Commodity Financialization and Information Transmission

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Abstract

We study how commodity financialization affects information transmission in a commodity futures market. The trading of financial traders injects both information and noise into the futures price. In consequence, price informativeness in the futures market first increases and then decreases with commodity financialization. When the priceinformativeness effect is negative, commodity financialization can aggravate the futures price bias. Financialization generally improves market liquidity in the futures market and strengthens the commodity-equity market comovement. Operating profits and producer welfare can move in opposite directions in response to commodity financialization. Our analysis provides important guidance for interpreting related empirical and policy studies.

Keywords: Commodity financialization, supply channel, price informativeness, feedback effect

JEL Classifications: D82, G14

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1 Introduction

Recent years have seen major changes in the nature of commodity futures markets. While traditionally these markets served mostly commodity producers and users looking to hedge their exposures and trade on their information, a trend of financialization started around 2004, whereby financial investors—such as commodity index traders, commodity trading advisers, and hedge funds—entered these markets and became dominant players in them (see, e.g., Cheng and Xiong, 2014; Basak and Pavlova, 2016; Bhardwaj, Gorton, and Rouwenhorst, 2016).

This trend has led to a surge of interest among researchers, practitioners, and regulators expressing concerns over the implications of financialization for commodity prices and real outcomes. The so-called "Masters Hypothesis" provided by hedge fund manager Michael W. Masters in his testimonies before the U.S. Congress and U.S. Commodity Futures Trading Commission (CFTC) claims that the large inflow of financial capital into commodity futures markets is responsible for the 2007-2008 spike in commodity futures prices (see Irwin, 2012; Irwin and Sanders, 2012).¹ An overview in the 2011 Report of the G20 Study Group on Commodities (p. 29) notes that "(t)he discussion centers around two related questions. First, does increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility? And second, does financial investment in commodity futures affect spot prices?". A burgeoning empirical literature tracks the effect of financialization on risk premia, market efficiency, correlations between commodity markets and equity markets, operating profits of commodity producers

¹This kind of complaints prompted CFTC to add Commodity Index Trader (CIT) position supplement to the traditional weekly Commitments of Traders (COT) reports, starting in 2007.

and users, and other variables. Interestingly, the various papers in this literature, many of them we mention below, often come up with conflicting messages on the implications of financialization.²

Given the stage of development of the empirical literature and the debates within it, there is need for theoretical frameworks providing a unified approach to understand the various mechanisms and help guide and interpret the empirical work. In this paper, we attempt to provide such a unified framework.³ Our model features trading that is based on information and hedging by both the traditional commodity producers (or users) and the newly arriving financial traders. It is built in the tradition of the classic papers by Danthine (1978) and Grossman and Stiglitz (1980), but tailored to address the question of how an increase in financialization of commodity futures markets affects the various parameters of commodity markets and real outcomes.

We focus on the combination of information-based trading and hedging-based trading by financial traders because the common accounts of the developments in futures markets depict financial traders as having these two motives (see, e.g., Cheng and Xiong, 2014). First, hedge funds and other financial traders have been investing a lot to acquire information on the fundamental developments of commodity demand and supply to guide their speculative trading in these markets. This is a main attraction for them in entering these markets, as they provide new opportunities for speculative gains. Second, for some financial traders, the main attraction of coming into these markets has been the ability to diversify and hedge exposures they have in other investments. Unlike commodity producers and users, they are not directly

 $^{^{2}}$ See Irwin and Sanders (2011), Fattouh, Kilian, and Mahadeva (2013) and Cheng and Xiong (2014) for excellent surveys on the empirical findings on commodity financialization.

³We review existing theoretical work and explain our distinct angle below.

involved with commodity spot markets, but rather attempt to gain higher efficiency on their portfolios by adding commodity futures to their other investments. Hence, in our model, we are exploring what increased financializatio—characterized by a bigger group of financial traders who participate in futures markets for hedging and speculation purposes—implies for the various parameters of commodity markets and their real effects.

Our setting features one commodity good and two periods (t = 0 and 1). The spot market of the commodity opens at date 1 and the spot price is determined based on the commodity supply and demand. The commodity demand is random, reflecting preference shocks to date-1 commodity consumers. The commodity supply is determined endogenously based on commodity producers' decisions, which are made at date 0 conditional on the equilibrium futures price. At date 0, the commodity futures market opens and the futures price is determined to clear the market. In our baseline model presented in Section 2, all commodity producers can trade futures contracts alongside financial traders and noise traders. Both commodity producers and financial traders have private information about the later commodity demand and thus they speculate on their information when trading futures. In addition, both types of traders trade futures for hedging purposes: commodity producers hedge the risk they are exposed to in their production, while financial traders hedge their positions in other assets such as stocks.

This setting offers a very direct "feedback effect" from financial markets to the real economy. There is a supply channel through which the current futures market affects the later spot market: a higher futures price induces commodity producers to supply more of the commodity, which in turn presses down the later spot price through the market-clearing mechanism in the spot market. Going back to the G20's second question quoted above, there is a very simple channel in our model through which developments in the futures market will affect spot prices and the real economy. Hence, it is indeed of high importance to understand the impact of financialization. Importantly, this feedback effect differs from that in the vast literature reviewed by Bond, Edmans, and Goldstein (2012), where market prices have a real effect through the information they provide. Here, the effect is more direct, as the futures price becomes the effective price considered by producers to determine their production decisions. As we discuss, our mechanism is related to but distinct from that in Leland (1992), where an increase in the stock price causes the firm to issue more equity shares and make more real investments, but the asset payoff is exogenous.

We then use our analysis to examine the implications of commodity financialization. An increase in commodity financialization is captured in our model as an increase in the population size of financial traders active in the futures market. Because of their dual trading motive, financial traders in our model bring both information and noise into the futures price. The former happens due to their speculation-based trade and the latter through their hedging-based trade.⁴ As a result, adding more financial traders can either improve or harm price informativeness. We show that the first effect dominates only when the size of the financial traders population is relatively small. Hence, a process of increased financialization first increases and then decreases price informativeness in our model. We discuss how this result can help reconcile mixed empirical findings that commodity financialization improved market efficiency in the U.S. crude oil futures market (Raman, Robe, and Yadav, 2017) but harmed market efficiency in broader commodity index markets (Brogaard, Ringgenberg, and

⁴Consistent with our model, Ready and Ready (2018) indeed find that commodity index investors are moving the market through their hedging trades.

Sovich, 2018).

We then explore the implications for the futures price bias, which, in our model, is captured by the deviation of the futures price from the expected later spot price. Commodity financialization affects the magnitude of the bias through two channels. First, adding more financial traders facilitates risk sharing, which tends to reduce the bias. Second, as mentioned above, commodity financialization also affects price informativeness, which affects the magnitude of the bias. When commodity financialization harms price informativeness, the negative informational effect can be strong enough such that the futures price bias increases with the mass of financial traders. This affirms the concern quoted from the G20 above that increased financialization can move the futures price away from fundamentals. Our model pinpoints the circumstances under which this will happen. We also show that commodity financialization generally contributes to greater market liquidity and an increase in the comovement between the commodity futures market and the equity market. The latter result is consistent with the empirical finding of Büyükşahin and Robe (2013, 2014) that the increased correlation between stocks and commodities is driven by the trading of hedge funds active in both futures and equity markets.

Considering real effects, we explore the implications of financialization for the profits and welfare of commodity producers. To gain full understanding of the effects, we extend the model to have two groups of commodity producers: one that trades futures contracts ("participating producers") and the other that does not ("nonparticipating producers"). This is consistent with real-world practices, whereby different producers participate in futures markets to different degrees. It is also a critical extension to understand the nature of the feedback effect and the implications for real outcomes. Specifically, as mentioned above, for participating producers the futures price is the effective selling price of their products, and so an increase in futures price directly induces them to increase production. In contrast, nonparticipating producers are only affected by the futures price for their production decisions to the extent that it provides information on fundamental commodity demand. This is the informational feedback as in the literature reviewed by Bond, Edmans, and Goldstein (2012).

Due to this distinction we show that commodity financialization has opposite welfare consequences on the two types of producers. Both of them see greater operating profits when the informativeness of the futures price improves. But, while the welfare of nonparticipating producers improves due to the more precise information to guide their decisions, the welfare of participating producers decreases in informativeness, as a result of the decrease in trading and risk sharing opportunities. These results are important for interpreting empirical evidence and for policy. Brogaard, Ringgenberg, and Sovich (2018) show that the decrease in informativeness that followed financialization led to a decrease in operating profits of commodity producers. While consistent with our model, we show that the welfare implications for the producers are only negative if they do not participate in the futures markets. Finally, in our analysis, increasing the population size of financial traders always lowers the welfare of existing financial traders. This result squares with Chen, Dai, and Sorescu's (2017) recent finding that commodity trading advisors are harmed by the ongoing financialization of commodity markets.

Related Literature Our paper is broadly related to two strands of literature. The first is the literature on commodity financialization, which is largely empirical and documents the trading behavior of financial traders in futures markets and their pricing impact. The theoretical research on the subject remains scarce. Basak and Pavlova (2016) construct dynamic equilibrium models to study how commodity financialization affects commodity futures prices, volatilities, and in particular, correlations among commodities and between equity and commodities. Fattouh and Mahadeva (2014) and Baker (2016) calibrate macrofinance models of commodities to quantify the effect of commodity financialization. Gorton, Hayashi, and Rouwenhorst (2012) and Ekeland, Lautier, and Villeneuve (2017) consider a combination of hedging pressure theory and storage theory to study commodity financialization. Knittel and Pindyck (2016) study a reduced-form setting of commodity financialization using a simple model of supply and demand in the cash and storage markets. Tang and Zhu (2016) model commodities as collateral for financing in a two-period economy with multiple countries and capital controls. Chari and Christiano (2017) develop a model to show that financial traders and traditional commodity traders insure each other. While these existing models offer important insights, they all feature symmetric information, and hence do not address the key channels of our model involving price informativeness and learning.

Three existing theoretical studies also analyze the effects of informational frictions in the context of commodity financialization. Sockin and Xiong (2015) focus on information asymmetry in the spot market. They show that a high spot price may further spur the commodity demand through an informational channel and that in the presence of complementarity, this informational effect can be so strong that commodity demand can increase with the price. Goldstein, Li, and Yang (2014) argue that financial traders and commodity producers may respond to the same fundamental information in opposite directions, such that commodity financialization may have a negative informational effect. Leclercq and Praz (2014) consider

how the entry of new speculators affects the average and volatility of spot prices. We view our paper as complementary to these papers since it highlights different channels through which financialization affects prices and real outcomes. In particular, the feedback effect from futures markets to the real economy in our model happens through the production decisions of commodity producers. Moreover, financial traders bring both information and noise to the futures market. These channels are empirically motivated and they generate very different implications, as our analysis demonstrates.

The second strand of related literature is the classic literature on futures markets (see Section 1.1 of Acharya, Lochstoer, and Ramadorai (2013) for a brief review of this literature). This literature has developed theories of "hedging pressure" (Keynes, 1930; Hicks, 1939; Hirshleifer, 1988, 1990) or "storage" (Kaldor, 1939; Working, 1949) to explain futures prices. Notably, the literature has also developed asymmetric information models on futures markets (e.g., Grossman, 1977; Danthine, 1978; Bray, 1981; Stein, 1987). However, because commodity financialization is just a recent phenomenon, these early models have focused on different research questions. The analysis in our model centers on the implications of increasing the population of financial traders, who inject both information and noise into the price, for various parameters in commodity markets and real outcomes. This question is very relevant in today's markets and has not been addressed by the older literature.

2 A Model of Commodity Financialization with Asymmetric Information

The model lasts two periods: t = 0 and 1. The timeline of the economy is described by Figure 1. At date 0, the financial market opens, where a mass μ of financial traders—such as hedge funds or commodity index traders—trade futures contracts against commodity producers and noise traders. We use parameter μ to capture financialization of commodities—i.e., the process of commodity financialization corresponds to an increase in μ . We normalize the mass of commodity producers as 1. Commodity producers make their decisions on commodity production at date 0, which in turn determine the commodity supply at the spot market that operates later at date 1. We describe the spot and futures markets in the following two subsections.

2.1 The Spot Market

There is one commodity good in our setting, such as oil or copper. The spot market opens at date 1. The supply of commodity will be determined by the production decisions of commodity producers, which we will discuss shortly in the next subsection. Following Hirshleifer (1988) and Goldstein, Li, and Yang (2014), we assume that the demand for the commodity is implicitly derived from the preference of some (unmodeled) consumers and it is represented by the following linear demand function:

$$y = \tilde{\theta} + \tilde{\delta} - \tilde{v}. \tag{1}$$

Here, \tilde{v} is the commodity spot price, which will be endogenously determined in equilibrium. Variables $\tilde{\theta}$ and $\tilde{\delta}$ represent exogenous shocks to consumers' commodity demand. Demand shocks $\tilde{\theta}$ and $\tilde{\delta}$ are normally distributed and mutually independent; that is, $\tilde{\theta} \sim N(\bar{\theta}, \tau_{\theta}^{-1})$ and $\tilde{\delta} \sim N(0, \tau_{\delta}^{-1})$, where $\bar{\theta} \in \mathbb{R}$, $\tau_{\theta} > 0$, and $\tau_{\delta} > 0.5$ We have normalized the mean of $\tilde{\delta}$ to 0 since its mean can be absorbed by the mean of $\tilde{\theta}$. We assume that financial traders and commodity producers can learn information about $\tilde{\theta}$ but not about $\tilde{\delta}$. The learnable component $\tilde{\theta}$ represents factors on which there are many sources of information available that traders can purchase and analyze. In contrast, the unlearnable component $\tilde{\delta}$ represents factors that are hard to predict given available data sources.

2.2 The Futures Market

At date 0, the financial market opens. There are two tradable assets: a futures contract on the commodity and a risk-free asset. We normalize the net risk-free rate as zero. The payoff on the futures contract is the date-1 spot price \tilde{v} of the commodity. Each unit of futures contract is traded at an endogenous price \tilde{p} . Commodity producers, financial traders, and noise traders participate in the financial market. Noise traders represent random transient demands in the futures market and they as a group demand $\tilde{\xi}$ units of the commodity futures, where $\tilde{\xi} \sim N(\bar{\xi}, \tau_{\xi}^{-1})$ with $\bar{\xi} \in \mathbb{R}$ and $\tau_{\xi} > 0$. We next describe in detail the behavior and information structure of commodity producers and financial traders.

2.2.1 Commodity Producers

There is a continuum [0, 1] of commodity producers, indexed by *i*. Commodity producers are risk averse so that they have hedging motives in the futures market. Specifically, commodity

⁵Throughout the paper, we use a tilde (~) to signify a random variable, where a bar denotes its mean and τ denotes its precision (the inverse of variance). That is, for a random variable \tilde{z} , we have $\bar{z} \equiv E(\tilde{z})$ and $\tau_z = \frac{1}{Var(\tilde{z})}$.

producer *i* derives expected utility from her final wealth W_i at the end of date 1; she has a constant-absolute-risk-aversion (CARA) utility over wealth: $-e^{-\kappa W_i}$, where $\kappa > 0$ is the risk-aversion parameter. Commodity producers make two decisions at date 0. First, they decide on the quantity of commodities to produce, which will in turn determine the commodity supply at the date-1 spot market. Second, they decide on the investment in futures contracts in the date-0 futures market. This investment serves to hedge their commodity production and to speculate on their private information. For now, we assume that all commodity producers can trade futures contracts. In Section 5, we will relax this assumption so that only a fraction of commodity producers will be able to trade futures contracts.

Commodity producers are endowed with private information about the fundamental θ in the demand function. Specifically, commodity producer *i* receives a private signal \tilde{s}_i which takes the following form:

$$\tilde{s}_i = \tilde{\theta} + \tilde{\varepsilon}_i. \tag{2}$$

Here, $\tilde{\varepsilon}_i \sim N(0, \tau_{\varepsilon}^{-1})$ (with $\tau_{\varepsilon} > 0$) and $(\{\tilde{\varepsilon}_i\}_i, \tilde{\theta}, \tilde{\delta})$ are mutually independent. The futures price \tilde{p} is observable to all market participants and thus, commodity producer *i*'s information set is $\{\tilde{s}_i, \tilde{p}\}$. When commodity producer *i* decides to produce x_i units of commodities, she pays a production cost:⁶

$$C(x_i) = cx_i + \frac{1}{2}x_i^2,$$
(3)

where c is a constant.

⁶The cost function $C(x_i)$ can be alternatively interpreted as an inventory cost. For instance, suppose that the date-0 commodity spot price is v_0 and carrying an inventory of x_i units of commodities incurs a cost of $cx_i + \frac{1}{2}x_i^2$. Then the total cost of storing x_i units of commodities is $C(x_i) = (c + v_0)x_i + \frac{1}{2}x_i^2$, which is essentially equation (3) with a renormalization of parameter c. However, this interpretation is made in a partial-equilibrium setting as the date-0 spot price v_0 is exogenous. We can fully endogenize this spot price at the expense of introducing one extra source of uncertainty, because otherwise the prices of futures and current spot price combine to fully reveal the shocks (see Grossman, 1977).

Commodity producer *i*'s problem is then to choose commodity production x_i and futures investment d_i (and investment in the risk-free asset) to maximize

$$E\left(-e^{-\kappa\tilde{W}_{i}}\middle|\tilde{s}_{i},\tilde{p}\right) \tag{4}$$

subject to

$$\tilde{W}_{i} = \tilde{v}x_{i} - C\left(x_{i}\right) + \left(\tilde{v} - \tilde{p}\right)d_{i}.$$
(5)

Here, $\tilde{v}x - C(x_i)$ is the profit from producing and selling x_i units of commodities: selling x_i units of commodities at a later spot price \tilde{v} generates a revenue of $\tilde{v}x_i$, which, net of the production cost $C(x_i)$, gives rise to the operating profit $\tilde{v}x - C(x_i)$. The term $(\tilde{v} - \tilde{p}) d_i$ is the profit from trading d_i units of futures contracts. Specifically, at date 0, buying a futures contract is equivalent to buying an asset that costs \tilde{p} and generates a payoff equal to the date-1 commodity spot price \tilde{v} . Thus, $(\tilde{v} - \tilde{p}) d_i$ is the profit from trading d_i units of futures contracts. In equation (5), we have normalized commodity producer i's initial endowment as 0, which is without loss of generality given the CARA preference.

To better connect our setup to previous models, we have followed the literature (e.g., Danthine, 1978) and interpreted commodity producers as commodity suppliers. In fact, a more precise interpretation of commodity producers should be "commercial hedgers". That is, the commodity producers in our model can be either commodity suppliers or commodity demanders. When $x_i > 0$ they are effectively suppliers, and when $x_i < 0$ they are effectively demanders, who are using or consuming the commodity, such as airlines using oil. In the latter case, the term $\tilde{v}x - C(x_i)$ in (5) should be interpreted as the utility from using $|x_i|$ units of commodities. The math and key results are the same in both cases, and our formulation allows for both in a tractable way.

2.2.2 Financial Traders

There is a mass $\mu \ge 0$ of financial traders who derive utility from their final wealth at the end of date 1. For simplicity, we assume that all financial traders are identical in preferences, investment opportunities, and information sets. They have a CARA utility with a riskaversion coefficient of $\gamma > 0$. We can show that in our setting, μ and γ affect the equilibrium only through the ratio $\frac{\mu}{\gamma}$ and thus, the comparative statics analysis in μ is equivalent to a comparative statics analysis in $\frac{1}{\gamma}$.

Like commodity producers, financial traders trade futures both for speculation and for hedging motives. However, their hedging needs are different. They are not exposed to the real production of commodities, but they hedge positions they have in other assets whose payoffs are correlated with the commodity market (and hence the payoffs on commodity futures). We follow Wang (1994), Easley, O'Hara, and Yang (2014), and Han, Tang, and Yang (2016) in modelling this hedging behavior of financial traders. Formally, we assume that at date 0, in addition to the risk-free asset and the futures contract, financial traders can invest in another asset or private technology. This can represent a stock index in which financial traders typically invest. Another real-world example is commodity-linked notes (CLNs) that are traded over the counter and have payoffs linked to the price of commodity or commodity futures. As documented by Henderson, Pearson, and Wang (2015), the regular issuers of CLNs are big investment banks, who often invest in commodity futures to hedge their issuance of CLNs. More broadly, introducing this additional asset is a modeling device that is meant to capture the important feature that financial traders trade futures partly for their own portfolio diversification and risk management goals, as emphasized by Cheng,

Kirilenko, and Xiong (2015).

The net return on the private technology is $\tilde{\alpha} + \tilde{\eta}$, where $\tilde{\alpha} \sim N(0, \tau_{\alpha}^{-1})$ and $\tilde{\eta} \sim N(0, \tau_{\eta}^{-1})$ with $\tau_{\alpha} > 0$ and $\tau_{\eta} > 0$. Similar to commodity demand shocks, the net return on the private technology also has two components. Variable $\tilde{\alpha}$ represents the forecastable component. It is independent of all other random variables and is privately observable to financial traders. Variable $\tilde{\eta}$ is the unforecastable component. Importantly, it is correlated with the unforecastable commodity demand shock $\tilde{\delta}$. We denote the correlation coefficient between $\tilde{\eta}$ and $\tilde{\delta}$ as $\rho \in (-1, 1)$. This correlation is the modeling ingredient that generates the hedging motive of financial traders in the futures market.

We assume that financial traders observe $\tilde{\theta}$ perfectly. The idea that they are more informed than regular commodity producers is realistic to the extent that financial traders, such as hedge funds, generally have more sophisticated information-processing capacities. With additional modeling complexity, we can allow that they observe less than perfect information.⁷ Of course, financial traders also observe the futures price \tilde{p} and thus, financial traders' information set is $\{\tilde{\theta}, \tilde{\alpha}, \tilde{p}\}$. Their problem is to choose investment d_F in futures and investment z_F in the private technology (and investment in the risk-free asset) to maximize

$$E\left[-e^{-\gamma[(\tilde{v}-\tilde{p})d_F+(\tilde{\alpha}+\tilde{\eta})z_F]}\middle|\,\tilde{\theta},\tilde{\alpha},\tilde{p}\right].$$
(6)

Here, $(\tilde{v} - \tilde{p}) d_F$ captures the profit from trading futures and $(\tilde{\alpha} + \tilde{\eta}) z_F$ captures the profit

⁷Our results are robust to a general assumption that financial traders observe a noisy version of $\tilde{\theta}$, for instance, $\tilde{s}_F = \tilde{\theta} + \tilde{\varepsilon}_F$. This alternative assumption will introduce noise $\tilde{\varepsilon}_F$ into the futures price $\tilde{\rho}$, which will complicate our analysis. Note that Stein (1987) relies on such an assumption to generate a negative informational externality. However, this alternative assumption will not be enough for negative informational consequences of commodity financialization in our setting (that is, in the absence of the noise $\tilde{\alpha}$ generated from the hedging motive of financial traders). This is because both the private information of commodity producers and that of financial traders are about the same fundamental $\tilde{\theta}$ in our framework. In contrast, in Stein's (1987) setting, financial traders and other traders have information about different variables, and financial traders' trading brings noise to the price, which impairs other traders' ability to make inferences based on current prices and their own information.

from investing in the private technology. Again, without loss of generality, we have normalized the initial endowment of financial traders to be zero.

3 Equilibrium

In our setting, $(\tilde{\theta}, \tilde{\delta}, \tilde{\xi}, \{\tilde{\varepsilon}_i\}_i, \tilde{\alpha}, \tilde{\eta})$ are the underlying random variables that characterize the economy. They are mutually independent, except that $\tilde{\delta}$ and $\tilde{\eta}$ are correlated with each other with correlation coefficient $\rho \in (-1, 1)$. The tuple $\mathcal{E} \equiv (\mu, \kappa, \gamma, c, \bar{\theta}, \bar{\xi}, \rho, \tau_{\theta}, \tau_{\delta}, \tau_{\varepsilon}, \tau_{\xi}, \tau_{\alpha}, \tau_{\eta})$ defines an economy. Given an economy, an equilibrium consists of two subequilibria: the date-1 spot-market equilibrium and the date-0 futures-market equilibrium. At date 1, the commodity demand clears the commodity supply provided by commodity producers at the prevailing spot price \tilde{v} . Because the commodity demand depends on demand shocks $(\tilde{\theta}, \tilde{\delta})$ and the commodity supply depends on producers' private information $\{\tilde{s}_i\}$ and the futures price \tilde{p} , we expect that the spot price \tilde{v} will be a function of $(\tilde{\theta}, \tilde{\delta}, \tilde{p})$. At date 0, we consider a noisy rational expectations equilibrium (NREE) in the futures market. Given that commodity producers have private information $\{\tilde{s}_i\}$, financial traders have private information $\{\tilde{\theta}, \tilde{\alpha}\}$, and noise trading is $\tilde{\xi}$, we expect that the futures price \tilde{p} will depend on $(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi})$. A formal definition of an equilibrium is given as follows:

Definition 1 An equilibrium consists of a spot price function, $v(\tilde{\theta}, \tilde{\delta}, \tilde{p}) : \mathbb{R}^3 \to \mathbb{R}$; a futures price function, $p(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi}) : \mathbb{R}^3 \to \mathbb{R}$; a commodity production policy, $x(\tilde{s}_i, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$; a trading strategy of commodity producers, $d(\tilde{s}_i, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$; a trading strategy of financial traders, $d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) : \mathbb{R}^3 \to \mathbb{R}$; and a strategy of financial traders' investment on the private technology, $z_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) : \mathbb{R}^3 \to \mathbb{R}$, such that: (a) At date 1, the spot market clears, i.e.,

$$\tilde{\theta} + \tilde{\delta} - v(\tilde{\theta}, \tilde{\delta}, \tilde{p}) = \int_0^1 x(\tilde{s}_i, \tilde{p}) \, di, \text{ almost surely;}$$
(7)

(b) At date 0, given that \tilde{v} is defined by $v(\tilde{\theta}, \tilde{\delta}, \tilde{p})$,

- (i) $x(\tilde{s}_i, \tilde{p})$ and $d(\tilde{s}_i, \tilde{p})$ solve for commodity producers' problem given by (4) and (5);
- (ii) $d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p})$ and $z_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p})$ solve financial traders' problem (6); and
- (iii) the futures market clears, i.e.,

$$\int_0^1 d\left(\tilde{s}_i, \tilde{p}\right) di + \mu d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) + \tilde{\xi} = 0, \text{ almost surely.}$$
(8)

We next construct an equilibrium in which the price functions $v(\tilde{\theta}, \tilde{\delta}, \tilde{p})$ and $p(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi})$ are linear. As standard in the literature, we solve the equilibrium backward from date 1.

3.1 Spot Market Equilibrium

The commodity demand is given by equation (1). The commodity supply is determined by commodity producers' date-0 investment decisions. The commodity producers' problem, given by (4) and (5), can be decomposed as follows:

$$\max_{x_i+d_i} \left[E\left(\tilde{v}-\tilde{p}|\tilde{s}_i,\tilde{p}\right)\left(x_i+d_i\right) - \frac{\kappa Var\left(\tilde{v}|\tilde{s}_i,\tilde{p}\right)}{2}\left(x_i+d_i\right)^2 \right] + \max_{x_i} \left[\tilde{p}x_i - C\left(x_i\right)\right].$$
(9)

Solving (9), we have:

$$x_i + d_i = \frac{E\left(\tilde{v} \mid \tilde{s}_i, \tilde{p}\right) - \tilde{p}}{\kappa Var\left(\tilde{v} \mid \tilde{s}_i, \tilde{p}\right)},\tag{10}$$

$$x_i = \tilde{p} - c. \tag{11}$$

The above expressions are similar to those in Danthine (1978). The intuition is as follows: since both real investment x_i and financial investment d_i expose a commodity producer to the same risk source \tilde{v} , her overall exposure to this risk is given by the standard demand function of a CARA investor, as expressed in (10). In it, the producer chooses a positive (negative) position when the expected spot price is above (below) the futures price, and the size of the position decreases in the risk it entails. Expression (11) says that after controlling the total exposure given by (10), financial producers essentially treat the futures price \tilde{p} as the commodity selling price when making real production decisions.

The second maximization problem in (9) and its solution in (11) demonstrate the feedback effect of the futures market on commodity producers' production activities. This effect says that an increase in the futures price \tilde{p} directly encourages commodity producers to supply more commodities. It is related to but distinct from the real effect of financial markets in Leland (1992). In Leland's setting, a firm who issues shares to maximize profits faces a similar problem as the second maximization problem in (9). As a result, an increase in the stock price causes the firm to issue more equity shares (and implicitly make more real investments). However, in Leland's setting, the asset payoff is exogenous; in contrast, in our setting, the payoff \tilde{v} on the futures contract is endogenously affected by the feedback effect, which is formalized below by Lemma 1.

Formally, aggregating (11) across all commodity producers delivers the aggregate commodity supply at the spot market:

$$\int_0^1 x_i di = \tilde{p} - c. \tag{12}$$

By the market-clearing condition (7) and equations (1) and (12), we can solve for the spot price \tilde{v} , which is given by the following lemma:

Lemma 1 (Spot prices) The date-1 spot price \tilde{v} is given by

$$\tilde{v} = \tilde{\theta} + \tilde{\delta} + c - \tilde{p}. \tag{13}$$

This lemma formally establishes a supply channel through which the date-0 futures price

 \tilde{p} affects the date-1 spot price \tilde{v} . The 2011 G20 Report on Commodities raised the following key question: "(D)oes financial investment in commodity futures affect spot prices?" In our setting, such an effect indeed exists because financial traders' investments in commodity futures will alter the futures price \tilde{p} , which in turn changes the later spot price \tilde{v} through equation (13). In other words, the futures market is not just a side show, and it has consequences for production and spot prices on the real side. Chen and Linn (2017) find that changes in oil and natural gas field investment measured by drilling rig use respond positively to changes in the futures prices of oil and natural gas. This finding is consistent with the supply channel in (11), which is behind the feedback effect in (13).

3.2 Futures Market Equilibrium

We conjecture the following linear futures price function:

$$\tilde{p} = p_0 + p_\theta \tilde{\theta} + p_\alpha \tilde{\alpha} + p_\xi \tilde{\xi}, \qquad (14)$$

where p_0, p_θ, p_α , and p_ξ are endogenous coefficients. We next compute the demand function of futures market participants and use the market-clearing condition to construct such a linear NREE price function.

By (10) and (11), commodity producer *i*'s demand for the futures contract is

$$d\left(\tilde{s}_{i},\tilde{p}\right) = \underbrace{\frac{E\left(\tilde{v}|\tilde{s}_{i},\tilde{p}\right) - \tilde{p}}{\kappa Var\left(\tilde{v}|\tilde{s}_{i},\tilde{p}\right)}}_{\text{speculation}} - \underbrace{\left(\tilde{p} - c\right)}_{\text{hedging}}.$$
(15)

As mentioned before, a commodity producer trades futures for two reasons. First, she hedges her real commodity production of $x_i = \tilde{p} - c$. Second, because she also has private information \tilde{s}_i on the later commodity demand and so the later spot price \tilde{v} , she speculates on this private information. The expressions in (15) show how the total demand of the producer in the futures market can be decomposed into these two motives.

By (14), the information contained in the futures price is equivalent to the signal \tilde{s}_p in predicting demand shock $\tilde{\theta}$:

$$\tilde{s}_p \equiv \frac{\tilde{p} - p_0 - p_{\xi}\xi}{p_{\theta}} = \tilde{\theta} + \pi_{\alpha}\tilde{\alpha} + \pi_{\xi}(\tilde{\xi} - \bar{\xi}), \text{ with } \pi_{\alpha} \equiv \frac{p_{\alpha}}{p_{\theta}} \text{ and } \pi_{\xi} \equiv \frac{p_{\xi}}{p_{\theta}},$$
(16)

which is normally distributed with mean $\tilde{\theta}$ and precision τ_p , where

$$\tau_p = \left(\frac{\pi_\alpha^2}{\tau_\alpha} + \frac{\pi_\xi^2}{\tau_\xi}\right)^{-1}.$$
(17)

Precision τ_p measures how informative the futures price \tilde{p} is about the later commodity demand "fundamental" $\tilde{\theta}$, and so we refer to τ_p as "price informativeness."

Using the expression of \tilde{v} in (13) and applying Bayes' rule to compute the conditional moments in commodity producer *i*'s demand function (15), we can obtain

$$d\left(\tilde{s}_{i},\tilde{p}\right) = \frac{\frac{\tau_{\theta}\bar{\theta}+\tau_{\varepsilon}\tilde{s}_{i}+\tau_{p}\tilde{s}_{p}}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}} + c - 2\tilde{p}}{\kappa\left(\frac{1}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}} + \frac{1}{\tau_{\delta}}\right)} - \left(\tilde{p}-c\right).$$
(18)

Solving financial traders' problem in (6), we can compute their futures demand as follows:

$$d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) = \underbrace{\frac{\tau_{\delta}(\tilde{\theta} + c - 2\tilde{p})}{\gamma (1 - \rho^2)}}_{\text{speculation}} - \underbrace{\frac{\rho \sqrt{\tau_{\delta} \tau_{\eta}}}{\gamma (1 - \rho^2)}}_{\text{hedging}} \tilde{\alpha}.$$
(19)

As discussed in Section 2, financial traders invest in futures contracts also for two reasons. First, they speculate on their private information, in particular, on their superior information about commodity demand shock $\tilde{\theta}$. Second, they have made informed investment on their private technology, whose payoff is correlated with the commodity market and thus, financial traders also trade futures to hedge their investment in the private technology.

Equation (19) reveals that the trading of financial traders injects both information $\tilde{\theta}$ (that is useful for predicting the later commodity demand) and "noise" $\tilde{\alpha}$ (that is orthogonal to commodity demand shocks) into the commodity futures market. Information $\tilde{\theta}$ is injected via financial traders' speculative trading, while noise $\tilde{\alpha}$ is injected via their hedging-motivated trading. This observation has important implications for price informativeness, as we will explore in Section 4.

We derive the equilibrium futures price function following the standard approach in the literature. That is, we insert demand functions (18) and (19) into the market-clearing condition (8) to solve the price in terms of $\tilde{\theta}$, $\tilde{\alpha}$, and $\tilde{\xi}$, and then compare with the conjectured price function in equation (14) to obtain a system defining the unknown *p*-coefficients. Solving this system yields the following proposition:

Proposition 1 (Futures market equilibrium) For any given mass $\mu \ge 0$ of financial traders, there exists a unique linear NREE where the futures price \tilde{p} is given by equation (14), where

$$p_{0} = D^{-1} \left[\frac{\frac{\tau_{\theta}\bar{\theta} - \tau_{p}\pi_{\xi}\bar{\xi}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + c}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + c + \frac{\mu\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)}c \right],$$

$$p_{\theta} = D^{-1} \left[\frac{\frac{\tau_{\varepsilon} + \tau_{p}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \frac{\mu\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)} \right],$$

$$p_{\alpha} = D^{-1} \left[\frac{\frac{\tau_{p}\pi_{\alpha}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} - \frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma \left(1 - \rho^{2}\right)} \right],$$

$$p_{\xi} = D^{-1} \left[\frac{\frac{\tau_{p}\pi_{\xi}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1 \right],$$

where

$$\begin{split} D &= \frac{2}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1 + \frac{2\mu\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)}, \\ \tau_{p} &= \left[\frac{\mu^{2}\rho^{2}\tau_{\delta}\tau_{\eta}}{\gamma^{2} \left(1 - \rho^{2}\right)^{2}\tau_{\alpha}} + \frac{1}{\tau_{\xi}}\right]^{-1}\pi_{\xi}^{-2}, \\ \pi_{\alpha} &= -\frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma \left(1 - \rho^{2}\right)}\pi_{\xi}, \\ with \ \pi_{\xi} \in \left(\left[\frac{\frac{\tau_{\varepsilon}}{\tau_{\theta} + \tau_{\varepsilon}}}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon}} + \frac{1}{\tau_{\delta}}\right)} + \frac{\mu\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)}\right]^{-1}, \left[\frac{\mu\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)}\right]^{-1}\right) \ being \ determined \ by \ the \ unique \ root \ to$$

the following equation:

$$\pi_{\xi} = \left[\frac{\frac{\tau_{\varepsilon}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \frac{\mu \tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)}\right]^{-1}$$

4 Price Informativeness, Asset Prices, and Welfare

4.1 **Price Informativeness**

As mentioned in Section 3.2, we use τ_p to measure price informativeness; it characterizes how much information the prevailing futures price \tilde{p} conveys about the futures contract's "fundamental," which is the commodity demand shock $\tilde{\theta}$ in our setting. Our price informativeness measure is broadly consistent with the concept of "market efficiency," which refers to the extent to which the prevailing market prices are informative about the future value of the traded assets.⁸ In this section, we examine the effect of the degree of financialization μ on price informativeness. This is a question that received large attention in the empirical literature, e.g., Raman, Robe, and Yadav (2017), and Brogaard, Ringgenberg, and Sovich (2018).

As shown by the demand function (19) of financial traders, their speculative trading injects information $\tilde{\theta}$ into the price \tilde{p} , while their hedging-motivated trading injects noise $\tilde{\alpha}$ into the price \tilde{p} . So, in general, adding more financial traders has an ambiguous effect on price informativeness. The following proposition characterizes the effect and how it depends on different parameters.

⁸For example, Brown, Harlow, and Tinic (1988, p. 355–356) write: "the efficient market hypothesis (EMH) claims that the price of a security at any point is a noisy estimate of the present value of the certainty equivalents of its risky future cash flows." Another relevant quote is: "A market in which prices always 'fully reflect' available information is called 'efficient." (Fama, 1970, p. 383). Due to its relation to information and prices, market efficiency is also termed as "informational efficiency" or "price efficiency."

Proposition 2 (Price informativeness)

(a) When the population size of financial traders is sufficiently small, commodity financialization improves price informativeness. That is, $\frac{\partial \tau_p}{\partial \mu} > 0$ for sufficiently small μ .

(b) Suppose that the precision level τ_{ε} of commodity producers' private signals is sufficiently high, then:

$$\frac{\partial \tau_p}{\partial \mu} > 0 \iff \mu < \frac{\kappa \gamma \tau_\alpha}{\tau_\delta \tau_\eta \tau_\xi} \left(\frac{1}{\rho^2} - 1\right). \tag{20}$$

Proposition 2 suggests that increasing the population size μ of financial traders first improves price informativeness and then harms price informativeness. To understand this result, we examine in detail the demand functions of commodity producers and financial traders, which are given by equations (18) and (19), respectively. We use ϕ_{θ} to measure the sensitivity of commodity producers' aggregate order flow to information $\tilde{\theta}$, i.e.,

$$\phi_{\theta} \equiv \frac{\partial \int_{0}^{1} d\left(\tilde{s}_{i}, \tilde{p}\right) di}{\partial \tilde{\theta}} = \frac{\frac{\tau_{\varepsilon}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)},$$

where the last equality follows from equation (18). Similarly, we define

$$\beta_{\theta} \equiv \frac{\partial d_F(\theta, \tilde{\alpha}, \tilde{p})}{\partial \tilde{\theta}} = \frac{\tau_{\delta}}{\gamma (1 - \rho^2)},$$
$$\beta_{\alpha} \equiv -\frac{\partial d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p})}{\partial \tilde{\alpha}} = \frac{\rho \sqrt{\tau_{\delta} \tau_{\eta}}}{\gamma (1 - \rho^2)}$$

to capture the sensitivities of financial traders' order flow to information θ and to noise $\tilde{\alpha}$.

Equipped with these notations and inserting the demand functions (18) and (19) into the market-clearing condition (8), we have

 $\underbrace{\phi_{\theta}\tilde{\theta}}_{\text{information from commodity producers}} + \underbrace{\mu\beta_{\theta}\tilde{\theta}}_{\text{information from financial traders}}$

$$\underbrace{\mu\beta_{\alpha}\tilde{\alpha}}_{\text{oise from financial traders}} + \underbrace{\xi}_{\text{exogenous noise trading}} - L\left(\tilde{p}\right) = 0, \quad (21)$$

where $L(\tilde{p})$ is a known linear function that absorbs all the other terms unrelated to information or noise in the order flows of market participants. In the above market-clearing condition, the speculative trading of commodity producers and of financial traders injects information $\tilde{\theta}$ into the aggregate demand, the hedging-motivated trading of financial traders injects endogenous noise based on the realization of $\tilde{\alpha}$ into the aggregate demand, and noise trading injects exogenous noise $\tilde{\xi}$ into the aggregate demand.

In (21), moving $L(\tilde{p})$ to the right-hand side and dividing both sides by $(\phi_{\theta} + \mu\beta_{\theta})$ lead to the following signal in predicting fundamental $\tilde{\theta}$:

$$\tilde{\theta} - \underbrace{\frac{\mu\beta_{\alpha}}{\phi_{\theta} + \mu\beta_{\theta}}}_{\text{region initiated here for each of the larger states}}_{\text{region ini$$

noise injected by financial traders exogenous noise trading This signal gives the informational content in the aggregate order flow. In equilibrium, it must coincide with \tilde{s}_p given by equation (16).

In equation (22), it is clear that increasing μ has two offsetting effects on the informativeness of \tilde{s}_p : first, it lowers the noise $\frac{1}{\phi_{\theta}+\mu\beta_{\theta}}\tilde{\xi}$ that is related to the exogenous noise trading; second, it increases the noise $\frac{\mu\beta_{\alpha}}{\phi_{\theta}+\mu\beta_{\theta}}\tilde{\alpha}$ brought in endogenously by financial traders. When μ is small—for instance, when $\mu \approx 0$ —the endogenous noise $\frac{\mu\beta_{\alpha}}{\phi_{\theta}+\mu\beta_{\theta}}\tilde{\alpha}$ added by financial traders is relatively small and thus, the main effect of increasing μ is to lower $\frac{1}{\phi_{\theta}+\mu\beta_{\theta}}\tilde{\xi}$. As a result, the price signal \tilde{s}_p becomes more informative about $\tilde{\theta}$ when μ increases from a very small value. In contrast, as μ becomes very large, the added noise $\frac{\mu\beta_{\alpha}}{\phi_{\theta}+\mu\beta_{\theta}}\tilde{\alpha}$ eventually dominates the noise $\frac{1}{\phi_{\theta}+\mu\beta_{\theta}}\tilde{\xi}$, and the price signal \tilde{s}_p becomes less informative about the fundamental $\tilde{\theta}$.

It is also useful to understand in detail the threshold value of μ in Part (b) of Proposition 2. A smaller threshold value implies that it is more likely for price informativeness to decrease with μ . First, when the correlation ρ between the private technology and the commodity demand is large in magnitude, the threshold value of μ is small, because a large $|\rho|$ implies that financial traders hedge more and so their trading brings more noise into the price. Second, when $\tau_{\delta}\tau_{\eta}$ is large, there is little residual uncertainty in both the private technology and the futures payoff and thus financial traders will trade more aggressively and hedge more, leading, for a similar reason, to a decrease in the threshold value of μ . Third, when $\frac{\tau_{\alpha}}{\tau_{\xi}}$ is small, the variance of the added noise by financial traders is large relative to the variance of the exogenous noise trading in the futures market, which means that the added noise is more powerful in diluting information, leading again to a decrease in the threshold value of μ . Fourth, when the risk aversion κ of commodity producers is small, commodity producers trade aggressively and their trading already injects a lot of information into the price. In this case, adding financial traders is more likely to adversely affect the aggregation of commodity traders' information, and so the threshold value of μ decreases. Finally, lowering risk aversion γ of financial traders is equivalent to scaling up the total order flow of financial traders and thus, the threshold value of μ decreases with γ as well.

Figure 2 provides a graphical illustration for the effect of μ on price informativeness τ_p . In this example, we set the parameter values as follows: $\tau_{\theta} = \tau_{\delta} = \tau_{\varepsilon} = \tau_{\xi} = \tau_{\alpha} = \tau_{\eta} = 1, \gamma = \kappa = 0.1$, and $\rho = 0.5$. The pattern is robust to the choice of parameter values. Indeed, we see that price informativeness τ_p first increases and then decreases with the mass μ of financial traders. This suggests that commodity financialization is beneficial to price informativeness if and only if the population size of new financial traders in the futures market is not too large.

This hump-shaped relation between μ and τ_p sheds light on recent empirical evidence finding mixed results on the question of how commodity financialization affects market efficiency. Raman, Robe, and Yadav (2017) document that the electronification of U.S. crude oil futures trading in 2006 brought about a massive growth in intraday activity by "noncommercial" institutional financial traders. In their sample, this financialization of intraday trading activity had a positive impact on price efficiency. In contrast, Brogaard, Ringgenberg, and Sovich (2018) examine the financialization of commodity index markets and find that financialization distorts the informational content in the futures price. One possibility to reconcile the two based on our findings is that the U.S. crude oil futures market is the world's largest commodity market, and so an influx of financial capital into this market corresponds to a relatively small value of μ , and so the positive effect on price informativeness in Raman, Robe, and Yadav (2017) is expected in our model. In other markets, μ may be relatively large and thus increasing μ lowers τ_p , as documented in Brogaard, Ringgenberg, and Sovich (2018).

4.2 Futures Price Biases

The literature has long been interested in "futures price bias," which is the deviation of the futures price from the expectation of the later spot price, $E(\tilde{v} - \tilde{p})$. A downward bias in the futures price is termed "normal backwardation," while an upward bias in the futures price is termed "contango."⁹ A major branch of literature on futures pricing has attributed bias to hedging pressures of commodity producers (e.g., Keynes, 1930; Hicks, 1939; Hirshleifer, 1988, 1990). Hamilton and Wu (2014) document that the futures price bias in crude oil futures on average decreased since 2005. Regulators are also very concerned about how commodity financialization affects the average futures price. As mentioned in the Introduction, the 2011

⁹In practice, the terms "normal backwardation" and "contango" are often used to refer to the bias between contemporaneous spot price and futures price. Capturing this definition exactly in our model would require us to extend the setting, and so, to keep it simple, we follow the literature such as Hirshleifer (1990) and define those terms as the difference between the current futures price and the expected value of the later spot price.

G20 Report on Commodities asked: "(D)oes increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility?". We now explore how the futures price bias is affected by financialization in our model in light of the risk sharing and information effects that we highlight.

We can compute the futures price bias $E(\tilde{v} - \tilde{p})$ as follows:

$$E\left(\tilde{v}-\tilde{p}\right) = \frac{\frac{\bar{\theta}-c}{2}-\bar{\xi}}{\underbrace{\frac{(\tau_{\theta}+\tau_{\varepsilon}+\tau_{p})\tau_{\delta}}{\kappa\left(\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}+\tau_{\delta}\right)}}_{\text{learning by commodity producers}} + \underbrace{\frac{\mu\tau_{\delta}}{\gamma\left(1-\rho^{2}\right)}}_{\text{risk sharing}} + \underbrace{\frac{2}{\gamma\left(1-\rho^{2}\right)}}_{\text{risk sharing}} + \underbrace{\frac{\mu\tau_{\delta}}{\gamma\left(1-\rho^{2}\right)}}_{\text{risk sharing}} + \underbrace{\frac{2}{\gamma\left(1-\rho^{2}\right)}}_{\text{risk sharing}} + \underbrace{\frac{2}$$

We can see that, depending on the sign of $\frac{\bar{\theta}-c}{2} - \bar{\xi}$, there can be either a downward bias or an upward bias in futures prices, that is, $E(\tilde{v}-\tilde{p}) > 0$ if and only if $\frac{\bar{\theta}-c}{2} > \bar{\xi}$. Intuitively, when the average commodity demand shock $\bar{\theta}$ is high relative to the production cost parameter c, commodity producers tend to produce more commodities and thus they will short more futures to hedge their commodity production. If their shorting pressure overwhelms the average demand $\bar{\xi}$ from noise traders, then on average, the futures price is depressed relative to its fundamental value, which leads to a downward bias in futures price (normal backwardation). By contrast, when $\frac{\bar{\theta}-c}{2}$ is small relative to $\bar{\xi}$, the futures price is biased upward, leading to a contango. Fama and French (1987) used 21 commodities to test the futures risk premium hypothesis, and indeed, they found that some markets feature "normal backwardation," while others feature "contango."

We are interested in the effect of the degree of financialization μ on the futures price bias. In equation (23), we can see that increasing μ does not affect the sign of the bias, but does affect its absolute magnitude $|E(\tilde{v} - \tilde{p})|$ in two ways. First, the newly added financial traders directly share more risk that is loaded off from the hedging needs of commodity producers. This tends to reduce the magnitude of the futures price bias. Second, increasing μ also affects price informativeness τ_p , which in turn changes the risk perceived by commodity producers through their learning from the futures price. As shown in Proposition 2, τ_p can either increase or decrease with μ . When τ_p increases with μ , the learning effect works in the same direction as the risk-sharing effect and thus the magnitude of the futures price bias $|E(\tilde{v} - \tilde{p})|$ decreases with μ . When τ_p decreases with μ , the learning effect works against the risk-sharing effect, which can generate a non-monotonic relation between $|E(\tilde{v} - \tilde{p})|$ and μ .¹⁰ The following proposition provides a full characterization.

Proposition 3 (Futures price bias)

(a) There is a downward bias (i.e., normal backwardation) in the futures price relative to the expected value of the later spot price if and only if $\frac{\bar{\theta}-c}{2} > \bar{\xi}$. That is, $E(\tilde{v}-\tilde{p}) > 0$ if and only if $\frac{\bar{\theta}-c}{2} > \bar{\xi}$.

(b) When price informativeness τ_p increases with the mass μ of financial traders, commodity financialization reduces the absolute magnitude of the futures price bias; that is, if $\frac{\partial \tau_p}{\partial \mu} > 0$, then $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial \mu} < 0$. In contrast, if $\frac{\partial \tau_p}{\partial \mu} < 0$, then it is possible that $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial \mu} > 0$.

Combining this result with that in Proposition 2, we get the following corollary.

Corollary 1 When the population size of financial traders is small, commodity financialization reduces the absolute magnitude of the futures price bias. That is, $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial \mu} < 0$ for sufficiently small μ .

¹⁰When financial traders' private information $\tilde{\alpha}$ has a nonzero mean $\bar{\alpha}$, the expression of $E\left(\tilde{v}-\tilde{p}\right)$ in (23) extends to $E\left(\tilde{v}-\tilde{p}\right) = \frac{\frac{\tilde{\theta}-c}{2}-\tilde{\xi}+\frac{\mu\rho\sqrt{\tau_{\delta}\tau_{\eta}}}{\gamma(1-\rho^2)}\bar{\alpha}}{\frac{(\tau_{\theta}+\tau_{\varepsilon}+\tau_{p})\tau_{\delta}}{\kappa(\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}+\tau_{\delta})}+\frac{\mu\tau_{\delta}}{\gamma(1-\rho^2)}+\frac{1}{2}}$. In this case, $E\left(\tilde{v}-\tilde{p}\right) > 0$ if and only if $\frac{\bar{\theta}-c}{2}+\frac{\mu\rho\sqrt{\tau_{\delta}\tau_{\eta}}}{\gamma(1-\rho^2)}\bar{\alpha} > 1$.

 $[\]bar{\xi}$. In consequence, an increase in the mass of financial traders may also change the sign of $E(\tilde{v} - \tilde{p})$, in addition to the change in the absolute magnitude due to the risk-sharing effect and the learning effect discussed here.

Figure 3 plots price informativeness τ_p and the magnitude of the futures price bias $|E(\tilde{v}-\tilde{p})|$ against the mass μ of financial traders. In the top panels, the parameters are the same as in Figure 2, that is, $\tau_{\theta} = \tau_{\delta} = \tau_{\varepsilon} = \tau_{\xi} = \tau_{\alpha} = \tau_{\eta} = 1, \gamma = \kappa = 0.1$, and $\rho = 0.5$. We have also set $\bar{\theta} = 2, c = 1$, and $\bar{\xi} = 0$, so that $E(\tilde{v}-\tilde{p}) > 0$ by Part (a) of Proposition 3. As we discussed in the previous subsection, price informativeness τ_p first increases and then decreases with μ in Panel a1. In Panel a2, the futures price bias $E(\tilde{v}-\tilde{p})$ monotonically decreases with μ , because the risk-sharing effect always dominates the learning effect in determining the overall effect of increasing μ on the futures price bias.

In the bottom panels of Figure 3, we have increased the values of $\tau_{\delta}, \tau_{\eta}$, and τ_{ξ} from 1 to 5. This change strengthens the negative effect on τ_p , because according to Part (b) of Proposition 2, the μ -threshold decreases with $\tau_{\delta}\tau_{\eta}\tau_{\xi}$. This can be seen from a left shift of the peak in Panel b1. In addition, we also increase the risk aversion γ of financial traders from 0.1 to 0.5 while still keeping the risk aversion κ of commodity producers at 0.1, so that commodity producers play a larger role in determining $E(\tilde{v} - \tilde{p})$ in equation (23). Both parameter changes can make it more likely for the learning effect to dominate the risksharing effect, so that the futures price bias can increase with μ . This is indeed the case: in Panel b2, $E(\tilde{v} - \tilde{p})$ first decreases with μ (as predicted by Corollary 1), then increases with μ (because the learning effect dominates), and finally decreases with μ again (because the risk-sharing effect will eventually dominate, i.e., $E(\tilde{v} - \tilde{p}) \rightarrow 0$ as $\mu \rightarrow \infty$ in (23)).

Overall, we can see that, while the futures price bias is often decreasing in the degree of financialization due to a strong risk sharing effect (which is sometimes assisted and sometimes weakened by the information effect), there are cases where the negative information effect is so strong that the bias increases following greater financialization. This provides some justification to the concerns voiced in policy circles.

4.3 Market Liquidity

Market liquidity refers to a market's ability to facilitate the purchase or sale of an asset without drastically affecting the asset's price. The literature has used the coefficient p_{ξ} in price function (14) to inversely measure market liquidity: a smaller p_{ξ} means that uninformed noise trading $\tilde{\xi}$ has a smaller price impact and thus that the market is deeper and more liquid. This measure of market liquidity is closely related to Kyle's (1985) lambda. Using Proposition 1, we can compute:

$$Liquidity \equiv \frac{1}{p_{\xi}} = \frac{1}{\frac{2\tau_{\delta}\left(\tau_{p} + \tau_{\theta} + \tau_{\varepsilon}\right)}{\kappa\left(\tau_{p} + \tau_{\theta} + \tau_{\varepsilon} + \tau_{\varepsilon}\right)}}{\frac{\tau_{\delta}\tau_{p}\pi_{\xi}}{\kappa\left(\tau_{\delta} + \tau_{\theta} + \tau_{\varepsilon} + \tau_{p}\right)}} + \frac{2\mu\tau_{\delta}}{\gamma\left(1 - \rho^{2}\right)} + 1}{\frac{\tau_{\delta}\tau_{p}\pi_{\xi}}{\kappa\left(\tau_{\delta} + \tau_{\theta} + \tau_{\varepsilon} + \tau_{p}\right)}} + 1}.$$
 (24)

Increasing the population size μ of financial traders has three effects on market liquidity $\frac{1}{p_{\xi}}$. The first effect is a direct positive effect denoted as "market making by financial traders" in (24). By submitting demand schedules, financial traders are effectively making the market to noise traders. So, the more financial traders are present in the market, the smaller is the price change induced by a change in the exogenous noise trading.

The other two effects are indirect and go through the effect of μ on price informativeness and the behavior of the commodity producers. To fix ideas, let us assume that price informativeness τ_p increases with μ , which is true when μ is small (see Proposition 2). First, when μ increases, commodity producers can learn more information from the futures price. This makes them face less uncertainty and trade more aggressively against the demands of noise traders, enhancing their market-making capacity. As a result, changes in noise trading are absorbed with a smaller price change. This positive effect is denoted as "market making by commodity producers" in (24). Second, a negative side effect of the increase in price informativeness resulting from an increase in μ is that commodity producers rely more on the price and end up making wrong inferences from the price change induced by noise trading, increasing adverse selection. This negative effect is denoted as "adverse selection of commodity producers" in (24).

The overall effect of increasing μ on market liquidity is determined by the interaction among the above three effects. The complexity of expression (24) precludes an analytical characterization. Nonetheless, we have conducted various numerical analyses and found that across all of them the positive effect dominates so that market liquidity $\frac{1}{p_{\xi}}$ generally increases with the mass μ of financial traders.

4.4 Commodity-Equity Market Comovement

The empirical literature has been actively debating whether commodity financialization strengthens the comovement between the commodity futures market and the equity market. Gorton and Rouwenhorst (2006) demonstrate that before 2004, commodity returns had negligible correlations with equity returns. Tang and Xiong (2012) document that the correlation between the Goldman Sachs Commodity Index (GSCI) and the S&P 500 stock returns rose after 2004, and was especially high in 2008, which is concurrent with the financialization of commodities. Cheng and Xiong (2014) suggest that commodity financialization has contributed to the sharp spike in the commodity-equity correlation during 2009–2011. Büyükşahin and Robe (2013, 2014) further link the increased correlation between commodities and stocks to the trading of hedge funds, especially those funds that are active in both equity and commodity futures markets. However, Bhardwaj, Gorton, and Rouwenhorst (2016) argue that the commodity-equity correlation falls back to its normal level after 2011, and they instead point to business cycles as the driving force of commodity-equity correlation patterns.

Our model can help shed light on this issue. We can interpret financial traders' additional investment opportunity in our setting as stocks. In line with Büyükşahin and Robe (2013, 2014), financial traders can represent hedge funds who hold positions in both equity and commodity futures markets. By construction, the return on stocks is simply $\tilde{\alpha} + \tilde{\eta}$ (investing one dollar at date 0 becomes $1 + \tilde{\alpha} + \tilde{\eta}$ dollars at date 1). We measure the return on futures by $\tilde{v} - \tilde{p}$: buying a futures contract at date 0 costs \tilde{p} ; the contract matures at date 1, and its date-1 price changes to \tilde{v} accordingly. Thus, this measure is effectively consistent with the empirical practice of constructing futures returns from the futures price data. We capture the commodity-equity comovement by the covariance $Cov(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta})$, and examine how $Cov(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta})$ changes with the mass μ of financial traders.

Proposition 4 (Commodity-equity market comovement)

(a) The covariance between stock returns $\tilde{\alpha} + \tilde{\eta}$ and futures returns $\tilde{v} - \tilde{p}$ is positive if and only if the correlation ρ between the unforecastable component $\tilde{\eta}$ in stock returns and the unforecastable component $\tilde{\delta}$ in commodity demand is positive. That is, $Cov(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta}) > 0$ if and only if $Cov(\tilde{\delta}, \tilde{\eta}) > 0$.

(b) When the population size μ of financial traders is sufficiently small, commodity financialization strengthens commodity-equity market comovement. That is, $\frac{\partial |Cov(\tilde{v}-\tilde{p},\tilde{\alpha}+\tilde{\eta})|}{\partial \mu} > 0$ for sufficiently small μ .

Intuitively, the hedging-motivated trades of financial traders injects the forecastable component $\tilde{\alpha}$ in stock returns into the futures price \tilde{p} , which leads to extra comovement between futures returns $\tilde{v} - \tilde{p}$ and stock returns $\tilde{\alpha} + \tilde{\eta}$. Our theory therefore predicts that financialization can indeed increase the commodity-equity correlation. Also note that in our setting, it is financial traders, active in both equity and commodity futures markets, who connect further these two markets. This is consistent with the empirical channel documented by Büyükşahin and Robe (2013, 2014).¹¹ Under our theory, the cyclicity of financialization can potentially drive the cyclicity of commodity-equity correlation. For instance, if the market first became financialized in 2009–2011 and then de-financialized afterwards, the commodity-equity correlation would exhibit the pattern documented by Bhardwaj, Gorton, and Rouwenhorst (2016). This provides a testable view complementary to the business-cycle based explanation suggested by Bhardwaj, Gorton, and Rouwenhorst (2016).

4.5 Welfare and Operating Profits

We now turn to analyze the effect of increased commodity financialization on the profits and welfare of financial traders and commodity producers. Such questions have been discussed in the empirical literature (e.g., Chen, Dai, and Sorescu, 2017; Brogaard, Ringgenberg, and Sovich, 2018). We use the ex ante certainty equivalents CE_F and CE_P to measure the

¹¹This view also complements Basak and Pavlova (2016) who obtain the increase in equity-commodity comovement through benchmarking institutional investors to a commodity index that serves as a new common factor on which all assets load positively.

welfare of financial traders and commodity producers, respectively:

$$CE_F \equiv -\frac{1}{\gamma} \ln \left[E(e^{-\gamma[(\tilde{v}-\tilde{p})d_F(\tilde{\theta},\tilde{\alpha},\tilde{p})+(\tilde{\alpha}+\tilde{\eta})z_F(\tilde{\theta},\tilde{\alpha},\tilde{p})]}) \right],$$

$$CE_P \equiv -\frac{1}{\kappa} \ln \left[E\left(e^{-\kappa[\tilde{v}x(\tilde{s}_i,\tilde{p})-C(x(\tilde{s}_i,\tilde{p}))+(\tilde{v}-\tilde{p})d(\tilde{s}_i,\tilde{p})]}\right) \right],$$

where $d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}), z_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}), d(\tilde{s}_i, \tilde{p})$, and $x(\tilde{s}_i, \tilde{p})$ are the equilibrium trading strategies and production policy. We also look at the operating profit of commodity producers, which is an easier object to analyze in empirical research:

Operating profits
$$\equiv E \left[\tilde{v} x_i - C \left(x_i \right) \right].$$

Due to the complexity of the expressions for the profits and welfare variables, we use Figure 4 to conduct a numerical analysis. Here, we plot price informativeness τ_p , operating profits $E[\tilde{e}x_i - C(x_i)]$, producers' welfare CE_P , and financial traders' welfare CE_F against the mass μ of financial traders. The parameter values are the same as those in Figure 2. The variable patterns are robust to different parameter choices. Panel a of Figure 4 simply reproduces Figure 2, that is, price informativeness first increases and then decreases with μ . Panel b of Figure 4 shows that operating profits of commodity producers exhibit a similar pattern. This is consistent with Brogaard, Ringgenberg, and Sovich (2018). They find that after the spike in commodity financialization in 2004, the information efficiency of futures index prices decreased and those firms using index commodities saw a decrease in their profits. The intuition is that higher price informativeness allows commodity producers to make more efficient production decisions, which leads to an increase in their operating profits.

However, a higher profit does not necessarily translate into a higher welfare for producers. In fact, Panel c of Figure 4 shows that the pattern of producer welfare is generally opposite to the pattern of operating profits. Specifically, producer welfare CE_P is U-shaped in μ , while both operating profits and price informativeness τ_p are hump-shaped in μ . The welfare pattern is a result of the effect of more informative prices on producers' trading opportunities. As futures prices become more informative, commodity producers have fewer opportunities to explore their information advantage and so their trading gains will deteriorate. In addition, their hedging and risk sharing opportunities are diminished when prices are more informative. This is related to the well-known Hirshleifer effect (1971).¹² These effects end up dominating the benefit from information in prices. To further examine this welfare result, we will devote the next section to analyzing an extension in which some commodity producers trade futures while others do not. In this extended setting, we find that for those commodity producers who do not trade futures, welfare and operating profits exhibit the same pattern as price informativeness, consistent with the intuition above that more informative futures prices allow more efficient production decisions.

Taken together, our analysis suggests that researchers should carefully differentiate among price efficiency, operating profits, and welfare when making normative statements. For instance, in Brogaard, Ringgenberg, and Sovich (2018), both price efficiency and operating profits deteriorate after 2004. This may suggest that in practice, commodity financialization harms those commodity producers who do not trade futures. However, for those commodity producers who do trade futures, they may actually benefit from commodity financialization. To make a welfare statement, a formal model such as ours is needed.

Finally, in Panel d of Figure 4, we see that the welfare CE_F of financial traders monoton-

 $^{^{12}}$ See Kurlat and Veldkamp (2015) and Goldstein and Yang (2017) for more discussions on the negative welfare effect of reduction in trading opportunities.

ically decreases with the the mass μ of financial traders. When more financial traders enter the futures market, there is more competition among them and in consequence, each existing financial trader sees a decrease in profits which translates to a decrease in welfare. This result is consistent with Chen, Dai, and Sorescu (2017) who suggest that the financialization in the commodities market harms commodity trading advisors at the individual fund level.

5 An Extension with Two Types of Producers

5.1 Setting and Equilibrium

We now analyze an extension in which only a fraction of commodity producers participate in the date-0 futures market. The motivation behind this analysis is threefold. First, while the results presented so far on the real effects of financialization relied on the premise that commodity producers participate in the futures market, it is certainly not the case that all of them do so in the real world. Commodity producers differ in their reliance on the futures market, and so we wish to understand the broader implications when only some commodity producers are active in the futures market. Second, on a theoretical basis, this extension naturally highlights two types of feedback effects from futures prices to production. Producers who participate in the futures market are affected directly by futures prices as these become the effective prices producers consider for their revenues, whereas producers who do not participate only see an indirect feedback via the information in the futures price (as was emphasized in the broad "feedback" literature surveyed by Bond, Edmans, and Goldstein (2012)). Third, as a result, we establish that commodity financialization indeed has opposite welfare consequences for commodity producers of the two types, depending on whether they participate in the futures market or not. This has important implications for the interpretation of empirical findings in the literature.

In the new setting, we divide the continuum of commodity producers into two groups: (1) "participating producers" (with mass $\lambda \in (0, 1)$, labeled with "P"), who trade futures and behave in the same way as the producers in the baseline model presented in Section 2; and (2) "nonparticipating producers" (with the remaining mass $1 - \lambda$, labeled with "N"), who do not participate in the futures market. Nonparticipating producers make production decisions at the same time as participating producers, and in particular, they still learn information from the futures price \tilde{p} . All of the other features of the baseline model in Section 2 remain unchanged. The baseline model corresponds to the degenerate case with $\lambda \to 1$.

In this extended economy, the equilibrium is still composed of the date-0 futures market equilibrium and the date-1 spot market equilibrium. Unlike the baseline model in which the two subequilibria can be solved sequentially, we now have to compute the two subequilibria simultaneously because the nonparticipating producers cannot take the futures price as the effective price for determining their revenues, but just use it in addition to their private information to update on the expected spot price. Formally, we conjecture the following price functions:

Date-1 spot market:
$$\tilde{v} = v_0 + v_\theta \theta + v_\delta \delta + v_p \tilde{p},$$
 (25)

Date-0 futures market:
$$\tilde{p} = p_0 + p_\theta \tilde{\theta} + p_\alpha \tilde{\alpha} + p_\xi \tilde{\xi},$$
 (26)

where the v-coefficients and p-coefficients are endogenous. We will compute these eight coefficients simultaneously.

Let us start with the date-1 spot market. The commodity supply comes from both groups of producers. The decision problem of participating producers is still the same as the baseline model. As a result, participating producer *i*'s optimal commodity production $x_P(\tilde{s}_i, \tilde{p})$ and futures investment $d_P(\tilde{s}_i, \tilde{p})$ are

$$x_P(\tilde{s}_i, \tilde{p}) = \tilde{p} - c, \tag{27}$$

$$d_P(\tilde{s}_i, \tilde{p}) = \frac{E\left(\tilde{v} \mid \tilde{s}_i, \tilde{p}\right) - \tilde{p}}{\kappa Var\left(\tilde{v} \mid \tilde{s}_i, \tilde{p}\right)} - \left(\tilde{p} - c\right), \qquad (28)$$

which correspond respectively to equations (11) and (15) in the main model. In particular, equation (27) illustrates the first type of feedback effect through which an increase in the futures price \tilde{p} directly encourages the production of participating producers who face a riskless production decision problem (i.e., the second maximization problem in (9)).

Nonparticipating producers do not trade futures and they only choose commodity production. Specifically, nonparticipating producer j's problem is

$$\max_{x_j} E\left(-e^{-\kappa \tilde{W}_j} \middle| \tilde{s}_j, \tilde{p}\right)$$

subject to

$$\tilde{W}_j = \tilde{v}x_j - cx_j - \frac{1}{2}x_j^2.$$

The FOC delivers the optimal production policy as follows:

$$x_N(\tilde{s}_j, \tilde{p}) = \frac{E\left(\tilde{v} \mid \tilde{s}_j, \tilde{p}\right) - c}{1 + \kappa Var\left(\tilde{v} \mid \tilde{s}_j, \tilde{p}\right)}.$$
(29)

Equation (29) illustrates the second type of feedback effect, which is based purely on information. Nonparticipating producers' production decision involves uncertainty, and they rely on the futures price \tilde{p} , in addition to their own signal, to make inference about the later spot price \tilde{v} and guide their production decisions (i.e., the expressions $E(\tilde{v}|\tilde{s}_j,\tilde{p})$ and $Var(\tilde{v}|\tilde{s}_j,\tilde{p})$ in (29) reflect the fact that nonparticipating producers use \tilde{p} and \tilde{s}_j to forecast \tilde{v}).

To both participating and nonparticipating producers, the date-0 futures price \tilde{p} is still equivalent to signal \tilde{s}_p given by equation (16). Using Bayes' rule, we compute the conditional moments in (27) and (29) to express out the production policies as functions of $\{\tilde{s}_i, \tilde{p}\}$. We then insert these production policies into the spot-market clearing condition

$$\int_{0}^{\lambda} x_{P}\left(\tilde{s}_{i},\tilde{p}\right) di + \int_{0}^{1-\lambda} x_{N}\left(\tilde{s}_{j},\tilde{p}\right) dj = \tilde{\theta} + \tilde{\delta} - \tilde{v},$$
(30)

to compute the implied spot-price function. Comparing the implied spot-price function with the conjectured spot-price function (25), we obtain the following four equations in terms of unknowns v's and p's:

$$v_{\delta} = 1, \tag{31}$$

$$v_{\theta} = 1 - (1 - \lambda) \frac{v_{\theta} \frac{\tau_{\varepsilon}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + v_{\delta}^{2} \frac{1}{\tau_{\delta}}\right)},$$
(32)

$$v_p = -\lambda - (1 - \lambda) \frac{v_\theta \frac{\tau_p \frac{1}{p_\theta}}{\tau_\theta + \tau_\varepsilon + \tau_p} + v_p}{1 + \kappa \left(v_\theta^2 \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_p} + v_\delta^2 \frac{1}{\tau_\delta}\right)},\tag{33}$$

$$v_0 = \lambda c - (1 - \lambda) \frac{v_0 + v_\theta \frac{\tau_\theta \theta + \tau_p \frac{\tau_\theta}{p_\theta}}{\tau_\theta + \tau_\varepsilon + \tau_p} - c}{1 + \kappa \left(v_\theta^2 \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_p} + v_\delta^2 \frac{1}{\tau_\delta}\right)}.$$
(34)

We next examine the date-0 futures market. In the futures market, participating commodity producers (with mass λ) and financial traders (with mass μ) trade against noise traders. The demand function $d_P(\tilde{s}_i, \tilde{p})$ of participating producers is given by equation (28). The decision problem of financial traders remains the same except that the futures payoff \tilde{v} now takes a more general form given by equation (25). In consequence, financial traders' demand function changes to

$$d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) = \frac{\tau_\delta}{\gamma \left(1 - \rho^2\right)} \left[v_0 + v_\theta \tilde{\theta} - \left(1 - v_p\right) \tilde{p} - \rho \sqrt{\frac{\tau_\eta}{\tau_\delta}} \tilde{\alpha} \right],\tag{35}$$

which extends equation (19) in the baseline model.

Inserting (28) and (35) into the futures-market clearing condition

$$\int_0^\lambda d\left(\tilde{s}_i, \tilde{p}\right) di + \mu d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) + \tilde{\xi} = 0,$$

we solve the implied futures-price function. We then compare the implied futures-price function with the conjectured futures-price function (26) to obtain the following four equations in terms of unknowns v's and p's:

$$p_{0} = \frac{\lambda \left[\frac{v_{0} + v_{\theta} \frac{\tau_{\theta} \bar{\theta} + \tau_{p} \frac{-p_{0} - p_{\alpha} \bar{\alpha} - p_{\xi} \bar{\xi}}{p_{\theta}}}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}} \right)} + c \right] + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} v_{0}}{\left[-v_{\theta} \frac{\frac{\tau_{p}}{p_{\theta}}}{-v_{0} + 1} - v_{0} + 1} \right]},$$
(36)

$$\lambda \left[\frac{\frac{-v_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} - v_{p} + 1}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}} \right)} + 1 \right] + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} \left(1 - v_{p} \right)$$
$$\lambda \frac{v_{\theta} \frac{\tau_{\varepsilon}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\lambda \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + 1 \right)} + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} v_{\theta}$$

$$p_{\theta} = \frac{\lambda \left[\frac{\nabla \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right) + \mu \gamma(1 - \rho^{2}) \nabla \theta}{\lambda \left[\frac{-v_{\theta} \frac{\tau_{p}}{\overline{p_{\theta}}} - v_{p} + 1}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1\right] + \mu \frac{\tau_{\delta}}{\gamma(1 - \rho^{2})} \left(1 - v_{p}\right)},$$
(37)

$$p_{\alpha} = -\frac{\mu \frac{\tau_{\delta}}{\gamma(1-\rho^2)} \rho \sqrt{\frac{\tau_{\eta}}{\tau_{\delta}}}}{\lambda \left[\frac{-v_{\theta} \frac{\tau_{p}}{p_{\theta}} - v_{p} + 1}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{e} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1\right] + \mu \frac{\tau_{\delta}}{\gamma(1-\rho^{2})} \left(1 - v_{p}\right)},$$
(38)

$$p_{\xi} = \frac{1}{\lambda \left[\frac{-v_{\theta} \frac{\tau_p}{\tau_{\theta} + \tau_{\varepsilon} + \tau_p} - v_p + 1}{\kappa \left(v_{\theta}^2 \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_p} + \frac{1}{\tau_{\delta}} \right)} + 1 \right] + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^2)} \left(1 - v_p \right)}$$
(39)

The eight unknowns $(v_0, v_\theta, v_\delta, v_p, p_0, p_\theta, p_\alpha, p_\xi)$ are jointly characterized by eight equations (31)–(34) and (36)–(39). We can further simplify the system to one equation in terms of one unknown $v_\theta \in (0, 1)$. After solving v_θ , we can compute the other seven unknowns accordingly.

Proposition 5 (Equilibrium in the extended economy) For any given $\lambda \in (0, 1)$, there exists an equilibrium with the date-1 spot-price function and the date-0 futures-price function given respectively by equations (25) and (26). The equilibrium is characterized by $v_{\theta} \in (0, 1)$, which is determined by

$$\frac{\tau_{\delta} v_{\theta} \left[\left(1 - \lambda \right) \tau_{\varepsilon} - \kappa v_{\theta} \left(1 - v_{\theta} \right) \right]}{\left(\kappa + \tau_{\delta} \right) \left(1 - v_{\theta} \right)} - \left(\tau_{\theta} + \tau_{\varepsilon} \right) = \frac{\left[\frac{\lambda \tau_{\varepsilon} (\kappa + \tau_{\delta}) (1 - v_{\theta})}{\kappa [\tau_{\varepsilon} (1 - \lambda) + \tau_{\delta} v_{\theta} (1 - v_{\theta})]} + \frac{\mu \tau_{\delta} v_{\theta}}{\gamma (1 - \rho^2)} \right]^2}{\frac{\mu^2 \rho^2 \tau_{\delta} \tau_{\eta}}{\gamma^2 (1 - \rho^2)^2 \tau_{\alpha}} + \frac{1}{\tau_{\varepsilon}}}, \tag{40}$$

which is equivalent to a 7th order polynomial of v_{θ} .

5.2 Results

The complexity of the extended model does not admit analytical solutions. Hence, we use Figure 5 to report the results on the effect of financialization for this extended economy based on numerical simulations. The parameter values we use are the same as in Panel b of Figure 3: $\tau_{\theta} = \tau_{\varepsilon} = \tau_{\alpha} = 1, \tau_{\delta} = \tau_{\eta} = \tau_{\xi} = 5, \gamma = 0.5, \kappa = 0.1, \bar{\theta} = 2, c = 1, \bar{\xi} = 0, \text{ and}$ $\rho = 0.5$. We also set $\lambda = 0.8$. The results are generally robust across different parameters. We start with repeating the positive analysis in Sections 4.1-4.4 and report the following variables for the extended model:

Price informativeness :
$$\tau_p$$
,
Futures price bias : $|E(\tilde{v} - \tilde{p})|$,
Market liquidity : $1/p_{\xi}$,
Commodity-equity market comovement : $Corr(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta})$.

We then move to the normative analysis of Section 4.5 and report the following variables for the extended model:

Welfare of participating producers CE_P : $-\frac{1}{\kappa} \ln \left[E(e^{-\kappa[\tilde{v}x_P(\tilde{s}_i,\tilde{p}) - C(x_P(\tilde{s}_i,\tilde{p})) + (\tilde{v} - \tilde{p})d_P(\tilde{s}_i,\tilde{p})]}) \right],$ Operating profits of participating producers : $E\left[\tilde{v}x_P\left(\tilde{s}_i,\tilde{p}\right) - C\left(x_P\left(\tilde{s}_i,\tilde{p}\right)\right)\right],$ Welfare of nonparticipating producers CE_N : $-\frac{1}{\kappa} \ln \left[E(e^{-\kappa[\tilde{v}x_N(\tilde{s}_j,\tilde{p}) - C(x_N(\tilde{s}_j,\tilde{p}))]}) \right],$ Operating profits of nonparticipating producers : $E\left[\tilde{v}x_N(\tilde{s}_j,\tilde{p}) - C\left(x_N(\tilde{s}_j,\tilde{p})\right)\right],$

Welfare of financial traders CE_F : $-\frac{1}{\gamma} \ln \left[E(e^{-\gamma[(\tilde{v}-\tilde{p})d_F(\tilde{\theta},\tilde{\alpha},\tilde{p})+(\tilde{\alpha}+\tilde{\eta})z_F(\tilde{\theta},\tilde{\alpha},\tilde{p})]}) \right].$

We find that our results remain robust in this extended economy. Specifically, financialization first increases and then decreases price informativeness τ_p ; futures price bias $|E(\tilde{v} - \tilde{p})|$ can be non-monotone in financialization exhibiting a decreasing-increasing-decreasing pattern; financialization generally improves market liquidity $1/p_{\xi}$ and strengthens the commodityequity market comovement $Corr(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta})$; and financialization harms the welfare of each existing financial trader.

The important new implications coming out of the extended model are that financialization has different normative implications for commodity producers depending on whether they participate in the futures market. For participating producers, their operating profits exhibit a similar pattern as price informativeness, but their welfare exhibits an opposite pattern. For nonparticipating producers, both their operating profits and their welfare are hump-shaped in financialization, which exhibits the same pattern as price informativeness. Overall, we can see that nonparticipating producers benefit from the greater informativeness of the price, which enables them to make more efficient decisions. While this is true also for participating producers, who see greater operating profits when price informativeness improves, the dominant effect for their welfare is that price informativeness decreases their trading and risk sharing opportunities, leading to an overall lower welfare.

6 Conclusion

Commodity futures markets have changed drastically in recent years. From markets that mostly serve commodity producers and users, they became markets that are also widely populated by financial traders. These financial traders find the commodity futures markets to be a fertile ground for speculative profits, based on information they produce on the fundamentals of the commodities, and also a good place to hedge and diversify other exposures. An emerging empirical literature has studied the consequences of financialization for market efficiency and for the profitability of firms exposed to commodities as producers or users. The topic has also been of wide concern to policymakers who wondered about distortions in prices and real outcomes.

We provide a unified framework to study the consequences of the financialization of commodity futures markets. We show that, due to the dual trading motive based on speculation and hedging, financialization injects both information and noise into futures prices. In our model, this translates into a very clear pattern, whereby an increase in financialization first increases and then decreases price informativeness. This non-monotone effect on informativeness combined with the effect on risk sharing can also lead to a non-monotone effect of financialization on the futures price bias. In general, commodity financialization seems to improve market liquidity in the futures market and increase the comovement between the commodity futures market and the equity market. Concerning real outcomes, our analysis highlights two types of feedback effects from futures prices to production decisions. Commodity producers who trade futures automatically increase production in the face of an increased futures price. Their operating profits increase when price informativeness increases, but their welfare can move in the opposite direction due to lost trading and risk sharing opportunities. Commodity producers who do not trade futures use the futures price as a source of information when making their production decisions. Their operating profits and welfare unambiguously increase when price informativeness increases.

The framework we develop here helps interpreting the wide range of empirical results in the literature and sheds light on normative implications of the financialization phenomenon. It also provides a basis for future quantitative work that can further explore the effects of information and risk sharing on commodity cycles.

Appendix A: Lemmas

In this appendix, we provide two lemmas that will be used for future proofs.

Lemma A1

$$\frac{\partial \pi_{\xi}}{\partial \mu} = -\frac{\pi_{\xi}^2 \left[\left(\frac{\tau_{\delta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_p + \tau_{\delta}} \right)^2 \frac{2\mu\rho^2 \tau_\eta \tau_{\varepsilon}}{\gamma^2 \kappa (1 - \rho^2)^2 \tau_{\alpha}} \tau_p^2 \pi_{\xi}^2 + \frac{\tau_{\delta}}{\gamma (1 - \rho^2)} \right]}{1 + \frac{2\tau_{\delta} \tau_{\varepsilon}}{\kappa (\tau_{\theta} + \tau_{\varepsilon} + \tau_p + \tau_{\delta})^2} \tau_p \pi_{\xi}} < 0,$$
(A1)

$$\frac{\partial \tau_p}{\partial \mu} = -\frac{2\mu\tau_\delta\tau_\eta\rho^2}{\gamma^2 \left(1-\rho^2\right)^2 \tau_\alpha} \tau_p^2 \pi_\xi^2 - 2\tau_p \pi_\xi^{-1} \frac{\partial \pi_\xi}{\partial \mu}; \quad and \tag{A2}$$

$$\frac{\partial \tau_p}{\partial \mu} > 0 \iff \frac{\mu}{\left[\frac{\mu^2 \rho^2 \tau_\eta \tau_\delta}{\gamma^2 (1-\rho^2)^2 \tau_\alpha} + \frac{1}{\tau_\xi}\right] \pi_\xi} < \frac{\gamma \tau_\alpha \left(1-\rho^2\right)}{\rho^2 \tau_\eta}.$$
 (A3)

Proof. We apply the implicit function theorem to equations (B2) and (B4) to compute equations (A1) and (A2). Inserting (A1) into (A2), we can show

$$\frac{\partial \tau_p}{\partial \mu} > 0 \Longleftrightarrow \mu \tau_p \pi_{\xi} < \frac{\gamma \tau_{\alpha} \left(1 - \rho^2\right)}{\rho^2 \tau_{\eta}}$$

We then use equation (B4) to express τ_p in terms of π_{ξ} on the left-hand-side (LHS) in the above condition to obtain (A3).

Lemma A2 As $\tau_{\varepsilon} \to \infty$, we have

$$\pi_{\xi} \to \left[\frac{\tau_{\delta}}{\kappa} + \frac{\mu\tau_{\delta}}{\gamma\left(1-\rho^{2}\right)}\right]^{-1} \text{ and } \tau_{p} \to \left[\frac{\mu^{2}\rho^{2}\tau_{\delta}\tau_{\eta}}{\gamma^{2}\left(1-\rho^{2}\right)^{2}\tau_{\alpha}} + \frac{1}{\tau_{\xi}}\right]^{-1} \left[\frac{\tau_{\delta}}{\kappa} + \frac{\mu\tau_{\delta}}{\gamma\left(1-\rho^{2}\right)}\right]^{2}.$$

Proof. These expressions are obtained directly from equations (B2) and (B4). ■

Appendix B: Proofs

Proof of Proposition 1

We plug demand functions (18) and (19) into the market-clearing condition (8) to write the equilibrium price \tilde{p} as a function of $(\tilde{\theta}, \tilde{\alpha}, \tilde{\xi})$. This gives the expressions of the *p*-coefficients in Proposition 1.

By the expressions of the *p*-coefficients, we have

$$\pi_{\alpha} \equiv \frac{p_{\alpha}}{p_{\theta}} = -\frac{\frac{\frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma(1-\rho^2)}}{\frac{\tau_{\varepsilon}}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}}}{\frac{\tau_{\varepsilon}}{\kappa\left(\frac{1}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}+\frac{1}{\tau_{\delta}}\right)} + \frac{\mu\tau_{\delta}}{\gamma(1-\rho^2)}},\tag{B1}$$

$$\pi_{\xi} \equiv \frac{p_{\xi}}{p_{\theta}} = \frac{1}{\frac{\tau_{\varepsilon}}{\overline{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}} \frac{1}{\kappa \left(\frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \frac{\mu \tau_{\delta}}{\gamma (1 - \rho^{2})}.$$
(B2)

Using these two equations, we can express π_{α} in terms of π_{ξ} as in Proposition 1:

$$\pi_{\alpha} = -\frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma\left(1-\rho^2\right)}\pi_{\xi}.$$
(B3)

Combining (17) and (B3), we have

$$\tau_{p} = \left[\frac{\mu^{2} \rho^{2} \tau_{\delta} \tau_{\eta}}{\gamma^{2} \left(1 - \rho^{2}\right)^{2} \tau_{\alpha}} + \frac{1}{\tau_{\xi}}\right]^{-1} \pi_{\xi}^{-2}.$$
 (B4)

Inserting (B4) into (B2) generates an equation that is defined in terms of a single unknown π_{ξ} . Now, we prove that there exists a unique solution of π_{ξ} .

First, by the intermediate value theorem, there exists a solution of π_{ξ} . To see this, note that when $\pi_{\xi} = 0$, we have $\tau_p = \infty$ by (B4), and so the right-hand-side (RHS) of (B2) is $\frac{\gamma(1-\rho^2)}{\mu\tau_{\delta}} > 0$. When $\pi_{\xi} = \infty$, we have $\tau_p = 0$ by (B4) and the RHS of (B2) is $\frac{\frac{1}{\tau_{\phi}+\tau_{\varepsilon}}}{\frac{\tau_{\phi}+\tau_{\varepsilon}}{\tau_{\phi}+\tau_{\varepsilon}}} < \infty$.

Second, note that the RHS of (B2) is increasing in τ_p . By equation (B4), τ_p is decreasing in π_{ξ} . Thus, the RHS of (B2) is decreasing in π_{ξ} . As a result, the solution of π_{ξ} is unique.

Finally, since the RHS of (B2) is increasing in τ_p , we set $\tau_p = 0$ and $\tau_p = \infty$ to generate the lower and upper bounds for the equilibrium value of π_{ξ} in Proposition 1.

Proof of Proposition 2

Part (a): We prove Part (a) by checking the sign of $\frac{\partial \tau_p}{\partial \mu}$ at $\mu = 0$. By (A1), we know that π_{ξ} decreases with μ . Thus, as $\mu \to 0$, π_{ξ} does not go to zero. As a result, condition (A3) is always satisfied at $\mu = 0$. That is, $\frac{\partial \tau_p}{\partial \mu}\Big|_{\mu=0} > 0$.

Part (b): Suppose $\tau_{\varepsilon} \to \infty$. Inserting the expression of π_{ξ} in Lemma A2 into condition (A3), we can show

$$\frac{\mu}{\left[\frac{\mu^2\rho^2\tau_\eta\tau_\delta}{\gamma^2(1-\rho^2)^2\tau_\alpha}+\frac{1}{\tau_\xi}\right]\pi_\xi} < \frac{\gamma\tau_\alpha\left(1-\rho^2\right)}{\rho^2\tau_\eta} \Longleftrightarrow \mu < \frac{\kappa\gamma\tau_\alpha\left(1-\rho^2\right)}{\tau_\delta\tau_\xi\tau_\eta\rho^2}.$$

Proof of Proposition 3

Part (a): By equation (23), it is straightforward to show that $E(\tilde{v} - \tilde{p}) > 0 \iff \frac{\bar{\theta} - c}{2} > \bar{\xi}$. Thus, the key is to compute equation (23). By demand functions (15) and (19) and the market-clearing condition (8), we can show

$$\left[\frac{1}{\kappa Var\left(\tilde{v}|\,\tilde{s}_{i},\tilde{p}\right)}+\frac{\mu\tau_{\delta}}{\gamma\left(1-\rho^{2}\right)}\right]E\left(\tilde{v}-\tilde{p}\right)=E\left(\tilde{p}-c\right)-\bar{\xi}.$$
(B5)

We then use the expression of \tilde{v} in (13) to obtain

$$E\left(\tilde{p}-c\right) = \frac{\bar{\theta}-c}{2} - \frac{1}{2}E\left(\tilde{v}-\tilde{p}\right).$$
(B6)

From equations (B5) and (B6), we can compute equation (23). **Part (b):** If $\frac{\partial \tau_p}{\partial \mu} > 0$, then $\frac{\partial}{\partial \mu} \frac{(\tau_{\theta} + \tau_{\varepsilon} + \tau_p)\tau_{\delta}}{\kappa(\tau_{\theta} + \tau_{\varepsilon} + \tau_p + \tau_{\delta})^2} = \frac{\tau_{\delta}^2}{\kappa(\tau_{\theta} + \tau_{\varepsilon} + \tau_p + \tau_{\delta})^2} \frac{\partial \tau_p}{\partial \mu} > 0$. Clearly, $\frac{\partial}{\partial \mu} \frac{\mu \tau_{\delta}}{\gamma(1 - \rho^2)} = \frac{\tau_{\delta}}{\gamma(1 - \rho^2)} > 0$. Thus, by equation (23), we have $\frac{\partial |E(\tilde{v} - \tilde{p})|}{\partial \mu} < 0$. If $\frac{\partial \tau_p}{\partial \mu} < 0$, Figure 3 constructs an example to show that $|E(\tilde{v} - \tilde{p})|$ first increases and then decreases with μ .

Proof of Corollary 1

The proof follows directly from combining Part (a) of Proposition 2 and Part (b) of Proposition 3.

Proof of Proposition 4

Part (a): By equations (13) and (14), we have

$$Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right) = Cov\left(\tilde{\alpha}+\tilde{\eta},\left(1-2p_{\theta}\right)\tilde{\theta}+\tilde{\delta}-2p_{\alpha}\tilde{\alpha}-2p_{\xi}\tilde{\xi}\right)$$
$$= Cov\left(\tilde{\alpha},-2p_{\alpha}\tilde{\alpha}\right)+Cov(\tilde{\eta},\tilde{\delta})$$
$$= -2p_{\alpha}\frac{1}{\tau_{\alpha}}+\frac{\rho}{\sqrt{\tau_{\eta}\tau_{\delta}}}.$$

By Proposition 1, we have

$$p_{\alpha} = D^{-1} \left[\frac{-\frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma(1-\rho^{2})} \pi_{\xi} \frac{\tau_{p}}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}} + \frac{1}{\tau_{\delta}}\right)} - \frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma \left(1-\rho^{2}\right)} \right],$$
$$= -\frac{\mu\rho\sqrt{\tau_{\eta}\tau_{\delta}}}{\gamma \left(1-\rho^{2}\right)} D^{-1} \left[\frac{\pi_{\xi} \frac{\tau_{p}}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}}{\kappa \left(\frac{1}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1 \right].$$

Thus,

$$Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right) = \frac{\rho}{\sqrt{\tau_{\eta}\tau_{\delta}}} \left(2\frac{\mu\tau_{\eta}\tau_{\delta}}{\gamma\left(1-\rho^{2}\right)}D^{-1}\left[\frac{\pi_{\xi}\frac{\tau_{p}}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}}{\kappa\left(\frac{1}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}+\frac{1}{\tau_{\delta}}\right)}+1\right]\frac{1}{\tau_{\alpha}}+1\right),$$

which implies that $Cov(\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p}) > 0$ if and only if $\rho > 0$. \mathbf{P}_{i}

art (b): Without loss of generality, let us assume
$$\rho > 0$$
. When $\mu = 0$, we have

$$Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)=\frac{\rho}{\sqrt{\tau_{\eta}\tau_{\delta}}}$$

When $\mu > 0$, we have

$$Cov\left(\tilde{\alpha}+\tilde{\eta},\tilde{v}-\tilde{p}\right)>\frac{\rho}{\sqrt{\tau_{\eta}\tau_{\delta}}}$$

Thus, it must be the case that $Cov(\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p})$ is increasing in μ at $\mu = 0$.

Proof of Proposition 5

We first establish that $v_{\theta} \in (0, 1)$ by equation (32). Suppose $v_{\theta} \leq 0$. Then, the left-hand-side (LHS) of (32) is nonpositive, while the right-hand-side (RHS) of (32) is strictly positive. A contradiction. Suppose $v_{\theta} \geq 1$. Then the LHS of (32) is weakly greater than 1, while the RHS of (32) is negative. Again, a contradiction.

We then characterize v_{θ} in a single equation, equation (40) in Proposition 5. Using (32), we can show

$$\tau_{\theta} + \tau_{\varepsilon} + \tau_{p} = \frac{(1-\lambda)v_{\theta}\tau_{\varepsilon} - (1-v_{\theta})\kappa v_{\theta}^{2}}{(1-v_{\theta})\left(1 + \frac{\kappa}{\tau_{\delta}}\right)}.$$
 (B7)

Using (37)–(39), we have

$$\frac{p_{\alpha}}{p_{\theta}} = -\frac{\mu \frac{\tau_{\delta}}{\gamma(1-\rho^2)} \rho \sqrt{\frac{\tau_{\eta}}{\tau_{\delta}}}}{\lambda \frac{v_{\theta} \frac{\tau_{\varepsilon}}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}}}{\kappa \left(v_{\theta}^2 \frac{1}{\tau_{\theta}+\tau_{\varepsilon}+\tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \mu \frac{\tau_{\delta}}{\gamma(1-\rho^2)} v_{\theta}},$$
(B8)

$$\frac{p_{\xi}}{p_{\theta}} = \frac{1}{\lambda \frac{v_{\theta} \frac{\tau_{\varepsilon}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} v_{\theta}}.$$
(B9)

Combining the above two equations with the expression of τ_p in (17), we obtain

$$\tau_p \left(\frac{\mu^2 \rho^2 \tau_\delta \tau_\eta}{\gamma^2 \left(1 - \rho^2\right)^2} \frac{1}{\tau_\alpha} + \frac{1}{\tau_\xi} \right) = \left[\lambda \frac{v_\theta \frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_p}}{\kappa \left(v_\theta^2 \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_p} + \frac{1}{\tau_\delta} \right)} + \mu \frac{\tau_\delta}{\gamma \left(1 - \rho^2\right)} v_\theta \right]^2.$$
(B10)

Inserting the expression of $\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}$ in (B7) into the RHS of (B10) and simplifying, we have

$$\tau_p = \frac{\left[\frac{\lambda\tau_{\varepsilon}(\kappa+\tau_{\delta})(1-\upsilon_{\theta})}{\kappa(\tau_{\varepsilon}(1-\lambda)+\tau_{\delta}\upsilon_{\theta}(1-\upsilon_{\theta}))} + \frac{\mu\tau_{\delta}}{\gamma(1-\rho^2)}\upsilon_{\theta}\right]^2}{\frac{\mu^2\rho^2\tau_{\delta}\tau_{\eta}}{\gamma^2(1-\rho^2)^2}\frac{1}{\tau_{\alpha}} + \frac{1}{\tau_{\xi}}}.$$
(B11)

The RHS of the above equation is the RHS of (40) in Proposition 5. Now, we use (B7) to express τ_p as a function of v_{θ} :

$$\tau_p = \tau_{\delta} \frac{v_{\theta}}{1 - v_{\theta}} \frac{(1 - \lambda) \tau_{\varepsilon} - \kappa v_{\theta} (1 - v_{\theta})}{(\kappa + \tau_{\delta})} - (\tau_{\theta} + \tau_{\varepsilon}), \qquad (B12)$$

which is the LHS of (40) in Proposition 5.

The existence of an equilibrium is obtained by applying the intermediate value theorem to (40). At $v_{\theta} = 0$, the LHS of (40) is negative, while the RHS of (40) is positive. At $v_{\theta} = 1$, the LHS of (40) is ∞ , while the RHS of (40) is finite.

Finally, once we figure out v_{θ} , we can compute the other price coefficients as follows:

$$p_{\theta} = \frac{\lambda \frac{\frac{v_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{\lambda \left[\frac{\frac{1}{\tau_{\varepsilon} + \frac{\tau_{p}}{1 + \frac{\tau_{\varepsilon}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{\lambda \left[\frac{\frac{1}{1 + \frac{1 - \lambda}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1}\right] + \frac{\mu \tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)} \left[1 + \frac{\lambda}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}\right],$$

$$v_{p} = -\frac{\lambda}{1 + \frac{\lambda}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} - \frac{\frac{(1 - \lambda)v_{\theta} \frac{\tau_{p}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \frac{1 - \lambda}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}\right)},$$

$$p_{\alpha} = -\frac{\mu \frac{\gamma \delta}{\gamma \left(1 - \rho^{2}\right)} \rho \sqrt{\frac{\tau_{\eta}}{\tau_{\delta}}}}{\lambda \frac{v_{\theta} \frac{\tau_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{\lambda \frac{v_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \mu \frac{\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)} v_{\theta}}}{p_{\theta}},$$

$$p_{\xi} = \frac{1}{\lambda \frac{v_{\theta}}{\frac{v_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \mu \frac{\tau_{\delta}}{\gamma \left(1 - \rho^{2}\right)} v_{\theta}}}{\rho_{\theta}},$$

$$p_{0} = \frac{\left[\left[\frac{\lambda}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} \right] \frac{\lambda c + (1 - \lambda) \frac{c}{1 + \kappa \left(v_{\theta}^{2} \frac{c}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \frac{1 - \lambda}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} + \lambda c \right]} \right]$$

$$p_{0} = \frac{\lambda c + \left(1 - \lambda\right) \frac{r_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} - v_{p} + 1}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} + \lambda c \right]}{\left(\lambda \left[\frac{-v_{\theta}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} - v_{p} + 1}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + 1 \right] + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} \left(1 - v_{p} \right)}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} \right)$$

$$- \left[\frac{\lambda}{\kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)} + \mu \frac{\tau_{\delta}}{\gamma (1 - \rho^{2})} \right] \frac{\left(1 - \lambda\right) \frac{v_{\theta} \frac{\tau_{p} \frac{1}{\tau_{\theta}}}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} \right)} + \lambda \frac{\lambda \frac{v_{\theta} \frac{\tau_{p} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} \right)} + \lambda \frac{\lambda \frac{v_{\theta} \frac{\tau_{p} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}} \right)$$
and $v_{0} = \frac{\lambda c + (1 - \lambda) \frac{c}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa \left(v_{\theta}^{2} \frac{1}{\tau_{\theta} + \tau_{\varepsilon} + \tau_{p}} + \frac{1}{\tau_{\delta}}\right)}}{1 + \kappa$

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Figure 1: Timeline



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This figure plots price informativeness τ_p against the population size μ of financial traders. The other parameters are: $\tau_{\theta} = \tau_{\delta} = \tau_{\varepsilon} = \tau_{\xi} = \tau_{\alpha} = \tau_{\eta} = 1, \gamma = \kappa = 0.1$, and $\rho = 0.5$.





This figure plots price informativeness τ_p and futures price biases $E(\tilde{v} - \tilde{p})$ against the population size μ of financial traders. In Panels a1 and a2, we set $\tau_{\theta} = \tau_{\delta} = \tau_{\varepsilon} = \tau_{\xi} = \tau_{\alpha} = \tau_{\eta} = 1, \gamma = \kappa = 0.1, \bar{\theta} = 2, c = 1, \text{ and } \rho = 0.5$. In Panels b1 and b2, we set $\tau_{\theta} = \tau_{\varepsilon} = \tau_{\alpha} = 1, \tau_{\delta} = \tau_{\eta} = \tau_{\xi} = 5, \gamma = 0.5, \kappa = 0.1, \bar{\theta} = 2, c = 1, \bar{\xi} = 0, \text{ and } \rho = 0.5$.



Figure 4: Operation Profits and Welfare

This figure plots price informativeness τ_p , operating profits $E[\tilde{v}x_i - C(x_i)]$, commodity producers' welfare CE_p , and financial traders' welfare CE_F against the mass μ of financial traders. The other parameters are: $\tau_{\theta} = \tau_{\delta} = \tau_{\varepsilon} = \tau_{\xi} = \tau_{\eta} = 1$, $\gamma = \kappa = 0.1$, $\bar{\theta} = 2$, c = 1, $\bar{\xi} = 0$, and $\rho = 0.5$.



Figure 5: Implications of Financialization in Extended Economies

This figure plots the implications of financialization in economies populated by two groups of commodity producers. A mass λ of commodity producers trade futures, while the remaining mass 1- λ of commodity producers do not. The parameter values are: $\tau_{\theta} = \tau_{\varepsilon} = \tau_{\alpha} = 1$, $\tau_{\delta} = \tau_{\eta} = \tau_{\xi} = 5$, $\gamma = 0.5$, $\kappa = 0.1$, $\bar{\theta} = 2$, c = 1, $\bar{\xi} = 0$, $\rho = 0.5$, and $\lambda = 0.8$.