Long-Run Trade Elasticity and the Trade-Comovement Puzzle *

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Abstract

We show that the *trade-comovement puzzle* – theory's failure to account for the positive relation between trade and business cycle synchronization – is intimately related to its counterfactual implication that short- and long-run trade elasticities are equal. Based on this insight, we show that modeling the disconnect between the low short- and the high long-run trade elasticity in consistency with the data is promising in resolving the puzzle. In a broader context, our findings are relevant for analyzing business cycle transmission in a large class of models and caution against the use of static elasticity models in cross-country studies.

Keywords: trade-comovement puzzle, elasticity puzzle, international business cycle synchronization

JEL codes: E32, F31

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1 Introduction

An extensive literature documents a tight link between bilateral trade intensities and business cycle comovement across countries.[1] These results seem to confirm the intuitive notion that shocks in one country, by demand complementarity, spill over to demand for goods produced in major trade partner countries, leading to an increased correlation of their GDPs. However, even though this effect is built into almost every international business cycle model – through imperfect substitutability of home and foreign goods – the theoretical literature has had very limited success providing a mechanism that accounts for this empirical pattern, leading Kose and Yi (2006) to coin the term *trade-comovement puzzle*.[2]

In this paper, we characterize the forces behind the puzzle and provide and quantitatively evaluate its resolution. Analytically, we show that a canonical international business cycle model necessarily implies a counterfactual negative trade-comovement link. To better understand the source of this result, we decompose the trade-comovement link into effects driven by short- and long-run trade responses – determined by short- and long-run trade elasticities, respectively. This decomposition highlights that a particular counterfactual implication of the canonical model – that the short- and long-run trade elasticities are equal (known in the literature as the *elasticity puzzle*)[3] – plays a central role in generating the trade-comovement puzzle. We demonstrate that disconnecting short-run and long-run trade elasticities in an otherwise

[1] By running cross-country regressions, Frankel and Rose (1998), Clark and van Wincoop (2001), Calderon, Chong and Stein (2002), Otto, Voss and Willard (2001), Baxter and Kouparitsas (2005), Kose and Yi (2006) and Inklaar, Jong-A-Pin and de Haan (2008) all find that, among bilateral country pairs, more trade is associated with more synchronized business cycle fluctuations. diGiovanni and Levchenko (2010) confirm these findings using disaggregated industry-level data. Related sectoral evidence can also be found in Johnson (2014).

[2] Endogeneity of business cycle synchronization has been stressed as relevant for policy because it puts into question policy prescriptions based on the initial degree of business cycle synchronization that are likely to affect trade, such as pegging exchange rates or forming monetary unions (Frankel and Rose (1998)).

[3] See the discussion in Ruhl (2008).

canonical model – in a way consistent with empirical estimates[4] – helps resolve the trade-comovement puzzle qualitatively. To establish the quantitative importance of this solution, we provide a micro-founded model of different short- and long-run elasticities, parameterize the model, and show that it comes close to quantitatively accounting for the trade-comovement link in the data. In a broader context, our findings caution against the use of static elasticity models in cross-country studies.

In the theoretical analysis of the puzzle, we begin by considering a simple canonical international business cycle model – two countries and labor as the only inputs – with identical short-run and long-run trade elasticities (*static elasticity model* henceforth). We show that the implied trade-comovement link in this model is necessarily negative for all parameter values. We identify and characterize analytically two opposing forces responsible for this result: the *demand complementarity channel* and the *risk-sharing channel*.[5]

Consider first the demand complementarity channel. Through this channel, a high level of trade has a *positive* effect on business cycle comovement. Intuitively, imperfect substitutability between domestic and foreign goods implies that as the foreign good becomes more abundant in the world, the reduction in its relative price makes both the domestic good and the domestic consumption basket more expensive. Because domestic consumption also includes the cheaper foreign good, the consumption basket's price goes up by less than the price of the domestic

[4]Numerous studies find that long-run elasticity of trade with respect to permanent tariff changes is very large (see for example Head and Ries (2001), Eaton and Kortum (2002), Clausing (2001), Anderson and van Wincoop (2004) or Romalis (2007)), and large values of elasticity are needed to account for the evolution of world trade in the last century (Yi (2003)). In contrast, business cycle frequency estimates point to much lower trade elasticities (Reinert and Roland-Holst (1992), Blonigen and Wilson (1999)). Standard international business cycle theory does not account for this discrepancy by predicting a single elasticity. Modeled via a CES aggregator, in a typical parameterization, standard models assume that both elasticities are low, as it helps the models match business cycle moments.

[5]The presence of these basic forces was first conjectured by Kose and Yi (2006).

good, and hence the real wage in the domestic country is driven up. This increases labor supply and production in the domestic country. Since these effects are stronger for high values of average trade between countries, the complementarity channel implies a *positive* link between trade and business cycle comovement.

Second, consider the risk-sharing channel. Intuitively, owing to risk sharing, after a positive shock in the foreign country, domestic households should work less and borrow to finance consumption, leading to *negative* output comovement. The implementation of this borrowing and lending arrangement – known as *resource-shifting motive* – is carried out by trade in goods, which is subject to trade costs. In particular, higher trade costs – by implying a lower long-run level of trade – make resource shifting less efficient, and by suppressing this motive lead to less comovement. Importantly, the strength of the negative effect of trade on comovement is tightly linked to the size of the reduction in trade costs required to induce a given change in the long-run level of trade. Accordingly, a lower long-run trade elasticity makes the risk-sharing channel stronger, as a larger reduction of trade costs is required to increase trade.

The analysis of the static elasticity model points to a natural resolution of the trade-comovement puzzle. On the one hand, the demand complementarity effect depends inherently on business cycle frequency responses to shocks, and its effect is stronger for a lower short-run trade elasticity after controlling for the effect of risk sharing. On the other hand, the impact of the risk-sharing channel depends on how permanent changes in trade costs affect long-run trade levels between countries, which by definition is weaker for a higher long-run trade elasticity. However, if the long-run trade elasticity is higher than the short-run trade elasticity, the complementarity effect may dominate.

We show that this is qualitatively the case in a prototypical *dynamic elasticity model* – in which the long-run trade elasticity exceeds the short-run elasticity owing to a simple convex adjustment cost introduced to slow down the response of trade shares to relative prices. Specifically, for a sufficiently large disconnect between the short- and long-run elasticities, we prove that a positive trade-comovement relationship arises. The basic logic here is that high long-run elasticity, by reducing the variation in trade costs needed to induce a certain variation in long-run trade levels between countries (steady-state trade levels), suppresses the effect of resource shifting on the trade-comovement relation, exposing the positive effect of the complementarity channel.

The second part of our paper explores the quantitative relevance of the above finding. To that end, we consider a multi-country setup with a micro-founded search friction that leads to a disconnect between the short- and long-run trade elasticities, as in Drozd and Nosal (2012). The main friction, which gives rise to the complementarity channel in the model, is that producers need to establish long-lasting relations with customers in order to sell goods, and the buildup of such relationships is time consuming. These vertical linkages introduce sluggishness in market shares of producers and thus sluggishness in the consumed ratio of domestic products and imports.[6] We match the strength of this friction to make the model consistent with business cycle frequency estimates of the short-run trade elasticity. Since the friction is not operational in the long run, the long-run response of trade is still determined by the intrinsic elasticity of substitution parameter in the CES aggregator, and hence we set it equal to a high value as implied by the high estimates of the long-run trade elasticity.

[6]Sluggish responses of trade in the short run (Eaton, Eslava, Kugler and Tybout (2008)), as well as anecdotal evidence on how international trade is organized (Hakansson (1982), Turnbull and Cunningham (1981), Egan and Mody (1993)), give support to our market share buildup friction over shorter horizons.

To establish the quantitative goal for the theory, we focus on the trade-comovement relationship between 20 OECD countries over the span of 1980-2011. We parameterize the model in order to reproduce bilateral trade intensities in the cross-section of 190 country pairs in our sample, while controlling for each country's trade openness. In the data, we establish a positive and statistically significant[7] link between a measure of bilateral trade intensity and the level of bilateral correlation of real GDPs, confirming existing results. We then run the exact same regressions on the model-generated data as in the empirical part of the paper.[8]

We find that the calibrated model implies regression coefficients that are between 40% and 60% of the data coefficients for GDP. Additionally, as in the data, the model exhibits a trade-comovement relationship that is much stronger for the top half of the bilateral trade intensity distribution. Specifically, for the high trade intensity half of the sample, we find that the model accounts for 70 to 80% of the data relationship. We show that it is the presence of dynamic trade elasticity that drives the result: an analogously parameterized static elasticity model implies no trade-comovement relationship, or even a negative relation.

Related literature. Our paper is related to a number of contributions in the literature. Most closely, it builds on Kose and Yi (2006), who use a frictionless business cycle framework to formally establish the quantitative failure of the frictionless theory of generating the trade-comovement relationship. In this context, we provide an analytic characterization of the sources of this quantitative failure and propose a resolution of the puzzle.

[7]Both in the data and subsequent model regressions, we include European Union dummies and country fixed effects.

[8]Including European Union dummies and country dummies.

Johnson (2014) documents the trade-comovement relationship on the sectoral level and considers a multi-country model with a rich input-output structure to address the data. He evaluates the potential of exogenous TFP correlations with trade on the sectoral level and concludes that the model featuring such correlations still fails to resolve the puzzle for aggregate output. Our exercise assumes exogenous productivities that do not vary with trade and deliver the resolution of the puzzle for aggregate output, following the approach in Kose and Yi (2006) and the standard theory of international business cycles.

Liao and Santacreu (2015) develop a model featuring productivity spillovers proportional to trade that generate positive trade-comovement relations. In contrast, we work with a standard business cycle transmission mechanism and show that it depends on the fact that short- and long-run trade elasticities are assumed equal, regardless of what is assumed about the TFP process.

The findings documented by Johnson (2014) put into question the role of TFP as being the sole driving force behind trade-comovement relations for output on the aggregate level. Nonetheless, our quantitative model provides an alternative channel of mismeasurement driving the TFP trade-comovement relationship. Although we do not consider it here, our model can generate different patterns across sectors stressed by Johnson (2014) by assuming heterogeneous markups and marketing technology.[9]

The rest of the paper is organized as follows. Section 2 sets up a simple prototype business cycle economy from which we derive our key theoretical results. Section 3 discusses data, presents our micro-founded quantitative model, describes parameterization, and discusses our

[9]Changing parameters governing markups and marketing spending can be introduced without changing our model's predictions for trade elasticity dynamics.

quantitative findings. Section 4 concludes.

2 Theory

We begin by considering a simple prototypical international business cycle model featuring a reduced-form friction that can flexibly disconnect the short-run trade elasticity from its long-run counterpart. We use this model to illustrate the puzzle and show how it can be resolved by modeling the dynamics of trade elasticity. Independently, our analysis highlights the importance of modeling the dynamics of trade elasticity for shock transmission in a broad class of macroeconomic models. To derive our results, we make several simplifying assumptions that we eventually relax in the quantitative model analysis. At the end of the section, and throughout, we comment on how these simplifying assumptions affect the results.

2.1 Setup

Time is discrete and the horizon is infinite. There are two symmetric countries, referred to as the home country and the foreign country. We denote the foreign analogs of the home country variables by using an asterisk. To streamline notation, whenever possible, we exploit symmetry and preset the setup from the home country's perspective.

Each country produces a differentiated tradable intermediate good: the home country produces good d and the foreign country produces good f. Moving goods across the border involves an iceberg transportation cost denoted by $\tau > 0$, which implies that $1 + \tau$ of the good must be shipped for one unit to arrive at a foreign destination. Production technology is linear in labor l and subject to a country-specific, exogenous, mean-reverting productivity process

A, implying that the home country output is

$$y = Al. \tag{1}$$

Feasibility requires that the total consumption of each type of good adds up to total output:

$$\begin{aligned} d + d^* + \tau d^* &= y \\ f + f^* + \tau f^* &= y^*. \end{aligned} \tag{2}$$

The key nonstandard feature of the model is that adjusting trade shares is subject to an adjustment cost

$$\Phi(d, f) = \phi \left(\frac{f}{d} \frac{\bar{d}}{\bar{f}} - 1 \right)^2 \tag{3}$$

borne by the consumer. For analytic tractability, we assume that the convex adjustment cost applies to deviations of f/d from the steady-state value $\bar{f}/\bar{d}$ calculated by setting $\Phi = 0$. Since the steady state is always calculated with $\Phi = 0$ in this model, this friction can be interpreted as representing the consumer's deep habit with respect to her long-run share of home and foreign goods in her basket.

The presence of the adjustment cost implies that the model with $\phi > 0$ features dynamic trade elasticity; that is, the response of trade to the terms of trade changes crucially depends on the horizon over which this adjustment takes place. In particular, the long-run response of trade to the trade cost is solely determined by ρ, where we associate the long-run levels with the (deterministic) steady state calculated by setting $\Phi = 0$. At the same time, in the short

run, or over the business cycle, adjustment is smaller, implying a lower elasticity. Accordingly, we refer to the model with $\phi = 0$ as *static elasticity model* and to the model with $\phi > 0$ as *dynamic elasticity model.*

There is a representative household in each country that derives utility from consumption of goods d and f and leisure. Goods are aggregated into final consumption via a standard symmetric CES aggregator:

$$G(d, f) = (\frac{1}{2}(d/2)^{\frac{\rho-1}{\rho}} + \frac{1}{2}(f/2)^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}, \tag{4}$$

which is part of household preferences. The utility function is logarithmic in consumption, $G(d, f)$, and quasi-linear in labor (l),

$$u(d, f, l) = \log(G(d, f)) - \Phi(d, f) - l. \tag{5}$$

and the adjustment cost is borne in the units of home labor.

Households supply labor l to firms at a competitive wage $w = A$, buy goods d and f at a relative price p, and trade a complete set of state-contingent bonds B in a world asset market at state-contingent price Q. Let s^t denote the history of shocks A, A^* up to and including period t. Formally, households in the home country maximize

$$\sum_t \sum_{s^t} \beta^t \{u(d(s^t), f(s^t), l(s^t))\} \tag{6}$$

subject to

$$d(s^t) + p(s^t)f(s^t)(1+\tau) + \sum_{s^{t+1}} Q(s^{t+1})B(s^{t+1}) = B(s^t) + w(s^t)l(s^t), \tag{7}$$

where good d is the numéraire (globally). The foreign household budget constraint is defined analogously and given by

$$p(s^t)f^*(s^t) + d^*(s^t)(1+\tau) + \sum_{s^{t+1}} Q(s^{t+1})B^*(s^{t+1}) = B^*(s^t) + p(s^t)w^*(s^t)l^*(s^t). \tag{8}$$

Market clearing requires that $B^*(s^t) + B(s^t) = 0$ and that firms make zero profit. Competitive equilibrium is defined in the usual way.[10]

2.2 Preliminaries

Our goal is to derive how trade between countries affects international transmission of foreign productivity shocks to home country output. To that end, we must define trade intensity between countries, introduce a measure of the strength of business cycle transmission (spillovers) within the model and define short- and long-run trade elasticities. We lay down these definitions below. At the end, we restrict parameter values to focus attention on parameterizations that are of practical importance.

[10]The formal definition of equilibrium and a complete list of equilibrium conditions can be found in the Appendix.

2.2.1 Definitions

Defining trade intensity requires us to assume how much of the trade cost τ is explicitly included in National Income Accounting and how much only enters the prices. The Bureau of Economic Analysis measures imports net of trade costs (insurance and freight, c.i.f.), while GDP and its components include these costs. In what follows, we assume that explicitly measured trade costs are a negligible fraction of total trade costs, and hence trade costs implicitly enter imports through prices. We consider the polar opposite in the Online Appendix and show that our results hold.[11] Specifically, we define trade intensity as the mean share of imports net of trade cost relative to domestic absorption:

$$x = \frac{\text{Real Imports}}{\text{Real Domestic Absorption}} = \frac{f(1+\tau)}{d + f(1+\tau)}, \tag{9}$$

where $1+\tau$ in the last expression is the overhead of the steady-state price of imports in the model.[12]

We define the baseline measure of the strength of business cycle transmission (*comovement* henceforth) $\mathcal{S}$ as

$$\mathcal{S} = \frac{d \log y}{d \log A^*}. \tag{10}$$

$\mathcal{S}$ measures the spillover of foreign productivity into domestic country output, starting from the deterministic steady state as the initial point. While the definition above simplifies the exposition, in the Online Appendix we show that all the results proven here carry over to a

[11] If trade costs were explicit, Bureau of Economic Analysis would subtract these costs from imports but not from domestic absorption (or GDP). In such a case only f would appear in the numerator and the denominator would remain unchanged. For details on national income accounting procedures in our data, see SNA (1993).

[12] In the quantitative model, we use GDP in place of domestic absorption. Note that, when trade is balanced, which is the case in the steady state of our model, GDP and domestic absorption are equal.

correlation coefficient, as the correlation coefficient is strictly monotonic in our more direct measure of comovement as shock spillover.

The focus of our paper is to characterize how the strength of shock transmission changes with the (deterministic) steady-state level of trade with $\Phi = 0$, which we denote by $\bar{x}$. Accordingly, we use the relationship between trade cost τ and the steady-state trade $\bar{x}$ implied by our model,[13]

$$\tau(\bar{x}) = (\frac{1}{\bar{x}} - 1)^{\frac{1}{\rho-1}} - 1, \tag{11}$$

in order to express trade cost as a function of the steady state trade in the model. Accordingly, we define our baseline measure of the *trade-comovement link* as

$$\mathcal{L} = \frac{d\mathcal{S}}{d\tau}\frac{d\tau(\bar{x})}{d\bar{x}} = \frac{d\mathcal{S}}{d\bar{x}}, \tag{12}$$

for $\tau(\bar{x})$ given by (11). $\mathcal{L}$ tells us how the spillover of foreign shock into domestic country output is affected by steady state trade $\bar{x}$.

Finally, we gauge the strength of the convex adjustment cost ϕ through the lens of the implied difference between the short- and long-run trade elasticity. We define the short-run elasticity in the model as the ratio of the standard deviation of equilibrium trade x and the standard deviation of terms of trade p:

$$SRE = \frac{std(\log x)}{std(\log p)}, \tag{13}$$

which we label *volatility ratio* following the approach by Drozd and Nosal (2012). Using the

[13] See the Appendix.

volatility ratio to measure short-run trade elasticity conveniently abstracts from this issue by focusing on the upper-bound of the regression coefficient between the two variables. In our analysis, the upper-bound suffices because our results are only reinforced by a lower short-run elasticity target. We measure the long-run elasticity as the elasticity of the long-run response of the steady-state level of trade $\bar{x}$ to a permanent change in the level of trade cost τ:

$$LRE = |\frac{d \log \bar{x}(\tau)}{d \log(1+\tau)}|, \tag{14}$$

where $\bar{x}(\tau)$ is the inverse of (11).[14] The key observation, summarized in the lemma below, is that the measured short-run elasticity is equal to the measured long-run elasticity for $\phi = 0$, and while the long-run elasticity is independent of ϕ, the short-run elasticity is strictly decreasing in ϕ.

Lemma 1 *The short- and long-run trade elasticities defined in* (13) *and* (14)*, respectively, are equal for* $\phi = 0$*, and while the long-run trade elasticity is independent of* ϕ*, the short-run trade elasticity is strictly decreasing to zero in* ϕ *as* $\phi \to \infty$*.*

2.2.2 Parameter values

We restrict the parameter domain to focus on the values that imply home-bias in the steady state ($\bar{x} < 1/2$) and a relationship between trade intensity and trade cost consistent with gravity in international trade. Specifically, given the definition of trade $\bar{x}$, we note that for a very low value of elasticity of substitution ρ, our model implies an inverse relation between

[14] Our measurement of the long-run trade elasticity in the model is consistent with the trade literature that relates steady-state outcomes in models to long-run changes in trade patterns occurring in response to (permanent) changes in tariffs.

trade cost τ and trade $\bar{x}$, implying that higher trade cost τ increases the steady-state level of trade $\bar{x}$. This follows from equation (11). Since this contradicts the evidence on gravity in international trade, we restrict our focus to $\rho > 1$.

Assumption 1 (Gravity) $\rho > 1$ *so that* $d\tau(\bar{x})/d\bar{x} < 0$ *in* (11).

Assumption 2 (Home-bias) $0 \leq \bar{x} < 1/2$.

2.3 Trade-comovement puzzle

We start from the analysis of the static elasticity model ($\phi = 0$), which implies that the short- and long-run trade elasticities are equal and monotonic in ρ, as shown in Lemma 1. We prove that this model implies negative trade-comovement for all parameter values (Proposition 1). That is, as trade $\bar{x}$ is increased by lowering trade cost τ, the foreign shock spills over *less* into domestic production. We next focus on analytically characterizing the forces behind this result.

Proposition 1 *(Trade-comovement puzzle)* $\mathcal{L} < 0$ *for* $\phi = 0$ *to a first-order approximation.*

To uncover the basic forces behind the puzzle, we consider a decomposition of the linearized model given by[15]

$$\hat{y}(s^t) = \alpha\hat{A} + \eta\hat{p}(s^t) + \chi R(s^t) \tag{15}$$

$$\hat{p}(s^t) = \pi(\hat{A}^* - \hat{A}) + \theta R(s^t) \tag{16}$$

$$R(s^t) = \mu(\hat{A}^* - \hat{A}), \tag{17}$$

[15]We log-linearize with respect to all variables except for R, with respect to which we linearize the system. Since the steady state value of R is zero, $\bar{R} = 0$, we use it as defined rather than in log terms as we do for the other variables.

where $\hat{}$ over a variable denotes log-deviations from the deterministic steady state, and $\alpha, \eta, \chi, \theta\, \pi, \mu$ – the coefficients of linearization – depend on model parameters (in particular, trade cost τ, and by (11), also trade). In the above decomposition,

$$R(s^t) = B(s^t) - \sum_{s^{t+1}|s^t} Q(s^{t+1})B(s^{t+1}) + (1 - p(s^t))(1 + \tau(\bar{x}))f(s^t), \tag{18}$$

and it corresponds to equilibrium risk-sharing transfers between the two countries. Risk sharing is implemented through asset trade (B) as well as the income effect associated with the endogenous terms of trade movements. It is determined by the risk-sharing condition,[16]

$$\frac{\partial u(d(s^t), f(s^t), l(s^t))}{\partial d(s^t)}(1 + \tau(\bar{x})) = \frac{\partial u(f^*(s^t), d^*(s^t), l^*(s^t))}{\partial d^*(s^t)}. \tag{19}$$

Intuitively, the risk-sharing condition implies that home and foreign households trade assets, determining risk-sharing transfers R, to equalize the marginal utility from each type of good across countries (net of trade costs and convex costs of adjusting trade shares). It is easy to see that risk-sharing transfers are zero-sum, as the budget constraints of the domestic and foreign household in (7) and (8) can be equivalently rewritten as

$$d + f(1 + \tau(\bar{x})) = wl + R \tag{20}$$

$$d^*(1 + \tau(\bar{x})) + f^* = w^*l^* - R. \tag{21}$$

The derivation of the decomposition (15)-(17) is detailed in the Appendix. We derive it by performing step-wise log-linearization of the equilibrium system to uncover intermediate

[16] See the Appendix for derivation and further discussion of this condition.

dependencies between variables. We set it up to isolate out the income effect of risk-sharing transfers R on the home country's household labor supply and hence output ($\chi\mu$). We distinguish it from the effect of equilibrium change in the terms of trade ($\eta(\pi+\theta\mu)$) on the home real wage w, and hence the home country's output and labor supply, since the home consumption basket comprises goods d and f, and home labor produces good d. Importantly, coefficients π and θ distinguish the *direct* effect of shocks on terms of trade from the *indirect* effect of shocks on the terms of trade via risk sharing, which, by affecting the allocation, affects the terms of trade.

Equipped with decomposition (15)-(17), we can decompose the international transmission of shocks

$$\mathcal{S} = \frac{d\hat{Y}}{d\hat{A}^*} = \frac{\partial \hat{Y}}{\partial \hat{p}} \frac{\partial \hat{p}}{\partial \hat{A}^*} + \frac{\partial \hat{Y}}{\partial R} \frac{\partial R}{\partial \hat{A}^*} \tag{22}$$

to the *complementarity channel*:

$$\mathcal{S}_C := \frac{\partial \hat{Y}}{\partial \hat{p}} \frac{d\hat{p}}{d\hat{A}^*} = \eta(\pi + \theta\mu), \tag{23}$$

and distinguish it from the *risk-sharing channel*:

$$\mathcal{S}_R := \frac{\partial \hat{Y}}{\partial R} \frac{dR}{d\hat{A}^*} = \chi\mu. \tag{24}$$

Accordingly, we can decompose the trade-comovement link $\mathcal{L}$ as follows:

$$\mathcal{L} = \mathcal{L}_C + \mathcal{L}_R, \tag{25}$$

where $\mathcal{L}_C = \frac{\partial \mathcal{S}_C}{\partial \bar{x}}$ measures the contribution of the complementarity channel and $\mathcal{L}_R = \frac{\partial \mathcal{S}_R}{\partial \bar{x}}$ measures the contribution of the risk-sharing channel. We next characterize these two channels of shock transmission.

Complementarity channel. The complementarity channel is brought about by the imperfect substitutability between goods d and f. It is the source of positive shock transmission, and it contributes positively to the trade-comovement relationship. Mechanically, this follows from the fact that $\eta(\pi + \theta\mu) = \bar{x}$, as derived in Lemma 2 below.

Lemma 2 *To a first-order approximation, if $\phi = 0$, the decomposition (15)-(17) gives $\mathcal{S}_C = \eta(\pi + \theta\mu) = \bar{x} > 0$, where $\eta = -\bar{x}$, $\pi = -\frac{1}{2}(\rho - \bar{x}(\rho - 1))^{-1}$, $\pi + \theta\mu = -1$, and hence $\mathcal{L}_C = 1$.*

Intuitively, if foreign productivity A^* rises, the terms of trade p falls, i.e., the price of good d rises relative to good f. Because the price of consumption at home comprises both goods, the price of the home consumption basket falls in terms of good d that is produced by domestic labor. This drives up the home country real wage in terms of consumption and leads to higher domestic labor supply, and hence higher output. This explains why $\eta = -\bar{x}$ in the decomposition and why both $\pi < 0$ and $\pi + \theta\mu < 0$. But how does the response of the terms of trade p change with trade? In our prototypical model, the net change in terms of trade turns out to be independent of trade ($\pi + \theta\mu = -1$), despite the fact that trade increases π in absolute value. As a general property, this happens because the risk-sharing condition (19) makes the terms of trade always equal to the marginal rate of transformation of producing goods d and f after factoring in differences in the value of leisure between countries, which

owing to the constant value of leisure here is given by $\hat{A}^* - \hat{A}$.[17] As a result, the amplifying effect of trade through π is entirely offset by the *indirect* effect of risk sharing on prices through the term $\theta\mu$. Although the exact offset depends on quasi-linearity of the utility function, the basic logic and the direction of the aforementioned effects still applies. See Section 2.5 for a discussion of nonseparability.

Risk-sharing channel. The risk-sharing channel is the source of negative shock transmission and it is the source of the negative trade-comovement link. Mechanically, this follows from the fact that $\chi = 1$ in the decomposition and μ is strictly increasing in trade $\bar{x}$, as shown in Lemma 3 below. Importantly, the effect of risk sharing always offsets the positive complementarity effect discussed above, as shown in Proposition 1.

Lemma 3 *To a first-order approximation, if* $\phi = 0$*, the decomposition* (15)-(17) *gives* $\mathcal{S}_R = \chi\mu < 0$*, where* $\chi = -1$*,* $\mu = \bar{x}\left((\rho - 1)\left(1 - 2\bar{x}\right) + \rho\right)$*, and hence* $\mathcal{L}_R = 1 + 2(\rho - 1)(1 - 2\bar{x}) < -1$*.*

Intuitively, risk sharing has a sizable impact on business cycle transmission because it enables countries to shift production toward the country with the highest productivity by borrowing and lending over the business cycle – an effect known as *resource-shifting* motive. Risk-sharing transfers, in our framework measured by R, cushion consumption from uneven production pattern across countries implied by offsetting movements of labor supply in response to relative productivity. The reason why trade affects the strength of this effect is that higher trade $\bar{x}$ is associated with a lower trade cost τ, and hence a lower deadweight loss from resource shifting with risk sharing that by definition requires goods to cross the border.

[17] See the discussion of risk-sharing condition and household first-order conditions in Appendix A.1 and in the Online Appendix E.

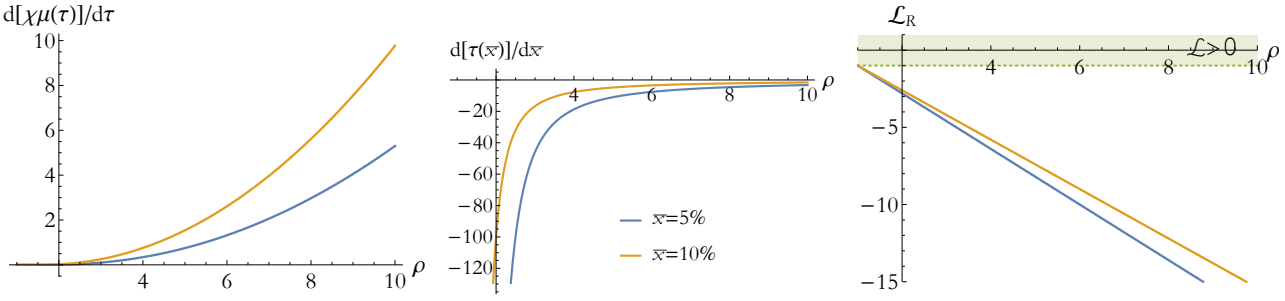

Figure 1: $\mathcal{L}_R = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}\frac{d\tau}{dx}|_{\tau(\bar{x})}$ as a function of elasticity $\rho \geq 1$, for $\phi = 0$.

We next formalize the above intuition by deriving the relevant coefficients of the decomposition (15)-(17) in Lemma 3. To highlight the "taxing" effect of trade costs, we recast $\chi\mu$ as a function of τ, which we obtain by using the inverse of (11):

$$\mathcal{L}_R = \frac{d\left(\chi\mu\right)}{d\bar{x}} = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}\frac{d\tau(\bar{x})}{d\bar{x}}. \tag{26}$$

In line with the intuition above, the term $\frac{d\tau(\bar{x})}{d\bar{x}}$ captures how much trade costs have to differ in order to vary the steady-state level of trade $\bar{x}$; the term $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ isolates out the effect of trade costs on the risk-sharing channel for a given level of steady-state trade $\bar{x}$.

Figure 1 plots these two terms for two distinct levels of trade intensity: $\bar{x} = .05$ and $\bar{x} = .1$. As we can see from the leftmost panel, the term $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ is always positive, confirming the intuition that higher trade costs attenuate the impact of risk sharing (resource shifting) on shock transmission. However, as the middle panel shows, by gravity, higher trade costs result in *lower* trade (lowers $\bar{x}$), and hence this channel implies a negative trade-comovement link (right panel).

The figure also shows that $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ is increasing in ρ, which turns out to dominate despite the fact that $\frac{d\tau(\bar{x})}{d\bar{x}}$ is decreasing in absolute value. The reason for this is that the marginal

utility of each good is more stable at a high level of elasticity, and so when elasticity is high, it is easier to shift production without adversely affecting the marginal utility from consumption. As we show below, modeling the low short-run trade elasticity plays an important role by suppressing this effect too.

2.4 Resolution of the trade-comovement puzzle

Below, we show that the convex adjustment cost $\phi > 0$ in our prototype economy leads to a positive trade-comovement link for sufficiently high values of ϕ. We relate this result to the disconnect between the short- and long-run trade elasticities implied by that.

Proposition 2 *(Resolution of the trade-comovement puzzle) There exists a cutoff value* $0 < \bar{\phi} \leq \bar{x}(1-\bar{x})$*, such that, to a first-order approximation,* $\phi > \bar{\phi}$ *implies* $\mathcal{L} > 0$*.*

Intuitively, as was the case with low elasticity ρ in the static elasticity model, the high value of the convex adjustment cost parameter ϕ suppresses the risk-sharing channel that was the source of the negative trade-comovement link in the static elasticity model. The difference is that the convex adjustment cost does not affect the long-run trade elasticity, nor the long-run relation between trade and trade costs $d\tau/d\bar{x}$ in (11). Importantly, low short-run elasticity also preserves the sensitivity of marginal utility from each good to changes in trade occurring over the business cycle, flattening the previously increasing risk-sharing term $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ with respect to ρ. As a result, for a high value of ρ, the effect of risk sharing on the trade-comovement link can be attenuated, and the still positive complementary effect can dominate.

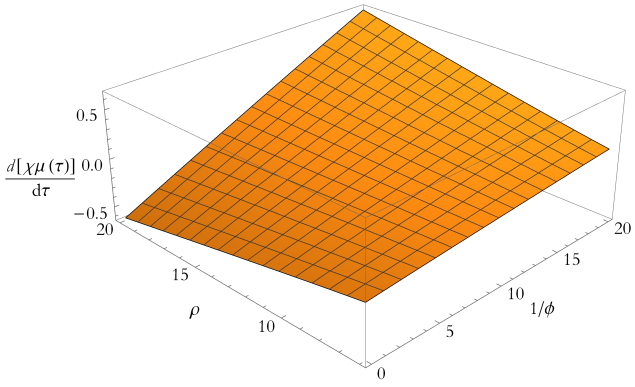

Figure 2: $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ as a function of $1/\phi$ and ρ, for $\bar{x} = .05$.

Lemma 4 *The decomposition* (15)-(17) *for* $\phi \geq 0$ *gives*

$$\begin{aligned} \mathcal{S}_C(\phi) &= \eta(\pi + \theta\mu) = \bar{x} \\ \mathcal{S}_R(\phi) &= \chi\mu = -\bar{x}\left(2\bar{x} + \frac{2\rho\bar{x}\,(1-\bar{x})^2}{(1-\bar{x})\bar{x} + 2\rho\phi} - 1\right) > \mathcal{S}_R(\phi = 0), \end{aligned}$$

and hence $\mathcal{L}_C(\phi) = 1$ *and* $\lim_{\phi\to\infty} \mathcal{L}_R(\phi) = 1 - 4\bar{x} > -1 > \mathcal{L}_R(\phi = 0)$.

Lemma 4 formalizes the above intuition by deriving the terms of the decomposition for the general case of $\phi > 0$. As is clear from the lemma, the complementarity is the same (i.e., $\mathcal{L}_C = 1$ for $\phi > 0$). As discussed above, perfect risk sharing equalizes the response of the terms of trade to the change in the marginal rate of transformation between goods d and f in production, here equal to $\hat{A} - \hat{A}^*$. Similarly as in the static elasticity model, the share of foreign goods in the home country basket, equal to $\bar{x}$, determines by how much the real wage rises in response to the foreign productivity shock.

What is different, however, is the offsetting effect of the risk-sharing channel on the trade-comovement link. The risk-sharing term $\frac{d(\chi\mu(\tau))}{d\tau}$ in (26) falls as ϕ increases (or $1/\phi$ is decreased),

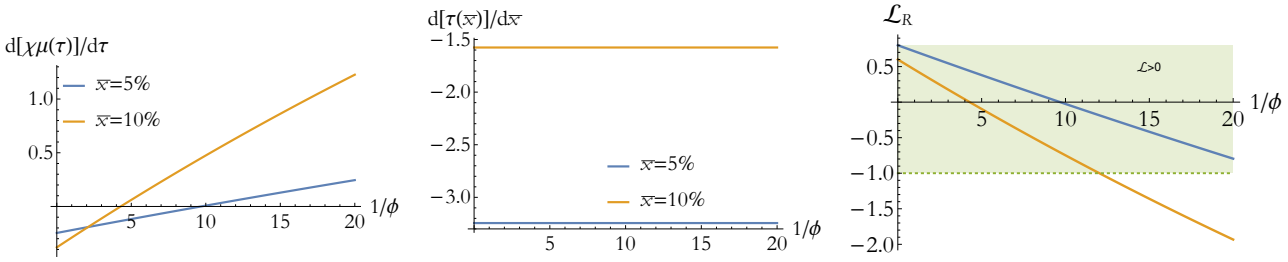

Figure 3: $\mathcal{L}_R = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}\frac{d\tau}{dx}|_{\tau(\bar{x})}$ as a function of $1/\phi$, for $\rho = 10$.

just like in the previous model it fell when ρ was decreased. At the same time, in the model with the adjustment cost, the term $\frac{d\tau}{dx}|_{\tau(\bar{x})}$ is independent from the assumed value of ϕ. Accordingly, for ρ sufficiently high, this term is arbitrarily small.

The fact that the previously increasing term $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ with respect to ρ is no longer as tightly connected to ρ reinforces this point. Note that the adjustment cost implies that the marginal utility from each good remains sensitive to quantities despite ρ being high. This property is illustrated in Figure 2, which shows that, for high ϕ (low $1/\phi$ in the figure), the relationship weakens between $\frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})}$ and ρ weakens and eventually changes direction.

Finally, Figure 3 illustrates the analog of the decomposition presented in Figure 1. The shaded region marks the area for which the model implies a positive trade-comovement link. As is clear from the comparison of Figures 1 and 3, the key difference is in the middle panel, which shows small and constant $d\tau/d\bar{x}$. As a result, there is always a high enough adjustment cost (i.e., lower $1/\phi$ on the graph) to imply that $\mathcal{L}_R$ crosses above -1, and hence the trade-comovement link $\mathcal{L}$ becomes positive (rightmost panel).

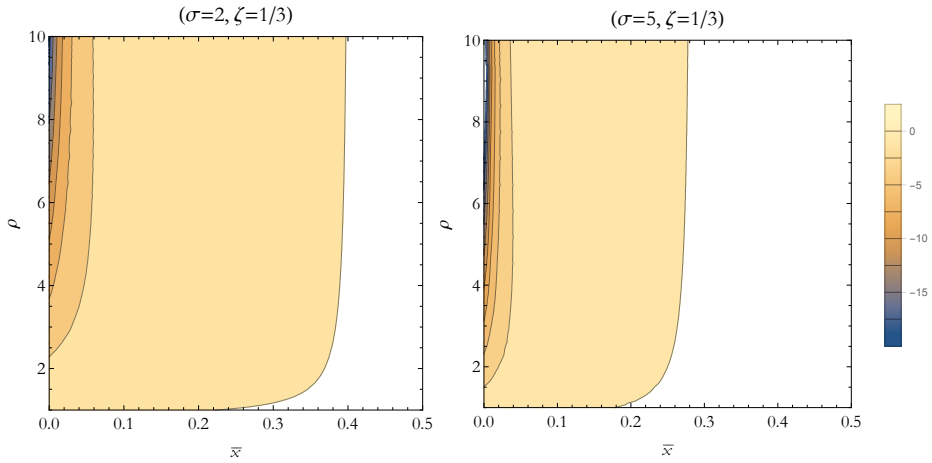

Figure 4: The negative region of the trade-comovement link $\mathcal{L}$ for nonseparable CRRA utility function, $\sigma = 2, \zeta = 1/3$ (left panel) and $\sigma = 5, \zeta = 1/3$ (right panel).

2.5 Generalizations and robustness of the results

We finish our discussion by noting that, while our model makes several stark assumptions, the results behind the resolution of the puzzle (Proposition 2) are more general. Conditional on the puzzle holding (Proposition 1), the analysis in Proposition 2 does not depend on the particular simplifying assumptions. The interaction between trade costs and risk sharing, and its general impact on quantities, is a feature shared by most macroeconomic models – including other types of shocks driving business cycle fluctuations. Similarly, the link between the level of trade costs and the implied long-run trade elasticity is in essence a tautology. Qualitatively, this is all that is needed for the resolution to work. While our analytic proof of the puzzle (Proposition 1) hinges on some of the particular assumptions we make to render the model tractable, Kose and Yi (2006) have shown numerically that the puzzle applies more broadly.

In the Online Appendix, we analyze the validity of the puzzle under nonseparable CRRA utility function in (36), as assumed by our quantitative model. We show that qualitatively it has the same implications as far as the decomposition goes. We also show that nonseparable

utility function has the potential to alleviate the puzzle. Specifically, for high enough levels of bilateral trade intensity (likely too high to be relevant empirically), nonseparability can even deliver a positive trade-comovement link. However, for the empirically plausible range of trade intensities, the model implies a negative trade-comovement link for all parameter values (including parameterizations featuring high levels of risk aversion). Figure 4 illustrates this property for two values of risk-aversion parameter $\sigma = 2$ and $\sigma = 5$. In general, the following result holds for nonseparable utility function in the limit under the restriction that $\sigma > 1$ and under Assumptions 1 and 2:

$$0 < \lim_{\bar{x} \to 0} \mathcal{L}_C = 1 + \zeta((\sigma - 1)(1 - \zeta) - 1) \quad < -\lim_{\bar{x} \to 0} \mathcal{L}_R = (1 - \zeta)(2\rho((\sigma - 1)\zeta + 1) - 1). \quad (27)$$

This result implies that for sufficiently low levels of bilateral trade, the model with nonseparable utility function exhibits trade-comovement puzzle for all parameter values. Our quantitative results and those documented by Kose and Yi (2006) seem to confirm that, in the relevant region of the parameter space, nonseparability implies a negative trade-comovement link.[18]

What is the intuition behind the impact of nonseparability on the trade-comovement link? Our analysis in the Online Appendix shows that nonseparability is a double-edged sword. On the one hand, it exacerbates the puzzle through the attenuating indirect effect of risk sharing ($\theta\mu$) on the dynamics of the terms of trade. This is because more equal consumption across countries, which high- and low-trade costs promote, dampens the terms of trade response due to the then more-pronounced offsetting movements in the relative value of leisure (that exhibits complementarity with consumption for nonseparable utility functions).[19] On the other

[18] See the Online Appendix for more details and an extended discussion of the nonseparable case.

[19] Recall our discussion of the complementarity effect. The offsetting indirect effect of risk sharing on terms

hand, complementarity between labor and leisure also dampens the direct effect of risk sharing associated with resource shifting, since resource shifting by definition leads to a disconnect between consumption and leisure that agents are then more averse to.

Finally, while our results point to the importance of risk sharing for the puzzle, they do not imply that simple forms of market incompleteness will resolve the puzzle. In fact, standard formulations of incompleteness of markets, including the case of financial autarky, have been found to only weaken the puzzle (see Kose and Yi (2006)). This is understandable given the classic result by Cole and Obstfeld (1991) that the income effect associated with the terms of trade fluctuations provides the bulk of risk sharing, even under the extreme assumption of financial autarky.[20] As we have shown, the terms of trade movements are amplified by trade, which then automatically leads to more risk sharing (through coefficient π in the decomposition).

3 Quantitative analysis

We next turn to the quantitative evaluation of the effects identified in the previous section. In what follows, we present our data target for the trade-comovement relationship and develop a three-country micro-founded model of dynamic elasticity that we calibrate to the data.

of trade through $\theta\mu$ is ultimately determined by the risk-sharing condition, which involves both the marginal rate of technical transformation and the relative value of leisure that is no longer equal to unity. See Appendix A.1. and Appendix E for more details.

[20] In the data, if agents modulate their exposure to exchange rates, they, in effect, are engaged in trades that modulate risk sharing. Foreign exchange markets' daily volume is among the largest in the world, estimated at about 5 trillion dollars by BIS (2016) at http://www.bis.org/publ/rpfx16.htm.

3.1 Data

We quantify the relation between trade and the business cycle comovement in the cross-country data using regression analysis, along the lines of Kose and Yi (2006).[21]

Our dataset comprises 20 industrialized countries over the period 1980Q1-2011Q4.[22] Countries in our sample constitute about 59% of world GDP and 53% of world trade (as of 2011). Our baseline specification of the cross-sectional regression is

$$corr(x_i, x_j) = \alpha + \beta_x trade_{ij} + X_i + X_j + E_{ij} + \varepsilon_{ij}, \tag{28}$$

where $corr(x_i, x_j)$ is the correlation between countries i and j of the logged and HP-filtered series of real GDP. X_i and X_j are country dummies, and E_{ij} is the European dummy, which takes the value of 1 if both countries in the pair are European countries. The variable $trade_{ij}$ is a symmetric measure of bilateral trade intensity of countries i and j, measured at the beginning of the sample[23] (in 1980), and given by the log of

$$\max\{\frac{IM_{ij}}{GDP_i}, \frac{IM_{ji}}{GDP_j}\}, \tag{29}$$

where IM_{ij} are nominal imports (in US dollars) by country i from country j and GDP_i is the nominal GDP (in US dollars) of country i, both measured in 1980.

[21] A number of studies have also explored alternative approaches as well as micro-data and have reached a similar conclusion regarding the link between trade and business cycle synchronization (see, for example, Clark and van Wincoop (2001), and more recently diGiovanni and Levchenko (2010) or Inklaar et al. (2008)).

[22] For a list of data sources, see the Online Appendix. Our country list includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[23] All of the results are robust to picking other years as the base year for the bilateral trade measure.

The measure of trade defined in (29) varies in our sample from 0.03% (Korea with Portugal) to 27% (Ireland with the United Kingdom). Notably, it is symmetric and yet robust to having trade partners of very different sizes. For example, if the United States is an important trading partner to Canada but the situation is less so vice versa, then the equation in (29) will nonetheless return a high number.[24]

Table 1 reports our results. We include OLS results as well as results from an IV regression in which the instruments are common border, common language and distance. As is clear from the table, both OLS and IV regressions give highly significant positive coefficients, which suggest a strong effect of bilateral trade on comovement of GDP. [25]

Columns 2 and 3 of Table 1 additionally consider a split sample to bottom and top halves of the bilateral trade distribution (the median trade intensity in our data is 0.85%). We find that the trade-comovement relationship is much stronger in the higher trade sample.

3.2 Quantitative model

We next describe the setup of our quantitative model and its parameterization. The model is based on Drozd and Nosal (2012), which we extend to include three countries of varying size.

Time is discrete and the horizon infinite. The world is composed of *three* countries. The

[24]This is in contrast to measures expressed as averages, for example $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$. Such measures give small numbers when trade partners have asymmetric sizes, i.e., small countries trade with big countries. For example, our measure is 8 times higher than $\frac{IM_{ij}+IM_{ji}}{GDP_i+GDP_j}$ for the Germany-Austria pair, 6 times higher for the US-Canada pair, and 15 times higher for the UK-Ireland pair. Our empirical results are robust to using this alternative measure – the results are available from the authors upon request.

[25]The estimates imply that moving from the 10th to the 90th percentile of the bilateral trade spectrum increases the GDP correlations by 0.21 (IV) or 0.11 (OLS). Relative to median GDP correlation of 0.52 in our sample, this is an economically significant effect.

Table 1: Regression results: trade-comovement in the data.

	Dependent Variable: GDP correlation		
	OLS	OLS bottom 50%	OLS top 50%
$trade_{ij}$	0.034^{**}	-0.017	0.055^{**}
	(0.016)	(0.031)	(0.025)
E_{ij}	0.060	0.311	-0.076
	(0.093)	(0.221)	(0.099)
R-squared	0.694	0.758	0.651
	IV	IV bottom 50%	IV top 50%
$trade_{ij}$	0.065^{***}	0.325	0.070^{**}
	(0.024)	(0.286)	(0.033)
E_{ij}	-0.028	-0.373	-0.103
	(0.106)	(0.661)	(0.107)

,* denote significance at the 5% and the 1% levels. Numbers in parentheses are standard errors.

first two countries, labeled *domestic* (D) and *foreign* (F), are symmetric and of equal size, and the third country, labeled *rest of the world* (W), is allowed to differ in size. The size of each country is determined by the population size of atomless households residing in the country. Labor and capital, supplied by the households, are assumed to be immobile across countries and are used by local producers to produce goods. Goods are differentiated by the country of origin and are tradable. Households in each country use these goods for consumption and investment in physical capital. Their preferences are characterized by imperfect substitutability between each type of good and a bias toward the locally produced good. Financial markets are complete. As before, the presentation of the model will be from the domestic country perspective, with the remaining countries' problems being analogous.

Production technology. Tradable goods are country specific and are produced by a unit measure of atomless competitive producers residing in each country. The good produced in the domestic country is labeled d, the good produced in the foreign country is labeled f, and the good produced in the rest of the world is labeled g. Producers employ local capital and labor to produce these goods using the technology available in their country of residence. Production technology is Cobb-Douglas, $Ak^{\alpha}l^{1-\alpha}$, and is subject to country-specific technology shock A, given by an exogenous AR(1) process

$$\log(A) = \psi \log(A_{-1}) + \varepsilon, \tag{30}$$

where $0 < \psi < 1$ is the shock persistence parameter, and ε is a normally distributed i.i.d. random variable with zero mean, and A_{-1} denotes the previous period value of A. For convenience, we summarize production constraints by an economy-wide marginal cost v_D, which, given competitive factor prices w and r, is

$$v_D \equiv \min_{k,l} \left\{ wl + rk \;\mid\; Ak^{\alpha}l^{1-\alpha} = 1 \right\}. \tag{31}$$

The model features an infinitely lived representative household, which trades a complete set of state-contingent bonds, accumulates physical capital, supplies labor and consumes. Household preferences for goods d, f, and g are described via a CES aggregator G:

$$G(d, f, g) = \left(\omega_d d^{\frac{\rho-1}{\rho}} + \omega_f f^{\frac{\rho-1}{\rho}} + \omega_g g^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}.$$

Distribution technology. The key feature of the model is that producers actively match and trade with the retailers, who then resell the purchased goods to local households.

Producers attract searching retailers by accumulating *marketing capital* m. Marketing capital accumulated by a given producer relative to the marketing capital held by other producers in the same market determines the fraction of searching retailers that meet with this producer. Formed matches are long lasting, with a separation rate δ_H. Accordingly, the law of motion for the customer base H_D of a domestic producer with marketing capital m_D^d accumulated in the domestic country is:

$$H_D = (1 - \delta_H) H_{D,-1} + \frac{m_D^d}{\bar{M}_D} h, \tag{32}$$

where $\bar{M}_D$ is the average marketing capital of all producers selling at home:[26] $\bar{M}_D = \bar{m}_D^d + \bar{m}_D^f + \bar{m}_D^g$.

The size of the customer base is critical for the producer, as it puts a limit on the amount of goods they can sell in each country. We assume that within each match one unit of the good can be traded per period, which gives the *sales constraint* of the producer of good d in the domestic country, $d \leq H_D$, a condition that always binds in equilibrium (due to positive markups).

The accumulation of marketing capital follows a standard capital-theoretic law of motion with depreciation rate δ_m and adjustment cost ϕ. Given last period's level of marketing capital $m_{D,-1}^d$ and the current level of instantaneous marketing input a_D^d, the current period's

[26] In equilibrium, $\bar{m}$ and m coincide, as there is measure one of producers in each country.

marketing capital is given by

$$m_D^d = (1-\delta_m)m_{D,-1}^d + a_D^d - \phi m_{D,-1}^d \left(\frac{a_D^d}{m_{D,-1}^d} - \delta_m \right)^2. \tag{33}$$

The accumulation of marketing capital is subject to a convex adjustment cost parameter by ϕ, which we refer to as the *market expansion friction parameter*. The convex adjustment cost captures the notion that the buildup of marketing-related assets, like brand awareness, reputation or distribution network, which determine market visibility, takes both time and resources. Importantly, this specification, together with the assumption that country-specific goods are closely substitutable, generates a high long- and low short-run price elasticity of trade flows – the dynamic elasticity we identified as crucial in our theoretical analysis.

Price formation. Producers from the domestic country sell goods in countries $i = D, F, W$ for wholesale prices $x_i p_i^d$, where x_i is the real exchange rate between country i and the domestic country. These prices are determined by bargaining with the retailer, who resells the good in a competitive domestic retail market for the price P_d determined by the domestic consumer's valuation of the good. We assume Nash bargaining with continual renegotiation over the continuation surplus from the match. As in Drozd and Nosal (2012), the solution is a simple static surplus splitting rule that gives:[27]

$$p_D^d = \theta(P_d - v_D) + v_D,$$

[27]For the case with tariffs, we maintain the static surplus splitting rule with the same proportion θ.

where θ is the Nash bargaining power of the producers. The competitive retail prices in the above equation come from the household problem and are given by the partial derivatives of the CES aggregator from the household's problem: $P_d = G_d(d, f, w)$.

Production sector. Given a customer base of a domestic producer in each country, H_D, H_F, H_G, the instantaneous profit function Π of the producer is given by the difference between the profit from sales in each market and the total cost of marketing the goods:

$$\Pi = \sum_{i=D,F,W} (x_i p_i^d - v_D) H_i - \sum_{i=D,F,W} x_i v_i a_i^d. \tag{34}$$

Dynamically, a representative producer from the domestic country, who enters period t in state s^t with the customer base (H_D, H_F, H_W), marketing capital m_D^d, m_F^d, m_W^d chooses the allocation of marketing expenditures a_D^d, a_F^d, a_W^d, period-t marketing capitals and customer bases, to satisfy the Bellman equation

$$V = \max \left\{ \Pi + EQV_{+1} \right\},$$

where Q is the stochastic discount factor implied by the household problem and the optimization is subject to the marketing technology constraints (33) and the laws of motion for customer base (32).

Retail sector. In each country, there is a sector of atomless retailers who purchase goods from producers and resell them to local households. Retailers who enter into the sector must incur an initial search cost χv_d in order to find a producer with whom they can match and

trade. The matching probabilities are taken as given by entering retailers but in equilibrium are determined in consistency with (32). Industry dynamics are governed by a free entry and exit condition, which endogenously determines the measure h of searching retailers at each date and state:

$$\frac{\bar{m}_D^d}{\bar{M}_D} J_D + \frac{\bar{m}_D^f}{\bar{M}_D} J_F + \frac{\bar{m}_D^g}{\bar{M}_D} J_W = \chi v_D, \tag{35}$$

where J_i is the value function associated with being matched with an i country producer given by $J_i = P_i - x_i p_i + (1 - \delta_H) EQ J_{i,+1}$. Equilibrium is defined in the usual way.

3.2.1 Parameterization

The baseline period length in the model and in the data is one quarter. We parameterize the utility function in a standard constant relative risk aversion specification:

$$u(c, l) = \frac{(c^\zeta (L - l)^{1-\zeta})^{1-\sigma}}{1 - \sigma}, \tag{36}$$

where $0 < \zeta < 1$. We set β so that the model is consistent with an annual risk free real interest rate of 4.1%, α to reproduce the constant labor share of 64% and the depreciation rate of δ of 2.5% (quarterly) as in Backus, Kehoe and Kydland (1995).[28] We use the standard value for the intertemporal elasticity of substitution/risk aversion parameter σ of 2 and arbitrarily set $\delta_h = 0.1$, implying that the matches in the economy last on average 2.5 years (10 quarters). We choose population sizes L_i to be 20 times larger in the rest of the world than for the symmetric country pair. Unless otherwise stated, we choose the same parameters for all three countries.

[28] It implies investment to GDP ratio of 25%. In the recent data, we find 20% in the US, 28% in Japan, 22% in Germany, and 21% in France. The OECD median is close to 20%. We adopt this number to make the model implications more comparable to the literature.

We set ρ equal to 15, which is close to the upper limit of the values reported in the trade literature (see Ruhl (2008)). We choose the market expansion friction parameter ϕ to match the measured value of the *volatility ratio*[29] in our cross-section of countries of 1.17.

The parameters ϕ, δ_m, χ, θ, ω_i^j, and ζ are calibrated jointly because each parameter influences more than one target. We set the following targets to determine these parameters: (i) producer markups of 10% as estimated by Basu and Fernald (1995); (ii) the volatility ratio of 1.17 equal to the median value in our sample; (iii) relative volatility of the real export price p_x to the real exchange rate x of 37% consistent with the data for the US; (iv) marketing expediture share in the GDP of 4.5%, which is halfway between estimates of marketing/sales of 7% reported by Lilien and Little (1976) and advertising/GDP of around 2% reported in Coen (June 2007); (v) the standard value for the share of market activities in the total time endowment of households equal to 30% (Cooley, ed (1995)); and finally (vi) our measure of bilateral trade between two symmetric countries of 0.85% and between a small country and the rest of the world of 19.03%, which are the median values in our sample.

The productivity process is country specific and follows the AR(1) process given by (30). The residuals ε_i of the process are assumed to be normally distributed with zero mean, standard deviation σ_i^2, and correlation coefficients r_{ij}. We set the parameters of the productivity process to be symmetric for the bilateral pair in the model (domestic and foreign) and set it to match the median behavior of real GDP in our sample of countries. For the rest of the world (third country), we set the productivity process so as to match the median behavior of real GDP for

[29]To construct the *volatility ratio* we use constant and current price values of imports and domestic absorption. Denoting the deflator price of domestic absorption by P_{DA} and the deflator price of imports by P_{IM}, the *volatility ratio* is then defined as $\sigma(\frac{IM}{DA})/\sigma(\frac{P_{DA}}{P_{IM}})$, where σ refers to the standard deviation of the logged and Hodrick-Prescott filtered quarterly time series. Notice that the *volatility ratio* places an upper bound on the regression coefficient between the two variables underlying its construction.

the relative rest of the world for country pairs in our sample. Specifically, we set $\psi_D = \psi_F$ and $\sigma_D = \sigma_F$ to match the median autocorrelation and the standard deviation of real GDP in our sample of 0.83 and 1.41%, respectively. For rest of the world, we set ψ_W and σ_W to match the autocorrelation of real GDP and standard deviations of 0.89 and 1.05%, respectively. To set the correlations of innovations, we target the median correlation or real GDPs within our bilateral pairs of 0.52 to set r_{DF} and target the median correlation of real GDPs of our sample countries with their relative rest of the world of 0.66 to set $r_{DW} = r_{FW}$.

To relate the model to the trade-comovement pattern in the data, we impose bilateral tariffs to induce deviations of trade out of the median. Tariffs work similarly to our iceberg cost, except that they are reimbursed back to the households. We target three values for each of our 190 country pairs: (*i*) the bilateral trade intensity as defined in equation (29) and (*ii* and *iii*) imports/GDP from the relative rest of the world of country 1 and country 2 from the pair.

The parameter values in the benchmark calibration are summarized in Table 2.[30]

Frictionless benchmark. For comparison, we report results from a three-country version of the frictionless international business cycle model (Backus et al. (1995)). The frictionless model is parameterized in the same way, whenever applicable. We report the values of the parameters in Table 3.

3.3 Findings

This section presents results from the benchmark model, as well as the predictions of a frictionless three-country business cycle model, and compares them to our empirical findings from

[30]For the details on national accounting in the model, see equation (37). More information can be found in the Online Appendix.

Table 2: Parameter values in the benchmark calibration.

Parameter		Value
	Preference Parameters	
ρ	elasticity of substitution	15
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.3929, 0.2917, 0.3154
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	0.2917, 0.3929, 0.3154
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.3154, 0.3154, 0.3691
ζ	leisure weight in utility	0.346
σ	risk aversion	2
β	time discount factor	0.99
	Technology Parameters	
α	capital share	0.36
δ	depreciation of physical capital	0.025
δ_H	match destruction rate	0.1
χ	search cost	1.13
δ_m	depreciation of marketing capital	0.0149
ϕ	adjustment cost of marketing capital	3.06
θ	bargaining power of producers	0.449
	Other Parameters	
ψ_1, ψ_2, ψ_3	persistence of the productivity shock	$0.54, 0.54, 0.71$
r_{12}	cross-correlation of productivity shocks	0.43
r_{13}, r_{23}	cross-correlation of productivity shocks	0.61, 0.61
L_D, L_F, L_W	population sizes	$1, 1, 20$

Section 3.1. We report regression coefficients based on model-generated data to quantitatively assess the trade-comovement relationship. In the Online Appendix, we additionally report median business cycle moments from our model to confirm the model's overall statisfactory performance vis-á-vis the standard theory.

3.3.1 Trade-comovement link

As described in the previous section, we choose bilateral tariffs to mimic exactly the trade patterns within the bilateral pair and of the pair countries with the rest of the world. The

Table 3: Parameter values in the frictionless model.

Parameter		Value
	Preference Parameters	
ρ	elasticity of substitution	1.17
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.8044, 0.0165, 0.1790
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	0.0165, 0.8044, 0.1790
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.1790, 0.1790, 0.6419
ζ	leisure weight in utility	0.332
σ	risk aversion	2
β	time discount factor	0.99
	Technology Parameters	
α	capital share	0.36
δ	depreciation of physical capital	0.025
	Other Parameters	
ψ_1, ψ_2, ψ_3	persistence of the productivity shock	$0.83, 0.83, 0.90$
r_{12}	cross-correlation of productivity shocks	0.52
r_{13}, r_{23}	cross-correlation of productivity shocks	0.67, 0.67
L_D, L_F, L_W	population sizes	$1, 1, 20$

exercise produces 190 data points of real GDP correlations and trade intensity within the bilateral pair, on which we then run the same regression as in the data.[31]

Table 4 presents results from regressions on model-generated data. The model implies a regression coefficient that is close to the data estimates. It accounts for 40 to 60% of the empirical slope in the overall sample. However, when we perform the split into high- and low-trade subsamples, the model turns out to feature the same kind of nonlinearity as the one we document in the data. In the top 50% of bilateral trade intensity subsample, the model accounts for 70 to 80% of the empirical relation.[32] In contrast, the frictionless model exhibits virtually no trade-comovement relationship.

[31] Including using the EU dummy for EU pairs.

[32] The results from the model differ between OLS and IV because, for the IV parameterization, we used theoretical trade levels implied by the first-stage regression for parameterizing trade.

Table 4: Regression results: data versus the benchmark model.

Dependent Variable: GDP correlation			
Coefficient β_{GDP}	OLS	OLS bottom 50%	OLS top 50%
Data	0.034^{**}	-0.017	0.055^{**}
Model	0.022	0.003	0.044
Model/data ratio	64%	–	81%
Frictionless model/data ratio	-0.3%	–	-1.0%
	IV	IV bottom 50%	IV top 50%
Data	0.065^{***}	0.325	0.070^{**}
Model	0.028	0.003	0.049
Model/data ratio	44%	–	70%
Frictionless model/data ratio	0.2%	–	-0.4%

,* denote significance at the 5% and the 1% levels for the data regression.

Finally, in Table 5, we show that it is indeed the case that disconnecting short-run and long-run elasticities is what is responsible for the resolution of the trade-comovement puzzle in the benchmark model. To that end, we shut down the key friction that disconnects short- and long-run trade elasticities in our setup and consider two parameterizations: one that assumes the elasticity ρ is set to the long-run target ($\rho = 15$), and one that assumes it is equal to our short-run trade elasticity target ($\rho = 1.17$).[33] These two parameterizations correspond to two ways of parameterizing a model with a single elasticity – by either targeting the data estimate of the long-run elasticity or the short-run elasticity.[34] As suggested by the analysis in Section 2, the tariff variation needed to induce the change in trade in the frictionless model

[33]To keep the exercise simple, we keep the other parameters at benchmark values. The numbers and conclusions do not change if we reparameterize the models.

[34]To generate Table 5, we consider two parameterizations of trade intensity. First, we consider a case in which trade with the rest of the world and bilateral trade are set to their medians (19% and 0.85%, respectively). Then, we change the trade costs to match the 90th percentile of the bilateral trade intensity (3.83%). We then report the implied regression coefficients for output and trade cost changes needed to implement the increased trade.

(73%) is an order of magnitude higher than the one needed in the benchmark model (11%). Reducing long-run elasticity to the short-run target brings our model's predictions in line with the frictionless model in terms of the required change in trade cost, and it also implies that the trade-comovement relationship implied by the model is counterfactually negative.

Table 5: Shutting down elasticity disconnect in the benchmark model.

	Implied regression coefficient	Ratio to the data	Required change in trade cost
Benchmark model	0.025	75%	11%
No elasticity disconnect, $\rho = 15$	-0.012	-36%	11%
No elasticity disconnect, $\rho = 1.17$	-0.031	-92%	82%
Frictionless model	0.0005	1.6%	73%

3.3.2 TFP correlations

A number of studies have found that an exogenous or endogenous relation between TFP correlations and trade is an important ingredient of the overall trade-comovement pattern (Kose and Yi (2006), Johnson (2014), and Liao and Santacreu (2015)). Our micro-founded model features endogenous measured TFP fluctuations due to variable markups and marketing capital frictions. In particular, our model delivers a positive trade-comovement relationship for TFP endogenously. Below, we provide empirical estimates of the trade-comovement relationship and report the estimates from the model. We then provide a discussion of the sources of the endogenous TFP movements in the model.

We use data on real GDP, capital and labor in order to construct measures of TFP for our 190 country pairs and then run the same cross-country regression as for real GDP correlations. The results are presented in Table 6.[35] Results from model-based regressions are presented in

[35]Both OLS and IV specifications give highly significant positive coefficients on bilateral trade, implying

Table 7. Our benchmark accounts for 50% to 60% of the empirical slope in the overall sample and 60% to 70% in the high-trade sample. We next discuss the sources of endogenous TFP movements in our model.

Table 6: Regression results: trade-comovement in the data.

Dependent Variable: TFP correlation			
	OLS	OLS bottom 50%	OLS top 50%
$trade_{ij}$	0.029**	−0.021	0.062***
	(0.013)	(0.032)	(0.018)
E_{ij}	0.011	0.170	−0.119
	(0.087)	(0.187)	(0.104)
R-squared	0.697	0.678	0.755
	IV	IV bottom 50%	IV top 50%
$trade_{ij}$	0.044**	0.180	0.053**
	(0.020)	(0.240)	(0.026)
E_{ij}	−0.033	−0.232	−0.104
	(0.103)	(0.537)	(0.109)

,* denote significance at the 5% and the 1% levels. Numbers in parentheses are standard errors.

Measured TFP in the quantitative model is different than the assumed productivity process A. The effect is subtle and comes from the specifics of the national accounting procedures. Specifically, in the data, TFP residuals are measured by subtracting the log of payments to labor and capital from the log of final output (real GDP). Since final output excludes intermediate inputs, and marketing expenditures are classified that way by the national accounting procedures System of National Accounts (SNA (1993)), TFP goes up when less marketing is needed for a given level of production/sales, as is the case after a positive productivity shock

that moving from the 10th to the 90th percentile of the bilateral trade distribution increases TFP correlations by 0.19 (IV) and 0.15 (OLS), which is high relative to a median TFP correlation of 0.44 in our sample. As with real GDP, the trade-comovement relationship is stronger in the high trade subsample.

Table 7: Regression results: data versus models.

Dependent Variable: TFP correlation			
Coefficient β_{TFP}	OLS	OLS bottom 50%	OLS top 50%
Data	0.029^{**}	-0.021	0.062^{***}
Model	0.018	0.003	0.036
Model/data ratio	62%	–	59%
Frictionless model/data ratio	–	–	–
	IV	IV bottom 50%	IV top 50%
Data	0.044^{**}	0.18	0.053^{**}
Model	0.023	0.003	0.039
Model/data ratio	52%	–	74%
Frictionless model/data ration	–	–	–

**,*** denote significance at the 5% and the 1% levels for the data regression.

abroad.

More specifically, real GDP in our model is given by

$$\begin{aligned} rGDP &= L_D(P_d d + P_f f + P_g g) + \sum_{i=F,W} x_i p_i^d L_i d_i - p_D^f L_D f - p_D^g L_D g \\ &\quad + v_D(a_D^f + a_D^g) - x_F v_F a_F^d - x_W v_W a_W^d, \end{aligned} \tag{37}$$

where all the prices are evaluated at their steady-state levels.

The above definition follows the actual national accounting procedures in our dataset and hence does not include domestic marketing expenditures a_D^d and the search effort of the retailer h. To obtain measured TFP, we use the resource constraint,

$$Af(K,L) = a_f^D + a_g^D + a_d^D + \sum_{i=D,F,W} L_i d_i + \chi h,$$

and calculate the Solow residual as $TFP = \frac{rGDP}{f(k,l)}$. The formula we obtain shows that the ratio

of (measured productivity TFP)/(exogenous shock A) in the model is

$$\Lambda \equiv \frac{rGDP}{a_f^D + a_g^D + a_d^D + \sum_{i=D,F,W} L_i d_i + \chi h},$$

where $rGDP$ is given by (37).

In the frictionless model, $P = p$, $h = 0$ and $a = 0$ so that Λ is a constant. In the benchmark model, Λ moves for three reasons: (i) shifts between d_D and d_W, d_F and the terms coming from net exports move Λ, because of differences in steady-state markups between retail and producer prices; (ii) movements of production between marketing investment a_i^j and goods production d_i; and (iii) shifts between search h and physical production d_i.

Table 8 quantifies the contribution of each of these components to the endogenous movements of the TFP defined by deviations of Λ_t from 1. In column 1, we report the overall equilibrium variance of Λ_t for the benchmark parameterization of around 1%. We then compute the counterfactual variance of Λ_t when we shut off all marketing investment (a_i^j) and search channels (h). The variance in this case comes solely due to the existence of retail-producer markups and amounts to only 0.2% (column 2). If we then turn on the marketing investment channel (a_i^j), but not the search channel (h), the variance of Λ goes up to 0.91% (column 3). The difference between column 3 and 1 is the contribution of the search channel.

Table 8: Sources of endogenous TFP movements.

	Benchmark	Markups Only	Markups + Marketing a_i^j
Volatility of Λ_t	1%	0.2%	0.91%

Exploring sectoral predictions of the model is beyond the scope of our paper, but we

conclude that our mechanism has the potential to relate to patterns documented in Johnson (2014) if the markups or the importance of marketing expenditures across sectors was allowed to vary across sectors.

4 Conclusions

We have shown how modeling the dynamics of trade elasticity can resolve the trade-comovement puzzle. The broader lesson from our analysis is that it cautions against drawing conclusions from models relying on a single (static) trade elasticity. The reason this is important is because it affects how we parameterize such models, which, as our analysis shows, crucially affects shock transmission. More specifically, whenever trade patterns between countries are taken into account, trade costs are calibrated to ensure that the level of trade in the model is consistent with the data. Across countries, this makes them an implicit function of trade and the assumed long-run elasticity. The calibrated level of trade costs, as we have shown, is not neutral for shock transmission in macroeconomic models.

References

Anderson, James E and Eric van Wincoop, "Trade Costs," Technical Report, National Bureau of Economic Research 2004.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, *International Business Cycle: Theory and Evidence*, In T. Cooley, Frontiers of Business Cycle Research, Princeton University Press, 1995.

Backus, D.K., P.J. Kehoe, and F.E. Kydland, "International Real Business Cycles," *Journal of Political Economy*, 1992, *100* (4), 174–1775.

Basu, S. and J. G. Fernald, "Are Apparent Productive Spillovers a Figment of Specification Error?," *Journal of Monetary Economics*, 1995, *36*, 165–188.

Baxter, M. and M. A. Kouparitsas, "Determinants of Business Cycle Comovement: A Robust Analysis," *Journal of Monetary Economics*, 2005, *52*, 113 – 157.

BIS, "Triennial Central Bank Survey," 2016. Bank for International Settlements.

Blonigen, B. A. and W. W. Wilson, "Explaining Armington: What Determines Substitutability Between Home and Foreign Goods," *Canadian Journal of Economics*, 1999, *32* (1), 1–22.

Calderon, C. A., A. E. Chong, and E. H. Stein, "Trade Intensity and Business Cycle Synchronization: Are Developing Countries Any Different?," *Manuscript, Inter-American Development Bank*, 2002.

Clark, T. E. and E. van Wincoop, "Borders and Business Cycles," *Journal of International Economics*, 2001, *55*, 59–85.

Clausing, K. A., "Trade Creation and Trade Diversion in the Canada-United States Free Trade Agreement," *Canadian Journal of Economics*, 2001, *34* (3), 677–696.

Coen, R., "Insider's Report," *UniversalMcCann*, June 2007.

Cole, Harold L. and Maurice Obstfeld, "Commodity Trade and International Risk Sharing: How Much do Financial Markets Matter?," *Journal of Monetary Economics*, 1991, *28* (1), 3–24.

Cooley, T., ed., *Frontiers of Business Cycle Research*, Princeton University Press, 1995.

diGiovanni, Julian and Andrei Levchenko, "Putting the Parts Together: Trade, Vertical Linkages and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, 2010.

Drozd, Lukasz A. and Jaromir B. Nosal, "Understanding International Prices: Customers as Capital," *American Economic Review*, 2012, *102* (1), 364–395.

Eaton, J. and S. Kortum, "Technology, Geography, and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Eaton, Jonathan, Marcela Eslava, Maurice Kugler, and James Tybout, "Export Growth in Colombia: Firm-Level Evidence," in "The Organization of Firms in the Global Economy," Elhanan Helpman, Dalia Marin, and Thierry Verdier, eds., 2008.

Egan, M. L. and A. Mody, "Buyer Seller Links in Export Development," *World Development*, 1993, *20* (3), 321–334.

Frankel, J. A. and A. Rose, "The Endogeneity of the Optimum Currency Area Criteria," *Economic Journal, Royal Economic Society*, 1998, *108* (449), 1009–25.

Hakansson, H., *International Marketing and Purchasing of Industrial Goods*, IMP Project Group, Wiley and Sons, 1982.

Head, K. and J. Ries, "Increasing Returns Versus National Product Differentiation as an Explanation for the Pattern of US-Canada Trade," *The American Economic Review*, 2001, *91* (4), 858–876.

Inklaar, Robert, Richard Jong-A-Pin, and Jakob de Haan, "Trade and Business Cycle Synchronization in OECD Countries – A Re-examination," *European Economic Review*, 2008, *52* (4).

Johnson, Robert C., "Trade in Intermediate Inputs and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, 2014, *6* (4).

Kose, M. A. and K-M. Yi, "Can the Standard International Business Cycle Model Explain the Relation between Trade and Comovement?," *Journal of International Economics*, 2006, *68* (2), 267–295.

Liao, Wei and Ana Maria Santacreu, "The Trade Comovement Puzzle and the Margins of International Trade," *Journal of International Economics*, 2015, *forthcoming.*

Lilien, G. L. and J. D. C. Little, "The ADVISOR Project: A Study of Industrial Marketing Budgets," *Sloan Management Review*, 1976, *17*, 17–31.

Otto, G., G. Voss, and L. Willard, "Understanding OECD Output Correlations," *RDP 2001-05, Reserve Bank of Australia*, 2001.

Reinert, K. A. and D. W. Roland-Holst, "Armington Elasticities for United States Manufacturing Sectors," *Journal of Policy Modeling*, 1992, *14* (6), 631–639.

Romalis, John, "NAFTA's and CUSFTA's Impact on International Trade," *Review of Economics and Statistics*, 2007, *89* (3), 416–435.

Ruhl, K. J., "The International Elasticity Puzzle," *University of Texas-Austin*, 2008, *unpublished manuscript.*

SNA, "System of National Accounts 1993," 1993. United Nations Statistics Division, OECD, World Bank, IMF, Commission of the European Communities.

Turnbull, P.W. and M. T. Cunningham, *International Marketing and Purchasing*, Palgrave Macmillan (April 1981), 1981.

Yi, K., "Can Vertical Specialization Explain the Growth of World Trade?," *Journal of Political Economy*, 2003, pp. 52–102.

Appendix

A Prototype model from Section 2

A.1 Equilibrium conditions and log-linearization

Here we derive equilibrium conditions for the model and log-linearize them to characterize model dynamic implications. Equilibrium comprises prices $p(s^t)$, $w(s^t)$, $w^*(s^t)$ and allocation $y(s^t)$, $l(s^t)$, $d(s^t)$,$f(s^t)$, $y^*(s^t)$, $l^*(s^t)$, $d^*(s^t)$, and $f^*(s^t)$, such that conditions (38) - (46) below hold.

Equilibrium conditions. The equilibrium must satisfy zero profit conditions,

$$w(s^t) = A(s^t) \tag{38}$$
$$w^*(s^t) = A^*(s^t), \tag{39}$$

feasibility conditions

$$d(s^t) + d^*(s^t)(1+\tau) = y(s^t) = A(s^t)l(s^t) \tag{40}$$
$$f^*(s^t) + f(s^t)(1+\tau) = y^*(s^t) = A^*(s^t)l^*(s^t), \tag{41}$$

and the first-order conditions derived from the home and foreign household problems, which imply

$$\frac{\partial[\log G(d(s^t), f(s^t)) - \Phi(d(s^t), f(s^t)) - l(s^t)]}{\partial d(s^t)} = A(s^t)^{-1} \tag{42}$$
$$\frac{\partial[\log G(d(s^t), f(s^t)) - \Phi(d(s^t), f(s^t)) - l(s^t)]}{\partial f(s^t)} = p(s^t)(1+\tau)A(s^t)^{-1} \tag{43}$$
$$\frac{\partial[\log G(f^*(s^t), d^*(s^t)) - \Phi(f^*(s^t), d^*(s^t)) - l^*(s^t)]}{\partial f^*(s^t)} = A^*(s^t)^{-1} \tag{44}$$
$$\frac{\partial[\log G(f^*(s^t), d^*(s^t)) - \Phi(f^*(s^t), d^*(s^t)) - l^*(s^t)]}{\partial d^*(s^t)} = (p\left(s^t\right) A^*(s^t))^{-1}(1+\tau), \tag{45}$$

and

$$\frac{\partial[\log G(d(s^t), f(s^t)) - \Phi(.,.)] - l^*(s^t)}{\partial d(s^t)}(1+\tau) = \frac{\partial[\log G(f^*(s^t), d^*(s^t)) - \Phi(.,.) - l^*(s^t)]}{\partial d^*(s^t)}. \tag{46}$$

The last condition determines risk sharing (risk-sharing condition hereafter). It implies that home and foreign households trade assets to, in effect, equalize the utility from each type of good across the border, net of trade costs of moving these goods across the border and the convex adjustment cost. To derive this particular formulation of the risk-sharing condition, we use the first-order conditions with respect to $B(s^{t+1})$ and $B^*(s^{t+1})$, which implies $\lambda(s^t) = \lambda^*(s^t)$ (the shadow value of good d at home and abroad must be equal in equilibrium), and combine it with the the first-order conditions with respect to d at home and d^* abroad to substitute our for the Lagrange multipliers. (Details can be found in the Online Appendix E.) Equivalently, the risk-sharing condition can be expressed as $p(s^t) = \frac{A^*(s^t)}{A(s^t)} \frac{u_l(s^t)}{u_l^*(s^t)}$, where the last terms represent the relatively marginal value of leisure across countries – which in the case of the separable utility function we use in text equals 1, implying

$p(s^t) = \frac{A(s^t)}{A^*(s^t)}$.

We augment the system defined by equations (40)-(46) by introducing an auxiliary variable R that measures risk-sharing transfers between countries as defined in (18). Since we want to substitute out for B, B^*, we use the home country's budget constraint in (7) to rewrite the condition for R in (18) as

$$R(s^t) = d(s^t) + f(s^t)(1+\tau) - w(s^t)l(s^t), \tag{47}$$

and include this equation as an equilibrium condition rather than the definition (18) that depends on Bs. Summarizing, the complete equilibrium system that isolates out risk-sharing transfers defined by R comprises (40)-(47).

As noted in text, risk-sharing transfers R are zero-sum across countries, implying the following budget constraint abroad: $d^*(1+\tau) + f^* = A^* l^* - R$, where, note, (47) represents the home country's budget constraint. To derive the above equation, we combine the foreign country's budget constraint (8) with the definition of R (18) and the feasibility condition for good f in (41) to substitute out for f. See the Online Appendix for more detailed derivations and the Mathematica notebook available online.

Steady state. We next calculate the deterministic steady state in the model. We do so by assuming $A = A^* = 1$ and, importantly, by setting $\Phi = 0$: $\bar{d} = \frac{1}{(\tau+1)^{1-\rho}+1}$, $\bar{f} = \frac{1}{(\tau+1)^{\rho}+\tau+1})^{-1}$, $l = 1$, $R = 0$.

Log-linearization. We linearize the system (40)-(47) with respect to R (which is zero in the steady-state) and log-linearize it with respect to other variables. We use the fact that $\bar{x} = \bar{f}(1+\tau)/(\bar{d} + \bar{f}(1+\tau))$ to link trade to trade costs in equation (11). We substitute out for trade cost τ throughout to obtain:

$$\hat{y} = \left(\frac{2\rho\,(\bar{x}-1)^2\,\bar{x}^2}{(1-\bar{x})\bar{x} + 2\rho\phi} + 2\bar{x}^2 - 2\bar{x} + 1 \right) \hat{A} + \left(\frac{2\,(\bar{x}-1)\,\bar{x}\,((\rho-1)\,(\bar{x}-1)\,\bar{x} + 2\rho\phi)}{(\bar{x}-1)\,\bar{x} - 2\rho\phi} \right) \hat{A}^* \tag{48}$$

$$R = \left(\frac{\bar{x}\,(2\rho\phi\,(2\bar{x}-1) + (\bar{x}-1)\,\bar{x}\,(2(\rho-1)\bar{x} - 2\rho + 1))}{(\bar{x}-1)\,\bar{x} - 2\rho\phi} \right) \hat{A} + \bar{x} \left(\frac{2\rho\bar{x}\,(\bar{x}-1)^2}{-\bar{x}^2 + \bar{x} + 2\rho\phi} + 2\bar{x} - 1 \right) \hat{A}^*. \tag{49}$$

See Mathematica notebook online for the complete list of linearized conditions.

A.2 Decomposition (15)-(17)

To derive decomposition (15)-(17), we start from the home country household problem in (55) and take first-order conditions with respect to d, f and l, plug in $w = A$ from (38), and similarly substitute out for the Lagrange multiplier λ using first-order conditions with respect to leisure l and l^*, respectively. That is, we use equations (42)-(43) derived in Section A.1, add equation (1) to define home country output, and also add the redefined relation for R in (47). We log-linearize this system with respect to A, y, d, f, l and p, and linearize it with respect to R as before (since it is zero in the steady state). After plugging in for $\tau(\bar{x})$ from (11), we obtain:

$$\hat{y} = -\hat{p}\bar{x} + \hat{A} - R.$$

The first equation corresponds to equation (15) of the decomposition. Since this equation depends on the equilibrium response of p and R, we add equation (49) derived in Section A.1 to complete the system, which gives equation (17) in the decomposition. We then add the equation for p by following the derivation in Section A.1 but without the risk-sharing condition in (46), under the assumption that the risk-sharing transfer R is an exogenous process. More precisely, we linearize the system composed of equations (40)-(45) and equation (47), and solve it in terms of $\hat{A}$, $\hat{A}^*$ and R (instead of only $\hat{A}$, $\hat{A}^*$ as in Section A.1). Equation (16) in the decomposition corresponds to the equation for p in this system. Finally, we obtain the following coefficients of decomposition (15)-(17):

$$\begin{aligned} \alpha &= 1 \qquad \eta = -\bar{x} \qquad \chi = -1 \\ \pi &= \frac{(\bar{x}-1)\,\bar{x} - 2\rho\phi}{2\bar{x}\left((\rho-1)\bar{x}^2 - 2\rho\bar{x} + \bar{x} + 2\rho\phi + \rho\right)} \qquad \theta = \frac{(\bar{x}-1)\,\bar{x} - 2\rho\phi}{2\bar{x}^2\left((\rho-1)\bar{x}^2 - 2\rho\bar{x} + \bar{x} + 2\rho\phi + \rho\right)} \\ \mu &= \bar{x}\left(\frac{2\rho\bar{x}\,(\bar{x}-1)^2}{-\bar{x}^2 + \bar{x} + 2\rho\phi} + 2\bar{x} - 1\right). \end{aligned} \tag{50}$$

Plugging in $\phi = 0$ and simplifying, we obtain

$$\pi = -\frac{1}{2}(\rho - (\rho-1)\bar{x})^{-1} \qquad \theta = \pi/\bar{x} \qquad \mu = -\bar{x}(1 + 1/\pi). \tag{51}$$

It is easy to verify that the above system is consistent with the one derived in Section A.1. More detailed derivations can be found in the Mathematica notebook available online.

A.3 Omitted proofs

Some of the lengthy algebraic manipulations have been omitted. Detailed derivations can be found in our Mathematica notebook available online.

Proof. [Lemmas 2 and 3] The result follows from the evaluation of the coefficients of the decomposition (15)-(17) derived in Section A.2. Specifically, see equation (51), which we derived by plugging in $\phi = 0$ to (50) and simplifying the resulting expressions. The last part follows from plugging in the values of the coefficients of the decomposition to definitions in (22)-(24) and (25). ■

Proof. [Proposition 1] Using definitions (22)-(25), decomposition (15)-(17), and Lemmas 2 and 3, we derive $\mathcal{S}(\phi = 0) = 2\bar{x}(1-\bar{x})(1-\rho)$ and

$$\mathcal{L}(\phi = 0) := \frac{d\mathcal{S}}{d\bar{x}} = 2(1-2x)(1-\rho).$$

It is clear that $\mathcal{L}(\phi = 0)$ is negative for all $0 < x < 1/2$, $\rho > 1$ as required by Assumptions 1-2. ■

Proof. [Lemma 4] The result follows from the coefficients derived in (50). The rest follows from plugging in (50) to (22)-(24) and (12), which gives the following expression for $\mathcal{L}_R$:

$$\mathcal{L}_R(\phi) = \frac{8\rho^3\phi^2\,(1-2\bar{x})}{\left(-\bar{x}^2 + \bar{x} + 2\rho\phi\right)^2} + 4(\rho-1)\bar{x} - 2\rho + 1.$$

After taking the limit $\phi \to \infty$, it can be verified that $\lim_{\phi\to\infty} \mathcal{L}_R(\phi) = 1 - 4\bar{x}$. ■

Proof. [Proposition 2] Using definitions (22)-(25), decomposition (15)-(17), and Lemma 4 (or coef-

ficients derived in Section A.2), we derive

$$\mathcal{S}(\phi) = \bar{x}\left(\frac{2\rho\bar{x}\left(\bar{x}-1\right)^2}{\left(\bar{x}-1\right)\bar{x}-2\rho\phi} - 2\bar{x} + 2\right),$$

and given (12), also

$$\mathcal{L}(\phi) = \frac{8\rho^2\phi^2\left(1-2\bar{x}\right) - 8(\rho-1)\rho\phi\left(\bar{x}-1\right)\bar{x}\left(2\bar{x}-1\right) + 2(\rho-1)\left(\bar{x}-1\right)^2\bar{x}^2\left(2\bar{x}-1\right)}{\left(\left(\bar{x}-1\right)\bar{x}-2\rho\phi\right)^2}.$$

Accordingly,

$$sign(\mathcal{L}(\phi)) \quad = \quad sign(8\rho^2\phi^2\left(1-2\bar{x}\right) - 8(\rho-1)\rho\phi\left(\bar{x}-1\right)\bar{x}\left(2\bar{x}-1\right) + 2(\rho-1)\left(\bar{x}-1\right)^2\bar{x}^2\left(2\bar{x}-1\right)).$$

It is easy to verify that the first derivative of the expression under $sign(.)$ with respect to ϕ, $8\rho\left(1-2\bar{x}\right)\left((\rho-1)\left(1-\bar{x}\right)\bar{x}+2\rho\phi\right)$, is strictly positive on the restricted domain by Assumptions 1-2 ($x \leq 1/2$, $\rho > 1$), and that limit as $\phi \to +\infty$ is $+\infty$. Given the function is smooth with respect to ϕ, this ensures the existence of a critical value on $\bar{\phi}$ such that for all $\phi > \bar{\phi}$ we have $sign(\mathcal{L}(\phi)) > 0$.

To calculate a lower bound for $\bar{\phi}$, we proceed as follows. We first note that $\mathcal{L}$ is strictly decreasing in ρ on the admissible domain by evaluating the partial derivative $\mathcal{L}$ wrt ρ. The derivative is

$$\frac{\partial\mathcal{L}}{\partial\rho} = -\frac{2\left(1-\bar{x}\right)^2\bar{x}^2\left(1-2\bar{x}\right)\left(\left(1-\bar{x}\right)\bar{x}+6\rho\phi\right)}{\left(\left(1-\bar{x}\right)\bar{x}+2\rho\phi\right)^3} > 0.$$

Hence, for any values of the parameters, we know that $sign(\lim_{\rho\to\infty}\mathcal{L}) \leq sign(\mathcal{L})$, implying that we can use the equation $\lim_{\rho\to\infty}\mathcal{L} = 0$ to calculate the sufficient lower bound for $\bar{\phi}$. Here recall that we previously found that $sign(\mathcal{L})$ is globally increasing in ϕ. The calculation of the limit gives

$$\lim_{\rho\to\infty}\mathcal{L} = \frac{2\left(1-2\bar{x}\right)\left(\phi-(1-\bar{x})\bar{x}\right)}{\phi},$$

and setting it equal to zero gives $\bar{\phi} = \bar{x}(1-\bar{x})$. ■

Long-run Trade Elasticity and the Trade-Comovement Puzzle

by Lukasz Drozd, Sergey Kolbin and Jaromir Nosal

ONLINE APPENDIX: NOT FOR PUBLICATION

A Relation of $\mathcal{S}$ to the correlation coefficient

To a first-order approximation, we have shown in Section A.1 that our model implies the following dynamic system:

$$\begin{aligned} \hat{y} &= (1-\mathcal{S})\hat{A} + \mathcal{S}\hat{A}^* \\ \hat{y}^* &= (1-\mathcal{S})\hat{A}^* + \mathcal{S}\hat{A}, \end{aligned}$$

where

$$0 < \mathcal{S} < 1/2.$$

(Note that the coefficients in (48) add up to one and that $\mathcal{S}$ corresponds to the one on $\hat{A}^*$ (to a first-order approximation).) For simplicity, normalize the variance of symmetric shock,

$$var(\hat{A}^*) = var(\hat{A}) = 1,$$

and note that

$$0 \leq cov(\hat{A}, \hat{A}^*) = corr(\hat{A}, \hat{A}^*) \leq 1.$$

Define

$$corr(y, \hat{y}) = \frac{cov\,(y, \hat{y}^*)}{var\,(\hat{y})},$$

and derive

$$\begin{aligned} cov\,(\hat{y}, \hat{y}^*) &= cov((1-\mathcal{S})\hat{A} + \mathcal{S}\hat{A}^*, (1-\mathcal{S})\hat{A}^* + \mathcal{S}\hat{A}) = \\ &= \left((1-\mathcal{S})^2 + \mathcal{S}^2\right) cov(\hat{A}, \hat{A}^*) + 2(1-\mathcal{S})\mathcal{S}, \end{aligned}$$

and

$$var\,(\hat{y}^*) = var\,(\hat{y}) = var((1-\mathcal{S})\hat{A} + \mathcal{S}\hat{A}^*) = ((1-\mathcal{S})^2 + \mathcal{S}^2 + 2(1-\mathcal{S})\mathcal{S}corr(\hat{A}, \hat{A}^*))$$

to obtain

$$corr\,(\hat{y}, \hat{y}^*) = \frac{\left((1-\mathcal{S})^2 + \mathcal{S}^2\right) corr(\hat{A}, \hat{A}^*) + 2(1-\mathcal{S})\mathcal{S}}{(1-\mathcal{S})^2 + \mathcal{S}^2 + 2(1-\mathcal{S})\mathcal{S}corr(\hat{A}, \hat{A}^*)}.$$

Observe that the above expression is strictly decreasing in $\mathcal{S}$ given Assumptions 1 and 2, since

$$\frac{\partial corr\left(\hat{y},\hat{y}^{*}\right)}{\partial \mathcal{S}}=-\frac{2(1-corr(\hat{A},\hat{A}^{*})^{2})(2\mathcal{S}-1)}{(1-2(1-corr(\hat{A},\hat{A}^{*}))(1-\mathcal{S})\mathcal{S}))^{2}}<0,$$

and hence that the correlation coefficient is strictly increasing in $\mathcal{S}$. For detailed derivations of the above expressions, refer to our *Mathematica* notebook available online.

B Alternative definition of trade in Section 2

Here we consider the polar case of all trade costs being explicit; that is, we assume that

$$x=\frac{f}{d+f(1+\tau)}.$$

In this case, gravity applies globally, and there is no restriction on ρ, although we will establish our main result by restricting $\rho>1$.

It is not possible to explicitly solve for $\tau(\bar{x})$ and hence we use implicit differentiation to calculate

$$\frac{d\tau(\bar{x})}{d\bar{x}}=\frac{\tau+1}{\bar{x}\left((\rho-1)(\tau+1)\bar{x}-\rho\right)}.$$

Note that the inverse of this expression is

$$\frac{d\bar{x}}{d\tau}=\frac{\bar{x}\left((\rho-1)(\tau+1)\bar{x}-\rho\right)}{\tau+1},$$

which implies that for all values of ρ we have a negative relation between trade cost and trade, and hence the restriction introduced in Assumption 1 no longer applies here. The comovement coefficient S in this case depends on both τ and $\bar{x}$, and so to capture the combined effect of trade we use the expression:

$$S(\bar{x},\tau)=(\tau+1)\bar{x}\left(\frac{2\rho\phi\left(2(\tau+1)\bar{x}-1\right)+(\tau+1)\bar{x}\left(\tau\bar{x}+\bar{x}-1\right)\left(2(\rho-1)(\tau+1)\bar{x}-2\rho+1\right)}{(\tau+1)\bar{x}\left(\tau\bar{x}+\bar{x}-1\right)-2\rho\phi}+1\right).$$

Accordingly, we define the trade-comovement link $\mathcal{L}$ as

$$\mathcal{L}=\frac{\partial S(\tau,\bar{x})}{\partial\bar{x}}+\frac{\partial S(\tau,\bar{x})}{\partial\tau}\frac{d\tau}{d\bar{x}}.$$

As is clear from the formula for S, analyzing this model is more challenging in this case as no explicit solution can be obtained. To make progress, we show that $\mathcal{L}$ is always negative for $\phi=0$, establishing the trade-comovement puzzle, and then we show that in the limit $\phi\to\infty$ it is strictly positive. Given the continuity of the expression, for sufficiently high ϕ, we conclude that the relationship between trade and comovement is (strictly) positive. It is easy to verify by plotting the function that it occurs for actually low values of ϕ, as shown in Figure 5. The figure plots $\mathcal{L}$ for all values of $\bar{x}$ and all values of ϕ (for $\tau=.38$ and $\rho=10$). We now prove this result formally.

Proposition 3 *For $\phi=0$, we have $\mathcal{L}<0$, implying the trade-comovement puzzle.*

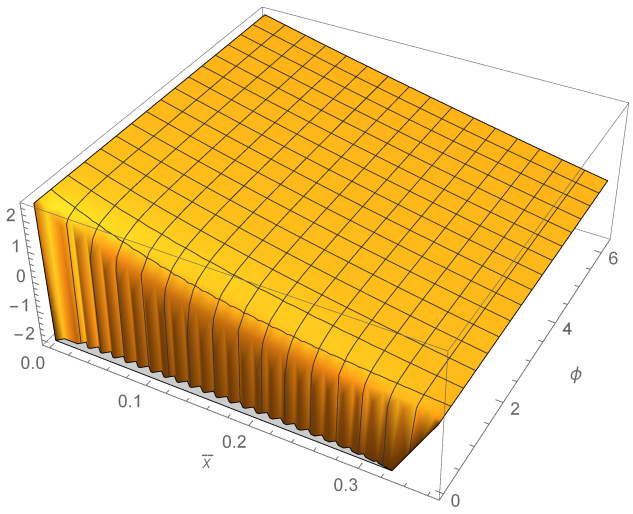

Figure 5: $\mathcal{L}$ as a function of $\bar{x}$ and ϕ, for $\tau = .38$ and $\rho = 10$.

Proof.

Note that after plugging in $\phi = 0$ we have:

$$\mathcal{L} = 2(\rho-1)(\tau+1)\left(2(\tau+1)\bar{x}-1\right)\left(\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}+1\right).$$

The sign of this expression is determined by

$$\begin{aligned} sign(\mathcal{L}) &= sign(-2(\rho-1)\left(\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}+1\right)) \\ &= sign(\frac{2(\rho-1)^2\left(1-(1+\tau)\bar{x}\right)}{(\rho-1)(\tau+1)\bar{x}-\rho}) \\ &= sign((\rho-1)(\tau+1)\bar{x}-\rho) \\ &= sign(\rho((\tau+1)x-1)-(\tau+1)\bar{x}), \end{aligned}$$

which is always negative. ■

Proposition 4 *There exists sufficiently large $\bar{\phi}$ so that for all $\phi > \bar{\phi}$ we have $\mathcal{L} > 0$ for any $\rho > 1$ (as a sufficient condition).*

Proof. Having shown that $\mathcal{L} < 0$ for $\phi = 0$, given the continuity of $\mathcal{L}$ with respect to ϕ, it suffices to show that the limit as $\phi \to \infty$ is strictly positive. We calculate the sign of the limit to obtain the following evaluation

$$\begin{aligned} sign\left(\lim_{\phi\to\infty}\mathcal{L}\right) &= sign\left(\left(\tau+1\right)\left(2-4(\tau+1)\bar{x}\right)\left(\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}+1\right)\right) \\ &= sign\left(1+\frac{1}{(\rho-1)(\tau+1)\bar{x}-\rho}\right), \end{aligned}$$

which is always positive for $\rho > 1$ (sufficient condition, not necessary). This is clearer by rewriting

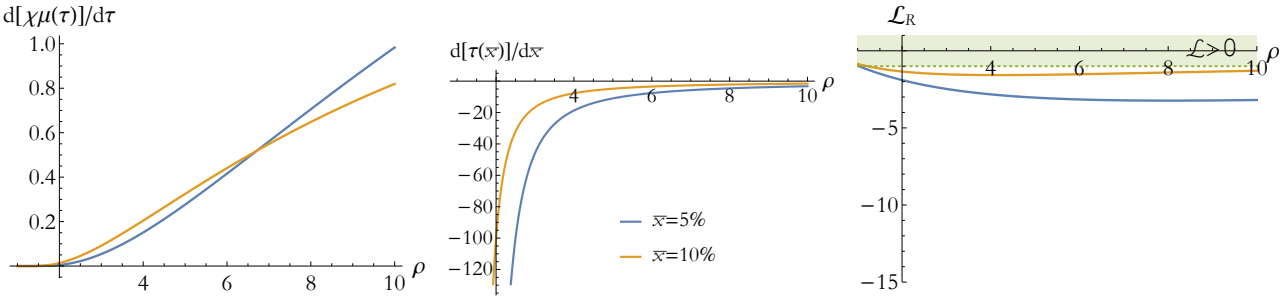

Figure 6: $\mathcal{L}_R = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})} \frac{d\tau}{dx}|_{\tau(\bar{x})}$ as function of elasticity $\rho \geq 1$ for CRRA utility function in (36) ($\sigma = 2$, $\zeta = 1/3$.)

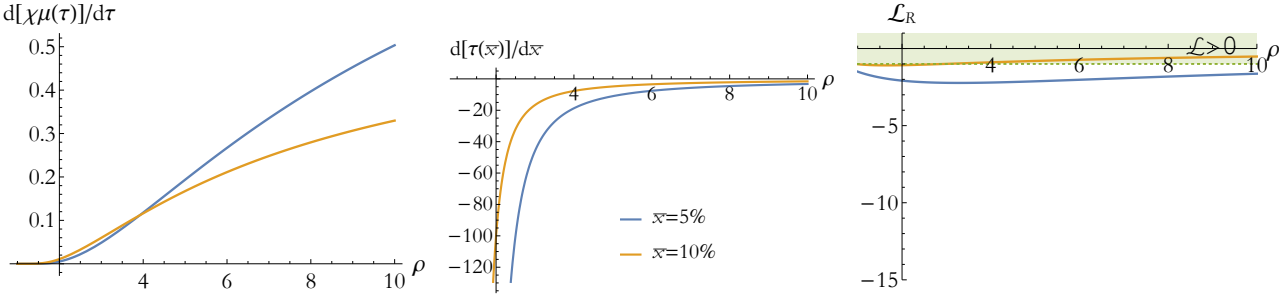

Figure 7: $\mathcal{L}_R = \frac{d(\chi\mu(\tau))}{d\tau}|_{\tau(\bar{x})} \frac{d\tau}{dx}|_{\tau(\bar{x})}$ as a function of elasticity $\rho \geq 1$ for CRRA utility function in (36) ($\sigma = 5$, $\zeta = 1/3$.)

the last term as

$$sign\left(1 - \frac{1}{\rho(1-(\tau+1)\bar{x}) + (\tau+1)\bar{x}}\right) > 0.$$

(It is possible to show that $\mathcal{L} > 0$ is increasing in ϕ for $\rho > 1$ (sufficient condition), and hence that $\mathcal{L}$ crosses zero only once, but we omit it.) ■ (Detailed algebraic derivations can be found in the *Mathematica* notebook available online.)

C Nonseparable utility function in Section 2

Here we characterize the forces behind the trade-comovement puzzle under the assumption that the utility function is as in (36) (with $\sigma > 1$) and $\Phi = 0$. We show that, qualitatively, the results of Section 2 stand, although we note that nonseparability alleviates the puzzle for an settings of bilateral trade intensity ($\bar{x}$) that are not plausible empirically. We derive the formulas and compare the results to those in the paper. We also prove Proposition 3 in the paper.

We begin by redoing Figure 1 from the paper for the utility function in (36) assuming $\sigma = 2$ and $\sigma = 5$ (and $\psi = 1/3$), which is illustrated in Figures 6 and 7. The low risk-aversion case of $\sigma = 2$ implies a negative trade-comovement link, while the high risk-aversion of $\sigma = 5$ implies a negative trade-comovement link for 5% trade intensity and essentially no trade-comovement link for 10%. Overall, nonseparable utility alleviates the puzzle but it does not resolve it.

The decomposition considered in the paper for nonseparable utility function implies the following values of the coefficients:

$$\alpha = 1 \qquad \eta = -(1-\zeta)\bar{x} \tag{52}$$

$$\pi = \frac{1}{2(\bar{x}(\rho+\zeta-1)-\rho)} \qquad \theta = \frac{1-2\zeta\bar{x}}{2\zeta\bar{x}(\bar{x}(\rho+\zeta-1)-\rho)} \tag{53}$$

$$\mu = \frac{\psi\bar{x}(-2\rho((\sigma-1)\zeta+1)+2\bar{x}(\rho(\sigma-1)\zeta+\rho+(\sigma-1)(\psi-1)\zeta-1)+1)}{4\zeta\bar{x}(-\rho\sigma+(\rho-1)\sigma\bar{x}+(\sigma-1)\zeta\bar{x}+1)-1}. \tag{54}$$

It can be verified that:

$$\begin{aligned}\mathcal{L}_R &= ((\psi-1)(-4\zeta\bar{x}^2(2(\rho-1)(\sigma-1)\zeta^2-(4\rho-3)(\sigma-1)\zeta+(\rho-1)(\sigma-2))- \\ &\quad 4\bar{x}(\rho(\sigma-1)\zeta+\rho+(\sigma-1)(\zeta-1)\zeta-1)+2\rho((\sigma-1)\zeta+1)-1))/ \\ &\quad (4\zeta\bar{x}\left(\bar{x}(\zeta-\sigma(\rho+\zeta-1))+\rho\sigma-1\right)+1)^2\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}_C &= (\zeta(4(\zeta-1)\bar{x}(\bar{x}((\sigma-1)\zeta^2((2\rho-1)\sigma-1)+ \\ &\quad \sigma\zeta(\sigma-3\rho(\sigma-1))-\rho\sigma+\sigma-\zeta)+(\sigma-1)(\zeta-1))+\sigma(-\zeta)+\sigma+\zeta-2)+1)/ \\ &\quad (4\zeta\bar{x}\left(\bar{x}(\zeta-\sigma(\rho+\zeta-1))+\rho\sigma-1\right)+1)^.\end{aligned}$$

Taking the limit $\bar{x} \to 0$, we obtain (27). This implies that the trade-comovement puzzle arises for low levels of trade for all parameter values. Figure 4 shows in detail the negative parameter-region of $\mathcal{L}$ for the two settings of σ.

Finally, Figure 8 plots all the coefficients of the decomposition and compares them side-by-side between the two specifications of the utility function. It is clear that, qualitatively, the relations are identical. The only difference is their relative magnitude, in particular the much weaker risk-sharing channel's connection to trade (slope of $\chi\mu$ in the bottom-left panel). But, this turns out to be a double-edged sword as far as the trade-comovement puzzle goes. On the one hand, it reduces the direct effect of risk sharing, but it also reduces its adverse effect on the complementarity channel through its indirect effect, making it weaker through $\eta\theta\mu$ (slope of bottom-right panel). Overall, nonseparability alleviates the puzzle but does not resolve it, as the puzzle still applies to empirically relevant ranges of bilateral trade intensity (e.g., 10% or lower).

(Derivations of the above expressions can be find in the Mathematica notebook available online.)

D Derivation of equilibrium system (38)-(46)

The Lagrangian of the domestic country household is

$$\begin{aligned}L &= \sum_{s^t} \Pr(s^t)\beta^t[\log G(d(s^t), f(s^t)) - \Phi(d(s^t), f(s^t)) - l(s^t) \\ &\quad - \lambda(s^t)((d(s^t)+p(s^t)f(s^t)(1+\tau)+Q(s^{t+1})B(s^{t+1})-w(s^t)l(s^t)-B(s^t))],\end{aligned} \tag{55}$$

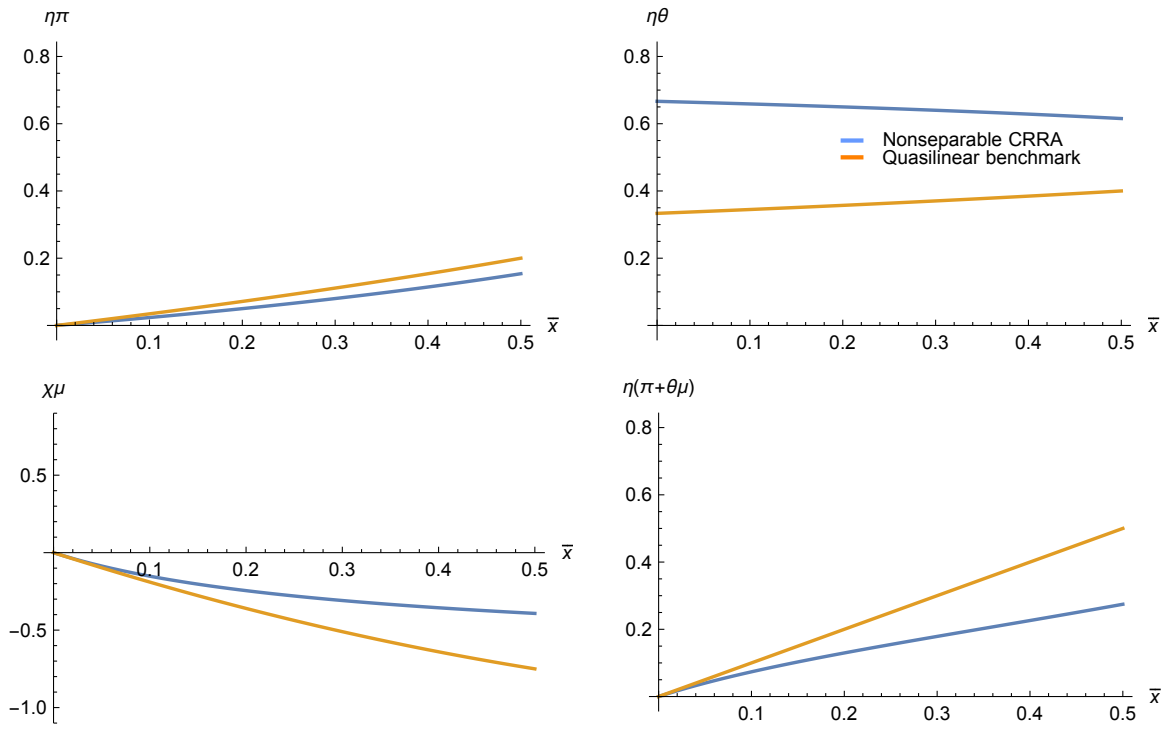

Figure 8: The negative region of the trade-comovement link $\mathcal{L}$ for nonseparable CRRA utility function, $\sigma = 2, \zeta = 1/3$ (left panel) and $\sigma = 5, \zeta = 1/3$ (right panel).

and the Lagrangian of the foreign country household is

$$\begin{aligned} L^* &= \sum_{s^t} \Pr(s^t)\beta^t[\log G(f^*(s^t), d^*(s^t)) - \Phi(f^*(s^t), d^*(s^t)) - l^*(s^t) \\ &\quad - \lambda^*(s^t)((p(s^t)f^*(s^t) + d^*(s^t)(1+\tau) + Q(s^{t+1})B^*(s^{t+1}) - p(s^t)w^*(s^t)l^*(s^t) - B^*(s^t))]. \end{aligned} \tag{56}$$

The equilibrium must satisfy the first-order conditions of the domestic country household Lagrangian with respect to $d(s^t), f(s^t), l(s^t), B(s^{t+1})$, and $\lambda(s^t)$, an analogous set of conditions for the foreign country, zero profit conditions,

$$w(s^t) = A(s^t) \tag{57}$$

$$w^*(s^t) = A^*(s^t), \tag{58}$$

and feasibility conditions

$$d(s^t) + d^*(s^t)(1+\tau) = y(s^t) = A(s^t)l(s^t) \tag{59}$$

$$f^*(s^t) + f(s^t)(1+\tau) = y^*(s^t) = A^*(s^t)l^*(s^t). \tag{60}$$

We use the first-order conditions for l and l^* to drop Lagrange multipliers, which, using (57) and

(58), are

$$\begin{aligned}\lambda(s^t) &= A(s^t)^{-1} \\ \lambda^*(s^t) &= (A^*(s^t)p(s^t))^{-1},\end{aligned}$$

and note that the remaining first-order conditions with respect to d,f,d^*,f^* imply

$$\frac{\partial[\log G(d(s^t), f(s^t)) - \Phi(d(s^t), f(s^t)) - l(s^t)]}{\partial d(s^t)} = A(s^t)^{-1} \tag{61}$$

$$\frac{\partial[\log G(d(s^t), f(s^t)) - \Phi(d(s^t), f(s^t)) - l(s^t)]}{\partial f(s^t)} = p(s^t)(1+\tau)A(s^t)^{-1} \tag{62}$$

$$\frac{\partial[\log G(f^*(s^t), d^*(s^t)) - \Phi(f^*(s^t), d^*(s^t)) - l^*(s^t)]}{\partial f^*(s^t)} = A^*(s^t)^{-1} \tag{63}$$

$$\frac{\partial[\log G(f^*(s^t), d^*(s^t)) - \Phi(f^*(s^t), d^*(s^t)) - l^*(s^t)]}{\partial d^*(s^t)} = (p\left(s^t\right) A^*(s^t))^{-1}(1+\tau), \tag{64}$$

and the first-order conditions with respect to B_{+1}, B^*_{+1} imply

$$\lambda(s^t) = \lambda^*(s^t). \tag{65}$$

Combining with the formulas for shadow values, we obtain

$$\frac{\partial[\log G(d(s^t), f(s^t)) - \Phi(.,.)] - l^*(s^t}{\partial d(s^t)}(1+\tau) = \frac{\partial[\log G(f^*(s^t), d^*(s^t)) - \Phi(.,.) - l^*(s^t)]}{\partial d^*(s^t)}. \tag{66}$$

E Business cycle implications of the quantitative model

To verify that our model accounts for the trade-comovement relationship without sacrificing the performance in other respects, we report a set of business cycle statistics generated from our model. The results, presented in Table 9, report median business cycle statistics from our simulated model, as well as medians in our dataset. As the inspection of the table shows, the model matches the statistics fairly well, at least as well as the frictionless model, and often better. One notable improvement is the prediction that output is more correlated internationally than consumption, addressing the so-called *quantity anomaly*.[36]

F Volatility ratio across countries in our sample

Table 10 presents estimates of the volatility ratio in our sample.

[36] Identified in Backus, Kehoe and Kydland (1992).

Table 9: Business cycle statistics: data versus models[a].

Statistic	Data Median[b]	Benchmark Median	Frictionless Median
A. Correlation			
domestic with foreign			
TFP (measured)	0.44	0.54	0.52
GDP	0.52	0.53	0.52
Consumption	0.41	0.45	0.57
Employment	0.42	0.46	0.54
Investment	0.50	0.38	0.45
GDP with			
Consumption	0.71	0.92	0.93
Employment	0.60	0.81	0.99
Investment	0.71	0.98	0.98
Net exports	-0.20	-0.63	-0.69
Terms of trade with			
Net exports	-0.31	-0.89	-0.54
B. Volatility relative to GDP			
Consumption	0.79	0.28	0.26
Investment	3.04	3.90	3.66
Employment	0.71	0.83	0.52
Net exports	0.59	0.20	0.14

[a]Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, the data column refers to the median in our sample of countries for the period 1980Q1-2011Q4.

G Definition of NIPA in the quantitative model

GDP in constant prices (steady-state prices) corresponds in our quantitative model to

$$L_D(P_d d + P_f f + P_g g) + \sum_{i=F,W} x_i p_i^d L_i d_i - p_D^f L_D f - p_D^g L_D g + v_D(a_D^f + a_D^g) - x_F v_F a_F^d - x_W v_W a_W^d,$$

consumption and investment in constant prices corresponds to[37]

$$L_D(P_{d,t} d_t + P_{f,t} f + P_{g,t} g_t) \frac{c_t}{G(d_t, f_t, g_t)}, \tag{67}$$

$$L_D(P_{d,t} d_t + P_{f,t} f + P_{g,t} g_t) \frac{i_t}{G(d_t, f_t, g_t)}, \tag{68}$$

[37]Consumption and investment in period zero prices are not equal to c and i. The reason is that the Euler's Law does not apply for period zero (steady-state) prices. However, quantitatively the difference is essentially zero.

Table 10: Volatility ratio in a cross-section of major industrialized countries.

	Detrending method	
Country	Hodrick-Prescott filter (1600)	Linearly detrended
Australia	0.88	0.78
Austria	2.76	2.31
Belgium	1.21	1.27
Canada	1.27	1.24
Denmark	1.17	1.52
Finland	1.67	1.31
France	0.77	0.86
Germany	1.38	1.36
Italy	1.07	1.12
Japan	0.68	0.63
Korea	0.59	0.65
Netherlands	0.99	0.77
Norway	1.18	1.21
Portugal	1.07	1.04
Spain	1.89	1.21
Sweden	1.59	2.14
Switzerland	1.05	0.87
United Kingdom	0.90	0.67
United States	1.20	0.88
Median	1.17	1.12

and the employment index corresponds to $l_{i,t}$. Notice that investment in marketing does not enter the expenditure side measurement of GDP. This assumption is consistent with the methodology of national income accounting, in which expenses on R&D, marketing, and advertising are all treated as intermediate inputs − see SNA (1993) Par. 1.49, 6.149, 6.163, 6.165. While $R\&D$ expenses have been capitalized in the U.S., this is the prevalent convention across countries in our sample.

H Data sources

Bilateral trade statistics were taken from the International Monetary Fund, Direction of Trade Statistics, 2005. From SourceOECD.org, Quarterly National Accounts: Gross Fixed Capital Formation ("P51: Gross fixed capital formation," "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"), GDP in constant prices ("B1_GE: Gross domestic product - expenditure approach," "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"). Our measure of labor is civilian employment or employment from the Quarterly National Accounts or the International Labor Orga-

nization (based on data availability). GDP is available from 1980Q1 to 2011Q4 for all countries in our sample. Employment data are missing for some countries for some years (see the Online Appendix for more details on what data we used). Since labor data are often not seasonally adjusted, we apply the X-12-ARIMA Seasonal Adjustment Program from census.gov.

Nominal GDP series come from World Development Indicators. Gross Fixed Capital Formation, GDP in constant prices and Civil Employment series come from SourceOECD.org, Quarterly National Accounts. The series for physical capital have been constructed using the perpetual inventory method with a constant depreciation of 2.5%. Aggregate GDP for blocks of countries has been computed from growth rates of GDP in constant prices (recent years, varies by country) weighted by the nominal GDP of each country in 2004 (we applied the growth rates backward).