Accept this Paper

Lones Smith Andrea Wilson Mavi Wilson

Wisconsin and Princeton

Best, Brightest and Rejected: Elite Colleges Turn Away Up to 95%

By **RICHARD PÉREZ-PEÑA** APRIL 8, 2014

Stanford University accepted 5 percent of applicants in the latest admissions season, a new low among elite colleges.
Thor Swift for The New York Times

Goal: Is Selectivity Excellence?

- Colleges advertise “selectivity”
  - U.S. News and World Report college rankings puts 12.5% weight on selectivity
  - The Princeton Review weights it as one of seven factors
- Should the best colleges have the highest rejection rates?
- Should the best journals have the highest rejection rates?
- Better journals have higher standards, but get better papers. Why should the former effect dominate?

Selectivity Need Not Be Excellence

- Short run — shrink your college and your rejection rate rises
- Chade, Lewis, and Smith "Student Portfolios and the College Admissions Problem" (*REStud*, 2014)
  - An elite college 1 and a safety college 2 respectively offer students a fixed high and low payoff
  - A continuum of heterogeneous students each choose to apply to stretch college 1, or safety college 2, or both, or neither; each application costs $c > 0$
  - Student evaluation is noisy: Colleges choose admission thresholds for random signals generated by students
  - Proposition: If college 2 shrinks its student capacity enough, (a) better students need not apply more ambitiously, and (b) college 1 has lower admission standards than college 2

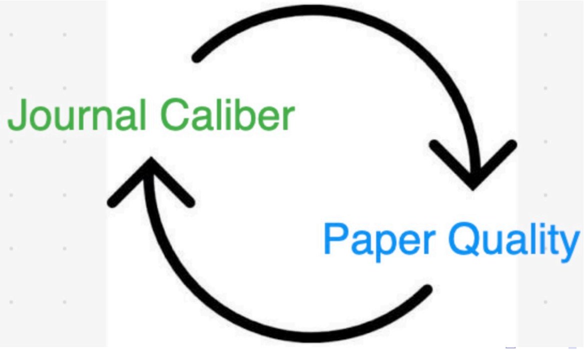
Static Game of Incomplete Information

Step 1 *An endogenous pool* of journals publicize and commit to standards and "calibers"

Step 2 As a function of his paper quality, each author submits to a single journal, seeking to maximize caliber $\times$ admission chance

Step 3 Rational expectations: Acceptance decisions ensure that *average acceptance quality equals advertised caliber*

- Similar to Bayesian persuasion's cheap talk with commitment



Benchmark Model: The Author Knows His Paper Quality

- Continuum Mass of Heterogenous Authors/Papers
  - Each has a unique paper with some *quality* x
  - Density of paper qualities on $[\underline{x}, \infty)$, where $0 < \underline{x} < \infty$

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- No Market Power: Continuum Mass of Journals
  - Journal *caliber* is the average quality of accepted papers
  - Caliber is in monetary value units: a quality v publication is worth v to the author
  - Free entry and exit of journals of any caliber
  - Knowing his paper quality, author picks a journal to submit to

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- Submission and Stochastic Evaluation
  - Journals see a noisy signal σ of the quality of any submitted paper, and choose whether to accept or reject it
  - Evaluation noise has location family: a quality x paper yields a signal realization σ, where $\sigma - x$ has a probability density g.
    - Example: Gaussian noise $g(\sigma - x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\phi^2}(\sigma - x)^2}$

A Robust Assumption on Signal Noise

- log-concave signal density g (eg. Gaussian, Gamma, uniform)

$\Rightarrow$ signal cdf G is log-concave (and thus continuous)

$\Rightarrow$ hazard rate $\frac{g(t)}{1-G(t)}$ is increasing.

$\Rightarrow$ The density is positive on a connected interval

$\Rightarrow$ No signal is perfectly revealing

- assume this interval has upper bound ∞
  $\Rightarrow$ every paper has a positive chance at every journal

Equilibrium Analysis

- Journal Motivations
  - Rational Expectations: promised caliber is realized
    - intuitive long-run steady-state with journal reputations
    - Short-run: Fly-by-night (or "predatory") journals reimburse authors for gap between their promised and delivered caliber
    - Journals publicly commit to acceptance standards

$\Rightarrow$ Journal v accepts when signal $\sigma \geq \theta(v)$, *acceptance threshold*

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  $\Rightarrow$ Journal v accepts when signal $\sigma \geq \theta(v)$, *acceptance threshold*
- Author Payoffs
  - Author's payoff is caliber times acceptance chance
    - $\Rightarrow$ subsumes dynamic case with resubmission and discounting when the author cares about $(1-\delta)$ times this
  - Author of quality x paper who submits to a caliber v journal with threshold θ gets payoff

$$(1 - G(\theta - x)) \cdot v$$

  $\Rightarrow$ acceptance threshold θ depends only on caliber v, for authors clearly submit to the lowest threshold journal for any caliber

Authors Play a Separating Equilibrium

Lemma

Every author submits to a journal equal to his caliber.

- Proof Sketch
  - Assume pooling occurs $\Rightarrow$ multiple papers go to same journal
  - Rational expectations $\Rightarrow$ some paper exceeds journal caliber: $x' > \kappa$

$\Rightarrow$ A journal κ' in (κ, x') can enter and skim off x' (log-concavity)

Separating Equilibrium Proof

- By rational expectations, it suffices to preclude pooling equilibria, where a journal v_1 (with threshold θ_1) attracts two or more paper qualities $x < v_1$ and $x' > v_1$.

$\Rightarrow$ **Claim:** *If so, a new journal skim off best papers at* v_1

- *Proof of Claim:* Let a new journal promise caliber $v_2 \in (v_1, x')$ and choose a threshold $\theta_2 > \theta_1$ that makes type x' indifferent, so that $[1 - G(\theta_2 - x')]v_2 = [1 - G(\theta_1 - x')]v_1$, then

$$\Rightarrow \frac{1 - G(\theta_2 - x')}{1 - G(\theta_1 - x')} = \frac{v_1}{v_2} \in (0, 1) \qquad (\clubsuit)$$

  $LHS = 1$ at $\theta_2 = \theta_1$, and continuously falls to zero as $\theta_2 \uparrow \infty$.
- By log-concavity of G, the left side of ($\clubsuit$) increases in x', since $\log(1 - G(\theta_2 - x')) - \log(1 - G(\theta_1 - x'))$ increases in x'
- Papers $x'' > x'$ prefer journal v_2, and papers $x'' < x'$ prefer v_1.
- Journal v_2 attracts only quality $x'' \geq x'$, but promise a caliber $v_2 < x'$, earning profits. Contradiction.

Journal Equilibrium: A Reduced Form Description

- A journal equilibrium is a threshold function $\theta(v)$ for which it is optimal for every author $x \in [\underline{x}, \infty)$ to submit to the same caliber journal $v = x$

The Worst Journal is not Selective

Lemma

The worst journal has caliber $\underline{x}$, and accepts all submissions.

- Proof: Since we ruled out pooling in equilibrium, the least caliber journal cannot exceed $\underline{x}$
- If the least journal $\underline{x}$ sometimes rejects, a new journal can enter, always accept, and attract all paper qualities just over $\underline{x} > 0$ (making profits) □

The Equilibrium First Order Condition

- *Author optimality*, given paper of quality x:

$$\max_v \,(1 - G(\theta(v) - x))\, v$$

- The interior FOC is

$$(1 - G(\theta(v) - x)) - g(\theta(v) - x)\theta'(v)v = 0$$

- By rational expectations, this must hold at $v = x$:

$$\Rightarrow \theta'(v) = \frac{1}{v} \cdot \frac{1 - G(\theta(v) - v)}{g(\theta(v) - v)} \qquad \text{[FOC*]}$$

- The SOC holds, given log-concavity

Journal Selectivity is Hump-Shaped

- equilibrium toughness $\tau(v) = \theta(v) - v$
- *equilibrium rejection rate* is $R(v) = G(\tau(v))$.

Proposition

(a) *There exists a unique equilibrium.*
(b) *The rejection rate is hump-shaped for all small* $\underline{x} > 0$.

- The rejection rate is hump-shaped if $\tau(v)$ is hump-shaped.

Proof of Hump-Shaped Toughness

- Recall equilibrium FOC:

$$\Rightarrow \theta'(v) = \frac{1}{v} \cdot \frac{1 - G(\theta(v) - v)}{g(\theta(v) - v)} \qquad \text{[FOC*]}$$

- Let's rewrite equilibrium FOC using τ:

$$\tau'(v) = \frac{1}{v} \cdot \frac{1 - G(\tau(v))}{g(\tau(v))} - 1 \qquad (\bigstar)$$

- First, $(\bigstar) \Rightarrow \tau'(\underline{x}) > 0$ for small enough $\underline{x}$
- By log-concavity, the reciprocal hazard rate $(1 - G)/g$ falls
- So $\tau(v)$ weakly rising implies $\tau'(v)$ strictly falling

$\Rightarrow$ any critical point is a max: $\tau'(v) = 0 \Rightarrow \tau''(v) < 0$

- But $\tau(v)$ cannot rise forever: For if so, the RHS of $(\bigstar)$ tends to -1, contradiction

An Intuition for the Hump-Shape

- Rewrite the equilibrium FOC with θ as independent variable:

$$\frac{v'(\theta)}{v(\theta)} = \frac{g(\theta - v(\theta))}{1 - G(\theta - v(\theta))} \qquad (\bigstar)$$

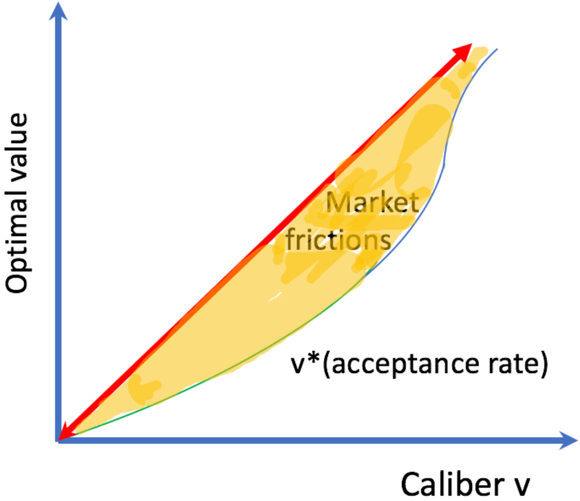
- Aside: The *rate* of increase in the journal caliber matches the (absolute) *rate* of fall of the acceptance rate in toughness:

$$[\log v(\theta)]' = -[\log(1 - G(t))]'\Big|_{t=\theta - v(\theta)}$$

- Whenever the rejection rate is increasing in θ
  $\Rightarrow$ equilibrium toughness $t(\theta) = \theta - v(\theta)$ is increasing in θ
  $\Rightarrow$ Differentiating, $0 < t(\theta) = 1 - v'(\theta)$
  $\Rightarrow$ If rejection rate always increases: $\frac{v'(\theta)}{v(\theta)} < \frac{1}{v(\theta)} \downarrow 0$ at high θ
- But an increasing rejection rate $G(\tau)$ in θ
  $\Rightarrow$ increasing $g(t)/[1 - G(t)]$, by log-concavity
  $\Rightarrow$ monotone increasing $v'(\theta)/v(\theta)$, by $(\bigstar)$
- So forever increasing rejection rate $\Rightarrow$ contradiction

Matching Frictions and Caliber

- The rejection rate is an informational market friction.
- Here, all rejections are mistakes.
- We plot the expected payoff for each caliber of paper.
- The sorting losses reflect the hump-shaped rejection rates



Solved Exponential Noise Example

- Assume $g(t) = \lambda e^{-\lambda t}$ and $G(t) = 1 - e^{-\lambda t}$
- The equilibrium FOC is

$$\theta'(v) = \frac{1}{v} \cdot \left(\frac{1 - G(\theta(v) - v)}{g(\theta(v) - v)} \right) = \frac{1}{\lambda v} \Rightarrow \theta(v) = \frac{1}{\lambda} \log v + C$$

- Boundary condition:
  Sure acceptance at journal $\underline{x} \Rightarrow \theta(\underline{x}) = \underline{x}$ and $C = \underline{x} - \frac{1}{\lambda} \log \underline{x}$

$\Rightarrow$ Journal threshold $\theta(v) = \underline{x} + \frac{1}{\lambda} \log \frac{v}{\underline{x}}$.

$\Rightarrow$ Rejection rate

$$R(v) = G(\theta(v) - v) = 1 - e^{-\lambda(\theta(v) - v)} = 1 - \frac{\underline{x}}{v} e^{\lambda(v - \underline{x})}$$

Solved Exponential Noise Example Grows Noisier

- Case 1: Precise signals: $\lambda > 1/\underline{x}$
  - corner solution $\theta(v) = v$, and zero rejection chance *in equilibrium* for all qualities.
- Case 2: Noisy signals: $\lambda < 1/\underline{x}$
  - A hump shape emerges

How Evaluation Noise Impacts Rejection Rates

- Old school: mean preserving spread. Not strong enough.
- *Dispersion* measures how "spread out" a distribution is
- G_2 is more dispersed than G_1
  $\Leftrightarrow G_2^{-1}(b) - G_2^{-1}(a) \geq G_1^{-1}(b) - G_1^{-1}(a)$ for any $b > a$
  $\Leftrightarrow g_2(G_2^{-1}(a)) < g_1(G_1^{-1}(a))$ for any $a \in (0, 1)$, with a density
- So the difference between any two quantiles (or percentiles) is higher under the more disperse distribution
- For many distributions, e.g. exponential and Gaussian, higher dispersion $\Longleftrightarrow$ higher variance

Proposition (Increasing Rejection Rates)

The rejection rate rises and peaks later if the evaluation noise G grows more disperse

Rejection Rate Rises in Evaluation Noise Dispersion

- The equilibrium FOC* is

$$\theta'(v) = \frac{1 - G(\theta(v) - v)}{vg(\theta(v) - v)} = \frac{1 - G(\tau(v))}{vg(\tau(v))}$$

- The rejection rate $R(v) = G(\tau(v))$ has slope

$$R'(v) = g(\tau(v))\tau'(v) = g(\tau(v))[\theta'(v) - 1]$$

$$\Rightarrow R'(v) = \frac{1 - R(v)}{v} - g(G^{-1}(R(v)) \qquad (\bigstar)$$

More Dispersion $\Rightarrow$ Higher Rejection Rates

- Assume density g_2 is more disperse than g_1
- Let rejection rates R_1, R_2 satisfy

$$R_i'(v) = \frac{1 - R_i(v)}{v} - g_i(G_i^{-1}(R(v))) \qquad (\bigstar)$$

- Claim: $R_1(v) = R_2(v) \Rightarrow R_2' > R_1'$
  - Apply $(\bigstar)$ and $g_2(G_2^{-1}(x)) < g_1(G_1^{-1}(x))\ \forall x$
  - $\Rightarrow$ $R_2(v)$ can only upcross through $R_1(v)$
  - $\Rightarrow$ Since $R^1(\underline{x}) = R^2(\underline{x}) = 0$, there is no no crossing

More Dispersion ⇒ Later Peak Rejection Rate

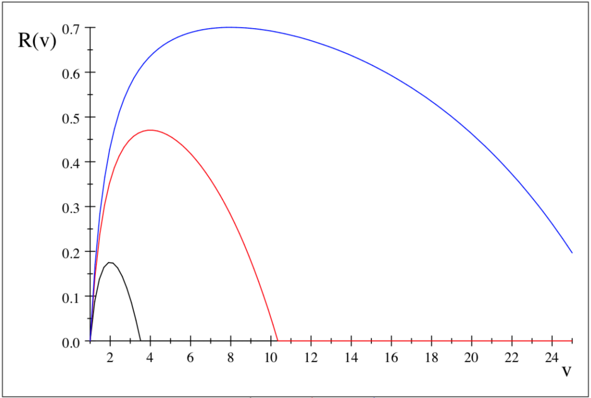
- Andrea show that $R_1' = R_2' \geq 0 \Rightarrow R_2'' > R_1''$

⇒ The peak of R_2 is right of the peak of R_1.

- But Andrea also claim she is not rejected:

Increasing Dispersion with Exponential Noise

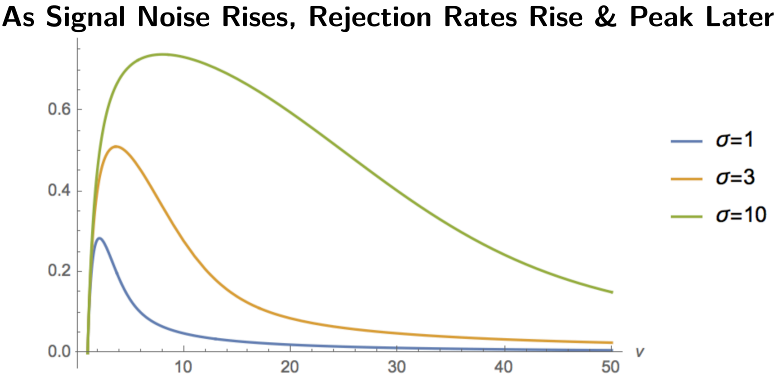
As Signal Noise Rises, Rejection Rates Rise & Peak Later



$\lambda = \frac{1}{2}$, $\lambda = \frac{1}{4}$, $\lambda = \frac{1}{8}$

The plots assume a worst paper $\underline{x} = 1$.

Gaussian Noise



What if Authors Do Not Know Paper Quality?

- Authors may be unsure of their paper's quality — just as a student may not know how good he is (e.g. Ramanujan)
- In this case, our one-shot model would not recur every period, but learning would occur.
- Our results should still inform what happens in the stage game, but it is a hard learning exercise.

General Model: Authors Do Not Know Their Paper Quality

- Journals see a noisy signal σ of the quality x of any submitted paper, where $\sigma - x$ has a density $g(\sigma - x)$.
- Each author sees a noisy signal ψ of his paper quality x, where $\psi - x$ has a density $h(\psi - x)$.

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- Each author sees a noisy signal ψ of his paper quality x, where $\psi - x$ has a density $h(\psi - x)$.
- Until now, the paper quality distribution was irrelevant for the conclusion, for neither authors nor journals needed Bayes rule
- Quality density f is log-concave on support $[\underline{x}, \infty)$ (say $\underline{x} = 1$)
- A journal equilibrium is an application strategy *and* acceptance threshold obeying author optimality and rational expectations
- Rational expectations trickier: *Each journal's caliber equals the expected average quality of papers it accepts*
- As before, authors don't mix and no one pools in equilibrium
  - Higher author types ψ are more ambitious: $V(\psi)$ is increasing
  - Better journals v set higher standards: $\theta(v)$ is increasing
  - both maps $V(\psi)$ and $\theta(v)$ are differentiable
  - Note: Since $V' > 0$, we instead find the inverse $\psi(v)$ of $V(\psi)$

General Journal Equilibrium

- $\theta(v)$ is the equilibrium threshold of journal v (with $\theta'(v) > 0$)
- Author type $\psi(v)$ submits to journal v (with $\psi'(v) > 0$)
- The *density of accepted paper qualities* x by journal v is:

$$\boxed{\alpha_v(x) \propto f(x)h(\psi(v) - x)(1 - G(\theta(v) - x))}$$

- The *rational expectations* (RE) condition is now more involved because journals publish a continuum of qualities:

$$\text{RE} \qquad v = \int_{\underline{x}}^{\infty} x\alpha_v(x)dx$$

- A *journal equilibrium* (ψ, θ) obeys (RE) and author optimality:

$$\text{FOC*} \qquad \frac{1}{v\theta'(v)} = \int_{\underline{x}}^{\infty} \frac{g(\theta(v) - x)}{1 - G(\theta(v) - x)}\alpha_v(x)dx$$

- This is the analogue of our earlier equilibrium FOC:

$$\theta'(v) = \frac{1}{v} \cdot \frac{1 - G(\theta(v) - v)}{g(\theta(v) - v)}$$

Equilibrium Rejection Rate

- The *density of submitted paper qualities* x at journal θ

$$\boxed{\zeta_v(x) \propto f(x)h(\psi(v) - x)}$$

- The *equilibrium rejection rate* is then

$$R(v) = \int_{\underline{x}}^{\infty} \zeta_v(x) G(\theta(v) - x) dx$$

- Higher-caliber journals
  - reject more often, with higher thresholds ($\theta \uparrow$)
  - get submissions from stochastically better papers ($\psi \uparrow$)
- The rejection rate is hump-shaped if first "direct effect" dominates at low qualities, and second "paper selection effect" at high qualities

Journal Equilibrium Equations, Reformulated

- *equilibrium toughness* $\tau(v) \equiv \theta(v) - v$ is again the excess of the journal threshold over its caliber
- *author's equilibrium sheepishness* $\xi(v) \equiv \psi(v) - v$ is the excess of the author's type over journal caliber he submits to
- Caliber-quality gap $z \equiv v - x$
- We can reformulate the accepted density in terms of sheepishness and toughness:

$$\begin{aligned} \alpha_v(v-z) &\propto f(x)h(\psi(v)-x)(1-G(\theta(v)-x)) \\ &\propto f(v-z)h(\xi(v)+z)(1-G(\tau(v)+z)) \end{aligned}$$

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- Equilibrium equations (recalling that paper quality $x \geq \underline{x} = 1$)

$$\begin{aligned} \text{RE} \quad & 0 = \int_{-\infty}^{v-1} \alpha_v(v-z) z dz \\ \text{FOC*} \quad & \frac{1}{v(\tau'(v)+1)} = \int_{-\infty}^{v-1} \alpha_v(v-z) \frac{g(\tau(v)+z)}{1-G(\tau(v)+z)} dz \end{aligned}$$

- RE requires a zero average accepted caliber-quality gap

Goals

1. Equilibrium toughness $\tau(v) \equiv \theta(v) - v$ is hump-shaped in journal caliber (as before)
2. Hump-shaped toughness $\Rightarrow$ hump-shaped rejection rate

Quasiconcave Toughness

- Say that a density μ is *decreasingly log-concave* if:

$$(\log \mu)'' \leq 0 \leq (\log \mu)'''$$

- Met by many typical log-concave distributions, eg Gaussian, exponential, uniform, Chi-squared, extreme value

Lemma

Assume densities f and h are decreasingly log-concave. Then equilibrium toughness is hump-shaped if author noise is not too dispersed, and otherwise toughness is increasing.

Comparative Statics Under Uncertainty Primer

- A function $\phi(x, y)$ is LSPM (log-supermodular) if $\log \phi$ is supermodular in (x, y) — ditto LSBM for log-submodular
- If ϕ and γ are LSPM / LSBM, then so too is the product $\phi\gamma$
- If a density γ is log-concave, then
  - Prekopa Theorem: Its cdf Γ and survivor $1 - \Gamma$ are log-concave
  - the kernel $\phi(v, x) = \gamma(v - x)$ is LSPM in (v, x)
  - the kernel $\phi(v, x) = \gamma(v + x)$ is LSBM in (v, x)
- Karlin and Rubin (1956): *The expectation $\int \phi(v, x)u(x)dx$ of an increasing function $u(x)$ with respect to a LSPM / LSBM kernel $\phi(v, x)$ is increasing / decreasing in v.*
- Proof:

$$\int [\phi(v_2, x) - \phi(v_1, x)]\, u(x)dx = \int \left(\frac{\phi(v_2, x)}{\phi(v_1, x)} - 1 \right) u(x)\phi(v_1, x)dx$$

- by Tchebyshev's inequality, this is positive if $u(x)$ and $\left(\frac{\phi(v_2,x)}{\phi(v_1,x)} - 1 \right)$ are comonotone, negative if reverse comonotone

Toughness Proof Sketch: A Failed Attempt

$$\text{FOC*} : \frac{1}{v(\tau'(v)+1)} \equiv \int_{-\infty}^{v-1} \alpha_v(v-z) \frac{g(\tau(v)+z)}{1-G(\tau(v)+z)} dz$$

- Equilibrium toughness is quasiconcave if any critical point is a maximum (sufficient condition)

$\Rightarrow$ it *suffices* that $\tau'(v)$ falls at a critical point $\tau'(v) = 0$

$\Rightarrow$ it *suffices* that $v(\tau'(v)+1)$ falls at a critical point $\tau'(v) = 0$

$\Rightarrow$ it *suffices* that the RHS of FOC* increases in v

- This is guaranteed if $\alpha_v(v-z)$ is LSPM in (v, z), since hazard rate increases in z by log-concavity of g, when $\tau'(v) = 0$
- But in that case, $\int \alpha_v(v-z) z dz \uparrow$ in v, violating RE (zero average accepted paper quality-caliber gap). Contradiction.

Getting Over the Hump for Hump-Shaped Toughness

- Recall the density of accepted paper qualities

$$\alpha_v(v-z) \propto f(v-z)h(\xi(v)+z)(1-G(\tau(v)+z))$$

  - First factor: f(v-z) is LSPM in (v, z)
  - Middle factor: $h(\xi(v)+z)$ is LSBM in $(\xi(v), z)$
  - Last factor: $(1-G(\tau(v)+z))$ is LSBM in $(\tau(v), v)$, by Prekopa
- So $\alpha_v(v-z)$ would be LSPM in (v, z) if
  - $\xi(v)$ is decreasing, and
  - $\tau'(v) = 0$ (namely, a critical point)
- But we just showed $\alpha_v(v-z)$ is not LSPM at a critical point

$\Rightarrow$ $\xi(v)$ must be increasing $\Rightarrow$ $h(\xi(v)+z)$ is LSBM

$\Rightarrow$ $\alpha_v(v-z)$ is a product of a LSPM and a LSBM function

Decreasingly Log-concave to the Rescue

- The density of accepted paper qualities

$$\alpha_v(v-z) \propto f(v-z)h(\xi(v)+z)(1-G(\tau(v)+z))$$

has cdf $A_v(x)$, i.e. $A'_v(x) = \alpha_v(x)$

Insight (★)

If f and h are decreasingly log-concave, then the cdf difference $A_{v_1}(v_1 - z) - A_{v_2}(v_2 - z)$ is upcrossing in z (though 0) for $v_2 > v_1$, and so is the slope $-\frac{d}{dv}A_v(v-z)$.

Comparative Statics Under Uncertainty Primer, Part II

Fact (The Folk Single Crossing Property for Integrals)

Let $a(x)$ be an upcrossing function with $\int a(x)dx = 0$. Then $\int a(x)b(x)dx \geq 0$ (or ≤ 0) if $b(x)$ is increasing (or decreasing).

Proof.

- Let $a(x)$ be upcrossing say at x_0, and $b(x)$ increasing

$$\begin{aligned}
\implies \int a(x)b(x)dx &= \int_{-\infty}^{x_0} \underbrace{a(x)}_{-} \underbrace{b(x)}_{\leq b(x_0)} dx + \int_{x_0}^{\infty} \underbrace{a(x)}_{\geq 0} \underbrace{b(x)}_{\geq b(x_0)} dx \\
&\geq b(x_0) \int a(x)dx = 0
\end{aligned}$$

□

Toughness is Quasiconcave: Easy Case

- If the *journal hazard rate* $r(x) := g(x)/(1 - G(x))$ is convex (eg Gaussian), then toughness is quasiconcave
- Integrate RE by parts and then differentiate in v:

$$0 = \frac{d}{dv} \int_{-\infty}^{v-1} \alpha_v(v-z) z dz = \int_{-\infty}^{v-1} -\frac{d}{dv} A_v(v-z) dz \qquad (1)$$

- Integrate FOC* by parts, & differentiate in v when $\tau'(v) = 0$:

$$\frac{d}{dv} \frac{1}{v(\tau'(v)+1)} = \int_{-\infty}^{v-1} -\frac{d}{dv} A_v(v-z) r'(\tau(v)+z) dz \qquad (2)$$

  - $-\frac{d}{dv} A_v(v-z)$ is upcrossing by Insight (★),
  - $-\frac{d}{dv} A_v(v-z)$ integrates to zero by (1)
  - The FOC derivative (2) is positive, by the folk SCP, since r' is increasing by convexity
- Hence, $\tau'(v) = 0 \Rightarrow v(\tau'(v)+1)$ is falling $\Rightarrow \tau''(v) < 0$

Toughness is Quasiconcave: Hard Case

- For most log-concave distributions g, the hazard rate r is convex-then-concave

Equilibrium Rejection Rates

- With known author types, hump-shaped toughness was necessary *and* sufficient for a hump-shaped rejection curve, via

$$R(v) \equiv G(\tau(v))$$

- For our unknown-types case, hump-shaped toughness is *necessary* (but not sufficient) for hump-shaped rejection rates:

$$1 - R(v) = \frac{\int f(v - z)h(\xi(v) + z)(1 - G(\tau(v) + z)dz}{\int f(v - z)h(\xi(v) + z)dz}$$

- We show that rising toughness $\Rightarrow$ rising rejection rates

Equilibrium Rejection Rates

- For our unknown-types case, hump-shaped toughness is a *necessary* (but not sufficient) condition for a hump-shaped equilibrium rejection rate:

$$\frac{1}{1 - R(v)} = \int_{-\infty}^{v-1} \alpha_v(v - z) \left(\frac{1}{1 - G(\tau(v) + z)} \right) dz$$

- Now, $\frac{1}{1-G(\tau(v)+z)}$ is convex in z, by log-concavity of G
- Mimicking earlier integration by parts analysis:
  - $(1 - R(v))^{-1}$ increases in v when $\tau'(v) = 0$
  - The derivative in v is $\int -\frac{d}{dv} A_v(v - z) \cdot \frac{d}{dz} \left(\frac{1}{1-G(\tau(v)+z} \right) dz$: the first term is upcrossing and integrates to zero, and the second increases by convexity
  - clearly, it is also increasing in $\tau(v)$
- **Increasing toughness $\Rightarrow$ increasing rejection rate**

Hump-Shaped Rejection Rates

Mavi's Ruff Result

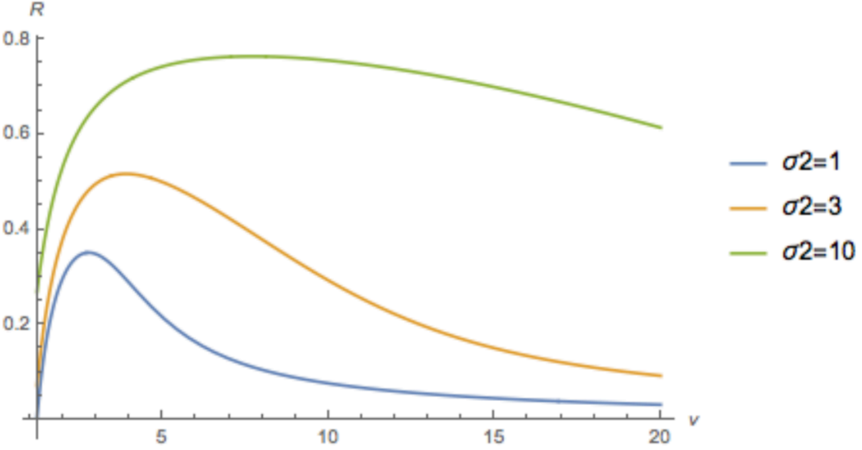
If the author signal is sufficiently less noisy than the journal signal, then the rejection rate $R(v)$ is hump-shaped; otherwise, it is everywhere increasing.

Mavi's Second Ruff Result

The rejection rate rises — and its peak shifts out — as the journal or author signal noise increases.

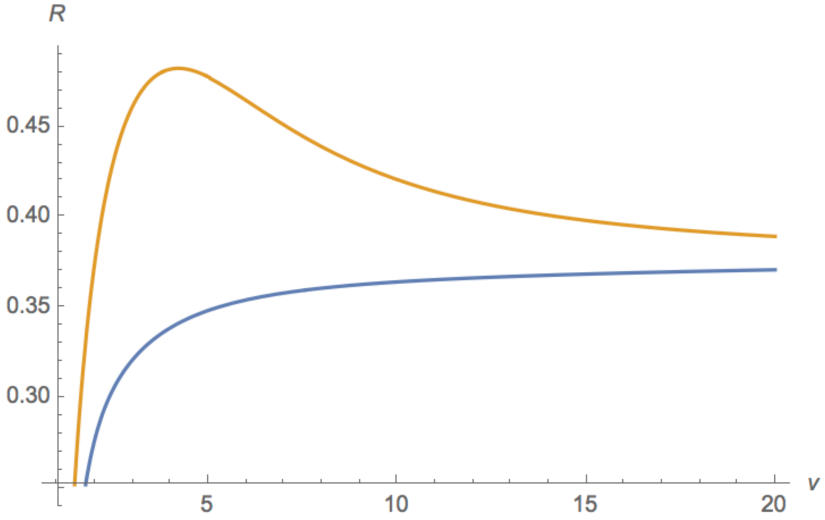
Gaussian Noise

As Signal Noise Rises, Rejection Rates Rise & Peak Later



Assume an improper uniform prior *f*, standard normal author signal distribution, and journal signal as above.

Humps Emerge with More Precise Author Information



- paper prior $f = \Gamma[2, 1]$, author signal $h = \Gamma[2, 1]$
- Blue journal signal $g = \Gamma[2, 1]$, orange $g = \Gamma[2, 2]$

Mavi's Sheep

Journal Rejection Rates

Hamermesh (2008), "How to Publish in a Top Journal"

- QJE 4%, JPE 5%, AER 7%, APSR 8%, JoLE 8%
- Econometrica 9%, EER 9%
- Journal of Human Resources 10%, Economica 11%
- RAND 11%, REStat 12%, Economics Letters 17%
- Canadian Journal of Economics 18%
- Industrial and Labor Relations Review 18%
- Journal of Monetary Economics 20%

Stanford University	CA	**5%**
Harvard University	MA	**5**
Columbia University	NY	**6**
Yale University	CT	**6**
Princeton University	NJ	**7**
California Institute of Technology	CA	**8**
Massachusetts Institute of Technology	MA	**8**
University of Chicago	IL	**8**
Brown University	RI	**9**
University of Pennsylvania	PA	**9**