Capital Accumulation and Dynamic Gains from Trade *

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Abstract

We compute welfare gains from trade in a dynamic, multi-country Ricardian model where international trade affects the factors of production in each period. Consistent with the data, our model allows for both the relative price of investment and the investment rate to depend on the world distribution of trade barriers. We calibrate the model for 93 countries and perform a counterfactual exercise to examine transition paths between steady-states after a permanent, uniform trade liberalization across countries. Our mechanism reveals the importance for quantifying the welfare gains from trade along the entire transition path.

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1 Introduction

How large are the welfare gains from trade? This is an old and important question. This question has been typically answered in a static setting by computing the change in real income from an observed equilibrium to a counterfactual equilibrium. In such computations, the factors of production and technology in each country are held fixed and the change in real income is entirely due to the change in each country's trade share that responds to a change in trade frictions. Recent examples include Arkolakis, Costinot, and Rodríguez-Clare (2012) who compute the welfare cost of autarky and Waugh and Ravikumar (2016) who compute the welfare gains from frictionless trade.

By design, the above computations cannot distinguish between static and dynamic gains from trade. We compute welfare gains from trade in a dynamic multi-country Ricardian model where international trade affects the capital stock in each period. Our environment is a version of Eaton and Kortum (2002) embedded into a two-sector neoclassical growth model. There is a continuum of tradable intermediate goods. The technology for producing the intermediate goods is country-specific and the productivity distribution is Fréchet. Each country is endowed with an initial stock of capital. Investment goods, produced using intermediate goods, augment the stock of capital. Final consumption goods are also produced using intermediate goods. Trade is subject to iceberg costs.

The model features two novel ingredients inspired by the data (i) endogenous relative price of investment, and (ii) endogenous investment rate.

We compute the steady state of the model for 93 countries and calibrate it to reproduce the observed trade flows across countries, prices, and output per worker in each country in 2011. We use this steady state as a baseline and conduct a counterfactual in which trade barriers are reduced simultaneously in every country. We then compute the dynamic path from the baseline steady state to the new steady state. Using the dynamic path, we compute the welfare gains using a consumption equivalent measure as in Lucas (1987).

We find that (a) the gains along the transition path account for about 60 percent of those measured by only comparing steady states and are three times larger than than those measured in a static model with capital held fixed, (b) countries that have lower GDP or higher trade frictions in the baseline experience the larger gains from trade liberalization, and (c) measured TFP and investment jump to the new steady state level almost immediately after trade liberalization while capital-labor ratio increases gradually.

We then show the importance of the main two features of our model to analyze dynamic welfare gains from trade. We find that the endogenous relative price of investment allows countries to attain permanently higher capital-output ratios, yielding higher output and consumption. Furthermore, the endogenous investment rate yields shorter half lives for capital accumulation, induced by temporarily high real rates of return to investment. As a result, the model delivers large gains from trade along the transition.

The predictions of our model are consistent with several features of the data. Wacziarg and Welch (2008) show that after a trade liberalization, GDP growth increases, the relative price of investment falls fast and real investment rates increase. All these are features of our model.

Our paper relates to two recent studies that examine dynamic trade models. Eaton, Kortum, Neiman, and Romalis (2015) and Caliendo, Dvorkin, and Parro (2015). Both papers compute the transitional dynamics of an international trade model by computing period-over-period changes in endogenous variables as a result of a changes in trade barriers (this is the so-called hat algebra approach). Our approach differs from theirs in several aspects. First, we solve for the transition of our model in levels; we do not use hat algebra. By solving the model in levels, we are able to validate the cross-sectional predictions of our baseline model. In particular, we find that our model is consistent with the cross-sectional distribution of capital and investment rates in the data. Second, computing the initial steady state in levels allows us to impose discipline on the particular type of trade liberalization we are interested in, which is not possible without knowing the initial levels. Finally, Eaton, Kortum, Neiman, and Romalis (2015) solve for the planner's problem and assume that the Pareto weights remain constant across counterfactuals. In our computation, however, each country's share in world consumption changes across counterfactuals and along the transition path.[1]

The paper closest to ours in methodology is Anderson, Larch, and Yotov (2015), who also study dynamic welfare gains and solve for transitional dynamics in levels. However, different from our paper, they study the gains in a one-sector growth model with log utility, Cobb-Douglas investment technology, and a constant exogenous relative price of investment. These features imply that anticipated changes to future trade frictions have no impact on current decisions or prices, so solving the dynamic model entails solving a sequence of static problems. In particular, each country's investment rate in their model is invariant to changes in trade

[1]Zylkin (2016) uses a similar approach to "hat algebra" to study how China's integration from 1993-2011 has had a effected on investment and capital accumulation in the rest of the world. His "hat algebra" approach differs from other papers in that he computes the change of the variable from from its baseline equilibrium value to its counterfactual equilibrium value, rather than computing period-over-period changes.

frictions. Our paper differs from theirs in that we account for all of the forward looking decisions and solve for the transitional dynamics where the both relative price of investment and the investment rate evolve endogenously in response changes in trade frictions. We present empirical evidence supporting both of these features.

Finally, recent studies have used "sufficient statistics" approaches to measure changes in welfare by looking at changes in the home trade share (Arkolakis, Costinot, and Rodríguez-Clare, 2012). The sufficient statistics formula, in our model, is only valid across steady states, but not along the transition path. We show that measuring changes in welfare using changes in consumption along the transition path yields very different implications than one would obtain by using sufficient statistics.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 describes the quantitative exercise. Section 4 reports the counterfactuals, and section 5 concludes.

2 Model

There are I countries indexed by $i = 1, \ldots, I$ and time is discrete, running from $t = 1, \ldots, \infty$. There are three sectors: consumption, investment, and intermediates, denoted by c, x, and m respectively. Neither consumption goods nor investment goods are tradable. There is a continuum of intermediate *varieties* that are tradable. Production of all the goods are carried out by perfectly competitive firms. As in Eaton and Kortum (2002), each country's efficiency in producing each intermediate variety is a realization of a random draw from a country- and time-specific distribution. Trade in intermediate varieties is subject to iceberg costs. Each country purchases each intermediate variety from its lowest-cost supplier and all of the varieties are aggregated into a *composite* intermediate good. The composite intermediate good, which is nontradable, is used as an input along with the stock of capital and labor to produce the consumption good, the investment good, and the intermediate varieties.

Each country admits a representative household. The representative household owns its country's stock of capital and labor, which it inelastically supplies to domestic firms, and purchases consumption and investment goods from the domestic firms.

2.1 Endowments

In each period, the representative household in country i is endowed with a labor force of size L_i, which is constant over time, and in the initial period is endowed with a stock of capital, K_{i1}.

2.2 Technology

There is a unit interval of varieties in the intermediates sector. Each variety within the sector is tradable and is indexed by $v \in [0, 1]$.

Composite good Within the intermediates sector, all of the varieties are combined with constant elasticity in order to construct a sectoral composite good according to

$$Q_{it} = \left[\int_0^1 q_{it}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)}$$

where η is the elasticity of substitution between any two varieties.[2] The term $q_{it}(v)$ is the quantity of good v used by country i to construct the composite good at time t. The resulting composite good, Q_{it}, is the quantity of the composite good available in country i to use as an intermediate input.

Individual varieties Each individual variety is produced using capital, labor, and the composite intermediate good. The technologies for producing each variety are given by

$$Y_{mit}(v) = z_{mi}(v) \left(K_{mit}(v)^{\alpha} L_{mit}(v)^{1-\alpha} \right)^{\nu_m} M_{mit}(v)^{1-\nu_m}$$

The term $M_{mit}(v)$ denotes the quantity of the composite good used by country i as an input to produce $Y_{mit}(v)$ units of variety v, while $K_{mit}(v)$ and $L_{mit}(v)$ denote the quantities of capital and labor employed.

The parameter $\nu_m \in [0, 1]$ denotes the share of value added in total output, while α denotes capital's share in value added. Each of these coefficients is constant both across countries and over time.

The term $z_{mi}(v)$ denotes country i's productivity for producing variety v. Following Eaton and Kortum (2002), the productivity draw comes from an independent country-specific Fréchet distributions with shape parameter θ and country-specific scale parameter T_{mi}, for $i = 1, 2, \ldots, I$. The c.d.f. for productivity draws in country i is $F_{mi}(z) = \exp(-T_{mi} z^{-\theta})$.

[2] The value η plays no quantitative role other than satisfying technical conditions which ensure convergence of the integrals.

In country i the expected value of productivity across the continuum is $\gamma^{-1}T_{mi}^{\frac{1}{\theta}}$, where $\gamma = \Gamma(1+\frac{1}{\theta}(1-\eta))^{\frac{1}{1-\eta}}$ and $\Gamma(\cdot)$ is the gamma function. As in Finicelli, Pagano, and Sbracia (2012), we refer to $T_{mi}^{\frac{1}{\theta}}$ as the fundamental productivity in country i.[3] If $T_{mi} > T_{mj}$, then on average, country i is more efficient than country j at producing intermediate varieties. The parameter $\theta > 0$ governs the coefficient of variation of the efficiency draws. A larger θ implies more variation in efficiency across countries and, hence, more room for specialization within each sector; i.e., more intra-sectoral trade.

Consumption good Each country produces a consumption good using capital, labor, and intermediates according to

$$Y_{cit} = A_{ci}\left(K_{cit}^{\alpha}L_{cit}^{1-\alpha}\right)^{\nu_c} M_{cit}^{1-\nu_c}$$

The terms K_{cit}, L_{cit}, and M_{cit} denote the quantity of capital, labor, and composite intermediate good used by country i to produce Y_{cit} units of consumption at time t. The parameters α and ν_c are constant across countries and over time. The term A_{cit} captures country i's productivity in the consumption goods sector—this term varies over time and across countries.

Investment good Each country produces an investment good using capital, labor, and intermediates according to

$$Y_{xit} = A_{xi}\left(K_{xit}^{\alpha}L_{xit}^{1-\alpha}\right)^{\nu_x} M_{xit}^{1-\nu_x}$$

The terms K_{xit}, L_{xit}, and M_{xit} denote the quantity of capital, labor, and composite intermediate good used by country i to produce Y_{xi} units of investment at time t. The parameters α and ν_x are constant across countries and over time. The term A_{xit} captures country i's productivity in the investment goods sector—this term varies over time and across countries.

[3] As discussed in Finicelli, Pagano, and Sbracia (2012), fundamental productivity differs from measured productivity because of selection. In a closed economy, country i produces all varieties in the continuum so its measured productivity is equal to its fundamental productivity. In an open economy, country i produces only the varieties in the continuum for which it has a comparative advantage and imports the rest. So its measured productivity is higher than its fundamental productivity, conditioning on the varieties that it produces in equilibrium.

2.3 Trade

All international trade is subject to barriers that take the iceberg form. Country i must purchase $d_{ij} \geq 1$ units of any intermediate variety from country j in order for one unit to arrive; $d_{ij} - 1$ units *melt* away in transit. As a normalization we assume that $d_{ii} = 1$ for all i.

2.4 Preferences

The representative household values consumption per capita over time, C_{it}/L_i, according to

$$\sum_{t=1}^{\infty} \beta^{t-1} L_i \frac{(C_{it}/L_i)^{1-1/\sigma}}{1-1/\sigma}$$

where $\beta \in (0,1)$ denotes the period discount factor and σ denotes the intertemporal elasticity of substitution. Both parameter are constant across countries and over time.

Capital accumulation Each period the representative household enters the period with K_{it} units of capital. A fraction δ, depreciates during the period, while new additions to the capital stock (gross capital formation) is denoted by X_{it}. The stock of capital is then carried over into the next period. The rate of depreciation is constant both across countries and over time. Thus, with $K_{i1} > 0$ given, the capital accumulation technology is

$$K_{it+1} = (1-\delta)K_{it} + X_{it}$$

Budget constraint The representative household earns income by supplying capital, K_{it}, and labor, L_i, inelastically to domestic firms earning a rental rate r_{it} on each unit of capital and a wage rate w_{it} on each unit of labor. The household purchases consumption at the price P_{cit} per unit and purchases investment at the price P_{xit} per unit. The period budget constraint is given by

$$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i$$

2.5 Equilibrium

A competitive equilibrium satisfies the following conditions: i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for accumulating capital, ii) taking prices as given, firms maximize profits subject to the available technologies, iii) intermediate varieties are purchased from their lowest-cost provider subject to the trade barriers, and iv) markets clear. At each point in time, we take world GDP as the numéraire: $\sum_i r_{it}K_{it} + w_{it}L_i = 1$ for all t. We describe each equilibrium condition in detail below.

2.5.1 Household optimization

The representative household chooses a path for consumption that satisfies the following Euler equation

$$C_{it+1} = \beta^{\sigma}\left(1 + \frac{r_{it+1}}{P_{ixt+1}} - \delta\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma} C_{it} \tag{1}$$

Combining the representative household's budget constraint together with capital accumulation technology and rearranging, implies the following

$$C_{it} = \left(1 + \frac{r_{it}}{P_{xit}} - \delta\right)\left(\frac{P_{xit}}{P_{cit}}\right)K_{it} + \left(\frac{w_{it}}{P_{cit}}\right)L_i - \left(\frac{P_{xit}}{P_{cit}}\right)K_{it+1} \tag{2}$$

2.6 Firm optimization

Markets are perfectly competitive, so firms set prices equal to marginal costs. Denote the price of variety v, produced in country j and purchased by country i, as $p_{mij}(v)$. Then $p_{mij}(v) = p_{mjj}(v)d_{ij}$, where $p_{mjj}(v)$ is the marginal cost of producing variety v in country j. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_{mi}(v) = \min_{j=1,\ldots,I}[p_{mjj}(v)d_{mij}]$. The price of the composite intermediate good in country i at time t is then

$$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{jt}d_{ij})^{-\theta}T_{mj}\right]^{-\frac{1}{\theta}} \tag{3}$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{jt}}{1-\nu_m}\right)^{1-\nu_m}$ is the unit cost for a bundle of inputs for intermediate-goods producers in country n at time t.

Next we define total factor usage in the intermediates sector by aggregating up across the individual varieties.

$$K_{mit} = \int_0^1 K_{mit}(v)dv, \qquad L_{mit} = \int_0^1 L_{mit}(v)dv,$$
$$M_{mit} = \int_0^1 M_{mit}(v)dv, \qquad Y_{mit} = \int_0^1 Y_{mit}(v)dv$$

The term $L_{mit}(v)$ denotes the quantity of labor employed in the production of variety v at time t. If country i imports variety v at time t, then $L_{mit}(v) = 0$. Hence, L_{mit} is the total quantity of labor employed in sector m in country i at time t. Similarly, K_{mit} is the total quantity of capital used, M_{mit} is the total quantity of intermediates used as an input, and Y_{mit} is the total quantity of output of intermediate goods.

Cost minimization by firms implies that, within each sector $b \in \{c, m, x\}$, factor expenses exhaust the value of output.

$$\begin{aligned} r_{it}K_{bit} &= \alpha\nu_b P_{bit}Y_{bit}, \\ w_{it}L_{bit} &= (1-\alpha)\nu_b P_{bit}Y_{bit}, \\ P_{mit}M_{bit} &= (1-\nu_b)P_{bit}Y_{bit} \end{aligned}$$

That is, the fraction $\alpha\nu_b$ of the value of each sector's production compensates capital services, the fraction $(1-\alpha)\nu_b$ compensates labor services, and the fraction $1-\nu_b$ covers the cost of intermediate inputs; there are zero profits.

2.6.1 Trade flows

The fraction of country i's expenditures allocated to intermediate varieties produced by country j is given by

$$\pi_{ijt} = \frac{(u_{mjt}d_{ijt})^{-\theta}T_{mj}}{\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}} \tag{4}$$

where u_{mjt} is the unit costs of a bundle of factors faced by producers of intermediate varieties in country j.

2.6.2 Market clearing conditions

We begin by describing the domestic factor market clearing conditions.

$$\sum_{b\in\{c,m,x\}} K_{bit} = K_{it}, \qquad \sum_{b\in\{c,m,x\}} L_{bit} = L_i, \qquad \sum_{b\in\{c,m,x\}} M_{bit} = Q_{it}$$

The first two conditions impose that the capital and labor market clear in country i at each time t. The third condition requires that the use of composite intermediate good equal its supply. It's use consists of intermediate demand by firms in each sector. Its supply is the quantity of the composite good which consists of both domestically- and foreign-produced varieties.

The next conditions require that goods markets clear.

$$C_{it} = Y_{cit}, \qquad X_{it} = Y_{xit}, \qquad \sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}$$

The first condition states that the quantity of consumption demanded by the representative household in country i must equal the quantity produced by country i. The second condition says the same for the investment good. The third condition imposes that the value of intermediates produced by country i has to be absorbed globally. Recall that $P_{mjt}M_{bjt}$ is the value of intermediate inputs that country i uses in production in sector b. The term π_{jit} is the fraction of country j intermediate-good expenditures sourced from country i. Therefore, $P_{mjt}M_{bjt}\pi_{jit}$ denotes the total value of trade flows from country i to country j.

Finally, we impose an aggregate resource constraint in each country: net exports equal zero. Equivalently, gross output equals gross absorption.

$$P_{mit}Y_{mit} = P_{mit}Q_{it}$$

The left-hand side denotes the gross output of intermediates in country i and the right-hand side denotes total expenditures on intermediates.

2.7 Welfare Analysis

We measure welfare using consumption-equivalent units to be consistent with the fact that utility in our model is defined over consumption. This is a departure from much of the literature in which welfare gains are computed in static models as changes in income. As such, as income changes along the transition we need to examine how the income is allocated to consumption and investment.

Measuring gains from trade across steady states We follow Lucas (1987) and compute the constant fraction, λ_i^{ss}, that consumers in country i must receive every period in the baseline steady state case to give them the same utility they obtain from the consumption in the counterfactual steady state. We refer to this measure of gains as "steady-state gains."

$$\sum_{t=1}^{\infty} \beta^{t-1} L_i \frac{\left(\left(1+\frac{\lambda_i^{ss}}{100}\right) C_i^{\star}/L_i\right)^{1-1/\sigma}}{1-1/\sigma} = \sum_{t=1}^{\infty} \beta^{t-1} L_i \frac{\left(C_i^{\star\star}/L_i\right)^{1-1/\sigma}}{1-1/\sigma}$$
$$\Rightarrow 1+\frac{\lambda_i^{ss}}{100} = \frac{C_i^{\star\star}}{C_i^{\star}} \tag{5}$$

where $C_i^{\star}$ is the (constant) consumption in the baseline steady state in country i, and $C_i^{\star\star}$ is the consumption in the the new (counterfactual) steady state. In our model consumption is proportional to income across countries in the steady state.[4]

In Appendix B we show that the steady-state income per capita can be expressed as

$$y_i \propto \underbrace{A_{ci}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1-\nu_c}{\theta\nu_m}}}_{\text{TFP contribution}} \underbrace{A_{xi}^{\frac{\alpha}{1-\alpha}}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}}_{\text{Capital contribution}} \tag{6}$$

In the steady state, all the change in income per capita resulting from changes in trade barriers are manifested in the home trade share as in Arkolakis, Costinot, and Rodríguez-Clare (2012) and Waugh (2010), augmented by the fact that capital is endogenous and it depends on trade barriers as in Anderson, Larch, and Yotov (2015) and Mutreja, Ravikumar, and Sposi (2014).

Measuring the dynamic gains from trade along the transition We follow Lucas (1987) and compute the constant fraction, λ_i^{dyn}, that consumers in country i must receive every period in the baseline case to give them the same utility they obtain from the consumption in the counterfactual. We refer to this measure as "dynamic gains."

[4]The formula for to ratio of consumption to income in country i is $\frac{C_i}{y_i L_i} = 1 - \frac{\alpha\delta}{\frac{1}{\beta}-(1-\delta)}$.

$$\sum_{t=1}^{\infty}\beta^{t-1}L_i\frac{\left(\left(1+\frac{\lambda_i^{dyn}}{100}\right)C_i^\star/L_i\right)^{1-1/\sigma}}{1-1/\sigma}=\sum_{t=1}^{\infty}\beta^{t-1}L_i\frac{\left(\tilde{C}_{it}/L_i\right)^{1-1/\sigma}}{1-1/\sigma}$$

$$\Rightarrow\left(1+\frac{\lambda_i^{dyn}}{100}\right)^{1-1/\sigma}=(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}\left(\frac{\tilde{C}_{it}}{C_i^\star}\right)^{1-1/\sigma} \quad (7)$$

where $C_i^\star$ is the (constant) consumption in the baseline steady state in country i, and $\tilde{C}_{it}$ is the consumption in the counterfactual at time t.[5]

This expression has been used by Anderson, Larch, and Yotov (2015) to measure dynamic welfare gains from trade, but they impose assumptions on preferences and technologies that yield a fixed investment rate. In addition, they assume the relative relative price of investment is 1. This restricts the household's ability to accumulate capital. Another drawback of an exogenous investment rate is that households do not respond to anticipated shocks. In our model, however, the dynamics of capital are governed by the Euler equation, which is key to analyze welfare along the transition. In particular, substituting equation (2), at periods t and $t+1$, into equation (1) yields the equilibrium law of motion for the stock of capital in country i

$$\begin{aligned}
&\left(1+\frac{r_{it+1}}{P_{xit+1}}-\delta\right)\left(\frac{P_{xit+1}}{P_{cit+1}}\right)\left(\frac{K_{it+1}}{L_i}\right)+\frac{w_{it+1}}{P_{cit+1}}-\left(\frac{P_{xit+1}}{P_{cit+1}}\right)\left(\frac{K_{it+2}}{L_i}\right)\\
&=\beta^\sigma\left(1+\frac{r_{it+1}}{P_{xit+1}}-\delta\right)^\sigma\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^\sigma\\
&\times\left[\left(1+\frac{r_{it}}{P_{xit}}-\delta\right)\left(\frac{P_{xit}}{P_{cit}}\right)\left(\frac{K_{it}}{L_i}\right)+\frac{w_{it}}{P_{cit}}-\left(\frac{P_{xit}}{P_{cit}}\right)\left(\frac{K_{it+1}}{L_i}\right)\right]
\end{aligned}$$

This is the key equation to analyze welfare along the transition, and it constitutes the main departure from the existing dynamic models analyzing welfare gains from trade with capital accumulation. Note that the dynamics of capital in country i depend on the capital stocks in all other countries since the prices are determined in the world economy in the presence of trade. Thus, the dynamics are pinned down by the solution to a system of I second-order, nonlinear difference equations. The optimality conditions for the firms combined with the relevant market clearing conditions pin down the prices as a function of the capital stocks across countries.

[5] We calculate sums using the counterfactual transition path solved from $t=1,\ldots,85$ and then set the counterfactual consumption equal to the new steady-state level of consumption for $t=86,\ldots,1000$.

3 Quantitative exercise

We describe in Appendix A the details of our algorithm for solving the model. Broadly speaking, we first reduce the infinite dimension of the problem down to a finite-time model with $t = 1, \ldots, T$ periods. We make T sufficiently large to ensure convergence to a new steady state. As such, this requires us to first solve for a terminal steady state to use as a boundary condition for the path of capital stocks. In addition, we take initial capital stocks as given by computing the initial steady state.

We define a steady state as a situation in which all endogenous variables are constant over time. Table A.1 provides the equilibrium conditions that describe the solution to the steady state in our model. Our technique for computing the steady state equilibria are standard, while our method for computing the equilibrium transition path between steady states is new.[6]

3.1 Calibration

We calibrate the initial parameters of the model to match data in 2011. Our assumption is that the world is in steady state at this time. Our model covers 93 countries (containing 91 individual countries plus 2 regional country groups). Table D.1 in the Appendix provides a list of the countries along with their 3-digit ISO codes. This set of countries accounts for 90 percent of world GDP as measured by the Penn World Tables, and for 84 percent of world trade in manufactures as measured by the United Nations Comtrade Database. Appendix C provides the details of our data.

Common parameters The values for the common parameters are reported in Table 1. Beginning with the trade elasticity, we appeal to recent estimates by Simonovska and Waugh (2014) and set $\theta = 4$. The value for η plays no quantitative role in the Eaton-Kortum model of trade other than satisfying the condition that $1 + \frac{1}{\theta}(1 - \eta) > 0$; we set $\eta = 2$.

[6] We solve for the competitive equilibrium of the model. This differs from Eaton, Kortum, Neiman, and Romalis (2015), who solve the planner's problem. In particular, they use the social planner's problem to solve for trade imbalances using fixed weights across counterfactuals. This implies that each countrys share in world consumption expenditures (i.e., the numeraire in their setting) is fixed across counterfactuals. In a decentralized economy, these shares would change, and still be efficient. We see this in our own counterfactuals. The second welfare theorem states that any social planner outcome can be replicated in a decentralized market with the appropriate transfers. In our context, this implies that the social planner weights would need to change in order to generate the same allocation as the decentralized economy without transfers (i.e., in our counterfactuals).

In line with the literature, we set capital's share in value added $\alpha = 0.33$ (from Gollin, 2002), the discount factor $\beta = 0.96$, the depreciation rate for capital $\delta = 0.06$, and the intertemporal elasticity of substitution $\sigma = 0.67$.

We compute $\nu_m = 0.28$ by taking the cross-country average of the ratio of value added to gross output of manufactures. We compute $\nu_x = 0.33$ by taking the cross-country average of the ratio of value added to gross output of investment goods. Computing ν_c is slightly more involved since there is not a clear industry classification for consumption goods. That is, all goods are consumed in the data. Instead, we infer this share by interpreting national accounts data through the lens of our model. We begin by noting that

$$r_i K_i = \frac{\alpha}{1-\alpha} w_i L_i$$

from the combination of firm optimization and the market clearing conditions for capital and labor. In steady state, multiplying the Euler equation by the capital accumulation technology we obtain

$$P_{xi} X_i = \frac{\delta \alpha}{\frac{1}{\beta} - (1-\delta)} \frac{w_i L_i}{1-\alpha} = \phi_x \frac{w_i L_i}{1-\alpha}$$

We compute ϕ_x by taking the cross-country average of the share of gross fixed capital formation in nominal GDP. The household's budget constraint then implies that

$$P_{ci} C_i = \frac{w_i L_i}{1-\alpha} - P_{xi} X_i = (1-\phi_x) \frac{w_i L_i}{1-\alpha}$$

As such, consumption in our model corresponds to the sum of private and public consumption, changes in inventories, and net exports. Now we can use the trade balance condition together with the firm optimality conditions and the market clearing conditions for sectoral output to obtain

$$P_{mi} Q_i = \left[(1-\nu_x)\phi_x + (1-\nu_c)(1-\phi_x)\right] \frac{w_i L_i}{1-\alpha} + (1-\nu_m) P_{mi} Q_i$$

where $P_{mi} Q_i$ is the total absorption of manufactures in country i and $\frac{w_i L_i}{1-\alpha}$ is the nominal GDP. We use a standard method of moments estimator to back out ν_c from the previous expression.

Country-specific parameters We set the workforce, L_i, equal to the total population. The remaining parameters A_{ci}, T_{mi}, A_{xi} and d_{ij}, for $(i, j) = 1, \ldots, I$, are not directly

Table 1: Common parameters

θ	Trade elasticity	4
η	Elasticity of substitution between varieties	2
α	Capital's share in value added	0.33
β	Annual discount factor	0.96
δ	Annual depreciation rate for stock of capital	0.06
σ	Intertemporal elasticity of substitution	0.67
ν_c	Share of value added in final goods output	0.91
ν_x	Share of value added in investment goods output	0.33
ν_m	Share of value added in intermediate goods output	0.28

observable. We parsimoniously back these out by linking structural relationships of the model to observables in the data.

Combining equations (3) and (4) we relate the unobserved trade barrier for any given country pair directly to the ratio of intermediate-goods prices in the two countries, and the trade shares between them as follows

$$\frac{\pi_{ij}}{\pi_{jj}} = \left(\frac{P_{mj}}{P_{mi}}\right)^{-\theta} d_{ij}^{-\theta} \tag{8}$$

Appendix C provides the details for how we construct the empirical counterparts to prices and trade shares. For observations in which $\pi_{ij} = 0$, we set $d_{ij} = 10^8$. We also set $d_{ij} = 1$ if the inferred value is less than 1.

Lastly, we derive three structural relationships that we use to pin down the productivity parameters A_{ci}, T_{mi}, and A_{xi}. The three equations relate observables—the price of consumption relative to intermediates, the price of investment relative to intermediates, income per capita, and home trade shares—to the unknown productivity parameters. These derivations appear in Appendix B. We set $A_{cU} = T_{mU} = A_{xU} = 1$ as a normalization, where the subscript

U denotes the U.S.

$$\frac{P_{ci}/P_{mi}}{P_{cU}/P_{mU}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}} / A_{ci}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}} / A_{cU}} \right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}} \right)^{\frac{\nu_c - \nu_m}{\nu_m}} \tag{9}$$

$$\frac{P_{xi}/P_{mi}}{P_{xU}/P_{mU}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}} / A_{xi}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}} / A_{xU}} \right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}} \right)^{\frac{\nu_x - \nu_m}{\nu_m}} \tag{10}$$

$$\frac{y_{mi}}{y_{mU}} = \left(\frac{A_{ci}}{A_{cU}} \right) \left(\frac{A_{xi}}{A_{xU}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}} \right)^{\frac{1-\nu_c+\frac{\alpha}{1-\alpha}(1-\nu_x)}{\nu_m}} \tag{11}$$

For each country i, system (9)–(11) yields three nonlinear equations with three unknowns: A_{ci}, T_{mi}, and A_{xi}. Information about constructing the empirical counterparts to $P_{ci}, P_{mi}, P_{xi}, \pi_{ii}$ and y_{mi} is available in Appendix C.

These equations are quite intuitive. The expression for income per capita provides a measure of aggregate productivity across all sectors: higher income per capita is associated with higher productivity levels, on average. The two expressions for relative prices tell us how to allocate the burden of productivity across sectors.

The expressions for relative prices boil down to two components. The first term reflects something akin to the Balassa-Samuelson effect: All else equal, a higher price of capital relative to intermediates suggests a low productivity in capital goods relative to intermediate goods. In our setup, the productivity for the traded intermediate good is partly endogenous, reflecting the degree of specialization as captured by the home trade share. The second term reflects the extent to which the two goods utilize intermediates with different intensities. If measured productivity is relatively high in intermediates, then the price of intermediate input is relatively low and the sector that uses intermediates more intensively will, all else equal, have a lower relative price.

3.2 Model fit

Our model consists of 8832 country-specific parameters: $I(I-1) = 8556$ bilateral trade barriers, $(I-1) = 92$ consumption-good productivity terms, $(I-1) = 92$ investment-good productivity terms, and $(I-1) = 92$ intermediate-goods productivity terms.

Calibration of the country-specific parameters utilizes 8924 independent data points. The trade barriers use up $I(I-1) = 8556$ data points for bilateral trade shares and $(I-1) = 92$ for ratio of absolute prices of intermediates. The productivity parameters use up $(I-1) = 92$ data points for the price of consumption relative to intermediates, $(I-1) = 92$ for the price of investment relative to intermediates, and $(I-1) = 92$ for income per capita.

As such, there 92 more data points than parameters so our model does not perfectly replicate the data. Another way to interpret this is that there is one equilibrium condition for each country that we did not impose on our identification:

$$P_{mi} = \gamma \left[\sum_{j=1}^{I} (u_{mj} d_{ij})^{-\theta} T_{mj} \right]^{-\frac{1}{\theta}}$$

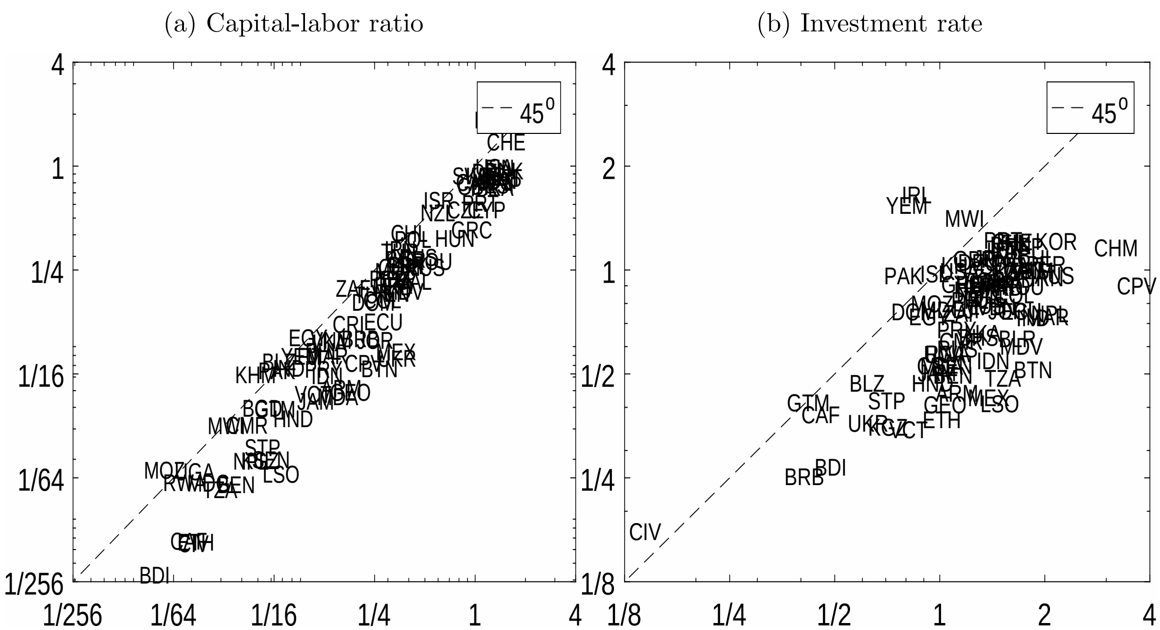
The model matches the data that we used well. The correlation between model and data is 0.96 for the bilateral trade shares, 0.97 for the absolute price of intermediates, 1.00 for income per capita, 0.96 for the price of consumption relative to intermediates, and 0.99 for the price of investment relative to intermediates.

Indeed, since we utilized relative prices of consumption and investment, not the absolute prices, matching the absolute prices is not necessarily a given. The correlation between model and data is 0.93 for the absolute price of consumption, and 0.97 for the absolute price of investment.

Implication for capital stock In our calibration we targeted income per capita. The burden is on the theory to disentangle what fraction of the cross-country income gap can be attributed to differences in capital and what fraction to differences in TFP.

Figure 1 shows that the model matches the data on capital-labor ratios across countries quite closely: the correlation is 0.93. It also shows that our model captures well the investment rate, $\frac{X_i}{y_i L_i}$, across countries in 2011. Note that we are imposing steady state in 2011, which implies that the investment rate is to proportional the capital-output ratio. Since our model matches GDP by construction, and also does well explaining capital stocks, our ability to replicate the investment rate is limited to the extent that the steady-state assumption is violated in the data.

Figure 1: Model fit: The vertical axis represents the model and the horizontal axis represents the data



4 Counterfactuals

In this section we implement a counterfactual trade liberalization via a one-time reduction in trade barriers. In this exercise we begin the world in the calibrated steady state. In the beginning of period $t = 1$ we reduce trade barriers uniformly across all countries so that the ratio of world trade to GDP increases from 50 percent to 100 percent across steady states, and keep all other parameters fixed at their baseline values. This amounts to reducing $d_{ij} - 1$ by 45 percent for each bilateral trade pair. All parameters are constant over time from $t = 1, \ldots, \infty$. The trade liberalization is unanticipated prior to the shock.

To perform these counterfactuals we need to solve for the transitional dynamics in levels. This differs from Eaton, Kortum, Neiman, and Romalis (2015) and Caliendo, Dvorkin, and Parro (2015) who solve for dynamics in terms of changes using hat algebra. We prefer our approach in the context of trade liberalization for two reasons. First, a crucial assumption in our model is that $d_{ij} \geq 1$. The hat-algebra approach does not make use of the initial levels of d_{ij} and hence cannot design an exercise in which $d_{ij} - 1$ is uniformly reduced; it can only analyze uniform changes to d_{ij}. It turns out that engineering a uniform trade liberalization by proportionally reducing d_{ij} results in a violation of the assumption that $d_{ij} \geq 1$ for many trade barriers. Second, the trade "barrier" is, by definition, $d_{ij} - 1$, as this is the proportion of goods that "melt away" in transit.

4.1 Welfare gains from trade

In each counterfactual we compute the welfare gains from trade in two ways. First we compute the consumption equivalent in terms of lifetime utility between the baseline and counterfactual steady states. Second, we take into account the transitional dynamics of the model and compute the dynamic gains from trade by computing the consumption equivalent of lifetime utility for the entire transition relative to the baseline.

4.1.1 Steady-state gains from trade

We compute the steady-state gains from trade using equation (5) and the dynamic gains from trade using equation (7). We find that the steady state gains from trade vary substantially across countries, ranging from 18 percent for the U.S. to 92 percent for Belize. The median change is 53 percent.

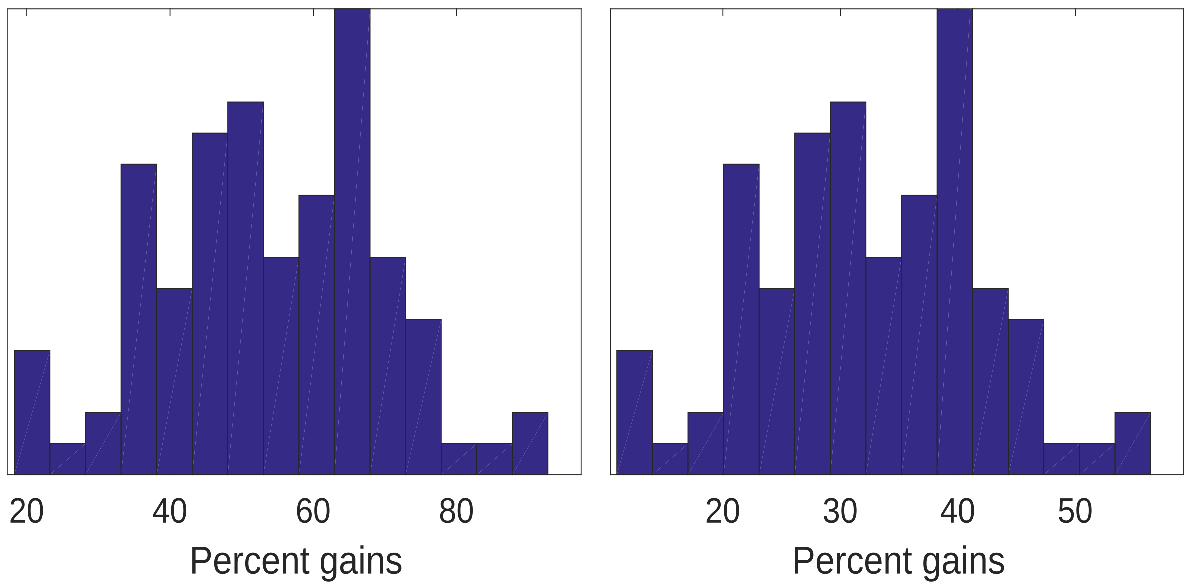
The steady-state gains from trade are identical to the change in income per capita across steady states in our model. Therefore, we exploit equation (6) to decompose the relative importance of changes in TFP and changes in capital in amounting for the gains. Equation (6) implies that the log-change in income that corresponds with a log-change in the home trade share is:

$$\frac{\partial \ln(y_i)}{\partial \ln(\pi_{ii})} = -\left(\underbrace{\frac{1-\nu_c}{\theta \nu_m}}_{\text{through TFP}} + \underbrace{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}_{\text{through capital}} \right)$$

Based on our calibration, the first term equals 0.08 while the second term equals 0.30. That is, given a change to trade barriers, 79 percent of the resulting change in income per capita across steady states can be attributed to change capital, and the remaining 21 percent to change in TFP. This number is constant across countries in our model since the elasticities $(\theta, \alpha, \nu_c, \nu_m, \nu_x)$ are all constant across countries. This does not imply that income per capita changes by equal proportions across countries, only that the relative contributions from TFP and capital are the same.

The larger contribution of capital to the steady-sate gains from trade reveals the impor-

Figure 2: Distribution of the gains from trade across countries: steady-state(left) and dynamic (right)



tance of modeling investment explicitly in trade models.

4.1.2 Dynamic gains from trade

The right panel of Figure 2 shows the distribution of the dynamic gains from trade liberalization across all countries. Gains for the median country are 32 percent. However, the differences are quite large across countries, ranging from 11 percent for the U.S., to 56 percent for Belize.[7]

The distribution of the dynamic gains from trade looks almost identical to the distribution of the steady-state gains (the distribution of steady-state gains are reported in the left panel of Figure 2). However, the dynamic gains are smaller in each country. The average ratio of dynamic gains to steady-state gains is 60.2 percent across countries, and varies from a minimum of 60.1 percent to a maximum of 60.5 percent.[8]

The proportionality of roughly 60 percent is a result of (i) the speed that consumption

[7] The gains from trade are systematically smaller for large countries, rich countries, and countries with smaller average export barriers. Each of these findings are consistent with existing literature.

[8] Desmet, Nagy, and Rossi-Hansberg (2015) consider, in a model of migration and trade, a counterfactual scenario that increases trade costs by 40% in the first period. They find that welfare decreases by around 34%.

converges to its new steady state and (ii) the rate at which future consumption is discounted. If consumption jumped to its new steady-state level on impact then this ratio would be close to 100 percent. If instead consumption declined significantly in the beginning and then converged to the new steady state after many years, then the ratio could be closer to 0 percent since there would be consumption losses in earlier periods, and future consumption gains would be highly discounted.

The Euler equation reveals the forces that influence consumption dynamics. A trade liberalization improves each country's terms of trade making more resources available for both consumption and investment. The allocation of output to consumption and investment is determined optimally by the household. In our model, a trade liberalization causes an initial drop in consumption, which occurs because the relative price of investment falls a lot, making investment very appealing. Household investment jumps and overshoots the steady state, see again Figure 3. The overshooting results from the fact the future real-rate of return (RRR), $1 + \frac{r_{it+1}}{P_{xit+1}} - \delta$, is higher than the steady-state RRR, $\frac{1}{\beta}$. As capital accumulates, the RRR returns to its original steady state level and investment settles down to its new (higher) steady-state level. Figure 4 shows the transition paths for the relative price of investment and the RRR in the U.S.

The initial drop in consumption, however, is limited by an offsetting force: the desire to smooth consumption. Since households are forward-looking and expect more resources in the future, there is an incentive to front-load consumption and invest less than what they would do otherwise. That is, although households can alter their investment to intertemporally allocate consumption, it is not optimal to fully front load consumption. In fact the opposite is true. Upon impact, consumption falls by 2.4 percent in the U.S. and by 6.3 percent in Belize. Immediately after the shock, consumption increases towards its new steady state as shown in Figure 3. Two housekeeping remarks are in order here. First, in the figures we index each series to 1 in the initial steady state. Second, the transition paths for every country exhibit similar characteristics to the U.S., but differ in their magnitudes: Belize is at one extreme and the U.S. is at the other extreme.[9]

4.1.3 Growth Accounting

The optimal intertemporal allocation of resources into consumption and investment by the household has implications for growth accounting. Income per capita in our model is driven

[9]We pick on the U.S. and Belize since they represent two extremes with respect to changes in income per capita across steady states.

Figure 3: Consumption (left) and investment (right) in the U.S.

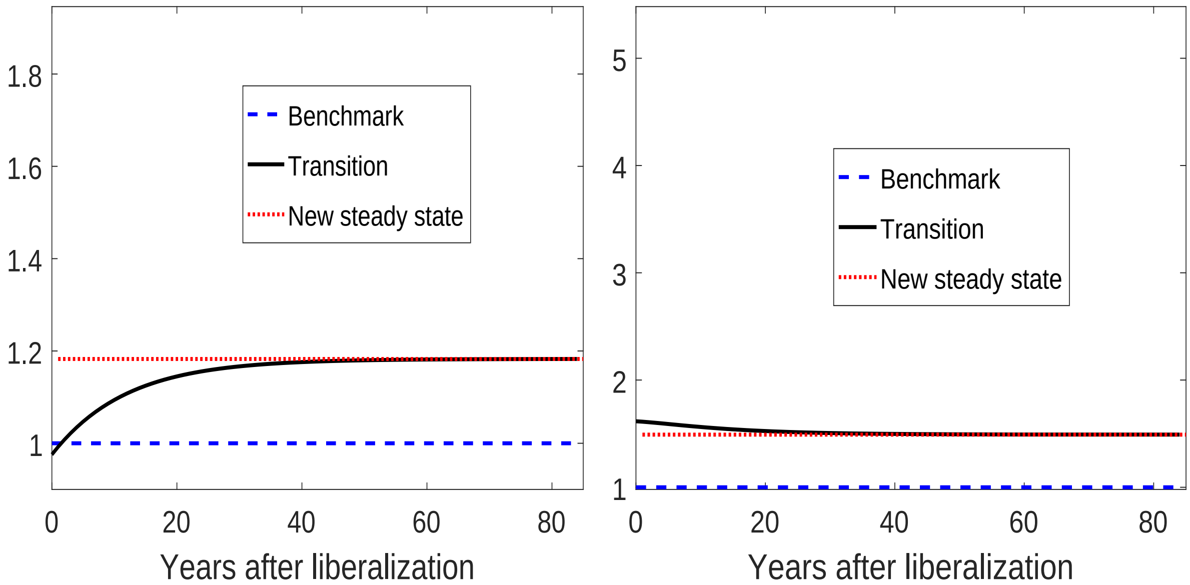
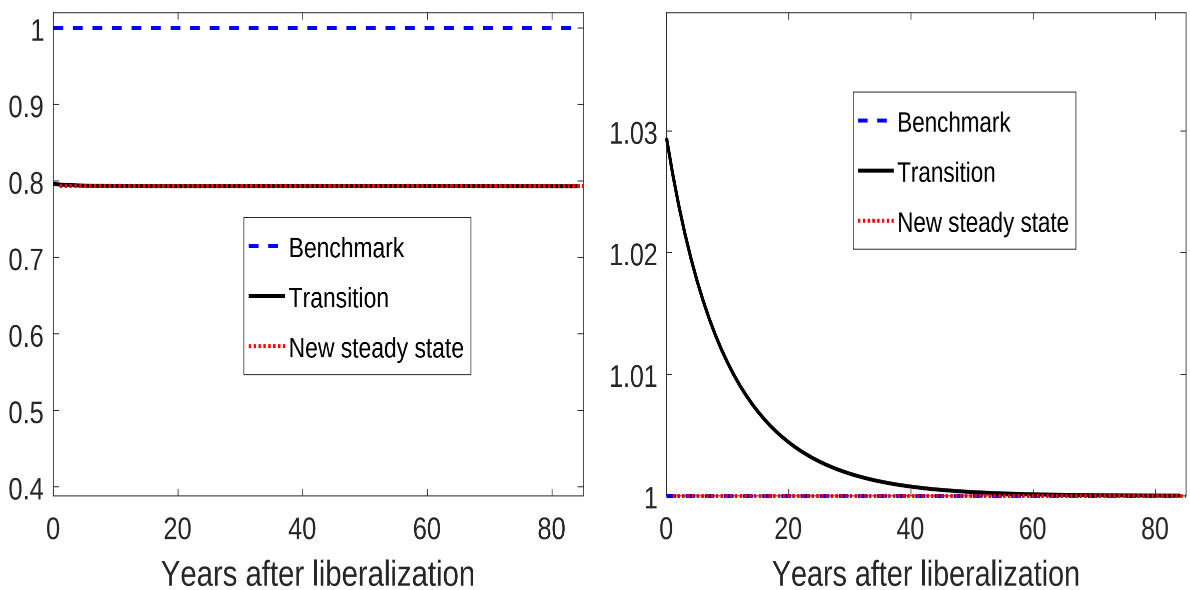


Figure 4: Relative price of investment (left) and real return (right) in the U.S.

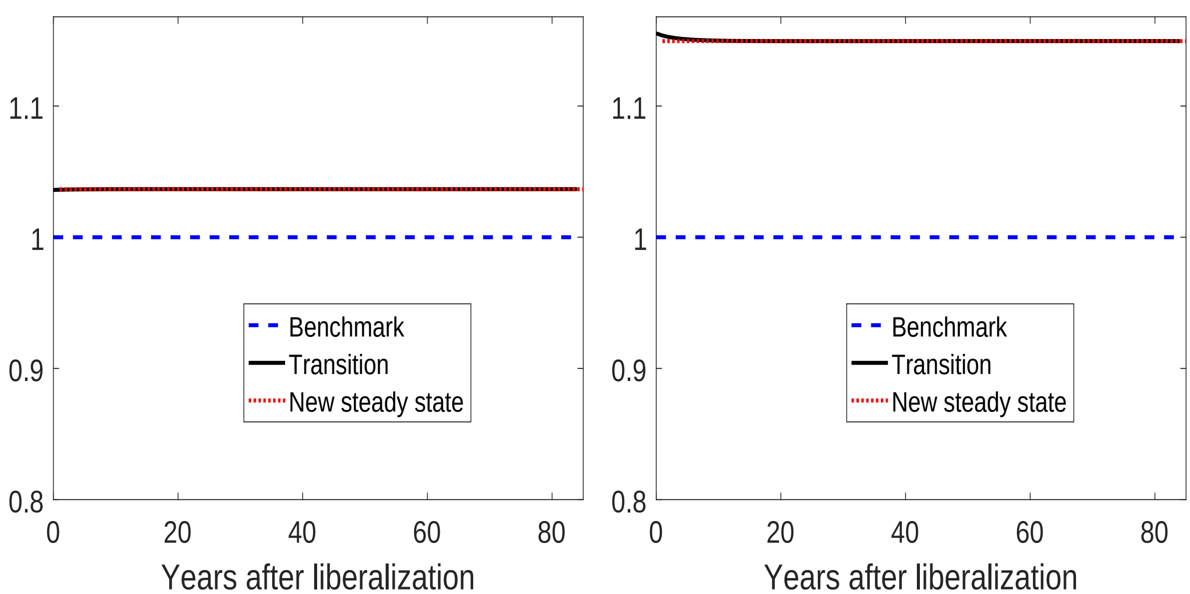


by two components: TFP and capital accumulation. In the Appendix we show the following:

$$y_{it} \propto \underbrace{A_{ci}\left(\frac{T_{mi}}{\pi_{iit}}\right)^{\frac{1-\nu_c}{\theta\nu_m}}}_{\text{TFP contribution}} \underbrace{\left(\frac{K_{it}}{L_i}\right)^{\alpha}}_{\text{Capital contribution}} \tag{12}$$

We use equation (12) to decompose the contributions of capital and TFP on income per capita following a trade liberalization. Figure 5 shows the transition paths for TFP. Note that the process for TFP is characterized by an initial jump close to its new steady-state value. The initial jump is larger for Belize than for the U.S. and reflects the improved terms of trade, captured by a decline in the home trade share. On impact, TFP slightly overshoots the new steady-state level in Belize, and slightly undershoots it in the U.S. The reason is because, after the initial shock, capital begins to adjust in period 2. Since capital grows faster in Belize than in the U.S., the home trade share increases in Belize and decreases in the U.S., implying a decrease in TFP in Belize and an increase in TFP in the U.S. Countries in which capital grows at the "average" rate do not experience either undershooting or overshooting. However, these changes in TFP beginning in period 2 are trivial compared to the jump in period 1.

Figure 5: TFP (U.S. left, Belize right)



As a result of the increased investment, the capital stock grows over time. Figure 6 shows the transition path for the capital stock in the United States and Belize. Begin by noting that in both the U.S. and Belize, the transition paths for capital are characterized by a gradual increase towards the new steady state with declining growth rates over time. In addition, capital grows at a much faster rate in Belize than in the U.S. as a result of having a higher investment rate along the transition. This difference is reflected in the relative contributions from capital and TFP to growth in income per capita along the transition.

Figure 6: Capital stocks (U.S. left, Belize right)

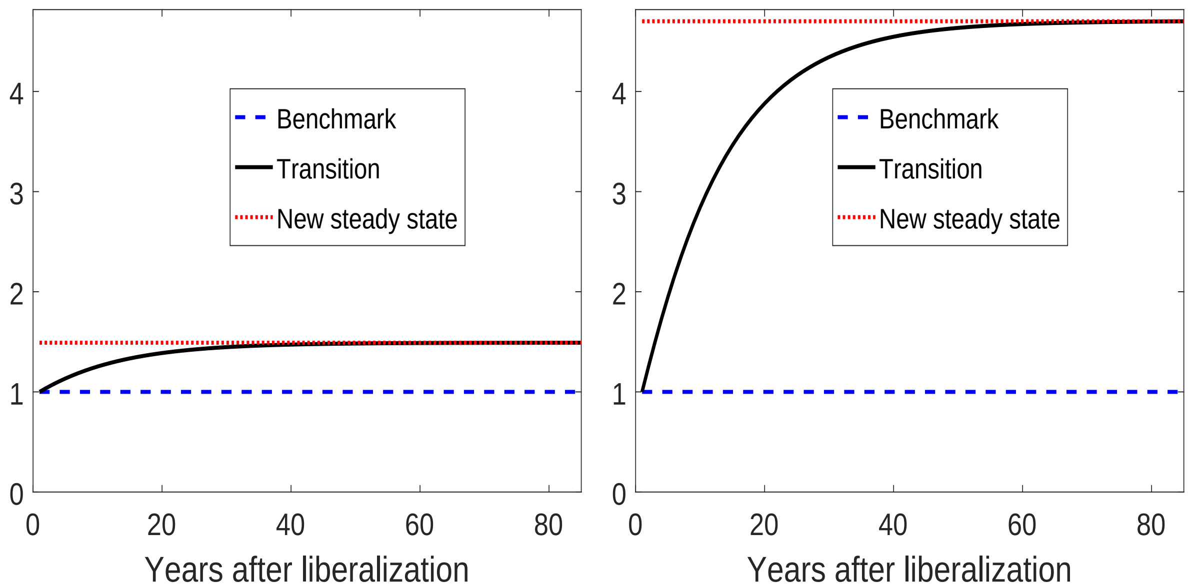
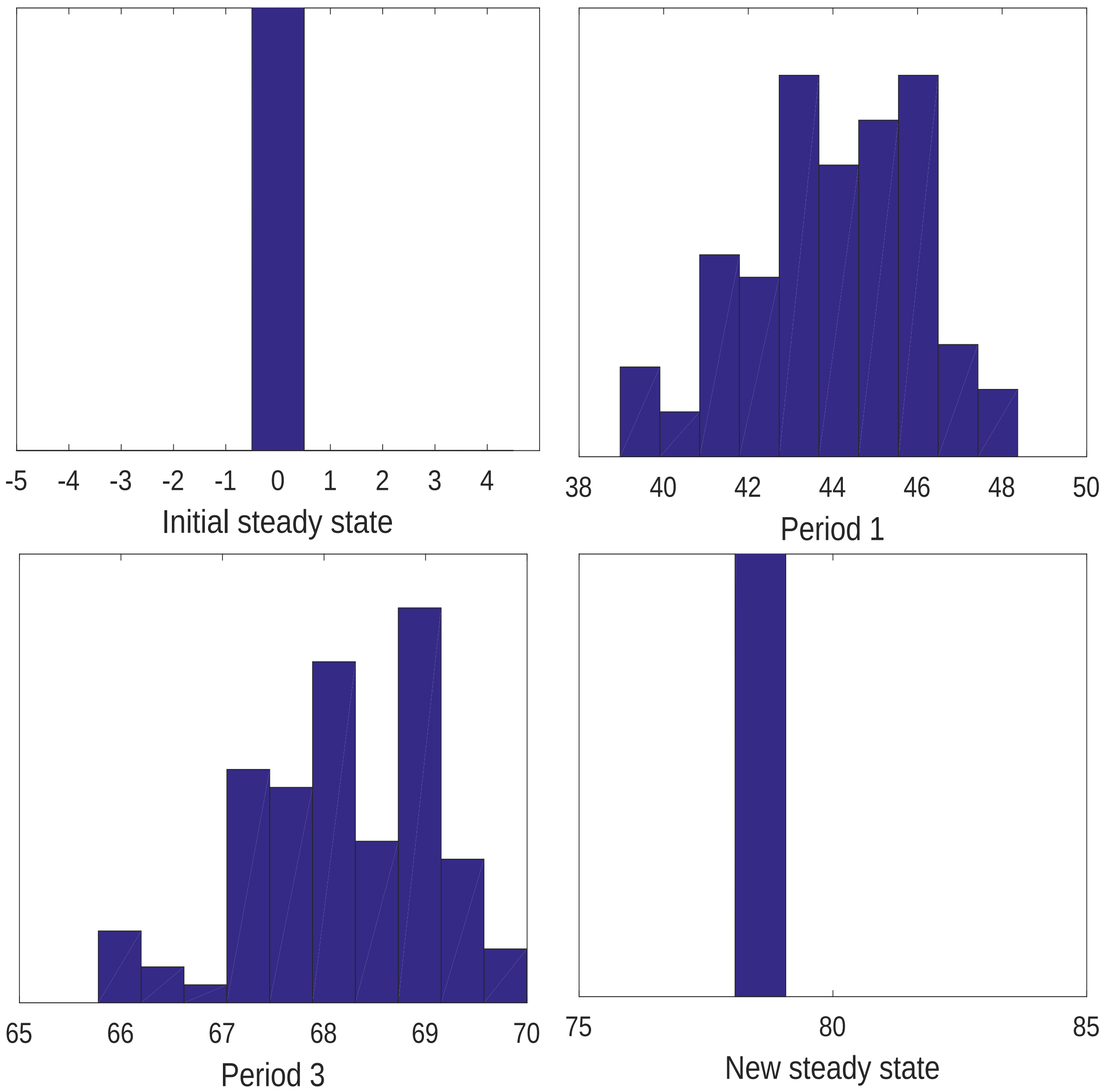


Figure 7 shows the contributions from capital stock to the change in income per capita in period 1, period 3, period 9, and the new steady state across all countries: $\frac{\log(K_{it}/K_i^\star)^\alpha}{\log(y_{it}/y_i^\star)}$. In period 1, 0 percent of the change in income per capita is accounted for by the change in capital in every country. This is no surprise since the capital stock in period 1 has not changed since it is inherited from the initial steady state.

By the third period, on average, changes in capital account for 44 percent of the change in income per capita, relative to the initial steady state. However, there is heterogeneity across countries, with contributions ranging from 39 to 48 percent. Countries like Belize have a relatively larger contribution from capital than countries like the U.S. This is merely a result of the fact that capital stocks are growing faster in countries like Belize.

After 9 periods, the average contribution from capital growth to income growth from the initial steady state is 68 percent, which is higher than in period 3, since TFP has not changed since period 3 but capital has continued to grow. The variation across countries is

Figure 7: Capital's contribution to income growth



much less than after 3 periods, ranging from 66 to 70 percent.

By the time the economy converges to the new steady state, capital's contribution settles down to 79 percent in every country as discussed in section 4.1.1.

4.1.4 A comparison to static gains from trade

Here we compare our dynamic gains from trade to those that would be obtained in a model with no capital accumulation (i.e., Arkolakis, Costinot, and Rodríguez-Clare (2012)). In a static model, welfare gains from trade would be driven entirely by changes in TFP. From equation 6, we obtained that TFP accounts for 21% of the steady-state gains from trade. This implies that the change in capital accounts for the remaining 79% of the gain across steady states. Recall that TFP is characterized by a one-time jump to its new steady-state level immediately following trade liberalization. This jump is unaffected by capital since the stock of capital does not change on impact. Therefore, the initial change in TFP corresponds to the welfare gains using the ACR formula in a model without capital, or in a model with capital taken exogenously. As a result, the static gains are 21 percent of the steady-state gains. We also know from our counterfactual exercise, that dynamic gains are around 60% of the steady-state gains in a model with capital accumulation. Therefore, in our dynamic model, dynamic gains are three times larger that static gains that would be obtained by ignoring transitional dynamics of capital.

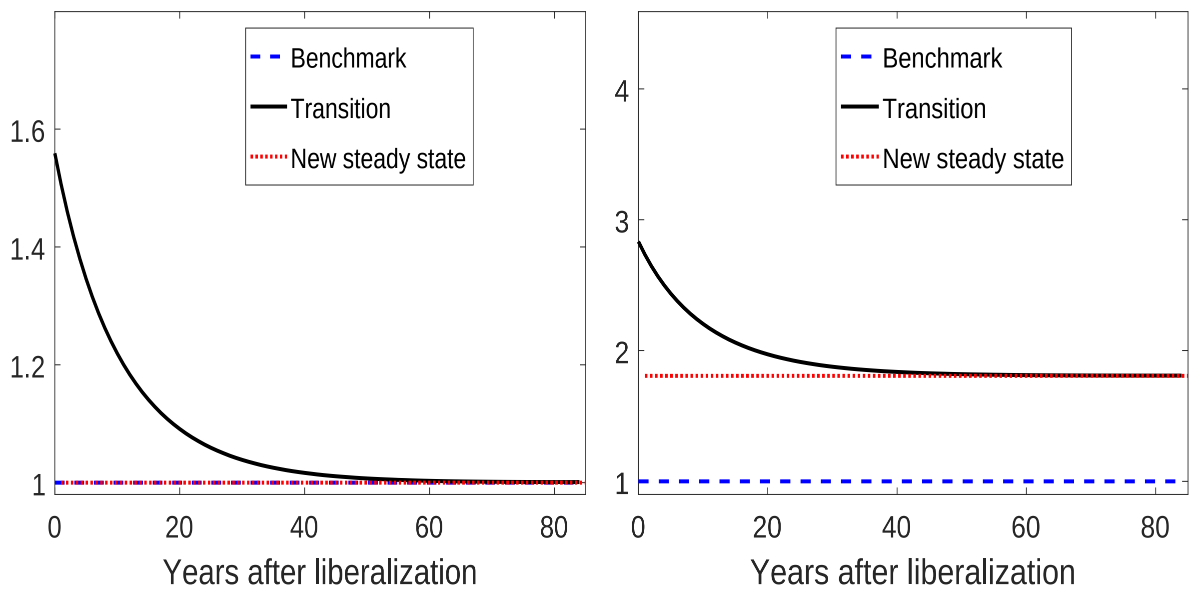
4.2 The mechanism

Some remarks are in order here regarding the importance of two features that distinguish our work from the literature: the endogenous relative price of capital and the endogenous investment rate.

In our model, the share of income that the household allocates towards investment expenditures is determined endogenously. That is, the nominal investment rate, $\frac{P_{xit}X_{it}}{L_i P_{cit} y_{it}}$ is not constant along the transition path as shown in the left panel of Figure 8. Combined with a decline the relative price of investment, the real investment rate, $\frac{X_{it}}{L_i y_{it}}$, increases substantially in response to trade liberalization as shown in the right panel of Figure 8. Indeed the real investment rate is permanently higher.

Alternative models To quantify the importance of the endogenous investment rate and endogenous relative price of investment, we solve versions of the model where we explic-

Figure 8: Transition path for investment rate in Greece: nominal (left) and real (right)



itly impose that $P_x/P_c = 1$ and/or that the nominal investment rate is exogenous. To do this we change only a couple of equations.

To eliminate the endogenous relative price of investment we introduce a final goods sector, denoted by f, and get rid of the separate sectors for consumption and investment. We force $A_{xi} = A_{ci} = A_{fi}$ and $\nu_x = \nu_c = \nu_f$. In the calibration we choose A_f to match the price of GDP relative to intermediates, and choose $\nu_f = 0.88$ to satisfy the national account equation, with all other parameters as in the baseline model.

To impose an exogenous nominal investment rate, the only feature that differs from the baseline model is that we eliminate the Euler equation and impose $P_{xit}X_{it} = \rho(w_{it}L_{it}+r_{it}K_{it})$, with $\rho = \frac{\alpha\delta}{1/\beta-(1-\delta)} = 0.1948$. That is, the household allocates an exogenous share, ρ, of its paycheck to investment expenditures. The value of ρ corresponds to the nominal investment rate that arises in the fully endogenous model in the steady state (which is constant across countries and across steady states).

We implement a similar trade liberalization in which barriers are uniformly reduced by 45 percent in every country. We report the numbers for Greece only, since Greece is the country that has the median gains from trade. All of the conclusions that we draw from Greece hold in every other country.

First, we find that an endogenous relative price governs the gap in capital between steady states. For instance, the left panel of Figure 9 shows that, in models 1 and 3, the capital stock converges to a higher steady-state level than in models 2 and 4. Indeed, having an endogenous relative price allows for higher steady-state capital-output ratio. To see why note that in a model with the the relative price of investment fixed, the real investment rate, $\frac{X_i}{y_i L_i}$, cannot adjust across steady states since $\rho = \frac{P_{xi} X_i}{y_i L_i} = \frac{\alpha\delta}{1/\beta-(1-\delta)}$ is constant. On the other hand, in models with endogenous relative price, the real investment rate converges to a higher steady-state level, see the right panel of Figure 9.

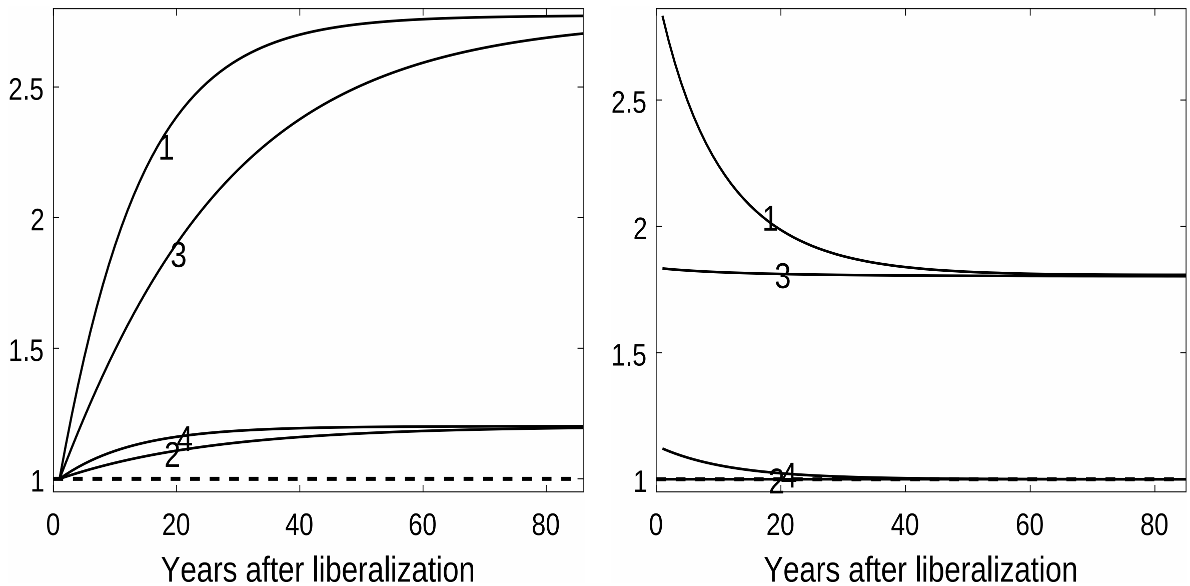
Second, endogenous investment rate affects the speed of capital accumulation, regardless of whether the relative price is exogenous or endogenous. For instance, the left panel of Figure 9 shows that, in model 1, capital converges faster to the new steady state than in model 3. Similarly, in model 4, capital converges faster to the new steady state than in model 2.

In sum, an endogenous relative price allows the economy to attain higher steady-state capital stocks, while an endogenous investment rate allows the economy to transition to the steady state faster. These features have implications for the path of consumption along the transition, and hence, for understanding the ratio of dynamic-to-steady-state gains from trade.

The ratio of dynamic-to-steady-state gains is a function of (i) the initial change in consumption and (ii) the speed of consumption growth, which depends on the half-life for capital. Table 2 shows that with endogenous relative price of investment (models 1 and 3), there is a larger difference between the steady-state and the dynamic welfare gains from trade. In models 1 and 4, the half lives for capital are similar, but in model 1 consumption drops on impact while it increases in model 4. As a result, dynamic gains, relative to steady-state gains, are lower in model 1 than in model 4. A similar argument applies to the comparison of models 2 and 3. Conversely, in models 3 and 4, the initial increase in consumption is similar, but the half lives for capital differ. In particular, in model 3 the half life is higher, meaning slower convergence, and hence the dynamic gains, relative to steady-state gains, are lower than in model 4.

In summary, while the endogenous investment rate has mainly an effect on the speed at what capital accumulates, the endogenous relative price of investment has mainly an effect on the gap between the initial and final steady-state. Consequently, the welfare gains from trade that result from accounting for the whole transitional path of the economy in a model in which both the investment rate and the relative price of capital are endogenous, are different from models that take one of them or both as exogenous.

Figure 9: Comparison of transitional dynamics across alternative models: capital stock (left) and real investment rate (right)



Notes: Model 1: endogenous relative price and endogenous nominal investment rate. Model 2: exogenous relative price and exogenous nominal investment rate. Model 3: endogenous relative price and exogenous nominal investment rate. Model 4: exogenous relative price and endogenous nominal investment rate. Here, relative price refers to $\frac{P_x}{P_c}$ and nominal investment rate refers to $\frac{P_x X}{P_c y L}$. We consider Greece since it is the country with the median gains from trade.

Empirical evidence in favor of our channels We have shown the importance of the two novel mechanims that we have introduced in the paper: endogenous relative price of investment and endogenous investment rate. Next we present some empirical evidence that supports the relevance of these mechanisms in the context of trade liberalizations.

Wacziarg and Welch (2008) identify dates that correspond to trade liberalization for 118 countries. They show that, on average, after trade liberalization, the relative price of investment falls and the real investment rate increases. These facts are both consistent with the mechanisms in our model, but are missing in Anderson, Larch, and Yotov (2015).

Hsieh (2001) provides evidence on these two channels via a contrast between Argentina and India. During the 1990s, India reduced barriers to imports that resulted in a 20 percent fall in the relative price of capital between 1990 and 2005. Consequently, the investment rate increased by 1.5 times during the same time period. After the Great Depression, Argentina restricted imports. From the late 1930s to the late 1940s, the relative price of capital doubled

Table 2: Outcomes in Greece from global 45% reduction in barriers

	Model 1	Model 2	Model 3	Model 4
Half life for capital	9.9 yrs	18.2 yrs	19.5 yrs	9.3 yrs
Initial change consumption	-5.1 %	13.1 %	9.9 %	9.8 %
Dynamic-to-SS gains	60.4 %	82.5 %	59.6 %	82.5 %

Notes: Model 1: endogenous relative price and endogenous nominal investment rate. Model 2: exogenous relative price and exogenous nominal investment rate. Model 3: endogenous relative price and exogenous nominal investment rate. Model 4: exogenous relative price and endogenous nominal investment rate. Here, relative price refers to $\frac{P_x}{P_c}$ and nominal investment rate refers to $\frac{P_x X}{P_c y L}$. We consider Greece since it is the country with the median gains from trade.

and the investment rate declined.

Using the liberalization dates obtained in Wacziarg and Welch (2008) and in Sachs and Warner (1995), we examine the transitional dynamics for India and Argentina. Figure 10 shows the real investment rates in Argentina and in India, before and after trade liberalization, which occurred in 1991 in Argentina, and in 1994 in India. Figure 10 shows the corresponding paths for the relative price of investment in both counties. We also constructed measures of TFP in the data for these countries: the TFPs increased significantly after trade liberalization.

Note that we are not calibrating the model to a particular trade liberalization. Instead, our counterfactual consists of a uniform and permanent reduction of trade barriers around the world that helps us understand the effect of our mechanisms on welfare gains from trade. Therefore there is not a one to one mapping between the empirical findings presented in this section and the results for the dynamics of capital, investment and TFP that we obtain in our counterfactual. However, we are able to generate patterns that are qualitatively similar to those that we observe in actual trade liberalizations, as the figures suggest.

Furthermore, our quantitative findings are also not too different from the empirical evidence. To shed light on the quantitative predictions of the model, we compare the elasticity of the real investment rate with respect to the trade-to-GDP ratio implied by our model to that observed in the data after a trade liberalization. We use the results from Wacziarg and Welch (2008). They find that, for the average country, the trade-to-GDP ratio increases from 15% to 30%, and the real investment rate increases from 18% to 21% after a trade liberalization. This implies an elasticity of investment of 0.58. In our counterfactual, the trade-to-GDP ratio increases from 36% to 56%, and the real investment rate increases from

Figure 10: Real investment rate before and after trade liberalization: Argentina (left) and India (right)

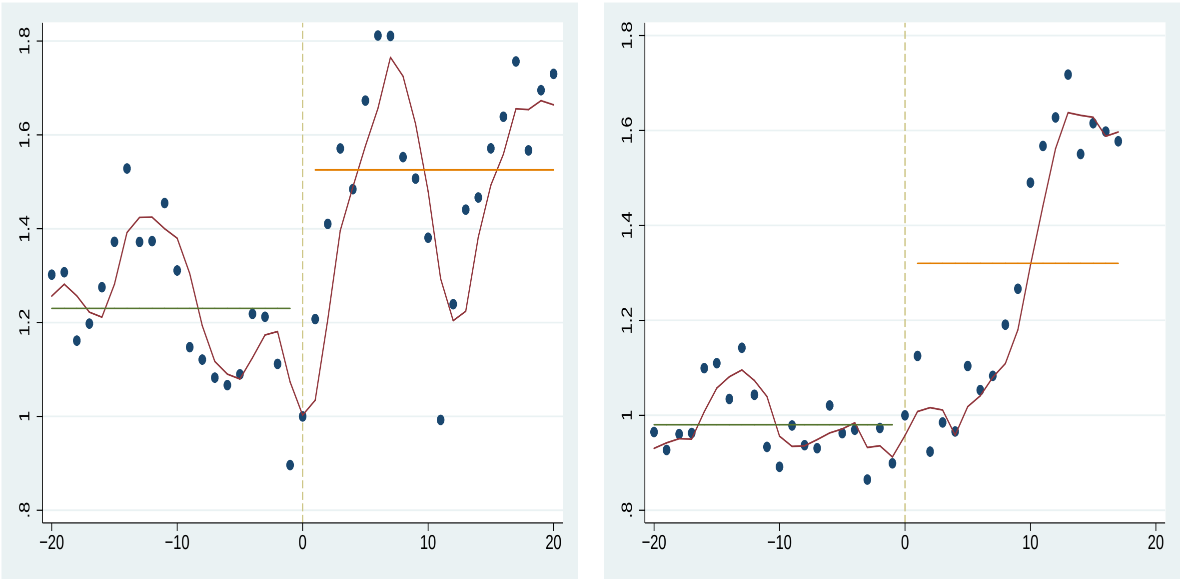
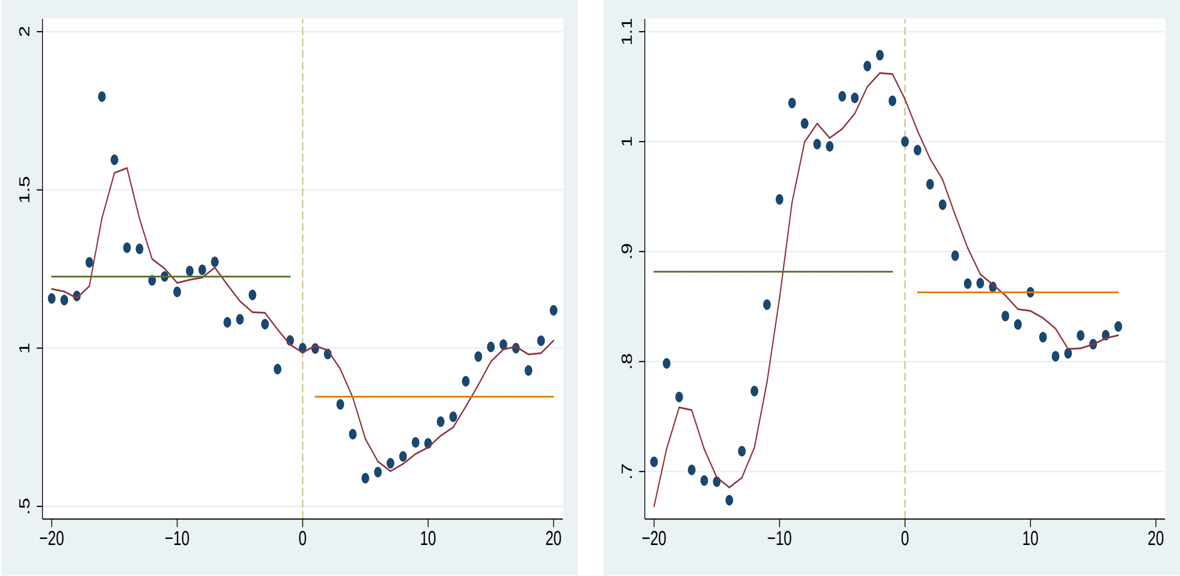


Figure 11: Relative price of investment before and after trade liberalization: Argentina (left) and India (right)



1% to 2.8%. This implies an elasticity of 0.8. The elasticity implied by our model is larger for a variety of reasons, including the fact that we study a simultaneous reduction in trade barriers rather than unilateral trade liberalizations as in Wacziarg and Welch (2008).

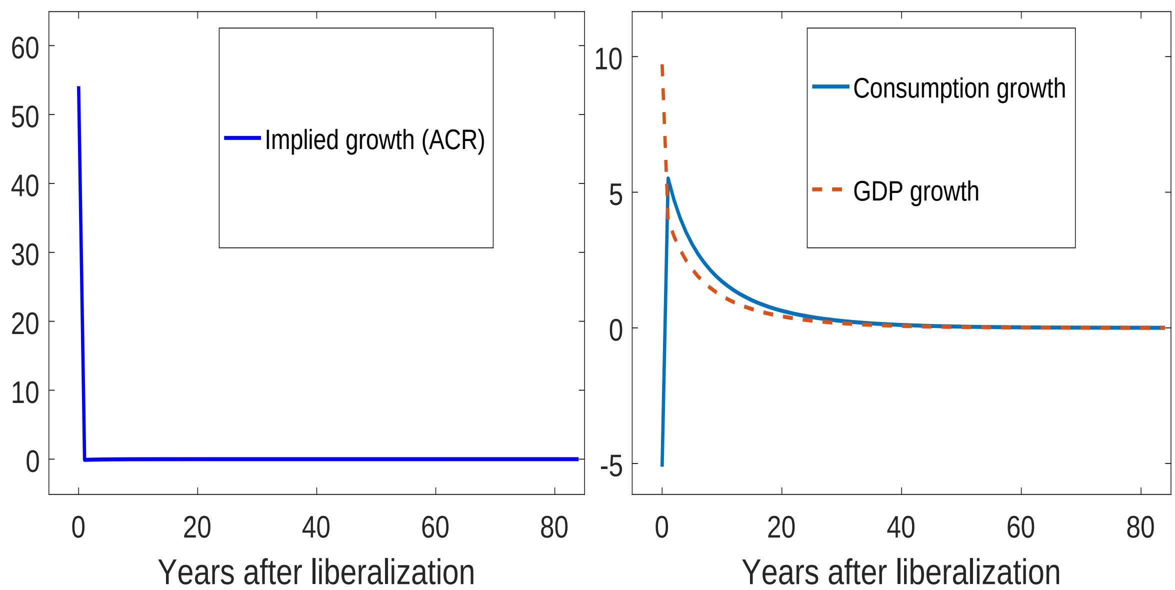
4.3 Relation to literature on welfare gains from trade

In this section, we compare our results on welfare to those that have been typically analyzed in the trade literature. In particular, we compare, period by period, welfare gains from trade using the same formula as in Arkolakis, Costinot, and Rodríguez-Clare (2012) in our model with capital (augmented ACR), to those resulting from comparing consumption growth period by period. The first measure is a"sufficient statistics" calculation in that it depends only on changes in the home trade share and an elasticity parameters. The sufficient-statistics calculation is equivalent to comparing welfare in a series of static exercises. The second measure captures the effect of capital accumulation on welfare gains from trade, and hence accounts for the whole transitional dynamics of the model after a trade liberalization, which cannot be summarized by a sufficient statistic. Figure 12 plots both measures.

In the augmented ACR case, all the gains from trade occur in the first period. The reason is that welfare gains occur through a decrease in the home trade share, which jumps upon impact and it reaches its new steady-state immediately. This is consistent with models that measure welfare gains from trade in a static context. If instead we take into account the transitional dynamics and compute consumption growth (or income growth) period by period, we observe that consumption drops upon impact and then it starts increasing, period by period, until it reaches its new steady-state. The initial drop is driven by a decrease in the relative price of investment, yielding a large increase in investment. But, after the initial period, consumption growth is positive, but diminishes toward zero as the economy converges to the new stay state. Income growth jumps on impact, and its growth rate gradually diminishes to zero over time. This exercise shows that measuring welfare using sufficient-statistics approaches, such as ACR, can yield a very different picture than the actual changes in consumption and income, when computed along the transition path.

As a final note, the sufficient-statistic formula is typically applied to assess the welfare costs of moving to autarky, since the home trade share in autarky is 1, and the current home trade share is observed in the data. In moving to free trade, even in a static model, one needs to solve for the home trade shares that arise under free trade. In our model there is no sufficient-statistic to compute the home trade share under free trade since it depends on

Figure 12: Comparison of welfare measures along the transition



the world distribution of capital.[10]

[10] In a one-sector model with no capital, Waugh and Ravikumar (2016) derive sufficient-statistics to measure the gains from moving to free trade. It involves the elasticities, the current home trade share, and current output.

5 Conclusion

We build a multi-country trade model with capital accumulation to study gains from trade. The model features endogenous investment rate and endogenous relative price of investment. We then solve for the transitional dynamics of a trade liberalization in levels rather than in differences. Our counterfactual suggests that dynamic gains are 60 percent of the gains across steady states, and three times larger than those implied by a static model with no capital accumulation. Furthermore, endogenous relative price of investment implies higher capital stocks whereas endogenous investment rate allows for faster convergence.

Our paper is a contribution to a large literature in international trade that focuses on measuring welfare gains from trade. Typically, these models are static and based on "sufficient statistics". We find large difference between changes in welfare in a model with endogenous capital accumulation and those measured by "sufficient statistics" along the transition, pointing at the importance on modeling dynamics explicitly when measuring the benefits of openness.

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A Solution algorithm

In this section of the Appendix we describe the algorithm for computing 1) the steady state and 2) the transition path. Before going further into the algorithms, we introduce some notation. We denote the steady-state objects using the $\star$ as a superscript, i.e., $K_i^\star$ is the steady state stock of capital in country i. We denote the cross-country vector of capital at a point in time using vector notation; $\vec{K}_t = \{K_{it}\}_{i=1}^I$ is the vector of capital stocks across countries at time t.

A.1 Computing the steady state equilibrium

The steady state equilibrium consists of 23 objects: $\vec{w}^\star$, $\vec{r}^\star$, $\vec{P}_c^\star$, $\vec{P}_m^\star$, $\vec{P}_x^\star$, $\vec{C}^\star$, $\vec{X}^\star$, $\vec{K}^\star$, $\vec{Q}^\star$, $\vec{Y}_c^\star$, $\vec{Y}_m^\star$, $\vec{Y}_x^\star$, $\vec{K}_c^\star$, $\vec{K}_m^\star$, $\vec{K}_x^\star$, $\vec{L}_c^\star$, $\vec{L}_m^\star$, $\vec{L}_x^\star$, $\vec{M}_c^\star$, $\vec{M}_m^\star$, $\vec{M}_x^\star$, $\vec{\pi}^\star$. Table A.1 provides a list of equilibrium conditions that these objects must satisfy.

We use the technique from Mutreja, Ravikumar, and Sposi (2014), which builds on Alvarez and Lucas (2007), to solve for the steady state. The idea is to guess at a vector of wages, then recover all remaining prices and quantities using optimality conditions and market clearing conditions, excluding the trade balance condition. We then use departures from the the trade balance condition in each country to update our wage vector and iterate until we find a wage vector that satisfies the trade balance condition. The following steps outline our procedure in more detail.

Table A.1: Equilibrium conditions in steady state

1	$r_i^\star K_{ci}^\star = \alpha\nu_c P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
2	$r_i^\star K_{mi}^\star = \alpha\nu_m P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
3	$r_i^\star K_{xi}^\star = \alpha\nu_x P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
4	$w_i^\star L_{ci}^\star = (1-\alpha)\nu_c P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
5	$w_i^\star L_{mi}^\star = (1-\alpha)\nu_m P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
6	$w_i^\star L_{xi}^\star = (1-\alpha)\nu_x P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
7	$P_{mi}^\star M_{ci}^\star = (1-\nu_c) P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
8	$P_{mi}^\star M_{mi}^\star = (1-\nu_m) P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
9	$P_{mi}^\star M_{xi}^\star = (1-\nu_x) P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
10	$K_{ci}^\star + K_{mi}^\star + K_{xi}^\star = K_i^\star$	$\forall(i)$
11	$L_{ci}^\star + L_{mi}^\star + L_{xi}^\star = L_i$	$\forall(i)$
12	$M_{ci}^\star + M_{mi}^\star + M_{xi}^\star = Q_i^\star$	$\forall(i)$
13	$C_i^\star = Y_{ci}^\star$	$\forall(i)$
14	$\sum_{j=1}^{I} P_{mj}^\star \left(M_{cj}^\star + M_{mj}^\star + M_{xj}^\star\right)\pi_{ji} = P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
15	$X_i^\star = Y_{xi}^\star$	$\forall(i)$
16	$P_{ci}^\star = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_i^\star}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_i^\star}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mi}^\star}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i)$
17	$P_{mi}^\star = \gamma\left[\sum_{j=1}^{I}(u_{mj}^\star d_{ij})^{-\theta} T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i)$
18	$P_{xi}^\star = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_i^\star}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_i^\star}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mi}^\star}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i)$
19	$\pi_{ij}^\star = \frac{(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}{\sum_{j=1}^{I}(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}$	$\forall(i,j)$
20	$P_{mi}^\star Y_{mi}^\star = P_{mi}^\star Q_i^\star$	$\forall(i)$
21	$P_{ci}^\star C_i^\star + P_{xi}^\star X_i^\star = r_i^\star K_i^\star + w_i^\star L_i^\star$	$\forall(i)$
22	$X_i^\star = \delta K_i^\star$	$\forall(i)$
23	$r_i^\star = \left(\frac{1}{\beta} - (1-\delta)\right) P_{xi}^\star$	$\forall(i)$

Note: $u_{mj}^\star = \left(\frac{r_j^\star}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_j^\star}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mj}^\star}{1-\nu_m}\right)^{1-\nu_m}$.

1. We guess a vector of wages $\vec{w} \in \Delta = \{w \in \mathbb{R}^I_+ : \sum_{i=1}^I \frac{w_i L_i}{1-\alpha} = 1\}$; that is, with world GDP as the numéraire.

2. We compute prices $\vec{P}_c, \vec{P}_x, \vec{P}_m$, and $\vec{r}$ simultaneously using conditions 16, 17, 18, and 23 in Table A.1. To complete this step, we compute the bilateral trade shares $\vec{\vec{\pi}}$ using condition 19.

3. We compute the aggregate capital stock as $K_i = \frac{\alpha}{1-\alpha}\frac{w_i L_i}{r_i}$, for all i, which derives easily from optimality conditions 1 & 4, 2 & 5, and 3 & 6, coupled with market clearing conditions for capital and labor 10 &11 in Table A.1.

4. We use condition 22 to solve for steady state investment $\vec{X}$. Then we use condition 21 to solve for steady state consumption $\vec{C}$.

5. We combine conditions 4 & 13 to solve for $\vec{L}_c$, combine conditions 5 & 14 to solve for $\vec{L}_x$, and use condition 11 to solve for $\vec{L}_m$. Next we combine conditions 1 & 4 to solve for $\vec{K}_c$, combine conditions 2 & 5 to solve for $\vec{K}_M$, and combine conditions 3 & 6 to solve for $\vec{K}_x$. Similarly, we combine conditions 4 & 7 to solve for $\vec{M}_c$, combine conditions 5 & 8 to solve for $\vec{M}_m$, and combine conditions 6 & 9 to solve for $\vec{M}_x$.

6. We compute $\vec{Y}_c$ using condition 13, compute $\vec{Y}_m$ using condition 14, and compute $\vec{Y}_x$ using condition 15.

7. We compute an excess demand equation as in Alvarez and Lucas (2007) defined as

$$Z_i(\vec{w}) = \frac{P_{mi}Y_{mi} - P_{mi}Q_i}{w_i}$$

(the trade deficit relative to the wage). Condition 20 requires that $Z_i(\vec{w}) = 0$ for all i. If the excess demand is sufficiently close to zero then we have an equilibrium. If not, we update our guess at the equilibrium wage vector using the information in the excess demand as follows.

$$\Lambda_i(\vec{w}) = w_i \left(1 + \psi \frac{Z_i(\vec{w})}{L_i}\right)$$

is be the updated guess to the wage vector, where ψ is chosen to be sufficiently small so that $\Lambda > 0$. Note that $\sum_{i=1}^I \frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = \sum_{i=1}^I \frac{w_i L_i}{1-\alpha} + \psi \sum_{i=1}^I w_i Z_i(\vec{w})$. As in Alvarez and Lucas (2007), it is easy to show that $\sum_{i=1}^I w_i Z_i(\vec{w}) = 0$ which implies that $\sum_{i=1}^I \frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = 1$, and hence, $\Lambda : \Delta \to \Delta$. We return to step 2 with our updated wage vector and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to zero. In our computations we find that our preferred convergence metric:

$$\max_{i=1}^{I} \{|Z_i(\vec{w})|\}$$

converges roughly monotonically towards zero.

A.2 Computing the equilibrium transition path

The equilibrium transition path consists of 23 objects: $\{\vec{w}_t\}_{t=1}^{\infty}$, $\{\vec{r}_t\}_{t=1}^{\infty}$, $\{\vec{P}_{ct}\}_{t=1}^{\infty}$, $\{\vec{P}_{mt}\}_{t=1}^{\infty}$, $\{\vec{P}_{xt}\}_{t=1}^{\infty}$, $\{\vec{C}_t\}_{t=1}^{\infty}$, $\{\vec{X}_t\}_{t=1}^{\infty}$, $\{\vec{K}_t\}_{t=1}^{\infty}$, $\{\vec{Q}_t\}_{t=1}^{\infty}$, $\{\vec{Y}_{ct}\}_{t=1}^{\infty}$, $\{\vec{Y}_{mt}\}_{t=1}^{\infty}$, $\{\vec{Y}_{xt}\}_{t=1}^{\infty}$, $\{\vec{K}_{ct}\}_{t=1}^{\infty}$, $\{\vec{K}_{mt}\}_{t=1}^{\infty}$, $\{\vec{K}_{xt}\}_{t=1}^{\infty}$, $\{\vec{L}_{ct}\}_{t=1}^{\infty}$, $\{\vec{L}_{mt}\}_{t=1}^{\infty}$, $\{\vec{L}_{xt}\}_{t=1}^{\infty}$, $\{\vec{M}_{ct}\}_{t=1}^{\infty}$, $\{\vec{M}_{mt}\}_{t=1}^{\infty}$, $\{\vec{M}_{xt}\}_{t=1}^{\infty}$, $\{\vec{\vec{\pi}}_t\}_{t=1}^{\infty}$ (we use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix in each period t). Table A.2 provides a list of equilibrium conditions that these objects must satisfy.

We reduce the infinite-dimensionality down to a finite-time problem from $t = 1, \ldots, T$, with T sufficiently large to ensure that the endogenous variables settle down to a steady state by T. As such, solving the transition first requires solving the terminal steady state. Also, it requires taking an initial stock of capital as given (either by computing an initial steady state or just taking it from data, for instance).

Our solution procedure mimics the idea of that for the steady state, but slightly modified to take into account the dynamic aspect as in Sposi (2012). Basically, we start with an initial guess for the entire sequence of wage vectors and rental rates (across countries and over time). Form these two objects we can recover all prices and quantities, across countries and throughout time, using optimality conditions and market clearing conditions, excluding the trade balance condition and the market clearing condition for the stock of capital. We then use departures from the the trade balance condition and the market clearing condition for the stock of capital at each point in time and in each country to update our wages and rental rates. Then we iterate until we find wages and rental rates that satisfy the trade balance condition and the market clearing condition for the stock of capital. We describe the steps to our procedure in more detail below.

1. We guess the entire path for wages $\{\vec{w}_t\}_{t=1}^{T}$ and rental rates $\{\vec{r}_t\}_{t=2}^{T}$ across countries, such that $\sum_i \frac{w_{it}L_i}{1-\alpha} = 1\,(\forall t)$. In period 1 set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1\vec{L}}{\vec{K}_1}\right)$ since the initial stock of capital is predetermined.

2. We compute prices $\{\vec{P}_{ct}\}_{t=1}^{T}, \{\vec{P}_{xt}\}_{t=1}^{T}$, and $\{\vec{P}_{mt}\}_{t=1}^{T}$ simultaneously using conditions

Table A.2: Equilibrium conditions along the transition

1	$r_{it}K_{cit} = \alpha\nu_c P_{cit}Y_{cit}$	$\forall(i,t)$
2	$r_{it}K_{mit} = \alpha\nu_m P_{mit}Y_{mit}$	$\forall(i,t)$
3	$r_{it}K_{xit} = \alpha\nu_x P_{xit}Y_{xit}$	$\forall(i,t)$
4	$w_{it}L_{cit} = (1-\alpha)\nu_c P_{cit}Y_{cit}$	$\forall(i,t)$
5	$w_{it}L_{mit} = (1-\alpha)\nu_m P_{mit}Y_{mit}$	$\forall(i,t)$
6	$w_{it}L_{xit} = (1-\alpha)\nu_x P_{xit}Y_{xit}$	$\forall(i,t)$
7	$P_{mit}M_{cit} = (1-\nu_c)P_{cit}Y_{cit}$	$\forall(i,t)$
8	$P_{mit}M_{mit} = (1-\nu_m)P_{mit}Y_{mit}$	$\forall(i,t)$
9	$P_{mit}M_{xit} = (1-\nu_x)P_{xit}Y_{xit}$	$\forall(i,t)$
10	$K_{cit} + K_{mit} + K_{xit} = K_{it}$	$\forall(i,t)$
11	$L_{cit} + L_{mit} + L_{xit} = L_i$	$\forall(i,t)$
12	$M_{cit} + M_{mit} + M_{xit} = Q_{it}$	$\forall(i,t)$
13	$C_{it} = Y_{cit}$	$\forall(i,t)$
14	$\sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}$	$\forall(i,t)$
15	$X_{it} = Y_{xit}$	$\forall(i,t)$
16	$P_{cit} = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_{it}}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_{it}}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mit}}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i,t)$
17	$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i,t)$
18	$P_{xit} = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_{it}}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_{it}}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mit}}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i,t)$
19	$\pi_{ijt} = \frac{(u_{mjt}d_{ij})^{-\theta}T_{mj}}{\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}}$	$\forall(i,j,t)$
20	$P_{mit}Y_{mit} = P_{mit}Q_{it}$	$\forall(i,t)$
21	$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i$	$\forall(i,t)$
22	$K_{it+1} = (1-\delta)K_{it} + X_{it}$	$\forall(i,t)$
23	$\left(\frac{C_{it+1}}{C_{it}}\right) = \beta^{\sigma}\left(1 + \frac{r_{it+1}}{P_{xit+1}} - \delta\right)^{\sigma}\left(\frac{P_{xt+1}/P_{ct+1}}{P_{xt}/P_{ct}}\right)^{\sigma}$	$\forall(i,t)$

Note: $u_{mjt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mjt}}{1-\nu_m}\right)^{1-\nu_m}$.

16, 17, and 18, in Table A.2. To complete this step, we compute the bilateral trade shares $\{\vec{\pi}_t\}_{t=1}^T$ using condition 19.

3. This step is slightly more involved. We show how we compute the path for consumption and investment by solving the intertemporal problem of the household. We do this in three parts. First we derive the lifetime budget constraint, second we derive the fraction of lifetime wealth allocated to consumption at each period t, and third we recover the sequence for investment and the stock of capital.

Deriving the lifetime budget constraint To begin, we compute the lifetime budget constraint for the representative household in country i. Begin with the period budget constraint from condition 21 and combine it with the capital accumulation technology in condition 22 to get

$$K_{it+1} = \left(\frac{w_{it}}{P_{xit}}\right) L_i + \left(1 + \frac{r_{it}}{P_{xit}} - \delta\right) K_{it} - \left(\frac{P_{cit}}{P_{xit}}\right) C_{it}.$$

We will iterate the period budget constraint forward through time and derive a lifetime budget constraint. At time $t = 1$ the stock of capital, $K_{i1} > 0$, is given. Next, compute the stock of capital at time $t = 2$.

$$K_{i2} = \left(\frac{w_{i1}}{P_{xi1}}\right) L_i + \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right) K_{i1} - \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1}$$

Similarly, compute the stock of capital at time $t = 3$, but do it so that it is in terms the initial stock of capital.

$$\begin{aligned}
K_{i3} &= \left(\frac{w_{i2}}{P_{xi2}}\right) L_i + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) K_{i2} - \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2} \\
\Rightarrow K_{i3} &= \left(\frac{w_{i2}}{P_{xi2}}\right) L_{i2} + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{w_{i1}}{P_{xi1}}\right) L_i \\
&+ \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right) K_{i1} \\
&- \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1} - \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2}
\end{aligned}$$

Continue to period 4 in a similar way

$$
\begin{aligned}
K_{i4} &= \left(\frac{w_{i3}}{P_{xi3}}\right) L_i + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) K_{i3} - \left(\frac{P_{ci3}}{P_{xi3}}\right) C_{i3} \\
\Rightarrow K_{i4} &= \left(\frac{w_{i3}}{P_{xi3}}\right) L_i + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(\frac{w_{i2}}{P_{xi2}}\right) L_i \\
&\quad + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{w_{i1}}{P_{xi1}}\right) L_i \\
&\quad + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right) K_{i1} \\
&\quad - \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1} \\
&\quad - \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2} - \left(\frac{P_{ci3}}{P_{xi3}}\right) C_{i3}
\end{aligned}
$$

Before we continue, it will be useful to define $(1 + R_{it}) = \prod_{n=1}^{t} \left(1 + \frac{r_{in}}{P_{xin}} - \delta\right)$.

$$
\begin{aligned}
\Rightarrow K_{i4} = &\frac{(1 + R_{i3}) \left(\frac{w_{i3}}{P_{xi3}}\right) L_i}{(1 + R_{i3})} + \frac{(1 + R_{i3}) \left(\frac{w_{i2}}{P_{xi2}}\right) L_{i2}}{(1 + R_{i2})} + \frac{(1 + R_{i3}) \left(\frac{w_{i1}}{P_{xi1}}\right) L_i}{(1 + R_{i1})} \\
&+ (1 + R_{i3}) K_{i1} \\
&- \frac{(1 + R_{i3}) \left(\frac{P_{ci3}}{P_{xi3}}\right) C_{i3}}{(1 + R_{i3})} - \frac{(1 + R_{i3}) \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2}}{(1 + R_{i2})} - \frac{(1 + R_{i3}) \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1}}{(1 + R_{i1})} \\
\Rightarrow K_{i4} = &\sum_{n=1}^{3} \frac{(1 + R_{i3}) \left(\frac{w_{in}}{P_{xin}}\right) L_{in}}{(1 + R_{in})} - \sum_{n=1}^{3} \frac{(1 + R_{i3}) \left(\frac{P_{cin}}{P_{xin}}\right) C_{in}}{(1 + R_{in})} + (1 + R_{i3}) K_{i1}
\end{aligned}
$$

By induction, for any time t,

$$
\begin{aligned}
K_{it+1} &= \sum_{n=1}^{t} \frac{(1 + R_{it}) \left(\frac{w_{in}}{P_{xin}}\right) L_i}{(1 + R_{in})} - \sum_{n=1}^{t} \frac{(1 + R_{it}) \left(\frac{P_{cin}}{P_{xin}}\right) C_{in}}{(1 + R_{in})} + (1 + R_{it}) K_{i1} \\
\Rightarrow K_{it+1} &= (1 + R_{it}) \left(\sum_{n=1}^{t} \frac{\left(\frac{w_{in}}{P_{xin}}\right) L_i}{(1 + R_{in})} - \sum_{n=1}^{t} \frac{\left(\frac{P_{cin}}{P_{xin}}\right) C_{in}}{(1 + R_{in})} + K_{i1} \right)
\end{aligned}
$$

Finally, observe the previous expression as of $t = T$ and rearrange terms to derive the lifetime budget constraint.

$$\sum_{n=1}^{T} \frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = \underbrace{\sum_{n=1}^{T} \frac{w_{in}L_i}{P_{xin}(1+R_{in})} + K_{i1} - \frac{K_{iT+1}}{(1+R_{iT})}}_{W_i} \tag{A.1}$$

In the lifetime budget constraint (A.1), we use W_i to denote the net present value of lifetime wealth in country i, and we take the capital stock at the end of time, K_{iT+1}, as given; in our case it will be the capital stock in the new steady state with T sufficiently large. Note that by imposing the terminal condition that $K_{iT+1} = K_i^\star$, the transversality condition is automatically satisfied since $\lim_{T\to\infty}(1 + R_{iT}) = \infty$ and $\lim_{T\to\infty} K_{iT+1} = K_i^\star$.

Solving for the path of consumption Next we compute how the lifetime consumption expenditures will be allocated throughout time. The Euler equation (condition 23) implies the following relationship between consumption in any two periods t and n:

$$C_{in} = \beta^{\sigma(n-t)} \left(\frac{(1+R_{in})}{(1+R_{it})}\right)^{\sigma} \left(\frac{P_{xin}}{P_{xit}}\right)^{\sigma} \left(\frac{P_{cit}}{P_{cin}}\right)^{\sigma} C_{it}$$

$$\Rightarrow \frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = \beta^{\sigma(n-t)} \left(\frac{P_{xin}(1+R_{in})}{P_{xit}(1+R_{it})}\right)^{\sigma-1} \left(\frac{P_{cin}}{P_{cit}}\right)^{1-\sigma} \left(\frac{P_{cit}C_{it}}{P_{xit}(1+R_{it})}\right)$$

Since equation (A.1) implies that $\sum_{n=1}^{T} \frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = W_i$, then we can rearrange the previous expression to obtain

$$\frac{P_{cit}C_{it}}{P_{xit}(1+R_{it})} = \underbrace{\left(\frac{\beta^{\sigma t} P_{xit}^{\sigma-1}(1+R_{it})^{\sigma-1} P_{cit}^{1-\sigma}}{\sum_{n=1}^{T} \beta^{\sigma n} P_{xin}^{\sigma-1}(1+R_{in})^{\sigma-1} P_{cin}^{1-\sigma}}\right)}_{\xi_{it}} W_i \tag{A.2}$$

That is, each period the household spends a share ξ_{it} of lifetime wealth on consumption, with $\sum_{t=1}^{T} \xi_{it} = 1$ for all i. Note that ξ_{it} depends only on prices.

Computing investment and the sequence of capital stocks Given paths of consumption, solve for investment $\{\vec{X}_t\}_{t=1}^{T}$ using the period budget constraint in condition 21. The catch here is that there is no restriction that household investment be non-negative up to this point. Looking ahead, there is no way that negative investment can satisfy market clearing conditions together with firm optimality conditions. As such, we restrict our attention to transition paths for which investment is always positive, which we find is the case for the equilibrium outcomes in our paper. However, off the equilibrium path, if during the course of the iterations any given value of X_{it} is negative, then set we it equal to a small positive number.

The last part of this step is to use condition 22 to compute the path for the stock of capital. $\{\vec{K}_t\}_{t=2}^{T+1}$. Note that $\vec{K}_1$ is taken as given and that $\vec{K}_{T+1}$ is by construction equal to the terminal steady-state value.

4. We combine conditions 4 & 13 to solve for $\{\vec{L}_{ct}\}_{t=1}^{T}$, combine conditions 5 & 14 to solve for $\{\vec{L}_{xt}\}_{t=1}^{T}$, and use condition 11 to solve for $\{\vec{L}_{mt}\}_{t=1}^{T}$. Next we combine conditions 1 & 4 to solve for $\{\vec{K}_{ct}\}_{t=1}^{T}$, combine conditions 2 & 5 to solve for $\{\vec{K}_{mt}\}_{t=1}^{T}$, and combine conditions 3 & 6 to solve for $\{\vec{K}_{xt}\}_{t=1}^{T}$. Similarly, we combine conditions 4 & 7 to solve for $\{\vec{M}_{ct}\}_{t=1}^{T}$, combine conditions 5 & 8 to solve for $\{\vec{M}_{mt}\}_{t=1}^{T}$, and combine conditions 6 & 9 to solve for $\{\vec{M}_{xt}\}_{t=1}^{T}$.

5. We compute $\{\vec{Y}_{ct}\}_{t=1}^{T}$ using condition 13, compute $\{\vec{Y}_{mt}\}_{t=1}^{T}$ using condition 14, and compute $\{\vec{Y}_{xt}\}_{t=1}^{T}$ using condition 15.

6. Until now we have imposed all equilibrium conditions except for two: The first being the trade balance condition 20, and the second being the capital market clearing condition 10.

Trade balance condition We compute an excess demand equation as in Alvarez and Lucas (2007) defined as

$$Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = \frac{P_{mit}Y_{mit} - P_{mit}Q_{it}}{w_{it}}$$

(the trade deficit relative to the wage). Condition 20 requires that $Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = 0$ for all i. If this is different from zero in at least some country at some point in time we update our guess at the wages as follows.

$$\Lambda_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = w_{it}\left(1 + \psi\frac{Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right)}{L_i}\right)$$

is the updated guess to the wages, where ψ is chosen to be sufficiently small so that $\Lambda^w > 0$.

Market clearing condition for the stock of capital We compute an excess demand equation defined as

$$Z_{it}^r \left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^T \right) = \frac{w_{it} L_i}{1-\alpha} - \frac{r_{it} K_{it}}{\alpha}$$

We have imposed, using conditions 1-6, that within each sector $\frac{r_{it} K_{bit}}{\alpha} = \frac{w_{it} L_{bit}}{1-\alpha}$. We have also imposed condition 11 that the labor market clear. Hence, the market for capital is in excess demand (i.e., $K_{cit} + K_{mit} + K_{xit} > K_{it}$) in country i at time t if and only if $\left(\frac{w_{it} L_i}{1-\alpha}\right) > \left(\frac{r_{it} K_{it}}{\alpha}\right)$ (it is in excess supply if and only if the inequality is $<$). If this condition does not hold with equality in some country at some point in time then we update our guess for rental rates as follows. Let

$$\Lambda_{it}^r \left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^T \right) = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{L_i}{K_{it}} \right) \Lambda_{it}^w \left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^T \right)$$

be the updated guess to the rental rates (taking into account the updated guess for wages).

We return to step 2 with our updated wages and rental rates and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to zero. In our computations we find that our preferred convergence metric:

$$\max_{t=1}^{T} \left\{ \max_{i=1}^{I} \left\{ \left| Z_{it}^w \left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^T \right) \right| + \left| Z_{it}^r \left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^T \right) \right| \right\} \right\}$$

converges roughly monotonically towards zero.

Along the equilibrium transition, $\sum_i w_{it} L_i + r_{it} K_{it} = 1$ $(\forall t)$; that is, we have chosen world GDP as the numéraire at each point in time.

The fact that $\vec{K}_{T+1} = \vec{K}^\star$ at each iteration is a huge benefit of our algorithm compared to algorithms that rely on shooting procedures or those that rely on using the Euler equation for updating. Such algorithms inherit the instability (saddle-path) properties of the Euler equation and generate highly volatile terminal stocks of capital with respect to the initial guess. Instead, we impose the Euler equation and the terminal condition for $\vec{K}_{T+1} = \vec{K}^\star$ at each iteration and use excess demand equations for our updating rules, just as in the computation of static models such as Alvarez and Lucas (2007). Another main advantage of using excess-demand iteration is that we do not need to compute gradients to choose step directions or step size, as is the case of most nonlinear solvers such as the ones used by Eaton, Kortum, Neiman, and Romalis (2015) and Kehoe, Ruhl, and Steinberg (2016). This saves a tremendous amount of computational time, particularly as the number of countries or the number of time periods is increased.

B Derivations

This section of the Appendix shows the derivations of key structural relationships. We refer to Table A.2 for the basis of the derivations and omit time subscripts to ease notation. We begin by deriving an expression for $\frac{w_i}{P_{mi}}$ that will be used repeatedly.

Combining conditions 17 and 19 we obtain

$$\pi_{ii} = \gamma^{-\theta} \left(\frac{u_{mi}^{-\theta} T_{mi}}{P_{mi}^{-\theta}} \right)$$

Use the fact that $u_{mi} = B_m r_i^{\alpha\nu_m} w_i^{(1-\alpha)\nu_m} P_{mi}^{1-\nu_m}$, where B_m is a collection of constants, then rearrange to obtain

$$\begin{aligned} P_{mi} &= \left(\frac{T_{mi}}{\pi_{ii}} \right)^{-\frac{1}{\theta}} \left(\frac{r_i}{w_i} \right)^{\alpha\nu_m} \left(\frac{w_i}{P_{mi}} \right)^{\nu_m} P_{mi} \\ \Rightarrow \frac{w_i}{P_{mi}} &= \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\theta}}}{\gamma B_m} \right)^{\frac{1}{\nu_m}} \left(\frac{w_i}{r_i} \right)^{\alpha} \end{aligned} \tag{B.1}$$

Note that this relationship holds in both the steady state and along the transition.

Relative prices We show how to derive the price of consumption relative to intermediates; the relative price of investment is analogous. Begin with condition 16

$$P_{ci} = \left(\frac{B_c}{A_{ci}} \right) \left(\frac{r_i}{w_i} \right)^{\alpha\nu_c} \left(\frac{w_i}{P_{mi}} \right)^{\nu_c} P_{mi}$$

where B_c is a collection of constants. Substitute equation (B.1) into the previous expression and rearrange to obtain

$$\frac{P_{ci}}{P_{mi}} = \left(\frac{B_c}{A_{ci}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{\nu_c}{\nu_m}} \tag{B.2}$$

Analogously,

$$\frac{P_{xi}}{P_{mi}} = \left(\frac{B_x}{A_{xi}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{\nu_x}{\nu_m}} \tag{B.3}$$

Note that these relationships hold in both the steady state and along the transition.

Capital-labor ratio We derive a structural relationship for the capital-labor ratio in the steady state only and make reference to conditions in Table A.1. Conditions 1-6 together with conditions 10 and 11 imply that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_i}{r_i}\right)$$

Using condition 23 we know that

$$r_i = \left(\frac{1}{\beta} - (1-\delta)\right) P_{xi}$$

which, by substituting into the prior expression implies that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{(1-\alpha)\left(\frac{1}{\beta} - (1-\delta)\right)}\right)\left(\frac{w_i}{P_{xi}}\right)$$

which leaves the problem of solving for $\frac{w_i}{P_{xi}}$. Equations (B.1) and (B.3) imply

$$\begin{aligned}\frac{w_i}{P_{xi}} &= \left(\frac{w_i}{P_{mi}}\right)\left(\frac{P_{mi}}{P_{xi}}\right) \\ &= \left(\frac{A_{xi}}{B_x}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{\nu_m}}\left(\frac{w_i}{r_i}\right)^{\alpha}\end{aligned}$$

Substituting in once more for $\frac{w_i}{r_i}$ in the previous expression yields

$$\left(\frac{w_i}{P_{xi}}\right)^{1-\alpha} = \left(\frac{1}{\beta} - (1-\delta)\right)^{-\alpha} \left(\frac{A_{xi}}{B_x}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{\nu_m}}$$

Solve out for the aggregate capital-labor ratio

$$\frac{K_i}{L_i} = \left(\frac{\frac{\alpha}{1-\alpha}}{\left(\frac{1}{\beta} - (1-\delta)\right)^{-\frac{1}{1-\alpha}}}\right) \left(\frac{A_{xi}}{B_x}\right)^{\frac{1}{1-\alpha}} \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{(1-\alpha)\nu_m}} \tag{B.4}$$

Note that we invoked steady state conditions so this expression does not necessarily hold along the transition path.

Income per capita We define (real) income per capita in our model as

$$y_i = \frac{r_i K_i + w_i L_i}{L_i P_{ci}}$$

We invoke conditions from Table A.2 for the remainder of this derivation. Conditions 1-6, 10, and 11 imply that

$$\begin{aligned} r_i K_i + w_i L_i &= \frac{w_i L_i}{1-\alpha} \\ \Rightarrow y_i &= \left(\frac{1}{1-\alpha}\right)\left(\frac{w_i}{P_{ci}}\right) \end{aligned}$$

To solve for $\frac{w_i}{P_{ci}}$ we use condition 16

$$\begin{aligned} P_{ci} &= \frac{B_c}{A_{ci}} \left(\frac{r_i}{w_i}\right)^{\alpha\nu_c} \left(\frac{w_i}{P_{mi}}\right)^{\nu_c} P_{mi} \\ \Rightarrow \frac{P_{ci}}{w_i} &= \frac{B_c}{A_{ci}} \left(\frac{r_i}{w_i}\right)^{\alpha\nu_c} \left(\frac{w_i}{P_{mi}}\right)^{\nu_c - 1} \end{aligned}$$

Substituting equation (B.1) into the previous expression, and exploiting the fact that $\frac{w_i}{r_i} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K_i}{L_i}\right)$ yields

$$
\begin{aligned}
y_i &= \left(\frac{1}{1-\alpha}\right)\left(\frac{w_i}{P_{ci}}\right) \\
&= \alpha^{-\alpha}(1-\alpha)^{\alpha-1}\left(\frac{A_{ci}}{B_c}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_c}{\theta\nu_m}}\left(\frac{K_i}{L_i}\right)^{\alpha}
\end{aligned}
\tag{B.5}
$$

Note that this expression holds both in the steady state and along the transition path.

The steady-state income per capita can be expressed more fundamentally by invoking equation (B.4) as

$$
y_i = \left(\frac{\left(\frac{1}{\beta}-(1-\delta)\right)^{-\frac{\alpha}{1-\alpha}}}{1-\alpha}\right)\left(\frac{A_{ci}}{B_c}\right)\left(\frac{A_{xi}}{B_x}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_c+\frac{\alpha}{1-\alpha}(1-\nu_x)}{\nu_m}}
\tag{B.6}
$$

C Data

This section of the Appendix describes the sources of data as well as any adjustments we make to the data to map it to the model.

C.1 Production and trade data

Mapping the trade dimension of our model to the data requires data on both production and international trade flows. Our focus is on manufactured intermediate goods. We interpret manufacturing broadly as defined by the International Standard Industrial Classification (ISIC).

We obtain production data from multiple sources. First, we utilize value added and gross output data from the (INDSTAT) which is reported at the two-digit Ievel using ISIC. This data countries extends no further than 2010, and even less for many countries. We turn to data on value added output in (UNIDO MEI) which reports value added output for 2011. For countries that report both value added and gross output in INDSTAT, we use the ratio from the year that is closet to 2011, and apply that ratio to the value added from UNIDO to recover gross output. For countries that are have no data on gross output in INDSTAT for any years, we apply the average ratio of value-added-to-gross output across all countries, and apply that ratio to the value added figure in UNIDO for 2011. In our data set, the ratio of value-added-to-gross output does not vary significantly over time, and is also not correlated with level of development or country size.

Our source of trade data is the UN Comtrade Database http://comtrade.un.org. Trade is reported for goods using revision 2 Standard International Trade Classification (SITC2) at the four-digit level. We make use of the correspondence tables created by Affendy, Sim Yee, and Satoru (2010) to map SITC2 to ISIC. We also omit any petroleum-related products from the trade data.

Using the trade and production data, we construct bilateral trade shares for each country pair by following Bernard, Eaton, Jensen, and Kortum (2003) as follows:

$$\pi_{ij} = \frac{X_{ij}}{ABS_{bi}},$$

where i denotes the importer and j denotes the exporter. X_{ij} denotes manufacturing trade flows from j to i, and ABS_i is country i's absorption defined as gross output less net exports of manufactures.

C.2 National accounts and price data

PPP GDP and population For our baseline calibration, we collect data on output-side real GDP at current PPPs (2005 U.S. dollars) from version 8.1 of the Penn World Tables (see Feenstra, Inklaar, and Timmer, 2015, (PWT from now on)) using the variable `cgdpo`.

We use the variable `pop` from PWT to measure the population in each country. The ratio $\frac{\texttt{cgdpo}}{\texttt{pop}}$ corresponds to GDP per capita, y, in our model.

In our counterfactuals, we compare changes over time to past trade liberalization episodes using national accounts data from the PWT: *rgdpna*, *rkna*, and *rtfpna*.

We take the price level of household consumption and the price level of capital formation (both relative to the price of output-side GDP in the U.S. in constant prices) from PWT using variables `pl_c` and `pl_i` respectively. These correspond to P_c and P_x in our model.

We construct the price of intermediate goods (manufacture) by using various data from the 2011 World Bank's International Comparison Program (ICP): http://siteresources.worldbank.org/ICPE
The data has several categories that fall under what we classify as manufactures: "Food and nonalcoholic beverages", "Alcoholic beverages, tobacco, and narcotics", "Clothing and foot wear", and "Machinery and equipment". The ICP reports expenditure data for these cat-

egories in both nominal U.S. dollars and unreal U.S. dollars. The conversion from nominal to real uses the PPP price, that is: the PPP price equals the ratio of nominal expenditures to real expenditures. As such, we simply compute the PPP for manufactures as a whole, for each country, as the sum of nominal expenditures across categories, divided by the sum of real expenditures across categories. For the RoW aggregate, we simply sum the expenditure across all of the countries that are not part of the 40 individual countries.

There is one mode step before we take these prices to the model. The data correspond to expenditures, thus include additional margins such as distribution. In order to adjust for this this, we first construct a price for distribution services. We assume that the price of distribution services is proportional to the overall price of services in each country and use the same method as above to compute the price across the following categories: "Housing, water, electricity, gas, and other fuels", "Health", "Transport", "Communication", "Recreation and culture", "Education", "Restaurants and hotels", and "Construction".

Now that we have the price of services in hand, we strip it away from the price of goods computed above to arrive at a measure of the price of manufactures that better corresponds to our model. In particular, let P_d denote the price of distribution services and let P_g denote the price of goods that includes the distribution margin. We assume that $P_g = P_d^{\rho} P_m^{1-\rho}$, where P_m is the price of manufactures. We set $\rho = 0.45$ which is a value commonly used in the literature.

D Additional figures and tables

Table D.1: Gains from trade – uniform reduction of barriers by 45%

Country	Isocode	Dynamic	Steady-state
Armenia	ARM	47.25	77.82
Australia	AUS	20.81	34.53
Austria	AUT	26.92	44.59
Bahamas	BHS	40.81	67.24
Baltics	BAL	32.89	54.41
Bangladesh	BGD	31.14	51.50
Barbados	BRB	52.67	86.61
Belarus	BLR	27.34	45.29
Belize	BLZ	55.73	91.58
Benin	BEN	43.71	72.05
Bhutan	BTN	40.39	66.54
Brazil	BRA	14.61	24.28
Bulgaria	BGR	37.31	61.65
Burundi	BDI	25.31	41.95
Cabo Verde	CPV	42.68	70.40
Cambodia	KHM	36.44	60.15
Cameroon	CMR	37.41	61.78

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Table D.1 – Continued

Country	Isocode	Dynamic	Steady-state
Canada	CAN	18.97	31.47
Central African Rep.	CAF	25.81	42.79
Chile	CHL	30.20	49.94
China, Hong Kong, Macao	CHM	11.46	19.08
Colombia	COL	27.75	45.93
Costa Rica	CRI	40.68	67.01
Cyprus	CYP	39.98	66.00
Czech Rep.	CZE	28.91	47.85
Cte d'Ivoire	CIV	40.18	66.28
Denmark	DNK	29.56	48.91
Dominican Rep.	DOM	23.65	39.18
Ecuador	ECU	39.05	64.38
Egypt	EGY	33.93	56.09
Ethiopia	ETH	35.74	59.06
Fiji	FJI	41.04	67.65
Finland	FIN	30.57	50.58
France	FRA	21.42	35.55
Georgia	GEO	46.64	76.82
Germany	DEU	20.71	34.39
Greece	GRC	32.31	53.46
Guatemala	GTM	27.82	46.05
Honduras	HND	37.68	62.14
Hungary	HUN	21.63	35.90
Iceland	ISL	47.46	78.15
India	IND	17.03	28.30
Indonesia	IDN	26.22	43.43
Iran	IRN	21.05	34.92
Ireland	IRL	29.62	49.00
Israel	ISR	37.70	62.21
Italy	ITA	21.90	36.35
Jamaica	JAM	41.79	68.88
Japan	JPN	13.12	21.83
Jordan	JOR	38.79	64.03
Kyrgyzstan	KGZ	40.43	66.69
Lesotho	LSO	30.60	50.59
Madagascar	MDG	36.88	60.90
Malawi	MWI	41.19	67.98
Maldives	MDV	46.18	76.10
Mauritius	MUS	44.76	73.78
Mexico	MEX	12.48	20.77
Morocco	MAR	38.71	63.89
Mozambique	MOZ	43.08	71.04
Nepal	NPL	29.50	48.79
New Zealand	NZL	33.12	54.73
Pakistan	PAK	21.47	35.62
Paraguay	PRY	40.87	67.35
Peru	PER	27.89	46.17
Philippines	PHL	29.50	48.80
Poland	POL	26.41	43.77
Portugal	PRT	30.65	50.71
Rep. of Korea	KOR	24.16	40.03
Rep. of Moldova	MDA	40.45	66.76
Romania	ROU	32.79	54.23
Russian Federation	RUS	23.85	39.55
Rwanda	RWA	22.45	37.25
Saint Vincent and the Grenadines	VCT	56.40	92.75
Sao Tome and Principe	STP	39.85	65.80
Senegal	SEN	46.60	76.78
South Africa	ZAF	29.62	49.01

Continued on next page...

Table D.1 – Continued

Country	**Isocode**	**Dynamic**	**Steady-state**
Spain	ESP	21.88	36.32
Sri Lanka	LKA	32.45	53.64
Sweden	SWE	28.09	46.51
Switzerland	CHE	28.71	47.51
TFYR of Macedonia	MKD	38.91	64.25
Thailand	THA	32.40	53.56
Tunisia	TUN	40.65	67.07
Turkey	TUR	26.77	44.35
USA	USA	10.97	18.26
Uganda	UGA	22.51	37.35
Ukraine	UKR	30.75	50.88
United Kingdom	GBR	23.83	39.51
United Rep. of Tanzania	TZA	44.10	72.70
Uruguay	URY	36.90	60.94
Venezuela	VEN	29.24	48.36
Viet Nam	VNM	36.46	60.19
Yemen	YEM	42.02	69.25