Fragile Coalitions

Under Social and Savings Frictions

H. Cole   D. Krueger   G. Mailath   Y. Park

November 2, 2015

QUESTION

- Risk sharing with limited commitment

    - Optimal insurance contract against idiosyncratic shock subject to limited enforcement

    - Seminal work : Kocherlakota (1996), Kehoe & Levine (1993, 2001), Alvarez & Jermann (2000)

    - Application : village insurance, consumption inequality, sovereign debt contract

- Typical assumption : individual deviation & no saving technology

- Exceptions :

    - Krueger and Uhlig (2006) : savings via financial intermediaries

    - Genicot and Ray (2003) : group deviations

Questions

- ▶ Question: How does does the possibility of group deviations affect risk sharing? Does the ability to save interact with this possibility?

  - ▶ We examine effects of social efficiency and financial efficiency with group deviations.

- ▶ Related work: Genicot and Ray (2003, Restud)

  - ▶ Consider impact of group size with sub-group deviations
  - ▶ Assume deviating coalition come only from original group.
  - ▶ Assume no savings
  - ▶ Look only at Markov arrangements.

- ▶ Kreuger and Uhlig : special case of our analysis

Social Efficiency and Saving Efficiency

Groups of Individuals form Coalitions to insure against risk.

[**Social Efficiency**]

- There can be barriers to formation.
- Social Efficiency = probability of successfully forming coalition
  - Social efficiency ↑⟹ Easier to form a coliation
  - Social efficiency ↑⟹ Easier to form an alternative coalition too
- How does greater social efficiency affect outcomes?

[**Savings Efficiency**]

- Societies can use storage to bond the coalition.
  - But alternative coalition can also use storage
- Savings Efficiency = return to storage
- How does greater savings efficiency affect outcomes?

CONTEXT
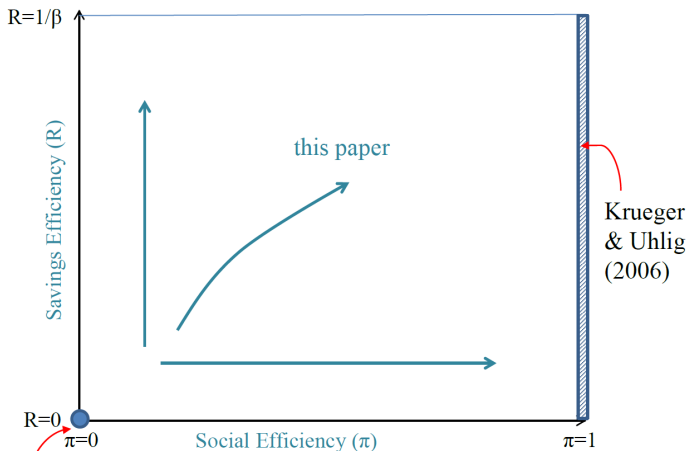
PREVIEW OF RESULTS

▶ Efficient risk-sharing allocation can be characterized by

  ▶ consumption floors

  ▶ decay rates

  ▶ public saving

▶ Social efficiency(π) $\uparrow\Rightarrow$ (Ex ante) Risk sharing $\frown$ (hump shape)

▶ Savings efficiency(R) $\uparrow\Rightarrow \begin{cases} \text{risk sharing}\downarrow \text{ (at low } R) \\ \text{risk-sharing}\uparrow \text{ (at high R)} \end{cases}$

▶ Interaction between π and R :

  ▶ substitues when the social efficiency(π) is very low

  ▶ complements at higher level of social efficency(π).

ECONOMY

Classic Insurance Economy

- Risk averse individuals subject to income risk.

$$E\left\{\sum_{t=1}^{\infty} u(c_t)\right\} \quad u \text{ is CRRA; } \gamma$$

  $y_t \in Y = \{y_1, ..., y_N\}$ i.i.d. across time and people

- Population is infinite.
- Period 0 is planning/coalition
- Income begins in period 1.

INSURANCE COALITIONS

- Coalition formation (Initial and Deviating) :

  - Initial coalition : formed with probability π

  - Deviating Colition:

    - Can exit from old colation to form new one with probability π.

    - Can include in new coalition people not in original one.

  - If (initial or deviating) coalition does not form then stuck in autarky forever.

- Saving (storage):

  - Gross interest rate of saving: $R \in [0, \beta^{-1}]$.

  - Saving can be done both on own or within group.

  - If by person, can take it when leave. By coalition cannot.

Optimal Allocation of Coalition

- Optimal allocation solves the social planning problem.

  - Maximize (utilitarian) social welfare of the coalition subject to RC and PC

  - Allocation of deviating coalition also solves the social planning problem.

- Individual weights in the planing problem :

  - Initial coalition : Ex ante identical $\Rightarrow$ equal weight

  - Deviating coalition : equal weight

    - In first period of deviating : cannot exit or undo.

    - From the second period : identical

PLAN OF TALK

1. Basics of Coalition and Storage Usage

   ▸ size of coalition, usage of storage

2. First Best Analysis

3. Optimal Coalition allocation characterization

4. Effects of π and R

   ▸ Is higher π good?

   ▸ Is higher R good?

   ▸ Are π and R substitutes or complements ?

Basics of Allocations

Coalition outcome is $\left\{c_i\left(y^{N,t}\right), s_i\left(y^{N,t}\right), S\left(y^{N,t}\right)\right\}$

- N is the number of members

- $y^{N,t}$ is the history of the vector of income realizations

- c_i and s_i are the consumption and savings of i

- S_t are the coalition level savings.

Basics of Coalitions

Proposition

Infinite coalitions (i.e. $N = \infty$) are always optimal. ▸ More

⇒ No aggregate risk

Proposition

Breakaway Coalitions will be homogeneous w.r.t. initial income.

▸ More

⇒ Deviating coalitions start with y - homogenous initial income.

BASICS OF STORAGE

▶ Individual level storage? No.

PROPOSITION

Storage within a coalition will only take place at level of the coalition
(i.e. $s_i\left(y^{N,t}\right) = 0$)

$\Rightarrow$ Coalition outcome is $\left\{c\left(y^t\right), S_t\right\}$

▶ Storage never be used in any complete efficient arrangement.

$$u'(E\{Y\}) \leq \beta R u'(E\{Y\}).$$

▶ Storage will not be used in autarky storage if

$$u'\left(\max\{y \in Y\}\right) \geq \beta R E\{u'(y_i)\}, \qquad \text{(N ST AUT)}$$

First Best

Assumption

Assume for now condition (N ST AUT) holds.

$\implies$ No storage in autarky

$\implies$ Autarky payoff:

$$V^A(y) = u(y) + \beta V^A, \text{ where } V^A = E\{u(y_i)\}/(1-\beta)$$

- First Best Ex Ante payoff:
  $$\pi \bar{V} + (1-\pi)V^A, \text{ where } \bar{V} = u(E\{Y\})/(1-\beta). \qquad \text{(V FB)}$$

- Payoff from deviating :
  $$u(y) + \beta \left[\pi \bar{V} + (1-\pi)V^A\right].$$

First Best

For FB to be feasible,

$$\frac{u(E\{Y\})}{1-\beta} \geq u(y) + \frac{\beta\left[\pi u(E\{y\}) + (1-\pi)E\{u(y)\}\right]}{1-\beta}. \qquad \text{(FB fes)}$$

When FB is possible at $\pi = 0$,

- Payoff conditional on initial coalition formation is weakly declining in π.
- Ex ante payoff is
  - strictly increasing in π at least up until FB is feasible
  - may or may not be increasing thereafter.

FIRST BEST

- ▶ Effects of Storage:

    - ▶ Increasing R up until β^{-1} no direct impact on FB (not used).

    - ▶ But it will raise $V^A(y)$. The FB can break down at lower π.

PROPOSITION

If FB possible with $R = 0$, then increasing R is always weakly bad. If the cut-off level of π at which storage breaks down is interior, i.e.

$$\frac{u(E\{Y\})}{1 - \beta} = \pi \left[u(y) + \frac{\beta u(E\{Y\})}{1 - \beta} \right] + (1 - \pi)V^A(y; R) \text{ for } \pi \in (0, 1),$$

this cut-off is strictly decreasing in R when (N ST AUT) does not hold and storage is used in autarky.

CHARACTERIZATION OF ALLOCATION:

ROADMAP

1. Consider outcomes when $R = 0$, and storage is never used.

   1.1 Develop approximation to optimal arrangement

   1.2 Characterize how outcomes depend upon π.

2. Consider outcomes when R is big enough so always used.

   2.1 Show approximiation is exact here.

   2.2 Characterize how outcomes depend upon R and π.

3. Consider outcomes when R may be used but not always.

   3.1 Especially possible in deviating coalitions.

   3.2 Also possible if storage starts to be used in transition.

   3.3 Extend approximation algorithm and characterize.

1. $R = 0$: Coalition Problem

$$F(\bar{V}, \pi) = \max_{c(y^t)} E \left\{ \sum_{t=1}^{\infty} \beta^t u(c_t(y^t)) \Pr(y^t) \right\}$$

subject to

$$\sum_{y^t} c_t(y^t) \Pr(y^t) = Y$$

$$\beta^t u(c_t^i(y^t)) + E \left\{ \sum_{n=t+1}^{\infty} \beta^k u(c(y^n)) | y^t \right\}$$

$$\geq \beta^t u(y_t(y^t)) + \beta^{t+1} \left[\pi \bar{V} + (1-\pi) V^A \right]$$

$$\text{for all } y^t, \tag{1}$$

1. $R = 0$: COALITION PROBLEM

PROPOSITION

- ► *The optimal continuation payoff $\bar{V} \in \mathcal{V} = \left[V^A, u(E\{Y\})/(1-\beta) \right].$*

- ► *The operator $F : \mathcal{V} \times [0,1] \to \mathcal{V}.$ defines the unique conditional payoff to our contracting problem, conditional the value of $\bar{V}$ in constraint (1).*

- ► *F is continuous and decreasing in $\bar{V}$ and π*

- ► *The optimum $\bar{V}^*$ is a fixed point of F, or*

$$\bar{V}^* = F(\bar{V}^*, \pi).$$

$$\mathcal{L} = \max_{\{c_t\}} \min_{\{\omega_t, \gamma_t\}} \sum_{t=1}^{\infty} \beta^t \sum_{y^t} \left\{ \underbrace{\left[1 + \sum_{n=1}^{t} \omega_n \left(y^n(y^t) \right) \right] u(c_t(y^t))}_{\equiv w_t(y^t)} + \gamma_t \left[Y - c_t^i \right] \right\} \Pr(y^t)$$

$$- E \sum_{t=1}^{\infty} \beta^t \sum_{y^t} \omega_t(y^t) \left\{ u(y_t(y^t) + \beta \left[\pi \bar{V} + (1 - \pi) V^A \right] \right\},$$

The f.o.c. for consumption :

$$w_t(y^t) u'(c_t(y^t)) = \gamma_t.$$

$$\implies c_t(y^t) = u'^{-1} \left(\frac{\gamma_t}{w_t(y^t)} \right).$$

1. $R = 0$: Characterizing Lagrangian

Since utility is CRRA with coefficient of risk aversion γ:

$$c_t(y^t) = \left(\frac{\gamma_t}{w_t(y^t)} \right)^{-1/\gamma}. \tag{2}$$

Resource constraint :

$$Y = \sum_{y^t} c_t(y^t) \Pr(y^t) = \gamma_t^{-1/\gamma} \sum_{y^t} w_t(y^t)^{1/\gamma} \Pr(y^t).$$

$\implies$ Shadow price of consumption:

$$\gamma_t = \left[\frac{Y}{\sum_{y^t} w_t(y^t)^{1/\gamma} \Pr(y^t)} \right]^{-\gamma},$$

$\implies$ growth rate of shadow price :

$$\frac{\gamma_t}{\gamma_{t-1}} = \left[\frac{\sum_{y^t} w_t(y^t)^{1/\gamma} \Pr(y^t)}{\sum_{y^{t-1}} w_{t-1}(y^{t-1})^{1/\gamma} \Pr(y^{t-1})} \right]^{\gamma} = g_t^{\gamma}.$$

1. $R = 0$: OPTIMAL ALLOCATION

▶ If the participation constraint does not bind,

$$c_t(y^t) = \left[\frac{\sum_{y^{t-1}} w_{t-1}(y^{t-1})^{1/\gamma} \Pr(y^{t-1})}{\sum_{y^t} w_t(y^t)^{1/\gamma} \Pr(y^t)} \right] c_{t-1}(y^{t-1}) = \frac{c_{t-1}(y^{t-1})}{g_t}$$

PROPOSITION

The efficient consumption allocation is determined by a sequence of consumption floors, $\{\bar{c}_t(y_1), ..., \bar{c}_t(y_N)\}$, and decay rates, $\{g_t\}$, where

$$c_t(y^t) = \begin{cases} c_{t-1}(y^{t-1})/g_t \text{ if } c_{t-1}(y^{t-1})/g_t \geq \bar{c}_t(y_t) \\ \bar{c}_t(y_t) \text{ o.w.} \end{cases} .$$

ASSUMPTION

Consumption allocation eventually becomes stationary, then

$$\bar{c}_t(y_i) \to \bar{c}(y_i) \text{ and } g_t \to g.$$

2. Adding Storage

► Autarky value with storage:

$$V^A(y; R) = \max u(c_1) + E\left\{\sum_{t=2}^{\infty} \beta^{t-1} u(c_t)\right\} \text{ subject to}$$

$$y_t + R s_{t-1} = s_t + c_t$$

with

$$y_1 = y, \ s_0 = 0, \text{ and } s_t \geq 0 \text{ for all } t \geq 1.$$

► The outside option payoff :

$$\pi \bar{V}(y) + (1 - \pi) V^A(y).$$

$\implies$ depends upon y through both Autarky and the deviatign coalition

2. ADDING STORAGE

▶ resource constraint :

$$\sum_{y^t} c_t(y^t) \Pr(y^t) = Y + RS_{t-1} - S_t,$$

$$S_0 = 0, \ S_t \geq 0 \text{ for all } t \geq 1.$$

▶ f.o.c. for S_t :

$$-\beta^t \gamma_t + \beta^{t+1} R \gamma_{t+1} \leq 0 \text{ and } = \text{ if } S_t > 0.$$

$$\implies g_{t+1}^\gamma = \frac{\gamma_{t+1}}{\gamma_t} \leq \ \frac{1}{\beta R} \text{ and } = \text{ if } S_t > 0.$$

▶ Implication:

PROPOSITION

Storage is used in the mechanism iff it can help to smooth the shadow price of consumption to planner.

$\implies$ Bonding may be an indirect benefit (not the primary motivation).

2. Adding Storage : Characterization

- ▶ Characterization of optimal coalition allocation

  - ▶ No storage case:

    - ▶ characterized by consumption floors and decay rates
    - ▶ The outside options pin down consumption floors
    - ▶ The resource constraint pin down the decay rates

  - ▶ Storage case:

    - ▶ characterized by consumption floors, decay rates, and savings
    - ▶ (time invariant) g is pinned down by the Euler equation
    - ▶ The resource constraint pin down the saving

Proposition

The ergodic ladder is the same as that of the optimal mechanism for any deviating coalition.

Two-State Case - No Storage

- Assumption: $y \in \{y_l, y_h\}$ with $prob(y_l) = prob(y_h) = \frac{1}{2}$

- Characterization of ergodic allocation:

  - Binding consumption levels : c_h, c_l

  $$y_h \geq c_h \geq Y \geq c_l \geq y_l, \text{ where } Y = .5(y_h + y_l)$$

  - decay rate in the ergodic ladder : $g \geq 1$

- Number of steps in the ladder between c_h and c_l

  $$T = \arg\max \left\{ t = 1, ..., \infty : c_h/g^{t-1} > c_l \right\}.$$

  - If $g > 1$, then $T < \infty$. If perfect insurance is possible $g = 1$, $T = \infty$, and $c_h = c_l = Y$.

  - This defines a consumption ladder with T steps

  $$\left\{ c_h, c_h/g, c_h/g^2, ..., c_h/g^{T-1}, c_l \right\}.$$

TWO-STATE CASE - NO STORAGE

- How to determine ergodic c_h, c_l, and g ?

    - c_h, c_l, and g should satisfy the resource constraint and participation constraints.

- Resource constraint w.r.t. c_h, c_l, and g :

    - The ergodic distribution on the T ladder steps :

    $$\Pr\left(c_h/g^{t-1}\right) = \left(\frac{1}{2}\right)^t$$

    $$\Pr\left(c_l\right) = 1 - \sum_{t=1}^{T}\left(\frac{1}{2}\right)^t = \left(\frac{1}{2}\right)^T.$$

    - Thus, RC should satisfy:

    $$\sum_{t=1}^{T}\left(\frac{1}{2}\right)^t c_h/g^{t-1} + \left(\frac{1}{2}\right)^T c_l = Y. \tag{3}$$

TWO-STATE CASE - NO STORAGE

- Participation constraints w.r.t. c_h, c_l, and g :

  - Determine the payoff conditional on the current consumption level: solving the following system of recursive equations.

  $$
  \begin{aligned}
  V_j &= u(c_h/g^{j-1}) + \beta \frac{1}{2} \left[V_{j+1} + V_1 \right] \text{ for all } j < T+1 \\
  V_{T+1} &= u(c_l) + \beta \frac{1}{2} \left[V_{T+1} + V_1 \right].
  \end{aligned}
  $$

  - PC for the high type:

  $$
  V_1(c_h, c_l, g) = u(y_h) + \beta \left[\pi \overline{V} + (1 - \pi) V^A \right]
  $$

  - PC for the low type :

  $$
  V_{T+1}(c_h, c_l, g) = u(y_l) + \beta \left[\pi \overline{V} + (1 - \pi) V^A \right]
  $$

$\implies$ PC's depend on endogenous $\overline{V}$ (payoff of coalition).

But how can we determine $\overline{V}$?

Two-State Case - No Storage

- ▶ Issues in determining $\overline{V}$:

    - ▶ In the transition: time-varying : $c_h(t)$ and g_t

    - ▶ Why? $g_t \nearrow g$, as weight of ladder promises press on RC.

- ▶ Remark: c_l is always a constant.

    - ▶ The conditional payoff at the bottom :

$$V_{T+1} = u(c_l) + \beta \frac{1}{2} \left[V_{T+1} + V_1 \right],$$

    - ▶ V_1 and V_{T+1} are pinned down by constant outside option.

$\implies$ To determine $\overline{V}$, we need to solve for $\{c_h(t), g_t\}_{t \geqslant 0}$ and c_l.

- ▶ **Approximation of** $\overline{V}$**:** approximate time varying $\{c_h(t), g_t\}_{t \geqslant 0}$

- **Assumption : As soon as get y_h start on ergodic ladder c_h.**

  - For $1 \leq t \leq T-1$, $\{y_l, y_{l,\dots}\}$ type gets residual to satisfy RC:

$$c_1 = 2\left[Y - \frac{1}{2}c_h\right]$$

$$c_t = 2^t\left[Y - \frac{1}{2}\sum_{j=0}^{t-1}\left(\frac{1}{2g}\right)^j c_h\right], \text{ for } 1 \leq t \leq T-1$$

  - In period T, everyone is on ergodic ladder ($\{y_l, y_{l,\dots}\}$ type hits c_l.).

$\implies$ Approximated allocation is characterized by ergodic c_h, c_l, and g.

- Allocation not optimal to the extent that decay rate of $\{y_l, y_{l,\dots}\}$ type not g.

- Can show decay rate becomes the same if T is large. ▸ Appendix

Two-State Case - No Storage

Approximation

- How to determine approximately optimal ergodic c_h, c_l, and g ?

    - c_h, c_l, and g should satisfy two PC's, and the RC.

    - RC: ergodic RC (3)

        - (Note): By construction, the RC for $t = 1, \cdots, T-1$ are satisfied.

    - PC:

        - Under the approximation assumption :

        $$\overline{V}(c_h, c_l, g) = \sum_{j=1}^{T} \left(\frac{1}{2}\right)^j \beta^{j-1} \left[V_1 + u(c_j)\right] + \left(\frac{1}{2}\right)^{T+1} \beta^T \left[V_1 + V_{T+1}\right]$$

    - $\implies$ PC's w.r.t. ergodic c_h, c_l, and g :

        $$V_1(c_h, c_l, g) = u(y_h) + \beta \left[\pi \overline{V}(c_h, c_l, g) + (1-\pi)V^A\right]$$

        $$V_{T+1}(c_h, c_l, g) = u(y_l) + \beta \left[\pi \overline{V}(c_h, c_l, g) + (1-\pi)V^A\right]$$

TWO-STATE CASE WITH STORAGE

We now consider the case where storage is always used.

[Payoff of initial coalition]

- How to determine ergodic c_h, c_l, g, and S ?
  - Storage fixes the decay rate to $g = (\beta R)^{-\frac{1}{\gamma}}$
  - Now outside options + res. const. determine c_h, c_l and ergodic S.
- How to determine initial c_1 and $\{S_t\}$?
  - Ergodic distribution of consumption in T steps so $S_T = S$ ergodic.
  - First period consumption of y_l, c_1 is determined by $S_T(c_1) = S$.

$$S_t = 1 - \sum_{j=1}^{t} \frac{c_h}{2^j g^{j-1}} - \frac{1}{2^t} \max\left\{ \frac{c_1}{g^{t-1}}, c_l \right\} + RS_{t-1} \text{ for } t = 1, \cdots T$$

- $c_h(t)$ and $g(t)$ are constant in the optimal mechanism. Thus, this algorithm is exact. (No approximation)

Two-State Case with Storage

[**Payoff of deviating coalition with** y_h]

- Consumption distribution over time:
    - $t = 1$: $c_1^d = y_h - s_1^d$, for all HH
    - $t = 2$: c_1^d/g , for all HH
    - $\cdots$
    - $t = T(c_1^d)$: $c_1^d/g^{T(c_1^d)-1}$, for all HH
    - $t = T(c_1^d) + 1$: $\{c_h, c_1^d/g^{T(c_1^d)}\}$ with prob $\{\frac{1}{2}, \frac{1}{2}\}$
    - $\cdots$
    - $t = T(c_1^d) + T(c_h)$: $\{c_h, c_h/g, \cdots, c_h/g^{T(c_h)-1}, \max\{c_1^d/g^{T(c_1^d)+T(c_h)-1}, c_l\}\}$
      with prob $\{\frac{1}{2}, \frac{1}{2^2}, \cdots \frac{1}{2^{T(c_h)}}, \frac{1}{2^{T(c_h)}}\}$
    - $t \geq T(c_1^d) + T(c_h) + 1$: ergodic distribution
- $S_t^d(c_1^d) = S$ at $t = T(c_1^d) + T(c_h) + 1$ pins down c_1^d
- We can deduce the value of outside option V_h .

Two-State Case with Storage

Payoff of deviaiton coalition with y_l similar to before

- Lowest possible income so no storage in period 1, $c_1 = y_l$.

- Afterwards follow original optimal mechanism.

- So $V_l = u(y_l) + \beta V = u(y_l) + \beta[\pi \overline{V} + (1 - \pi)V^A]$ optimal coalition payoff.

Constant $g(t)$ and $c_h(t)$ are optimal in the deviating coalition. Thus, this algorithm is exact for the payoff of deviation coalition also.

TWO-STATE CASE WITH STORAGE

Now two-dimensional fixed point to determine allocation

- Given outside values V_h and V_l, solve for c_h and c_l which determines S.

- Solve for $\overline{V}$ implied by this ladder, which determines $V_l = u(y_l) + \beta[\pi\overline{V} + (1-\pi)V^A]$ and iterate until convergence of V_l.

- Solve for V_h implied by this ladder and iterate until convergence of V_h.

Two-State Case w. Temporary Storage

Storage may be used but not always. We extend the approximation algorithm.

- Deviating coalition with y_h will go to same ergodic ladder.
  - So if no storage erodically in optimal mechanism no long run storage for them either.
  - But will store temporarily if

  $$u'(y_h) < \beta R u'(c_1).$$

- Decay rate g_t of consumption for $\{y_l, y_l, ...\}$ in approximately optimal allocation increase with t.
  - Never used in the optimal mechanism if not used ergodically.
  - If used ergodically, it may start to be used in the transition.

Two-State Case w. Temporary Storage

For deviating coalition, add simple savings dimension to no storage approximation if not used on ergodic ladder.

▶ Treat c's as transfers and allocation problem:

$$
\max_{c_1^d, \{\tau\}} u(c_1^d) + \sum_{t=2}^{T(c_h)} \beta^{t-1} \left[\sum_{j=1}^{t-1} u(c_h/g^{j-1} + \tau_{t,j}) \left(\frac{1}{2}\right)^j + u(c_t + \tau_t) \left(\frac{1}{2}\right)^{t-1} \right]
$$

$$
+ \sum_{t=T(c_h)+1}^{\infty} \beta^{t-1} \left[\sum_{j=1}^{T(c_h)} u(c_h/g^{j-1} + \tau_{t,j}) \left(\frac{1}{2}\right)^j + u(c_l + \tau_t) \left(\frac{1}{2}\right)^{T(c_h)} \right]
$$

$$
\text{s.t. } c_1^d + \sum_{t=1}^{T(c_h)} \frac{1}{R^{t-1}} \left\{ \sum_{j=1}^{t-1} \tau_{t,j} \left(\frac{1}{2}\right)^j + \tau_t \left(\frac{1}{2}\right)^{t-1} \right\}
$$

$$
+ \sum_{t=T(c_h)+1}^{\infty} \frac{1}{R^{t-1}} \left\{ \sum_{j=1}^{T(c_h)} \tau_{t,j} \left(\frac{1}{2}\right)^j + \tau_t \left(\frac{1}{2}\right)^{T(c_h)} \right\} = y_h, \text{ and } \tau \geq 0.
$$

TWO-STATE CASE W. TEMPORARY STORAGE

For the optimal mechanism which uses storage ergodically but not initially, we apply the no storage approximation for initial k peirods.

- ▶ Ergodic storage fixes the ergodic decay rate to $g = (\beta R)^{-1}$.
- ▶ Outside options + res. const. determine ergodic c_h, c_l and S.
- ▶ For initial $k - 1$ periods where storage is not used, we apply the no storage approximation.
    - ▶ As soon as get y_h start on ergodic ladder c_h.
    - ▶ $\{y_l, y_{l,...}\}$ type just gets residual from the resource constraint: $\{c_t\}_{t=1}^{k-1}$
- ▶ Once storage is used, decay rate is constant $g_t = (\beta R)^{-1}$. Period-k consumption of y_l, c_k^* is determined by RC and need to hit S at $T - 1$.
- ▶ Determine k by increasing k until it satisfies $S_{T-1}(c_k) < S$. (c_k: residual consumption)

Quantitative Analysis

We take the two-state numerical example with :

- $y_h = 2$, $y_l = 1$
- $\beta = 0.9$
- $\gamma = 1$
- $R = 0$ (no storage) and $R \in [1.01, 1.10]$ (temporary and always)
  (Note: $\beta R < 1$)

We first present the result for $R = 0$ (no storage). Then, we will present the result for storage case.

No storage case. Effects of increasing π :
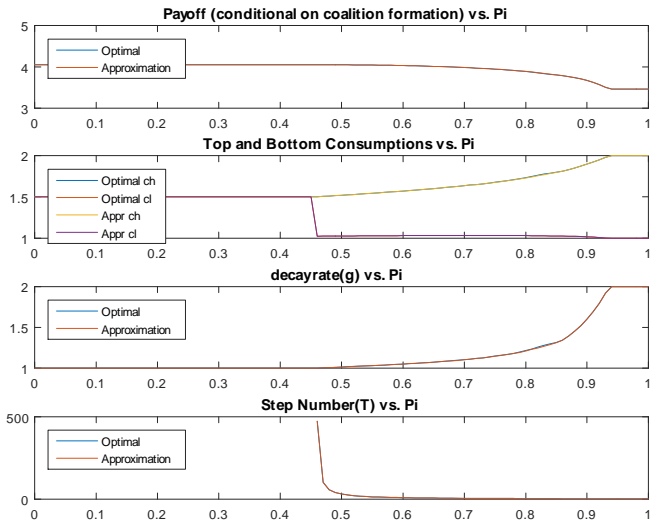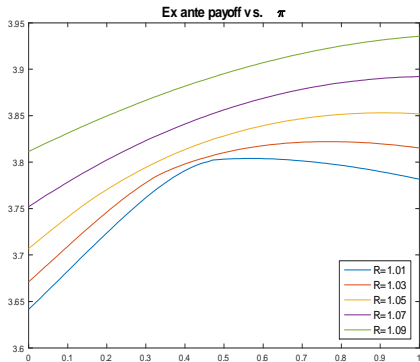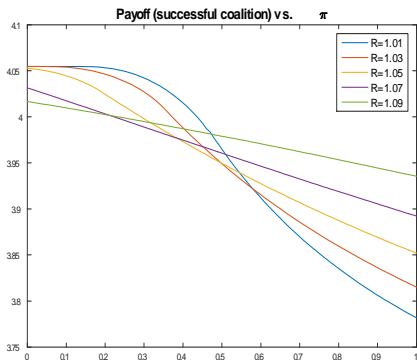
QUANTITATIVE ANALYSIS ($R = 0$)

- Risk-sharing is declining in π.

  - Payoff conditional on coalition formation weakly decreasing in π.

  - Consumption dispersion increases with π.

  - Decay rate of consumption weakly increasing in π. Hence growth rate of shadow cost of consumption weakly increasing in π.

  - Number of steps from in ladder is weakly decreasing in π.

- Risk sharing can happen even with $\pi = 1$.

  - Comparing y_h followed by c_h or c_1 vs. c_h followed by c_h/g.

- The approximation error is very small.

- ▶ Effects of increasing π

  - ▶ Payoff conditional on coalition formation is weakly declining in π.

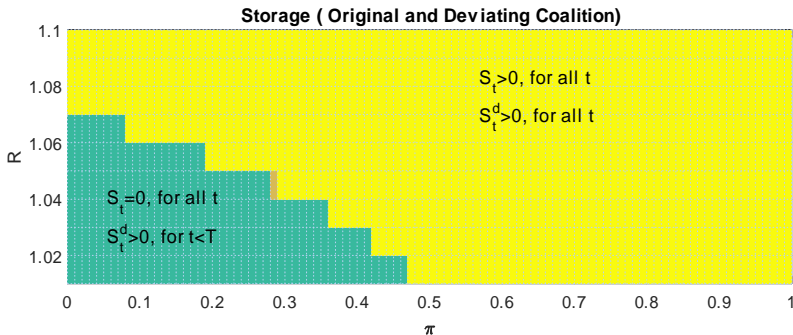 - ▶ Ex ante payoff $(\pi \bar{V}(\pi) + (1-\pi) EV^A)$ has hump shape in π when R is low. It is increasing in π when R is high.

    $(\pi \uparrow : +, \quad \bar{V}(\pi) \downarrow : -)$

# QUANTITATIVE ANALYSIS ($R > 0$): PAYOFF

Payoff (successful coalition) vs. R
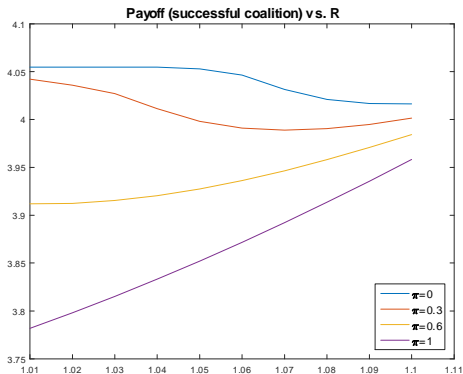
▶ Effects of increasing R

    ▶ For low R, storage is only used in outside values, thus increasing R is bad. Once initial coalition uses storage, increasing R is good.

    ▶ For high π, initial coalition is more likely to use storage at lower R. Thus, the turning point R $(- \to +)$ is decreasing in π

QUANTITATIVE ANALYSIS ($R > 0$): STORAGE

Storage (Original and Deviating Coalition)

Plot with R on the vertical axis (from 1.02 to 1.1) and π on the horizontal axis (from 0 to 1).

- $S_t>0$, for all t
- $S_t^d>0$, for all t
- $S_t=0$, for all t
- $S_t^d>0$, for $t<T$

- ▶ At low R, storage is only used in deviating coalition and autaky.

- ▶ Storage is used in the initial coalition, if R is sufficiently high.

- ▶ When π is high, shadow price of consumption grows faster over time, and storage can help smoothing shadow prices. Thus, starting point $R(\pi)$ of using storage is decreasing in π.

Quantitative Analysis ($R > 0$): Storage

Ergodic Saving vs.π
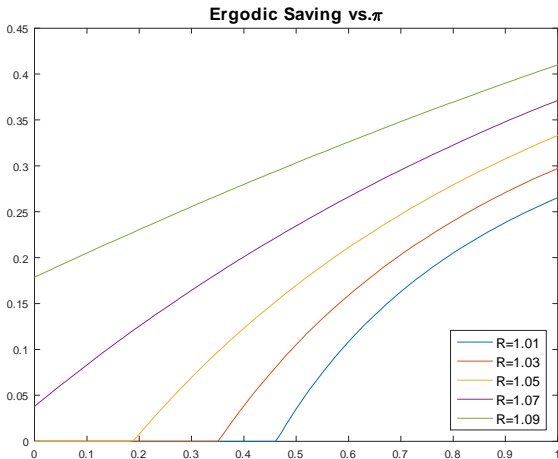
- Saving is weakly increasing in π and R

SOCIAL EFFICIENCY AND SAVINGS EFFICIENCY

▶ Social efficiency and savings efficiency are substitutes when social efficiency (π) is very low.

▶ Social efficiency and savings efficiency are complements when social efficiency (π) is very high.

▶ In the intermediate level of social efficiency, social and savings efficiency can be either substitutes or complements (substitutes for low R, complements for high R)

QUANTITATIVE ANALYSIS ($R > 0$):

ALLOCATION

consumption for the binding guys, decay rate, number of steps:
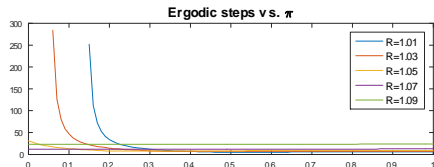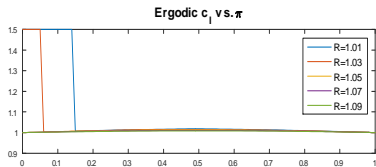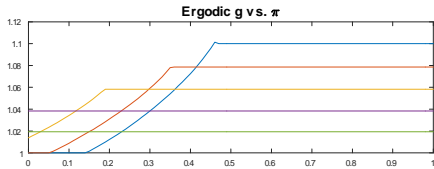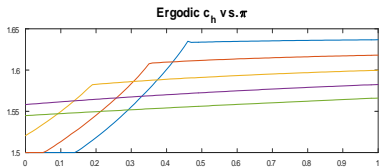
Basics of Coalitions

Proposition

Infinite coalitions (i.e. $N = \infty$) are always optimal.

▶ Initial coalition everyone ex ante identical - no income in period 0.

  ▶ so can replicate N-member outcome with $2N$ hence weakly better.

▶ For deviating coalitions

  ▶ Members' start conditional on y_t. but initial incomes matter only because of impact on per capita Y.

  ▶ Replicate N-member coalition with same per capita income so weakly better.

▶ Infinite coalition strictly better since greater insurance.

Basics of Coalitions

Proposition

Breakaway Coalitions will be homogeneous w.r.t. initial income.

- Prefer coalition members with higher income.

- Since true for everyone, get positive assortitive matching in coalition formation - high with highs forces mediums with mediums which in turn forces low with low.

Two-State Case - No Storage

Almost optimal if # of steps is large

Remark

It then follows that

$$\frac{c_{t+1}}{c_t} = \frac{2^{t+1}\left[Y - \sum_{j=1}^{t+1}\left(\frac{1}{2}\right)^j c_h/g^{j-1}\right]}{2^t\left[Y - \sum_{j=1}^{t}\left(\frac{1}{2}\right)^j c_h/g^{j-1}\right]} \textit{ for all } t < T.$$

Then note that

$$Y - \sum_{j=1}^{t}\left(\frac{1}{2}\right)^j c_h/g^{j-1} = \sum_{j=t+1}^{T}\left(\frac{1}{2}\right)^t c_h/g^{j-1} + \left(\frac{1}{2}\right)^T c_l, \quad (4)$$

where

$$c_l : c_h/g^{T-1} \geq c_l \geq c_h/g^T.$$

Remark

$$\frac{c_{t+1}}{c_t} = \frac{1}{g} + \frac{2^{t+1}\left[\left(\frac{1}{2}\right)^T c_l - \left(\frac{1}{2}\right)^{T+1} c_l/g - \left(\frac{1}{2}\right)^{T+1} c_h/g^T\right]}{2^t\left[\sum_{j=t+1}^T \left(\frac{1}{2}\right)^t c_h/g^{j-1} + \left(\frac{1}{2}\right)^T c_l\right]}$$

$$> \frac{1}{g}.$$

However, note that

$$\frac{2^{t+1}\left[\left(\frac{1}{2}\right)^T c_l - \left(\frac{1}{2}\right)^{T+1} c_l/g - \left(\frac{1}{2}\right)^{T+1} c_h/g^T\right]}{2^t\left[\sum_{j=t+1}^T \left(\frac{1}{2}\right)^t c_h/g^{j-1} + \left(\frac{1}{2}\right)^T c_l\right]} \simeq 0$$