Screening in Vertical Oligopolies

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Motivation

- Screening is central in economic theory and empirical work
  - Mussa and Rosen (1978), Maskin and Riley (1984), Rothschild-Stiglitz (1976)
- This paper $\rightarrow$ screening with oligopolistic competition
- Important questions:
  - What do equilibria look like? Do pure-strategy equilibria exist?
  - Who does asymmetric information help or hurt?
  - What are the equilibrium effects of entry, or of mergers?

Main Results

- Necessary conditions for equilibrium
- Sufficiency conditions and existence
- Welfare, entry, mergers

Literature on Oligopolistic Screening

- Spulber (1989)
- Champsaur and Rochet (1989)
- Biglaiser and Mezzetti (1993)
- Stole (1995)
- Jullien (2000)

The Model

Principals and Agents

- Unit measure of agents (customers or workers)
  - $\theta \in [0,1]$, $\theta \sim H$, $\mathcal{C}^1$ density $h > 0$
  - H and $1 - H$ strictly log-concave
- Each agent chooses an observable action $a \geq 0$
  - Utility $\mathcal{U}(a) + a\theta - t$, $\mathcal{U}$ is $\mathcal{C}^2$, $t \in \mathbb{R}$
- N principals (firms)
  - Profit $\mathcal{V}^n(a) + t$, $\mathcal{V}^n(a)$ strictly spm (a, n), $\mathcal{C}^2$ in a
  - Profits additively separable across agents served
- Match surplus between n and θ who chooses a is $V^n(a) + a\theta$
  - $V^n(a) = \mathcal{V}^n(a) + \mathcal{U}(a)$, V^n strictly concave Examples

Contracts, Surplus, and Profits

- Firm n offers a menu (α^n, t^n) (pair of functions)
  - $\alpha^n(\theta) \rightarrow$ action required of θ by n
  - $t^n(\theta) \rightarrow$ transfer to θ by n
- Contracts are exclusive, cannot depend on offers of other firms
- Equivalently, $v^n(\theta) = \mathcal{U}(\alpha^n(\theta)) + \alpha^n(\theta)\theta - t^n(\theta)$, and Firm n offers (α^n, v^n)
  - $v^n(\theta) \rightarrow$ surplus offered to θ by n
- Menu (α^n, v^n) incentive compatible iff
  - α^n increasing
  - $v^n(\theta) = v^n(0) + \int_0^\theta \alpha^n(\tau) d\tau$
- $S^n \rightarrow$ incentive compatible menus $s^n = (\alpha^n, v^n)$, $S = \times_{n=1}^N S^n$

Contracts, Surplus, and Profits

- Profit to n on θ if surplus to agent is v_0 and action is a is

$$\pi^n(\theta, a, v_0) \equiv V^n(a) + a\theta - v_0$$

- Write $\pi^n(\theta, \alpha, v)$ for $\pi^n(\theta, \alpha(\theta), v(\theta))$
- Assumption (Relevance) $\rightarrow$ Each firm is uniquely best at serving some type:
  - For each n there is a θ such that

$$v_*^n(\theta) \equiv \max_a (V^n(a) + a\theta) > \max_{n' \neq n} v_*^{n'}(\theta)$$

  - By strictly spm V^n, there is then an interval of actions (a_e^{n-1}, a_e^n), with $V^n(a_e^n) = V^{n+1}(a_e^n)$, such that n is most efficient at action a in the interval

The Game and Equilibrium

- Firms simultaneously post menus $s^n = (\alpha^n, v^n)$
  - Notation: s^{-n}, s, $v^{-n}(\theta) = \max_{n' \neq n} v^{n'}(\theta)$, a^{-n} slope of v^{-n}
  - v^{-n} and a^{-n} summarize everything n cares about
- Agents sort themselves to the most advantageous firm and announce types
- Firm n wins θ if $v^n(\theta) > v^{-n}(\theta)$ and loses if $v^n(\theta) < v^{-n}(\theta)$
- Ties broken equiprobably

The Game and Equilibrium

- $\Pi^n(s) = \int \pi^n(\theta, \alpha^n, v^n)\varphi^n(\theta, s)h(\theta)d\theta$
- $BR^n(s) = \arg\max_{s^n \in S^n} \Pi^n(s^n, s^{-n})$
- A strategy profile s is a Nash equilibrium of $(S^n, \Pi^n)_{n=1}^N$ if $\forall n$, $s^n \in BR^n(s)$
    - Pure strategies
    - Refinement: No Extraneous Offers (*NEO*)
        - Equilibrium exhibits *NEO* if α^n is continuous with actions in $[a_e^{n-1}, a_e^n]$

Necessary Conditions

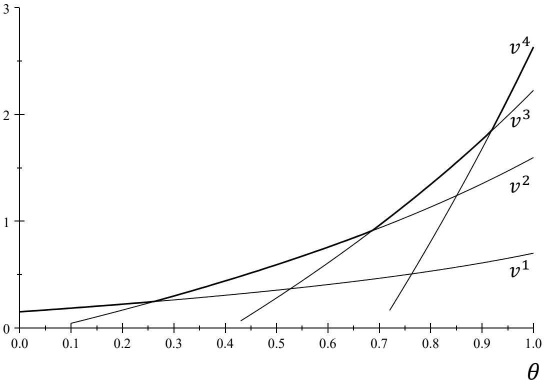
Positive Profits (PP) and No Poaching (NP)

- $PP \rightarrow$ Probability Firm n serves a type on whom it strictly loses money is 0
  - Intuition: given any $s^n \in S^n$, get rid of losing money contracts (private values)
  - Implications: strictly positive expected profits, no cross-subsidization
- $NP \rightarrow$ For all θ, $v^O(\theta) \geq V^{(2)}(a^O(\theta)) + a^O(\theta)\theta$
  - $v^O(\cdot) = \max_n v(\cdot)$, $a^O(\cdot)$ associated actions, $V^{(2)}(a)$ second largest $V^n(a)$
  - Imitating θ's equilibrium contract is unprofitable

Positive Sorting (PS)

- $PS \to$ Ordered intervals (θ_l^n, θ_h^n)
  - $\varphi^n = 1$ on (θ_l^n, θ_h^n)
  - $\varphi^n = \frac{1}{2}$ on $[\theta_h^{n-1}, \theta_l^n]$ and $[\theta_h^n, \theta_l^{n+1}]$
- Intuition: Single-crossing
- $SPS \to \theta_h^{n-1} = \theta_l^n$ for all n (no overlap)
- Implications:
  - Competition between adjacent firms can lead to overlap
  - Complete profit dissipation on overlapped types
  - If firms are differentiated enough, then SPS and gaps in actions
  - α^n continuous where $v^n \geq v^{-n}$ (property of best response)

Strict PS



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Internal Optimality (IO)

- Define $\gamma^n(\cdot, \kappa)$ by
$$\pi_a^n(\theta, \gamma^n(\theta, \kappa), v) = \frac{\kappa - H(\theta)}{h(\theta)}$$
where $\kappa \in [0, 1]$ and $\pi_a^n = V_a^n + \theta$

- $IO \rightarrow$ for each n there is $\kappa^n \in [H(\theta_l^n), H(\theta_h^n)]$, $\kappa^1 = 0$ and $\kappa^N = 1$, such that $\alpha^n = \gamma^n(\cdot, \kappa^n)$ on $[\theta_l^n, \theta_h^n]$

- Implications,
  - Firm 1 distorts actions upwards, Firm N downwards
  - A middle firm n distorts actions downward below $\theta_0^n = H^{-1}(\kappa^n)$, upwards above

- Proof relies on solution of a relaxed problem Relaxed

Optimal Boundaries (OB)

$$OB \to \begin{cases} \pi^n(\theta_h^n, \alpha^n, v^n) + \pi_a^n(\theta_h^n, \alpha^n, v^n)(a^{-n}(\theta_h^n) - \alpha^n(\theta_h)) = 0 \\ \pi^n(\theta_l^n, \alpha^n, v^n) - \pi_a^n(\theta_l^n, \alpha^n, v^n)(\alpha^n(\theta_l^n) - a^{-n}(\theta_l^n)) = 0 \end{cases}$$

- Discard the second condition for Firm 1, and the first one for Firm N
- Intuition (with SPS) pic
- In contrast to NP, OB is about local changes in who is served
- Implications:
  - For $n \neq \{1, N\}$, $\kappa^n \in (H(\theta_l^n), H(\theta_h^n))$, so upward/downward distortions bite
  - Most profitable type is interior for $n \neq \{1, N\}$
  - π^n strictly single peaked at θ_0, $\pi^n > 0$ on (θ_l^n, θ_h^n), and on $[\theta_l^n, \theta_h^n]$ if SPS

Sufficiency and Existence

Stacking and Main Result

- *Stacking* $\to$ for all $n < N$, $\gamma^{n+1}(\cdot, 1) > \gamma^n(\cdot, 0)$
  - Eliminates ties at boundaries; holds if firms are differentiated enough ▸ Example
  - v^n and v^{n+1} cross strictly; set of types served change continuously in s^n
- Given s^{-n} and n, s^n and $\hat{s}^n$ are *equivalent* if they differ only where neither wins; strategy profiles s and $\hat{s}$ *equivalent* if equivalent for each n

Theorem

Assume stacking. Then any strategy profile satisfying PS, IO, and OB is equivalent to a Nash equilibrium, and a Nash equilibrium exists.

- Result affords easy numeric analysis: $3N - 3$ equations and unknowns ▸ num
- Sufficiency is hard since $\Pi^n(\cdot, s^{-n})$ not quasiconcave
- Existence is hard since Π^n not continuous, $\Pi^n(\cdot, s^{-n})$ not quasiconcave

Sufficiency

- Move from choice by n of s^n to a two-dimensional problem
  - By *IO*, $\alpha^n(\cdot) = \gamma^n(\cdot, \kappa^n)$, and can focus on optimal choice of θ_l^n, θ_h^n
- We restrict menus as follows:
  - **C1** α^n continuous, $\alpha^n(\theta) \in [\gamma^n(\theta, 1), \gamma^n(\theta, 0)]$ for all θ
  - **C2** $v^n \leq v_*^n$
- We can then relate n's original problem with $\max_{\theta_l, \theta_h} r(\theta_l, \theta_h)$

Proposition

Assume stacking. Fix n and s^{-n} satisfying $C1$ and $C2$. Then, r has a maximum (θ_l, θ_h), and $\hat{s}$ is a maximum of $\Pi^n(\cdot, s^{-n})$ if and only if for some maximum (θ_l, θ_h) of r, $\hat{s}$ is the single winner on (θ_l, θ_h), and $\hat{s}$ and $\tilde{s}(\theta_l, \theta_h)$ are equivalent.

- Does r have a unique maximum? Yes. Most of the work is here

Sufficiency

- Outline of the proof of sufficiency:
  - Let $\hat{s}$ satisfy stacking, *PS*, *IO*, *OB*
  - Fix n, let $\hat{s}^n = (\hat{\alpha}, \hat{v})$ with $\hat{\kappa}$
  - *IO* $\Rightarrow$ **C1** on (θ_l, θ_h), and with *OB* $\Rightarrow \pi^n > 0$ for all $\theta \in [\theta_l, \theta_h] \Rightarrow$ **C2** holds
  - Redefine $(\hat{\alpha}, \hat{v})$ outside $[\theta_l, \theta_h]$ so **C1** and **C2** hold as well $\rightarrow$ equivalent (α, v)
  - Do the same for all n to obtain strategy profile s
  - Unique maximum property (where profits are positive) of r yields best response property of $s^n = (\alpha, v)$ against s^{-n}
  - Thus, $\hat{s}$ is equivalent to a Nash Equilibrium

Existence

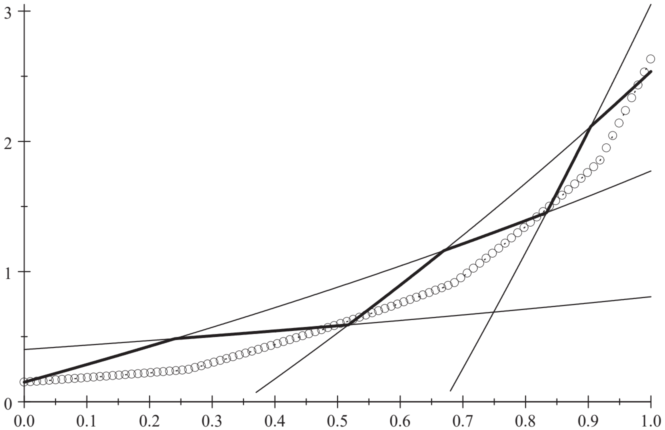
- Outline of the proof of existence:
  - Restrict strategy space so that continuity and convexity of best responses hold
  - **C3** uniform bound on γ and its slope; **C4** lower bound of surplus at $\theta = 1$
  - For each n define $S^n_R \subset S^n$ s.t. **C1–C4** hold
  - If $s^{-n} \in S^{-n}_R$, then $BR^n(s^{-n}) \cap S^n_R$ (sufficiency is key here)
  - $(S^n_R, \Pi^n)_{n=1}^N$ has a Nash Equilibrium (all the conditions of Kakutani-Fan-Glicksberg Theorem are satisfied; sufficiency is key here)

Implications and Applications

Welfare Effects

- Consider the complete information version of the model
- In a monopoly world,
  - Agents lose all information rents
  - Allocation becomes efficient
  - Firm is unambiguously better off
- In our setting,
  - Agents again lose information rents
  - But poaching is easier and so v^{-n} increases
  - Agents near the "boundaries" are unambiguously better off
  - All agents can be strictly better off

Welfare Effects



Multiplant Monopoly and Mergers

- What are the effects of mergers in our setting?
- Building block $\rightarrow$ Multiplant monopoly case
  - Single firm M controls technologies $V^{n_l},..,V^{n_h}$
  - Faces a type dependent outside option $\bar{u}$, first "shallow" then "steep" (stacking)
  - All previous results apply (M serves $[\theta_l^M, \theta_h^M]$, IO with single κ, OB)
  - Finite number of jumps in γ^M ▸ Multiplant

Multiplant Monopoly and Mergers

- Oligopoly $n_l, ..., n_h$ versus multiplant monopoly M
  - Fixed span: both serve $[\theta_l, \theta_h]$ ("must-serve" condition imposed on M)
- All types in (θ_l, θ_h) are strictly worse off under M
  - An interval of low types receive a strictly lower action than before
  - An interval of high types receiving a strictly higher action than before
- Intuition $\rightarrow$ more interior types to extract rents from
- Must-serve condition not enough to protect consumers after a merger

Multiplant Monopoly and Mergers

- Without legal constraint, M will not only lower surplus but also shed types

Theorem

Let M optimally serve $[\theta_l^M, \theta_h^M]$. Then $[\theta_l^M, \theta_h^M] \subset [\theta_l, \theta_h]$. All types in (θ_l, θ_h) are strictly worse off compared to oligopoly.

- What if M is just a subset of all firms?
- There are countervaling forces ▸ Merger
  - M lowers surplus and sheds types $\rightarrow$ incentives for other firms to lower surplus
  - Adjacent firms to M can gain types "cheaply" $\rightarrow$ incentives to increase surplus
  - All computed examples show first effect dominates, and also that it is better to have a merger than to let a firm exit ("failing-firm" defense)

Conclusion

- Screening among heterogeneous oligopolists
  - Higher-index firms serve higher intervals of types
  - Equilibrium pinned down by intuitive local conditions
  - Implications for welfare, mergers, and entry
- Many open questions
  - Horizontal differentiation
  - Common values
  - Moral hazard

Competitive Limit

- Forces that affect equilibrium surplus of any given type:
  - Action is distorted; firm and type mismatched; firm that serves type earns profits
- As number of firms grows we obtain efficiency and all surplus goes to agents
  - Firms enter at a cost $F > 0$ and choose $z \in [\underline{z}, \overline{z}]$, $V(\cdot, z)$
  - For any N, there is $[z_l, z_h]$ s.t. $z_l \leq z^1 < \cdots < z^N \leq z_h$, so $V^n(a) = V(a, z^n)$
  - Equilibrium with endogenous entry (EEE): $\Pi^n \geq F$, no new entrant can do so

Theorem

In any EEE with NEO, there is $\rho \in (0, \infty)$ s.t. $1/(\rho F^{1/3}) \leq N \leq (\rho/F^{1/3}) + 2$, while π, and difference between $v(\theta)$ earns and $v_*(\theta)$ are each of order $1/N^2$.

Examples

- Product market with quality differentiation:
  - $\mathcal{V}^n(a) = -c^n(a)$, c^n cost to Firm n of quality a, c^n is convex, strictly sbm
  - $\mathcal{U}(a) + a\theta = \sqrt{\rho + a} + a\theta$, $\rho > 0$ small, be the value to θ of product quality a
  - $V^n(a) = \sqrt{\rho + a} - c^n(a)$
- Labor market:
  - $\mathcal{V}^n(a) = \zeta^n + \beta^n \log(\rho + a)$, $\rho > 0$ small, β^n is strictly increasing in n
  - Worker's effort disutility $c(a) - a\theta$, c convex, and thus $\mathcal{U}(a) + a\theta = -c(a) + a\theta$
  - $V^n(a) = \zeta^n + \beta^n \log(\rho + a) - c(a)$ Back Stacking

Relaxed Problem

$$r(\theta_l, \theta_h) = \max_{(\alpha, v)} \int_{\theta_l}^{\theta_h} \pi(\theta, \alpha, v) h(\theta) d\theta$$

$$s.t. \;\; v(\theta_l) \geq v^{-n}(\theta_l)$$

$$v(\theta_h) \geq v^{-n}(\theta_h), \text{ and}$$

$$v(\theta) = v(0) + \int_0^{\theta} \alpha(\tau) d\tau \text{ for all } \theta.$$

- Solution is unique on $[\theta_l, \theta_h]$ and with the IO form
- Elsewhere set $\alpha(\theta) = \alpha(\theta_h)$ for all $\theta \geq \theta_h$, $\alpha(\theta) = \alpha(\theta_l)$ for all $\theta \leq \theta_l$ Back

Numeric Analysis

- Unknowns: N $v^n(0)$'s, $N-1$ θ^n's, $N-2$ κ^n's, so $3N-3$ unknowns
- Equations:

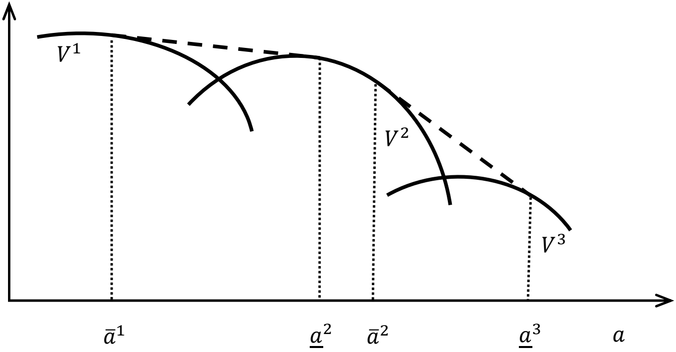
$$v^n(\theta^n) - v^{n+1}(\theta^n) = 0$$

$$\pi^n(\theta^n, \gamma^n(\cdot, \kappa^n), v^n) + (\kappa^n - \theta^n)(\gamma^{n+1}(\theta^n) - \gamma^n(\theta^n)) = 0$$

$$\pi^{n+1}(\theta^n, \gamma^{n+1}(\cdot, \kappa^{n+1}), v^{n+1}) + (\kappa^{n+1} - \theta^n)(\gamma^n(\theta^n) - \gamma^{n+1}(\theta^n)) = 0$$

- N equal surplus at boundaries, $2(N-1)$ *OB*, so $3N-3$ equations Back

Multiplant Monopoly and Mergers



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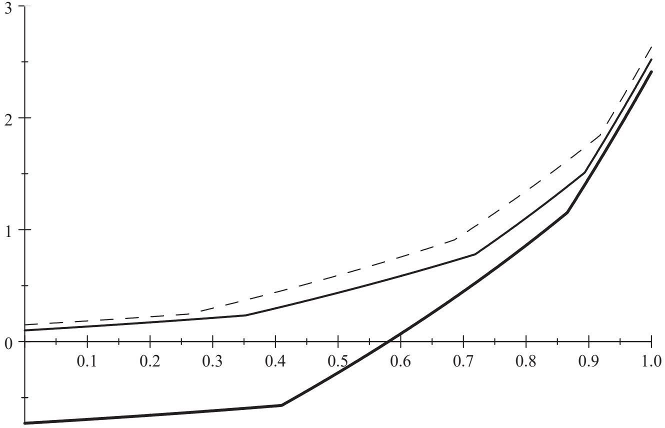
Multiplant Monopoly and Mergers



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