International Trade, Income Distribution and Welfare

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January 2016

Abstract

This paper studies the relationship between income distribution and international integration in a canonical trade setting with one change. In the standard model prices are solely a function of (constant) marginal costs and (constant) elasticities, implying that information on consumer income is of no value to a typical firm. To allow a more realistic role for consumer level information, a firm's strategy space is expanded to include non-linear prices (i.e. potential to offer product lines). In equilibrium firms use information on income distribution to design a product for each income class, with prices that induce each group to optimally select their intended product. Equilibrium designs involve some items below the first best while others exceed it. When countries with differing income distributions integrate, this has implications for the size of these distortions, influencing the gains from trade both within and across countries. These implications are quantified and shown to be potentially significant factors affecting welfare outcome from integration – with the consequences more pronounced at lower trade costs. The structure of trade, expenditure patterns and prices which emerge also match a range of empirical patterns. These results are driven by firm strategy based on income difference alone as preferences are assumed to be identical and homothetic across countries, placing the distribution of income at the center of the analysis.

Key Words: Intra-industry trade, monopolistic competition, Inequality
JEL Classifications: F12, F15, F60

1 Introduction

Models of international trade have traditionally used richness/heterogeneity on the supply side to gain insight into why countries trade and the likely implications of integration. Any heterogeneity on the demand side is usually suppressed by imposing identical and homothetic preferences on consumers. While analytically convenient, this assumption leads models of international trade to ignore one of the most pronounced differences across individuals, regions and countries: income and expenditure patterns. How to incorporate this variation and analyze its implications represents a persistent challenge to the literature.

To address this issue the typical approach starts by relaxing the assumption of homotheticity, freeing up expenditure shares to depend not just on relative prices but also income levels.[1] In essence this assumes that individuals with different incomes are hardwired to make different choices – reducing the problem to choosing an appropriate preference specification. However, by focusing on preference structure the literature has overlooked an alternative possibility – firms may also be interested in income variation across consumers and try to exploit this variation to increase profits. The set of techniques a firm can employ to do this is relatively rich but can be broadly summarized as a form of discrimination – charging different prices, offering different qualities and/or selling different sizes. Whether or not preferences are homothetic, firms have an incentive to *induce* consumers with different incomes to make different choices. It is then entirely possible for high and low income consumers to face exactly the same offerings from a firm but end up choosing differently. These choices can lead to natural variation in expenditure patterns, even among consumers with homothetic preferences. Moreover, discrimination generally does have implications for welfare outcomes. The open question is whether international integration tends to enhance the positive aspects of discrimination or magnify the negative ones.

The objective of this paper is to answer this question and explore the implications of income differences both within and across countries for international trade. In contrast to the non-homothetic literature, preferences will have the standard features of being identical

[1] Dingel (2015), Fajgelbaum and Khandelwal (2015), Simonovska (2015), Caron et al. (2014), Markusen (2013), Fajgelbaum et al. (2011), Fieler (2011), Hallak (2010), Choi et al. (2009) and Mitra and Trindade (2005) to list only some of the recent contributions.

and homothetic for all consumers. This is done solely to distinguish the analysis from the previous literature.[2] To further highlight the differences and simplify the analysis, the single sector structure of Krugman (1980) is adopted.[3]

The key feature that differentiates this paper from the previous literature is a focus on how a firm views and evaluates information relating to the distribution of income.[4] In the standard analysis firms are assumed to use linear prices, implying they are only interested in the curvature of the residual demand function when formulating their optimal strategies. Moreover, with Spence-Dixit-Stiglitz (SDS) preferences the elasticity of residual demand is constant and the same for all consumers. The combination of these two assumptions has relatively extreme implications for how firms respond as we enrich their information set. For example, if a firm was suddenly able to observe the income levels of each consumer, the best they could do under linear pricing is implement third degree price discrimination. However, with the elasticity of demand independent of income and the same for all consumers, a firm would not change their behavior, continuing to charge the same price per unit to all types. Contrary to what might be imagined, this additional consumer level information would essentially be of no value to a firm.

To allow for a more plausible reaction a firm's strategy space is expanded to include non-linear prices. We follow the typical approach by assuming that a firm knows the distribution of income but not an individual consumer's income. More formally this is a setting where a firm implements indirect discrimination (aka second degree price discrimination). If a firm optimally chooses to exploit this information, it does so through the design of a menu of options offered to a consumer (or more broadly a product line).[5]

A particularly neat illustration of a product line is the iPad range.[6] The initial offerings

[2]The mechanism developed below can also be analyzed with non-homothetic preferences.

[3]All firms have the same technology and are considered symmetrically by consumers.

[4]See Antras et al. (2015) for an analysis where the government has preferences over the distribution of income.

[5]This product line is associated with goods of different characteristics/quality and prices that induce consumers with different income to select different options from the product line. In this sense a firm offers multiple products. However, this view of multi-product firms is much narrower than usual perspective employed in the international trade literature. See for example Bernard et al. (2011).

[6]Since Apple launched the iPad in 2010 there has been a proliferation of firms supplying tablet computers, all of them using product lines. The website www.tabletcompare.net lists the top 14 brands that supply over 100 different tablets between them.

only had one dimension of variation, memory: 16GB, 32GB and 64GB. For the first two sizes the prices are \$499 and \$599. If we use these prices to linearly project the price of a 64GB machine we arrive at \$399 + \$6.25(64) = \$799, which is \$100 more than the actual price of \$699.[7] What's behind this pricing behavior – differences in cost, elasticity or something else? Industry sources confirm that the marginal cost of a GB is constant, so costs can't explain the variation. Similarly the prices imply that the elasticity of demand is increasing in memory size, contrary to the typical assumption.[8] Using the implied elasticity from the 16GB machine suggests that the 64GB iPad would be priced over \$1100. Evidently a simple mark-up formula isn't employed, leaving scope for more sophisticated pricing strategies underlying product menus and their design. Moreover, the widespread use of product lines raises a general question about their welfare implications, not only for a single product, but also at an aggregate level. A natural way to capture the broader welfare consequences of indirect discrimination is through a general equilibrium framework – an approach which we adopt.[9]

An important characteristic of indirect discrimination is that firm behavior and the resulting monopolistically competitive equilibrium is now not just a function of the curvature of the demand functions but also their position. Specifically, the profit maximizing menu trades off the desire to extract rents from an income group (by offering a design close to the first best) against the cost that this provides an enhanced outside option for another income group/s. This trade-off is resolved by the relative size and frequency of incomes groups. As a consequence the distribution of income is a fundamental determinant of the design of the equilibrium product line. A feature of this equilibrium is that product design is distorted relative to the first best. In general, products designed for low income types are below the first best, while the products targeted to the high income groups are above the first best.[10] It then follows that welfare differences are more exaggerated than income differences.

[7] To put this number in context, the additional assembly cost of onshoring the closely related iPhone has been estimated at around \$65, "How the US lost out on iPhone work," *The New York Times*, 21 January, 2012.

[8] See Fajgelbaum et al. (2011) or Hummels and Lugovskyy (2009).

[9] Aside from electronics, many other sectors use product lines but untangling cost and markup changes is often not straightforward. Another example where marginal cost is likely to be constant is the perfume industry. For example, Chanel N^o 5 – the best selling perfume in the world – is sold in three sizes, with the price per oz of the largest bottle 35% lower than the smallest bottle. This translates to a saving of \$175 for buying the larger bottle.

[10] Monopoly models of indirect discrimination predict the first result but not the second. See for example Maskin and Riley (1984).

The critical role of the distribution of income in this outcome immediately implies that the integration of two countries with different income distributions alters product line design and consequently welfare. Insight into the implications are clearest when countries can be ranked in terms of income distribution. In particular, if a country's income distribution dominates the global distribution then the gains from free trade will be larger than predicted by the sufficient static measure developed by Arkolakis et al. (2012) (henceforth ACR).[11] Moreover, these gains are disproportionately concentrated at the bottom end of the income distribution. In this case, trade reduces the distortions from indirect discrimination and the benefits are felt across the entire distribution of income. The opposite occurs in a country whose income distribution is dominated by the global distribution, as trade adds to the distortions from indirect discrimination. Since these distortions are not present in the standard model of international trade they represent a new dimension of the welfare analysis.

Another insight follows from decomposing the gains from trade into those derived from additional varieties and those associated with the design of the menu of choices. Critically, these two components respond differentially to the level of trade costs with important implications for trade liberalization. In particular, when trade barriers are relatively high, incremental liberalization is primarily about reducing the costs of serving a market and has little impact on menu design. Thus, for high trade barriers the gains from gradual liberalization follow a pattern familiar from the standard model and consistent with ACR. However, once trade barriers become sufficiently low, the potential for international arbitrage triggers a convergence in product design across countries.[12] Since not all types in all countries gain from design convergence, there is potential for a gradual process of trade liberalization to stall – at the margin the negative effects for product design in one country can outweigh further savings from lower trade costs.

To examine the potential relevance of this mechanism, the model is quantified on the same data utilized by Costinot and Rodriguez-Clare (2014) (hereafter CRC). The focus of CRC was on moving from an observed trade equilibrium to autarky. The commonality of the two

[11] Given the primitives of the model are from Krugman (1980), ACR predict that a sufficient static for welfare gains can be constructed based on the domestic expenditure share and the trade elasticity.

[12] See Mrázová and Neary (2014) for a discussion of market integration/segmentation in a linear price setting.

frameworks is evident when trade costs are relatively high since both models provide the same welfare outcomes. However, we are also interested in the consequences of additional liberalization: moving from the observed trade equilibrium to one of full integration. That is, an exercise concerning greater integration, not moving to autarky. Moving trade costs in this direction generates differences between the two frameworks. In particular, using country level income distribution information reveals that in only 5 countries will all income groups benefit ambiguously from design changes induced by integration. In these five countries gains are magnified relative to ACR. In contrast, the remaining 27 countries all have at least one income group that could be adversely affected by the negative consequences of menu redesign.

The relevance of these negative consequences depend on whether they offset the positive gains associated with the change in the domestic expenditure share. For 13 countries, the change in the domestic expenditure share would need to exceed 10 percentage points – requiring a larger change than observed for any of these countries between 1995 to 2008. Consequently, in these countries there is at least one income group that would prefer the initial trade equilibrium to full integration. Moreover the findings are similar when additional sectors are introduced. These results suggest that if the negative consequences of standardizing global product lines are disproportionally associated with future liberalization, then a number of countries may resist efforts to fully integrate markets through reduction in trade barriers and/or harmonization of standards/regulations.

The model also has a number of predictions for observable outcomes that allow it to be evaluated relative to empirical findings. The first relates to the equilibrium price distribution, which is shown to be increasing and concave in income. This is consistent with previous findings in the literature – outcomes which have been interpreted as inconsistent with existing trade models (see Manova and Zhang (2012)).[13] The second prediction relates to the specification of the gravity equation. In particular, the model predicts higher trade between countries with similar per capita income (holding dispersion constant) and higher trade between countries with similar income dispersion (holding per capita income constant). These

[13] Also see Choi et al. (2009), Bekkers et al. (2012) and Hummels and Lugovskyy (2009).

"Linder" type predictions contrast with the existing non-homothetic literature which does not provide an aggregate gravity prediction. After appropriately controlling for endogeneity, these predictions are confirmed in a sample based on the World Input Output Database (the same as used in CRC).

To develop these results the paper is broken into 3 sections. Section 2 constructs a general equilibrium monopolistically competitive model of indirect discrimination with three income types. This framework facilitates comparisons with both the previous trade literature based on general equilibrium models with linear pricing and also the partial equilibrium monopoly literature that analyzes indirect discrimination. Section 3 considers integration between countries with different income distributions, and examines the consequences of gradual liberalization while also providing empirical evidence on the observable predictions of the model. The final section quantifies the welfare effects of integration in a world where firms indirectly discriminate through product lines in both single and multiple sector settings.

2 Model

The main elements of the model are familiar from Krugman (1980): one factor (inelastically supplied), monopolistic competition between a set of symmetric firms with a constant marginal cost (and unit labor requirement), w, and a firm level fixed cost, wF. There is a single sector where consumers have the same SDS preferences over products:

$$U = \left[\sum_i q_i^\rho\right]^{1/\rho} \qquad \text{and} \quad 0 < \rho < 1 \tag{1}$$

To add within country income variation, these basic features are augmented by including three types of workers who differ in terms of labor endowment. The middle type, M, has an endowment normalized to unity, and the low type, L, has an endowment of $1-\alpha$ while the high type, H, possesses $1+\alpha$, with $\alpha \in (0,1)$. Letting β_i^I denote the fraction of population of country i that is type $I \in \{L, M, H\}$, then country i has an aggregate endowment of $L_i = 1 + \alpha(\beta_i^H - \beta_i^L)$. Normalizing the population in a given country to unity implies that there

is variation within countries due to individual endowment differences as well as variation across countries due to aggregate differences in endowments.

This three type set-up has the advantage of being rich enough to allow for the first and second moments of the income distribution to be varied independently, yet still keep the model relatively tractable. Note that simply adding within country income variation to Krugman (1980) does not alter any of that models results, the key departure involves allowing firms to utilize information on income distribution when setting non-linear prices.

These non-linear prices are implemented as a menu of options offered to consumers, $\{T(q), q\}$, where $T(q)$ is the payment required for a product with attribute q. While a firm would like to extract all the surplus from a consumer, it is constrained by the fact it only knows the distribution of income and not the income of any individual. From the literature on indirect discrimination, we know in this setting a firm designs the menu subject to a set of incentive compatibility (each income group prefers the option designed for them) and participation constraints (a consumer's net pay-off has to be non-negative).

These constraints accommodate a wide range menus, including the option to use linear prices, as in the standard formulation of the model. In this case, a firm would offer three options: $\{T^I(q^I) = \frac{w}{\rho}q^I,\ q^I\}$, where q^I corresponds to the quantity demanded by an individual with income I when confronted with a per unit price of $\frac{w}{\rho}$.[14] Which menu a firm offers depends on how they anticipate a consumer will behave when confronted with a menu. Thus, to solve the model, we start by considering the consumer choice problem when presented with a discrete set of options by a firm.

2.1 Budget Constraint

To analyze consumer choice, it is useful to start by examining the budget constraint. To maximize utility a consumer will exhaust their budget: $m^I = \sum_i T_i^I(q_i^I)$. To transform the budget constraint into one more familiar from standard utility maximization, note that an incentive compatible menu has an equivalent representation as a two-part tariff. That is, T_i^I

[14]Since a consumer choosing the appropriate option from this menu always receives a positive surplus, the menu satisfies all participation constraints. Moreover, selecting a different option can't raise the pay-off of a consumer since the per-unit price is the same across bundles. Hence, incentive compatibility is also satisfied.

can be decomposed into a fixed/access component, A_i^I and a usage fee $p_i^I q_i^I$, so that $T_i^I = A_i^I + p_i^I q_i^I$. The marginal price, p_i^I, can be read off the inverse demand function (derived below) given the firm's choice of q_i^I, which implies $A_i^I = T_i^I - p_i^I q_i^I$. From a modeling perspective this has the advantage that A_i^I acts like a lump sum tax, allowing the budget constraint to be expressed in a relatively familiar form. Therefore, a consumer with gross income m^I has net income:

$$\bar{m}^I = m^I - \sum_i A_i^I = \sum_i p_i^I q_i^I$$

Hence, the main modification to the model is in relation to net income. In the standard model (i.e. linear prices) there is no difference between net and gross income ($\bar{m}^I = m^I$). However, under non-linear prices net income can diverge from gross income.

2.2 Consumer Optimization

Apart from using net income rather than gross income, the utility maximization program results in familiar expressions with the inverse demand for a variety targeted at consumer I by firm i:

$$p_i^I = \theta q_i^{I(\rho-1)} \qquad \text{with} \;\; \theta^I = \frac{\bar{m}^I}{Q^{I\rho}} \;\; \text{and} \;\; Q^I = \left[\sum_i q_i^{I\rho}\right]^{1/\rho} \tag{2}$$

Facing these residual demand curves a typical firm evaluates the surplus from serving consumer I in the following way:

$$S_i^I(q) = \theta^I \int_0^{q_i^I} z^{\rho-1} dz = \frac{\theta^I q_i^{I\rho}}{\rho}$$

2.3 Profit Maximizing Product Lines

Using the surplus functions from above and the information on the distribution of types in the population, a typical monopolistically competitive firm chooses a menu of $\{T^I, q^I\}$, $I \in$

$\{L, M, H\}$ to maximize

$$\pi = \sum_I \beta^I (T^I - wq^I) - wF$$

subject to

$$\theta^I \frac{q^{I\rho}}{\rho} - T^I \geq \theta^I \frac{q^{K\rho}}{\rho} - T^K, \ \forall I \neq K \tag{3}$$

$$\theta^I \frac{q^{I\rho}}{\rho} - T^I \geq 0, \ \forall I \tag{4}$$

where (3) are the incentive compatibility constraints while (4) are the participation constraints. In a monopoly non-linear pricing problem the ordering of the $\theta's$ is enough to ensure that the single crossing property holds – implying that only three of these constraints bind, the incentive constraint for the high and middle types and the participation constraint for the low type.[15] However, since the θ's are determined as part of an equilibrium outcome we cannot simply take for granted that $\theta^H > \theta^M > \theta^L$. Nevertheless, we conjecture that this ordering holds (it is in fact satisfied in equilibrium) allowing the relevant constraints to be rewritten as:

$$T^L = \theta^L \frac{q^{L\rho}}{\rho} \tag{5}$$

$$T^M = \left(\theta^M \frac{q^{M\rho}}{\rho} - \theta^M \frac{q^{L\rho}}{\rho}\right) + T^L = \theta^M \frac{q^{M\rho}}{\rho} - (\theta^M - \theta^L)\frac{q^{L\rho}}{\rho} \tag{6}$$

$$T^H = \theta^H \frac{q^{H\rho}}{\rho} - \theta^H \frac{q^{M\rho}}{\rho} + T^M = \theta^H \frac{q^{H\rho}}{\rho} - (\theta^H - \theta^M)\frac{q^{M\rho}}{\rho} - (\theta^M - \theta^L)\frac{q^{L\rho}}{\rho} \tag{7}$$

These prices imply total revenues, along with total costs, of:

$$\begin{aligned} TR &= \beta^L T^L + \beta^M T^M + \beta^H T^H \\ &= (\theta^L - (1-\beta^L)\theta^M)\frac{q^{L\rho}}{\rho} + ((\beta^M + \beta^H)\theta^M - \beta^H\theta^H)\frac{q^{M\rho}}{\rho} + \beta^H\theta^H\frac{q^{H\rho}}{\rho} \end{aligned} \tag{8}$$

$$TC = \beta^L wq^L + \beta^M wq^M + \beta^H wq^H + wF \tag{9}$$

[15] See Maskin and Riley (1984).

Taking first order conditions with respect to q^I defines optimal behavior of a firm:

$$\theta^H q^{H\rho-1} = w \tag{10}$$

$$((\beta^M+\beta^H)\theta^M-\beta^H\theta^H)q^{M\rho-1} = \beta^M w \tag{11}$$

$$(\theta^L-(1-\beta^L)\theta^M)q^{L\rho-1} = \beta^L w \tag{12}$$

The value function is derived by observing that (8) is homogeneous of degree ρ in the vector of production designs, q^I, which implies $\sum_I \frac{\partial TR}{\partial q^I} q^I = \rho TR$. Since marginal revenue of any design equals (constant) marginal cost it follows from (10)–(12) that the value function can be written as $\frac{1-\rho}{\rho}\sum_I \beta^I w q^I - wF$. Setting this equal to zero confirms that free entry output/characteristics must satisfy:

$$\sum_I \beta^I q^I = F(\sigma-1) \tag{13}$$

where $\sigma = \frac{1}{1-\rho}$ is the elasticity of demand. This implies that the average attributes of a firm's product line is the same as chosen by a social planner and also coincides with what arises in the standard model with linear prices (see for example Mrázová and Neary (2014)). Given the aggregate endowment of labor is fixed, this implies the equilibrium number of firms, n_i, is the same across all three scenarios in this single sector setting.[16]

2.4 Equilibrium

Having derived the equilibrium attributes of each firm, the second issue is the allocation across income groups. To determine this, start by combining (10) and (12):

$$(\theta^L-(1-\beta^L)\theta^M)q^{L\rho-1} = \beta^L\theta^H q^{H\rho-1} \;\Rightarrow\; \left(\frac{\theta^L}{\theta^H}-(1-\beta^L)\frac{\theta^M}{\theta^H}\right)\phi^{LH(\rho-1)} = \beta^L$$

Now combine (10) and (11):

$$\left((\beta^M+\beta^H)\frac{\theta^M}{\theta^H}-\beta^H\right)\phi^{MH(\rho-1)} = \beta^M$$

[16] A multi-sector model is considered below.

where $\phi^{IK} \equiv \frac{q^I}{q^K}$ and $\overline{\alpha}^{IK} \equiv \frac{\bar{m}^I}{\bar{m}^K}$ which implies $\frac{\theta^I}{\theta^K} = \frac{\overline{\alpha}^{IK}}{\phi^{IK\rho}}$.

We will focus specifically on the relative design of products, ϕ^{IK}. Using these expressions, the equilibrium conditions for relative design can be written as:

$$\beta^H \phi^{MH\rho} + \beta^M \phi^{MH} = (\beta^M + \beta^H)\overline{\alpha}^{MH} \tag{14}$$

$$(1-\beta^L)\phi^{LM\rho} + \beta^L \phi^{LM} \frac{\phi^{MH}}{\overline{\alpha}^{MH}} = \overline{\alpha}^{LM} \tag{15}$$

$$\text{(or equivalently)} \qquad (1-\beta^L)\phi^{LH\rho} \frac{\overline{\alpha}^{MH}}{\phi^{MH\rho}} + \beta^L \phi^{LH} = \overline{\alpha}^{LH} \tag{16}$$

To complete the analysis of the equilibrium we need to derive the net incomes. For the low type net income follows from (5):

$$\rho T^L = \theta^L q^{L\rho} = \frac{\bar{m}^L}{n} \Rightarrow \rho n T^L = \rho m^L = \bar{m}^L \tag{17}$$

For the middle income group (6) implies:

$$\begin{aligned} \rho T^M &= \theta^M q^{M\rho}(1-\phi^{LM\rho}) + \rho T^L \\ \Rightarrow \bar{m}^M &= \frac{\rho(m^M - m^L)}{1-\phi^{LM\rho}} = \frac{\rho(m^M - m^L)}{\phi^{MH\rho} - \phi^{LH\rho}} \phi^{MH\rho} \end{aligned} \tag{18}$$

While (7) gives:

$$\bar{m}^H = \frac{\rho(m^H - m^M)}{1-\phi^{MH\rho}} \tag{19}$$

So the equilibrium product designs, $\{\phi^{LH}, \phi^{MH}\}$, must satisfy:

$$\beta^H \phi^{MH\rho} + \beta^M \phi^{MH} = (\beta^M + \beta^H) \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{1-\phi^{MH\rho}}{\phi^{MH\rho} - \phi^{LH\rho}}\right) \phi^{MH\rho} \tag{20}$$

$$(1-\beta^L)\phi^{LH\rho} \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{1-\phi^{MH\rho}}{\phi^{MH\rho} - \phi^{LH\rho}}\right) + \beta^L \phi^{LH} = \left(\frac{\alpha}{1-\alpha}\right) (1-\phi^{MH\rho}) \tag{21}$$

Inspecting this system it is immediately apparent that any solution is determined solely by the distribution of income, $\{\beta^I\}$, holding α and ρ constant.[17] The following proposition characterizes the nature of the equilibrium.

PROPOSITION 1. *An autarky equilibrium exists and is unique.*

(See appendix for proof.)

A distinctive aspect of this equilibrium is the distribution of output/quality across the income groups. As identified above, the aggregate feature of each firm is first best, but this property doesn't carry over to the products offered to each income group.

PROPOSITION 2. *For any non-degenerate income distribution, each firm always designs a menu that induces the low income group to purchase a product below the first best while offering the high income group a product that is above the first best. The product designed for the middle income group can be above, equal or below the first best depending on the characteristics of the income distribution.*

This proposition is the basis of the difference between the current model and the previous trade literature and also helps to distinguish between partial and general equilibrium models of indirect discrimination. With this in mind there are three issues to highlight.

First, in contrast to a model with linear pricing – which generates a first best allocation in a single sector setting – distortions exist in equilibrium. These distortions are a result of a firm's differential ability to extract rents from the various income groups. Second, market power distorts output decisions both above and below the first best. While the usual downward monopoly distortion is evident for the low income group, it is always the case that the

[17] This equilibrium also has the feature that an increase in marginal cost would lead to a proportional reduction in all elements of the product line. This is consistent with the type of changes documented for non-durable goods. For example, facing an increase in the cost of cocoa, Mondelez reduced the size of its chocolate bars proportionately, 200g to 180g and also 100g to 90g – "Bitter chocolate slab to swallow," *Sunday Times*, June 15, 2014. Nevertheless, the price ratio was maintained with the larger block offering a 23% discount per gram.

high income group receives a product above the first best i.e. distortions are bi-directional. Outcomes above the social optimum provide a stark contrast to the linear price model. Note also that this result doesn't arise in the canonical monopoly model of indirect discrimination where the high type always receives the first best outcome (see Maskin and Riley (1984)). In that setting the positions of the residual demand curves are exogenous (i.e. θ^I is given) and there is a single firm. Relaxation of either of these aspects can play a role in the result that a high income type is offered a product above the first best. In this single sector setting, the position of a residual demand curve is influenced by the net income of a consumer type. For the high income types, the capture of information rents raises their net income and consequently shifts their residual demand function out relative to the first best. Conditional on the position of this residual demand curve a firm has no incentive to distort a high type's design since this product doesn't concede information rents to any other type. Instead the problem is the residual demand curve of the high type is in the "wrong" position. The final point to emphasize is that welfare differences are more exaggerated than income differences. To see this, note from the symmetry of menus a consumer's welfare is linear in product design i.e. $U^I = n^{\frac{1}{\rho}} q^I$. Since product design is above the first best for high types but below the first best for low types, it follows that differences in welfare outcomes must be more pronounced than income differences.

To underscore this last point and to help facilitate the analysis to come, consider the indirect utility function. A key step in deriving this function relates to marginal price for group I:

$$p^I = \theta^I q^{I(\rho-1)} = \frac{\overline{\alpha}^{IH}}{\phi^{IH}} w \tag{22}$$

Indirect utility is then given by:

$$\begin{aligned} U^I &= n^{\frac{1}{\rho}} \left(\frac{\bar{m}^I}{np^I} \right) = n^{\frac{1-\rho}{\rho}} \left(\frac{\bar{m}^H}{w} \right) \phi^{IH} \\ &= \phi^{IH} U^H \end{aligned} \tag{23}$$

$$U^H = \rho n^{\frac{1-\rho}{\rho}} \left(\frac{\alpha}{1-\phi^{MH\rho}} \right) \tag{24}$$

We are now in a position to reflect on the implications of firms using non-linear prices when the only source of variation across consumers is the income they possess. Apart from expanding a firm's strategy space in a plausible way, all of the other assumptions of the standard general equilibrium model of monopolistic competition are retained - especially the assumption of homothetic preferences and a constant elasticity of residual demand. Nevertheless the differences in product design and welfare outcomes are striking. A key take-away is that the distribution of income is the primary determinant of the size of distortions and consequently welfare outcomes. Given countries differ substantially in their income distributions, this suggests if we start from an autarky situation, the size and relevance of the distortions will also vary considerably across countries. How does trade affect these distortions? How do these distortions affect the gains from trade? It is to these questions we now turn our attention.

3 Implications of International Trade

3.1 Free Trade in the Standard Model

As a benchmark consider the standard model where technology and preferences are as described above but firms are constrained to use linear prices. Since welfare of an income group is proportional to their share of income we only need to consider aggregate demand for a variety and the number of varieties (which are a function of endowments). Consequently, for

a country with an endowment of L_i that has access to n_j varieties we have:

$$q_i = \frac{p^{-\sigma} w L_i}{n_j p^{1-\sigma}} = \frac{\rho L_i}{n_j} \quad \& \quad n_j = \frac{L_j}{\sigma F} \qquad \Rightarrow \quad U_i \;=\; \rho n_j^{\frac{1}{\sigma-1}} L_i$$

Autarky is then a situation where $n_j = n_i$ and free trade involves $n_j > n_i$. Using $\mathcal{F}$ to denote free trade and $\mathcal{A}$ for autarky it follows that the gains from trade in the standard model for country i have the form:

$$\frac{U_{i\mathcal{F}}}{U_{i\mathcal{A}}} \;=\; \left(\frac{L_w}{L_i}\right)^{\frac{1}{\sigma-1}} = \overline{GFT}_i \tag{25}$$

where L_w is the size of the labor endowment of the integrated countries. Whenever this country engages in free trade with another country the sole mechanism for welfare change is through the number of varieties. This makes relative size the only determinant of the gains from free trade: the more varieties accessed under free trade, the larger the gains from free trade. In this setting differences in income distribution just translate into differences in relative size.

3.2 Free Trade with Indirect Discrimination

Against this benchmark consider the integration of two countries with potentially different income distributions. While the nature of a country's income distribution is critical for the design of the product line in autarky, it is the characteristics of the global income distribution that shapes design under free trade. If countries have very different income distributions, then there will be very pronounced differences in product design across countries in autarky. To understand the implications of eliminating this variation through integration we'll focus on two dimensions that are commonly emphasized when comparing income distributions (i) mean income, and (ii) income dispersion.

To trace through the consequences of integrating with another country use (23), (24) and

(25) to derive:

$$GFT_i^H = \left(\frac{1-\phi_{i\mathcal{A}}^{MH\rho}}{1-\phi_{\mathcal{F}}^{MH\rho}}\right)\overline{GFT}_i \tag{26}$$

$$GFT_i^I = \left(\frac{\phi_{\mathcal{F}}^{IH}}{\phi_{i\mathcal{A}}^{IH}}\right)GFT_i^H \tag{27}$$

Naming the two countries, Home, h, and Foreign, f, and assuming that Home has a higher per capita income than Foreign, leads to the following claim.

PROPOSITION 3. *If the likelihood ratio of Home's income distribution dominates that of Foreign, then Home's gains from free trade are greater than the standard model while the opposite holds in the Foreign country. Furthermore, within the Home country, the proportional gain follows a rank that is inversely related to income. Consequently, the lowest income group in the Home country gains the most from trade. The converse holds in the Foreign country.*

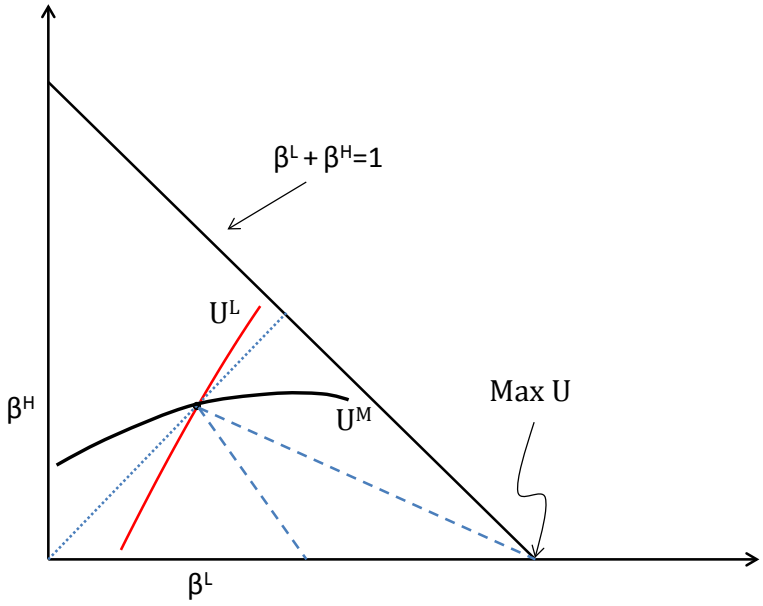


Figure 1: The distribution of the gains from free trade

Proposition 3 can be understood with the aid of Figure 1 which represents the set of income distributions in our three type setting. The vertical distance from the x-axis measures the share of high income types in the the population while the horizontal distance from the y-axis measures the fraction of low income types. The fraction of middle income types is

then implicitly defined as $\beta^M = 1 - \beta^L - \beta^H$ or either the vertical or horizontal distance from any point in the triangle to the $\beta^L + \beta^M = 1$ line. If we consider a Home country with an income distribution given by $\{\beta^L,\ \beta^H\}$ then all distributions with the same average income are given by the dotted line starting at the origin. Any distribution with lower mean income lies south-east of this line (all iso-mean-income lines are parallel). Indifference curves for the low and middle income types are plotted, with higher welfare for a type below a given indifference curve.[18] The relative slopes of the indifference curves at $\{\beta^L,\ \beta^H\}$ can be derived from (23) and (24). Expressing these relationships in terms of proportional changes we have:

$$\widehat{U}^H = \left(\frac{\rho\phi^{MH\rho}}{1-\phi^{MH\rho}}\right)\hat{\phi}^{MH} \tag{28}$$

$$\widehat{U}^M = \left(\frac{1-(1-\rho)\phi^{MH\rho}}{1-\phi^{MH\rho}}\right)\hat{\phi}^{MH} \tag{29}$$

$$\widehat{U}^L = \hat{\phi}^{LM} + \left(\frac{1-(1-\rho)\phi^{MH\rho}}{1-\phi^{MH\rho}}\right)\hat{\phi}^{MH} \tag{30}$$

Along an indifference curve these changes equal zero. This implies that for a given $\Delta\beta^L > 0$, the $\Delta\beta^H$ required for $\widehat{U}^L = 0$ (i.e. $\hat{\phi}_{MH} < 0$) is greater than required for $\widehat{U}^M = 0$ (i.e. $\hat{\phi}_{MH} = 0$).[19] Hence, at any common point the slope of the low types indifference curve is greater than that of the middle or high types indifference curve.

Using Figure 1 we can now see why a straight ranking of mean incomes is generally not sufficient to predict the distribution of the gains from trade across countries. If the lower mean income country also has a relatively small fraction of low income types, then the low types in the higher mean income country are served a free trade product that is degraded relative to autarky. This occurs for income distributions below the dotted income line but above U^L. A similar conclusion follows for distributions below the dotted income line but above U^M, although product design for the middle type is now relatively degraded because there are too few middle types. Restricting the comparison to income distributions that can be ranked according likelihood ratios allows more definitive predictions to be made. The set

[18] The indifference curve for the high type has the same slope as the middle income type. This is evident from comparing (28) and (29).

[19] To see this suppose that the change in $\beta^M < 0$ and $\beta^H > 0$ is such that (20) remains satisfied for the same ϕ^{MH}. It them follows from (21) that $\phi^{LH} > 0$.

of distributions dominated by $\{\beta^L,\ \beta^H\}$ is given by the triangle bounded by the dashed lines and the horizontal axis. Integration by a Home country with $\{\beta^L,\ \beta^H\}$ and a country within this set will deliver amplified gains from trade for the Home country, with the largest gains for the lowest income groups.

Proposition 3 provides a contrast to the standard model where homothetic preferences and linear pricing ensure that firms only focus on aggregate demand and not its composition: all consumers receive the same proportional gains from trade within a country. The above proposition reveals that once firms are able to utilize information on income distribution, the distribution of the gains from free trade can vary significantly across income groups within a country. The ordering imposed by likelihood ratio dominance provides a sufficient condition for magnification of the gains from free trade relative to the standard model for the country with the higher GDP per capita. However, this ordering is also typically associated with a change in the dispersion of income. To consider these components separately, hold the GDP per capita constant across countries but vary income dispersion. A particularly neat parametrization is achieved by setting $\beta_i^L = \beta_i^H = \beta_i$ where $i \in \{h, f\}$. This implies mean income in both countries is unity and variance of income is given by $2\alpha^2\beta_i$.

PROPOSITION 4. *If Home's income distribution is a mean preserving spread of Foreign's income distribution (i.e. $\beta_h > \beta_f$), then the low income group in the Home country receives gains from free trade that are lower than the standard model, while the gains for the middle and high income groups are higher. The converse holds in the Foreign country.*

Once again the intuition for this proposition is captured in Figure 1. Moving along the dotted income line from the origin increases the dispersion of income while holding mean income constant. For a Home country with income distribution $\{\beta^L,\ \beta^H\}$, integrating with a low dispersion country offers better product design for the middle income group resulting in higher welfare for both the middle and high income groups. However, the smaller fraction of low types in the global economy leads to an inferior product for the low type and reduced gains from trade for the low type in Home. This logic is reversed when integration occurs with a country that has higher income dispersion. In this case there are now relatively more of both low and high income types in the global economy but fewer middle income types.

This facilitates an improved design for the low income group, and amplified gains from free trade, but a less attractive product for the middle income type. The poorer design of the middle income product reduces the outside option for the high income group, which diminishes their gains from integration.

Together these propositions reveal that the gains from trade are fundamentally changed by indirect discrimination. In the standard model, relative size is the sole determinant of the gains from trade: the smaller the country, the larger the gains from trade. In our three type model, this implies the country with the lower average income would gain the most from trade. With indirect discrimination, relative size is no longer enough to completely characterize the gains from trade. In fact a smaller country may have their variety gains from trade dramatically diminished by inferior product design. The main mechanism operates through the desire of firms to customize products to extract rents – better products generate more surplus but also concede information rents to higher income groups. This trade-off is resolved with reference to the distribution of income. The critical factor shaping the gains from trade is then the extent and nature of the difference between the national and global income distributions. Pronounced differences give rise to big differences not just between the number of varieties available but also between the menu of choices offered in autarky and free trade.

Under free trade the menu of choices is common to all countries and this implies that prices paid for a specific product will also be the same. However, since the distribution of income varies across countries the distribution of transaction prices will also vary. There is now an growing literature documenting the association between country characteristics and import prices. What does the indirect discrimination model imply about the distribution of import prices and to what extent is this consistent with the patterns observed in the trade data?

3.3 Comparing Prices Across Destination Markets

The prices implied by the free trade menu, T^I, have the property that they are increasing and concave in income: $T^L < T^M < T^H$ and that $\frac{dT^I}{dq^I} = p^I$. The concavity of the price schedule

follows from the marginal price declining in income:

$$p^L = \frac{\bar{\alpha}^{LH}}{\phi^{LH}} w > p^M = \frac{\bar{\alpha}^{MH}}{\phi^{MH}} w > p^H = w$$

A typical finding in the empirical literature is that conditional on exporter-product pairs, import unit values are increasing in destination per-capita income.[20] Moreover the per-capita income elasticity is less than unity - suggesting a concave pricing function. Studies that consider within country income dispersion are less common, with Choi et al. (2009) and Bekkers et al. (2012) among the few that examine this dimension. Choi et al. (2009) find that differences in the dispersion of income across countries is associated with a less than proportional increase in the dispersion of import prices. In the context of their model they find this result puzzling but it is consistent with a concave price schedule of the type implied by indirect discrimination. While Choi et al. (2009) consider all HS 6 import prices for 26 importers, Bekkers et al. (2012) narrow their focus to 1260 HS 6 categories of *final* goods but expand the sample to over 100 countries. The motivation for this narrower set of products is a tighter mapping to consumer income. They find a negative correlation between within country inequality and per unit import prices.[21] Consequently, the evidence tends to suggest that the distribution of import prices is concave with respect to per-capita income – as predicted by the model of indirect price discrimination.

3.4 Gradual Trade Liberalization

While the autarky/free-trade dichotomy offers a useful benchmark, it is typically not the case that trade costs are characterized by either of these extremes. Nevertheless, under the standard iceberg interpretation, a lowering of the trade costs monotonically increases welfare for all countries.[22] While it is tempting to assume that a similar monotonicity applies in the

[20] This finding is documented across a range of countries and also appears in data disaggregated to the firm level, see Manova and Zhang (2012).

[21] Bekkers et al. (2012) conclude that is consistent with a model of non-homothetic hierarchic demand but inconsistent with non-homothetic models of quality or ideal variety. Each of these models employ the assumption of linear pricing.

[22] The absence of tariff revenue implies the optimal trade cost is zero for all countries. For an analysis of trade policy with general pricing behavior see Antràs and Staiger (2012) and McCalman (2010).

indirect discrimination model, the following proposition reveals that all of the differences from the standard model arise only once trade barriers are sufficiently low.

PROPOSITION 5. *Let $\tau \geq 1$ represent the iceberg transport cost between the Home and the Foreign country. Then there exists a transport cost $\bar{\tau}$ such that the gains from trade for a high income type are:*

$$GFT_i^H(\tau) = \begin{cases} \overline{GFT}_i(\tau) & \text{for } \tau \geq \overline{\tau} \\ \left(\frac{1-\phi_{i\mathcal{A}}^{MH\rho}}{1-\overline{\phi_{i\tau}^{MH}}^{\frac{1}{\rho}}}\right) \overline{GFT}_i(\tau) & \text{for } \tau < \overline{\tau} \end{cases}$$

and for income type $I \neq H$

$$GFT_i^I(\tau) = \begin{cases} \overline{GFT}_i(\tau) & \text{for } \tau \geq \overline{\tau} \\ \left(\frac{\overline{\phi_{i\tau}^{IH}}}{\phi_{i\mathcal{A}}^{IH}}\right) GFT_i^H(\tau) & \text{for } \tau < \overline{\tau} \end{cases}$$

where $\overline{GFT}_i(\tau) = \left(1 + \frac{L_j}{L_i}\left(\frac{\tau w_j}{w_i}\right)^{1-\sigma}\right)^{\frac{1}{\sigma-1}}$ denotes the gains from trade in the standard model and $\overline{\phi_{i\tau}^{IH}} = \left(\frac{n_i}{\tilde{n}_i}\phi_{ii}^{IH\rho} + \frac{d_i n_j}{\tilde{n}_i}\phi_{ij}^{IH\rho}\right)^{\frac{1}{\rho}}$ is the average product design.

This says that when trade barriers are relatively high, the indirect discrimination model delivers the same proportional gains from trade as the standard model. Thus, all the changes described in Propositions 3 and 4 occur only after trade barriers are below $\bar{\tau}$.

This demarcation can have important implications for a process of gradual trade liberalization. To see this consider the scenario described in Proposition 3 where the low average income country gains from trade due to an increase in the number of varieties but has these gains diminished by an inferior product design relative to autarky (lower ϕ^{IH} for all $I \in \{L, M\}$). According to Proposition 3 the reduction in design is most pronounced for the low income group. Since all of the change in design occurs for trade barriers below $\bar{\tau}$ this suggests the possibility that the decline in design may completely offset the standard gains from trade associated with further liberalization. If so, the gains from trade for a low income type may reach a maximum before free trade. An essential ingredient for such a result is a relatively large share of low income types in the population, since the decline in design is more pronounced the larger the share of low types.

This interaction between trade barriers and welfare highlights a potential downside to gradual trade liberalization: beyond a point one country simply may not benefit from further trade liberalization. Once again the root cause is differences across countries in the distribution of income and the associated design of products. When markets are segmented, access to additional varieties is the only source of gains from trade.[23] As trade barriers fall, markets become more deeply integrated and product design becomes more universal. As we have seen, this standardizing of products doesn't bring unambiguous gains to all consumers in all countries. Understanding the potential magnitude of this mechanism motivates the quantification exercise of section 4.

3.5 Gravity Equation

The potential for welfare outcomes to vary dramatically from the standard model raises the question of whether there is a similarly pronounced analogue for an observable outcome like trade flows. To explore this issue, let $d_{ij} = (\frac{\tau_{ij} w_j}{w_i})^{1-\sigma}$ and $\tilde{n}_i = \sum_j n_j d_{ij}$, and note that:

$$T_{ij}^H = d_{ij}\frac{\bar{m}_i^H}{\rho \tilde{n}_i}(1 - \phi_{ij}^{MH\rho}) + T_{ij}^M, \quad T_{ij}^M = d_{ij}\frac{\bar{m}_i^M}{\rho \tilde{n}_i}\frac{(\phi_{ij}^{MH\rho} - \phi_{ij}^{LH\rho})}{\overline{\phi_i^{MH\rho}}} + T_{ij}^L, \quad T_{ij}^L = d_{ij}\frac{\bar{m}_i^L}{\rho \tilde{n}_i}\frac{\phi_{ij}^{LH\rho}}{\overline{\phi_i^{LH\rho}}}$$

Then bilateral trade between importer i and exporter j is given by:

$$\begin{aligned} X_{ij} &= n_j\left(\beta_i^H T_{ij}^H + \beta_i^M T_{ij}^M + \beta_i^L T_{ij}^L\right) \\ &= \frac{n_j d_{ij}}{\tilde{n}_i}\frac{\bar{m}_i^H}{\rho}\left(\beta_i^H + \left((\beta_i^H + \beta_i^M)\overline{\alpha}_i^{MH} - \beta_i^H\overline{\phi_i^{MH\rho}}\right)\frac{\phi_{ij}^{MH\rho}}{\overline{\phi_i^{MH\rho}}} + \left(\overline{\alpha}_i^{LH} - (1-\beta_i^L)\frac{\overline{\phi_i^{LH\rho}}}{\overline{\phi_i^{MH\rho}}}\overline{\alpha}_i^{MH}\right)\frac{\phi_{ij}^{LH\rho}}{\overline{\phi_i^{LH\rho}}}\right) \end{aligned}$$

Note that

$$X_i = \sum_j X_{ij} = \frac{\bar{m}_i^H}{\rho}\left(\beta_i^H(1 - \overline{\phi_i^{MH\rho}}) + (\beta_i^H + \beta_i^M)\frac{\overline{\alpha}_i^{MH}}{\overline{\phi_i^{MH\rho}}}(\overline{\phi_{ij}^{MH\rho}} - \overline{\phi_{ij}^{LH\rho}}) + \overline{\alpha}_i^{LH}\frac{\overline{\phi_i^{LH\rho}}}{\overline{\phi_i^{LH\rho}}}\right)$$

[23] The choice of intellectual property rights regime can also be influenced by the nature of market segmentation. For analysis of this issue in a linear price setting see Saggi (2013).

This implies the following expenditure shares for country i:

$$\frac{X_{ij}}{X_i} = s_{ij}\frac{\left(\beta_i^H + \left[(1-\beta_i^L)\overline{\alpha}_i^{MH} - \beta_i^H\overline{\phi_i^{MH\rho}}\right]\frac{\phi_{ij}^{MH\rho}}{\overline{\phi_i^{MH\rho}}} + \left[\overline{\alpha}_i^{LH} - (1-\beta_i^L)\frac{\overline{\phi_i^{LH\rho}}}{\overline{\phi_i^{MH\rho}}}\overline{\alpha}_i^{MH}\right]\frac{\phi_{ij}^{LH\rho}}{\overline{\phi_i^{LH\rho}}}\right)}{\left(\beta_i^H + \left((\beta_i^H+\beta_i^M)\overline{\alpha}_i^{MH} - \beta_i^H\overline{\phi_i^{MH\rho}}\right) + \left(\overline{\alpha}_i^{LH} - (1-\beta_i^L)\frac{\overline{\phi_i^{LH\rho}}}{\overline{\phi_i^{MH\rho}}}\overline{\alpha}_i^{MH}\right)\right)} \quad (31)$$

$$s_{ij} = \frac{n_j d_{ij}}{\sum n_k d_{ik}}$$

Since s_{ij} captures the standard gravity equation it is apparent that trade flows will deviate from this to the extent that products from country j diverge from the typical designs consumed in country i, $\frac{\phi_{ij}^{IH}}{\overline{\phi_i^{IH}}}$, and on the signs of $\left((\beta_i^H+\beta_i^M)\overline{\alpha}_i^{MH} - \beta_i^H\overline{\phi_i^{MH\rho}}\right)$ and $\left(\overline{\alpha}_i^{LH} - (1-\beta_i^L)\frac{\overline{\phi_i^{LH\rho}}}{\overline{\phi_i^{MH\rho}}}\overline{\alpha}_i^{MH}\right)\Big)$. The interaction of these components will determine whether trade is above or below that predicted by the standard model. Before getting to the implications of the interaction, we'll characterize behavior of each of these terms.

Starting with relative product design, there are two situations in which $\phi_{ij}^{IH} = \overline{\phi_i^{IH}}$ for all j. First when $\tau \geq \bar{\tau}$ – markets are segmented. As we established above this coincides with the gains from trade in the standard model. This should not be too surprising since trade barriers in this set satisfy all of the primitive assumptions as well as the macro-level restrictions R1-R3 of Arkolakis et al. (2012). The second environment where $\phi_{ij}^{IH} = \overline{\phi^{IH}}$ is under free trade. This is more interesting since it also satisfies all the restrictions of Arkolakis et al. (2012). In fact, s_{ij}, coincides with the form of R3' when $\tau_{ij} = 1$. However, as outlined in Propositions 3 and 4, knowledge of the domestic expenditure share and the trade elasticity, $\sigma - 1$, aren't sufficient to calculate the gains from trade. So even though all the assumptions are met, the results from Arkolakis et al. (2012) no longer hold under indirect discrimination. Consequently, the volume of trade will be exactly as predicted by the standard model, but the mapping to welfare will be fundamentally different.

What happens to trade when $\tau \in (1, \bar{\tau})$? To characterize the rank of ϕ_{ij}^{IH} for markets that are partially integrated consider the scenario of Proposition 3 where autarky relative designs in Foreign are uniformly superior to Home. In this case consumers in the Home country will prefer the designs offered in the Foreign market if trade barriers are low enough. In

this sense, f, serves as the reference market. Since cross-hauling isn't part of an equilibrium, product design in the L and M segments is shaped by the no arbitrage constraint. That is, the design of q^I_{fj} will pin-down the design of q^I_{hj} for $I \in \{L,\ M\}$.

To illustrate the implications of partial integration consider product design in the low income segment when trade barriers are sufficiently low for only this market segment to be constrained by the potential for arbitrage. This implies that a middle income type in the Home country must be indifferent between purchasing the local variant from a firm's product line and cross-hauling the product designed for the Foreign low income type by the same firm. That is:

$$\theta^M_h \frac{q^{L\rho}_{hj}}{\rho} - T^L_{hj} \;=\; \theta^M_h \frac{\left(q^L_{fj}/\tau\right)^\rho}{\rho} - T^L_{fj}$$

Since neither of the low types have an outside option $T^L_{hj} = \theta^L_h \frac{q^{L\rho}_{hj}}{\rho}$ and $T^L_{fj} = \theta^L_f \frac{q^{L\rho}_{fj}}{\rho}$, this implies:

$$q^{L\rho}_{hj} \;=\; \frac{\tau^{-\rho}(\theta^M_h - \tau^\rho \theta^L_f)}{(\theta^M_h - \theta^L_h)} q^{L\rho}_{fj} = \tau^{-\rho}\gamma^L q^{L\rho}_{fj} \quad \Rightarrow \quad q^L_{hj} = \tau^{-1}\gamma^{L\frac{1}{\rho}} q^L_{fj}$$

This constraint implies that design for the low income market must satisfy the following first order conditions:

$$\begin{aligned}
\left((\theta^L_f - (1-\beta^L_f)\theta^M_f) + \tau^{-\rho}(\theta^L_h - (1-\beta^L_h)\theta^M_h)\gamma^L\right) q^{L\rho-1}_{fh} &= \left(\beta^L_f + \gamma^{L\frac{1}{\rho}}\beta^L_h \tau^{-2}\right)\tau w_h \\
\left((\theta^L_f - (1-\beta^L_f)\theta^M_f) + \tau^{-\rho}(\theta^L_h - (1-\beta^L_h)\theta^M_h)\gamma^L\right) q^{L\rho-1}_{ff} &= \left(\beta^L_f + \gamma^{L\frac{1}{\rho}}\beta^L_h\right) w_f
\end{aligned}$$

The ratio of these conditions imply $\frac{q^L_{fh}}{q^L_{ff}} = \left(\left(\frac{\beta^L_f + \gamma^{L\frac{1}{\rho}}\beta^L_h \tau^{-2}}{\beta^L_f + \gamma^{L\frac{1}{\rho}}\beta^L_h}\right)\frac{\tau w_h}{w_f}\right)^{-\sigma} < \left(\frac{\tau w_h}{w_f}\right)^{-\sigma}$ – which says that countering within product line arbitrage requires less of an adjustment for the Home firms. The intuition is relatively straightforward: under segmentation transport costs already ensure $q^L_{fh} < q^L_{ff}$, so a Home firm's product concedes fewer information rents, hence they have less to lose from international arbitrage and make less of an adjustment to counter cross-hauling. In addition, since the high end of the product line is never susceptible to cross-hauling, its design is never subject to integration which implies designs in the reference

market have the following rank:

$$\frac{\phi_{fh}^{LH}}{\phi_{ff}^{LH}} = \left(\frac{\beta_f^L + \gamma^{L\frac{1}{\rho}} \beta_h^L}{\beta_f^L + \gamma^{L\frac{1}{\rho}} \beta_h^L \tau^{-2}} \right)^{\sigma} \geq 1$$

To recover the rank in the non-reference market note that:

$$\frac{\phi_{hh}^{LH}}{\phi_{hf}^{LH}} = \frac{\phi_{fh}^{LH}}{\phi_{ff}^{LH}} \frac{q_{fh}^H}{q_{ff}^H} \frac{q_{hf}^H}{q_{hh}^H} = \frac{\phi_{fh}^{LH}}{\phi_{ff}^{LH}} \frac{1}{\tau^{2\sigma}} \leq 1$$

Similar arguments can be constructed for the middle income market segment that show $\phi_{fh}^{MH} \geq \phi_{ff}^{MH}$ and $\phi_{hf}^{MH} \geq \phi_{hh}^{MH}$. These results imply that the local design is always weakly inferior to the overseas design when the likelihood ratio of Home's income distribution dominates Foreign's. Whether this translates into higher or lower trade flows than the standard model depends on the sign of the second component of the interaction terms.

To see that these terms are capable of being either positive or negative, consider what happens as we approach free trade (i.e. $\tau \to 1$). In this case variation in design across sources becomes relatively compressed so that $\phi_{ij}^{IH} \to \phi^{IH}$. The signs of interest then depend on the following comparisons:

$$\overline{\alpha}^{MH} = \frac{\beta_w^H}{\beta_w^H + \beta_w^M} \phi^{MH\rho} + \frac{\beta_w^M}{\beta_w^H + \beta_w^M} \phi^{MH} \lessgtr \frac{\beta_i^H}{\beta_i^H + \beta_i^M} \phi^{MH\rho} \tag{32}$$

$$\overline{\alpha}^{LH} = (1 - \beta_w^L) \phi^{LH\rho} \frac{\overline{\alpha}^{MH}}{\phi^{MH\rho}} + \beta_w^L \phi^{LH} \lessgtr (1 - \beta_i^L) \phi^{LH\rho} \frac{\overline{\alpha}^{MH}}{\phi^{MH\rho}} \tag{33}$$

where β_w^I denotes the fraction of the world population with income I. It is clear that when $\beta_i^I = \beta_w^I$ then the LHS will be greater than the RHS and all the interaction terms will be positive. In this case the volume of trade under partial integration will be greater than predicted by the standard model.

In contrast, when (32) and (33) are both negative trade is below the standard gravity prediction. When is this most likely to occur? When the difference in per capita income is greatest. To see this consider $\beta_f^L = 1$ and ask how the value of trade changes as we vary β_h^H

when we start from $\beta_h^L = 1$. The omission of the middle income type implies $\phi^{MH\rho} = \overline{\alpha}^{MH}$ and reduces the equilibrium condition for low type design to $(1 - \beta_w^L)\phi^{LH\rho} + \beta_w^L \phi^{LH} = \overline{\alpha}^{LH}$. In a world with two countries we can construct the global income distribution as $\beta_w^I = \frac{\beta_h^I + \beta_f^I}{2}$. As a result the sign of the interaction term now depends on $\phi^{LH} \lesseqgtr \beta_h^H(\phi^{LH\rho} + \phi^{LH})$. If both countries have the same income distribution (i.e. $\beta_h^H = 0$) then the LHS is greater than the RHS and the interaction term is positive. However, as we increase β_h^H the RHS increases faster than the LHS and at $\beta_h^H = 1$ the interaction term is negative.[24] This provides us with the following proposition

PROPOSITION 6. *When income distributions can be ordered by likelihood ratio dominance and trade barriers are low enough for markets to be partially integrated, the indirect discrimination model predicts that the deviation from the standard gravity model can be either positive or negative. More trade is predicted if the difference in GDP per capita is not too large. However, if the difference is relatively large, then the indirect discrimination model predicts lower trade volumes than the standard gravity model.*

This result resembles the "Linder Hypothesis" in that it relates the volume of trade to differences in per-capita income: similarity in per capita income gives rise to augmented trade flows but relatively large differences reduce the volume of trade. To date it has been asserted that such a trade pattern can only be explained by non-homothetic preferences. What is interesting about the above result is that preferences are not only identical and homothetic, but they also impose the additional restriction that the elasticity of demand is constant. Nevertheless, simply allowing firms to maximize profits by exploiting information on income distribution in a relatively plausible way results in a positive correlation between similarity in income per-capita and trade. While there is variation across consumers on the demand-side, it is purely in terms of income rather than hardwired into preferences. The intuition is also relatively direct. When markets are partially integrated, a location that delivers a product design better than the average in an importing country faces two competing forces that shape trade flows. First, an above average design allows more rents to be extracted from the low

[24] At $\beta_h^H = 0$ we have $\frac{d\phi^{LH}}{d\beta_h^H} < 0$ while the derivative of the RHS is $(\phi^{LH\rho} + \phi^{LH}) > 0$.

types simply because a better product generates more rents. Second, a better product design allows the higher types to capture more information rents, which tends to suppress the volume of trade by lowering prices for the higher types. When β_h^L is relatively high (given $\beta_f^L = 1$), the first effect dominates and trade flows are higher than predicted by the gravity equation. However, when β_h^H is relatively high, the second effect dominates and trade flows tend to be smaller than the standard model would predict.

While a focus on the correlation between trade and per capita income differences is natural in this setting, the model also has implications for the volume of trade as income dispersion varies across countries. As in Proposition 4 consider a setting where Home's income distribution is parameterized through β_h to be a mean preserving spread of Foreign's income distribution (i.e. $\beta_h > \beta_f$). To isolate the central mechanism let $\beta_f = 0$ – the Foreign country has no income heterogeneity. This last characteristic means that under segmentation foreign consumers receive no information rents, while under partial integration they capture information rents by having the product offered to the low income consumer in Home as an outside option. Having an outside option implies that Home's exports to Foreign can be expressed as:[25]

$$n_h T_{fh}^M = \frac{\rho(m_f^M - m_h^L)}{\tilde{n}_f} \left[\left(\frac{\gamma_M \phi_{hh}^{MH\rho} + \gamma_L \phi_{hh}^{LH\rho}}{\gamma_M \overline{\phi_h^{MH\rho}} + \gamma_L \overline{\phi_h^{LH\rho}}} \right) \left(\frac{1 - \widehat{\phi_{hh}^{LM}}/\tau^\rho}{1 - \widehat{\phi_h^{LM}}/\tau^\rho} \right) \right] + \frac{\rho m_h^L}{\tilde{n}_h} \left[\frac{\phi_{hh}^{LH\rho}}{\overline{\phi_h^{LH\rho}}} \right] \tag{34}$$

To characterize trade flows as β_h is varied, start by considering $\beta_h \approx \beta_f$. For partial integration to occur in this setting trade barriers must be relatively small, i.e. $\tau \approx 1$. This combination implies that $\phi_{hj}^{IK} \approx \phi_h^{IK}$, so there is little variation in design across locations and the terms in square brackets are both approximately unity. Consequently, trade flows are similar to the standard gravity prediction.

If we examine the other extreme, $\beta_h \to \frac{1}{2}$ a different result emerges. Now the Home country houses only high and low income types. Once again the foreign middle income type only captures information rents if trade barriers are sufficiently small to make the Home low's product a viable outside option. This only occurs when $\theta_f^M/\tau^\rho \geq \theta_h^L$ – a condition that

[25] See appendix for derivation.

is met simultaneously for firms located in both countries, ensuring $\phi_{hj}^{LH} = \phi_{h}^{LH}$. In contrast, the trade cost that implies the home High income type prefers the foreign middle's product as an outside option within a foreign firm's product line is higher than the threshold trade cost for a similar incentive to arise within a home firm's product line: $\phi_{hf}^{MH} > \phi_{hh}^{MH}$. Combining these two features implies $\phi_{hh}^{LM} > \phi_{hf}^{LM}$ which gives:

$$\left(\frac{\gamma_M \phi_{hh}^{MH\rho} + \gamma_L \phi_{h}^{LH\rho}}{\gamma_M \overline{\phi_{h}^{MH\rho}} + \gamma_L \phi_{h}^{LH\rho}}\right)\left(\frac{1 - \widehat{\phi_{hh}^{LM}}/\tau^{\rho}}{1 - \overline{\widehat{\phi_{h}^{LM}}}/\tau^{\rho}}\right) < 1$$

Hence, trade flows are lower than predicted by the standard gravity model. We can summarize these results as follows.

PROPOSITION 7. *If Home's income distribution is a mean-preserving spread of Foreign's ($\beta_h \geq \beta_f$) and trade costs are sufficiently small, then trade volume declines as the difference in income dispersion in the Home country increases (i.e. increase β_h).*

This proposition augments the "Linder Hypothesis" by identifying differences in income dispersion as a characteristic that diminishes trade flows. The intuition derives from the enhanced ability of consumers to look abroad for outside options as trade costs fall, constraining the ability of firms to extract rents. For a given trade barrier, this mechanism is stronger the greater the difference in income dispersion across countries.

3.6 Augmented Gravity

To connect the analysis with the empirical literature on gravity, note that equation (31) has a multiplicative form consistent with the Head and Mayer (2013) definition of general gravity. This implies the indirect discrimination framework fits squarely within the structural gravity literature. Consequently, as emphasized by Egger and Nigai (2015), the validity of inference depends fundamentally on the specification of bilateral trade costs since any unobserved trade costs will inevitably bias the estimates of observed trade costs and the importer and

exporter fixed effects. To see this note that the structural gravity model can be written as:

$$X_{ij} = exp(\zeta_j + \delta_{ij} + \mu_i) \tag{35}$$

where δ_{ij} reflects country pair bilateral trade costs and ζ_j and μ_i are respectively exporter and importer specific variables. The latter two are implicit functions of bilateral trade costs through the resource constraint (with deficit parameter D_i):

$$\sum_{i=1}^{J} X_{ij} = \sum_{i=1}^{J} X_{ji} + D_j \tag{36}$$

which delivers the structural country parameters:

$$exp(\zeta_j) = \frac{\sum_{i=1}^{J} exp(\zeta_i + \delta_{ji} + \mu_j) + D_j}{\sum_{i=1}^{J} exp(\delta_{ij} + \mu_i)}; \qquad exp(\mu_i) = \frac{\sum_{j=1}^{J} exp(\zeta_i + \delta_{ji} - \mu_j) - D_j}{\sum_{i=1}^{J} exp(\zeta_j + \delta_{ij})} \tag{37}$$

Hence, a typical fixed effect approach produces unbiased estimates of the structural model only if (35)-(37) are satisfied and δ_{ij} is measured without error.

To account for this issue we follow the approach of Egger and Nigai (2015) who build on the work of Silva and Tenreyro (2006) and Fally (2015). In particular, Egger and Nigai (2015) propose a two step procedure to estimate gravity models, with the first stage employing a dummy variable model to provide an unbiased decomposition of trade costs into an exporter effect, an importer effect and a bilateral effect. This is achieved by disciplining parameter estimates to satisfy (35)-(37) – constrained analysis of variance (CANOVA). Bilateral trade costs can then be further decomposed in a second stage. They show that the standard one-step methodology is associated with pronounced bias in parameter estimates which can be minimized by the CANOVA procedure.

Table 3 provides a set of results to evaluate the predictions of Propositions 6 and 7. The estimates are based on the 40 countries included in the World Input Output Database.[26]

[26]The appendix documents the data and sources used. This is the same base dataset as used by Costinot and Rodriguez-Clare (2014) and Fajgelbaum and Khandelwal (2015).

The approach is motivated by equation (31) which suggests a standard gravity formulation augmented by terms to reflect deviations from the typical specification due to both differences across trade partners in per capita income and income dispersion. Note that this direct link between the model and the aggregate gravity specification contrasts with the previous literature which has typically adopted ad hoc formulations at the aggregate level or relied on non-homothetic preferences that generate predictions about sectoral rather than aggregate trade volumes (see Hallak (2010), Fieler (2011) and Caron et al. (2014)).[27]

Table 3 decomposes the bilateral trade costs estimated in the first stage, $exp(\delta_{ij})$ for the years 1995 and 2005.[28] The specification includes the typical list of candidate measures of trade costs. It adds to the standard list of bilateral trade costs by including the absolute differences in log per capita income between trading partners ("Linder Income") and also differences in income dispersion as measured by absolute value of differences in the Gini coefficient ("Linder Gini"). Table 1 provides summary statics for these new variables.

Table 1
Summary statistics for 2005

Variable	Mean	P50	Std. Dev.
Linder Income	0.72	0.58	0.63
Linder Gini	0.07	0.05	0.06

Table 3 considers two specifications. In columns (1) and (3) the usual gravity factors are used, augmented by the Linder terms. By construction the parameter estimates are independent of country specific effects, but they are nevertheless susceptible to bias due to correlation with trade cost factors not included in the specification. This is a legitimate concern since the basic specification employed in (1) and (3) only explains 53% and 41% of the observed variation, respectively. To improve the fit, importer, exporter and intra-country trade effects are included. In both years this has a pronounced impact of the fraction of variation explained. In light of this difference, we will focus on the results in columns (2) and (4).

[27] Fajgelbaum and Khandelwal (2015) is an exception since they derive an aggregate relationship. However, they estimate a trade share equation as a function of gravity variables along with the interaction of inequality adjusted real income of an importer and the income elasticity of an exporter.

[28] This method uses J^2 observations on bilateral trade flows and accounts for them by $J(J-1)$ country-pair specific indicators for all pairs $i \neq j$, J exporter indicators and J importer indicators, subject to the GE constraints in (37). The parameters on country-pair specific indicators on intra-national trade costs are normalized to zero ($\delta_{ii} = 0$).

Consistent with Proposition 6 the coefficient on the Linder Income variable is negative and significant across both years. The results are also economically meaningful. Evaluated at the median difference in per capita income for 2005, trade flows are reduced by around 9% due to differences in per capita income. This increases to 11% when evaluated at the mean difference in per capita income. Similarly the negative and significant coefficient on Linder Gini matches the prediction of Proposition 7. Differences in income distribution also have an important impact on trade flows. Trading partners which differ in their Gini coefficients by 0.05 units in 2005 have trade volumes that are 6% lower than trading partners that have the same Gini. While the impact on trade flows seems to be considerable, it also suggests that the welfare impact of market integration can also be pronounced. The next section provides a quantitative assessment of the potential role of this new mechanism.[29]

4 Quantifying the Gains from Integration

Proposition 5 identifies a threshold level of trade costs above which markets are segmented. With segmented markets, the gains from trade coincide with those defined by ARC and computed by CRC. In particular, these papers define the gains from trade as the proportional change in welfare from an observed point to the counterfactual of autarky. This definition allows the parsimonious and elegant sufficient static for the gains from trade to be employed with no additional parameters or equilibrium calculations. In contrast, moving from one trading equilibrium to another typically involves solving for the change in the equilibrium set of factor prices. For a broad class of models ACR show that this calculation only depends on trade data and the trade elasticity for a given vector of changes in trade costs. Nevertheless, conditional on a particular proportional change in the domestic expenditure share, $\widehat{\lambda}_{ii}$, the gains from trade can be computed. Consequently, the ACR framework and the indirect discrimination model predict the same gains from trade if markets are segmented in the sense of Proposition 5. However, we are also interested in the predictions concerning additional liberalization/integration. It is in this dimension that the two models diverge and the need

[29]Note that it is important to correct for bias using the CANOVA methodology. The typical one step procedure generates a positive and significant coefficient on "Linder Income", matching the finding of Hallak (2010).

for quantification is most apparent.

Consider a reduction in trade barriers such that complete integration of markets is achieved by λ_{ii}^{Int}, then the ARC measure calculates the gains to be $\widehat{W}_i = \left(\frac{\lambda_{ii}^{Int}}{\lambda_{ii}}\right)^{\frac{1}{1-\sigma}} = \hat{\lambda}_{ii}^{\frac{1}{1-\sigma}}$. Proposition 5 says that if firms utilize a strategy of indirect discrimination this simple formula will no longer be sufficient to capture the welfare change. In particular, the welfare change is modified by the position of an individual in the local and the global income distribution. For income groups $I \in [1, ..., \bar{I}]$ the following measures apply:

$$\widehat{W}_i^{\bar{I}} = \left(\frac{1-\phi_{i\tau}^{k\bar{I}\rho}}{1-\phi^{k\bar{I}\rho}}\right)\hat{\lambda}_{ii}^{\frac{1}{1-\sigma}} = (Adj_{\bar{I}})\,\hat{\lambda}_{ii}^{\frac{1}{1-\sigma}}, \qquad \widehat{W}_i^{I} = \left(\frac{\phi^{I\bar{I}}}{\phi_{i\tau}^{I\bar{I}}}\right)\widehat{W}_i^{\bar{I}} = (Adj_I)\,\widehat{W}_i^{\bar{I}} \tag{38}$$

where $k = \bar{I} - 1$ and a subscript τ on ϕ implies a design based on the trade cost vector τ. Similar to ACR/CRC the calculation of the gains from integration requires information on the share of domestic expenditure, λ_{ii}^{Int}. The new dimension relates to changes in designs. If a change in trade costs doesn't induce changes in relative design within a market, then (38) collapses to the usual ACR equation. Since we are interested in additional liberalization, we will consider the relative design for each income group at the initial equilibrium ($\phi_{i\tau}^{I\bar{I}}$) and under complete integration ($\phi^{I\bar{I}}$).

To make progress assume that the initial trade equilibrium is characterized by market segmentation – each country has a set of designs that are based on the national income distribution.[30] In addition we follow CRC and consider 2008 as the benchmark year. Under the assumption of segmentation, we need to calculate the designs for each country. In contrast, under full integration we only need to determine the designs at the global level.

The design in any equilibrium depends on the distribution of income and the elasticity σ. Information on the income distribution for a large number of countries is compiled by Lakner and Milanovic (2015). These data have the virtue that they are constructed for the purpose of international comparison and also to facilitate the derivation of the global distribution of income. A national income distribution is represented by population deciles and the mean

[30]While this assumption facilitates the analysis it is also consistent with Dvir and Strasser (2014) who find that car manufacturers discriminate by manipulating the menu of included car features available across countries over 2003-2011.

income associated with each decile. Utilizing the information on population for each income bin, the global income distribution is represented by population percentiles and the mean income in each bin. To reduce the dimensionality of the design calculation and to translate the information into a form more pertinent to a firm, the global income distribution is broken into five bins, each with the same total income. The population distribution for each quintile is given in Table 2.

Table 2

Global Income Distribution 2008

	Q1	Q2	Q3	Q4	Q5
Population Share	74	13	7	4	2

Under full integration a firm designs products based on the global income distribution. However, under segmented markets firms use the local distribution. This is derived as the fraction of the national population which falls into each bin. Such a formulation ensures that the national income distributions aggregate to the global distribution.[31] The equilibrium set of designs then must satisfy:

$$\phi^{IJ\rho}\frac{\beta^I}{\sum_I^{\bar{I}}\beta^i}+\phi^{IJ}\frac{\phi^{J\bar{I}}}{\bar{\alpha}^{J\bar{I}}}\frac{\sum_J^{\bar{I}}\beta^i}{\sum_I^{\bar{I}}\beta^i}=\bar{\alpha}^{IJ} \qquad J>I \tag{39}$$

where $\bar{\alpha}^{IJ}=\frac{\bar{m}^I}{\bar{m}^J}$, $\bar{m}^I=\frac{\rho(m^I-m^k)}{1-\phi^{kI\rho}}$, $m_0=0$ and $\phi_{01}=0$. Hence the difference between the segmented and the integrated equilibrium is determined exclusively by the difference between the national and global income distribution.

Table 4 presents the adjustment factors defined by (38) utilizing the designs implied by (39) for the set of countries analyzed by CRC. Columns (1)-(5) are the adjustments required when moving from segmentation to full integration. A void implies that a country has no mass in that part of the distribution. These factors suggest that the USA is likely to have its gains from integration understated the most by the ACR measure. In particular, the lowest income group will have gains up to $\frac{1}{3}$ higher if design considerations are included in

[31] We follow Fajgelbaum and Khandelwal (2015) and assume that the extent of integration doesn't alter the income distribution.

welfare calculations. In fact, every income group in the USA is predicted to have gains from integration augmented by design improvements. However, this is a relatively uncommon outcome, with only four other countries having all income groups gain unambiguously from beneficial design changes. The majority of countries have at least one income group that is subject to the negative consequences of design change – for many this also applies to the majority of the population.

To help put these adjustments in context column 6 lists (the inverse of) the proportional change in domestic expenditure share required for all income groups to unambiguously gain from integration. If full integration occurs before this change in expenditure share is achieved then at least one income group in country i will be better off in the initial trading equilibrium. For some countries the reduction in domestic expenditure share is relatively small and it is plausible that no income group in these countries would be adversely affected by further integration. These tend to be either relatively rich and/or open economies. However, for other countries this gap is relatively daunting, with 13 countries facing at least a 10 percentage point decline in domestic expenditure share before the traditional gains from trade are sufficient to offset the design consequences of integration.

A sense of the adjustment required is given by the change in the domestic share for each country over the period 1995-2008. This offers the potential for an alternative interpretation. Suppose, in contrast to the maintained assumption, that 2008 reflects a fully integrated equilibrium. Then assuming that 1995 represents a segmented equilibrium, and provided (7) $>$ (6), it would be the case that all income groups gain from integration. However, for 18 countries this condition fails. Consequently, either set of assumptions suggest that design consideration have the potential to appreciably alter the gains from trade liberalization.

4.1 Multiple Sectors

By employing a one factor/one sector framework the results in Table 4 demonstrate, among other things, that non-homothetic preferences are not necessary for the gains from trade to vary by income group within a country. However, there is no reason to restrict the analysis to a single sector. To incorporate multiple sectors, once again we follow convention by assuming

a two-tier utility function in which the upper level is Cobb-Douglas and the lower level is SDS. Specifically assume $U = \Pi_{s=1}^{S} Q_s^{\gamma_s},\ \gamma_s \in (0,1)\ \ \&\ \ \sum \gamma_s = 1$. Since the only variation across consumers within a country is income, we have:

$$Q_s^I = \frac{\gamma_s \bar{m}^I}{P_s^I} \quad \text{where } P_s^I = \left[\sum_i p_{i,s}^{I\,\frac{\rho_s}{\rho_s - 1}} \right]^{\frac{\rho_s - 1}{\rho_s}}$$

While these expressions have a familiar form there is an additional feature to highlight. The Cobb-Douglas specification normally results in fixed budget shares, ruling out cross price effects. However, since demand in each sector is a function of $\bar{m}^I$, a model with indirect discrimination potentially has cross price effects and non-constant budget shares from gross income. In particular, this framework can mimic the non-linear Engel curves that motivate the use of non-homothetic preferences.

In this expanded model demand for variety i in sector s targeted to consumer I, is given by:

$$p_{i,s}^I = \theta_s^I q_{i,s}^{\rho_s - 1} \qquad \text{with } \ \theta_s^I = \frac{\gamma_s \bar{m}^I}{Q_s^{I\rho_s}}$$

The equilibrium conditions for product design are a straightforward extension of (39) only requiring the substitution of ρ_s and ϕ_s^{IJ} in the obvious places. A more involved calculation is associated with the derivation of $\bar{m}^I$. Nevertheless this also has a relatively familiar form:

$$\bar{m}_I = \frac{(m_I - m_k)}{\sum_s \frac{\gamma_s}{\rho_s} \left(1 - \phi_s^{kI\rho_s}\right)} \quad , \ \ m_0 = 0 \quad \& \ \ \phi_s^{01} = 0 \tag{40}$$

The analogue to (38) is given by

$$\widehat{W}_i^{\bar{I}} = \Pi_{s=1}^{S} \left(\frac{1 - \phi_{i\tau,s}^{k\bar{I}\rho_s}}{1 - \phi_s^{k\bar{I}\rho_s}} \right) \left(\frac{\hat{\lambda}_{ii,s}}{\hat{r}_{i,s}} \right)^{\frac{\gamma_s}{1-\sigma_s}}, \qquad \widehat{W}_i^{I} = \Pi_{s=1}^{S} \left(\frac{1 - \phi_{i\tau,s}^{k\bar{I}\rho_s}}{1 - \phi_s^{k\bar{I}\rho_s}} \right) \left(\frac{\phi_s^{I\bar{I}}}{\phi_{i\tau,s}^{I\bar{I}}} \right)^{\gamma_s} \left(\frac{\hat{\lambda}_{ii,s}}{\hat{r}_{i,s}} \right)^{\frac{\gamma_s}{1-\sigma_s}} \tag{41}$$

where $r_{i,s}$ is the share of total revenues in country i generated from sector s. Once again the welfare measures augment the one defined by ACR/CRC: $\Pi_{s=1}^{S} \left(\frac{\hat{\lambda}_{ii,s}}{\hat{r}_{i,s}} \right)^{\frac{\gamma_s}{1-\sigma_s}}$. The welfare implications of allowing for multiple sectors now feature within sector design components to

the welfare changes.

To illustrate the implications of this set-up, consider a two sector version of the model. In particular, split the sectors between traded (sector 1) and non-traded products (services, sector 2). We follow CRC and assume that the service sector plays a relatively passive role in terms of trade liberalization. We implement this by assuming that the ACR/CRC measure in sector 2 equals 1.[32] As highlighted above, if $\sigma_1 \neq \sigma_2$, then the expenditure shares will also differ from γ_s. Aggregating the WIOD data into two sectors generates the typical non-linear Engel curve across countries: the expenditure share on traded goods declines with per capita GDP. Such a non-linearity can be generated when $\sigma_1 < \sigma_2$. To explore this issue a range of values for σ_1 and σ_2 are considered. In addition, to discipline the exercise, γ_1 is selected to match the average expenditure on tradables across country of 0.30.

Table 5 presents the net changes in domestic expenditure and comparative advantage necessary for every income group to unambiguously gain when moving from an initial trading equilibrium. Column 1 provides a direct extension from Table 4 with the threshold change now net of the non-traded sector. Not surprisingly this translates to larger changes required before all income groups within a country gain unambiguously. Context is provided by column (6) which can be interpreted as either a measure of capacity for change (assuming markets are segmented in 2008) or a sense of whether sufficient benefits have been derived from liberalization (if 1995 is viewed as segmented and 2008 as integrated). A comparison of (1) and (6) reveals that for 22 countries column (1) is greater than column (6) – in these countries at least one income group would prefer the initially segmented equilibrium compared to a fully integrated equilibrium.

Moving from left to right the relative differential between σ_1 and σ_2 increases, with the service sector becoming relatively more elastic. This generates a more pronounced non-linear Engel curve, with the associated γ_1 decreasing to maintain an average expenditure share of 0.30 across countries. This reduction in γ_1 lowers the weight on sector 1 in (41), leading to an associated reduction in the compensatory changes required in the ACR/CRC measure. The parameters in column (5) generate the most plausible expenditure patterns and also predict

[32]This assumption is most consistent with the indirect discrimination framework when σ_2 is relatively large.

the most optimistic outcomes for all income groups. Nevertheless, the gains required from the ACR/CRC sources are still greater than those realized between 1995 and 2008 for over a third of the countries. This suggests that design changes are still likely to have an appreciable impact on the gains from trade, even in a multi-sector setting.

5 Conclusion

This paper considers the implications of allowing firms to be sophisticated enough to design product lines. This makes them interested in consumer level information, and the distribution of income in particular. Enriching firm behavior in this way results in a tractable model and provides a link between the distribution of income and the gains from trade.

This link arise as firms indirectly discriminate between the various income classes, which in equilibrium results in a product line that differs from the first best allocation. Since the distortions have the largest negative welfare impact at the lower end of the income distribution, this is where the consequences of international integration are also most pronounced. However, trade can mitigate these welfare losses in countries whose income distribution dominates the global distribution, while amplifying them in countries that are dominated.

These findings imply even more magnified changes under a process of gradual liberalization since the variety and design dimensions of welfare change respond differentially to the level of trade barriers. In particular, design changes occur disproportionately at lower trade barriers, with the potential to derail the process of trade liberalization. Quantifying the relative importance of this mechanism suggests that it is a legitimate issue that could significantly complicate future integration efforts.

Table 3

Decomposition of CANOVA Bilateral Trade Factors: $exp(\delta_{ij})$

	(1) 1995	(2) 1995	(3) 2005	(4) 2005
Linder Income	-0.63***	-0.38***	-1.41***	-0.16***
	(0.14)	(0.07)	(0.25)	(0.06)
Linder Gini	-2.44***	-1.70***	-15.93***	-1.18**
	(0.87)	(0.63)	(3.27)	(0.57)
Distance	-0.30***	-0.75***	-0.34***	-0.98***
	(0.10)	(0.09)	(0.08)	(0.06)
Contiguity	0.28	0.21*	-1.02***	0.16
	(0.24)	(0.13)	(0.21)	(0.10)
Language	-0.49	0.25*	-0.38	0.45***
	(0.42)	(0.14)	(0.30)	(0.10)
Colonial	0.92***	0.22**	-0.03	0.12
	(0.27)	(0.11)	(0.43)	(0.11)
PTA	0.83***	0.23***	-0.37*	-0.12
	(0.13)	(0.08)	(0.22)	(0.08)
Legal	0.38**	0.17**	1.44***	-0.08
	(0.16)	(0.09)	(0.13)	(0.06)
Currency	3.92***	1.11***	1.75***	0.32***
	(0.23)	(0.31)	(0.13)	(0.08)
Pseudo R^2	.53	.77	.41	.85
Observations	1,600	1,600	1,600	1,600
domestic sales fe	n	y	n	y
importer fe	n	y	n	y
exporter fe	n	y	n	y
Specification	PPML	PPML	PPML	PPML

Pseudo R^2 = squared correlation between (log) calibrated trade costs and predicted trade costs.
Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4

Modified Welfare Outcomes from Trade Liberalization

	(1) Adj_1	(2) Adj_2	(3) Adj_3	(4) Adj_4	(5) Adj_5	(6) $\left(\frac{1}{\hat{\lambda}}\right)$	(7) $\left(\frac{1}{\hat{\lambda}}\right)_{95}^{08}$
AUS	1.21	1.05	1.02	0.99		1.03	1.02
AUT	1.23	1.06	1.02	1.00		1.02	1.09
BEL	1.20	1.04	1.00	0.98		1.08	1.05
BRA	0.98	0.98	0.97			1.16	1.02
CAN	1.31	1.14	1.09	1.04	1.02		
CHN	0.95	0.95				1.32	1.06
CZE	1.08	0.98	0.97			1.16	1.08
DEU	1.24	1.08	1.04	1.03	1.02		1.11
DNK		1.06	1.02	1.00		1.02	1.09
ESP	1.16	1.02	1.00	0.98		1.09	1.04
FIN	1.20	1.04	1.00	0.98		1.08	1.06
FRA	1.24	1.08	1.04	1.03	1.02		
GBR	1.30	1.13	1.08	1.04	1.02		
GRC	1.12	1.01	1.00	0.98		1.10	1.09
HUN	0.99	0.95				1.31	1.22
IDN	0.93					1.48	1.03
IND	0.93					1.48	1.06
IRL	1.21	1.05	1.02	0.99		1.03	1.06
ITA	1.20	1.04	1.00	0.98		1.08	1.03
JPN	1.17	1.04	1.00	0.98		1.08	1.05
KOR	1.19	1.05	1.02	0.99		1.03	1.09
MEX	0.98	0.98	0.97			1.16	1.03
NLD		1.08	1.03	1.00		1.01	1.04
POL	0.99	0.95				1.31	1.12
PRT	1.08	0.99		0.99		1.06	1.05
ROM	0.95	0.95				1.32	1.09
RUS	1.00	0.97	0.97			1.18	1.01
SVK	0.99	0.95				1.31	1.16
SVN	1.17	1.01	1.00	0.98		1.09	1.08
SWE	1.23	1.06	1.02	1.00		1.02	1.07
TUR	0.97	0.94				1.33	1.05
TWN	1.19	1.03	1.00	0.98		1.09	1.09
USA	1.32	1.15	1.09	1.06	1.03		

Table 5

Changes required in Domestic Expenditure Share and Comparative Advantage

	$\sigma_1 = 6$ $\sigma_2 = 6$ $\gamma_1 = 0.30$	$\sigma_1 = 5$ $\sigma_2 = 7$ $\gamma_1 = 0.29$	$\sigma_1 = 4$ $\sigma_2 = 8$ $\gamma_1 = 0.275$	$\sigma_1 = 3$ $\sigma_2 = 9$ $\gamma_1 = 0.25$	$\sigma_1 = 2$ $\sigma_2 = 10$ $\gamma_1 = 0.20$	
	(1)	(2)	(3)	(4)	(5)	(6)
	$\overline{\left(\frac{\hat{r}_1}{\hat{\lambda}_1}\right)}$	$\overline{\left(\frac{\hat{r}_1}{\hat{\lambda}_1}\right)}$	$\overline{\left(\frac{\hat{r}_1}{\hat{\lambda}_1}\right)}$	$\overline{\left(\frac{\hat{r}_1}{\hat{\lambda}_1}\right)}$	$\overline{\left(\frac{\hat{r}_1}{\hat{\lambda}_1}\right)}$	$\left(\frac{\hat{r}_1}{\hat{\lambda}_1}\right)_{95}^{08}$
AUS	1.10	1.07	1.05	1.03		1.40
AUT	1.08	1.06	1.04	1.02		1.40
BEL	1.30	1.23	1.17	1.12	1.06	1.15
BRA	1.62	1.46	1.35	1.27	1.18	0.95
CAN						1.05
CHN	2.50	2.06	1.79	1.60	1.44	1.33
CZE	1.65	1.48	1.36	1.27	1.17	1.45
DEU						1.09
DNK	1.07	1.06	1.04	1.02		0.96
ESP	1.35	1.27	1.20	1.15	1.08	1.09
FIN	1.30	1.23	1.17	1.12	1.06	1.04
FRA						0.79
GBR						0.97
GRC	1.37	1.28	1.21	1.16	1.09	1.68
HUN	2.44	2.02	1.75	1.55	1.35	1.01
IDN	3.66	2.78	2.27	1.94	1.65	0.96
IND	3.66	2.78	2.27	1.94	1.65	0.69
IRL	1.10	1.07	1.05	1.03		1.00
ITA	1.30	1.23	1.17	1.12	1.06	1.16
JPN	1.31	1.23	1.18	1.13	1.06	1.21
KOR	1.10	1.07	1.05	1.03		1.05
MEX	1.62	1.46	1.35	1.27	1.18	1.19
NLD	1.04	1.03	1.02	1.00		1.18
POL	2.44	2.02	1.75	1.55	1.35	1.03
PRT	1.23	1.17	1.13	1.10	1.05	0.90
ROM	2.50	2.06	1.79	1.60	1.44	0.95
RUS	1.73	1.54	1.41	1.31	1.21	1.47
SVK	2.44	2.02	1.75	1.55	1.35	1.31
SVN	1.32	1.25	1.19	1.14	1.07	1.14
SWE	1.08	1.06	1.04	1.02		0.95
TUR	2.59	2.12	1.82	1.60	1.40	1.27
TWN	1.33	1.25	1.19	1.14	1.08	0.95
USA						1.21

Parameter values generate an average expenditure on tradables of 0.30 across all countries in the initial equilibrium using 2008 data.

6 Appendix

6.1 Proof of Propostion 1

It is slightly simpler to work with $\{\phi^{LM}, \phi^{MH}\}$, which must satisfy:

$$\beta^H \phi^{MH\rho} + \beta^M \phi^{MH} = (\beta^M + \beta^H)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-\phi^{MH\rho}}{1-\phi^{LM\rho}}\right) \tag{42}$$

$$(1-\beta^L)\phi^{LM\rho} + \beta^L \phi^{LM}\phi^{MH}\left(\frac{1-\phi^{LM\rho}}{1-\phi^{MH\rho}}\right) = \left(\frac{1-\alpha}{\alpha}\right)(1-\phi^{LM\rho}) \tag{43}$$

Equation (42) implicitly defines a function $M(\phi^{LM}, \phi^{MH}) \equiv 0$ and similarly for equation (43), $L(\phi^{LM}, \phi^{MH}) \equiv 0$. Begin by noting that the slope of the first condition is positive while the slope of the second is negative. That is,

$$\frac{d\phi^{MH}}{d\phi^{LM}}\Big|_{M(\phi^{LM},\phi^{MH})=0} = -\frac{\partial M/\partial \phi^{LM}}{dM/\partial \phi^{MH}} > 0$$

where

$$\begin{aligned}
\frac{\partial M}{\partial \phi^{LM}} &= -\rho\phi^{LM\rho-1}(1-\beta^L)\overline{\alpha}_{MH} < 0 \\
\frac{\partial M}{\partial \phi^{MH}} &= (\beta^M + \beta^H \rho \phi^{MH\rho-1})(1-\phi^{LM\rho}) + \rho\phi^{MH\rho-1}(1-\beta^L) > 0
\end{aligned}$$

$$\frac{d\phi^{MH}}{d\phi^{LM}}\Big|_{L(\phi^{LM},\phi^{MH})=0} = -\frac{\partial L/\partial \phi^{LM}}{dL/\partial \phi^{MH}} < 0$$

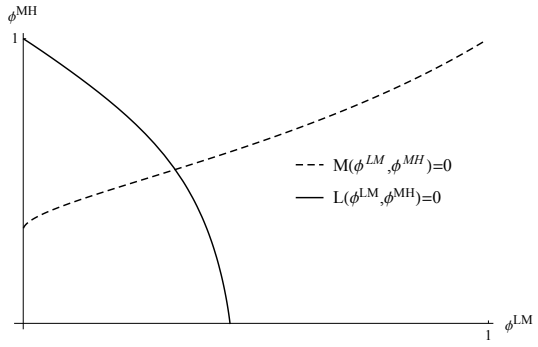
where

$$\begin{aligned}
\frac{\partial L}{\partial \phi^{LM}} &= (1-\beta^L)\rho\phi^{LM\rho-1} + \frac{(1-\beta^L)\rho\phi^{LM2\rho-1}}{1-\phi^{LM\rho}} + \frac{(1-\beta^L)\phi^{MH}(1-\phi^{LM\rho})}{1-\phi^{MH\rho}} > 0 \\
\frac{\partial L}{\partial \phi^{MH}} &= \rho\phi^{MH\rho-1}\beta^L\phi^{LM}\phi^{MH}(1-\phi^{LM\rho}) > 0
\end{aligned}$$

Furthermore, $L(\phi^{LM}, \phi^{MH}) = 0$ implies that when $\phi^{MH} = 0 \Rightarrow \phi^{LM\rho} = \frac{(1-\alpha)}{(1-\alpha)+\alpha(1-\beta^L)} \leq 1$, while $\phi^{LM} = 0 \Rightarrow \phi^{MH} = 1$. Using, $M(\phi^{LM}, \phi^{MH}) = 0$ implies that when $\phi^{LM} = 0 \Rightarrow \phi^{MH\rho} + \phi^{MH} = \frac{(1-\beta^L)}{(1-\beta^L-\beta^H)} \leq 1$. Implying a unique intersection with $\phi^{LM}, \phi^{MH} \in (0,1)$. Which can also be confirmed by plotting the conditions:

6.2 Proof of Proposition 2

The first best product for income group I is $q_1^I = \frac{\rho m^I}{n\bar{w}}$. The product offered under indirect discrimination to type I is $q^I = \frac{\bar{m}^I}{np^I}$. From (17) and (22) we have $q^L = \frac{\rho(1-\alpha)}{n}\left(\frac{\phi^{LH}}{\overline{\alpha}^{LH}}\right)$. From (16) it follows that $\overline{\alpha}^{LH} \in [\phi^{LH}, \phi^{LH\rho}]$ where the lower bound only arises if $\beta^L = 1$. Hence, $q_1^L > q^L$ for $\beta^L < 1$.



Since $q_1^L > q^L$, it must be the case that $q^H > q_1^H$ and/or $q^M > q_1^M$ (since firm scale is first best under indirect discrimination). Note that $q^H > q_1^H$ requires $\frac{\rho\alpha}{n(1-\phi^{MH\rho})} > \frac{\rho(1+\alpha)}{n}$ which implies $1 - \phi^{MH\rho} - \alpha\phi^{MH\rho} < 0$. In contrast, $q^M > q_1^M$ requires $\frac{\rho\alpha}{n(1-\phi^{LM\rho})}\left(\frac{\phi^L M}{\bar{\alpha}^{LM}}\right) > \frac{\rho}{n}$ which implies $1 - \phi^{MH\rho} - \alpha\phi^{MH} < 0$. So whenever $q^M > q_1^M$ then it must be the case that $q^H > q_1^H$ - which occurs if β^H is sufficiently small. Finally, if $q^M \leq q_1^M$ then $q^H > q_1^H \ \ \forall\ \beta^H \in [0,1)$.

6.3 Proof of Proposition 3

Start by defining the set of distributions $\{b^L, b^M, b^H\}$ that are likelihood ratio dominated (LRD) by $\beta \equiv \{\beta^L, \beta^M, \beta^H\}$. LRD requires $\frac{\beta^L}{b^L} \leq \frac{\beta^M}{b^M} \leq \frac{\beta^H}{b^H}$. The first part of this equality implies:

$$\frac{\beta^L}{b^L} \leq \frac{\beta^M}{b^M} \qquad \Rightarrow b^H \geq 1 - \left(\frac{1-\beta^H}{\beta^L}\right) b^L$$

While the second part implies:

$$\frac{\beta^M}{b^M} \leq \frac{\beta^H}{b^H} \qquad \Rightarrow b^H \leq \frac{\beta^H}{1-\beta^L}(1-b^L)$$

This set is illustrated by the dashed lines in Figure 1.
Note that (28)-(30) implies the following differences:

$$\begin{aligned} \hat{U}^L - \hat{U}^M &= \hat{\phi}^{LM} \\ \hat{U}^M - \hat{U}^H &= \hat{\phi}^{MH} \end{aligned}$$

Consequently the proposition requires that $\hat{U}^L > 0$ and $\hat{U}^H > 0$ when a country with an income distribution β integrates with an country in the LRD set. To characterize the behavior of ϕ^{MH} consider the level sets $\phi^{MH}(\beta^L, \beta^H)$. The slope of a contour is given by:

$$\frac{d\beta^H}{d\beta^L} = -\frac{d\phi^{MH}/d\beta^L}{d\phi^{MH}/d\beta^H}$$

To evaluate the RHS use the system:

$$\begin{aligned} M(\beta,\phi) &\equiv \beta^H\phi^{MH\rho} + (1-\beta^L-\beta^H)\phi^{MH} - (1-\beta^L)\left(\frac{1-\phi^{MH\rho}}{1-\phi^{LM\rho}}\right) = 0 \\ L(\beta,\phi) &\equiv (1-\beta^L)\phi^{LM\rho} + \beta^L\phi^{LM}\phi^{MH}\left(\frac{1-\phi^{LM\rho}}{1-\phi^{MH\rho}}\right) - \left(\frac{1-\phi^{LM\rho}}{\alpha}\right)(1-\alpha) = 0 \end{aligned}$$

Therefore

$$\begin{aligned}\frac{d\beta^H}{d\beta^L} &= -\left(\frac{-M_{\beta^L}L_{\phi^{LM}} + L_{\beta^L}M_{\phi^{LM}}}{M_{\beta^H}L_{\phi^{LM}}}\right) \\ &= -\frac{\beta^H}{1-\beta^L} + \frac{L_{\beta^L}M_{\phi^{LM}}}{M_{\beta^H}L_{\phi^{LM}}}\end{aligned}$$

Since $L_{\beta^L} < 0, M_{\phi^{LM}} < 0$ while $M_{\beta^H} > 0, L_{\phi^{LM}} > 0$, the second term on the RHS is positive, which implies the slope of the contour is decreasing in β^L but always greater than $-\frac{\beta^H}{1-\beta^L}$. Consequently, integration with any country which is LRD will result in a global income distribution that is also LRD and consequently lead to an increase in ϕ^{MH} relative to autarky.

Since LRD implies $\hat{\phi}^{MH} > 0$, confirming that the gains for the middle type are greater than the gains for the high type. The ranking of the low and middle types requires integration to yield $\hat{\phi}^{LM} > 0$. To see that this is also implied by LRD, consider that the slope of the ϕ^{LM} contour is given by:

$$\begin{aligned}\frac{d\beta^H}{d\beta^L} &= -\left(\frac{-L_{\beta^L}M_{\phi^{MH}} + M_{\beta^L}L_{\phi^{MH}}}{M_{\beta^H}L_{\phi^{MH}}}\right) \\ &= -\frac{\beta^H}{1-\beta^L} + \frac{L_{\beta^L}M_{\phi^{MH}}}{M_{\beta^H}L_{\phi^{MH}}}\end{aligned}$$

Since $L_{\beta^L} < 0$, this implies that the second term is negative and less than $-\frac{\beta^H}{1-\beta^L}$. This implies that moving along the constraint $b^H = \frac{\beta^H}{1-\beta^L}(1-b^L)$ from β results in higher ϕ^{LM}. However, if the slope of the contour is greater than $-\frac{1-\beta^L}{\beta^H}$, moving along $b^H = 1 - \left(\frac{1-\beta^H}{\beta^L}\right) b^L$ results in lower ϕ^{LM}. To see that this doesn't occur, consider the slope of the contour when $b^H = 0$. This implies $\phi^{MH} = \bar{\alpha}^{MH}$ and the slope of the contour line when $b^H = 0$ can be expressed as:

$$-\left(\frac{1-\beta^H}{\beta^L}\right)\left(\frac{1-\beta^L-\beta^H}{1-\beta^H}\right)\left(\frac{\phi^{LM\rho}(b^L) - \phi^{LM}(b^L)}{\phi^{LM\rho}(b^L)\phi^{LM}(b^L)}\right)$$

To ensure that this is less than $-\frac{1-\beta^H}{\beta^L}$ requires $\left(\frac{1-\beta^L-\beta^H}{1-\beta^H}\right)\left(\frac{\phi^{LM\rho}-\phi^{LM}}{\phi^{LM\rho}\phi^{LM}}\right) \geq 1$.

Writing this condition as:

$$(1-\beta^H)(\phi^{LM\rho} - \phi^{LM} - \phi^{LM\rho}\phi^{LM}) \geq \beta^L(\phi^{LM\rho} - \phi^{LM})$$

This holds when $\beta^L = 0$. Holding β^H constant, as β^L increases the LHS decreases while the RHS increases. This means if this condition fails for any β^L it will also fail for $\beta^L = (1-\beta^H)$. To evaluate this limit note that as $\beta^L \to 1-\beta^H$ we know from $M(\beta,\phi) \equiv 0$ that $\phi^{LM} \to 1$. This implies that there are really only two types in this setting, L and H, and it is straightforward to show that the low types welfare increases proportionally more than the high types as β^L increases.

Hence LRD implies that the dominant country gains more than the standard model and the gains are proportionately higher the lower is the income.

6.4 Proof of Proposition 5

The central claim is that there exists a trade cost, $\bar{\tau}$, such that above this trade cost the gains from trade are equivalent to the standard model and below that level the gains are manifestly different. Begin by considering trade costs that are sufficiently high that markets are segmented. In order for the gains from trade to be the same as the standard model we require that relative product design is not altered by trade barriers. From Proposition 1 we know that each isolated market has a unique equilibrium with the relative design in each market given by $\{\phi_{i\mathcal{A}}^{IH}\}$. If firms from j ship to i then the first order conditions under segmentation are:

$$\frac{\partial \pi_j}{\partial q_{ij}^H} = \theta_i^H q_{ij}^{H\rho-1} - \tau w_j = 0 \tag{44}$$

$$\frac{\partial \pi_j}{\partial q_{ij}^M} = ((\beta_i^M + \beta_i^H)\theta_i^M - \beta_i^H \theta_i^H) q_{ij}^{M\rho-1} - \beta_i^M \tau w_j = 0 \tag{45}$$

$$\frac{\partial \pi_j}{\partial q_{ij}^L} = (\theta_i^L - (1-\beta_i^L)\theta_i^M) q_{ij}^{L\rho-1} - \beta^L \tau w_j = 0 \tag{46}$$

However, it is immediately apparent that combining (44) and (45) and along with (44) and (46) reproduces the equilibrium conditions (14) and (15) which is solely a function of the distribution of income in country i and ρ. Consequently, under segmentation firms from both locations offer the product line $\{\phi_{i\mathcal{A}}^{IH}\}$ in country i.

To show the existence of $\bar{\tau}$, note that it is the location of the outside option which is relevant – i.e. for the income class immediately below type I, is the next best option within a product line local or not (this also implies that we should focus on the incentive constraints). Under segmentation the next best option is always strictly the local option. To illustrate the existence of $\bar{\tau}$ consider a setting where $\phi_f^{LM} > \phi_h^{LM}$ (as would arise under the conditions of Proposition 3) which implies a that within a Foreign firm's product line the product designed for the low income consumer in the Foreign country is superior to the product designed for the low income consumer in the Home country. Since the information rents of the middle income consumer are determined by the product offered to low type, the relevant no arbitrage condition is:

$$\begin{aligned}
\theta_h^M \frac{q_{hf}^{L\rho}}{\rho} - T_{hf}^L &> \theta_h^M \frac{q_{ff}^{L\rho}}{\rho\tau^\rho} - T_{ff}^L \\
(\theta_h^M - \theta_h^L)\frac{q_{hf}^{L\rho}}{\rho} &> (\frac{\theta_h^M}{\tau^\rho} - \theta_f^L)\frac{q_{ff}^{L\rho}}{\rho} \\
\frac{(\theta_h^M - \theta_h^L)}{\theta_h^H}\phi_h^{LH\rho}\theta_h^H \frac{q_{hf}^{H\rho}}{\rho} &> \left(\frac{\theta_h^M}{\theta_h^H}\frac{\theta_h^H}{\theta_f^H}\frac{1}{\tau^\rho} - \frac{\theta_f^L}{\theta_f^H}\right)\phi_f^{LH\rho}\theta_f^H \frac{q_{ff}^{L\rho}}{\rho} \\
\frac{(\theta_h^M - \theta_h^L)}{\theta_h^H}\phi_h^{LH\rho}\theta_h^H \tau &> \left(\frac{\theta_h^M}{\theta_h^H}\frac{\theta_h^H}{\theta_f^H}\frac{1}{\tau^\rho} - \frac{\theta_f^L}{\theta_f^H}\right)\phi_f^{LH\rho}
\end{aligned} \tag{47}$$

Under segmentation the relative positions of the residual demand curves within a market are invariant to the trade cost, which implies the LHS of (47) is increasing in τ. Using the

balanced trade condition it can be shown that $\frac{\theta_h^H}{\theta_f^H}\frac{1}{\tau^\rho} = \left(\frac{1-\phi_f^{MH\rho}}{1-\phi_h^{MH\rho}}\right)^{1-\rho}\left(\frac{w_f}{\tau w_h}\right)^\rho$ which is decreasing in τ. Given the LHS is monotonically increasing in τ while the RHS is monotonically decreasing, this implies that there must exist a $\bar{\tau}$ such that a middle income Home consumer will find it attractive to arbitrage within a product line and purchase the product designed for the low income Foreign consumer. However, this implies that below $\bar{\tau}$ the low income markets in the Foreign firm's product line are no longer segmented. The optimal design for low income groups are now linked for a Foreign firm as (47) binds for $\tau < \bar{\tau}$, resulting in $q_{hf}^{L\rho} = \left(\frac{\frac{\theta_h^M}{\tau^\rho} - \theta_f^L}{\theta_h^M - \theta_h^L}\right) q_{ff}^{L\rho}$.

Note that it is not necessarily the case that at $\bar{\tau}$ the low income markets are integrated within a Home firm's product line – it is possible that they are still segmented. This implies that product design below $\bar{\tau}$ will vary by income group, location and firm nationality – so the relative design for income group I, in country i, by firm j is ϕ_{ij}^{IH}.

The gains from trade for a member of a high income group are:

$$\frac{U_{i\mathcal{T}}^H}{U_{i\mathcal{A}}^H} = \frac{\tilde{n}_i^{\frac{1}{\rho}} q_{ii}^H}{n_i^{\frac{1}{\rho}} q_{i\mathcal{A}}^H} = \frac{\tilde{n}_i^{\frac{1}{\rho}} \frac{\bar{m}_i^H}{\tilde{n}_i} w_i}{n_i^{\frac{1}{\rho}} \frac{\bar{m}_{i\mathcal{A}}^H}{n_i}}$$

To complete the derivation requires the net income for a high type.

$$\rho T_{ii}^H = (1-\phi_{ii}^{MH\rho})\frac{\bar{m}_i^H}{\tilde{n}_i} + \rho T_{ii}^M \quad \& \quad \rho T_{ij}^H = (1-\phi_{ij}^{MH\rho})\frac{d_i \bar{m}_i^H}{\tilde{n}_i} + \rho T_{ij}^M$$

$$\begin{aligned}
\rho(n_i T_{ii}^H + n_j T_{ij}^H) &= \bar{m}_i^H \left(\frac{n_i}{\tilde{n}_i} + \frac{d_i n_j}{\tilde{n}_i}\right) - \bar{m}_i^H \left(\frac{n_i}{\tilde{n}_i}\phi_{ii}^{MH\rho} + \frac{d_i n_j}{\tilde{n}_i}\phi_{ij}^{MH\rho}\right) + \rho m_i^M \\
\Rightarrow \bar{m}_i^H &= \frac{\rho(m_i^H - m_i^M)}{1 - \overline{\phi_{i\tau}^{MH}}^{\frac{1}{\rho}}}
\end{aligned}$$

Hence

$$\frac{U_{i\mathcal{T}}^H}{U_{i\mathcal{A}}^H} = \left(\frac{1-\phi_{i\mathcal{A}}^{MH\rho}}{1-\overline{\phi_{i\tau}^{MH}}^{\frac{1}{\rho}}}\right)\left(\frac{\tilde{n}_i}{n_i}\right)^{\frac{1-\rho}{\rho}}$$

While the gains for someone with income I are:

$$\begin{aligned}
\frac{U_{i\mathcal{T}}^I}{U_{i\mathcal{A}}^I} &= \frac{(n_i q_{ii}^{I\rho} + n_j q_{ij}^{I\rho})^{\frac{1}{\rho}}}{n_i^{\frac{1}{\rho}} q_{i\mathcal{A}}^I} = \frac{(n_i \phi_{ii}^{IH\rho} q_{ii}^{H\rho} + n_j \phi_{ij}^{IH\rho} q_{ij}^{H\rho})^{\frac{1}{\rho}}}{n_i^{\frac{1}{\rho}} \phi_{i\mathcal{A}}^{IH} q_{i\mathcal{A}}^H} \\
&= \frac{\overline{\phi_{i\tau}^{IH}}\,\tilde{n}_i^{\frac{1}{\rho}} q_{ii}^H}{\phi_{i\mathcal{A}}^{IH} n_i^{\frac{1}{\rho}} q_{i\mathcal{A}}^H}
\end{aligned}$$

6.5 Derivation of equation (34)

Assume that trade costs are sufficiently small that all markets are partially integrated – home high views foreign middle as an outside option and foreign middle views home lows product as an outside option. This implies that the foreign middle income consumer captures information rents:

$$\theta_f^M q_{fj}^{M\rho} - \rho T_{fj}^M = \theta_f^M \left(\frac{q_{hj}^L}{\tau}\right)^\rho - \rho T_{hj}^L$$

Which must also be reflected in the information rents captured by the home high income group:

$$\begin{aligned}
\theta_h^H q_{hj}^{H\rho} - \rho T_{hj}^H &= \theta_h^H \left(\frac{q_{fj}^M}{\tau}\right)^\rho - \rho T_{hj}^M \\
\left(\theta_h^H - \theta_h^M\right) q_{hj}^{M\rho} + \left(\theta_h^M - \theta_h^L\right) q_{hj}^{L\rho} &= \left(\theta_h^H/\tau^\rho - \theta_f^M\right) q_{fj}^{M\rho} + \left(\theta_f^M/\tau^\rho - \theta_h^L\right) q_{hj}^{L\rho} \\
\Rightarrow q_{fj}^{M\rho} &= \frac{(\theta_h^H - \theta_h^M)}{(\theta_h^H/\tau^\rho - \theta_f^M)} q_{hj}^{M\rho} + \frac{(\theta_h^M - \theta_f^M/\tau^\rho)}{(\theta_h^H/\tau^\rho - \theta_f^M)} q_{hj}^{L\rho} \\
&= \gamma_M q_{hj}^{M\rho} + \gamma_L q_{hj}^{L\rho}
\end{aligned}$$

$$\begin{aligned}
Q_f^{M\rho} &= n(q_{ff}^{M\rho} + q_{fh}^{M\rho}) \\
&= n(\gamma_M q_{hf}^{M\rho} + \gamma_L q_{hf}^{L\rho} + \gamma_M q_{hh}^{M\rho} + \gamma_L q_{hh}^{L\rho}) \\
&= \gamma_M Q_h^{M\rho} + \gamma_L Q_h^{L\rho} \\
&= Q_h^{H\rho}\left(\gamma_M \overline{\phi_h^{MH}}^{\frac{1}{\rho}} + \gamma_L \overline{\phi_h^{LH}}^{\frac{1}{\rho}}\right)
\end{aligned}$$

$$m_f^M = \frac{\rho(m_f^M - m_h^L)}{(1 - \overline{\widehat{\phi_h^{LM}}}/\tau^\rho)}$$

where $\widehat{\phi_{hj}^{LM}} = \dfrac{\phi_{hj}^{LM}}{\gamma_M + \gamma_L \phi_{hj}^{LM}}$, $\quad \overline{\widehat{\phi_h^{LM}}} = s_f \widehat{\phi_{hf}^{LM}} + (1 - s_f)\widehat{\phi_{hh}^{LM}}$

$$
\begin{aligned}
T_{fh}^{M} &= \theta_f^M q_{fh}^{M\rho} - \theta_f^M \left(\frac{q_{hh}^L}{\tau}\right)^{\rho} + T_{hh}^L \\
&= \frac{\bar{m}_f^M}{Q_f^{M\rho}} \left(q_{fh}^{M\rho} - \left(\frac{q_{hh}^L}{\tau}\right)^{\rho}\right) + T_{hh}^L \\
&= \frac{\bar{m}_f^M}{\tilde{n}_h q_{hh}^{H\rho}} \frac{q_{fh}^{M\rho}}{\left(\gamma_M \overline{\phi_h^{MH\rho}} + \gamma_L \overline{\phi_h^{LH\rho}}\right)} \left(1 - \left(\frac{q_{hh}^L}{q_{fh}^M \tau}\right)^{\rho}\right) + T_{hh}^L \\
&= \frac{\rho(m_f^M - m_h^L)}{\tilde{n}_h (1 - \widehat{\phi_h^{LM}}/\tau^{\rho})} \frac{\left(\gamma_M \phi_{hh}^{MH\rho} + \gamma_L \phi_{hh}^{LH\rho}\right)}{\left(\gamma_M \overline{\phi_h^{MH}}^{\frac{1}{\rho}} + \gamma_L \overline{\phi_h^{LH}}^{\frac{1}{\rho}}\right)} \left(1 - \widehat{\phi_{hh}^{LM}}/\tau^{\rho}\right) + T_{hh}^L
\end{aligned}
$$

6.6 Data Appendix

All trade data are from the World Input-Output Database (WIOD), Timmer et al. (2012) and relate to the year 2005. Bilateral trade flows are generated by aggregating across all sectors. Countries included: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Taiwan, Turkey, UK, USA.

Gravity data is taken from Head and Mayer (2013). GDP per capita is from the Penn World Tables. The gini coefficients are from "All the Ginis" (version November 2014), web reference http://econ.worldbank.org/projects/inequality. Lakner-Milanovic World Panel Income Distribution database contains a panel of country-deciles covering the twenty year period 1988-2008, expressed in common currency and prices (2005 Purchasing Power Parity (PPP) dollars derived from the 2005 International Comparison Project). The database allows comparisons of average incomes by decile both across time and across countries – Lakner and Milanovic (2015).

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