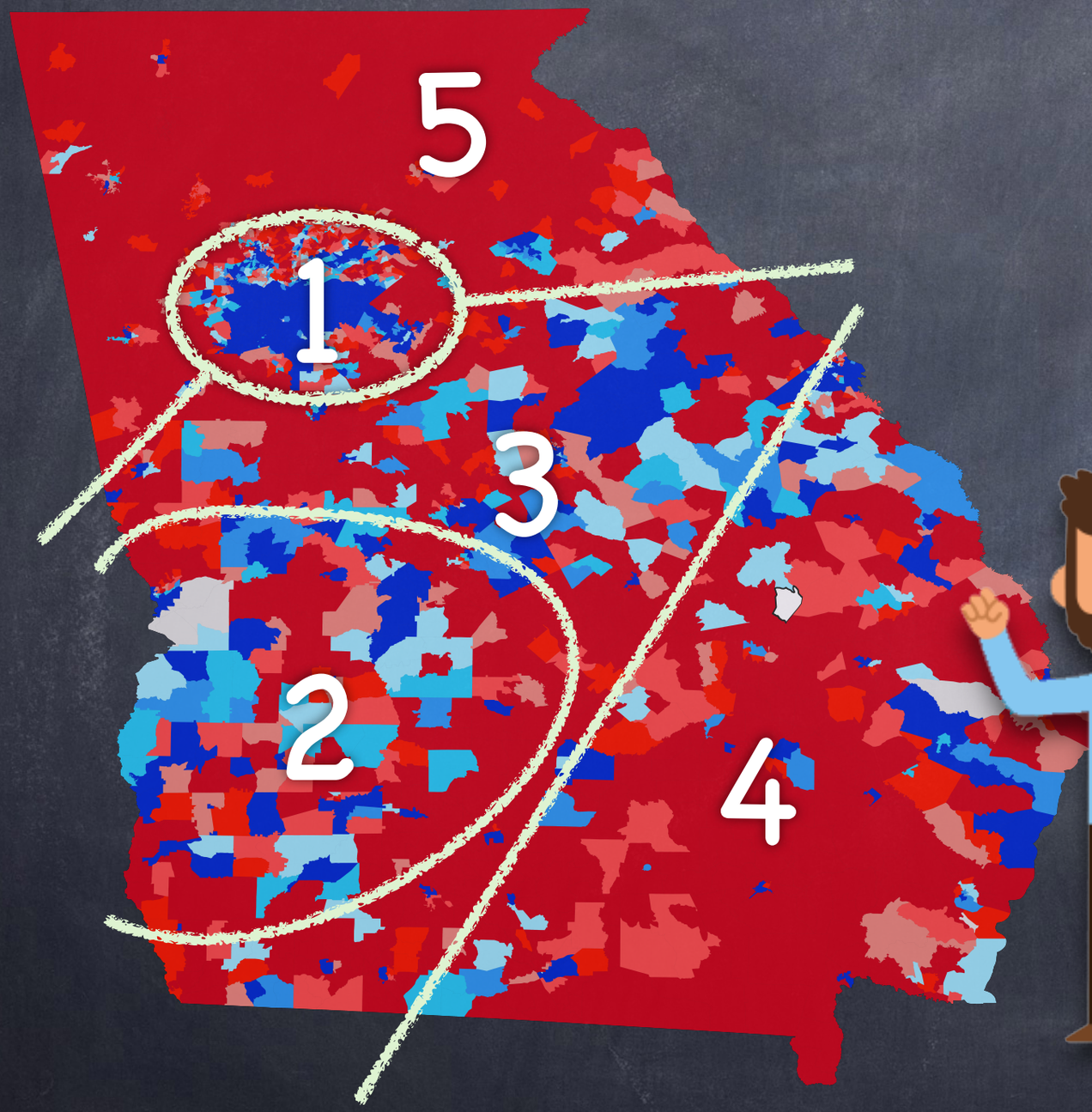
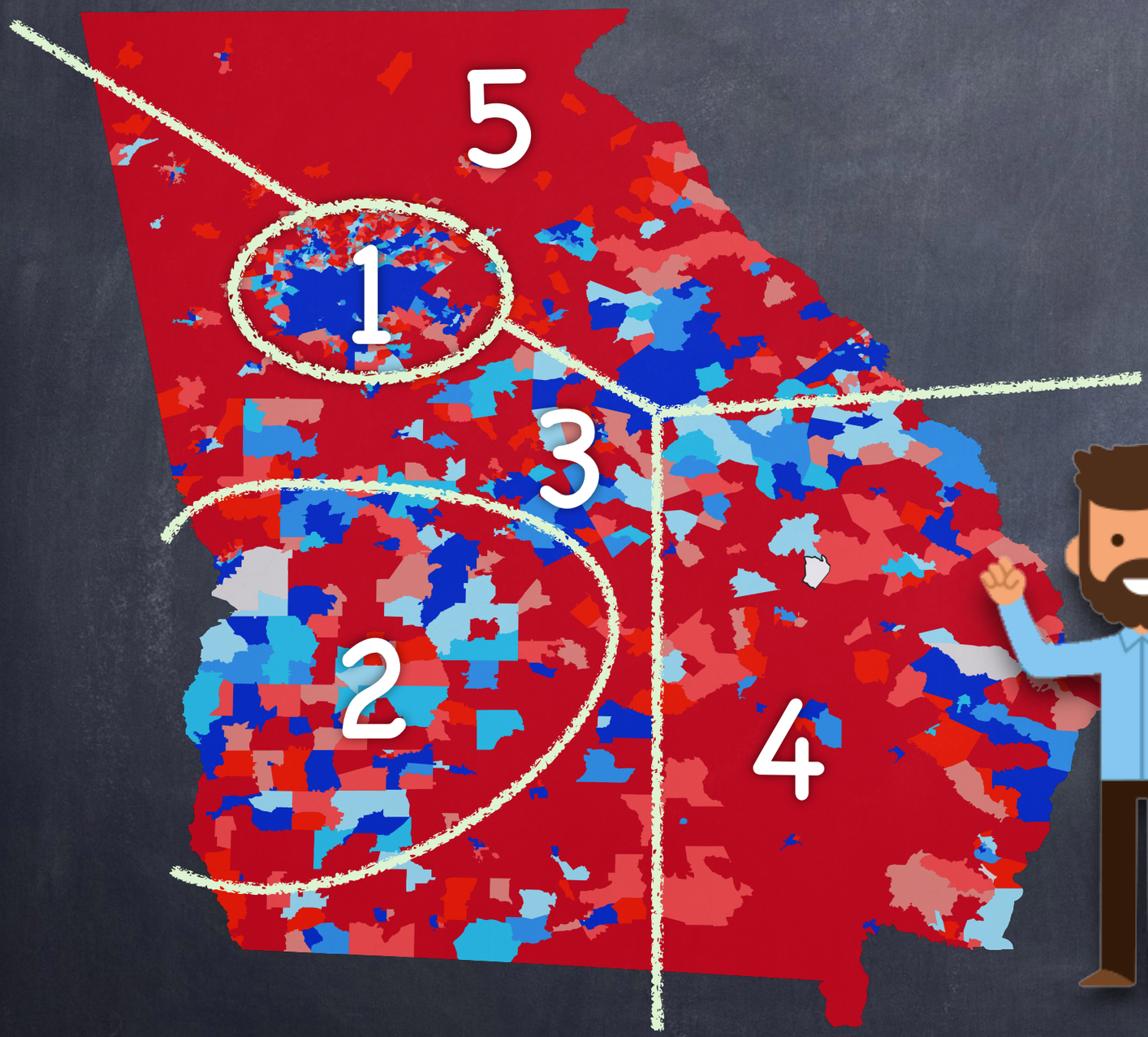


median legislator

Georgia



median legislator

1 2 3 4 5

Looking at the map,
what was the districter's intention?

Electoral Maldistricting

Andrei Gomberg
Romans Pancs
Tridib Sharma

ITAM

PennTheOn
26 March 2021

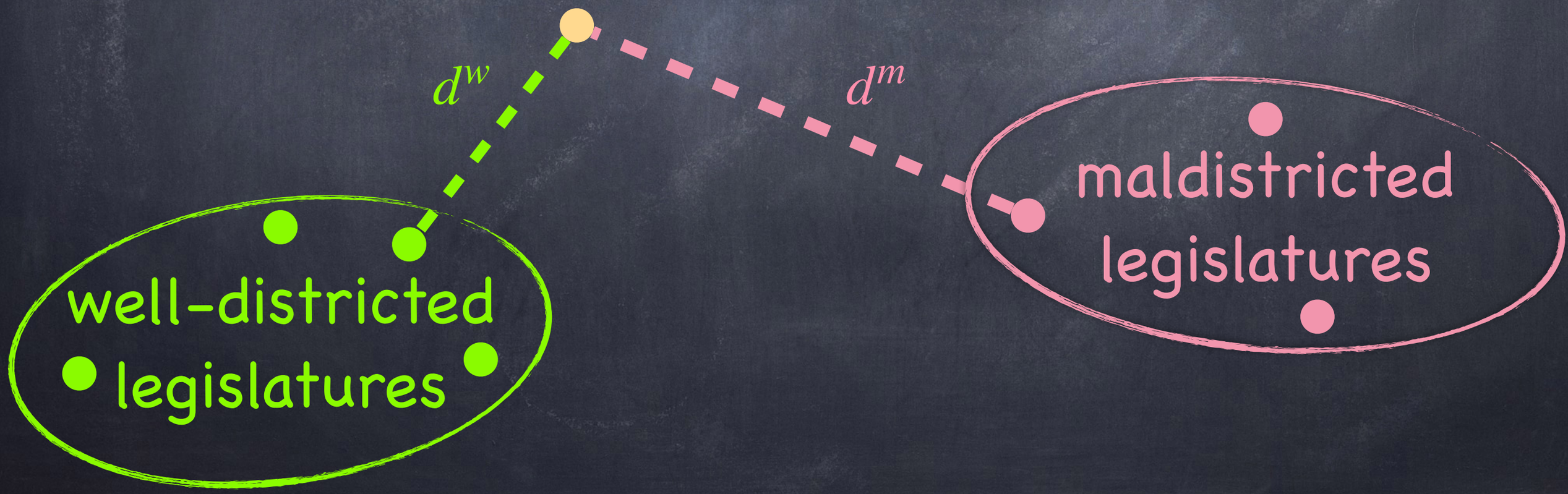
Question

- Friedman and Holden (2008): How to district to meet a partisan objective?
- Coate and Knight (2007): How to district to maximise welfare?
- Us: Given a district map, what was maximised?

Answer

$$\text{index of maldistricting} = \frac{1}{1 + d^m / d^w}$$

an observed legislature



Damage vs Intent

- Damage: How much does a map hurt?
- Intent: Was a map likely designed to hurt?
- In law, intent often determines guilt.

Model

- Voters: uniform on $[0,1]^2$.
- Ideology: 0 or 1 for each voter.
- Affiliation function: share $\rho(l)$ of ideology-1 voters at location $l \in [0,1]$.
- District map: partitions $[0,1]$ into K equisized districts.
- Legislature: ideology means (representatives' ideologies) $(r_1, r_2, \ldots, r_K)$ for the K districts.
- Policy: $p = \text{median}\{r_1, r_2, \ldots, r_K\}$.

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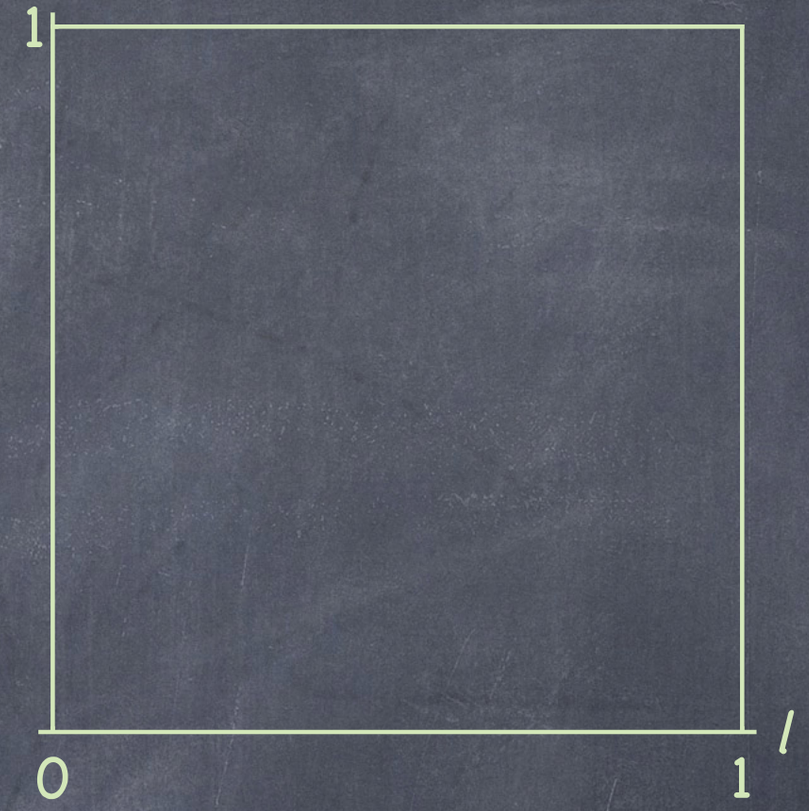

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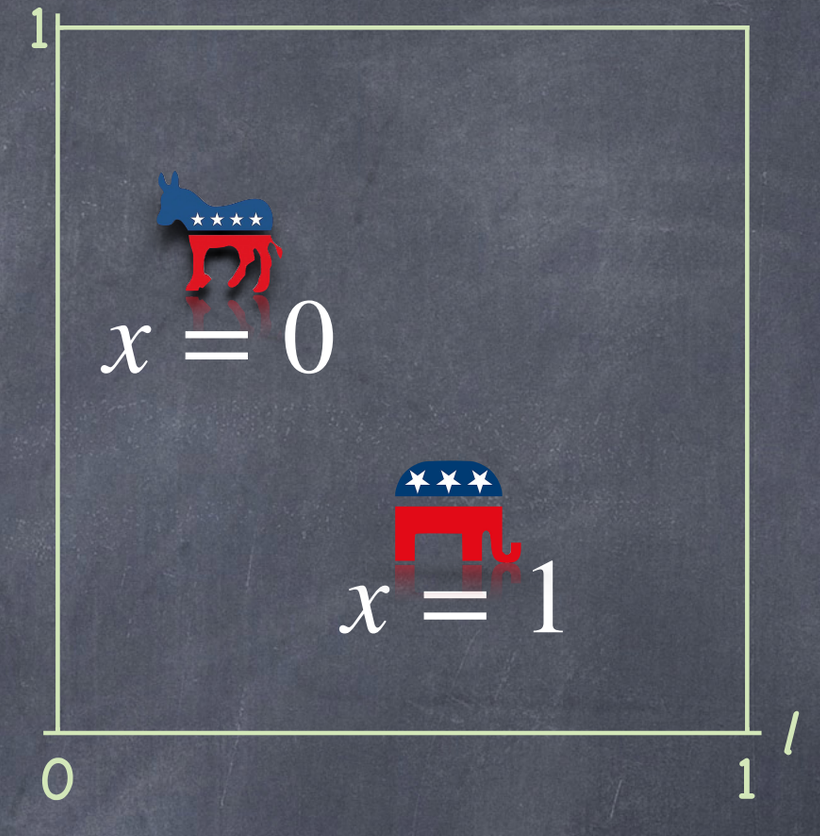

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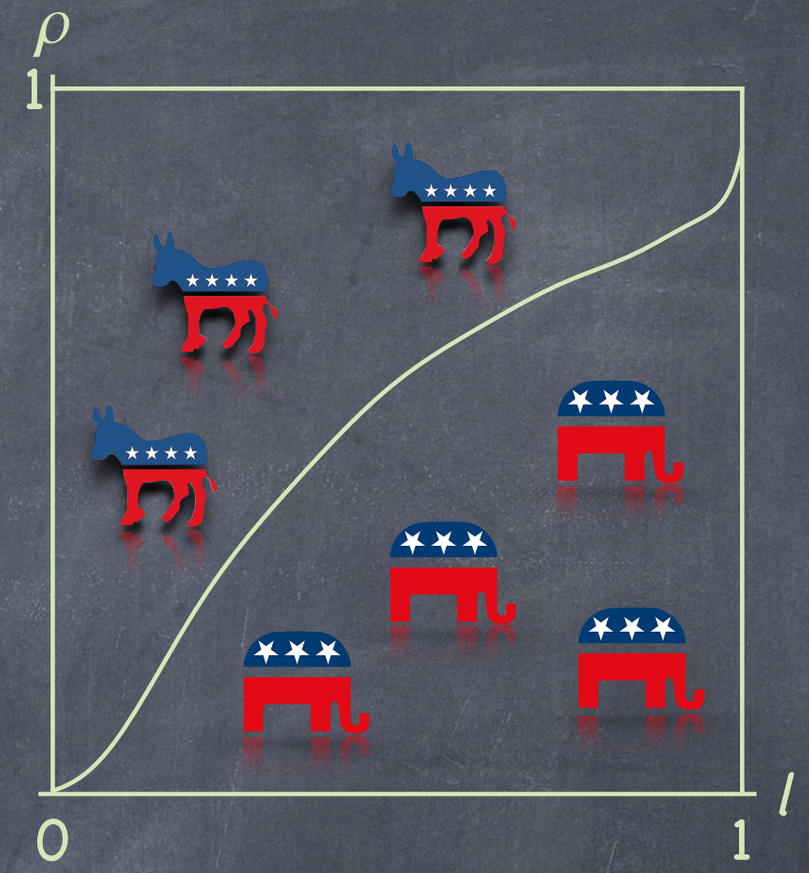

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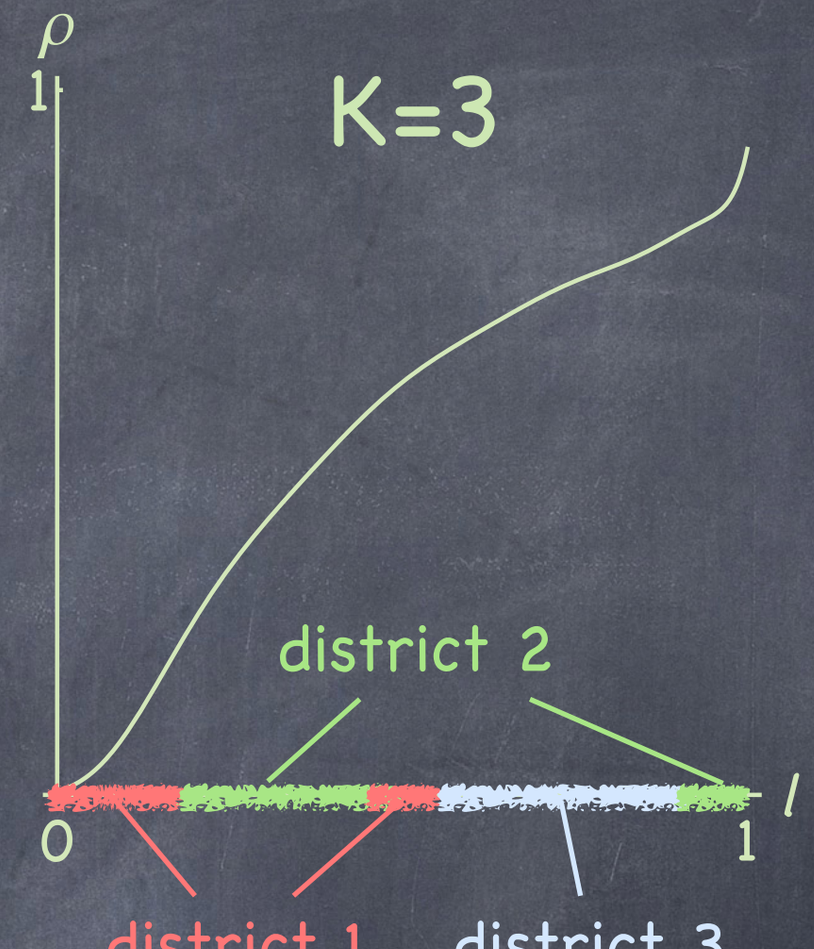

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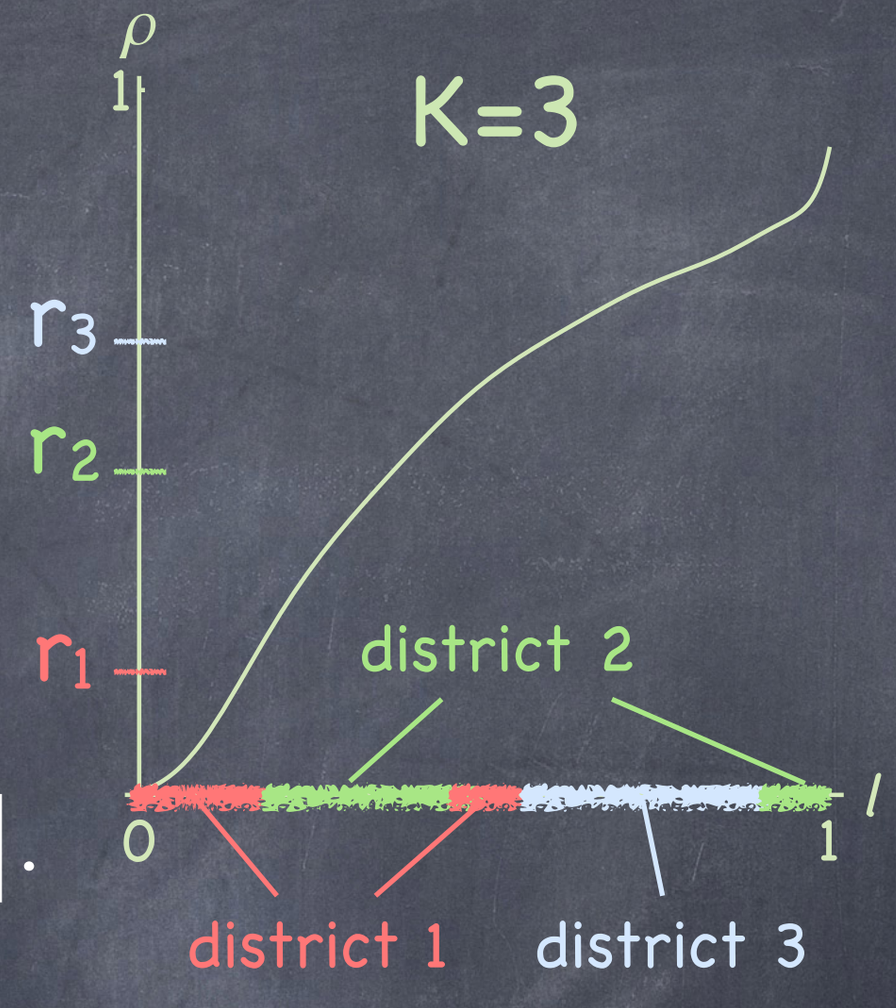

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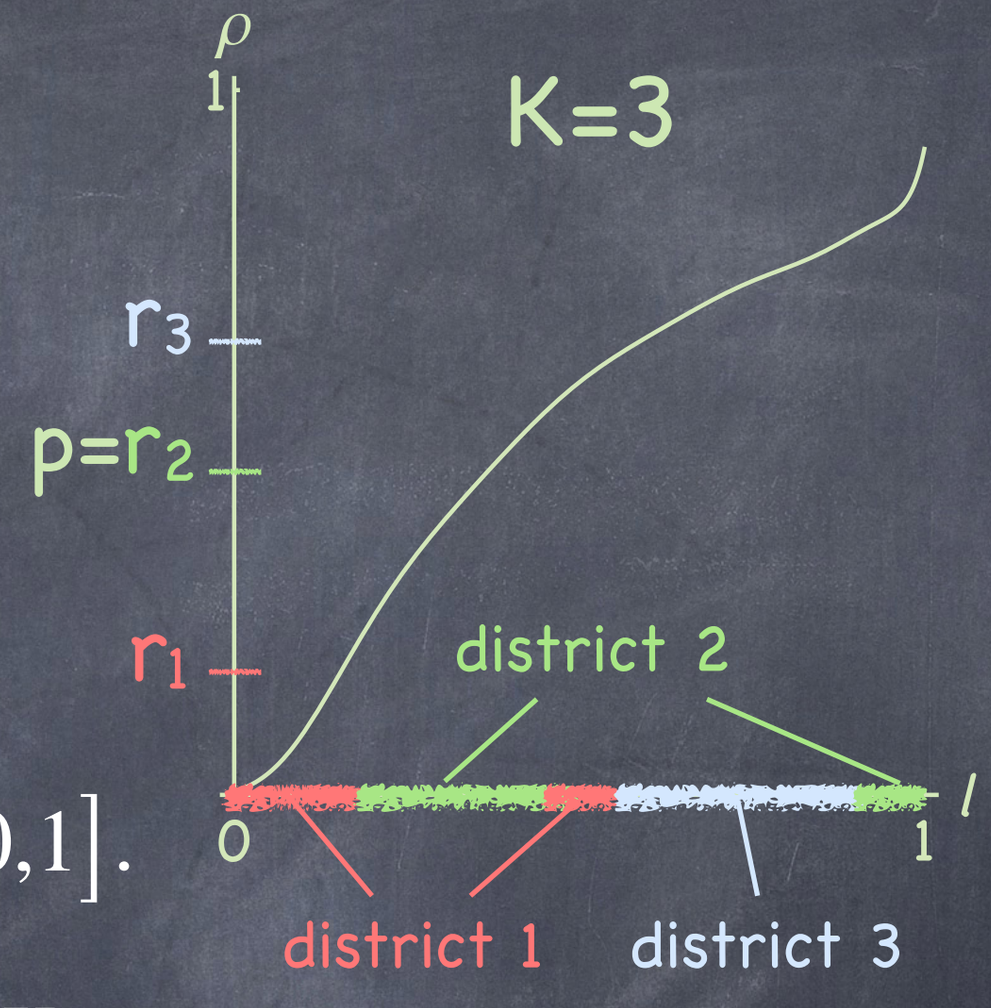

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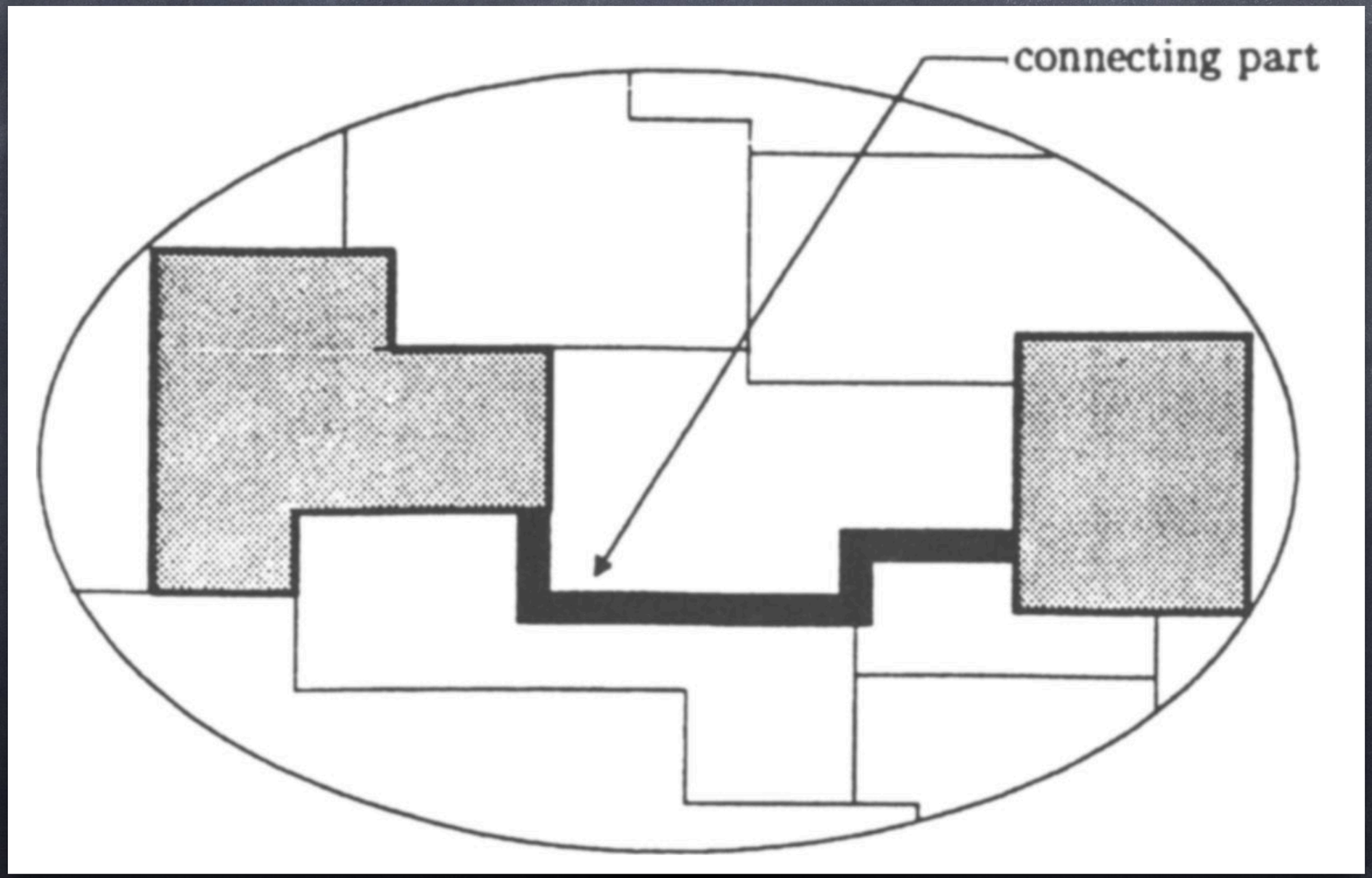
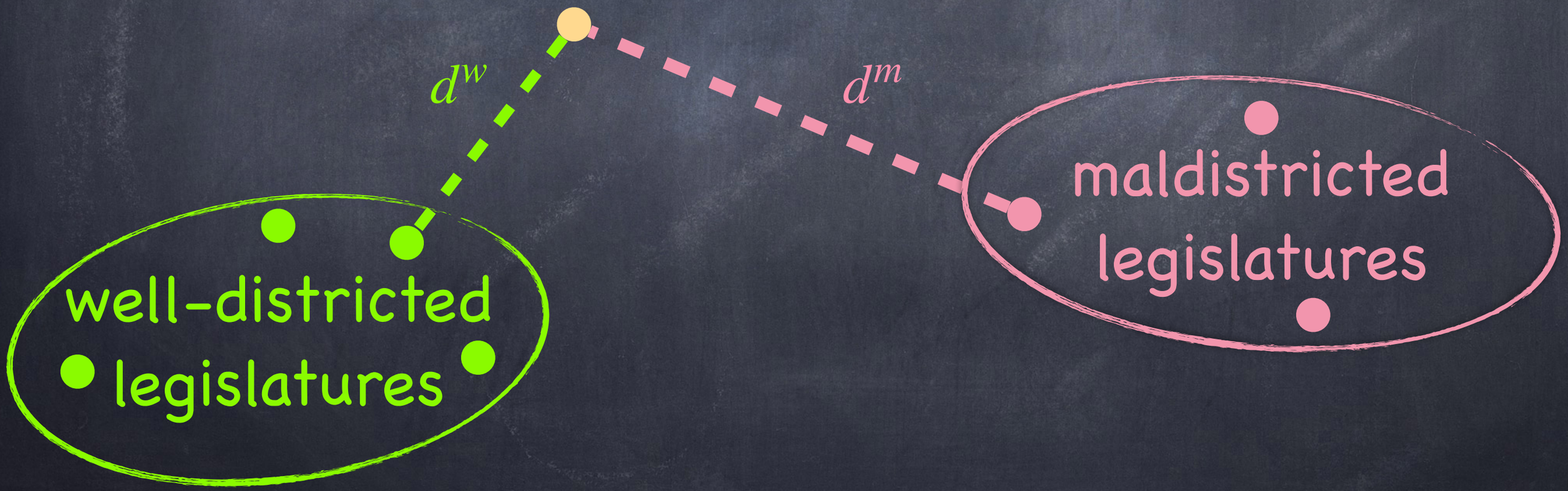
Georgia in 1D

Sherstyuk (1998) on contiguity-irrelevance in "How to gerrymander: A formal analysis"



$$\text{Index of Maldistricting} = \frac{1}{1 + d^m / d^w}$$

an observed legislature



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implementable

legislatures

Implementable Legislatures

Definition A legislature $(r_1, r_2, \ldots, r_K)$ is implementable if it is induced by a district map $g : [0,1] \to \{1,2,\ldots,K\}$ that maps locations into equisized districts.

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cf. Gentzkow and Kamenica (2016)
"Rothschild-Stiglitz Approach to Bayesian Persuasion"

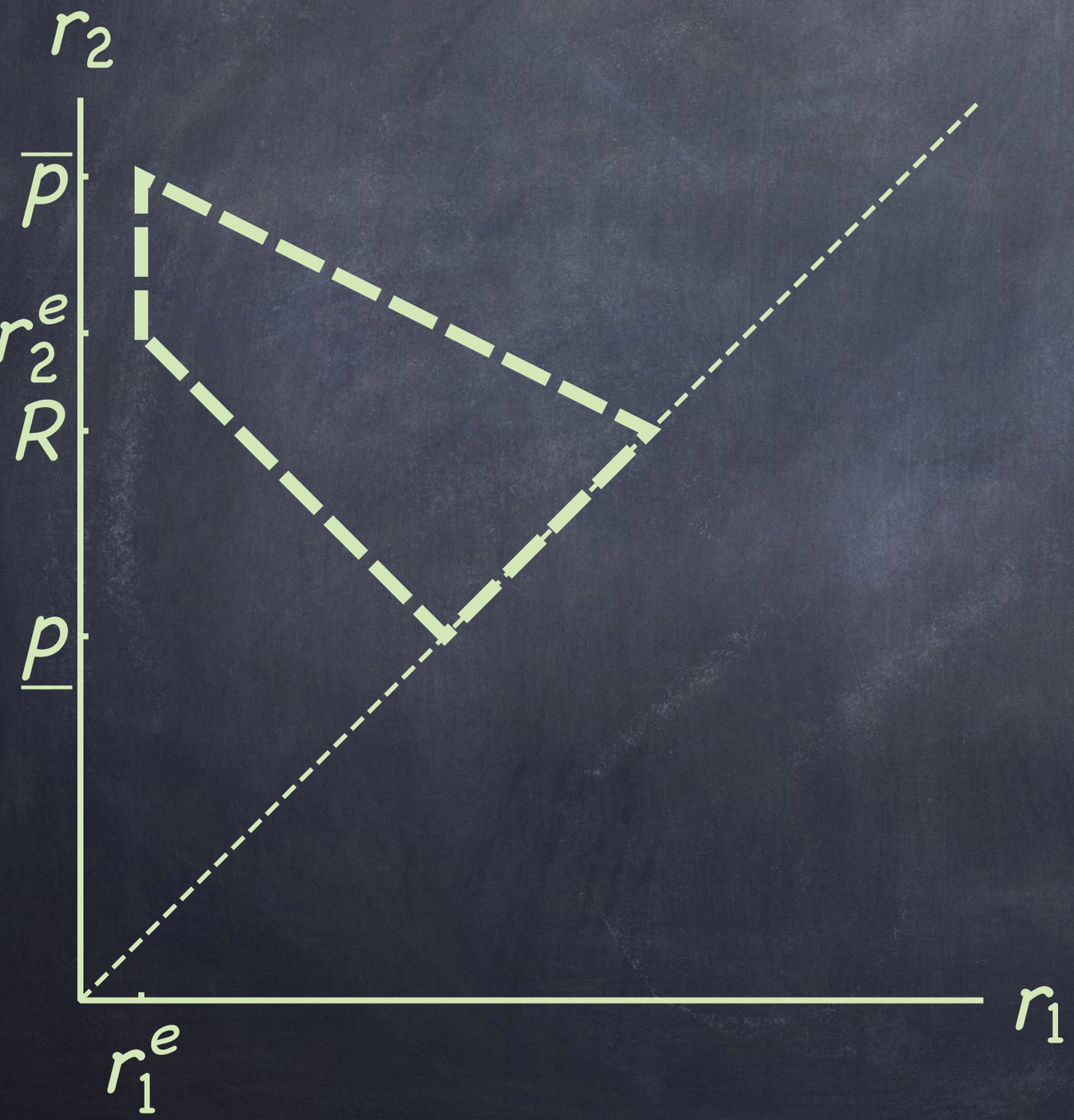
contemporaneous: Kolotilin and Wolitzky (2020)

"The Economics of Partisan Gerrymandering"

Normalisation

$$r_1 \leq r_2 \leq \ldots \leq r_K$$

(Projected) Polytope of Implementable Legislatures, K=3



$\bar{p}$: maximal policy

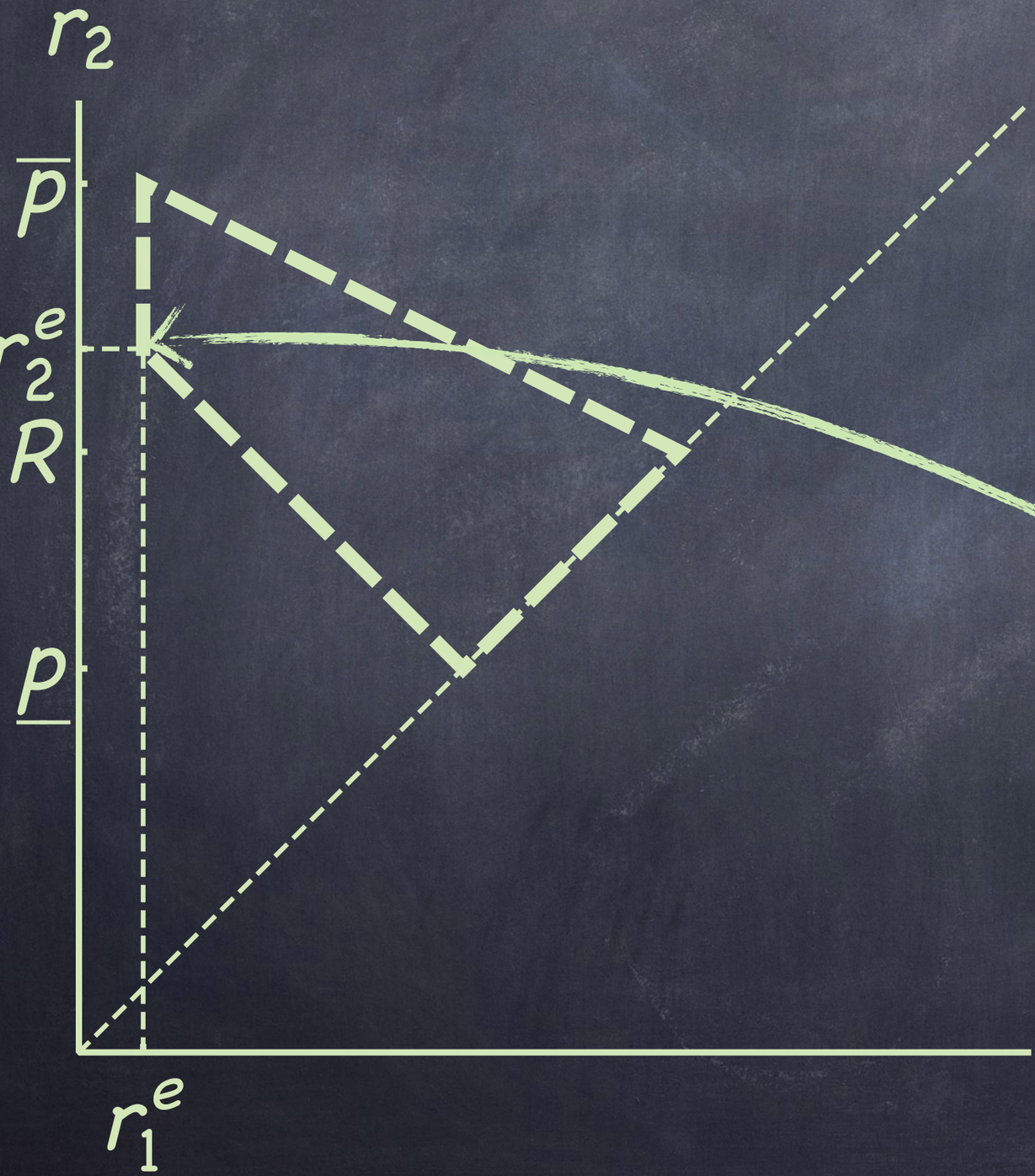
$\underline{p}$: minimal policy

R: (state) mean ideology

$r_3 = 3R - r_1 - r_2$

r^e: extreme legislature

(Projected) Polytope of Implementable Legislatures, K=3



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$r_3 = 3R - r_1 - r_2$

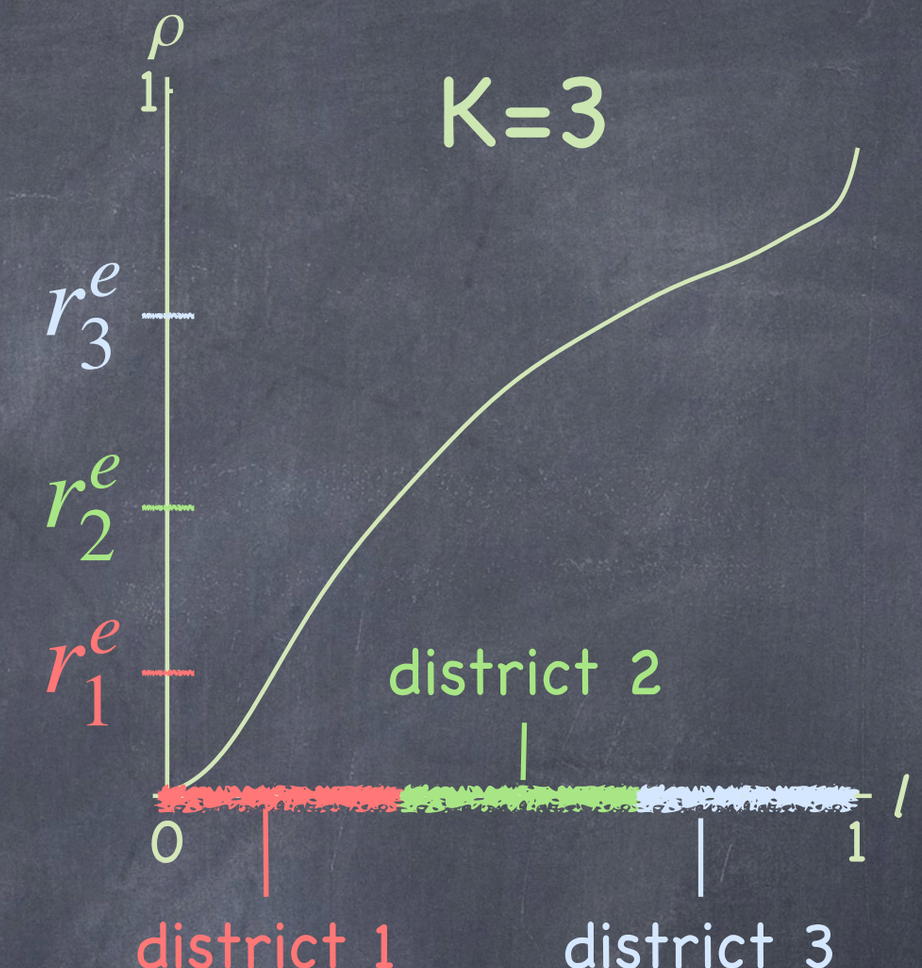
r^e: extreme legislature

Characterization

Proposition The following are equivalent:

1. r is an implementable legislature.
2. r is "less informative" about voter ideology than the extreme legislature.
3. r is majorized by the extreme legislature.
4. r lies in a 2^{K-1}-vertex polytope.

Proposition's part (1) $\Longleftrightarrow$ part (2) A legislature is implementable if and only if it is less informative about voter ideology than the extreme legislature.



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A voter's district is a less precise signal about his ideology.

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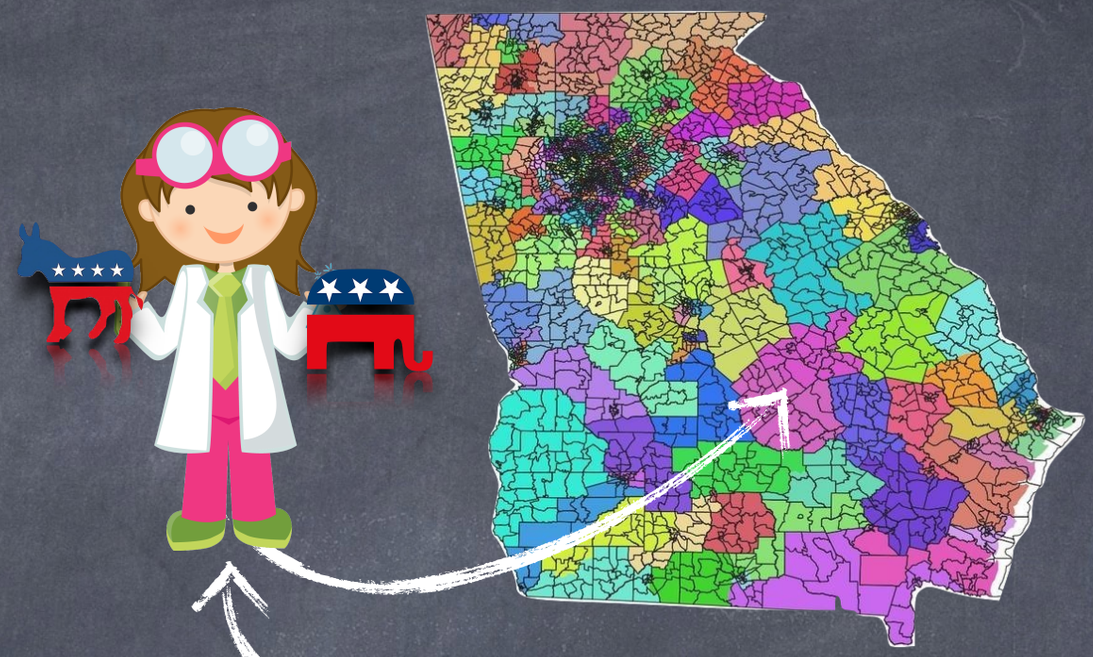

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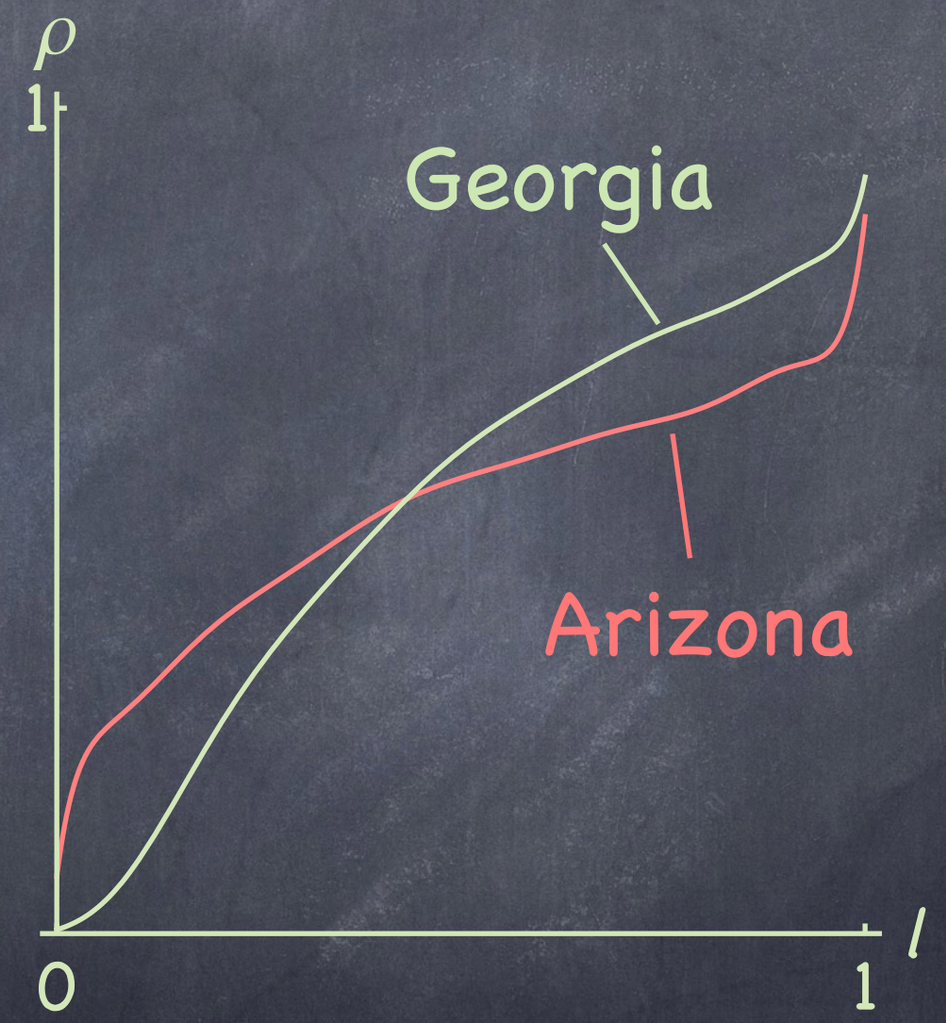
Proposition's part (1) $\iff$ part (2) A legislature is implementable if and only if it is less informative about voter ideology than the extreme legislature.

vary (by varying ρ)
for comparative statics

Comparative Statics

Corollary More is implementable when a location is more informative about ideology (SOSD shift in ρ).

Comparative Statics



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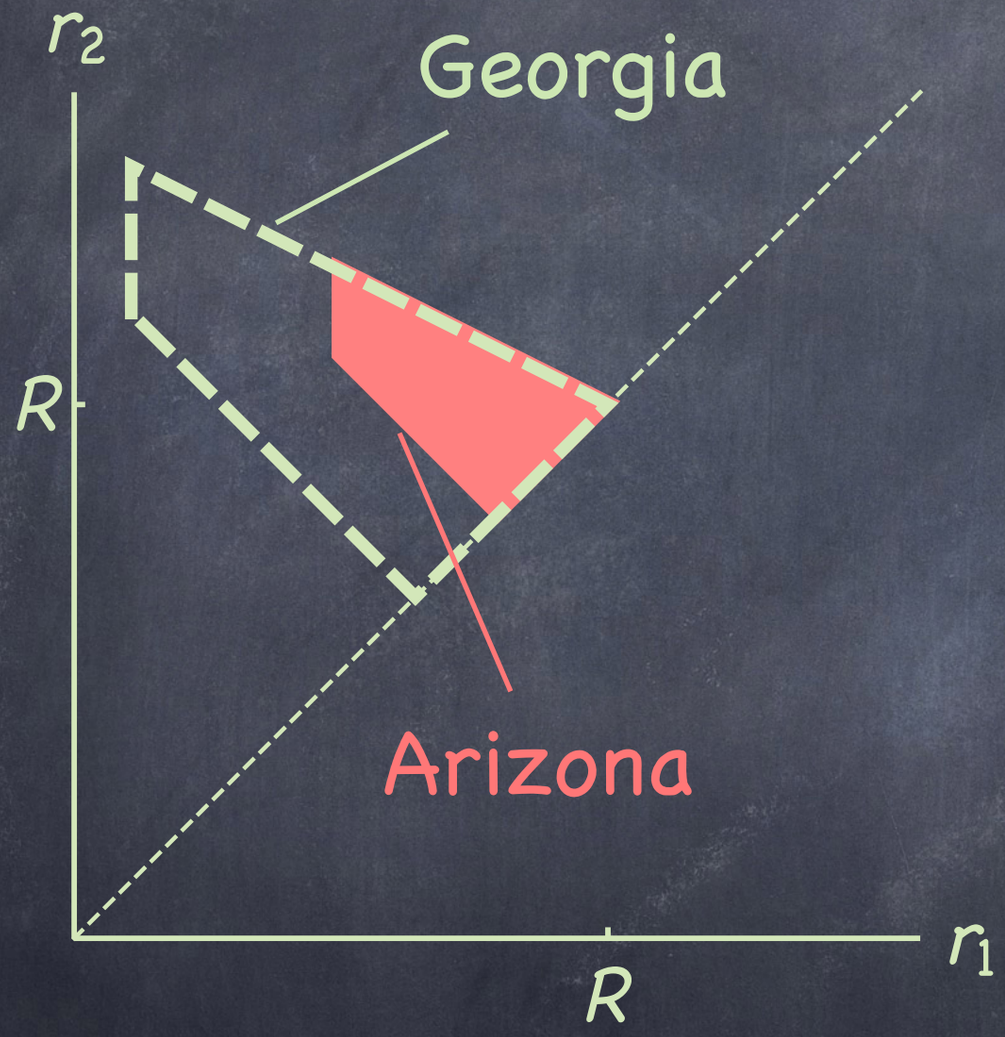
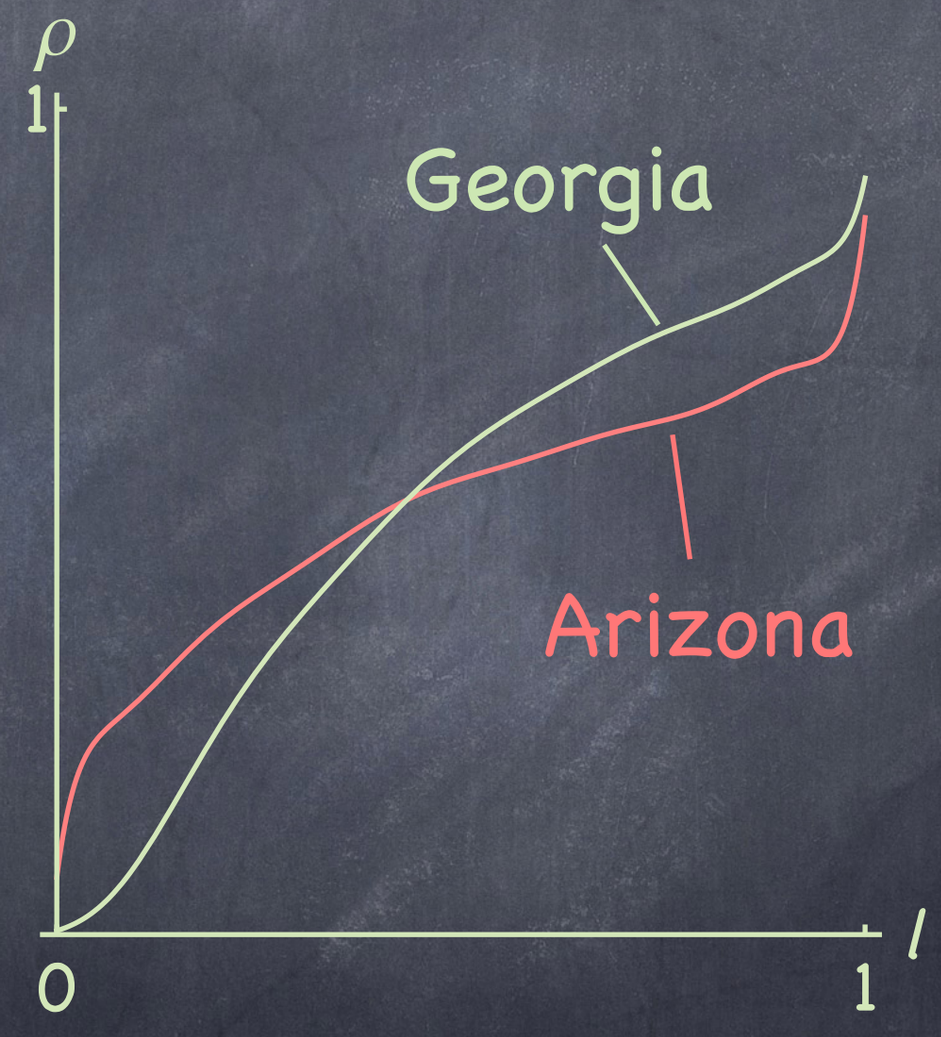
Comparative Statics





Corollary More is implementable when a location is more informative about ideology (SOSD shift in ρ).

Proposition's part (1) $\Longleftrightarrow$ part (3) r is implementable if and only if r is majorized by r^e:

$$\sum_{k=1}^{K} r_k = \sum_{k=1}^{K} r_k^e$$

$$\sum_{k=1}^{m} r_k \geq \sum_{k=1}^{m} r_k^e, \quad m < K\,.$$

part (3) $\Longleftrightarrow$ part (4) by Hoffman (1969)

Proposition's part (1) $\iff$ part (4) r is implementable if and only if r lies in a 2^{K-1}-vertex polytope.

Example Partition $\{1,2,\ldots,K\}$ for $K=3$:

$$\{\{1\},\{2\},\{3\}\} \to \text{vertex } \left(r_1^e, r_2^e, r_3^e\right)$$

$$\{\{1,2\},\{3\}\} \to \left(\frac{r_1^e + r_2^e}{2}, \frac{r_1^e + r_2^e}{2}, r_3^e\right)$$

$$\{\{1\},\{2,3\}\} \to \left(r_1^e, \frac{r_2^e + r_3^e}{2}, \frac{r_2^e + r_3^e}{2}\right)$$

~~$\{\{1,3\},\{2\}\}$~~

$$\{\{1,2,3\}\} \to \left(\frac{r_1^e + r_2^e + r_3^e}{3}, \frac{r_1^e + r_2^e + r_3^e}{3}, \frac{r_1^e + r_2^e + r_3^e}{3}\right)$$

$$\text{Index of Maldistricting} = \frac{1}{1 + d^m / d^w}$$

an observed legislature

d^w

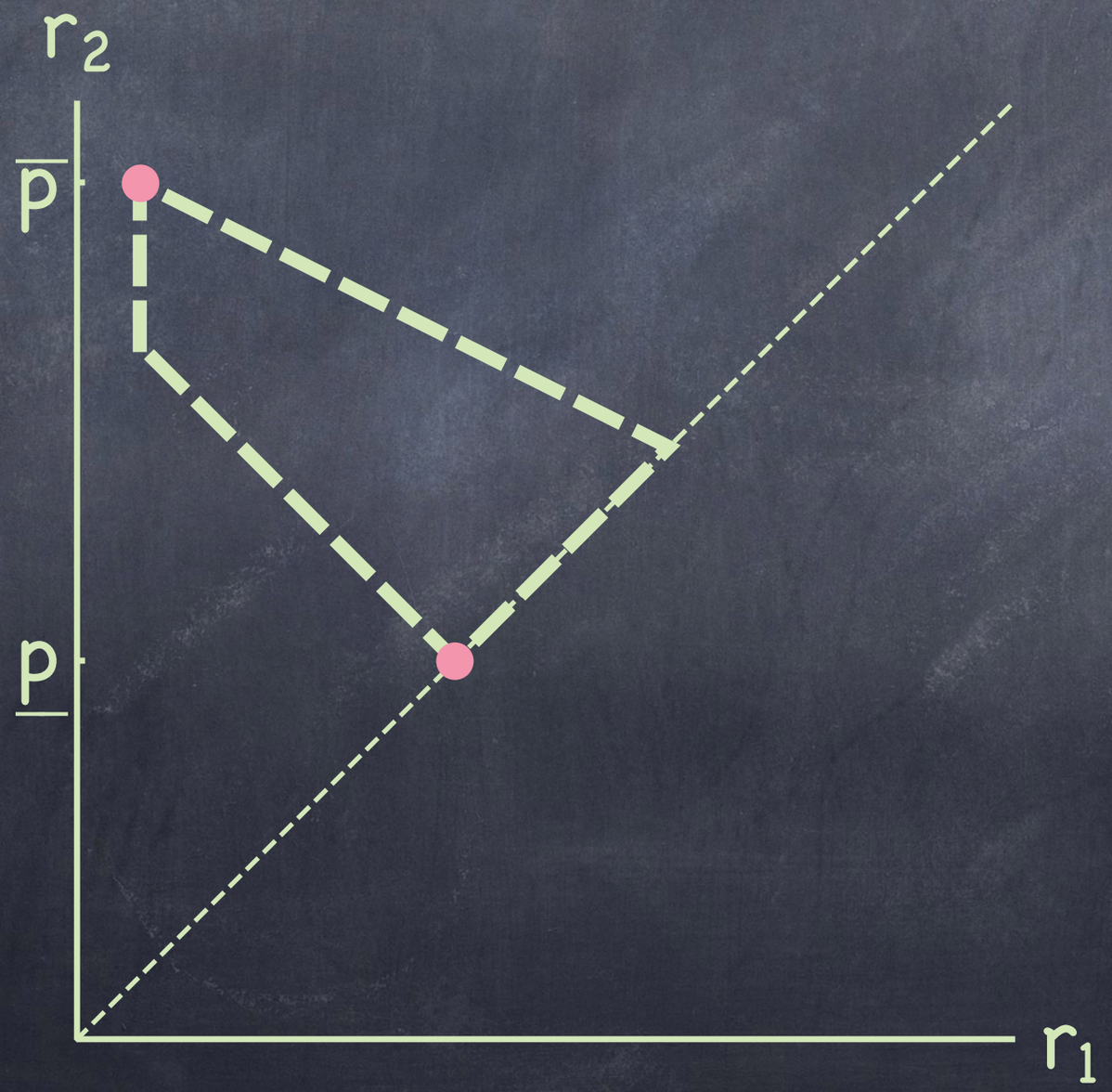
d^m

well-districted legislatures

maldistricted legislatures

Maldistricted Legislatures

extremize the policy: $p \in \left\{ \bar{p}, \underline{p} \right\}$



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an observed legislature

d^w

d^m

well-districted legislatures

maldistricted legislatures

Well-Districted Legislatures

maximise utilitarian welfare for some $\gamma \in [0,1]$:

$$-\gamma \int \left[\rho(l) (1-p)^2 + (1-\rho(l)) p^2 \right] dl$$

$$-(1-\gamma) \int \left[\rho(l) \left(1 - r_{g(l)}\right)^2 + (1-\rho(l)) r_{g(l)}^2 \right] dl .$$

$r_{g(l)}$: district representative's ideology at l

Well-Districted Legislatures

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disutility from policy

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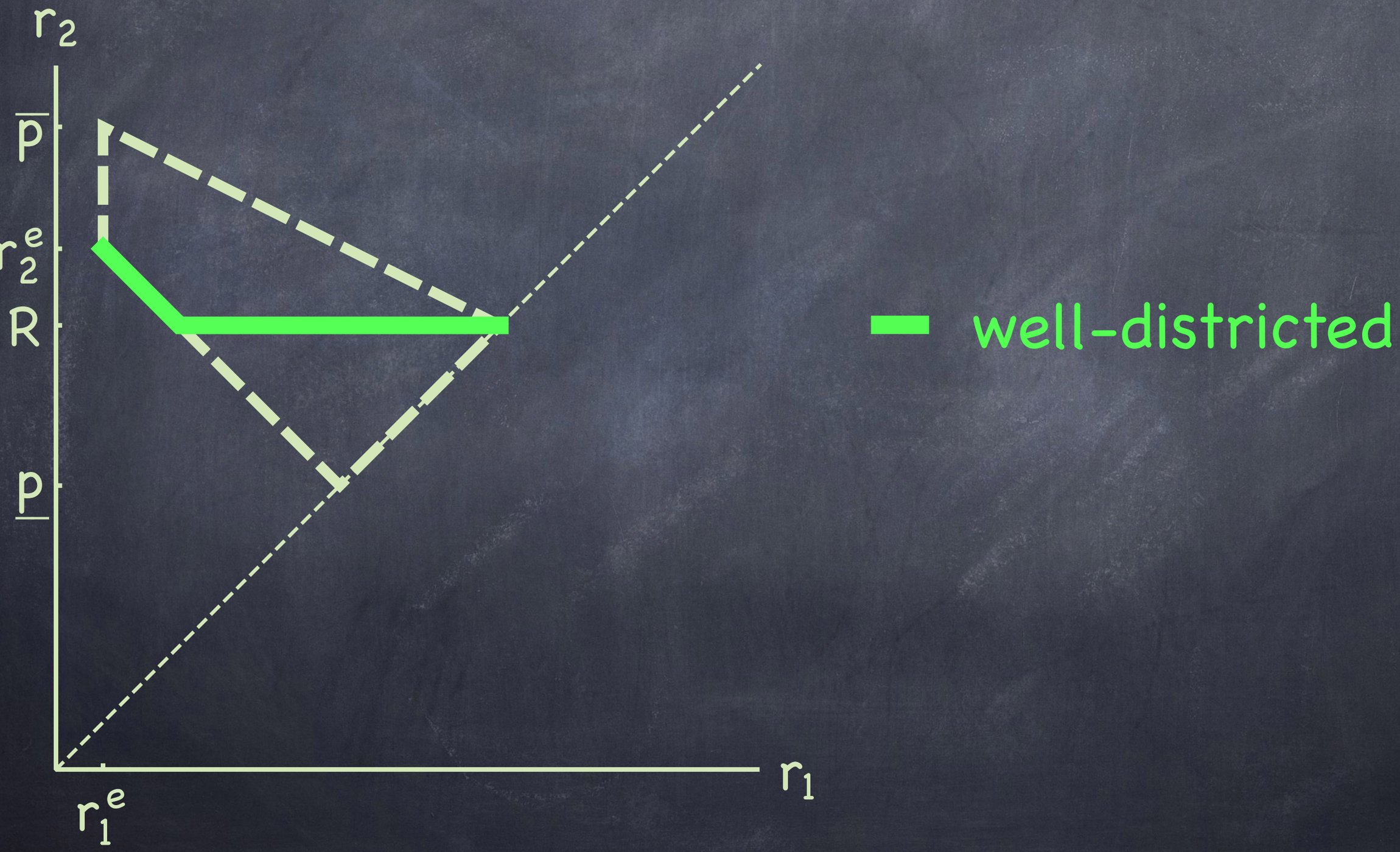
disutility from misrepresentation

$r_{g(l)}$: district representative's ideology at l

Well-Districted Legislatures



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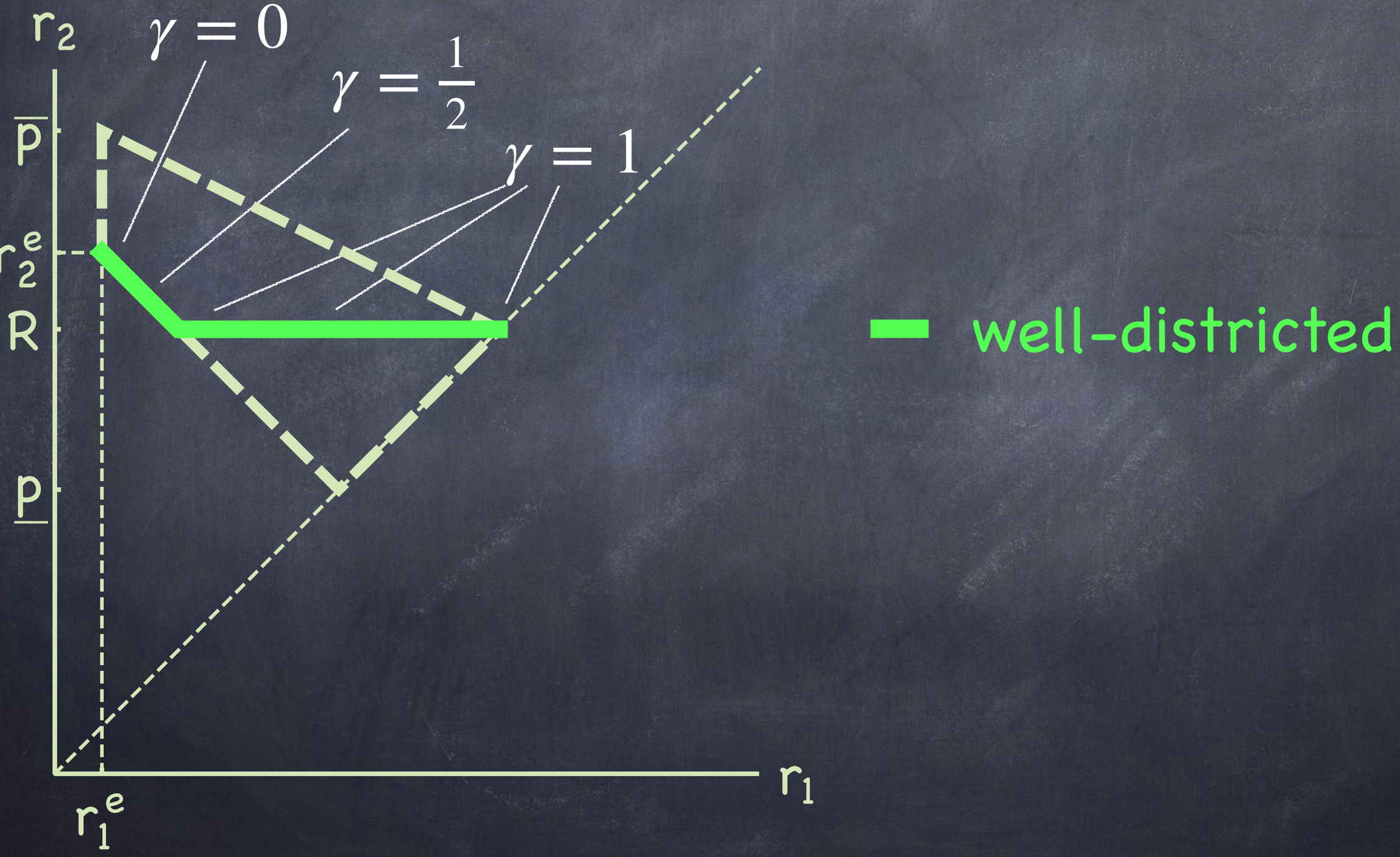
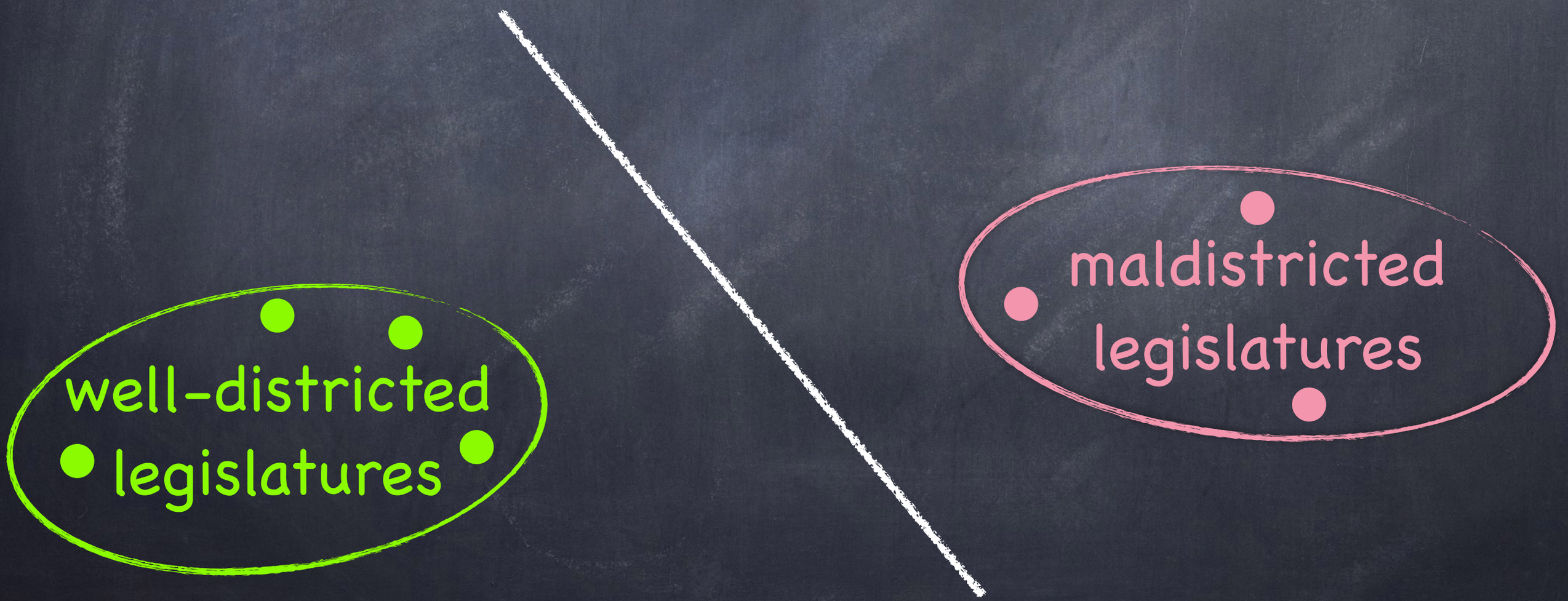


Fact: Disjoint Sets



$$\text{Index of Maldistricting} = \frac{1}{1 + d^m / d^w}$$

an observed legislature

d^w

d^m

well-districted legislatures

maldistricted legislatures

Distance Between Legislatures

Idea Observed legislature
= intended legislature + noise.

Story

1. The districter draws a map g, intending either a well-districted or a maldistricted legislature.

2. Voters move randomly.

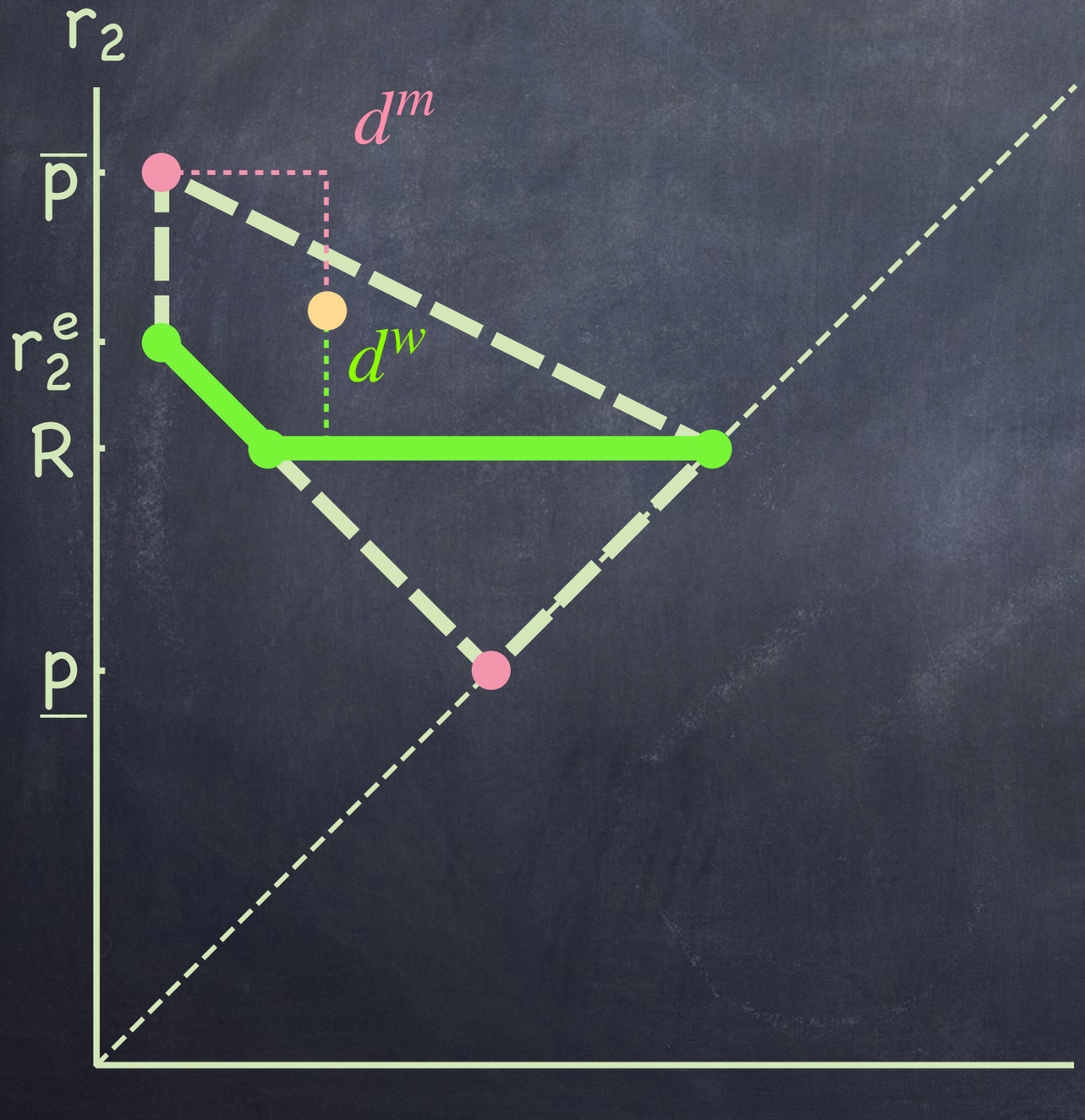
3. Voters vote. A legislature is observed.

Distance Between Legislatures

Definition The distance $d(r, r')$ between legislatures r and r' is the minimal (over g) measure of voters who must move for the district map g to induce r instead of r'.

Fact $$d(r, r') = \sum_{k=1}^{K} \left| r_k - r'_k \right|$$ (L_1-distance).

The Index Illustrated



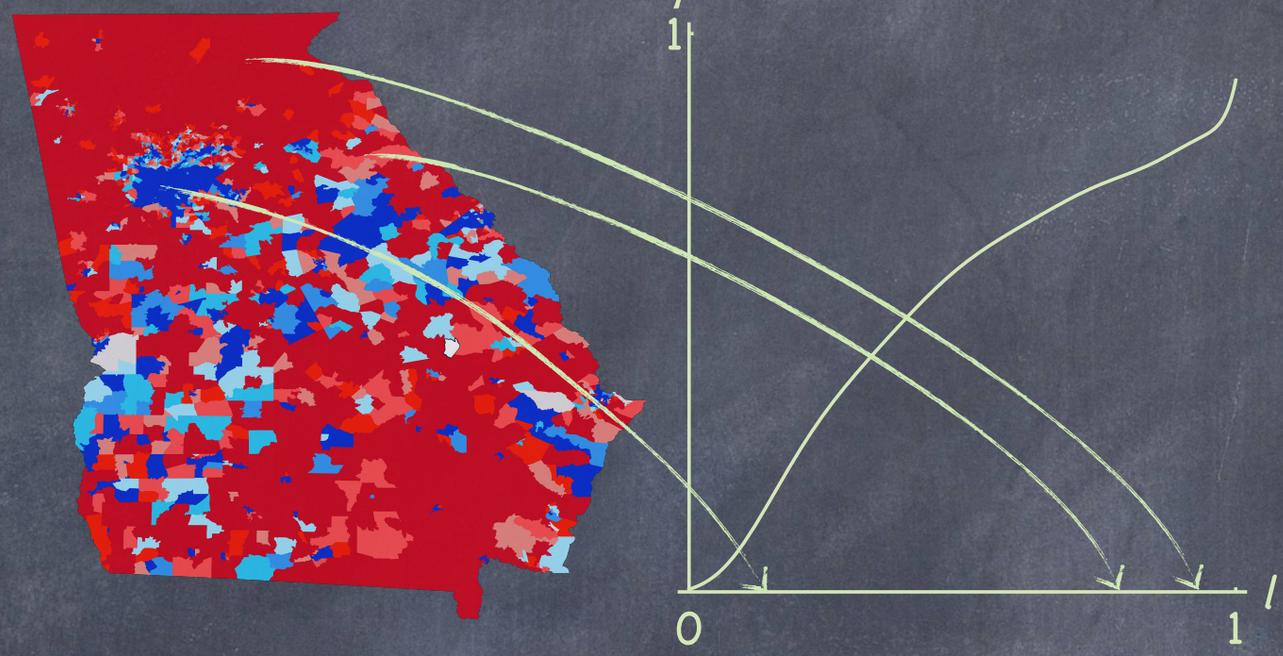
- maldistricted
- observed
- well-districted

$$\text{index} = \frac{1}{1 + d^m / d^w}$$

More in the Paper

- characterisation of well-districted legislatures
- alternative motives to maldistrict:
  - incumbent protection
  - party seat maximization

Empirical Analysis



l = electoral precinct

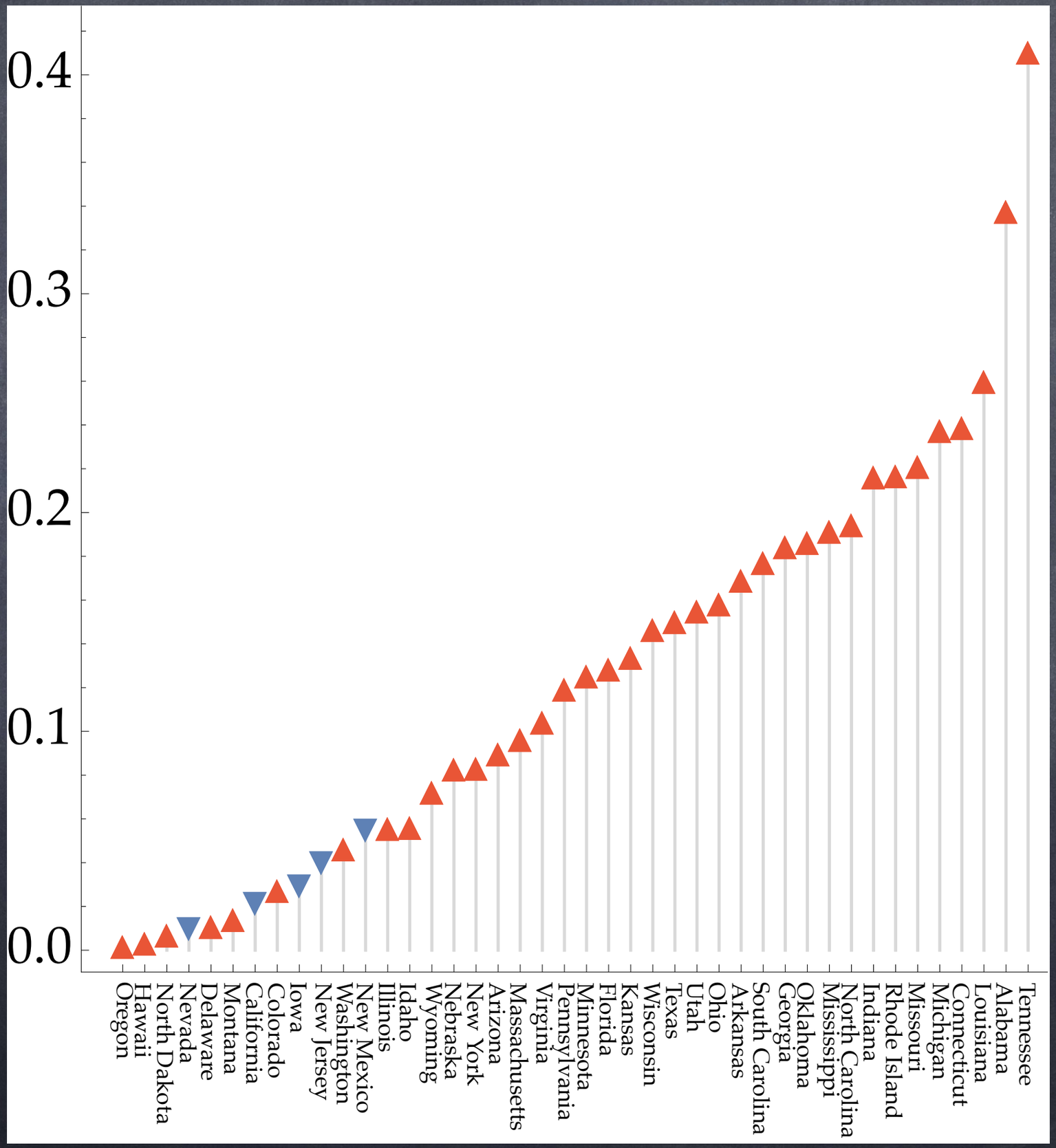
$$r_k = \frac{\#(\text{McCain votes})}{\#(\text{McCain votes}) + \#(\text{Obama votes})}$$

$(r_1, r_2, \ldots, r_K)$ = the house of representatives

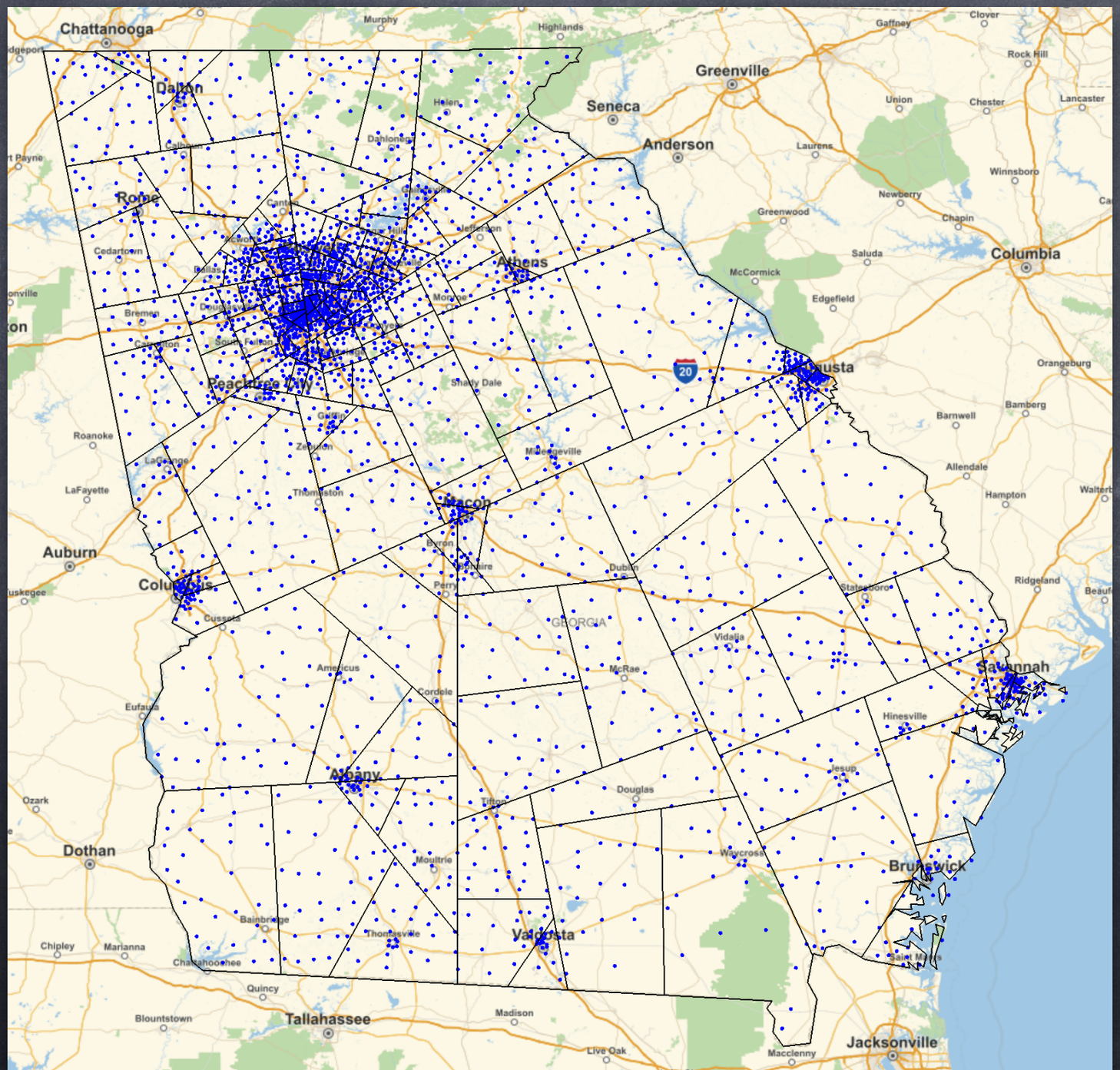
Findings

- Observed maps have a Republican bias.
- So do "natural maps" (defined shortly).
- Courts demand maps redrawn $\Longrightarrow$ index $\searrow$.

Index for Observed Maps



A Natural Map (via the shortest splitline)



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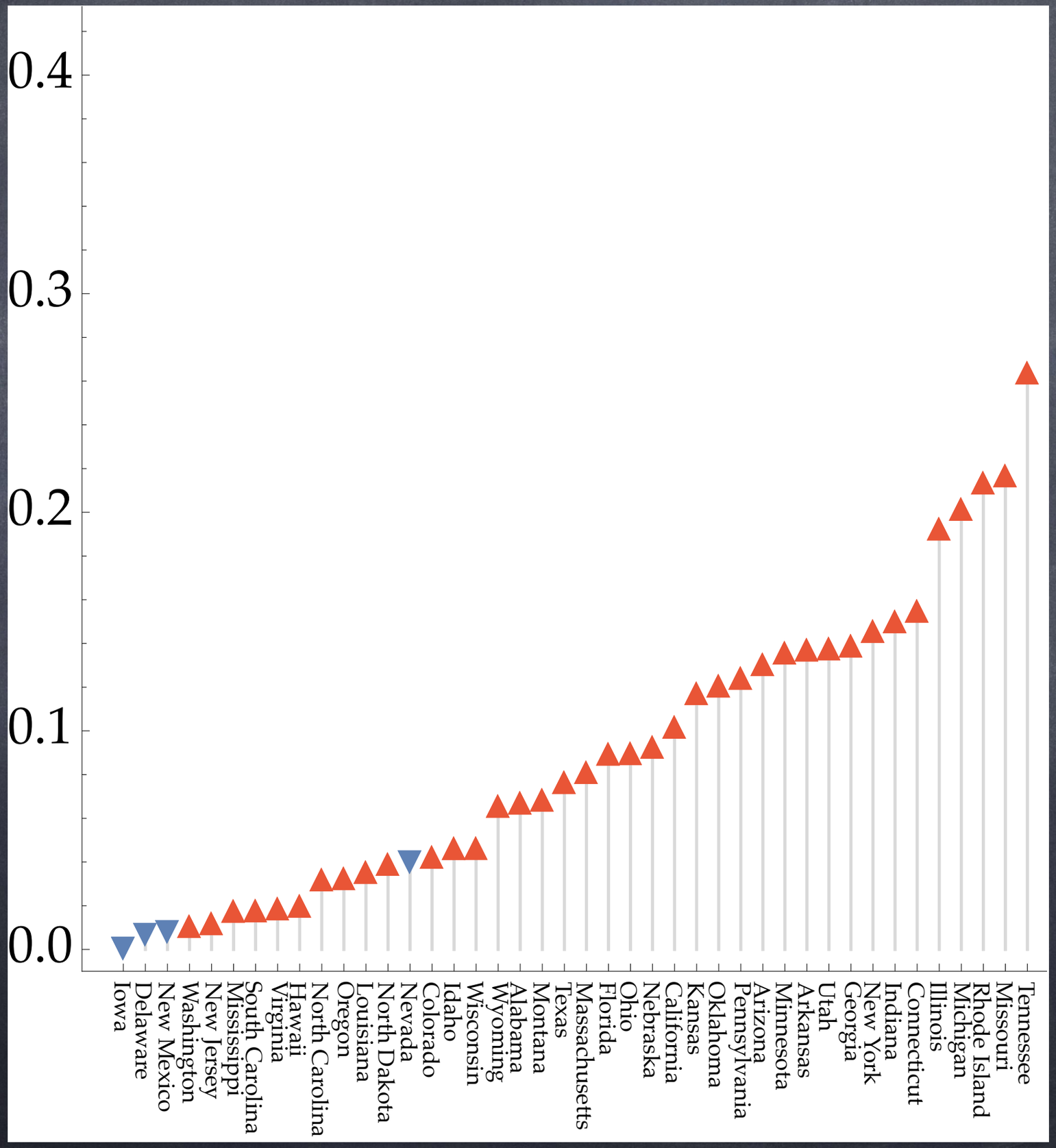


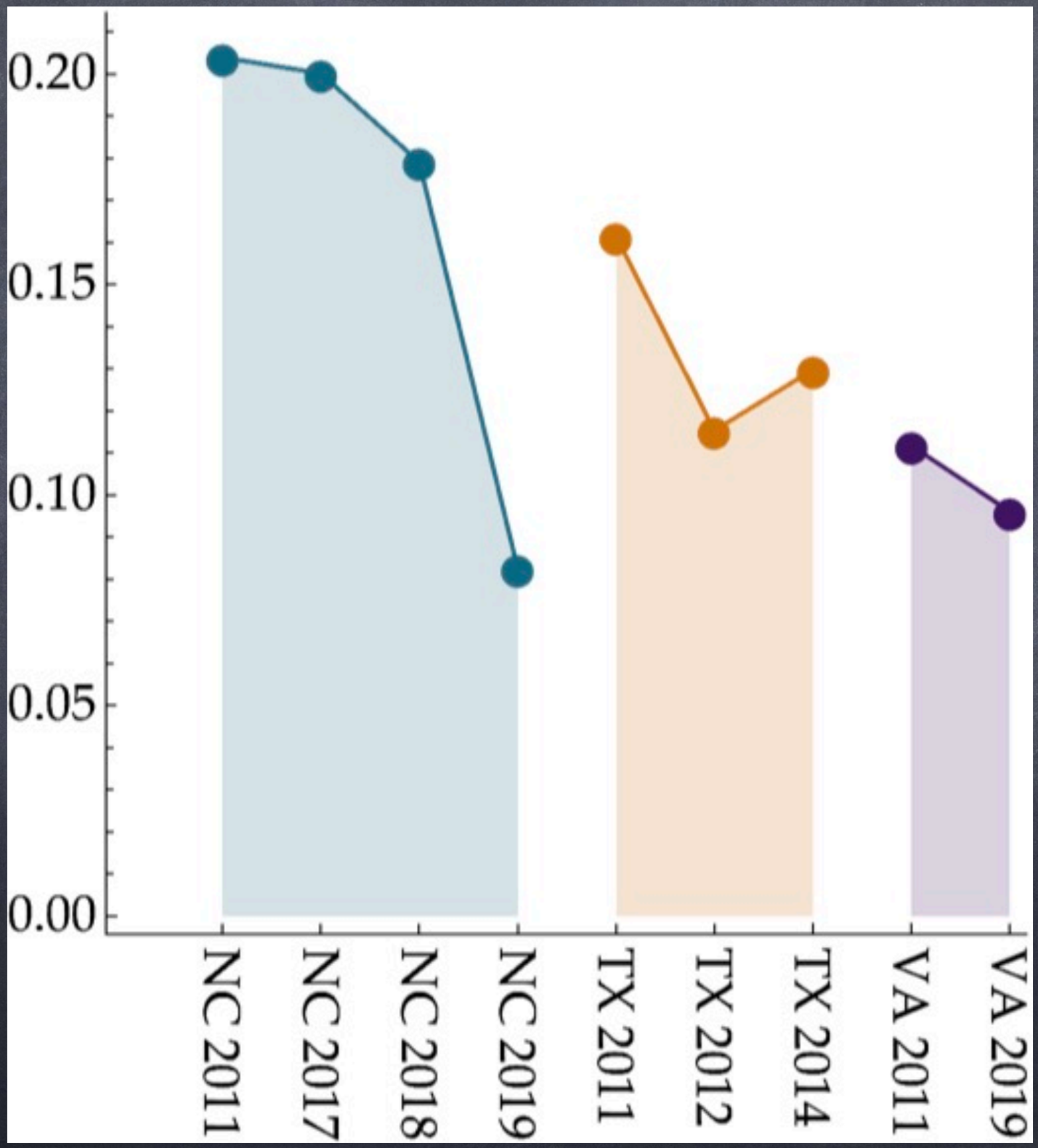
Intuition

1. Ideologically similar precincts are adjacent.
2. Republican precincts are more ideologically alike than Democratic ones (convex ρ).

$\Longrightarrow$

Natural maps favour Republicans.

Litigated Maps



Concluding Remarks

- A tractable model, whose parameters have direct counterparts in data
- A modular index
- Modules in our paper:
  - maldistricting = {party welfare}
  - well-districting = {voter welfare}

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	party welfare	voter welfare
policy	Gilligan and Matsusaka (2006)	Coate and Knight (2006)
repre-sentation	Owen and Grofman (1988) Gilligan and Matsusaka (1999) Friedman and Holden (2008)	Chamberlin and Courant (1983) Monroe (1995)

Thank You