# Bargaining as a Struggle Between Competing Attempts at Commitment *

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**Abstract**

The strategic importance of commitment in bargaining is widely acknowledged. Yet disentangling its role from key features of canonical models, such as proposal power and reputational concerns, is difficult. This paper introduces a model of bargaining with strategic commitment at its core. Following Schelling (1956), commitment ability stems from the costly nature of concession and is endogenously determined by players' demands. Agreement is immediate for familiar bargainers, modelled via renegotiation-proofness. The unique prediction at the high concession cost limit provides a strategic foundation for the Kalai bargaining solution. Equilibria with delay feature a form of gradualism in demands.

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# 1 Introduction

If two agents seek to divide some surplus, what division will they agree on and when and how? This set of questions, that I collectively label the bargaining problem, is key to a vast range of economic interactions. Economic models rely on the strategic theory of bargaining to resolve it, either directly or indirectly by informing the appropriate choice of a bargaining solution.

Strategic models of bargaining that allow negotiations to unfold over time typically have at their core either the alternating-offers model of Rubinstein (1982) or the reputation model of Abreu and Gul (2000). Schelling (1956, 1960) proposed a third approach. As summarized in Crawford (1982), Schelling views the bargaining process as a struggle between players to commit themselves to —that is, to convince their opponent of their inability to retreat from —advantageous bargaining positions. Schelling's own treatment of his approach was impressionistic and by way of examples. Subsequent work has either developed the theory in static environments or focused on evaluating the role of commitment while relying on one of the two canonical models mentioned above to resolve the underlying bargaining problem.[1]

This paper presents a formalization of Schelling's theory with an infinite-horizon model of bargaining with complete information. The objective is to characterize the extent to which this theory, built on the use of strategic commitments, resolves the bargaining problem and how, and furthermore establish conditions under which the model's predictions are adequately summarized by some bargaining solution.

The model builds on two key elements of Schelling's theory. First, a bargainer may find it costly to back down from a stated demand and this is the source of her commitment ability. Second, the commitment ability is nevertheless endogenous, in that it depends on the demands. A less aggressive demand weakens the opponent's commitment ability by allowing more room for her to back down. By contrast, a demand that leaves an opponent's back against the wall only ensures the latter's commitment.

In the model, the bargainers simultaneously announce demands. If the demands are compatible, bargaining ends on those terms. If incompatible, the players decide

[1] See for example, Crawford (1982), Muthoo (1996), Ellingsen and Miettinen (2008) and Dutta (2012) for the first and Fershtman and Seidmann (1993), Compte and Jehiel (2004), Wolitzky (2012) and Basak and Deb (2020) for the second. Ellingsen and Miettinen (2014) consider a dynamic model of a hybrid nature that I discuss in detail in section 5.

whether to stick to their demand or concede to the opponent's offer. Concession incurs an additional cost which is increasing in the conceded amount. If neither player concedes, then the current period of bargaining ends and the next period begins with a fresh round of demands. The game proceeds in this manner until either compatible demands or a concession following incompatible demands. The bargainers are impatient, as captured by constant discount factors. I focus on subgame perfect equilibria with pure strategies in the demand stage (henceforth SPE).

The model can be seen as a variant of the infinite horizon version of the Nash Demand Game (henceforth IH-NDG). While in the latter, incompatible demands end the current round of bargaining, in the present model bargainers get a chance to concede. Indeed, if the concession costs are made arbitrarily high, then concession is effectively ruled out and the IH-NDG obtains at the limit.

The model predictions depend on the two sets of model parameters, namely the discount factors and concession cost functions. In any SPE outcome, the bargainers eventually agree upon an efficient division of the surplus, following some delay, if any. In contrast to common dynamic bargaining models, the range of efficient divisions of the surplus that can arise in equilibrium is linked to the maximum delay the equilibrium accommodates following any history.

Renegotiation-proof SPE, used to model familiar bargainers, feature no delay and an exact characterization obtains for the corresponding set of surplus divisions. This leads to the key finding of the paper. As the marginal concession costs are made arbitrarily high, the set of renegotiation-proof SPE outcomes converges to selecting a unique efficient outcome in the limiting IH-NDG. This outcome is identical to that of the Kalai bargaining solution (see Kalai (1977)) with its proportion determined by the discount factors and a limit ratio of the concession cost functions. Therefore, not only does the formalization of Schelling's theory fully resolve the bargaining problem, it also provides a strategic foundation for the Kalai bargaining solution. Furthermore the parameters of the non-cooperative model select the appropriate bargaining solution from the family of solutions characterized in Kalai (1977).

Markov perfect equilibria (which may violate renegotiation-proofness) can exhibit delay. There is a bound to the length of delay and it depends on the two sets of model parameters. In a natural way, such equilibria with delay yield a form of gradualism, the feature in which bargainers start with extreme demands that soften over time. Finally, the set of stationary Markov perfect equilibrium outcomes coincides with the set of renegotiation-proof SPE outcomes.

As Binmore, Osborne and Rubinstein (1992) states, *The ultimate aim of what is now called the "Nash program" (see Nash 1953) is to classify the various institutional frameworks within which negotiation takes place and to provide a suitable "bargaining solution" for each class.* This paper contributes to this literature by making a case for the Kalai bargaining solution in environments in which commitment ability due to concession costs is salient.[2] Binmore, Rubinstein and Wolinsky (1986) establish a robust connection between the alternating-offers model and the Nash bargaining solution. Studies on commitment that rely on the alternating-offers model, such as Muthoo (1996), find similar support for the (asymmetric) Nash bargaining solution. Relying on the struggle to commit itself to resolve the bargaining problem, as the current paper shows, leads instead to the Kalai bargaining solution. This is an important distinction. The appropriate choice of a bargaining solution is not merely a game-theoretic curiosity. Aruoba, Rocheteau and Waller (2007), for instance, show that the choice of bargaining solution matters both qualitatively and quantitatively for questions of first-order importance in monetary economics.

To the best of my knowledge, Dutta (2012) and Hu and Rocheteau (2020) are the only other papers that provide strategic bargaining foundations for the Kalai bargaining solution. Dutta (2012) is the static (one-period) version of the current model and captures a qualitatively similar role for the concession costs, in that higher costs benefit the bargainer. It shares the unrealistic feature of the Nash demand game in ruling out future negotiations following a single round of disagreement, and as a result has no role for discount factors. Hu and Rocheteau (2020) rely on the alternating-offers model. They show that if the surplus is divided into $N$ parts and in each of $N$ rounds players engage in Rubinstein bargaining over one of these parts, then the outcome corresponds to the Kalai bargaining solution as $N$ tends to infinity.

The rest of the paper is as follows. In section 2, I introduce the general model and show how all SPEs have a simple structure. In section 3, I focus on a linear specification of the model, which allows for closed form characterizations. Here I derive the set of renegotiation-proof SPE outcomes along with all the Markov perfect equilibria results. In section 4, I return to the general model, characterize the set of renegotiation-proof SPE outcomes and establish the link with the Kalai bargaining

[2]This occurs, for instance, in negotiations between political leaders who face domestic audiences, as discussed in Fearon (1994) and Martin (1993).

solution. Finally in section 5, I discuss some related literature.

## 2 The Model

Two players, 1 and 2, play an infinite horizon game to split a pie of size 1. In period $t \in \mathbb{N} \equiv \{1, 2, 3, \ldots\}$, if the bargaining problem is still unresolved, each player $i \in \{1, 2\}$ announces a demand $z_i \in [0, 1]$. The announcements are simultaneous. For a given demand profile $z = (z_1, z_2)$, let $d(z) = z_1 + z_2 - 1$. If the demands are compatible ($d(z) \leq 0$) then the game ends with both players receiving their own demands. The resulting payoff profile is $(u_1(z_1), u_2(z_2))$, where $u_i$ is the payoff function for player $i$.

Following incompatible demands ($d(z) > 0$), the bargainers enter a concession stage. Here the players simultaneously decide whether to stick to their demands or back down and accept the other's offer. Backing down comes at a cost which is a function of the conceded amount, the difference between the initial demand and the accepted amount, $z_i - (1 - z_{-i}) = d(z)$, and is captured by the *concession cost function* $c_i$. If both players stick to their demand then the bargaining problem remains unresolved and moves to the next period. This concession stage game is represented in the table below.

Table 1: Concession Stage following Incompatible Demand Profile $z$

| | *Accept* (A) | *Stick* (S) |
|---|---|---|
| *Accept* (A) | $u_1(1 - z_2) - c_1(d(z)), u_2(1 - z_1) - c_2(d(z))$ | $u_1(1 - z_2) - c_1(d(z)), u_2(z_2)$ |
| *Stick* (S) | $u_1(z_1), u_2(1 - z_1) - c_2(d(z))$ | $u_1(0), u_2(0)$ |

As long as some player chooses $A$ the game ends this period with the associated payoffs in the table, otherwise it moves to period $t + 1$. The following assumptions hold throughout the paper.

**Assumption 1** *For $i \in \{1, 2\}$, $u_i$ is a strictly increasing, concave and continuously differentiable function with $u_i(0) = 0$.*

**Assumption 2** *For $i \in \{1, 2\}$, $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and continuously differentiable function with $c_i(0) = 0$.*

A history of play that leads to the beginning of period $t+1$ with $t \in \mathbb{N}$, denoted as $h^t$, is a sequence of $t$ incompatible demand profiles with $(S, S)$ in the corresponding concession stages, $(z^1, SS, z^2, SS, \ldots, z^t, SS)$. Let $H^t$ be the set of all such $t$-period histories, with the null history $H^0 = \{h^0\}$ and $H = \cup_{t=0}^{\infty} H^t$. A history of play that leads to the concession stage in period $t$, denoted as $h^{t'}$, is an element of $H^{t-1}$ followed by an incompatible demand profile $z^t$. Let $H^{t'}$ be the set of all such $t$-period histories and $H' = \cup_{t=1}^{\infty} H^{t'}$. A pure strategy for player $i$ is a function $\sigma_i : H \cup H' \rightarrow [0,1] \cup \{A, S\}$ such that $\sigma_i(h) \in [0,1]$ for $h \in H$ and $\sigma_i(h) \in \{A, S\}$ for $h \in H'$. The subgame following history $h \in H \cup H'$ is labeled $g(h)$.

Given a history $h^t \in H$, a strategy profile $\sigma = (\sigma_1, \sigma_2)$ determines the period $n > t$ when bargaining ends in the subgame $g(h^t)$, with payoffs in that period of $y = (y_1, y_2)$, where $y = (0, 0)$ if $n = \infty$. Call $(y, n - t)$ the outcome of the game $g(h^t)$ under $\sigma$. A strategy profile $\sigma$ with outcome $(y, n - t)$ in the subgame $g(h^t)$ yields the discounted payoff of $\delta_i^{n-t-1} y_i$ to player $i$ at the beginning of the subgame, where $\delta_i \in (0, 1)$ is player $i$'s discount factor.

## 2.1 Subgame Perfect Equilibria

To analyze its content, I focus on *pure strategy subgame perfect equilibria* of the model. Subsequently, for expositional ease, I will refer to these simply as subgame perfect equilibria or SPE. Infinite horizon games with simultaneous moves typically feature a vast multiplicity of SPE with a sense of *anything goes*. The current model features multiplicity too. Nevertheless, the following straightforward yet useful lemma shows that all such equilibria have a simple structure. Exactly compatible demands imply $d(z) = 0$.

**Lemma 1** *A subgame perfect equilibrium at any period must feature either*
*(a) exactly compatible demands, or*
*(b) incompatible demands followed by both players choosing* Stick*.*

**Proof.** Consider a period in which incompatible demands $(z)$ are followed by some action profile other than $(S, S)$ in the concession stage. Then, as the payoff matrix in table 1 shows, there must be some player $i$ who receives a payoff strictly less than $u_i(1 - z_{-i})$ and is strictly better off by deviating to the compatible demand $1 - z_{-i}$ instead of the original $z_i$.

Next, given a period with compatible demands that add up to less than 1, the player with the lower demand, say $i$, is strictly better off demanding $1 - z_{-i}$ instead.

■

In other words, any SPE involves some rounds of delay, if any, via incompatible demands, followed by an agreement on an efficient division of the surplus.

Dynamic bargaining games featuring multiple SPE typically have the following feature.[3] The range of efficient SPE outcomes constitutes the first-order multiplicity. These rely on history-dependent strategies but do not require strategy profiles involving delay. This first-order multiplicity is used, through appropriate history-dependent strategies, to generate varying lengths of delay, the second-order multiplicity. In the current model the first-order multiplicity resides in delay. Limiting the length of delay permissible in an SPE limits the range of efficient outcomes that can arise in equilibrium. The following classification of SPEs helps clarify this feature.

**Definition 1** *An SPE $\sigma$ is called an SPE with maximum delay $m$ if for any subgame $g(h^t)$, $h^t \in H$, it generates an outcome $(y, n-t)$ where $n-t-1 \leq m$.*

The characterization results that follow rely on the stationary structure of the model. To this end, for any $h \in H$, let $O^m(h)$ denote the set of outcomes of SPE with maximum delay $m$ in the subgame $g(h)$. Now define

$$B^m \equiv \left\{ z | (u(z), t) \in O^m(h^0) \right\}$$

to be the set of all surplus divisions that can arise as the outcome of some SPE with maximum delay $m$ in the bargaining game. Due to the stationary structure of the game, it follows that

$$B^m = \{ z | (u(z), t) \in O^m(h) \}, \qquad \textit{for all } h \in H.$$

Finally observe that by lemma 1, $z \in B^m \Rightarrow z_1 + z_2 = 1$.

# 3 The Linear Model

In this section, I analyze the following specification of the bargaining model.

$$\forall i \in \{1,2\}, \qquad u_i(z_i) = z_i \qquad \textit{and} \qquad c_i(d(z)) = k_i d(z) \textit{ for some } k_i > 0.$$

This linear specification retains the strategic tradeoffs of the general model while allowing for closed-form characterizations of equilibrium outcomes.

[3] See, for instance, Sutton (1986), Avery and Zemsky (1994) and Merlo and Wilson (1995).

The following lemma captures a key restriction that the strategic considerations about commitment impose on the set of compatible demands that can arise in equilibrium. In particular, the compatible demands must be such that neither bargainer can raise her own demand and force her opponent to back down in the resulting concession stage. "Forcing" here would require the (unique) dominance solvable outcome of the concession game to consist of the deviator sticking to her demand while her opponent concedes, irrespective of the equilibrium continuation strategy profile.

**Lemma 2** *Suppose $\sigma$ is a pure strategy profile with $\sigma(h^{t-1}) = z$ and $\sum_{i=1}^{2} z_i = 1$ for some $h^{t-1} \in H$. If for some $i \in \{1, 2\}$, there exists $z_{-i} < \hat{z}_{-i} \leq 1$ such that*

$$1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < \delta_{-i}^n \tilde{z}_{-i} \tag{1}$$

*and*

$$1 - \hat{z}_{-i} - k_i(z_i + \hat{z}_{-i} - 1) > \delta_i^n \tilde{z}_i \tag{2}$$

*for all $\tilde{z} \in B^{n^*}$ and $1 \leq n \leq n^* + 1$ then $\sigma$ is not an SPE with maximum delay $n^*$.*

**Proof.** Without loss of generality set $i = 1$. Now note that bargaining failure in period $t$ leads to $g(h^t)$ beginning in the next period. Since $\sigma$ is an SPE with maximum delay $n^*$, and by lemma 1, the outcome $(x, m)$ of this subgame must satisfy $x \in B^{n^*}$ and $m \leq n^* + 1$. Suppose one such continuation outcome is given by $(\tilde{z}, n)$. Now consider a deviation $\hat{z}_2$ from the compatible profile $z$ which satisfies both inequalities 1 and 2 for this continuation profile.

Table 2: Augmented Concession Game following deviation $\hat{z}_2$ from Profile $z$

| | $A$ | $S$ |
|---|---|---|
| $A$ | $1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1), 1 - z_1 - k_2(z_1 + \hat{z}_2 - 1)$ | $1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1), \hat{z}_2$ |
| $S$ | $z_1, 1 - z_1 - k_2(z_1 + \hat{z}_2 - 1)$ | $\delta_1^n \tilde{z}_1, \delta_2^n \tilde{z}_2$ |

The deviation leads to the augmented game above in the concession stage, with $(S, S)$ yielding a discounted payoff consistent with the continuation outcome $(\tilde{z}, n)$. Due to inequality 1, in this concession stage $S$ strictly dominates $A$ for player 2. Inequality 2 in turn ensures that given player 2's choice of $S$, player 1 strictly prefers to play $A$. In other words, the unique dominance solvable outcome in the augmented concession game is $(A, S)$. Furthermore this outcome gives player 2 a strictly higher payoff than $z_2$. So, if there exists a $\hat{z}_2$ such that no matter what the continuation profile (consistent with $\sigma$ being an SPE with maximum delay $n^*$) the two inequalities above are always satisfied, then $\hat{z}_2$ is a profitable deviation from $z$ and therefore $\sigma$ is not an SPE. ■

To see how the constraint identified in lemma 2 has bite, consider the compatible demand profile $(1, 0)$. Fix any set of discount factors and marginal concession costs. Notice that the highest payoff player 1 gets if bargaining fails this period is $\delta_1$. By choosing a $\hat{z}_2 > 0$ close enough to 0, player 2 can ensure that conditional on 2 choosing $S$, 1 would rather concede and get a payoff arbitrarily close to 1 rather than settle for the lower amount of $\delta_1$. By contrast, player 1 has no room for backing down since any concession leads to a negative payoff. So irrespective of the continuation strategy, following a deviation to $\hat{z}_2$, her dominant strategy would be $S$. In summary player 2's deviation from $(1, 0)$ guarantees her a positive payoff. This rules out $(1, 0)$ as an equilibrium outcome.

To obtain a more complete characterization, the exercise above is extended to identify the most favourable equilibrium surplus division (element of $B^{n^*}$) for player $i$.[4] The restriction to SPE with maximum delay $n^*$ ensures that in any continuation game the eventually agreed upon division must also belong to $B^{n^*}$. This structure turns out to be sufficient to obtain the characterization below.

**Proposition 1** *If $(z, t)$ is the outcome of a subgame perfect equilibrium with maximum delay $n^*$, then*

$$\frac{1-\delta_1}{1-\delta_2^{n^*+1}} \frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1^{n^*+1}}{1-\delta_2} \frac{1+k_2}{k_1}. \tag{3}$$

**Proof.**

Let $z_i^* = \sup_{z \in B^{n^*}} z_i$. Now suppose for some exactly compatible demand profile $z$, there exists $\hat{z}_2$ such that

$$1 - z_1 - k_2(z_1 + \hat{z}_2 - 1) < \delta_2^{n^*+1}(1 - z_1^*) \tag{4}$$

and

$$1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1) > \delta_1 z_1^*. \tag{5}$$

[4]This step is in the spirit of the approach taken in Shaked and Sutton (1984) to solve the alternating-offers model.

Then such a $\hat{z}_2$ also satisfies inequalities 1 and 2 for all $\tilde{z} \in B^{n^*}$ and $1 \leq n \leq n^* + 1$, since for any such $\tilde{z}$ and $n$ it follows that $\delta_2^{n^*+1}(1 - z_1^*) \leq \delta_2^n \tilde{z}_2$ and $\delta_1 z_1^* \geq \delta_1^n \tilde{z}_1$. Therefore, by lemma 2, $z$ cannot arise in any SPE (i.e., $z \notin B^{n^*}$).

Now since $z_i^* = \sup_{z \in B^{n^*}} z_i$ it must be that we cannot find such a $\hat{z}_2$ for the compatible profile $z = (z_1^*, 1 - z_1^*)$. So there cannot be a $\hat{z}_2 > 1 - z_1^*$ which satisfies both

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^{n^*}(1 - z_1^*), \qquad \textit{and}$$

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1 z_1^*.$$

These inequalities simplify to

$$\hat{z}_2 > \frac{(1 - z_1^*)(1 + k_2 - \delta_2^{n^*})}{k_2} \qquad \textit{and} \qquad \hat{z}_2 < 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

Therefore such a $\hat{z}_2$ cannot exist only if

$$\begin{aligned} &\frac{(1 - z_1^*)(1 + k_2 - \delta_2^{n^*})}{k_2} \geq 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1} \\ \Rightarrow &\frac{(1 - z_1^*)(1 - \delta_2^{n^*})}{k_2} \geq \frac{z_1^*(1 - \delta_1)}{1 + k_1} \\ \Rightarrow &\frac{1 - \delta_1}{1 - \delta_2^{n^*}} \frac{k_2}{1 + k_1} \leq \frac{1 - z_1^*}{z_1^*} \end{aligned}$$

A symmetric argument establishes

$$\frac{1 - \delta_2}{1 - \delta_1^{n^*}} \frac{k_1}{1 + k_2} \leq \frac{1 - z_2^*}{z_2^*}$$

which transforms to

$$\frac{z_2^*}{1 - z_2^*} \leq \frac{1 - \delta_1^{n^*}}{1 - \delta_2} \frac{1 + k_2}{k_1}.$$

To conclude the proof note that

$$z \in B^{n^*} \Rightarrow \frac{1 - z_1^*}{z_1^*} \leq \frac{z_2}{z_1} \leq \frac{z_2^*}{1 - z_2^*}.$$

■

The result confirms Schelling's insight about weakness being a strength, in that higher marginal concession costs generate better equilibrium outcomes for the bargainer. Greater patience is similarly beneficial. This preserves a key implication of the canonical bargaining models.

It is clear from proposition 1 that additional conditions that restrict the maximum delay allowed in an SPE, as a result, also shrink the set of compatible demand profiles that can obtain in equilibrium. I study three such conditions in the following subsections.

## 3.1 Renegotiation-Proofness

Negotiators who are familiar with each other should, in the presence of multiple equilibria, be able to avoid the strictly Pareto dominated ones. This is especially so, if the Pareto dominating equilibrium is one they anticipate to play following some history. Since the game is identical following any history $h \in H$, the negotiators would see the incongruence of taking an efficient path following one such history and an inefficient one following another. Given their familiarity they need not take their cues from some possibly inefficient norm, but rather count on renegotiating away from such inefficient equilibria. The notions of *weak renegotiation proofness* in Farrell and Maskin (1989) and *internal consistency* in Bernheim and Ray (1989) capture this idea in the context of repeated games. While not a repeated game, the present model shares its key feature that following any number of rounds (of failed bargaining), the continuation game looks the same. Relying on this stationarity, I import an appropriate notion of renegotiation-proofness for the current setting.

Let $\psi(\sigma; h^t)$ be the continuation payoff (profile) implied by $\sigma$ given history $h^t \in H$ and let

$$\Psi(\sigma) = \cup_{h^t \in H} \psi(\sigma; h^t)$$

be the set of all continuation payoffs under $\sigma$.

**Definition 2** *An SPE $\sigma$ is renegotiation-proof if for no $x, y \in \Psi(\sigma)$ is $x \gg y$.*

Note that renegotiation-proofness does not rule out history dependent strategies. Consider, for instance, the construction due to Avner Shaked reported in Sutton (1986). It supports any efficient division of the surplus as a subgame perfect equilibrium outcome of a 3-person Rubinstein bargaining game for high enough discount factors. The construction relies heavily on the history-dependence of the strategy profile. Imposing an appropriate version of renegotiation-proofness has no effect on the result since all continuation outcomes are efficient. The severe multiplicity persists. In the current model, however, renegotiation-proofness sharply restricts the set of equilibrium outcomes.

**Proposition 2** $(z, t)$ *is the outcome of a renegotiation-proof subgame perfect equilibrium if and only if* $t = 1$ *and*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}. \tag{6}$$

I now sketch the argument behind this result. The detailed proof is in the appendix. Given the structure of SPE identified in lemma 1, renegotiation-proofness simply rules out any delay. The necessity of inequality 6 then follows immediately from proposition 1. To establish sufficiency, I construct the following stationary strategy profile, which I show to be subgame perfect for any $z$ satisfying inequality 6 in lemma 4 in the appendix.

**Construction 1** *Consider the following stationary strategy profile,* $\sigma$*. Fix* $z$ *such that* $d(z) = 0$*. For all* $h^t \in H$*, set* $\sigma_i(h^t) = z_i$*. If player* $i$*, for some* $i \in \{1, 2\}$*, in period* $t$ *deviates to a higher demand,* $\hat{z}_i > z_i$*, then in the concession stage game* $(S, S)$ *is played if it is a Nash equilibrium and otherwise* $(A_i, S_{-i})$ *is played. For all other* $h \in H'$ *some pure strategy Nash equilibrium of the concession stage game is played.*

The strategy profile $\sigma$ above satisfies renegotiation-proofness, since following any history $h \in H$ the continuation outcome is efficient and consists of agreeing on the compatible demand profile $z$.

Proposition 2 offers a preview of the limit uniqueness result in section 4. Consider a sequence of these linear bargaining games parametrized by marginal concession costs $\{k_1^n, k_2^n\}_{n=1}^{\infty}$ such that $k_1^n = \gamma k_2^n$ for all $n$ and $k_2^n \to \infty$ as $n \to \infty$. Observe first that at the limit, it is too costly for any bargainer to concede following any incompatible demand. The model therefore reduces to the IH-NDG. However, in contrast to the acute multiplicity of SPE in the IH-NDG, the set of renegotiation-proof SPE as characterized in proposition 2 converges to a singleton at the limit. At this unique limit outcome, the bargainers agree on the compatible profile $z$ with

$$\frac{z_2}{z_1} = \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma}.$$

## 3.2 Markov Perfect Equilibria and Gradualism

Negotiations often take place between strangers or relatively inexperienced bargainers. The assumption of renegotiation-proofness may not be appropriate in such cases. A different assumption, routinely made in applied work, requires players to use Markov strategies. Maskin and Tirole (2001) discusses some of the theoretical considerations that support its use. In this section I focus on SPE in Markov strategies.

**Definition 3** *$\sigma_i$ is a Markov strategy for player $i$ if for all $h, \tilde{h} \in H^t$*
*(i) $\sigma_i(h) = \sigma_i(\tilde{h})$ and*
*(ii) $\sigma_i(h, z^{t+1}) = \sigma_i(\tilde{h}, z^{t+1})$.*

In words, under the Markov requirement, player $i$'s demand in period $t$ must be invariant to the specific $t-1$ demand profiles rejected in the past. Further, the concession stage decision in period $t$ should depend upon the period $t$ demand profile alone. Note, however, that it allows demands and concession stage behaviour to depend on calendar time. For instance, a strategy in which the demands get less and less extreme over the first $m$ periods of bargaining is permitted. Indeed, such strategies can be shown to generate delay in equilibrium.

**Proposition 3** *Let $j \in \{1, 2\}$ such that $\delta_j \geq \delta_{-j}$. If $(z, t)$ is a Markov perfect equilibrium outcome then $t \leq n^*$ and*

$$\frac{1-\delta_{-j}^{n^*}}{1-\delta_j^{n^*}} \frac{k_j}{1+k_{-j}} \leq \frac{z_j}{z_{-j}} \leq \frac{1-\delta_{-j}}{1-\delta_j} \frac{1+k_j}{k_{-j}},$$

*with*

$$n^* - 1 = \left\lfloor \frac{\ln \frac{k_1+k_2}{k_1+k_2+k_1 k_2}}{\ln \delta_j} \right\rfloor.$$

The proof is in the appendix. Here I discuss the key steps. First, there is an upper bound to the delay that can arise in any MPE. To see why, notice that to sustain delay in equilibrium the bargainers must make incompatible demands that neither wishes to deviate from. To ensure that a unilateral deviation to a compatible profile is not profitable, the demands simply need to be sufficiently aggressive. For instance, both players demanding the entire surplus works, no matter the length of delay. It is more demanding to rule out profitable unilateral deviations to an incompatible profile. A bargainer in such a deviation makes a lower but still incompatible demand, which nevertheless forces her opponent to concede in the subsequent concession game. Sufficient delay makes such deviations feasible even for very aggressive demands. A long expected delay lowers the payoff from disagreement and makes concession more palatable. This feature limits the amount of delay that can arise in an MPE.

The upper bound to the length of delay, restricts the set of equilibrium continuation outcomes. In the remainder of the proof, similar to that of proposition 2, I characterize the best compatible demand profile that can arise for each player, relying on the recursive structure of the game.

As for the result itself, observe first that if the two bargainers are equally impatient, then the set of MPE outcomes coincides with the renegotiation-proof SPE outcomes. So not only is the larger set of equilibrium surplus agreements dependent on the delay allowed by MPE, it relies on bargainers having different degrees of impatience. Second, suppose the maximum delay under MPE is *derived* to be $m$ under proposition 3, the bounds on MPE surplus divisions are tighter than the bounds that arise for SPE with maximum delay $m$, as derived in proposition 1. The lack of history-dependent strategies delivers a sharper prediction.

A final interesting feature of the Markov environment is the nature of incompatible demands in an MPE with delay. Gradualism is a commonly observed feature of bargaining in which players gradually lower their demands, starting with very aggressive ones and ending with a compatible profile.[5] MPEs with delay yield gradualism in a natural way. The following proposition characterizes this feature.

**Proposition 4** *If $(y, m)$ is the outcome of a Markov perfect equilibrium with a delay of $m-1 > 0$ periods then the incompatible demand profiles $z^t$ for $1 \leq t \leq m-1$ must satisfy*

$$z_i^t \geq \frac{(1 - \delta_{-i}^{m-t} y_{-i})(1 + k_i) - \delta_i^{m-t} y_i}{k_i}.$$

In words, the smallest (incompatible) demand that can arise in an MPE is higher the further away (in periods) it is made from the eventual agreement. Two separate features contribute to this. The obvious one is that for neither player to want to deviate to simply accepting the others implicit offer (by making a compatible demand) it must be that the offers are worse than accepting the delayed agreement. The longer the delay the worse the offers need to be, and therefore higher demands. The less

[5]See, for instance, Backus, Blake, Larsen and Tadelis (2020).

obvious feature is that a bargainer may find it profitable to deviate to a lower but still incompatible demand profile that forces the other player to concede. To rule out such a deviation, the incompatible demands need to be even higher than the level required to rule out deviations to compatible profiles. Further, this threshold is higher the more periods that remain to agreement.

I end this section by characterizing the set of stationary MPE outcomes. Stationarity does not allow strategies to depend on calendar time. It requires

$$\sigma_i(h) = \sigma_i(\tilde{h}) \qquad \forall h, \tilde{h} \in H.$$

**Proposition 5** $(z, t)$ *is a stationary Markov perfect equilibrium outcome if and only if* $t = 1$ *and*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

Notice that the set of stationary MPE outcomes coincides exactly with the set of renegotiation-proof SPE outcomes. In general dynamic games where both concepts apply the two are typically not the same. Take an infinitely repeated prisoner's dilemma game with high enough discount factors, for instance. The unique MPE outcome involves both parties defecting forever. On the other hand, cooperation can be sustained as a weak renegotiation-proof SPE, as shown in Farrell and Maskin (1989).[6]

# 4 Strategic Foundation for the Kalai Bargaining Solution

In this section I return to the general model to establish the main finding of this study. When concession cost functions are very high, effectively ruling out concession following incompatible demands, then the renegotiation-proof SPE outcome of the general model is well approximated by a Kalai bargaining solution. To show this I first obtain an exact characterization of the set of renegotiation-proof SPE outcomes for any payoff and cost functions that satisfy the assumptions in section 2.

Let $\mathcal{U}$ and $\mathcal{C}$ be the set of all (pairs of) functions that satisfy assumptions 1 and 2, respectively. Fix some $u \in \mathcal{U}$ and $c \in \mathcal{C}$. Now for $i \in \{1, 2\}$, let the incompatible

[6] See also van Damme (1989).

demand profile $(z_i^M, \hat{z}_{-i}(z_i^M))$ be defined by the following pair of equations.[7]

$$u_{-i}(1 - z_i^M) - c_{-i}(z_i^M + \hat{z}_{-i}(z_i^M) - 1) = \delta_{-i} u_{-i}(1 - z_i^M) \tag{7}$$

$$u_i(1 - \hat{z}_{-i}(z_i^M)) - c_i(z_i^M + \hat{z}_{-i}(z_i^M) - 1) = \delta_i u_i(z_i^M). \tag{8}$$

**Proposition 6** *In the general model, $(y, t)$ is the outcome of a renegotiation-proof subgame perfect equilibrium with $y_i = u_i(z_i)$ for $i \in \{1, 2\}$, if and only if $t = 1$, $d(z) = 0$ and*

$$\frac{1 - z_1^M}{z_1^M} \leq \frac{z_2}{z_1} \leq \frac{z_2^M}{1 - z_2^M}. \tag{9}$$

This result significantly generalizes proposition 2. A key step in the proof (in the appendix) is to show that the solution to equations 7 and 8 indeed always exists. The assumption of concavity of the payoff functions plays a role at this step.

Making the concession cost functions steeper makes it progressively harder for the bargainers to back down from their demands. At the limit, with arbitrarily high marginal concession costs, the infinite horizon version of the Nash demand game obtains. Neither player can back down from incompatible demands. Any efficient payoff profile can be supported as an SPE outcome of the IH-NDG, as pointed out in Binmore (1987). Infinite delay can also be supported in SPE by each bargainer always demanding the entire surplus. Chatterjee and Samuelson (1990) show that this acute multiplicity further survives trembling hand perfection (see Selten (1975)). The limit set of renegotiation-proof SPE outcomes, in sharp contrast, is a singleton.

For any $u \in \mathcal{U}$ and $c \in \mathcal{C}$, let $g^c(u)$ denote the game described in section 2, where $u_i$ and $c_i$ are player $i$'s payoff and concession cost functions, respectively, for $i \in \{1, 2\}$. The corresponding set of renegotiation-proof SPE payoff profiles is denoted by $\xi(g^c(u))$. $g^c$ therefore maps any pair of payoff functions in $\mathcal{U}$ to its corresponding infinite horizon bargaining game. Consider a sequence of such mappings $\{g^{c^n}\}_{n=1}^{\infty}$ with $c^n \in \mathcal{C}$ for all $n$, such that as $n \to \infty$, $c_i^{n\prime}(0+) \to \infty$ (the right derivative of the concession cost functions at 0 becomes arbitrarily large). Further, assume that $\exists \epsilon > 0$ and some integer $N$ such that $\forall n > N$, $\forall d \in (0, \epsilon)$, $c_1^n(d)/c_2^n(d)$ is a constant. Let

$$\xi_\gamma^*(u) = \lim_{n \to \infty} \xi(g^{c^n}(u)), \qquad \textit{where } \gamma = \lim_{n \to \infty} c_1^n(0+)/c_2^n(0+).$$

[7] The dependence of $(z_i^M, \hat{z}_{-i}(z_i^M))$ on $u$ and $c$ is suppressed for expositional ease.

The limit set of renegotiation-proof SPE is therefore captured by $\xi^*_\gamma(u)$. It is parameterized by $\gamma$, which is the ratio of the concession cost functions evaluated at the limit as the amount conceded becomes vanishingly small.

Kalai (1977) introduces a family of bargaining solutions parametrized by a single variable, a proportion. Any bargaining solution that is monotonic, in that an increase in possible bargaining outcomes never hurts either bargainer, is a Kalai (or proportional) bargaining solution and vice versa. The family of solutions is exactly characterized by the axioms of independence of irrelevant alternatives, individual monotonicity and continuity. In addition to being compelling theoretically, the solutions are used extensively and in a variety of fields. Recently, for instance, it is used increasingly in the field of monetary economics.[8] [9]

I now introduce some notation in order to define the Kalai bargaining solution. Let $\Pi(u) = \{y | y_i = u_i(z_i), z_i \geq 0, \forall i \in \{1,2\} \textit{ and } z_1 + z_2 \leq 1\}$ denote the set of feasible payoffs that can arise from some allocation of the surplus. Set $u^d = (u_1(0), u_2(0)) = (0,0)$ to be the disagreement point. Combined, $(\Pi(u), u^d)$ represents a bargaining problem. Finally let $\mathcal{B} = \{(\Pi(u), u^d) | u \in \mathcal{U}\}$ be the set of all bargaining problems that can arise from payoff functions that satisfy assumption 1.

The Kalai Bargaining Solution with proportions $(\theta, 1)$, denoted by $\mathcal{K}_\theta$, is defined as $\mathcal{K}_\theta(\Pi, u^d) = \lambda(\Pi, u^d)(\theta, 1), \forall \Pi \in \mathcal{B}$ where $\lambda(\Pi, u^d) = \max\{q | q(\theta, 1) \in \Pi\}$.

**Proposition 7** *For all $u \in \mathcal{U}$, $\xi^*_\gamma(u) = \mathcal{K}_\theta(\Pi(u), u^d)$ where $\theta = \gamma(1-\delta_2)/(1-\delta_1)$.*

It is clear in Kalai (1977) that while the family of bargaining solutions is a compelling one, finding the relevant proportion needs information beyond what is modelled in a standard bargaining problem (an element of $\mathcal{B}$). In proposition 7 the degree of impatience of the bargainers and their relative concession costs constitute this information. So not only does this formalization of Schelling's theory provide a strategic foundation for the Kalai bargaining solution, it also selects the appropriate proportion.

# 5 Discussion

Ellingsen and Miettinen (2014) (henceforth EM) extend the static model of Ellingsen and Miettinen (2008) to a fairly involved dynamic model. Formalizations of Schelling's

[8]See, for instance, Lagos, Rocheteau and Wright (2017).

[9]Duffy, Lebeau and Puzzello (2021) find that the Kalai bargaining solution better fits the behaviour of bargainers in the laboratory facing liquidity constraints.

ideas are usually closely related to the Nash demand game. The EM model has elements of both the Nash demand game (simultaneous demands) and the generalized Rubinstein bargaining framework. As examples of the latter, (a) following demands that are more than compatible, a single responder is selected randomly to accept or reject the other's offer and (b) following a choice of flexibility by both bargainers, a single player is randomly selected to make an offer for that period. The key difference with the current formalization, however, is that in EM (as well as Ellingsen and Miettinen (2008)) commitment ability is exogenous and independent of the actual demands made by the players. It does not matter whether a bargainer is offered a lot of room to back down or none at all, her commitment ability is pinned down by an exogenous randomization device. This distinction is critical, since in the current study the strategic feature that resolves the bargaining problem, is precisely the ability of bargainers to affect each other's commitment ability by choosing appropriate demands.

The delay obtained in Markov perfect equilibrium in section 3.2 is neither the result of money burning as in Avery and Zemsky (1994) nor due to strategic uncertainty as in Friedenberg (2019). In a sense, as Sakovics (1993) puts it, the delay is wholly ritualistic and can be expected in settings where bargainers take their cues from norms or traditions that are perhaps optimal in some larger context but offer an inefficient prescription in the specific bargaining instance. Similar equilibria also arise in Perry and Reny (1993) and Sakovics (1993), who study a generalization of the Rubinstein model with less restriction on when offers can be made and responded to. A key finding in both is that allowing for simultaneous demands generates an acute multiplicity of equilibria including those with delay. While not their focus, the SPE with delay in these models feature a milder form of the gradualism that appears in the current study. The further away the anticipated agreement, the further apart the incompatible demands need to be to deter deviation to a compatible profile. As stated earlier, in the current study the incompatible demands need to be even further apart to rule out deviations to incompatible profiles. Compte and Jehiel (2004) provides a wholly different rationale for gradualism. Players always have access to outside options whose values depend on past offers. If more favourable offers increase the value of the opponent's outside option, then bargainers find it optimal lower their demand gradually in equilibrium.

# A Appendix

**Lemma 3** *$\sigma$ cannot be an SPE in the general model, if for some $h \in H$, $\sigma(h) = z$ such that $z_i = 1$ and $z_{-i} = 0$ for some $i \in \{1, 2\}$.*

**Proof.** Suppose under $\sigma$, in the subgame $g(h)$, the two players make the compatible demands $z_i = 1$ and $z_{-i} = 0$, and player $-i$ obtains a payoff of $u_{-i}(0) = 0$. The highest payoff player $i$ could get if bargaining broke down this period is $\delta_i u_i(1)$. Notice that $u_i(1 - \hat{z}_{-i}) - c_i(z_i + \hat{z}_{-i} - 1)$ is a continuous (decreasing) function of $\hat{z}_{-i}$. It takes a value of $u_i(1)$ at $\hat{z}_{-i} = 0$, which is strictly greater than $\delta_i u_i(1)$. Therefore there exists $\hat{z}_{-i} > 0$ such that $u_i(1 - \hat{z}_{-i}) - c_i(z_i + \hat{z}_{-i} - 1) > \delta_i u_i(1)$. Now, if player $-i$ were to deviate to this $\hat{z}_{-i}$ instead of demanding 0, then in the subsequent concession game the dominance solvable outcome would involve player $i$ playing $A$ and $-i$ playing $S$. Since this is a profitable deviation, the strategy profile $\sigma$ cannot be an SPE.

■

**Lemma 4** *The strategy profile $\sigma$ described in Construction 1 is an SPE if*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

**Proof.** The payoff to player $i$ from $\sigma$ at any subgame $g(h)$ with $h \in H$ is simply $z_i$. A lower demand would only lower the payoff. A higher demand would lead to either $(S, S)$ and a continuation payoff of $\delta_i z_i$ or $(A_i, S_{-i})$ leading to a payoff strictly lower than $z_i$ due to the resulting concession cost. Therefore no player has an incentive to deviate in the demand stage of any period.

To verify subgame perfection, therefore, it is sufficient to show that in the concession stage game following an incompatible demand profile $(\hat{z}_i, z_{-i})$, if $(S, S)$ is not a Nash equilibrium then $(A_i, S_{-i})$ is. To establish this result, in turn, it is sufficient to show the following,

$$1 - \hat{z}_i - k_{-i}(\hat{z}_i + z_{-i} - 1) > \delta_{-i} z_{-i} \Rightarrow 1 - z_{-i} - k_i(\hat{z}_i + z_{-i} - 1) > \delta_i z_i$$

which is equivalent to

$$1 - \frac{\delta_{-i} z_{-i} + k_{-i} z_{-i}}{1 + k_{-i}} > \hat{z}_i \Rightarrow \frac{(1 - z_{-i})(1 + k_i - \delta_i)}{k_i} > \hat{z}_i.$$

A sufficient condition for this is simply

$$
\begin{aligned}
&\frac{(1-z_{-i})(1+k_i-\delta_i)}{k_i} > 1 - \frac{\delta_{-i}z_{-i}+k_{-i}z_{-i}}{1+k_{-i}} \\
\Leftrightarrow\ &\frac{(1-z_{-i})(1+k_i-\delta_i)}{k_i} > \frac{z_{-i}(1-\delta_{-i})}{1+k_{-i}} \\
\Leftrightarrow\ &\frac{1-\delta_i}{1-\delta_{-i}}\frac{1+k_{-i}}{k_i} > \frac{z_{-i}}{1-z_{-i}}.
\end{aligned}
$$

Requiring the above inequalities to hold for $i \in \{1,2\}$ make them equivalent to

$$
\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.
$$

■

**Proof for Proposition 2.** Lemma 3 above establishes that, even in the general model, compatible demand profiles in which one player demands the entire surplus cannot arise in an SPE. This combined with lemma 1 implies that if $(z,t)$ is the outcome of an SPE then $d(z)=0$ and $z_i \in (0,1)$ for $i \in \{1,2\}$. This in turn means that if $\sigma$ is a renegotiation-proof SPE with outcome $(z,t)$ then $t=1$. To see why, suppose instead that $t>1$. Then

$$
\psi(\sigma;h^0) = (\delta_1^{t-1}z_1, \delta_2^{t-1}z_2) \ll (z_1,z_2) = \psi(\sigma,\tilde{h}^t),
$$

where $\tilde{h}^t$ is the history that occurs on the equilibrium path with the $t-1$ periods of incompatible demands with neither player conceding in the subsequent concession games. Therefore by proposition 1, inequality 6 is a necessary condition for renegotiation-proof SPE outcomes. Lemma 4 establishes sufficiency by constructing stationary SPE strategies with outcome $(z,t)$ for any $z$ satisfying inequality 6 and $t=1$. Fix one such $z$ and its corresponding stationary SPE strategy profile, $\sigma$. Notice that $\sigma$ satisfies renegotiation-proofness since by construction $\psi(\sigma;h)=(z_1,z_2)$ for all $h \in H$. ■

**Lemma 5** *If $(y,m)$ is the outcome of a Markov perfect equilibrium and $\delta_j \geq \delta_{-j}$ for some $j \in \{1,2\}$, then*

$$
m-1 \leq \left\lfloor \frac{\ln \frac{k_1+k_2}{k_1+k_2+k_1k_2}}{\ln \delta_j} \right\rfloor.
$$

**Proof.** Suppose $\sigma$ is a Markov perfect equilibrium with outcome $(y,m)$ that features delay and so $m > 1$. Consider the demand profile $z^1 = \sigma(h^0)$, which must be incompatible, $d(z^1) > 0$. Let the continuation payoff profile following such demands be $(w_1, w_2)$, which results from the outcome $(y, m)$ of the subgame $g(h^0)$. In particular, $w_i = \delta_i^{m-1} y_i$. Further $y$ is an exactly compatible demand profile by lemma 1, as in $y_1 + y_2 = 1$. $w_1 + w_2 < 1$ follows from $m > 1$.

First note that to be in equilibrium requires $z_i^1 \geq 1 - w_{-i}$ for $i \in \{1, 2\}$. Otherwise player $-i$ would be strictly better off making the compatible demand $1 - z_i^1$ in period $t$. Set $D = \{z | z_i \geq 1 - w_{-i}, \forall i \in \{1, 2\}\}$.

Next, observe that the equation $1 - y_i - k_{-i}(y_1 + y_2 - 1) = w_{-i}$ is satisfied at $y = (1 - w_{-i}, w_{-i})$. It follows that if $y_i \geq 1 - w_{-i}$ and $y_{-i} > w_{-i}$ then $1 - y_i - k_{-i}(y_1 + y_2 - 1) < w_{-i}$. Therefore for any $z \in D$ and any $\hat{z}_{-i} > w_{-i}$ the following inequality holds

$$1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < w_{-i}.$$

This implies that for any incompatible profile in $D$ played in period 1, player $-i$ can deviate to a demand arbitrarily close to $w_{-i}$ and in the resulting concession stage game her action $S$ would strictly dominate $A$. If following such a deviation player $i$ preferred $A$ to $S$, then $-i$ would indeed be better off with the deviation since she would obtain a higher payoff than $w_{-i}$.

For player $i$ to prefer $A$ to $S$ following an incompatible demand profile $y$ requires $1 - y_{-i} - k_i(y_1 + y_2 - 1) > w_i$. If this inequality is satisfied for $y$ with $y_i = 1$ then it will be satisfied for all $(x_i, y_{-i})$ with $1 - w_{-i} \leq x_i \leq 1$. So if the inequality

$$1 - w_{-i} - k_i(1 + w_{-i} - 1) > w_i \tag{10}$$

holds then for any incompatible profile in $D$, player $-i$ can deviate to an appropriate demand greater than $w_{-i}$ such that the unique dominance solvable outcome in the concession stage has $i$ playing $A$ and $-i$ playing $S$, with a payoff greater than $w_{-i}$ to $-i$. So for $D$ to contain some incompatible demand profile that can support the delay in period 1 under $\sigma$ with continuation payoff $w$ requires

$$\frac{1}{k_i} \leq \frac{w_{-i}}{1 - w_1 - w_2}, \qquad \forall i \in \{1, 2\}.$$

A necessary condition for this is

$$\frac{1}{k_1} + \frac{1}{k_2} \leq \frac{w_1 + w_2}{1 - w_1 - w_2},$$

which simplifies to $w_1 + w_2 \geq (k_1 + k_2)/(k_1 + k_2 + k_1 k_2)$. Since $\delta_j \geq \delta_{-j}$, it follows that $\delta_j^{m-1} \geq (k_1 + k_2)/(k_1 + k_2 + k_1 k_2)$ is necessary in turn. The result follows. ■

**Lemma 6** *If $\delta_j \geq \delta_{-j}$ then*

$$\frac{1 - \delta_j^n}{1 - \delta_{-j}^n} \leq \frac{1 - \delta_j^{n+1}}{1 - \delta_{-j}^{n+1}}.$$

**Proof.**

$$\begin{aligned}
\frac{1 - \delta_j^n}{1 - \delta_{-j}^n} \leq \frac{1 - \delta_j^{n+1}}{1 - \delta_{-j}^{n+1}} &\Leftrightarrow \frac{1 - \delta_{-j}^{n+1}}{1 - \delta_{-j}^n} \leq \frac{1 - \delta_j^{n+1}}{1 - \delta_j^n} \\
&\Leftrightarrow \frac{1 + \delta_{-j} + \delta_{-j}^2 + \ldots + \delta_{-j}^n}{1 + \delta_{-j} + \delta_{-j}^2 + \ldots + \delta_{-j}^{n-1}} \leq \frac{1 + \delta_j + \delta_j^2 + \ldots + \delta_j^n}{1 + \delta_j + \delta_j^2 + \ldots + \delta_j^{n-1}} \\
&\Leftrightarrow \frac{\delta_{-j}^n}{1 + \delta_{-j} + \delta_{-j}^2 + \ldots + \delta_{-j}^{n-1}} \leq \frac{\delta_j^n}{1 + \delta_j + \delta_j^2 + \ldots + \delta_j^{n-1}} \\
&\Leftrightarrow \frac{1}{\delta_j^n} + \frac{1}{\delta_j^{n-1}} + \ldots + \frac{1}{\delta_j} \leq \frac{1}{\delta_{-j}^n} + \frac{1}{\delta_{-j}^{n-1}} + \ldots + \frac{1}{\delta_{-j}} \\
&\Leftrightarrow \delta_j \geq \delta_{-j}.
\end{aligned}$$

■

**Proof for Proposition 3.** Let $C^M$ denote the set of compatible demand profiles that can arise in some Markov perfect equilibrium. Let $z_i^* = \sup_{z \in C^M} z_i$. First I show that there cannot exist a $\hat{z}_2 > 1 - z_1^*$ such that

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^n (1 - z_1^*) \tag{11}$$

and

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1^n z_1^* \tag{12}$$

for all $1 \leq n \leq n^*$ where $n^* - 1 = \left\lfloor \frac{\ln \frac{k_1 + k_2}{k_1 + k_2 + k_1 k_2}}{\ln \delta_j} \right\rfloor$. Consider a Markov perfect equilibrium in which $(z_1^*, 1 - z_1^*)$ is agreed upon in the first period. The equilibrium must specify a continuation payoff profile if the current period instead ended in an impasse. This must be some fixed $(\delta^n z_1, \delta^n z_2)$ where $z \in C^M$ and, by lemma 5, $1 \leq n \leq n^*$. It is fixed in the sense that the payoff is independent of the exact incompatible demands made, due to Markov perfection. If there exists a $\hat{z}_2$ that satisfies inequalities 11 and 12 for all $1 \leq n \leq n^*$ then it must also satisfy

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^n(1 - z_1) \qquad \textit{and}$$

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1^n z_1$$

for any such $z \in C^M$ and $1 \leq n \leq n^*$, since $\delta_1^n z_1 \leq \delta_1^n z_1^*$ and $\delta_2^n(1 - z_1) \geq \delta_2^n(1 - z_1^*)$. This means that following the incompatible demand profile $(z_1^*, \hat{z}_2)$ the (unique) dominance solvable outcome in the concession game is $(\textit{Accept}, \textit{Stick})$, bringing player 2 the higher payoff of $\hat{z}_2$. So in this case, the compatible demand profile $(z_1^*, 1 - z_1^*)$ cannot arise in a Markov perfect equilibrium. The same argument applies to compatible demand profiles arbitrarily close to $(z_1^*, 1 - z_1^*)$. Therefore, it must be that no such $\hat{z}_2 > 1 - z_1^*$ exists that satisfies inequalities 11 and 12 for all $1 \leq n \leq n^*$.

Inequalities 11 and 12 simplify to

$$\hat{z}_2 > \frac{(1 - z_1^*)(1 + k_2 - \delta_2^n)}{k_2}, \qquad \textit{and}$$

$$\hat{z}_2 < 1 - \frac{(k_1 + \delta_1^n) z_1^*}{1 + k_1}.$$

Therefore a $\hat{z}_2$ satisfying inequalities 11 and 12 for a given $1 \leq n \leq n^*$ cannot exist only if

$$\begin{aligned} &\frac{(1 - z_1^*)(1 + k_2 - \delta_2^n)}{k_2} \geq 1 - \frac{(k_1 + \delta_1^n) z_1^*}{1 + k_1} \\ \Rightarrow &\frac{(1 - z_1^*)(1 - \delta_2^n)}{k_2} \geq \frac{z_1^*(1 - \delta_1^n)}{1 + k_1} \\ \Rightarrow &\frac{1 - \delta_1^n}{1 - \delta_2^n} \frac{k_2}{1 + k_1} \leq \frac{1 - z_1^*}{z_1^*}. \end{aligned}$$

Finally then a $\hat{z}_2$ satisfying inequalities 11 and 12 for *all* $1 \leq n \leq n^*$ cannot exist only if

$$\min_{n \leq n^*} \frac{1 - \delta_1^n}{1 - \delta_2^n} \frac{k_2}{1 + k_1} \leq \frac{1 - z_1^*}{z_1^*}.$$

A symmetric argument establishes

$$\min_{n \le n^*} \frac{1-\delta_2^n}{1-\delta_1^n}\frac{k_1}{1+k_2} \le \frac{1-z_2^*}{z_2^*}$$
$$\Rightarrow \frac{z_2^*}{1-z_2^*} \le \max_{n \le n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{1+k_2}{k_1}.$$

Since $z \in C^M$ implies that $(1-z_1^*)/z_1^* \le z_2/z_1 \le z_2^*/(1-z_2^*)$, it follows that

$$\min_{n \le n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{k_2}{1+k_1} \le \frac{z_2}{z_1} \le \max_{n \le n^*} \frac{1-\delta_1^n}{1-\delta_2^n}\frac{1+k_2}{k_1}.$$

By lemma 6, if $\delta_1 \ge \delta_2$ then

$$\frac{1-\delta_1^n}{1-\delta_2^n} \le \frac{1-\delta_1^{n+1}}{1-\delta_2^{n+1}}.$$

So if $\delta_1 \ge \delta_2$ then

$$\min_{n \le n^*} \frac{1-\delta_1^n}{1-\delta_2^n} = \frac{1-\delta_1}{1-\delta_2} \qquad \textit{and} \qquad \max_{n \le n^*} \frac{1-\delta_1^n}{1-\delta_2^n} = \frac{1-\delta_1^{n^*}}{1-\delta_2^{n^*}}$$

and the result follows. A symmetric argument works for $\delta_2 \ge \delta_1$.

■

**Proof for Proposition 4.** Let $\sigma$ be the Markov perfect equilibrium with the outcome $(y, m)$. Let $z^t = \sigma(h^{t-1})$ for $h^{t-1} \in H$ and $1 \le t \le m-1$. By assumption $z^t$ is an incompatible demand profile. In the subgame $g(h^{t-1})$, player $i$'s payoff from following $\sigma$ is $\delta_i^{m-t} y_i$. Then it must be that $z_i^t \ge 1 - \delta_{-i}^{m-t} y_{-i}$. Otherwise player $-i$ would do better by making the compatible demand $1 - z_i^t$. Set $D = \{z | z_i \ge 1 - \delta_{-i}^{m-t} y_{-i}\}$. So $z^t \in D$.

Next, there cannot exist $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ such that

$$1 - z_i^t - k_{-i}(z_i^t + \hat{z}_{-i} - 1) < \delta_{-i}^{m-t} y_{-i}$$

and

$$1 - \hat{z}_{-i} - k_i(z_i^t + \hat{z}_{-i} - 1) > \delta_i^{m-t} y_i.$$

Otherwise, player $-i$ in period $t$ would deviate to the incompatible demand $\hat{z}_{-i}$ and the dominance solvable outcome of the resulting concession game would be $(A_i, S_{-i})$

with the higher payoff of $\hat{z}_{-i}$. It is already shown in the proof for lemma 5 that for all $z \in D$ there exists $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ such that $1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < \delta_{-i}^{m-t} y_{-i}$.

Finally, requiring $1 - \hat{z}_{-i} - k_i(z_i^t + \hat{z}_{-i} - 1) \geq \delta_i^{m-t} y_i$ to hold for all $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ implies that

$$z_i^t \geq \frac{(1 - \delta_{-i}^{m-t} y_{-i})(1 + k_i) - \delta_i^{m-t} y_i}{k_i}.$$

■

**Proof for Proposition 5.** A stationary MPE must feature either immediate agreement or perpetual delay. Perpetual delay is ruled out since either player would deviate in the first period to making an arbitrarily small demand. This would either lead to a compatible demand profile, or if incompatible force the opponent to concede. Therefore stationary MPEs feature no delay. The result then follows from lemma 4 and proposition 1. ■

**Proof for Proposition 6.** By lemma 1, any SPE at any history $h \in H$ will involve exactly compatible demands or incompatible ones followed by $(S, S)$. SPE that further satisfy renegotiation-proofness cannot permit delay. To see this, consider a strategy profile, $\sigma$ with outcome $(y, t)$ where $t > 1$. By lemma 1, $y_i = u_i(z_i)$ with $d(z) = 0$. By lemma 3, $y_i > 0$. Now, $c(\sigma; h^0) = (\delta_1^t y_1, \delta_2^t y_2)$ while $c(\sigma; h^t) = (y_1, y_2)$. Since $c(\sigma; h^t) \gg c(\sigma; h^0)$, $\sigma$ is not renegotiation-proof. This concludes the argument for why $t = 1$ if $(y, t)$ is the outcome of a renegotiation-proof SPE in the general model.

Let $O^{RP}(h)$ be the set of all renegotiation-proof SPE outcomes of the game $g(h)$ and let

$$B^{RP} \equiv \left\{ z | (u(z), t) \in O^{RP}(h^0) \right\}.$$

Given the stationary structure of the game it follows that

$$B^{RP} = \left\{ z | (u(z), t) \in O^{RP}(h) \right\}, \qquad \textit{for all } h \in H.$$

*Necessity*

Let $z_i^* = \sup_{z \in B^{RP}} z_i$. Then there cannot exist a deviation $\hat{z}_2 > 1 - z_1^*$ such that

$$u_2(1 - z_1^*) - c_2(z_1^* + \hat{z}_2 - 1) < \delta_2 u_2(1 - z_1^*) \tag{13}$$

and

$$u_1(1 - \hat{z}_2) - c_1(z_1^* + \hat{z}_2 - 1) > \delta_1 u_1(z_1^*). \tag{14}$$

To see why, suppose that $\sigma(h^0) = (z_1^*, 1 - z_1^*)$ and there exists $\hat{z}_2$ that satisfies the inequalities above. Then it must be that

$$u_2(1 - z_1^*) - c_2(z_1^* + \hat{z}_2 - 1) < \delta_2 u_2(z_2)$$

and

$$u_1(1 - \hat{z}_2) - c_1(z_1^* + \hat{z}_2 - 1) > \delta_1 u_1(z_1)$$

for all $z \in B^{RP}$ since for all such $z$, $z_1 \leq z_1^*$ and $z_2 = 1 - z_1 > 1 - z_1^*$. In other words, irrespective of the continuation strategy profile, following such a deviation, in the resulting concession stage game, the dominance solvable outcome would be $(A, S)$, giving player 2 the payoff $u_2(\hat{z}_2)$ which is strictly greater than $u_2(1 - z_1^*)$. Therefore, if such a deviation were to exist then $(z_1^*, 1 - z_1^*) \notin B^{RP}$. The same argument ensures that $z \notin B^{RP}$ for $z$ arbitrarily close to $(z_1^*, 1 - z_1^*)$, which in turn contradicts $z_1^* = \sup_{z \in B^{RP}} z_1$.

Next, for a given compatible demand profile $z$, consider the following equation.

$$u_2(1 - z_1) - c_2(z_1 + \tilde{z}_2 - 1) = \delta_2 u_2(1 - z_1) \tag{15}$$

$\tilde{z}_2$ here is the smallest demand that leads to incompatibility and ensures that 2 prefers $S$ in the resulting concession stage, assuming that in the next period the compatible demand profile $z$ is announced.

Similarly, for a compatible demand profile $z$ consider the following,

$$u_1(1 - \tilde{\tilde{z}}_2) - c_1(z_1 + \tilde{\tilde{z}}_2 - 1) = \delta_1 u_1(z_1). \tag{16}$$

$\tilde{\tilde{z}}_2$ is the largest demand that leads to incompatibility and ensures that in the resulting concession stage, conditional on 2 choosing $S$, 1 prefers $A$, again assuming that in the next period the compatible demand profile $z$ is announced.

By the implicit function theorem, equations 15 and 16 deliver $\tilde{z}_2$ and $\tilde{\tilde{z}}_2$ as functions of $z_1$, denoted $\tilde{z}_2(z_1)$ and $\tilde{\tilde{z}}_2(z_1)$. Now, it cannot be that $\tilde{\tilde{z}}_2(z_1^*) > \tilde{z}_2(z_1^*)$ since then a deviation that satisfies inequalities 13 and 14 would exist; any $\hat{z}_2 \in (\tilde{z}_2(z_1^*), \tilde{\tilde{z}}_2(z_1^*))$ would suffice.

$\tilde{z}_2(z_1)$ is a strictly decreasing function with slope

$$\frac{\partial \tilde{z}_2(z_1)}{\partial z_1} = -\frac{(1-\delta_2)u_2'(1-z_1)}{c_2'(z_1+\tilde{z}_2-1)} - 1 < -1.$$

$\tilde{\tilde{z}}_2(z_1)$ is also a strictly decreasing function with slope

$$\frac{\partial \tilde{\tilde{z}}_2(z_1)}{\partial z_1} = -\frac{\delta_1 u_1'(z_1) + c_1'(z_1+\tilde{\tilde{z}}_2'-1)}{u_1'(1-\tilde{\tilde{z}}_2) + c_1'(z_1+\tilde{\tilde{z}}_2'-1)} > -1$$

by the concavity of $u_i$. Observe that $\tilde{z}_2(1) = 0$ while $\tilde{\tilde{z}}_2(1) > 0$. Also, $\tilde{z}_2(0) > 1$ while $\tilde{\tilde{z}}_2(0)$. Therefore the function $\tilde{z}_2(z_1) - \tilde{\tilde{z}}_2(z_1)$ is positive at $z_1 = 0$, negative at $z_1 = 1$, continuous and (from the slope inequalities above) strictly decreasing over the interval $[0, 1]$ Then again by the intermediate value theorem, there exists a unique $z_1^M$ such that $\tilde{z}_2(z_1^M) - \tilde{\tilde{z}}_2(z_1^M) = 0$. Let $\hat{z}_2(z_1^M) \equiv \tilde{z}_2(z_1^M) = \tilde{\tilde{z}}_2(z_1^M)$.

Since $\tilde{\tilde{z}}_2(z_1) > \tilde{z}_2(z_1)$ for any $z_1 > z_1^M$, it must be that $z_1^* \leq z_1^M$. A symmetric argument establishes that $z_2^* \leq z_2^M$.

*Sufficiency*
Consider the equations

$$u_{-i}(1-z_i) - c_{-i}(z_i+\tilde{z}_{-i}-1) = \delta_{-i}u_{-i}(1-z_i) \tag{17}$$

and

$$u_i(1-\tilde{\tilde{z}}_{-i}) - c_i(z_i+\tilde{\tilde{z}}_{-i}-1) = \delta_i u_i(z_i). \tag{18}$$

As in the necessity argument, by the implicit function theorem, these equations deliver $\tilde{z}_{-i}$ and $\tilde{\tilde{z}}_{-i}$ as functions of $z_i$, denoted $\tilde{z}_{-i}(z_i)$ and $\tilde{\tilde{z}}_{-i}(z_i)$. Similarly, it follows that the function $\tilde{z}_{-i}(z_i) - \tilde{\tilde{z}}_{-i}(z_i)$ is positive at $z_i = 0$, negative at $z_i = 1$, continuous and strictly decreasing over the interval $[0, 1]$. It is clear that the $z_i^M$ corresponding to equations 7 and 8, also satisfies $\tilde{z}_{-i}(z_i^M) = \tilde{\tilde{z}}_{-i}(z_i^M)$.

I first show that $z_1^M \geq 1 - z_2^M$. To see this, consider equations 7 and 8 with $i = 1$. The solution is $(z_1^M, \hat{z}_2(z_1^M))$. This implies the following pair of inequalities

$$u_2(1-z_1^M) - c_2(z_1^M + \hat{z}_2(z_1^M) - 1) < \delta_2 u_2(\hat{z}_2(z_1^M))$$

and

$$u_1(1 - \hat{z}_2(z_1^M)) - c_1(z_1^M + \hat{z}_2(z_1^M) - 1) > \delta u_1(1 - \hat{z}_2(z_1^M))$$

since $\hat{z}_2(z_1^M) > 1 - z_1^M$. Comparing these inequalities with equations 17 and 18 with $i = 2$, it follows that

$$\tilde{z}_1(\hat{z}_2(z_1^M)) > z_1^M \qquad \textit{and} \qquad \tilde{\tilde{z}}_1(\hat{z}_2(z_1^M)) < z_1^M.$$

This in turn implies that $\tilde{z}_1(\hat{z}_2(z_1^M)) - \tilde{\tilde{z}}_1(\hat{z}_2(z_1^M)) > 0$. Therefore it must be that $z_2^M > \hat{z}_2(z_1^M)$. Since $\hat{z}_2(z_1^M) > 1 - z_1^M$ it follows that $z_2^M > 1 - z_1^M$.

Now, fix some $z$ such that $d(z) = 0$ and $z_i \leq z_i^M$ for $i \in \{1, 2\}$. Consider the following stationary strategy profile, $\sigma$. For all $h^t \in H$, $\sigma_i(h^t) = z_i$. If player $i$ in period $t$ deviates to making a higher demand, $\hat{z}_i > z_i$, then in the concession stage game $(S, S)$ is played if it is a Nash equilibrium and otherwise $(A_i, S_{-i})$ is played. For all other $h \in H'$ some pure strategy Nash equilibrium of the concession stage game is played.

Given the strategy profile $\sigma$, it is clear that making a lower demand at any period is never profitable. Making a higher demand for player $i$ also yields her a lower payoff, since it either leads to $(S, S)$ in the concession game and a continuation payoff of $\delta_i u_i(z_i)$ or $(A_i, S_{-i})$ with a payoff strictly less than $u_i(z_i)$ due to the concession cost. Hence no profitable deviation exists in any demand stage. To verify subgame perfection, therefore, it is sufficient to verify that following an incompatible demand profile $(z_i, \hat{z}_{-i})$, if $(S, S)$ is not a Nash equilibrium then $(S_i, A_{-i})$ is. For this it is sufficient to show that $\tilde{z}_{-i}(z_i) \geq \tilde{\tilde{z}}_{-i}(z_i)$.

Recall that $\tilde{z}_{-i}(z_i)$ as defined in equation 17 corresponds to the smallest demand by $-i$ that leads to incompatibility and ensures that $-i$ prefers $S$ over $A$ in the subsequent concession stage game, assuming that in the next period the compatible profile $z$ is announced. $\tilde{\tilde{z}}_{-i}(z_i)$, as defined in equation 18 in turn is the largest demand by $-i$ that leads to incompatibility and ensures that in the subsequent concession game, $i$ prefers $(A_i, S_{-i})$ to $(S, S)$, assuming that in the next period $z$ is announced. So if $\tilde{z}_{-i}(z_i) \geq \tilde{\tilde{z}}_{-i}(z_i)$ then following any incompatible demand $\hat{z}_{-i}$, if $(A_i, S_{-i})$ is a Nash equilibrium, then it must be that $\hat{z}_{-i} \leq \tilde{z}_{-i}(z_i)$ and therefore $(S_i, A_{-i})$ is a Nash equilibrium too. Since $(A, A)$ is never a Nash equilibrium, this shows that with $\tilde{z}_{-i}(z_i) \geq \tilde{\tilde{z}}_{-i}(z_i)$ if $(S, S)$ is not a Nash equilibrium then $(S_i, A_{-i})$ must be.

Finally observe that $\tilde{z}_{-i}(z_i) \geq \tilde{\tilde{z}}_{-i}(z_i)$ since $z_i \leq z_M$. ■

**Proof for Proposition 7.** It follows from proposition 6 that

$$\xi(g^{c^n}) = \left\{ y = u(z) \middle| \frac{1 - z_1^{Mn}}{z_1^{Mn}} \leq \frac{z_2}{z_1} \leq \frac{z_2^{Mn}}{1 - z_2^{Mn}} \ \textit{and} \ d(z) = 0 \right\},$$

where the incompatible demand profile $(z_i^{Mn}, \hat{z}_{-i}^{Mn})$ for $i \in \{1, 2\}$ is characterized by the equations,

$$u_{-i}(1 - z_i^{Mn}) - c_{-i}^n(z_i^{Mn} + \hat{z}_{-i}^n(z_i^{Mn}) - 1) = \delta_{-i}u_{-i}(1 - z_i^{Mn}) \tag{19}$$

$$u_i(1 - \hat{z}_{-i}^n(z_i^{Mn})) - c_i^n(z_i^{Mn} + \hat{z}_{-i}^n(z_i^{Mn}) - 1) = \delta_i u_i(z_i^{Mn}). \tag{20}$$

Set $z_i^{M*} = \lim_{n\to\infty} z_i^{Mn}$. Notice that since $u_{-i}$ is bounded above and $c_{-i}^{n'}(0+) \to \infty$ as $n \to \infty$, it follows from equation 19 that $\lim_{n\to\infty} \hat{z}_{-i}(z_i^{Mn}) = 1 - \lim_{n\to\infty} z_i^{Mn} = 1 - z_i^{M*}$.

Now equations 19 and 20 together imply

$$\frac{(1 - \delta_{-i})u_{-i}(1 - z_i^{Mn})}{u_i(1 - \hat{z}_{-i}^n(z_i^{Mn})) - \delta_i u_i(z_i^{Mn})} = \frac{c_{-i}^n(z_i^{Mn} + \hat{z}_{-i}(z_i^{Mn}) - 1)}{c_i^n(z_i^{Mn} + \hat{z}_{-i}^n(z_i^{Mn}) - 1)}.$$

Taking limits on both sides of this equation as $n \to \infty$ gives

$$\frac{(1 - \delta_{-i})u_{-i}(1 - z_i^{M*})}{(1 - \delta_i)u_i(z_i^{M*})} = \lim_{n\to\infty} \frac{c_{-i}^n(z_i^{Mn} + \hat{z}_{-i}(z_i^{Mn}) - 1)}{c_i^n(z_i^{Mn} + \hat{z}_{-i}^n(z_i^{Mn}) - 1)}.$$

The right hand side is equal to $\gamma$ for $i = 2$ and $1/\gamma$ for $i = 1$. Therefore,

$$\frac{(1 - \delta_2)u_2(1 - z_1^{M*})}{(1 - \delta_1)u_1(z_1^{M*})} = \frac{1}{\gamma} \ \textit{and} \ \frac{(1 - \delta_1)u_1(1 - z_2^{M*})}{(1 - \delta_2)u_2(z_2^{M*})} = \gamma.$$

Now $y \in \xi_\gamma^*(u)$ implies that $y = u(z)$ such that $d(z) = 0$ and

$$\frac{u_2(1 - z_1^{M*})}{u_1(z_1^{M*})} \leq \frac{u_2(z_2)}{u_1(z_1)} \leq \frac{u_2(z_2^{M*})}{u_1(1 - z_2^{M*})}$$

$$\Leftrightarrow \frac{1 - \delta_1}{1 - \delta_2}\frac{1}{\gamma} \leq \frac{u_2(z_2)}{u_1(z_1)} \leq \frac{1 - \delta_1}{1 - \delta_2}\frac{1}{\gamma}.$$

■

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