Who Cares More? Allocation with Diverse Preference Intensities

Pietro Ortoleva Evgenii Safonov Leeat Yariv

Idea

Common **allocation problems**:

Public housing

Medical Appointments

School choice

Restaurant lines

Often:

Same **Ordinal** Pref

Different **Intensity**

No transfers

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Often:

Same **Ordinal** Pref

Different **Intensity**

No transfers

Our Question

What is **optimal way** for a social planner to allocate?

Best **incentive compatible** mechanism?

Main Insights

With **observable** preferences intensities:

- Generally: give best items to those who want them most
- But: sometimes **involves a lottery**

  Chance of very desirable and not desirable

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With **observable** preferences intensities:

- Generally: give best items to those who want them most
- But: sometimes **involves a lottery**
  Chance of very desirable and not desirable

With **Unobservable** preference intensities:

- Optimal incentive-compatible mechanism: full separation
- Always involves lotteries
- **May coincide with First-Best** (with lotteries)
- May involve **artificial disposal** of services

Plan for today

- Lit (brief)
- Framework
- First-best: observable intensities
- Second-best: unobservable intensities
- N types
- Market alternative
- Variants

Literature

- Matching with incomplete information
  - Decentralized (Static and Dynamic)
    Liu *et al* 14, Agranov *et al.* 20, Ferdowsian Niederle Yariv 20
  - Centralized (Static and Dynamic)
    Fernandez Rudov Yariv 20, Leshno 19
- Different preference intensities
    Abdulkadiroglu Che Yasuda 11, 15
- Timing as a screening device
    Dimakopoulos Heller 18, Ely Szydlowski 17, Leshno 17
- Screening time-inconsistent agents
    Della Vigna Malmendier 06, Eliaz Spiegler 06
- Screening risk-averse agents (with prices)
    Rothschild Stiglitz 76, Maskin Riley 84, …
- Disposal can help selection:
    Alatas *et al* 06, Austen-Smith Banks 00

framework

A Simple Model of Service Allocation

- Continuum of goods: $[0, T]$
  - Public housing provided at different times
  - Schools varying in quality
  - Doctor appointments varying in physician's expertise or dates

- $\diamond$ denotes the outside option

- Supply $f(t)$
  - Continuous density $f(t)$, CDF F
  - Support $[0, T]$

Agents

- 2 types: P and I, masses $\mu_P, \mu_I > 0$
- Each consumes single indivisible good
- Same **ordinal** preferences: $u_k(\cdot)$ **decreasing** on $[0, T]$
- Difference **Cardinal** ones:
$$\frac{u_P''(t)}{u_P'(t)} > \frac{u_I''(t)}{u_I'(t)} \quad \text{for all } t \in [0, T]$$
  Ranked in terms of curvature:
  - I **care more** about getting high quality
  - P more risk-averse than I
- u can have any shape, as long as monotone and ranked
- $u_k(0) = 1$, $u_k(T) \geqslant u_k(\diamond) = 0$

Examples

1. Heterogenous goods ranked identically
    - Colleges and U.S. News and World Report ranking
    - CRRA or CARA utilities with different parameters ranked

2. Identical goods with different **delivery date**
    - Many examples

      Public housing, Medical Appointments, Restaurants
    - Normalize good "value" at 1
    - **Patient** (P) discount rate r_P : $u_P(t) = e^{-r_P t}$

      Impatient (I) discount rate r_I: $u_I(t) = e^{-r_I t}$

      $0 < r_P < r_I$
    - **Lead example for today**

Lotteries, Allocations, Welfare

- **Allocation** $q = (q_P, q_I)$, where q_k is density on $[0, T] \cup \{\diamond\}$
- **Feasibility**: $\mu_P q_P(t) + \mu_I q_I(t) \leqslant f(t)$
- **Assume Sufficient supply** (today): $\mu_P + \mu_I \leqslant F(T)$
- **Expected payoff**: $V_k(q_k) = \int_0^T u_k(t) q_k(t) dt \quad k \in \{P, I\}$
- **Welfare**: $W(q) = \mu_P V_P(q_P) + \mu_I V_I(q_I)$
  - Here: equal weights on all types
  - In paper: arbitrary weight allowing under-weighting P-agents

A Note on Storage

- Some applications (e.g., public housing) allow storage
- If Q is CDF of lottery :

  Feasibility with storage: $\mu_P Q_P(t) + \mu_I Q_I(t) \leqslant F(t)$

- Results is the same: storage never used

first-best

The First-Best Solution

- First, suppose utilities/types are observable
- Relevant for some applications
  - Urgency of appointment seekers
  - BMI of individuals waiting for food
- If timing allocation:

Do you give goods to impatient first?

Computing the First Best

- Benefit of allocating to P relative to I at time t :

$$g(t) = e^{-r_P t} - e^{-r_I t}$$

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  - Give to I when $g(t)$ is **low**
  - Give to P when $g(t)$ is **high**
- $g(0) = 0 \Rightarrow$ give to I initially

Computing the First Best

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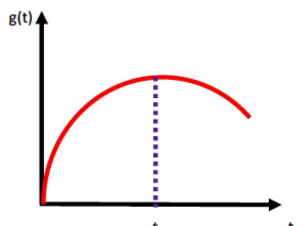
$$g(t) = e^{-r_P t} - e^{-r_I t}$$

- Want to
  - Give to I when $g(t)$ is **low**
  - Give to P when $g(t)$ is **high**
- $g(0) = 0 \Rightarrow$ give to I initially
- But $g(t)$ is **single-peaked**



Computing the First Best

- Let $\overline{T}$ be minimal time to service everyone:

$$\overline{T} = \inf\{t \mid F(t) \geqslant \mu_P + \mu_I\}$$

- Let $\overline{t}$ be minimal time to service only I:

$$\overline{t} = \inf\{t \mid F(t) \geqslant \mu_I\}$$

When to Service all I-agents First?

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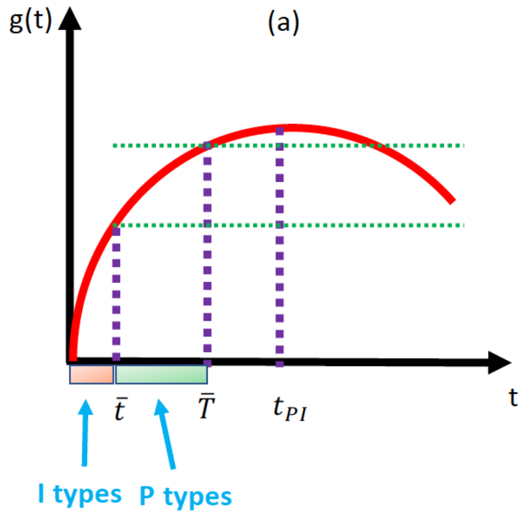
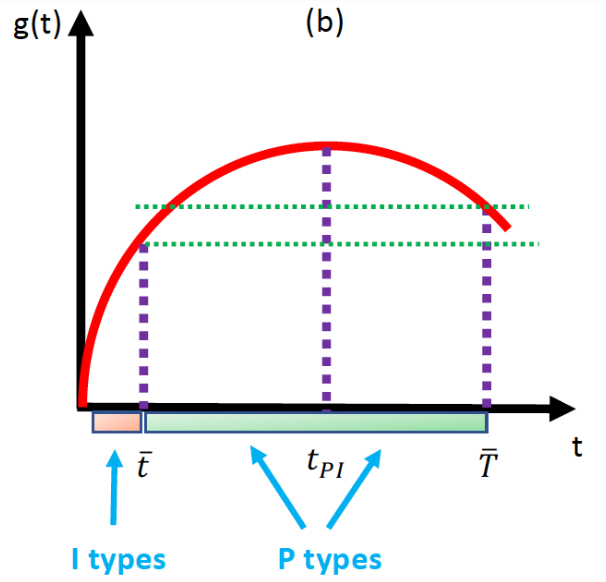
- When costs are low up to $\bar{t}$: $g(\bar{t}) \leqslant g(\bar{T})$



When to Service all I-type Agents First?



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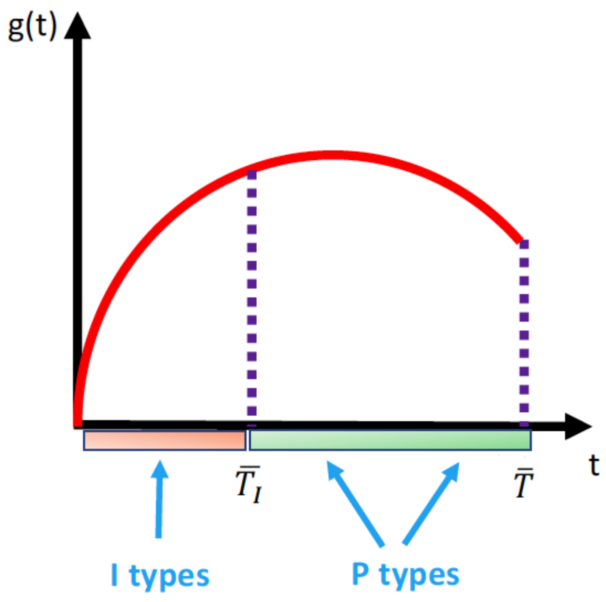

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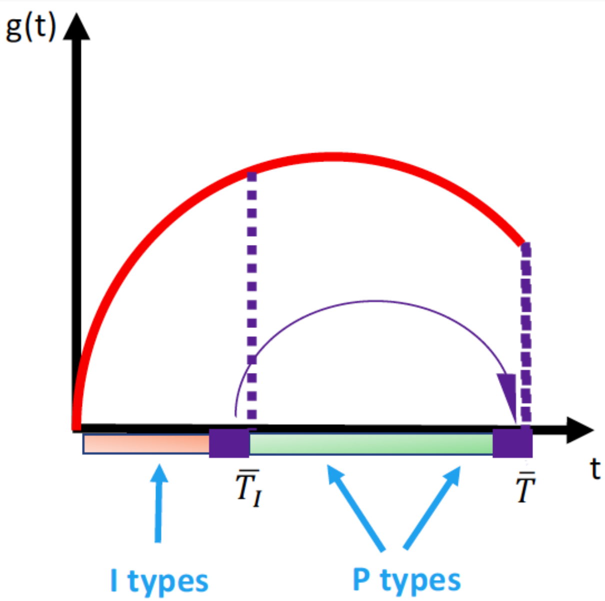
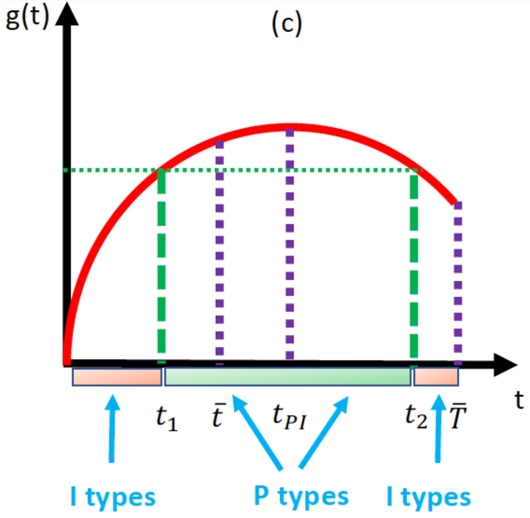


When to Serve I-agents with a "Lottery"?



Lotteries in the First-best Allocation

Proposition

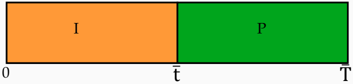
First best exists and is unique. Moreover,

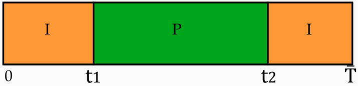
- *when* $g(\bar{t}) \leqslant g(\bar{T})$*, First-Best gives*
  - $[0, \bar{t})$ *to* I*;*
  - $(\bar{t}, \bar{T}]$ *to* P*;*

- *when* $g(\bar{t}) > g(\bar{T})$*, First-Best gives*
  - $[0, t_1) \cup (t_2, \bar{T}]$ *to* I
  - $[t_1, t_2]$ *to* P

  where t_1 *and* t_2 *are unique and identified by*

$$g(t_1) = g(t_2) \quad \text{and} \quad F(t_2) - F(t_1) = \mu_P.$$





Intuition: Risk Attitudes

- Expected Discounted Ut. ⇒ **risk seeking over time lotteries**
- Compare $t = 2$ for sure vs. $t = 1$ or $t = 3$ with equal chances

$$\begin{aligned} \beta^2 u(x) &< \frac{1}{2}\beta^1 u(x) + \frac{1}{2}\beta^3 u(x) \\ \beta^2 &< \frac{1}{2}\beta^1 + \frac{1}{2}\beta^3 \end{aligned}$$

- β^t is **convex** → **risk seeking**

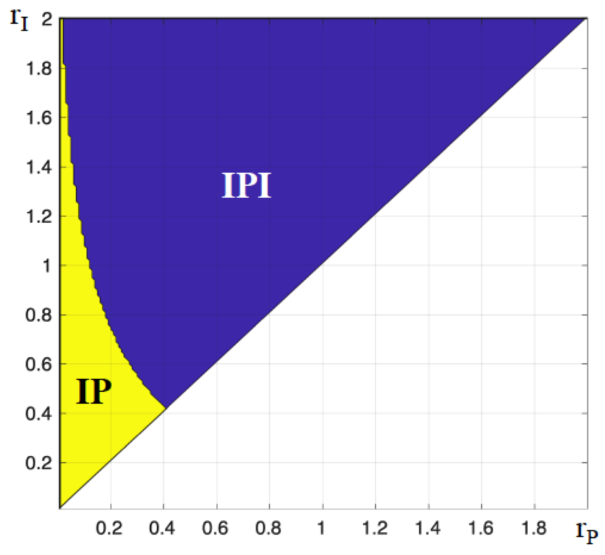
  [Dejarnette Dillenberger Gottlieb Ortoleva 2020]

- More discounting ⇒ more risk seeking

- I strictly more risk seeking than P
- I benefit from lottery that places high probability on early

The First-best Allocation

$\mu_I = \mu_P = 1/2$, uniform supply



incentive-compatible mechanism

Incentive-Compatible Mechanism

- What if intensity is unobserved?
- Relevant for many settings:
  - Family circumstances of public-housing customers
  - Urgency in need of attention in scheduling settings
  - Restaurants..
- One obvious mechanism: give randomly
- **Can I do better**?

Mechanism Designer Problem (Formal Statement)

$$\max_{q(t)\geqslant 0}\left[\sum_{k=P,I}\mu_k\int_0^{\infty}u_k(t)q_k(t)dt\right]$$

such that

$$IC_{kj} \quad : \quad \int_0^{\infty}u_k(t)q_k(t)dt\geqslant\int_0^{\infty}u_k(t)q_j(t)dt \quad \forall k,j=P,I$$

$$\text{Feasibility} \quad : \quad \sum_{k=P,I}\mu_k q_k(t)\leqslant f(t) \quad \forall x\in[0,\infty)$$

Can it be First Best?

- When I are serviced before P:
    - P want to imitate I
    - Cannot be incentive compatible

    $\implies$ SB $\neq$ FB

Can it be First Best?

- When I are serviced before P:
  - P want to imitate I
  - Cannot be incentive compatible

    $\implies \text{SB} \neq \text{FB}$

- When I-agents receive a lottery?
  - Not obvious any more
  - Could it be that FB is incentive compatible?
  - Could it be that SB = FB?

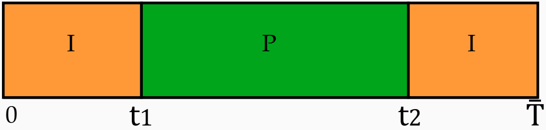
Second-best = First-best

Proposition

For a positive measure of discount factors, the first-best allocation is incentive compatible.

Intuition

- Suppose FB has lottery: type IPI
  - otherwise no hope



- I served in $[0, t_1) \cup (t_2, \bar{T}]$, P served in $[t_1, t_2]$
- I really care about early service

  $\Rightarrow$ more willing to take risk, prefer lottery
- P doesn't mind waiting

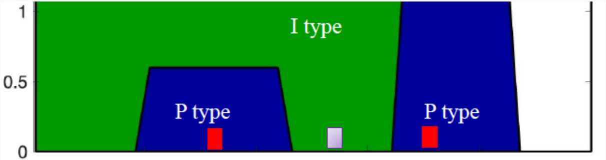
  $\Rightarrow$ less willing to take risks, prefer $[t_1, t_2]$

Second-best Allocation more Generally

- If all type-k served before all type-m
  $\Rightarrow$ type-m want to imitate type-k
- Therefore, we cannot have ‘dominance’
- **Need** lotteries
- What can we say?
- Let’s proceed in steps
- [Note: sloppy formal statements in slides]

Step 1: No "Inverted Spreads"

Definition: **Inverted Spread** if "some I served between some P," or some P not served at all



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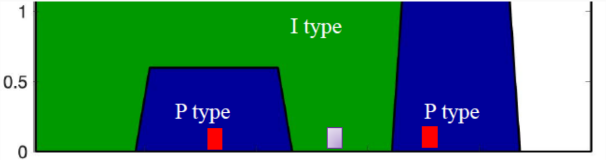
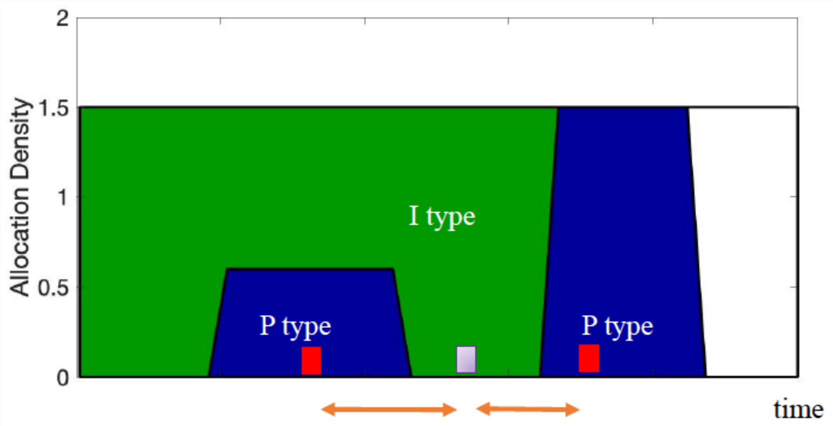


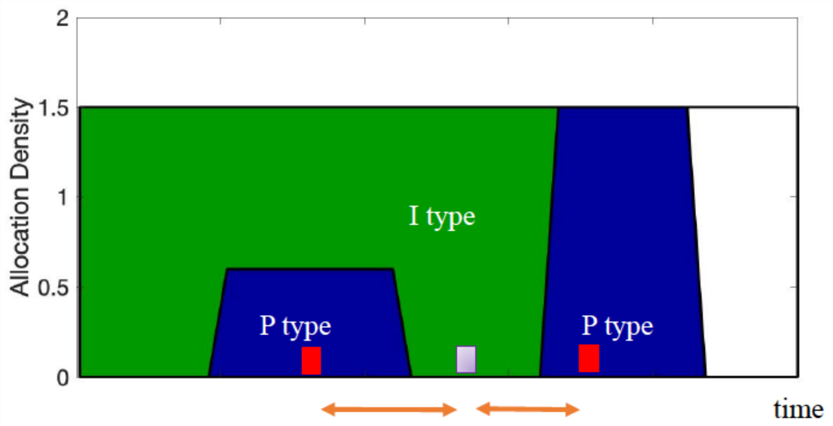
Lemma

No Solution of the MD problem exhibits Inverted spread.

Intuition of No Inverted Spreads



Intuition of No Inverted Spreads



- P indifferent between δ_t and $\lambda\delta_{t'} + (1-\lambda)\delta_{t''}$
  $\Rightarrow$ I strictly prefers lottery
- Trade increases welfare, preserves incentive constraints

Corollaries

Corollary

Full separation: in each t, either all to P or all to I.

Strong form of separation

Corollaries

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All Ps receive a good, that is, $q_P(\diamond) = 0$.

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Full separation: in each t, either all to P or all to I.

Strong form of separation

Corollary

All Ps receive a good, that is, $q_P(\diamond) = 0$.

Corollary

In the solution of the MD problem, IC_{IP} and IC_{PI} cannot be both binding.

- If both bind, both types indifferent between both allocations
- Then, also indifferent with any convex combination
- Thus: convex comb incentive compatible and same welfare
- Must also be solution—but not fully separating!

Step 2: P agents served in one block

Lemma

In all solutions of the MD problem, there is an interval $[x_1, x_2]$ such that all supply given to P agents, who are only served there.

Step 2: P agents served in one block

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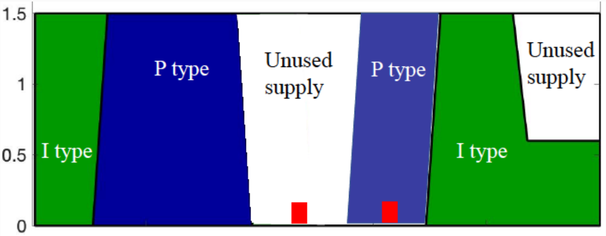
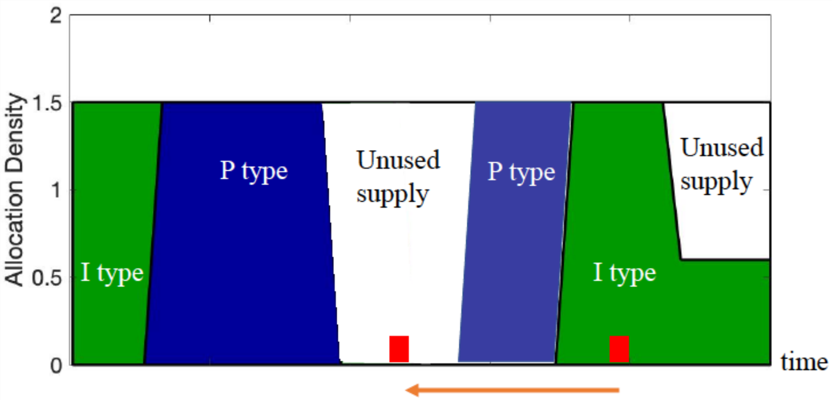
That is, we don't have



Intuition



- We know we can have both IC binding
- If IC_{PI} does not bind: shift small mass of I forward
- If IC_{IP} does not bind: shift small mass of P forward

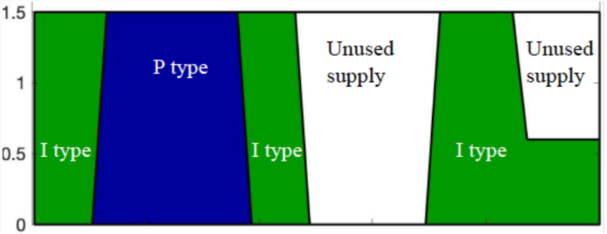
Step 3: Only disposal is denial of goods to I

Definition: An allocation exhibits **disposal** if some types do not receive goods while some are available, or unused higher quality

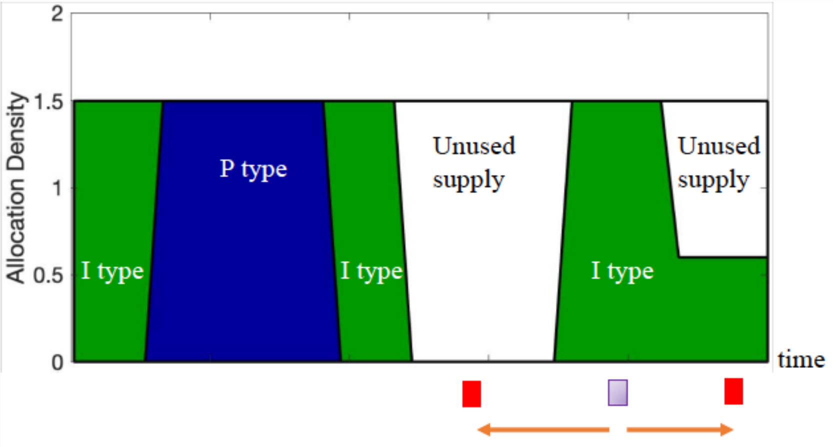
Lemma

"Only disposal" is $q_I(\diamond) > 0$.

That is, don't have:



Intuition



- Take mass in later usage
- Spread in a way that keeps P indifferent: maintains IC
- $\Rightarrow$ I strictly better off

Implications for the Second-best Allocation

- Results above together $\Longrightarrow$
  - P: single time block $[x_1, x_2]$
  - I: two blocks $[0, x_1], [x_2, x_3] + \diamond$

- Feasibility: $F(x_2) - F(x_1) = \mu_P \;\Rightarrow\; x_2 = x_2(x_1)$

- Two degrees of freedom remain:
  - x_1: controls distribution of early service between agent types
  - x_3: controls probability of service for I

- Transform complex problem into simple 2 dimensional problem

Solution of the MD-problem

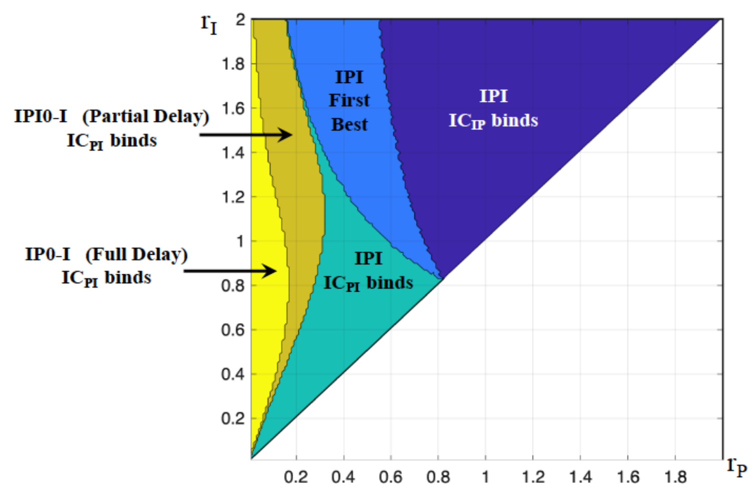
Proposition

The second-best allocation is (gen.) unique and fully separating. Moreover, there exist x_1, x_2, x_3 it such that:

- q_P *has support* $[x_1, x_2]$*;*
- q_I *has support* $[0, x_1] \cup [x_2, x_3]$ *and in some cases* $\diamond$*;*

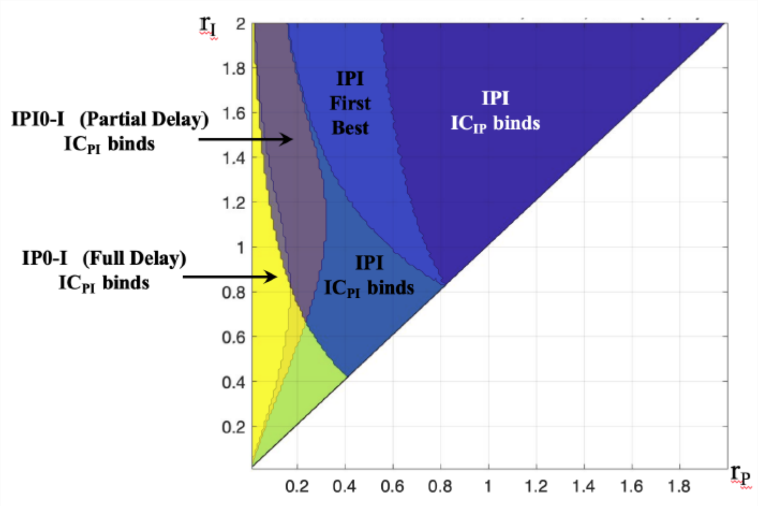
- **Full separation**: each type of good to different type of agent
- All solutions of the form *IPI*
- Always a lottery for *I*
- *P* served in one block
- Block for *P* ‘in between’ *I*
- Lottery for *I* may involve not receiving a good

All Allocations



Uniform distribution, equal masses

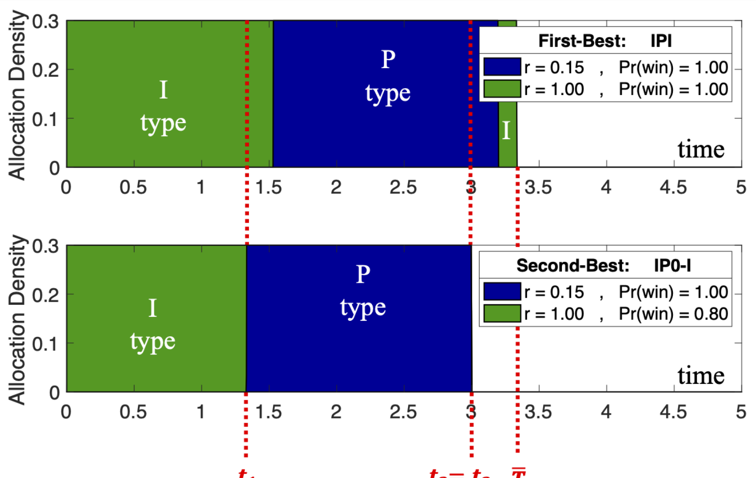
Comparison with First-best Allocation



Uniform distribution, equal masses, $T = 5$

Why disposal?

- We have seen sometimes disposal of service
- Why? Take a case in which IC_{PI} binds
- How to solve it? Cheap way: worsen q_I



Welfare of second-best: Who Gains?

Benchmark: uniform allocation (pooling)

$$\begin{aligned} &\text{First-best = second-best} && \implies && P\uparrow, I\uparrow \\ &IC_{IP} \text{ binds} && \implies && P\uparrow, I = \\ &IC_{PI} \text{ binds, no disposal} && \implies && P =, I\uparrow \\ &IC_{PI} \text{ binds, disposal} && \implies && P\downarrow, I\uparrow \end{aligned}$$

If your IC constraint binds, welfare not higher than pooling n types

Proposition

The first-best exhibits 1) no inverted spread and 2) no disposal.

In a sense, “complete” characterization.

N Types

Proposition

The first-best exhibits 1) no inverted spread and 2) no disposal.

In a sense, "complete" characterization.

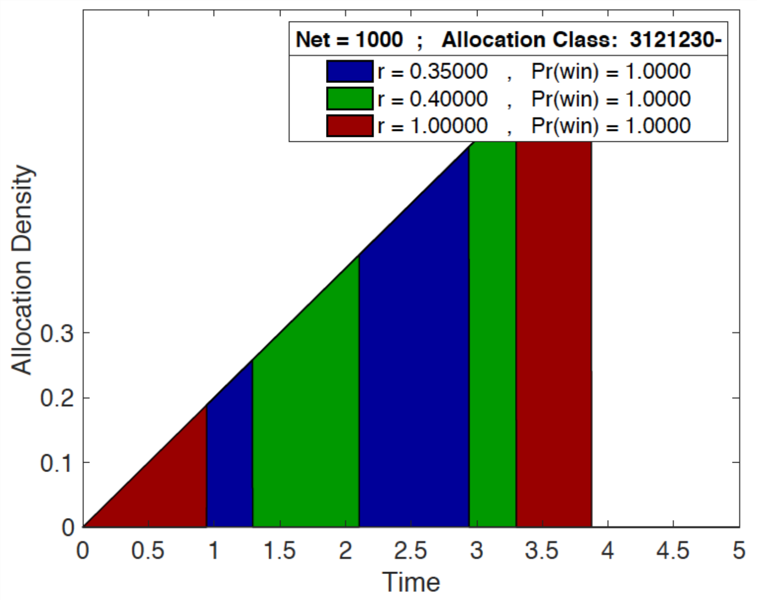
Proposition

With N types, a solution of the MD problem exists and:

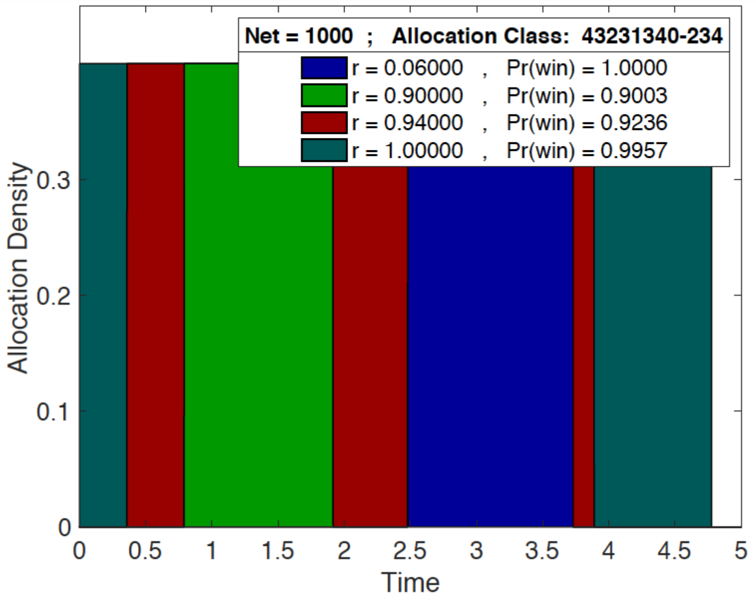
- *is unique;*
- *exhibits "full separation;"*
- *the graph of binding IC constraints has no directed cycles.*

In Second-Best can get Inverted spread!

N Types – Example 1



N Types – Example 2



N Types – Partial improvement

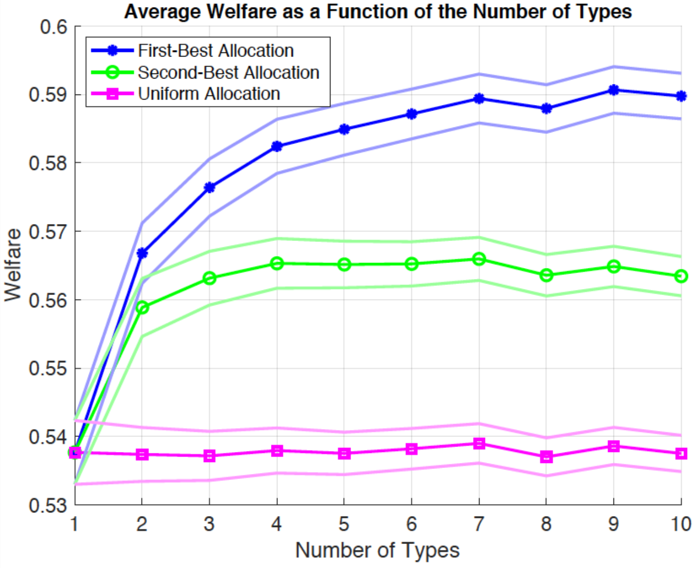
- With N, we can think about 'third-best'
- IC mechanism that is not second-best but still improves
- E.g.: divide you types in 2 groups, and 'pool in groups'

- Still better than general pooling
- You can show: **pooling is worst IC allocation**

Welfare as a Function of Number of Types

- N agents, discount rates distributed $U[0, 1]$
- Simulate resulting welfare from first-best, second-best, and uniform (pooling) allocation

Uniform supply, equal masses



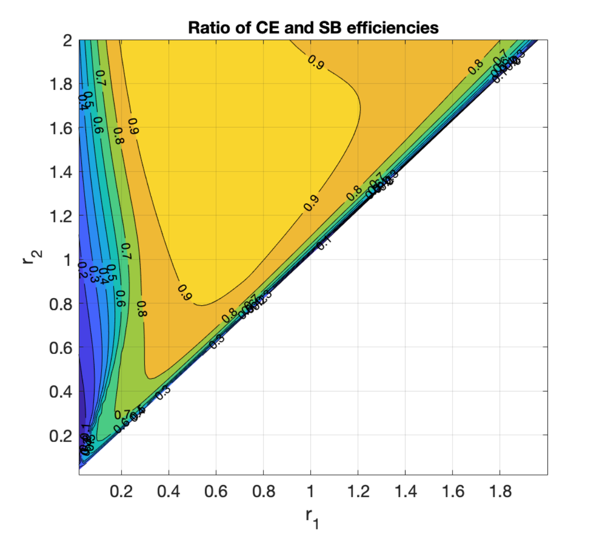
a market solution

A Market for Lotteries

- Instead of mechanism, market
- Endow all agents with equal shares of supply and allow trade
- Find competitive equilibrium: price, demand functions
- Reminiscent of Hylland and Zeckhauser 79
- Equilibrium is **unique**
- Can solve also for N
- **No disposal and no inverted spread**! For any N!
- First Welfare Theorem $\Rightarrow$ outcome Pareto-efficient
- But: need not coincide with SB—generically it won't

Efficiency of Market Outcomes relative to SB

Uniform sufficient supply, $\mu_P = \mu_I = 1/2$, consider $\frac{W^{CE} - W^U}{W^{SB} - W^U}$



variants

Extension 1 – Storage

- Suppose goods can be stored
- Or: damage quality
- Relevant for some applications: housing, etc.
- Result: storage never used

Extension 2 – Bounds to Disposal

- Suppose all agents must get a good if available
- Or even: no disposal allowed
- Solution is similar:
  - Again IPI
  - Use disposal/damaging as much as possible

conclusion

Conclusions

- Allocation problem with:
    - Same **ordinal** ranking
    - But: different **cardinal** preference/intensities
    - Focus on case when well ordered
- First-Best may involve lotteries
- Incentive Compatible Mechanism
    - Easy to characterize
    - **May coincide with First-Best**
    - **May involve disposal**
- Also solve for market solution: different

additional slides

Related Literature

- Dynamic allocation problems: Baccara Lee Yariv 19, Bloch 17, etc.
- Link between discounting and risk attitudes: Dejarnette Dillenberger Gottlieb Ortoleva 19
- Using timing as a screening device: Dimakopoulos Heller 18, Ely Szydlowski 17, Leshno 19
- Screening of time-inconsistent agents: Della Vigna Malmendier 06, Eliaz Spiegler 06
- Adding costs can help with selection: Alatas *et al.* 06

EndExpansion

Why and When to Dispose

- A marginal tradeoff:

$$\underbrace{g(t_2) - g(t_1)}_{\text{welfare increase}} = \underbrace{\lambda \left(\frac{1}{\mu_P} + \frac{1}{\mu_I} \right) \left(e^{-r_P t_1} - e^{-r_P t_2} \right)}_{\text{cost of incentive constraint}}$$

Why and When to Dispose

- A marginal tradeoff:

$$\underbrace{g(t_2) - g(t_1)}_{\text{welfare increase}} = \underbrace{\lambda\left(\frac{1}{\mu_P} + \frac{1}{\mu_I}\right)\left(e^{-r_P t_1} - e^{-r_P t_2}\right)}_{\text{cost of incentive constraint}}$$

- $h(t)$ – net benefit of servicing I-agents at t relative to delay
- The delay tradeoff:

$$h(t) = \underbrace{e^{-r_I t}}_{\text{welfare increase}} - \underbrace{\lambda \frac{1}{\mu_I} e^{-r_P t}}_{\text{cost of incentive constraint}}$$

Second-best Allocation with Distorted Lotteries

- **Suppose IC_{IP} is violated in the FB**

Second-best Allocation with Distorted Lotteries

- **Suppose IC_{IP} is violated in the FB**
- $\Longrightarrow$ Need to compensate further I-agents
- $\Longrightarrow$ No point in delaying service for I-agents
- $\Longrightarrow$(Proposition 3a) **Generate lottery in which I-agents are serviced for a longer period initially relative to FB**

- **Suppose IC_{PI} is violated in the FB**
- Recall: cannot have both ICs binding

Second-best Allocation with Delay

- **Suppose IC_{PI} is violated in the FB**
- Recall: cannot have both *IC*s binding
- $\implies$ Need to compensate further *P*-agents
- $\implies$ Can generate lottery in which *P*-agents are serviced sooner relative to FB
- (Proposition 3b) **Could also generate delay for *I*-agents, possibly not serving some at all**

Efficiency of SB relative to Market Outcomes

Uniform sufficient supply, $\mu_P = \mu_I = 1/2$, consider $\frac{W^{SB} - W^{CE}}{W^{FB} - W^{CE}}$