Information Processing: Contracts versus Communication

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Contracts versus communication

- A principal expects to receive private information
- The principal relies on an agent acting on that information
- Ideally, complete control in advance:
  actions pre-specified for all contingencies
- Difficult to exercise this degree of control
- Role for non-binding *ad hoc* communication (cheap talk)

Questions:

When to commit (to instructions) and when to communicate?

How do contract and cheap talk interact?

Timeline of the game

Stage 1: Principal writes contract

- Codifes language that makes conditions (sets of states) and actions (instructions) verifiable to third parties
- Commits principal to provide instructions (not state)
- Commits agent to follow instructions
- Incompleteness: effort to make conditions verifiable
    - Finitely many instructions and potential gaps

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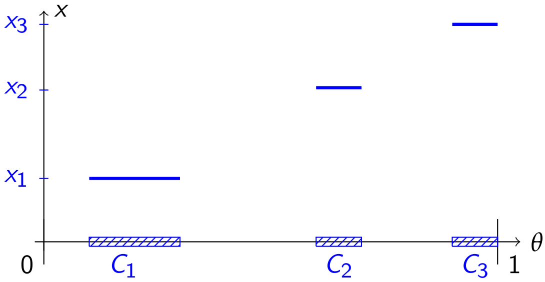


Timeline of the game

Stage 2: State realizes and is privately observed by principal

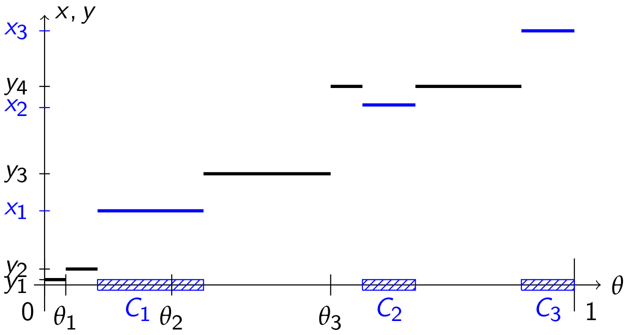
Stage 3: Principal communicates with agent:

- Sends instruction (contract)
- Sends cheap talk recommendation (gap)

Stage 4: Agent takes action

- Follows instruction (contract)
- Chooses action (gap)

Timeline of the game



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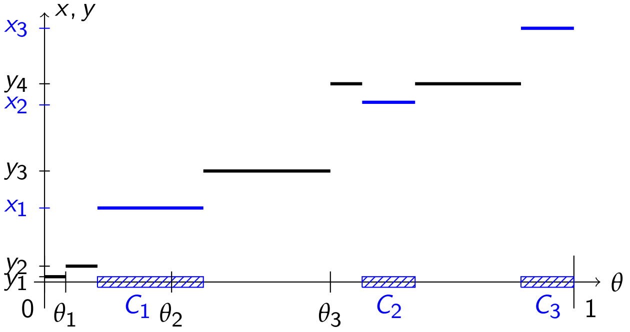


Stage 5: States in the contract become verifiable

- Contract establishes language that makes states in the contract verifiable
- Informal communication cannot be verified

Take-aways

Tradeoff: ex ante commitment versus ex post discretion

General preferences:

- Small disagreement: communication dominates
- Many clauses: contracting dominates

Uniform-quadratic:

- Contracts relax incentive constraints in communication
- Benefits from contract
  - direct: shifts control to principal
  - indirect: more actions in communication
    more equalized communication intervals

Example:

- Contracts cover states with more conflict
- Contracts cover states that are more likely

Related literature

- Crawford and Sobel (1982) (Cheap talk)
- Dye (1985), Battigalli & Maggi (2002) (Writing cost)
- Shavell (2006), Heller & Spiegler (2008), Schwartz & Watson (2013) (Contract interpretation)
- Aumann & Hart (2003), Krishna & Morgan (2004), Golosov, Skreta, Tsyvinski, & Wilson (2014) (Endogenous communication thresholds)

Model

Model

- Players:
  - Sender (Principal)
  - Receiver (Agent)
- Receiver's action $y \in \mathbb{R}$
- State $\theta \in [0, 1]$ with $\theta \sim F, f(\theta) > 0$
- Payoffs: standard concave loss functions satisfying positive mixed-partial condition
  - Sender $U^S(y, \theta, b)$
  - Receiver $U^R(y, \theta)$
  - positive sender-bias $b > 0$

Timing of the contract writing game G

1. Sender writes a contract – $\mathcal{C}$
   - simple – not fully detailed complete
   - gaps – potentially not obligationally complete
2. Sender observes the state
   - contract induces action
   - gap induces communication
3. Communication subgame – $\Gamma^{\mathcal{C}}$
   - sender sends message
   - receiver takes action

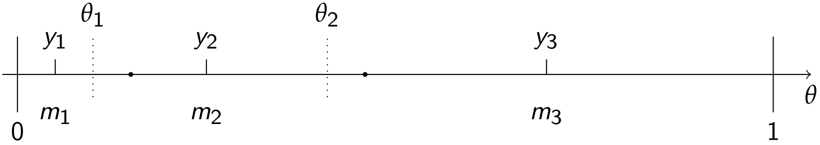
Goal: characterize sender-optimal SPEa

Contract writing game $G(\widehat{K}, b)$

- Sender writes contract $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{K}$
- Clauses (C_k, x_k), $k = 1, \ldots, K$
- Conditions are intervals in the state space, $C_k \subseteq [0, 1]$
- Instructions $x_k \in \mathbb{R}$
- $K \leq \widehat{K}$
- **Commitment**:
  If $\theta \in C_k$, action x_k implemented

Communication subgame $\Gamma^{\mathcal{C}}$

- Gap in the contract: $\mathcal{L}(\mathcal{C}) := [0,1] \setminus \bigcup_{k=1}^{K} C_k$
- Sender strategy (messages) $\sigma : \mathcal{L}(\mathcal{C}) \to \Delta(M)$
- Receiver strategy (actions) $\rho : M \to \mathbb{R}$
- No commitment
- Partitional equilibria
    - critical types θ_i
    - "steps" = induced actions y_i
- Γ^0 induced by $\mathcal{C}^0$ is a CS game:



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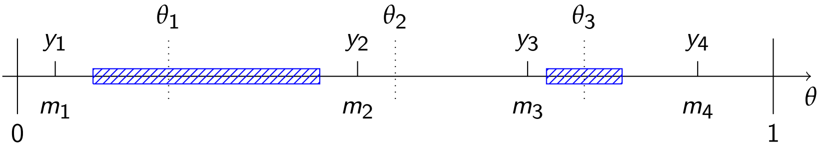


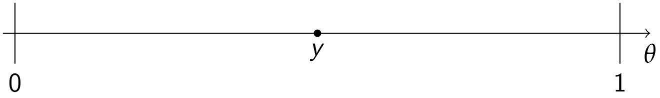
Example

Example: $\widehat{K} = 1$ and $b = \frac{1}{3}$

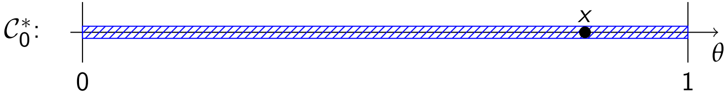
- Uniform distribution, quadratic payoffs, constant bias
  - Sender: $U^S(y, \theta, b) = -(\theta + b - y)^2$
  - Receiver: $U^R(y, \theta) = -(\theta - y)^2$
- There cannot be an equilibrium with more than two steps
- We compare:
  no contract, 0-step, 1-step, and 2-step optimal contracts

Example: no contract, 0-step

No contract = CS communication:

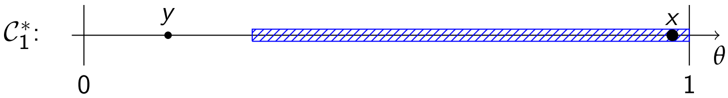


Obligationally complete contract = no communication (0-step):

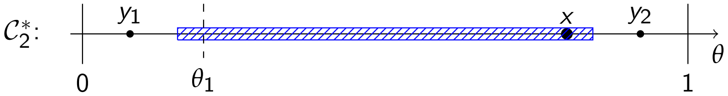


Example: 1-step and 2-step

Allowing for 1-step communication:



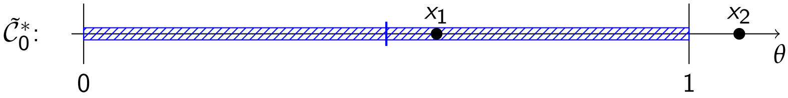
Allowing for 2-step communication:



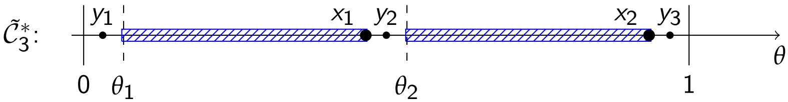
The sender's payoffs are ordered:
no contract $\prec$ obl. complete contract $\prec$ 1-step $\prec$ 2-step contract

Example with different parameters

Increase number of clauses to $\widehat{K} = 2$, keeping $b = \frac{1}{3}$:



Increase number of clauses to $\widehat{K} = 2$ and decrease bias to $b = \frac{1}{5}$ (recall: without contract maximally two actions in equilibrium):



Example: take-aways

- More clauses improve payoff
- More clauses can drive out communication
- Communication can replace contracting for smaller bias
- More communication actions with contract compared to CS:

  Contract relaxes incentive constraints in communication

General results

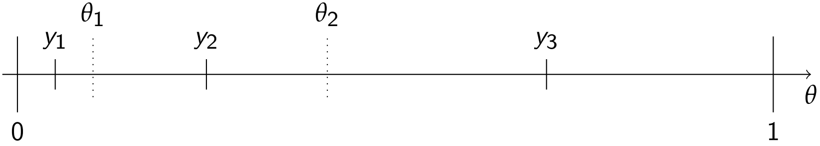
Maximal use of clauses

The sender optimally uses as many clauses as possible:

Proposition 1

If $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{K}$ *is an optimal contract in* $G(\widehat{K}, b)$, *then* $K = \widehat{K}$.

- Intuition:
    - Replace communication interval: sender imposes her bias
    - Split existing clause: actions more precise



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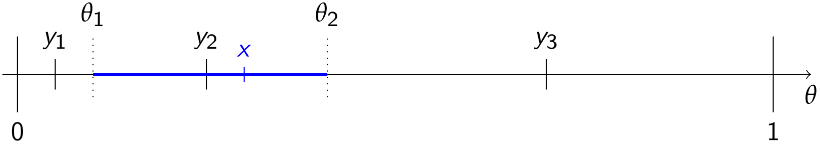


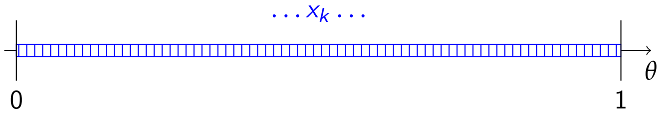
Many clauses

If the maximal number of clauses goes to infinity, contracting drives out communication:

Proposition 2

For any sequence of $\{\mathcal{L}_{\widehat{K}}\}_{\widehat{K}=1}^{\infty}$ *of gaps arising in sender-optimal equilibria* $e(\widehat{K}, b)$ *of contract-writing games* $G(\widehat{K}, b)$, $\widehat{K} = 1, 2, \ldots,$

$$\lim_{\widehat{K}\to\infty} \mathrm{Prob}(\mathcal{L}_{\widehat{K}}) = 0.$$



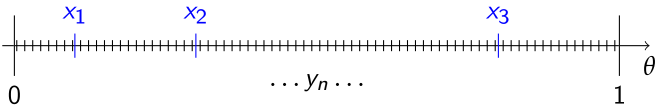
Decreasing bias

If the bias goes to zero, communication drives out contracting:

Proposition 3

Suppose that the continuity property holds for the games $\Gamma^0(b_i)$. For any sequence $\{\mathcal{L}_i\}_{i=1}^{\infty}$ of gaps in sender-optimal equilibria $e(b_i)$ of games $G(\widehat{K}, b_i)$ with $\lim_{i\to\infty} b_i = 0$,

$$\lim_{i\to\infty} \mathrm{Prob}(\mathcal{L}_i) = 1.$$



Results for uniform-quadratic environment

Results for uniform-quadratic environment

(Not necessarily optimal) contracts can increase the number of steps in communication:

Proposition 4

For any b, there exist a $\widehat{K}$ and a contract $\mathcal{C}$ such that there is an equilibrium of the communication subgame $\Gamma^{\mathcal{C}}$ with n induced actions if and only if $n < 1 + \frac{1}{2b}$.

- Comparison to CS for $b < \frac{1}{2}$:

$$\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{2b}} < 1 + \frac{1}{2b}$$

- Example $b = \frac{1}{10}$: $N_{CS} = 2$ and $\hat{N} = 5$

Many clauses: no communication

Sufficiently many clauses – relative to the bias – result in no communication:

Proposition 5

If $\widehat{K} > \frac{1}{2b}$, then any optimal contract will cover $[0, 1]$.

Contracts versus communication

An optimal contract relaxes incentive constraints:

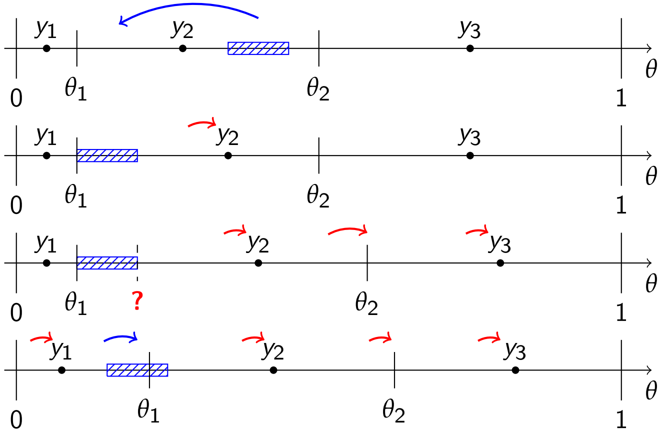
- Every "condition cluster" contains a critical type (equivalently, no condition cluster belongs to the interior of a communication interval)
- For meaningful communication, there is a condition cluster with a critical type that is not 0 or 1

Proposition 6

Suppose that the contract $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{\widehat{K}}$ is optimal in the contract-writing game G, and the equilibrium $e^{\mathcal{C}}$ is sender-optimal in the communication subgame $\Gamma^{\mathcal{C}}$. Then, for every condition cluster $\boldsymbol{C}$, there is a critical type θ with $\boldsymbol{C} \cap \{\theta\} \neq \emptyset$. If, in addition, the equilibrium $e^{\mathcal{C}}$ induces at least two communication actions, then there is a condition cluster $\boldsymbol{C}$ and a critical type $\theta \neq 0, 1$ with $\boldsymbol{C} \cap \{\theta\} \neq \emptyset$.

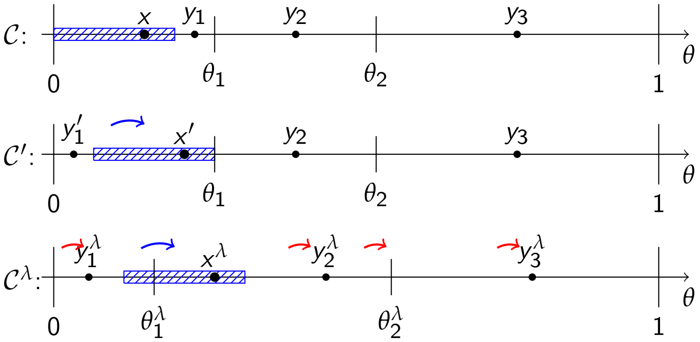
No condition in interior

Intuition:



Condition has interior type

Intuition:



Structure of optimal contracts

- Equilibrium is partitional and monotonic

Corollary 7

Suppose that the contract $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{\widehat{K}}$ *is part of a sender-optimal equilibrium* e^G *in the contract-writing game* G *and induces a sender-optimal n-step equilibrium* $e^{\mathcal{C}}$ *in the communication subgame* $\Gamma^{\mathcal{C}}$*. Then, the equilibrium* e^G *is*

1. *partitional – there is a partition* $\mathcal{P} = \{P_1, P_2, \ldots, P_{\widehat{K}+n}\}$ *of the type space* $[0, 1]$ *into intervals such that each* $P \in \mathcal{P}$ *is either a condition of* $\mathcal{C}$ *or a communication interval in* $e^{\mathcal{C}}$*; and,*
2. *monotonic – for any two* $P, P' \in \mathcal{P}$, $P \neq P'$*, with* $\inf(P') \geq \sup(P)$*, the actions* $a(P')$ *and* $a(P)$ *taken for states in* P' *and* P *satisfy* $a(P') > a(P)$.

Structure of optimal contracts

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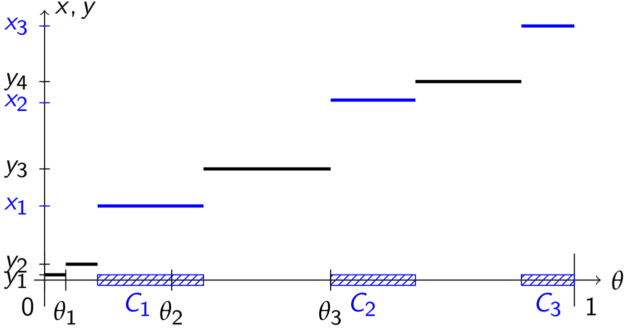
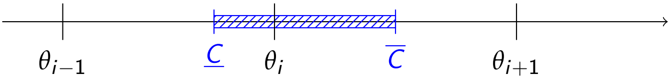


Equalizing communication intervals

- Contracts relax incentive constraints
- Lengths of communication intervals can be equalized

Corollary 8

Suppose $\widehat{K} = 1$, the contract $\mathcal{C}$ with condition $[\underline{C}, \overline{C}]$ is optimal, and $\mathcal{C}$ induces at least two communication actions in the sender-optimal equilibrium $e^{\mathcal{C}}$ of the communication subgame $\Gamma^{\mathcal{C}}$. If θ_{i-1}, θ_i, and θ_{i+1} are critical types in the equilibrium $e^{\mathcal{C}}$ with $\theta_i \in [\underline{C}, \overline{C}]$, then $|\theta_{i+1} - \overline{C}| < |\underline{C} - \theta_{i-1}| + 4b$; and, if $\theta_i \in (\underline{C}, \overline{C})$, then $|\theta_{i+1} - \overline{C}| \leq |\underline{C} - \theta_{i-1}|$.



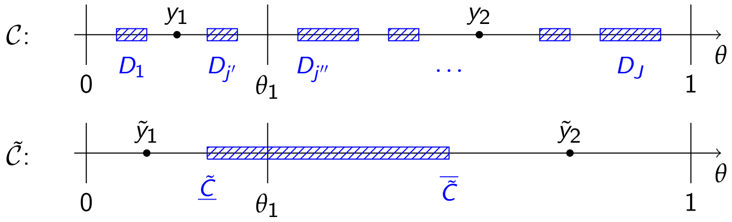
Extensions

Finite unions of disjoint closed intervals as conditions

- It is never optimal to split the condition into finitely many disjoint intervals.

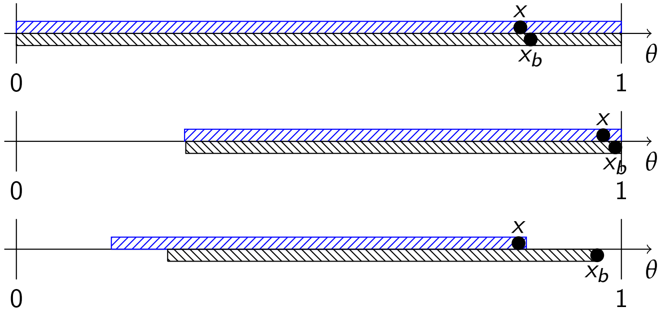
Proposition 9

Suppose that we allow contracts with conditions C that are finite unions of disjoint closed intervals. Then, for $b > \frac{1}{4}$ and $\widehat{K} = 1$, any optimal contract is nonempty and the condition in that contract is a single interval.



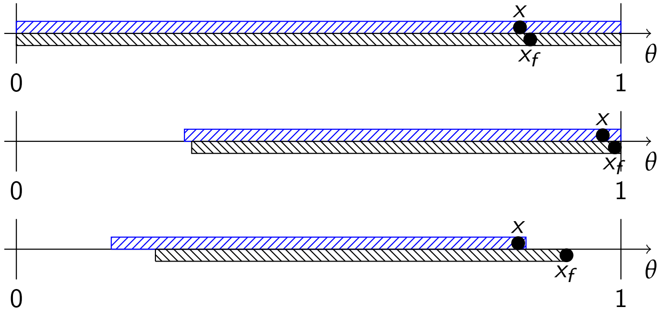
Example: nonconstant bias

- Assume $b(\theta) = \frac{1}{3} + \frac{1}{30}\theta$
- Optimal contract covers states with relatively higher bias



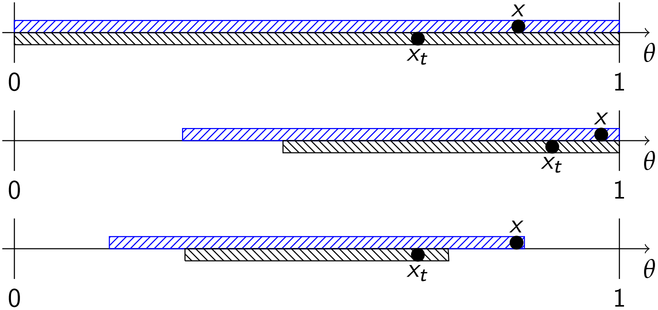
Example: nonuniform distribution

- Assume $f(\theta) = \frac{9}{10} + \frac{2}{10}\theta$
- Optimal contract covers more likely states



Example: transfers

- Sender: $U^S(y, \theta, b, w) = -(\theta + b - y)^2 - w$
- Receiver: $U^R(y, \theta, w) = -\alpha(\theta - y)^2 + (1 - \alpha)w$
- Sender maximizes:
  $\mathbb{E}U^S(y, \theta, b, w)$ s.t. $\mathbb{E}U^R(y, \theta, w) = \overline{u}^R$
- Optimal contract covers fewer states



Extensions: take-aways

- Some motivation for assuming intervals
- Some robustness with respect to: bias, distribution, transfers
- Contract covers states with higher conflict
- Contract covers states that are more likely
- Transfers reduce the set of states in the contract

Conclusions

Concluding remarks

- Model of interaction between contracts and communication
- Tradeoff: *ex ante* commitment versus *ex post* discretion
- At the extremes:
  - Small disagreement: communication dominates
  - Many clauses: contracting dominates
- Insight: two benefits from contracts
  - direct: shift control to principal
  - indirect: relaxation of incentive constraints
    $\rightarrow$ potential for more actions induced by communication
    $\rightarrow$ potential for more equalized communication intervals
- Equilibria are partitional and monotonic
- Optimal contracts
  - cover states that have more conflict
  - cover states that are more likely
  - with transfers cover smaller sets of states

Thank you!

Continuity assumption

Nothing unexpected happens for $b \to 0$:

Continuity Property. *For any sequence of biases* $\{b_i\}_{i=1}^{\infty}$ *with* $b_i \to 0$ *and any sequence* $\{e(b_i)\}_{i=1}^{\infty}$ *of sender-optimal equilibria in the games* $\{\Gamma^0(b_i)\}_{i=1}^{\infty}$*, the sender's payoffs in those equilibria converge to* $\int_{[0,1]} U^S(y^S(\theta), \theta, 0) dF(\theta)$.

back