Firms' Choices of Wage-Setting Protocols in the Presence of Minimum Wages*

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Abstract

We study the formation of wages in a frictional search market where firms can choose either to bargain with workers or post non-negotiable wage offers. Workers can secure wage increases for themselves by engaging in on-the-job search and either moving to firms that offer higher wages or, when possible, leveraging an outside offer into a higher wage at the current firm. We characterize the optimal wage posting strategy of non-negotiating firms and how this decision is influenced by the presence of renegotiating firms. We quantitatively examine the model's unique implications for efficiency, wage dispersion, and worker welfare by estimating it using data on the wages and employment spells of low-skill workers in the United States. In the estimated steady state of the model, we find that more than 10% of job acceptance decisions made while on the job are socially sub-optimal. We also find that, relative to a benchmark case without renegotiation, the presence of even a small number of these firms increases the wage dispersion attributable to search frictions, deflates wages, and reduces worker welfare. Moving to a general equilibrium setting, we use the estimated model to study the impact of a minimum wage increase on firm bargaining strategies and worker outcomes. Our key finding is that binding minimum wages lead to an increase in the equilibrium fraction of renegotiating firms which, relative to a counterfactual in which this fraction is fixed, significantly dampens the reduction in wage dispersion and gains in worker welfare that can typically be achieved with moderate minimum wage increases. Indeed, the presence of endogenous bargaining strategies reverses the sign of the average welfare effect of a $15 minimum wage from positive to negative.

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1 Introduction

Using data collected from a sample of recent hires, Hall and Krueger (2012) show that in setting initial compensation, some firms specify a fixed, non-negotiable wage or salary, while other firms negotiate with the new employee over compensation levels. Approximately one-third of sample members report having bargained with their employers at the time of their initial hiring, with bargaining more likely to have occurred for highly-educated workers. In these cases, they found that their current employers had often learned their compensation in earlier jobs before making the compensation offer in the current job.

These findings suggest that employers may use different strategies when hiring workers, with some essentially following a wage-posting paradigm, while others actively engage in bargaining. Although Hall and Krueger find evidence that there is a systematic relationship between the characteristics of the worker and the propensity to bargain, within any class of workers they find cases in which wages were bargained over and others in which they were not. This heterogeneity in wage determination methods has not been examined within the vast majority of partial and general equilibrium models of labor market search. In models of wage posting, employers make take-it-or-leave-it offers to applicants, which the applicant either accepts or rejects. Perhaps the most well-known models of wage posting are Albrecht and Axell (1984) and Burdett and Mortensen (1998). In these models, firms offer fixed wages to all applicants they encounter, and it is often assumed that all applicants are equally productive. In the Burdett and Mortensen model, workers of homogeneous productivity are offered different wages by ex ante identical firms. Their model produces an equilibrium wage offer distribution and steady state wage distribution that are nondegenerate, even though all workers and firms are assumed to be ex ante identical.[1]

Most wage bargaining models estimated using individual-level data are based on an assumption of ex ante heterogeneity in worker and/or firm productivities. Most typically, some sort of cooperative bargaining protocol is assumed, such as Nash bargaining or simply surplus division. In the cases in which on-the-job (OTJ) search is introduced, assumptions are made regarding the amount of information available to the worker and firm during the bargaining process. In one extreme case, firms are assumed to know not only the worker's current (or potential) productivity at their firm, but also the value of the worker's best alternative productivity match (e.g., Postel-Vinay and Robin (2002), Dey and Flinn (2005), Cahuc et al. (2006)). An alternative assumption is that employers either do not know the employee's outside option or that they simply don't respond to such information when making an offer.

In this paper we consider a world in which there exists a positive measure of firms that rene-

[1] These assumptions are relaxed when taking the model to data, as is they imply a monotone increasing density function on the support of the wage distribution. For the estimation of such a model, see Bontemps et al. (2000))

gotiate and a positive measure of firms that do not, with the proportion of firms of both types determined within an equilibrium model of vacancy posting. Firms that renegotiate have informational advantages with respect to those that don't, but also agree to a surplus division policy that commits them to increasing wages whenever the worker's outside option improves, even if this option is dominated by the current employment contract. Wage-posting firms issue wage contracts that are functions of the employee's productivity at the firm, and are fixed over time.[2] Wage-posting firms are cognizant of the existence of renegotiating firms, and make conditional (on productivity) strategic wage offers that are functions of the measure of renegotiating firms in the labor market. In this environment, an employee of a non-renegotiating (N) firm who encounters a renegotiating (R) firm may leave the current employer even though their productivity is lower at the type R firm. In this world, there exists inefficient mobility, a phenomenon that does not exist if all firms are type N or if all firms are type R.

We assume that the vacancy posting costs are different for jobs at R- or N-type firms. One rationale for this assumption is that R-type firms must invest in verifying an applicant's current outside option, including what their productivity level is if they are currently employed by another firm and what type of firm their current employer is (i.e., R or N). Since we assume that all firms are identical ex ante, we characterize the unique equilibrium in which firms are indifferent between posting an R- or an N-type vacancy, and where the expected value of a vacancy of either type is 0. We denote the equilibrium proportion of type R firms by p_R. One of our goals is to determine how the mixture of firm types is affected by changes in the labor market environment. Our policy application will be to assess the impact of minimum wages on labor market outcomes. The proportion of firms of type R when the minimum wage is set at m is given by $p_R(m)$, and our interest is in determining the equilibrium effects of a minimum wage change through this channel. Our estimates indicate that at realistic values of a (binding) minimum m_0, $\partial p_R(m_0)/\partial m > 0$. This type of equilibrium impact of changes in the minimum wage has not previously been investigated, and we find that this mechanism plays a significant role within the general equilibrium framework that we develop.

There are a small set of papers that consider firms' choices of method of compensation in an equilibrium setting. The two papers most similar to ours are Postel-Vinay and Robin (2004) and Doniger (2015). In Postel-Vinay and Robin (2004), the authors assume an absolutely continuous distribution of firm productivities z given by Γ and workers of homogeneous productivity, with the marginal product of labor at a firm of type z equal to z. Employees search on and off the job, and the rate of contact with potential employers is a function of the search effort that they supply. Firms that commit to matching outside offers have the benefit of increasing the likelihood

[2] These wages could change if the model allowed for productivity shocks over the course of the job spell, but we abstract from these in this paper.

of recruiting and retaining employees, conditional on search effort, but this policy also gives rise to a moral hazard problem as employees devote increased effort to generating offers to which the firm must respond. Under certain restrictions on primitives, they find a separating equilibrium in which firms with productivities in the set $z > z^*$ agree to renegotiation and those with $z \leq z^*$ refuse. The intuition behind this result is that good firms have a higher valuation of a filled vacancy and thus adopt a negotiating strategy that is more likely to lead to employee retention, even if the value of retaining an employee will be reduced through the increased number of offers generated under this policy.

Doniger (2015) abstracts from the endogenous effort/moral hazard problem, but otherwise largely follows the modeling assumptions of Postel-Vinay and Robin (2004) with the exception of assuming that it is costly to the firm to be able to verify and respond to job offers. This is modeled as a constant flow cost to the firm, with the net flow productivity reduced by $c > 0$. Once again, since highly productive firms have a larger value of a filled vacancy, they are the ones that absorb this cost, and the equilibrium is of the separating type as in Postel-Vinay and Robin (2004). The assumptions required for this result are, of course, different in the two papers since the mechanism generating the differential valuations of the two types of jobs across firm types are different. Doniger emphasizes the point that there is no inefficient mobility in this case. Since firms with some productivity level greater than z^* are the renegotiating type, all competitions between firms who renegotiate will result in the worker accepting the offer of the highest productivity firm. Among the wage-posting firms, the wage offer is increasing in the firm's productivity value z, so that any competition between them results in the highest z firm acquiring the worker. Finally, any competition between a wage-posting and renegotiating firm will result in the renegotiating firm winning since it can respond to any outside offer and it has, given the properties of the separating equilibrium, greater productivity than the competing wage-posting firm. This will also be true in the Postel-Vinay and Robin (2004) framework under the conditions required to produce a separating equilibrium of this type.

Our modeling framework differs along a number of dimensions from the ones employed in these two papers, for reasons related to the type of data utilized and the minimum wage application that is the focus of our empirical work. Unlike these two papers, we use U.S. data taken from the Survey of Income and Program Participation (SIPP) that includes very limited information on firm characteristics. We assume homogeneous firms, but allow an individual's productivity at any firm to be the product of a time-invariant individual productivity parameter a and a match-specific productivity draw θ, with the individual's total productivity at a firm given by $y = a\theta$. The distribution of a, F_a, can be estimated non-parametrically in the baseline specification of the model, and we assume that θ draws are independently and identically distributed across employees and firms according to a parametric F_θ. It is necessary to allow for variability in individual productivity levels since we expect the impact of large minimum wage changes to have notably different impacts across the ability distribution.

The results we obtain are also relevant to recent policy proposals and laws not related to minimum wages. For example, in order to confront gender and racial discrimination, various states and municipalities have instituted laws limiting a potential employer's ability to ask and receive information on an applicant's salary history. For example, Bill S.2119 passed by the Senate of the State of Massachusetts and signed by the the Governor in August 2016 requires that potential employers refrain from asking for salary information from an applicant, and prohibit a current employer from providing such information to another firm. The objective of the law is to break a cycle of discriminatory practices by firms in setting wages for women or minorities. Within our framework, this law essentially bans the creation of a type R vacancy, and the effects of such a law can be determined using our model estimates, with or without a binding minimum wage.

There is an argument to be made for such laws on equity grounds, whether or not systematic discrimination takes place within a frictional labor market.[3] Consider two individuals of equal productivity y at a type R firm. The wage of each individual will depend on the best outside option that she has had during the current employment spell. As a result, their wages will differ as long as the values of their outside options differ. Instead, if both are employed at a type N firm, their wages will be identical. Thus, from a normative point of view, having a greater proportion of firms of type N may be considered more equitable in that wages are not a function of the sample path of offers over the current employment spell, which has no effect on the individuals' current productivities.[4,5]

In Section 2 we describe the model and present some results. Section 3 introduces a minimum wage into the model. In Section 4 we discuss our choice of data that will inform an empirically plausible parameterization of the model, which we arrive at through a minimum distance estimation procedure. Section 5 describes and presents the results of this procedure. The resulting estimates allow us to quantitatively explore some implications of the model in steady state and partial equilibrium. In Section 6 we examine, in a general equilibrium setting, the implications of the model for the labor market's response to a minimum wage increase. Section 7 concludes.

[3]For a consideration of the impact of discrimination on labor market outcomes in a model with search frictions, see Flabbi (2010).

[4]Since individuals are assumed to be risk neutral, we are not necessarily arguing that the agents themselves would have a preference for having more type N firms in the labor market environment. Searchers' attitudes to the mix of R- and N-type firms will be a function of the primitive parameter vector Ω.

[5]In a model with endogenous search intensity (e.g., Postel-Vinay and Robin (2004)) or on-the-job human capital investment (e.g., Flinn et al. (2017)), the history will impact the likelihood of a given productivity level on the current job. However, in the model estimated here neither of these phenomena are present.

2 Model

2.1 Setup and Preliminaries

The model is set in continuous time, with all agents on the supply side of the market distinguished by their (time-invariant) ability, a. Upon meeting any firm in the market, individuals draw a match-specific productivity realization, θ. The random variables a and θ are distributed on a subset of the positive real line with c.d.f.s F_a and F_θ, respectively. It is necessary for us to assume that F_θ is continuous on its domain, while we do not, in principle, require any such restriction on F_a. The value θ, as well as ability a, are perfectly observed by both (potential) employees and firms, and match-productivity realizations θ are independently distributed across employee-employer pairs. An employee with ability a at a firm with match θ produces a flow output $a\theta$, while an unemployed worker of ability a enjoys flow utility ab, where b is a scalar of unrestricted sign.

Firms in the market are *ex ante* homogeneous except for the manner in which they interact with potential or current employees in setting wages, the only utility-yielding characteristic of the employment contract to the worker. The firm's bargaining type is indicated by j, with $j \in \{N, R\}$. A type R firm is a "renegotiator," and this type of employer bargains over wage contracts with employees at the beginning and over the course of their tenure at the firm. A type N firm is a "non-renegotiator," which is a firm that makes a one-time take-it-or-leave-it wage offer to a potential employee based upon the individual's ability, a, and potential match-productivity at the firm, θ. In the remainder of this paper, we adopt the semantic convention of referring to a single firm as an "R-firm" or an "N-firm". The value to workers of being at either type of firm is summarized by the value function V_j, $j \in \{N, R\}$.[6]

In the steady state, unemployed workers meet firms at rate λ_U, while workers encounter alternative employers at a rate λ_E. Matches are exogenously destroyed at a constant rate δ. When meeting a potential employer, the probability is p_R that it is of type R. In section 6 we describe how the contact rates λ_U and λ_E and the proportion, p_R are determined in general equilibrium. A critical assumption that our solution requires is the *free entry condition*: the expected return to market entry (achieved by purchasing and posting a vacancy) for either type of vacancy is equal to zero.

In the remainder of this section we focus our attention on how to solve for several important endogenous objects in equilibrium. We proceed by:

[6]While we refer to the bargaining type as characterizing a firm, this need not strictly be the case. If all firms have only one worker, then the equivalence is obvious. However, when firms have more than one worker, some may be hired into a job in which wages are set using negotiation, while others may have their wages set at the time of their hiring with no negotiation at any point in their tenure at the firm. For simplicity, we will typically refer to a firm type instead of a job type.

1. Introducing the wage-bargaining framework for R-type firms (Section 2.2).

2. Solving for the worker's value functions, V_N and V_R, and mobility decisions, holding fixed the wage-offer strategies of N-type firms (Section 2.3).

3. Solving for the distribution of workers in the steady state across employment states (Section 2.4).

4. Fixing the above endogenous objects, we solve the wage-offer problem faced by an N-firm. To close the model, N-firms' optimal wage offer strategies must be in concordance with those we fixed in step (2). We show how to solve the model under this equilibrium restriction (Section 2.5).

5. Considering the implications of adding a binding minimum wage to the model (Section 3).

A note on heterogeneity in the model To simplify exposition, we suppress dependence of the model's value functions and wages on ability, a. Since wages at both firm types can be conditioned on ability, the reader can think of the following model solution as applying for fixed a. [7]

2.2 Wage-Setting At R-type Firms

While N-type firms make non-negotiable, fixed offers, R-type firms make wage offers based on the worker's productivity and best outside offer. Specifically, we assume R-type firms commit to surplus division with the employee. Wages are set so that the value afforded to workers is equal to their private outside option plus a share, α, of the *joint surplus* generated by the match. Let $T_R(\theta)$ denote the total value of a match θ at an R-firm. Since the outside option value of a firm with an unfilled vacancy is 0, then the surplus from the match is $T_R(\theta)$ minus the outside option value of the individual. For example, when hiring a worker from an N-type firm where she is paid a wage w, the wage offer that is bargained produces a continuation value to the worker of

$$\begin{aligned} V_{R,N}(\theta, w) &= V_N(w) + \alpha(T_R(\theta) - V_N(w)) \\ &= \alpha T_R(\theta) + (1-\alpha)V_N(w), \end{aligned}$$

where the surplus in this case is $S_R(\theta, V_N(w)) = T_R(\theta) - V_N(w)$. In general, the value of an employment contract at a type R firm where the match value is θ and the outside option has value

[7] All values determined in equilibrium can be derived without explicit reference to the distribution F_a. However, if there exists a binding minimum wage, the equilibrium values of the model cannot be determined without reference to F_a. This issue is discussed further in Section 3.

ω is equal to $\omega + \alpha(T_R(\theta) - \omega)$. Then hiring the worker out of unemployment, the value of the employment contract is

$$V_{R,U}(\theta) = V_U + \alpha(T_R(\theta) - V_U) = \alpha T_R(\theta) + (1-\alpha)V_U.$$

When, during the bargaining process, the worker currently has a job at an R-type firm with match θ' and draws a match value at the new potential employer of $\theta > \theta'$, we assume that both firms are drawn into Bertrand competition. In this setting, the outcome is identical to that in Dey and Flinn (2005) and Cahuc et al. (2006), where the losing firm is willing to pay a wage up to, but not exceeding, the value of the match, θ'. Thus, the worker's outside option in this case is the total value of the match $T_R(\theta')$ and the value of her employment contract is

$$V_R(\theta, \theta') = \alpha T_R(\theta) + (1-\alpha)T_R(\theta').$$

For completeness, we note that

$$V_{R,U}(\theta) = V_{R,R}(\theta, \theta^*),$$

where θ^* will be the reservation match value, the one at which the individual is indifferent between employment and continued search.

2.3 Values and Match Surplus Equations

Before describing the full set of mobility patterns that can occur in equilibrium, it will be useful to write down and investigate the properties of the total value function, T_R, at R-type firms and the worker's value function, V_N, at N-type firms. We will let Φ denote the endogenous distribution of offers received from non-negotiating firms (the shape of which will be determined in equilibrium, in a manner described below). Taking Φ as given, the value to a worker at an N-firm can be written as

$$(\rho+\delta)V_N(w) = w + \underbrace{\lambda_E p_R \int \alpha[T_R(x) - V_N(w)]^+ dF_\theta(x)}_{(1)} + \underbrace{\lambda_E(1-p_R)\int [V_N(x) - V_N(w)]^+ d\Phi(x)}_{(2)} + \delta V_U, \quad (1)$$

where $[x]^+ \equiv \max\{x, 0\}$. Term (1) is the expected increase in the continuation value realized when the worker meets an R-firm, which occurs at a rate $\lambda_E p_R$. If the surplus attainable at this firm exceeds the value of remaining, a fraction α of the difference is obtained through the surplus division process. Term (2) is the expected continuation value derived when meeting another N-firm, which is accepted only when the value of the offered wage exceeds the value of remaining.

This term includes the rate of meeting an N-type firm, which is $\lambda_E(1-p)$. Such a meeting is only relevant when it increases the worker's value of employment.[8]

The total value function associated with a job at an R-type firm, T_R, can be derived in a similar fashion. This object, which is the total value to both the worker and the R-firm from the match, is useful because in our framework we have assumed perfectly transferable utility (i.e., a linear Pareto frontier). However, it may help the reader to imagine that $T_R(\theta)$ is the value to the worker when their wage is equal to total match output (and hence they have captured the full surplus from the match), so that

$$(\rho+\delta)T_R(\theta) = \theta + \underbrace{\lambda_E p_R \int \alpha[T_R(x) - T_R(\theta)]^+ dF_\theta(x)}_{(1)} \\ + \underbrace{\lambda_E(1-p_R)\int [V_N(x) - T_R(\theta)]^+ d\Phi(x)}_{(2)} + \delta V_U \quad (2)$$

Once again, term (1) shows what happens when the worker meets another R-firm at which $x > \theta$. Bertrand competition bids up the worker's outside option to $T_R(\theta)$ and an additional fraction α of the difference at the new firm is obtained through this bargaining procedure. The firm receives a value of 0, by virtue of the free entry condition. In term (2), the worker meets an N-firm and additional surplus is only generated if the value from the wage offer exceeds $T_R(\theta)$. In this case, the firm once again receives 0.

When comparing equations (1) and (2), we see that setting $V_N = T_R$ allows both recursive equations to hold. This establishes the following useful lemma.

Lemma 1. *A worker that has claimed the full surplus of the match θ at an R-firm achieves the same value as a worker earning wage θ at an N-firm:*

$$T_R(\theta) = V_N(\theta)$$

Lemma 1 is useful because it allows us to write a dynamic program solely in terms of T_R:

$$(\rho+\delta)T_R(\theta) = \theta + \lambda_E p_R \int \alpha[T_R(x)-T_R(\theta)]^+ dF_\theta(x) + \lambda_E(1-p_R)\int [T_R(x)-T_R(\theta)]^+ d\Phi(x) + \delta V_U.$$

It then follows, using standard recursive arguments, that this function is *strictly increasing* in its sole argument, θ. This permits us to define the *reservation match value*, θ^*, according to

$$V_U = T_R(\theta^*) = V_N(\theta^*) \quad (3)$$

This concept defines which matches (and wage offers from N-firms) are acceptable to workers when being hired out of unemployment.

[8] Standard recursive logic will permit us to show that V_N is increasing in the wage and, hence, this occurs when $x > w$.

In words, the equivalence $T_R(w) = V_N(w)$ stems from the fact that when the full value of the match is given to the worker, there is no further scope for the firm to respond to any change in the outside option of the worker, so that the wage is effectively fixed at w, just as it is at an N-type firm where the individual is paid a wage of w. At any $w < \theta$ at an R-type firm, the firm does have scope to respond to changes in the outside option of the individual, and for this reason $T_R(\theta) > V_N(w)$ whenever $\theta > w$.

This lemma provides the basis for simplifying the job mobility decisions of workers. We define the *maximum available wage,* $\bar{w}$, at a firm as the maximum wage attainable over the set of possible outside options that the worker could possess. When an individual is employed at an N-type firm where she is paid a wage of w, since the firm does not respond to any outside offers, the wage is fixed at w over the tenure of the job, and $\bar{w} = w$. At an R-type firm, the maximum wage attainable is the one corresponding to the full productivity value of the match, so that $\bar{w} = \theta$. This wage offer is available to the worker whenever she meets another R-type firm at which her match productivity is $\theta' \geq \theta$ or when she meets any N-type firm for which her wage offer $w' \geq \theta$.

It is clear that knowledge of this firm-specific value at any job is sufficient for describing mobility patterns to and from it. In particular, let the maximum wage available at two jobs be given by $\bar{w}$ and $\bar{w}'$. Then if $\bar{w} \geq \bar{w}'$, the individual will accept the job associated with $\bar{w}$. Conversely, if $\bar{w} < \bar{w}'$,[9] the individual will accept the job associated with $\bar{w}'$.

We can also use the maximum available wage to parsimoniously characterize the value function for workers at R-firms, since it is a sufficient statistic for the best outside option the worker could solicit from a competing firm. Recall that this best offer is equal to the match value of the losing firm when this firm is of R-type (due to our assumption of induced Bertrand competition) and equal to the wage offer made by the losing firm when it is of N-type. This coincides exactly with the definition of $\overline{w}$ at the losing firm. We can therefore write the value function V_R as

$$V_R(\theta, \bar{w}) = \alpha T_R(\theta) + (1 - \alpha) T_R(\bar{w}), \;\; \theta \geq \bar{w}. \tag{4}$$

The state $\bar{w}$ is sufficient for describing when an individual who remains at an R-firm with a θ match will have their wage increased. This will occur whenever she receives new potential employment opportunity associated with a maximum wage of $\bar{w}'$, where $\bar{w} < \bar{w}' \leq \theta$. In this case, the new value of the employee's problem will be

$$V_R(\theta, \bar{w}') = \alpha T_R(\theta) + (1 - \alpha) T_R(\bar{w}'),$$

with the new wage determined through this equality. We can explore this wage function, denoted

[9] Without loss of generality, we assume that individuals remain at the incumbent firm in the case of a tie. We will see that the probability of this event is zero, a direct consequence of the distribution of θ being absolutely continuous on its support.

by $\phi(\theta, \bar{w})$, in more detail by explicitly writing the function $V_R(\theta, \bar{w})$ as

$$
\begin{aligned}
&(\rho + \delta + \lambda_E p_R \tilde{F}_\theta(\bar{w}) + \lambda_E(1-p_R)\tilde{\Phi}(\bar{w}))V_R(\theta, \bar{w}) = \phi(\theta, \bar{w}) \\
&\quad + \lambda_E p_R \left[\underbrace{\int_{\bar{w}}^{\theta} [(1-\alpha)T_R(x) + \alpha T_R(\theta)] dF_\theta(x)}_{(1)} + \underbrace{\int_{\theta} [\alpha T_R(x) + (1-\alpha)T_R(\theta)] dF_\theta(x)}_{(2)} \right] \\
&\qquad + \lambda_E(1-p_R) \left[\underbrace{\int_{\bar{w}}^{\theta} [\alpha T_R(\theta) + (1-\alpha)V_N(x)] d\Phi(x)}_{(3)} + \underbrace{\int_{\theta} V_N(x) d\Phi(x)}_{(4)} \right] + \delta V_U
\end{aligned}
$$

Here the wage $\phi(\theta, \bar{w})$ is set such that the surplus split defined in (4) is achieved. Gains in the worker's value can arise from four different outcomes. The event (1) occurs when the worker meets another R-type firm at which the match value is greater than the current $\bar{w}$ but is less than the match value at the current firm. In this case, the wage is renegotiated upward at the current firm and the worker remains. In situation (2), the worker meets another type R-firm at which her match productivity is greater than at the current firm. In this case her outside option becomes the incumbent firm, and the value of remaining there is $T_R(\theta)$. In case (3), the worker meets a type N-firm and obtains a wage offer of x, where $\theta \geq x > \bar{w}$. Through the surplus-sharing arrangement, her wage is increased due to the increase in the value of her outside option, and she remains with her current employer. In the last case (4), the worker meets an N-firm and draws a non-negotiable wage offer x that is greater than the best available offer from the incumbent firm, which is θ. The wage function ϕ is derived in detail in Appendix D.3.

We conclude this subsection by deriving the value of unemployed search, V_U. First, note that when hiring a worker out of unemployment, this is equivalent to hiring a worker from a firm with match productivity θ^*, and hence the worker's value can be written as $V_R(\theta, \theta^*)$ in this case. We obtain

$$
\rho V_U = b + \lambda_U p_R \int_{\theta^*} \alpha(T_R(x) - V_U) dF_\theta(x) + (1-p_R) \int_{\theta^*} (V_N(x) - V_U) d\Phi(x). \tag{5}
$$

Using (3) with the above, and substituting $T_R(\theta^*) = V_N(\theta^*)$, we can define the reservation match quality θ^* by the relation:

$$
\theta^* = b + (\lambda_U - \lambda_E) \left[p_R \int_{\theta^*} \alpha(T_R(x) - T_R(\theta^*)) dF_\theta(x) + (1-p_R) \int_{\theta^*} (T_R(x) - T_R(\theta^*)) d\Phi(x) \right] \tag{6}
$$

With the definition of θ^* now in hand, the following lemma will prove useful in the next section, so we introduce it here.

Lemma 2. *Define* $\underline{w} = \inf\{x : \Phi(x) > 0\}$. *Then* $\underline{w} = \theta^*$.

Proof. See appendix. ∎

This result follows immediately by noting that all matches $\theta > \theta^*$ are profitable for N-firms, while wage offers above the match value are not profitable.

2.4 Steady State

In the previous section we characterized the conditions under which (and the rate at which) workers move between employment states. In particular, we showed that the rule defining job-to-job mobility is *unidimensional*, in that we were able to define a single variable (the maximum attainable wage) that dictates mobility decisions. This greatly enhances the tractability of the model, since it allows us to now analytically derive the steady state distribution of workers across labor market states.[10]

First, normalizing the mass of workers in the economy to 1, we let M_E and M_U $(= 1 - M_E)$ denote the steady state mass of workers in employment and unemployment, respectively. We know that the flow rate out of unemployment is $h_U = \lambda_U \tilde{F}_\theta(\theta^*)$, where $\tilde{F}_\theta = 1 - F_\theta$. [11] The flow rate into unemployment is simply δ. Thus, we can write:

$$M_E = \frac{\lambda_U \tilde{F}_\theta(\theta^*)}{\delta + \lambda_U \tilde{F}_\theta(\theta^*)}, \qquad M_U = \frac{\delta}{\delta + \lambda_U \tilde{F}_\theta(\theta^*)}.$$

In the last section we showed that the value of a given employment opportunity could be represented by a scalar sufficient statistic, the *maximum attainable wage*. Let G be the steady state distribution of workers across this state. We can see how this object is required knowledge when an N-firm makes its wage offer, since it must consider the probability that a given wage will be acceptable to the prospective employee. For a randomly-sampled employed worker, the probability that a wage, w, is acceptable is given by $G(w)$, which is the likelihood that its current maximal acceptable wage is no greater than w.

Let $G(x, R)$ and $G(x, N)$ be the measure of workers at R and N firms with maximum attainable wage less than or equal to x. We know that the marginal distribution, $G(x)$, is simply

$$G(x) = G(x, R) + G(x, N).$$

Finally, let $H(\cdot|x)$ be the conditional distribution of most recent competing offers for workers at R firms with match x. For example, $H(q|x)$ is the probability that a worker at a firm with match x used an outside offer less than or equal to q for their most recent wage-bargain. Clearly, $q \leq x$, and $H(x|x) = 1$.

[10] This is one reason we do not consider more complex (and potentially more efficient) labor market contracts for N-firms than the fixed wage offer. Offering more complex contracts that resemble, for example, a wage-tenure profile break this key simplification of the model.

[11] The survivor function of any c.d.f. H will be denoted $\tilde{H}$ throughout the paper.

Balancing flow equations in Appendix D.1 gives the following closed-form expressions for each of the steady state distributions which are being considered, namely

$$\begin{aligned} G(x) &= \frac{p_R(F_\theta(x) - F_\theta(\theta^*)) + (1-p)(\Phi(x) - \Phi(\theta^\star))}{\delta + \lambda_E p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)} \frac{M_U}{M_E} && (7) \\ g(x,R) &= \frac{\lambda_U p_R f_\theta(x)(\delta + \lambda_E \tilde{F}_\theta(\theta^*))}{\Psi(x)^2} \frac{M_U}{M_E} && (8) \\ g(x,N) &= \frac{\lambda_U (1-p_R)\phi(x)(\delta + \lambda_E \tilde{F}_\theta(\theta^*))}{\Psi(x)^2} \frac{M_U}{M_E} && (9) \\ H(q|x) &= \left(\frac{\Psi(x)}{\Psi(q)}\right)^2 && (10) \end{aligned}$$

where

$$\Psi(x) = \delta + \lambda_E p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)$$

This final term, $\Psi(x)$, defines the flow rate of exit of a worker from a firm whose highest possible wage offer is x.

With these expressions we have fully characterized, given the endogenous wage offer distribution Φ, how workers are distributed across firms and wages in steady state. We complete the model solution by deriving the distribution of wage offers from type N firms in a manner that is consistent with the equilibrium conditions presented thus far.

2.5 The Type N Firm's Problem

We have shown how the value functions of workers and R-type firms are determined conditional on a distribution of wage offers from N-type firms. In this section we solve for this function given the model structure that has been developed. As was the case for R-type firms, we assume that when a potential employee encounters the firm, a match quality value is drawn from F_θ.[12] At this point, the N-type firm makes a *non-negotiable* wage offer, w, that the worker will accept or reject. If the wage is rejected, the individual continues in their current state, and the firm continues with an unfilled vacancy. We assume that the firm makes its offer with no information about the worker's current employment state. In particular, the firm does not know whether the individual is currently unemployed, employed at another N-type firm at some wage w', or is employed at an R-type firm where her match productivity is θ'. While this assumption may appear overly restrictive, it is interesting to note that the intent of many recent laws is to promote exactly this type of wage-setting environment, as we discussed in the introduction. It is also consistent with a modified version of the wage-posting model of Burdett and Mortensen (1998) in their homogeneous worker case. Firms made (different) fixed wage offers to equally productive workers in their case, whereas here firms make fixed wage offers to workers of heterogeneous (match and idiosyncratic)

[12] We also assume that when a searcher encounters either type of firm, that individual's general ability a is also perfectly observable, as we have been assuming throughout.

productivity levels that are solely based on the worker's productivity and are independent of their current labor market status.

When a firm encounters an individual with match productivity θ, the firm's expected discounted profit can be written as:

$$J(\theta, w) = \underbrace{\text{HireProb}(w)}_{(1)} \times \underbrace{\frac{\theta - w}{\rho + \delta + \lambda_E\{p_R \tilde{F}_\theta(w) + (1 - p_R)\tilde{\Phi}(w)\}}}_{(2)}.$$

Term (1) is simply the probability that the wage offer w is accepted by the worker, while term (2) is the expected present value of profits to the firm. The numerator of this term is the firm's flow profit, while the denominator reflects the effective discount rate, which incorporates the rate at which the firm loses the worker to another employer. This occurs whenever the worker meets an R-firm and a match is drawn that exceeds w or whenever the worker meets an N-firm and a wage is drawn that exceeds w.

Restricting our attention to wage offers that satisfy $w \geq \theta^*$ (since wage offers less than θ^* are unacceptable to any searcher), the hiring probability can be written as

$$\text{HireProb}(w) = \underbrace{\frac{\lambda_U M_U}{\lambda_U M_U + \lambda_E M_E}}_{(1)} + \underbrace{\frac{\lambda_E M_E}{\lambda_U M_U + \lambda_E M_E}}_{(2)} G(w).$$

Term (1) gives the probability that the searcher encountered is unemployed, in which case any feasible wage offer is accepted. Term (2) is the probability that a currently employed individual is encountered multiplied by the probability that a wage offer of w is acceptable to the individual. As we have shown, the wage offer w will be accepted as long as the maximum attainable wage at her current job is no greater than w, the probability of which is given by $G(w)$.

Using our derivation of G from the previous section, we can then write the firm's expected profit as

$$J(\theta, w) = \frac{\lambda_U M_U \lambda_E \tilde{F}_\theta(\theta^*)}{\lambda_U M_U + \lambda_E M_E} \frac{\theta - w}{\Psi(w)(\rho + \Psi(w))} = \Gamma(w)(\theta - w) \tag{11}$$

Given a match draw θ, the firm solves the problem

$$\max_w J(\theta, w). \tag{12}$$

Notice that if this problem identifies a unique wage, w, for each match level, θ, then this defines a function $\varphi : \mathbb{R}^+ \mapsto \mathbb{R}^+$. If this function, φ, is strictly increasing, then we can write

$$\Phi(w) = F_\theta(\varphi^{-1}(w)).$$

To clarify the role played by Φ as a functional parameter of the maximization problem, let us re-write (12) as

$$\max_w \{\Gamma(\Phi(w), F_\theta(w))(\theta - w)\}. \tag{13}$$

Given a monotonically increasing offer function, φ, this defines an operator:

$$[\mathcal{T}\varphi](\theta) = \arg\max_{w} \left\{ \Gamma(F_\theta(\varphi^{-1}(w)), F_\theta(w))(\theta - w) \right\}.$$

In equilibrium, it must be that the wage offer function φ is a fixed point of this operator $\mathcal{T}$, i.e. $\mathcal{T}\varphi = \varphi$. Before we show how such a fixed point can be found, the following result guarantees that searching for such a deterministic, monotonic function is appropriate in this setting.

Proposition 1. *Firms' optimal wage offer strategies are given by a deterministic function φ that is (1) monotonically increasing; (2) lower semi-continuous; (3) almost everywhere differentiable; and (4) satisfies $\varphi(\theta^*) = \theta^*$.*

Proof. The proof is given as a combination of Lemmas 2-9 in the Appendix. ∎

We now describe a parsimonious computational strategy for finding the fixed point, φ. Notice that, if such a solution is found, the model in partial equilibrium (i.e. with fixed values of p_R, λ_U, and λ_E) is solved. Inspecting equation (11) reveals the trade-off that firms face: both hiring and worker retention probabilities are *increasing* in w, while flow profits are decreasing in w. These are, as usual, reflected in the first-order condition

$$\frac{d}{dw}\Gamma\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big)(\theta - w) - \Gamma(F_\theta(\varphi^{-1}(w), F_\theta(w)) = 0.$$

Using Γ_1, Γ_2, to denote the derivative of Γ in its first and second arguments, we find

$$\Big[\Gamma_1\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big) f_\theta(\varphi^{-1}(w))/\varphi'(\varphi^{-1}(w)) \\ + \Gamma_2\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big) f_\theta(w)\Big](\theta - w) - \Gamma\big(F_\theta(\varphi^{-1}(w)), F_\theta(w)\big) = 0 \quad (14)$$

Finally, by imposing that in equilibrium we must have $\varphi(\theta) = w$, this condition becomes

$$\Big[\Gamma_1\big(F_\theta(\theta), F_\theta(w)\big) f_\theta(\theta)/\varphi'(\theta) + \Gamma_2\big(F_\theta(\theta), F_\theta(w)\big) f_\theta(w)\Big](\theta - w) - \Gamma\big(F_\theta(\theta), F_\theta(w)\big) = 0 \quad (15)$$

This can be rearranged into a first order differential equation:

$$\varphi'(\theta) = \frac{\Gamma_1\big(F_\theta(\theta), F_\theta(w)\big) f_\theta(\theta)}{\Gamma\big(F_\theta(\theta), F_\theta(w)\big)/(\theta - w) - \Gamma_2\big(F_\theta(\theta), F_\theta(w)\big) f_\theta(w)} \quad (16)$$

which, when combined with the boundary condition $\varphi(\theta^*) = \theta^*$, is readily solved numerically. Notice that Proposition 1 does not guarantee that the first order condition uniquely identifies the optimal wage offer, and in fact there may be discontinuities in φ. In Appendix D.2 we show how to leverage the properties of the wage solution into a robust numerical algorithm.

Inspection of equation (16) reveals three properties of the wage function that hold for any choice of model parameters. First, we see that the first term in the denominator becomes indefinitely large as $w \to \theta$. Since Lemma 2 requires that the limit of $\varphi(\theta)$ as $\theta \to \theta^*$ is equal to θ^*, we know that φ must be increasingly flat in the region close to the reservation match value θ^*. Second, we see that the density of the match distribution $f_\theta(\theta)$ appears in the numerator, suggesting that (all else being equal) φ will be more steep where the density is high, and more flat where the density is low (in the tails of the distribution, typically). Related to this characteristic is a third property, which is that the wage offer function will become increasingly flat in the right tail of the match productivity distribution. In Appendix D.2 we present analytic expressions for Γ, Γ_1, and Γ_2 from which one can verify that these properties hold. Turning to Figure 5, which shows the solution for φ given our parameter estimates obtained in Section 5, one can easily discern the features we have just described. In the background of this figure we have plotted the match density f_θ. Although φ is flat in the region near θ^*, it quickly increases as competition intensifies (as $f(\theta)$ increases). In the tails, where the likelihood of competing offers decreases again, φ once more begins to flatten out.

2.6 Inefficiency of Job Mobility Decisions

One important implication of our modeling setup is that, when R and N firms compete for workers, an R-type firm with a lower match value may win the worker simply because of the N-type firm's unwillingness or inability to renegotiate. We refer to this phenomenon as *inefficient mobility.* It should be noted that most undirected search models (e.g., Postel-Vinay and Robin (2002); Cahuc et al. (2006); Flinn (2006); Dey and Flinn (2005)) generically exhibit inefficient mobility in the sense that workers will make socially undesirable job acceptance decisions out of unemployment,[13] which is a feature shared by this model. However, those models typically imply efficient job mobility decisions: when a worker at a firm with match θ meets another potential employee with match θ', a job switch can only transpire if $\theta' \geq \theta$. In this model, however, this condition may be violated when the two employers differ in the wage-setting protocols that they use. For example, if the current employer is a type N firm and the alternative employer is type R, it may be the case that the individual leaves her current employer even though $\theta \geq \theta'$. In the case of the strict inequality, we say that this is an inefficient job acceptance (or mobility) decision.

To elaborate on this point, let $\theta_R < \theta_N$ denote the match values at an R firm and an N firm, respectively. Now, suppose that $\varphi(\theta_N) < \theta_R$. Efficiency requires that the worker always go to the firm at which she is most productive. However since Lemma 1 implies that $T_R(\theta_R) > V_N(\varphi(\theta_N))$, the R firm wins the competition for the worker. This decision, in which the worker ends up at the firm where her match-specific and total productivity is lower, is inefficient both when the N firm is the incumbent and loses the worker, and when the R firm is the incumbent and succeeds in retaining the worker. Given a wage offer function, φ, for each match value θ_N there is an entire range of values for θ_R: $(\varphi(\theta_N), \theta_N)$ for which the worker will either suboptimally move to

[13] We thank Ilse Lindenlaub for making this observation.

an R firm or suboptimally reject the N firm's offer. To graphically illustrate, Figure 9, using the function φ derived from our model estimates, displays the entire set of combinations of (θ_R, θ_N) that result in an inefficient job-to-job mobility decision.

The severity of this problem can be measured as the fraction of on-the-job wage offers that result in either a suboptimal rejection or acceptance of an offer, which we call the *rate of inefficient mobility*. In Section 5.1.3 we will revisit this issue and calculate the rate of inefficient mobility in the steady state of our estimated model.

2.7 Heterogeneous Ability of Workers

So as to simplify the description of the model solution, we have ignored time-invariant individual productivity differences. By assuming that the production technology is given by $y = a\theta$ and by assuming that the flow utility associated with the unemployment state is given by ba, where b is a common scalar parameter in the population, it is straightforward to show that all the functions $\xi(\theta)$ for which we have solved in the previous subsections become $a\xi(\theta)$ when individual heterogeneity in ability is reintroduced. In particular, we have

$$\begin{aligned}
T_R(a, \theta) &= aT_R(\theta) \\
V_R(a, \theta, \bar{w}) &= aV_R(\theta, \bar{w}) \\
\phi(a, \theta, \bar{w}) &= a\phi(\theta, \bar{w}) \\
\varphi(a, \theta) &= a\varphi(\theta) \\
V_U(a) &= aV_U
\end{aligned}$$

The above properties also imply that the reservation match value θ^* is invariant in a. Although the modeling framework was more straightforward, the same type of assumptions on the production function and flow utility in unemployment were utilized in Flinn and Mullins (2015), and are often used in search models with no match heterogeneity but only worker and firm heterogeneity (e.g., Postel-Vinay and Robin (2002), Bartolucci (2013)). Unfortunately, in the case of a binding minimum wage, this "neutrality" property is lost.

2.8 General Equilibrium: The Determination of λ_U, λ_E, and p_R

The model is completed by specifying the manner in which contact rates between workers and firms are determined. Under our random meeting assumption, unemployed individuals contact type R firms at rate $\lambda_U p_R$ and type N firms at rate $\lambda_U(1 - p_R)$, while employed individuals meet R-type and N-type vacancies at rates $\lambda_E p_R$ and $\lambda_E(1 - p_R)$, respectively. We utilize the standard matching function and competitive vacancy creation assumptions (Mortensen and Pissarides, 1994) to solve for these rates. Our only innovation on this score is the inclusion of a marginal indifference condition to determine the proportion of R-type vacancies created, p_R.

The total measure of vacancies created in the steady state is given by $v = v_R + v_N$, where v_R and v_N are the measures of type R and type N vacancies created, respectively. The total stock of search effort on the supply side of the market is given by $M = M_U + \mu_E M_E$. The parameter $\mu_E \in (0, 1]$ is the search efficiency of employed individuals relative to unemployed ones. Then define labor market tightness by

$$\kappa = \frac{v}{M} = \frac{v_R + v_N}{M_U + \mu_E M_E}.$$

Assuming that the matching function has a Cobb-Douglas representation and with the TFP parameter set to 1, the rate of arrivals of contacts to searchers is

$$\begin{aligned} \lambda_U &= \kappa^\gamma \\ \lambda_E &= \mu_E \kappa^\gamma, \end{aligned}$$

so that $\lambda_E/\lambda_U = \mu_E$. The rate of arrivals of applicants to firms is $q(\kappa) = \kappa^{\gamma-1}$.

Following Lise and Robin (2017), we assume that the cost of creating either kind of vacancy, an N or an R, is an increasing function of the measure of these types of vacancies that are created, with this cost function given by $c_j(v_j)$, $j = R, N$.[14] The free entry conditions for both types of vacancies, R and N, are given by

$$0 = -c'_R(v_R) + q(\kappa) \int a \int \int (1-\alpha)[T_R(\theta) - T(x)]^+ dF_\theta(\theta) d\hat{G}(x) dF_a(a) \tag{17}$$

$$0 = -c'_N(v_N) + q(\kappa) \int a \int \int J\big(\theta, \varphi(\theta)\big) \mathbf{1}\{\varphi(\theta) > x\} dF_\theta(\theta) d\hat{G}(x) dF_a(a), \tag{18}$$

where $\hat{G}$ is the distribution of maximum attainable offers, G, augmented to include the probability of meeting an unemployed worker, for whom the maximum outside option is θ^*. The first term on the right side of these first order conditions is the marginal cost of posting a type j vacancy. The second term is the expected value of posting each type of vacancy, which is given by the rate of arrival of applicants to the firm, $q(\kappa)$, multiplied by the expected value of a meeting (with the meetings resulting in no employment contract implicitly assigned a value of 0). In solving the model, we will assume that

$$c_j(v_j) = \frac{c_j}{1+\psi} v_j^{1+\psi} \text{ for } j \in \{R, N\},$$

[14] This differs from the usual assumption of constant marginal costs, in which case we would merely have c_R and c_N. As in Lise and Robin, the main motivation for this assumption is for conducting counterfactual policy experiments. In some cases, depending on the primitive parameter values used, assuming constant costs may not allow satisfaction of the free entry condition for one type of vacancy.

as did Lise and Robin (2017). Given the solutions to these first order conditions, v_R^* and v_N^*, we have that the equilibrium proportion of type R vacancies is given by

$$p_R^* = \frac{v_R^*}{v_R^* + v_N^*}.$$

Equilibrium in this model is summarized by the pair of values (κ^*, p_R^*) given our assumption of undirected (i.e., random) search.

3 The Introduction of a Minimum Wage

We now extend the model to allow for a binding minimum wage, m. There exist a number of analyses of the impact of minimum wages in a labor market in which there are search frictions. These include models in which all firms are wage-posters (i.e., $p_R = 0$) and others in which all firms renegotiate (i.e., $p_R = 1$). We will briefly describe the impact of minimum wages on labor market outcomes under these two assumptions, and we will attempt to motivate the manner in which a minimum wage will affect equilibrium in the labor market when there exists a positive measure of both types of firms.

We first consider the case of minimum wage impacts in the Albrecht and Axell (1984) and Burdett and Mortensen (1998) wage-posting frameworks. Within both models, workers were considered to be of equal productivity. In the case of Albrecht and Axell, firm productivity was one-dimensional and continuously-distributed. Searchers on the supply side of the market were assumed to differ in their value of leisure, with there being high- and low-leisure value types. Firms chose to make one of two wage offers in equilibrium, with high-wage and high-productivity firms attracting both types of workers and low-productivity firms making low wage offers and only attracting workers who had low leisure valuations. The equilibrium determined the types of firms operating in the market and the steady state wage offer and employment distributions.

Eckstein and Wolpin (1990) took this model to data, after generalizing it somewhat so as to improve its potential to fit the individual-level labor market data they utilized. They estimated a model that allowed for more than two types of employees, and that made the likelihood of receiving a job offer a function of the number of active firms in the market.[15] Although the restrictions imposed by the theory implied a poor fit of the data, the authors did conduct an illustrative minimum wage policy experiment. Based on their estimates, they tentatively concluded that the reduction in the offer probabilities that comes with the minimum wage reducing the measure of firms operating in the market dominated its beneficial selection effect on the firm productivity

[15] They noted that this modification to the Albrecht-Axell model, where offer probabilities were independent of the measure of active firms, was necessary in order to investigate minimum wage impacts. Otherwise, a minimum wage set near the top of the firm productivity distribution, thereby excluding all but the top firms from participating in the labor market, would be a costless way to increase workers' welfare.

distribution. In order to maximize worker welfare, they concluded that it was not optimal to impose a binding minimum wage.

The Burdett and Mortensen (1998) framework is more elegant theoretically in that even in the case of homogeneous workers and firms the equilibrium wage distribution is shown to be nondegenerate and absolutely continuous.[16] In particular, they proved that there could be no mass points on the finite support of the equilibrium wage distribution, which was the interval $[\underline{w}, \overline{w}]$.

Although elegant, the Burdett and Mortensen model produces the implication that the wage density defined on $[\underline{w}, \overline{w}]$ is monotone increasing, an implication easily rejected by any casual inspection of wage data. When this model is taken to data, it typically is modified to allow (at least) firm heterogeneity in productivity. This route is followed in van den Berg and Ridder (1998) and Bontemps et al. (1999, 2000), for example. In the case of the imposition of a binding minimum wage, that is, $m > \underline{w}$, the new equilibrium wage offer distribution is also nondegenerate, with support $[m, \overline{w}(m)]$. Whether or not there exists a binding minimum wage, the equilibrium wage offer distribution remains absolutely continuously distributed on its support.[17] In particular, when there exists a binding minimum wage, the lowest wage offer is given by m and there is no mass point (i.e., "spike") at m. In the empirical analysis of van den Berg and Ridder (1998), where they allow for extensive segmentation of the labor market, they find that the uniform minimum wage imposed in their analysis eliminates a number of low-productivity segments. They perform no formal welfare analysis, per se. The analysis of the minimum wage in Bontemps et al. (1999, 2000) focuses primarily on the impact of the minimum wage on the support of the distribution of equilibrium wages and its empirical implications. As in van den Berg and Ridder (1998), the emphasis is not on setting an "optimal" minimum wage using a social welfare function or efficiency criterion.

When all firms in the labor market negotiate with their workers, the situation is quite different. In the case of no on-the-job search and with only match-specific productivity following an absolutely-continuous distribution, surplus division produces a wage distribution that is also absolutely continuous. Flinn (2006) views the introduction of a binding minimum wage into this bargaining protocol as creating a side-constraint on the surplus-division problem solved by the worker and firm. Introducing a binding minimum wage of m has the direct impact of truncating the wage distribution from below at m, and has an additional impact on the wage distribution that is caused by the change in the value of unemployed search, each worker's outside option.

[16]The original Albrecht and Axell model and the generalized version estimated in Eckstein and Wolpin assume no on-the-job search. On-the-job search is required by Burdett and Mortensen to generate the equilibrium that they describe.

[17]Of course, the minimum wage imposed must not exceed worker productivity in order for the labor market to continue to exist.

This creates a "spillover" effect that ripples throughout the entire wage distribution. A binding minimum wage imposed in a labor market with continuously-distributed match productivity produces a mixed continuous-discrete wage distribution, with a mass point at m and a continuous distribution of wages for all $w > m$.

Further analyses of minimum wage impacts using generalizations of this framework that also include individual-level heterogeneity in productivity and that allow for human capital investment appear in Flinn and Mullins (2015) and Flinn et al. (2017). These papers focus on the impact of minimum wages on investment in schooling (2015) and investment in general and specific human capital over the labor market career (2017), and both include general equilibrium effects through firms' vacancy-creation decisions. Generally speaking, it was found that moderate increases in minimum wages are beneficial for low-skilled individuals, but that high values of the minimum wage cause large welfare decreases for low-skilled individuals, as they become "priced-out" of the labor market.

In all of these models based on wage-setting through surplus division, minimum wages generate mass points at the (common) minimum wage m, at least for some ability types. In our setting, in which there exist a positive measure of both wage-posting and negotiating firms, mass points in the wage distribution for individuals of certain ability levels can only be produced by the wage-setting behavior of negotiating firms. As we will show, the equilibrium proportion of firms of type R, p_R, will be a function of the minimum wage level. Thus, changes in the minimum wage will affect the size of the mass point at m in two ways. Fixing $p_R(m)$ at its initial level, an increase in m will result in a (weakly) increasing proportion of individual ability types (a) directly impacted by the minimum wage constraint and will increase the size of the set of match productivity values (θ) for which the minimum wage constraint is binding for each of the a subject to the minimum wage constraint. However, the minimum wage will also increase the measure of R-type firms in equilibrium, as will be shown below. This effect will also lead to an increase in the mass of workers at the minimum wage. However, the effect of this change on worker welfare is ambiguous, and it can only be evaluated with estimates of the parameters characterizing the labor market environment.

Before proceeding to the formal description of the model when there exists a binding minimum wage, we begin with a brief overview of how a minimum wage of m impacts the labor market outcomes of a type a individual. Consider the reservation match θ^* and the wage equation ϕ when there is not a binding minimum wage. If $a\theta^* < m$, then the minimum wage renders unprofitable previously acceptable matches in the interval $[\theta^*, m/a]$. In the case of the wage set when the individual is employed at an R-type firm where her match productivity is θ and her outside option is θ', if $a\phi(\theta, \theta') < m$, then the minimum wage binds in the bargaining problem. Thus, if either inequality holds, then the wage equations ϕ and φ are no longer solutions to the model.

In order to determine if the minimum wage binds, we do not need to know both a and m, but rather just their ratio the *effective minimum wage* in ability (a) units, $\tilde{m} = m/a$. We formalize this notation by introducing the following definition.

Definition 1. *Fixing ability, a, and minimum wage, m, let the effective minimum wage be the match value at which flow output is exactly equal to m:*

$$a\tilde{m} = m, \qquad \tilde{m} = \frac{m}{a} \tag{19}$$

In other words, when a match productivity value $\theta = \tilde{m}$ for an individual of ability a, the flow productivity is equal to the minimum wage m. Although $\tilde{m}$ is technically a function of both m and a, we suppress this dependence in order to avoid notational clutter. The following lemma is immediate.

Lemma 3. *Fixing a parameterization of the model, and letting ϕ and φ be the solutions to the wage equations in the unrestricted case, an effective minimum wage binds iff either:*

1. *$\theta^* < \tilde{m}$; or*

2. *There exists a pair $(\theta, \theta') \in [\theta^*, \infty) \cap supp(F_\theta)$ such that $\phi(\theta, \theta') < \tilde{m}$.*

Since the minimum wage is applied uniformly across ability levels, the effective minimum wage changes with ability, and so the model must be solved separately at each individual ability level. However, to simplify notation, we will continue to write the wage equations and value functions in efficiency units, and index them by the effective minimum wage, $\tilde{m}$. The model can thus be solved for each a by first finding the effective minimum wage that applies to that ability level, then solving the model objects in ability units, then later multiplying wages and values through by a, for each ability level.

Let $M(\theta; \tilde{m})$ be the value to a worker of earning the minimum wage, $\tilde{m}$, at an R-firm with match θ. Noting that values are monotonically increasing in the wage earned, we see that the minimum wage will be paid whenever $\phi(\theta, y; \tilde{m}) < \tilde{m}$, which occurs whenever

$$\alpha T_R(\theta; \tilde{m}) + (1-\alpha) T_R(y; \tilde{m}) < M(\theta; \tilde{m}),$$

and so the value function M can be written recursively as:

$$\begin{aligned}(\rho+\delta)M(\theta;\tilde{m}) &= \tilde{m} + \lambda_E p_R \int_{\tilde{m}}^{\theta} \max\{(1-\alpha)T_R(x;\tilde{m}) + \alpha T_R(\theta;\tilde{m}) - M(\theta;\tilde{m}), 0\} dF_\theta(x) \\ &+ \lambda_e p_R \int_\theta \max\{M(x;\tilde{m}) - M(\theta;\tilde{m}), \alpha T_R(x;\tilde{m}) + (1-\alpha)T_R(\theta;\tilde{m}) - M(\theta;\tilde{m})\} dF_\theta(x) \\ &+\lambda_e(1-p_R)\int \max\{(1-\alpha)V_N(x;\tilde{m}) + \alpha T_R(\theta;\tilde{m}) - M(\theta;\tilde{m}), V_N(x;\tilde{m}) - M(\theta;\tilde{m}), 0\} d\Phi(x) + \delta V_U(\tilde{m})\end{aligned}$$

When a worker, earning minimum wage at an R-type firm with match θ, meets another firm with best available wage x, there can be three potential outcomes, each of which corresponds to an integral on lines one through three of the implicit function for M written above.

1. With probability p_R, the new firm is willing to negotiate, and if $\tilde{m} < x < \theta$, our usual bargaining assumptions apply, with the worker negotiating a new wage offer $\phi(\theta, x; \tilde{m})$. However, if the newly negotiated wage does not exceed $\tilde{m}$, then the wage will remain at $\tilde{m}$ and the worker gets no wage increase from the new potential employer. Whether or not a wage increase occurs, the individual always remains at the current firm with match value θ.

2. With probability p_R the firm is willing to negotiate and, if the competing firm is willing to negotiate and $x > \theta$, then the wage offer $\phi(x, \theta; \tilde{m})$ exceeds the best available offer from the incumbent, and the worker moves. However, it may still be the case that $\phi(x, \theta; \tilde{m}) < \tilde{m}$, in which case the worker may still be paid the minimum wage. In such a case, although the worker does not improve her current wage, she does improve her future (potential) bargaining position. Once again, whether or not a wage increase occurs, the worker will move to the new firm.

3. With probability $(1 - p_R)$ the firm is a type N firm, and it will make a non-negotiable offer of x. In this case, the worker may accept a newly bargained wage at the incumbent firm, $\phi(\theta, x; \tilde{m})$, she may accept the non-negotiable wage x and switch firms, or she may prefer to keep her minimum wage offer at the incumbent firm.

We must also re-write value function T_R in order to incorporate the possibility that future wages are constrained by $\tilde{m}$. This can occur due to the nonmonoticity of the ϕ function. The new functional equation is

$$
\begin{aligned}
(\rho + \delta) T_R(\theta; \tilde{m}) = \theta + \lambda_E p_R \int_\theta \max\{M(x; \tilde{m}) - T_R(\theta; \tilde{m}), \alpha[T_R(x; \tilde{m}) - T_R(\theta; \tilde{m})]\} dF_\theta(x) \\
+ \lambda_E (1 - p_R) \int_\theta (V_N(x; \tilde{m}) - T_R(\theta; \tilde{m})) d\Phi(x) + \delta V_U(\tilde{m}) \quad (20)
\end{aligned}
$$

We can, as before, define the reservation match value by the relation $T_R(\theta^*; \tilde{m}) = V_U(\tilde{m})$. Since there is no guarantee that $\theta^* \geq \tilde{m}$, we must define in addition $\theta^* = \max\{\theta^*, \tilde{m}\}$ which gives the lowest match value that will result in an employment contract. In turn, this defines the lower bound of the offer distribution for N-type firms. Adding the restriction below on reservation matches to those above is sufficient to determine the solution:

$$
\begin{aligned}
\rho T_R(\theta^*; \tilde{m}) = b + \lambda_E p_R \int_{\theta^\star} \max\{M(x; \tilde{m}), \alpha(T_R(x; \tilde{m}) - T_R(\theta^*; \tilde{m})), 0\} dF_\theta(x) \\
+ \lambda_E (1 - p_R) \int_{\theta^\star} (V_N(x; \tilde{m}) - T_R(\theta^*; \tilde{m})) d\Phi(x). \quad (21)
\end{aligned}
$$

3.1 Modifications Required in General Equilibrium

As has been shown, when there exists a binding minimum wage for some values of a in the support of the distribution F_a, it is no longer possible to write the solution for an acceptable match productivity independently of the value of a. When this is the case, it is possible to estimate the distribution F_a nonparametrically. Without this property, however, the model must be solved for each value of a in the support, and it is no longer feasible to allow F_a to be absolutely continuous. Instead, we will assume that F_a is discrete, with a finite number of points of support, $a_1, a_2, ..., a_M$, and with $\pi_j \equiv \Pr(a = a_j)$.

With this change, the conditions that determine the vacancy postings of type R and type N and the proportion of type R vacancies are given by

$$\begin{aligned}
0 &= -c_R'(v_R) + q(\kappa)\sum_{j=1}^{M} a_j \int (1-\alpha)[T_R(\theta; m/a_j) - T_R(x; m/a_j)]^+ dF_\theta(\theta) d\hat{G}(x|a_j)\hat{\pi}_j \\
0 &= -c_N'(v_N) + q(\kappa)\sum_{j=1}^{M} a_j \int J\big(\theta, \varphi(\theta; m/a_j)\big)\mathbf{1}\{\varphi(\theta) > x\} dF_\theta(\theta) d\hat{G}(x|a_j)\hat{\pi}_j \\
p_R &= \frac{v_R}{v_R + v_N}.
\end{aligned}$$

An important implication of the minimum wage is that it potentially produces different reservation match values for workers of different ability levels. Thus, the endogenous distribution of best available offers, $\hat{G}$, must be conditioned on ability, a. In addition, since workers of different ability levels are differently selected into employment, the distribution of draws from the undirected search technology is no longer equal to π; instead we use $\hat{\pi}$ to denote this endogenous model object.

3.2 When Minimum Wages Bind

We next revisit the question of when minimum wages bind in this model, where by "bind," we mean that the minimum wage results in a mass point in the steady state wage distribution. In a sense, minimum wages affect the distribution of wage offers and the steady state distribution of wages at type N firms, but they never produce a mass point at m, since the conditions of Lemma 2 continue to apply in this case. A minimum wage can only shift the support of the distribution of wage offers from these firms.

The case of R-type firms is different. Figure 2 shows two scenarios in which the minimum wage will bind at R-type firms. For illustrative purposes, we set the minimum wage at \$5.50 an hour and assume that worker ability $a = 1$. We fix the match productivity value at the "winning" firm at \$6 per hour, and consider what happens to $\phi(6, y)$ as y decreases. We can see in the left panel of Figure 2 that the bargained wage hits the lower bound of \$5.50 for lower values of y. In addition, we see that the presence of a binding minimum wage inflates *all* wages above what they would be in the non-binding case. In the right panel of Figure 2, we see what happens when the outside option is fixed at \$6 per hour and the winning match y increases: For values immediately above \$6, the wage is greater than the minimum wage. As y increases, and the future bargaining advantage associated with employment at the firm increases, the flow wage rate is equal to the minimum wage of \$5.50. Eventually, flow productivity becomes sufficiently large that the flow wage payment exceeds the minimum wage. Thus the minimum wage is paid at a job with intermediate values of match productivity, but not at a job with the lowest (but acceptable) match productivity values. This pattern can be observed when there exists a binding minimum wage that is not exceedingly high within a particular labor market environment. At very high values of the minimum wage, however, the minimum wage will be paid even at the lowest acceptable match value.

In general, we can solve for the set of match value pairs (θ, θ') at which the minimum wage is greater than the bargained wage in its absence (while still allowing for minimum wage impacts on outside option values). In these case, either $\phi(\theta, \theta') < m$ or $\phi(\theta', \theta) < m$. Using our model estimates of the labor market environment and continuing to assume that $m = \$5.50$, in Figure 3 we trace out the set of (θ, θ') values for which the minimum wage constraint is binding within the surplus sharing protocol. Both Figures 3 and 2 suggest that large sections of the match pair space will be directly affected by binding minimum wages and, due to the non-monotonicity of the wage offer function $\phi(\cdot, x)$, it is not solely lower match productivities that generate wages equal to m at R-type firms.

4 Data

Although Hall and Krueger (2012) provide compelling evidence that wages are set both through bargaining and non-negotiable offers, the data they collect is not informative about the other primitive parameters characterizing our model. In estimating the model, we primarily utilize data from the *Survey of Income and Program Participation* (SIPP), which has been used successfully in the past to estimate models that include OTJ search and bargaining (Dey and Flinn, 2005; Flinn and Mullins, 2015). The SIPP is a nationally representative, household-based survey comprised of longitudinal panels. Each panel lasts for four years in total. The survey is administered in four-month waves, and at each survey date information on labor market events that have occurred since the last interview is collected retrospectively. As a result, each panel contains 12 waves of the survey. Our data is constructed from waves 3 through 8, [18] yielding data on employment status and wages for a 24 month window, from 2004 to 2006.[19]

Since our principal application of the model developed in the previous section will be to the introduction of a binding minimum wage, we focus our attention on a subpopulation most likely

[18] See Appendix E for the reasons that motivate our choice of these waves

[19] Because of the SIPP's rotating wave structure, the beginning and ending months of each wave are not identical for all survey members

to be affected by this change: individuals between the age of 21 and 30, with 12 or fewer years of schooling. This selection criterion also alleviates concerns that the allowance for individual-level heterogeneity within the model may be too limited, entering as it does only through the scalar a. Of course, the reader should bear in mind that our estimates of model parameters and the policy inferences drawn using these estimates only apply to this particular subpopulation. Individuals who otherwise satisfy the criteria but who had missing information on key variables were also excluded from the final estimation sample.

In Appendix E we offer more precise details on how these data are constructed. Here we give sufficient detail for the reader to understand the analysis that follows. The data used in estimation for sample member i is a panel D_i consisting of the information

$$D_i = \{(e_{i,s}, w^m_{i,s,0}, w^m_{i,s,1}, t_{i,s}),\ s = 1, 2, ..., S_i\} = \{D_{i,s},\ s = 1, 2, ..., S_i\}$$

where $e_{i,s} \in \{0, 1\}$ indicates the employment status of the worker in spell s, $t_{i,s}$ is the duration (in months) of the spell, while S_i is the number of spells observed for worker i. We use $D_{i,s}$ to denote the relevant data for spell s of worker i over the 24-month period.

If employed ($e_{i,s} = 1$), then $w^m_{i,s,0}$ is the wage measured[20] at the beginning of the spell, and $w^m_{i,s,1}$ is the wage at the end of the spell. If unemployed ($e_{i,s} = 0$), wage entries take null values. Spells at the beginning and end of the 24-month period are truncated, in which case we record the truncated durations, letting $w^m_{i,1,0}$ and $w^m_{i,S_i,1}$ be the measured wage at the beginning and end of the sample window, respectively. Since our key solution concept is the notion of *steady state equilibrium*, we must assume that the economy is in steady state when we draw our sample. Under this assumption, employment status and wages in the first observed spell, $\{e_{i,0}, w^m_{i,0}\}$, which is taken at the beginning of the observation window, can be thought of as a random draw from the steady state distribution. The wage distribution associated with the employment spells in progress at the beginning of the sample window can be considered to be a consistent estimator of the steady state wage distribution.

In Table 2 we present some descriptive statistics from this data set. Perhaps the most striking result is the length of time that sample members spend in unemployment spells, 5.847 months on average. Accordingly, the steady state rate of "unemployment" is high, around 20 percent. There are two things to take into account when interpreting this statistic. First, a higher unemployment rate than in the population as a whole is to be expected for this selected sample of low-skill individuals.[21] Second, we are classifying "unemployment" somewhat differently from traditional

[20]This notation allows us to assume later that the true wage $w_{i,s,j}$ and the measured wage in the data are separated by measurement error.

[21]The Bureau of Labor Statistics reports that the unemployment rate for individuals with only a high school diploma was 5.2 percent in 2016, compared to an overall average of 4.0. The unemployment rate of individuals with a Bachelor's degree was 2.7 percent, in comparison.

studies of labor-market flows, where sample members must report actively looking for work (in the previous four weeks as of the interview date) to be considered to be unemployed. We designate unemployment to be any observed absence of an employment spell. Given the documented pattern of movement between traditional definitions of unemployment and the designation of being "out of the labor force,"(Flinn and Heckman, 1983) the high unemployment rate in our data is not inconsistent with results in the literature.

Despite the relative slackness of this labor market, we find that OTJ transitions are not uncommon. On the contrary, we document that 31.8 percent of the employment spells that end in our sample (i.e. that are not truncated by the 24-month cut-off) are ended by a transition to a new employer.

We also make note of the average wage for workers in this sample. At roughly \$14 an hour in 2016 dollars,[22] this implies that many of our sample workers would be affected by recent minimum wage proposals, which have ranged between \$10 and \$15 an hour. Conversely, a \$15 minimum wage in 2016 dollars corresponds to a minimum wage of roughly \$11.80 in our sample. From Figure 1, which shows the distribution of workers' hourly wages, we can see that a sizeable fraction of the sample would indeed be affected by such a change.

To conclude this section, we revisit the statistics provided by the survey of Hall and Krueger (2012), which are presented in Table 1. We separate workers by those who have obtained a high school diploma or less, and those who have attended some college. The pattern of increasing bargaining rates for higher-skilled workers is clearly observable, with college-attending workers doing so at a rate of 40.8 percent, whereas among high school workers only 22.7 percent report bargaining. In order to make an appropriate comparison with our selected SIPP sample, we also examine the 74 workers in this survey who report having no more than a high school diploma and are between 21 and 30 years of age. While this adds considerable imprecision to the estimate of the bargaining proportion, we feel that 15.5 percent is a reasonable fraction for this population, and uncertainty around this number can easily be accommodated in our estimation procedure. Finally, while college workers enjoy the highest premium to bargained wages (nearly \$7 an hour), a small, yet significant, premium can be detected amongst the full sample of high school-educated workers.

5 Estimation

Using the panel dataset constructed from the SIPP in the previous section, we proceed now to the problem of estimating the structural parameters of the model. As is common (see, e.g.,

[22] A back of the envelope calculation using the Bureau of Labor Statistics' Consumer Price Index (CPI) series. See `https://download.bls.gov/pub/time.series/cu/`

Flinn (2006), Flinn and Mullins (2015)), we employ a two-step estimation procedure to recover all of the estimable parameters of the general equilibrium model. We begin by estimating all of the parameters required to fully characterize the partial equilibrium version of the model. In this case, we estimate contact rates (λ_U and λ_E) and treat them as fixed, which they are in equilibrium. In a second step of the estimation, considered below, we utilize the estimates of the partial equilibrium model to estimate a subset of parameters characterizing firms' vacancy-posting decisions. We will denote the vector of primitive parameters required to characterize the partial equilibrium model by Ω.

In order to estimate Ω, we utilize various statistics from the data set, which are denoted by $\mathcal{S}_N$, where the N subscript denotes the number of individuals in the sample. Corresponding to each of these sample statistics, we construct a vector of model-based analogs, $\mathcal{S}(\Omega)$. Given the complexity of the model, there are no closed-form expressions for the $\mathcal{S}(\Omega)$, so that we resort to simulation to approximate this vector. The estimator based on the sample of size N and the selected sample statistics is defined by

$$\hat{\Omega}_N = \arg\min(\mathcal{S}_N - \mathcal{S}(\Omega))'\mathbf{W}(\mathcal{S}_N - \mathcal{S}(\Omega)),$$

which is essentially a Simulated Minimum Distance (SMD) estimator, elsewhere known as Indirect Inference (Gourieroux et al., 1993). Imposing standard regularity conditions on the asymptotic properties of $\mathcal{S}_N$ as sample size $N \to \infty$ guarantee that the estimator $\hat{\Omega}_N$ is itself consistent and uniformly asymptotically normal for any positive-definite weighting matrix W (Gourieroux et al., 1993). In practice, for W we use the inverse of a diagonal matrix, where the ith component of the diagonal is equal to the variance of the ith component of $\mathcal{S}_N$.

Although any sample characteristics vector $\mathcal{S}_N$ of sufficient dimension (at least $\#\Omega$) and with an associated matrix of first partials of the analog vector ($\partial\mathcal{S}(\Omega)/\partial\Omega$) that are linearly independent can be used to form this estimator, in practice a judicious choice of sample characteristics is vitally important to obtain precise parameter estimates in samples of the size that we use. Estimation of the rate parameters $\{\lambda_U, \lambda_E, \delta\}$ is straightforward using employment and duration data, following the analyses of Flinn and Heckman (1982) and Postel-Vinay and Robin (2002). We use the mean duration of employment and unemployment spells which (fixing other parameters) are tightly linked to the contact rate in unemployment, λ_U, and the total arrival rate of potentially spell-ending events, $\lambda_E + \delta$. We target as a third moment the proportion of completed employment spells that result in a job-to-job transition.

In order to estimate the remaining parameters in Ω, we consider the distribution of log-wage changes under a set of employment histories. Noting that, in our model, $w_{i,s,j} = a_i\omega_{i,s,j}$ where $\omega_{i,s,j}$ is either equal to $\phi(\theta_{i,s}, q_{i,s,j})$, or $\varphi(\theta_{i,s})$, depending on the firm that currently employs worker i, we see that:

$$\log(w_{i,s,j}) - \log(w_{i,t,k}) = \omega_{i,s,j} - \omega_{i,t,k} \qquad \forall i, s, t, j, k.$$

Examining log-wage differences eliminates individual ability, a_i, and allows us to separately estimate the remaining primitive parameters without considering the distribution of ability, F_a.[23] We also want to account for the possibility of measurement error in wages, given that wages are self-reported in the SIPP. Thus, we assume the following relationship between wages in the data and the true wage:

$$w^m_{i,s,j} = \epsilon_{i,s,j} w_{i,s,j}$$

where the measurement error, $\epsilon_{i,s,j}$, is drawn independently and identically over time from the distribution F_ϵ, with the restriction that $\mathbb{E}[\epsilon_{i,s,j}] = 1$. Assuming multiplicative measurement error, we get:

$$\log(w^m_{i,s,j}) - \log(w^m_{i,t,k}) = \omega_{i,s,j} + \log(\epsilon_{i,s,j}) - \omega_{i,t,k} - \log(\epsilon_{i,t,k}) \qquad \forall i, s, t, j, k.$$

With the addition of measurement error, there are three distributions to estimate: F_a, F_θ, and F_ϵ. Let $h = \{d_1, d_2, ..., d_K\}$ be a particular employment history (a sequence of employment transitions). Our identification strategy relies on the fact that, given values of the subset of primitive parameters $\{\lambda_u, \lambda_e, \delta, \alpha, p_R, b\}$, conditioning on a particular employment history, h, provides a log-wage change distribution defined by a parametric operator on the match distribution:

$$F_{\Delta \log(w)|H_i=h} = \mathcal{L}_{h,\Omega}[F_\theta]$$

Let us denote the corresponding characteristic function of $F_{\Delta \log(w)|h}$ as $\zeta_{\Delta\omega|h}$. Once our observations are overlayed with measurement error, we have the characteristic function decomposition:

$$\zeta_{\Delta \log(w^m)|h}(t) = \zeta_{\Delta\epsilon}(t)\zeta_{\Delta\omega|h}(t)$$

Thus we estimate the model by attempting to match the distribution of wage changes under three different employment histories:

$$\log(w^m_{i,s+1,0}) - \log(w^m_{i,s,1})|e_{i,s} = 1, e_{i,s+1} = 1 \tag{EE}$$

$$\log(w^m_{i,s+2,0}) - \log(w^m_{i,s,1})|e_{i,s} = 1, e_{i,s+1} = 0, e_{i,s+2} = 1 \tag{EUE}$$

$$\log(w^m_{i,s,1}) - \log(w^m_{i,s,0}) \mid e_{i,s} = 1,\ t_{i,s} = 24, w^m_{i,s,0} \neq w^m_{i,s,1} \tag{EE24}$$

The last line reflects the length of the panel we use, which is 24 months. Identification now rests on these three histories being sufficiently informative to recover F_θ and $F_{\Delta \log(\epsilon)}$. Assuming that

[23] We note that these statements all assume the absence of a binding minimum wage. In this case, job acceptance decisions are made without reference to the individual's value of a, and the wages are expressed as given in the text. When there is a binding minimum wage, it is no longer possible to factor wages and consider mobility decisions without reference to a given value of a.

$F_{\log(\epsilon)}$ is symmetric, the latter is sufficient to identify it up to a location normalization, which we have made already. In the absence of measurement error, it is conceivable that even using only one of these histories would be sufficient, equivalent to the operator $\mathcal{L}_{h,\Omega}$ being invertible for a given history h. Since each distribution is convoluted with $\Delta\log(\epsilon)$ however, we add two more histories to ensure that there are adequate restrictions on the model.

Typically, unless distributions can be directly recovered from the data,[24] nonparametric identification under these conditions requires assumptions of invertibility on the operator defined by the model. It is typically difficult to prove such properties, so we do not undertake this exercise in this paper. We will, however, find that estimation permits a relatively flexible specification of the distributions F_θ and F_ϵ. Specifically, we allow each to be a mixture of two independent log-normal distributions. F_θ therefore has parameters $\mu_1^\theta, \mu_2^\theta, \sigma_1^\theta, \sigma_2^\theta$ with mixing parameter π^θ, and F_ϵ has an equivalent set of parameters. We normalize $\mu_1^\theta = \mu_2^\theta = 0$, [25] while we choose μ_1^ϵ and μ_2^ϵ such that $\mathbb{E}[\epsilon_{i,s,j}] = 1$.

Given these assumptions on the distributions of θ and ϵ, and consistent estimates of all other primitive parameters in Ω except for F_a, it is possible to nonparametrically estimate F_a from the steady state distribution of wages. This follows from the observation that

$$\zeta_{\log(w^m)}(t) = \zeta_a(t)\zeta_{\log(\epsilon)}(t)\zeta_\omega(t)$$

Where the last two characteristic functions on the right hand side come directly from the estimates of the model parameters. Since we will later consider the case of binding minimum wages, in which the model for each ability type must be solved separately, we choose to discretize the distribution of a. Noting that any distribution F_a can be approximated arbitrarily well with K equiprobable types (as K grows large), we will estimate a K-point distribution in this space. That is, we will fix the probability weights $\pi_k = 1/K$, for $k = 1, 2, ..., K$, and estimate the vector of ability levels $a_1, a_2, ..., a_K$.

In practice, we find that a 5 point distribution of this type is sufficient to accurately fit the deciles of the steady state wage distribution, and so we adopt $K = 5$ in the estimation and simulation procedures throughout this paper. [26]

[24] Postel-Vinay and Robin (2002) is a relevant example of such a case

[25] Technically, setting $\mu_1^\theta = 0$ is the only "free" normalization, while further requiring $\mu_1^\theta = \mu_2^\theta$ is a restriction to the space of log-symmetric distributions.

[26] Here we have used the approximation argument as $K \to \infty$ to justify our choice of distribution, but it should be noted that this procedure would not be consistent if we assumed that the "true" model involved 5 types of unknown probability weight and unknown ability.

5.1 Estimates

We estimate the model using transition moments and deciles of the log wage change distributions described above. The estimates of the parameters characterizing the partial equilibrium version of the model are presented in Table 3. Given the relatively long unemployment spells in the data, it is not surprising that our estimated offer arrival rates are low. Our estimate of the meeting rate from the searchers' perspective is 0.115 when they are unemployed and is 0.026 when they are employed. This means that contacts only occur every 8.7 months on average when unemployed and only every 38.5 months on average when employed. Although our estimate of λ_E is low, it is still larger than the estimated rate at which jobs end exogenously, which is $\hat{\delta} = 0.020$. That said, all offers that arrive whether the individual is unemployed or employed are not accepted. Our estimated hazard out of unemployment is $\hat{h}_U$ =0.113 .

Turning to model fit, Tables 4 and 5 demonstrate that the model does a good job of matching the features of the data that we deem to be important and informative from the perspective of the estimation of primitive parameters. Of additional interest, however, are the implications of the model estimates regarding the three underlying probability distributions (and densities, when they exist) for individual ability, match productivity, and measurement error. Figure 4 displays the estimated densities of match productivity, θ, and measurement error, ϵ. These two random variables appear to show a comparable level of dispersion, with the measurement error distribution being slightly more concentrated around its mean value. The estimates suggest that variation in match quality is non-trivial, and it is not uncommon for the quality of the match to diminish or enhance output by as much as 20 percent.

In order to help with a difficult identification problem, the ability distribution has been specified as a five point multinomial distribution, and we have constrained the probabilities of the types to be equal (0.2). Thus, all of the differences in the probability distribution are due to the values of the mass points. Recall that due to our sample selection criteria, we would expect lower amounts of dispersion in general, time-invariant productivity since all of our sample members have 12 or fewer years of schooling. Even in this case, we see large differences in ability, with the most-productive type being approximately three times as productive as the least-productive type. That said, the fourth highest type is slightly less than twice as productive as the least productive type, so the highest ability type accounts for much of the dispersion in the distribution.

Our estimate that the proportion of existing vacancies that involve bargaining, p_R, is 0.074, about half the proportion of bargained wages observed in steady state, 0.155. This can be rationalized by the fact that R-firms are able to retain workers more effectively than are N-firms.

5.1.1 Wage offers at N-firms

In equilibrium, how do N-type firms adjust their wage offers with match quality? In Figure 5, we plot wage offers as a function of match quality, $\varphi(\theta)$. As discussed in Section 2.5, there are several properties of note. First, for low matches close to the reservation match value, θ^*, the wage offer function is flat. Accordingly, for lower values of matches, N-type firms are able to obtain a greater fraction of the total match surplus. Second, the wage offer gets steeper as the match density increases. Higher match densities reflect a greater probability that winning offers close to the current match are drawn, which requires the equilibrium offer to be steeper in this region. For higher match values, where the probability of a better match being drawn is low, φ is once again flat. This implies that N-type firms can also extract a greater fraction of the surplus for higher match values. We can, in fact, compute an implied bargaining share to the workers when an N-type firm is met, that is given by

$$\alpha_N(\theta) = \frac{T_R(\varphi(\theta)) - V_U}{J_N(\theta, \varphi(\theta)) + T_R(\varphi(\theta)) - V_U} \tag{22}$$

The term in the numerator gives the surplus of the match to the worker, while the denominator gives the total surplus of the match to both parties. Figure 6 shows $\alpha_N(\theta)$ using the model solution implied by our estimates. At the reservation match, since the only acceptable offer is θ^*, the worker claims all the surplus. However, since $\varphi(\theta)$ is flat in this region, the implied share to workers quickly decreases, and then increases as competition to retain the match intensifies in the part of the probability distribution in which there is more mass. We see a decrease once again for higher match values, when the local density associated with F_θ decreases. For reference, we also plot the estimated worker's surplus-sharing parameter α at R-type firms in Figure 6, which takes the value 0.192. We see that the implied surplus share of individuals at type N firms frequently exceeds α.. This finding is consistent with the worker strictly preferring meeting an N-type firm for certain productivity draws.

5.1.2 Wage Inequality

A recurrent question in the empirical literature on search frictions concerns the extent to which search frictions can account for observed inequality in worker's wages (Postel-Vinay and Robin, 2002; Hornstein et al., 2011; Flinn et al., 2017). Our extension of standard OTJ search models to include both negotiating and non-negotiating firms has unique implications for this question. Recall that, ignoring measurement error and assuming no binding minimum wage, the log wage rate can be written as $\log(w_i) = \log(a_i) + \log(\omega_i)$, where ω_i is equal to $\phi(\theta_i, q_i)$ if the worker is at an R firm and is equal to $\varphi(\theta_i)$ if she is at an N firm. Thus, overall wage inequality can be conveniently decomposed into a component derived from ability, a_i, and a component derived from search frictions, ω_i. In Figure 7 we plot the distribution of the component ω, which we will refer to as "residual wages" in the steady state at N and R firms. We see that, while workers at both types of firms display dispersion in residual wages, residual wages at R-type firms exhibit much more variance. Equivalently, we can say that the existence of N-type firms compresses the dispersion in wages attributable to search frictions. The logic behind this result is as follows. For lower match values, while N-type firms are forced to offer wages at or near the reservation match θ^*, R-type firms can offer much lower wages, which the worker is willing to accept given her expectation that wage increases are more likely in the future. At higher match values, workers at R-type firms are able to obtain consistently higher wages (and greater fractions of match productivity) through encounters on the job with other firms. At N-firms, on the other hand, the wage offer function φ is relatively flat in this range, and encounters with other firms to not result in large wage increases.

We calculate that the variance of $\log(\omega_i)$, the log-wage residual, is 0.0301 overall, 0.0173 at N-type firms, and 0.0793 at R-type firms. When compared to the variance in log-ability, 0.1142, we find that search frictions in this population account for a reasonable fraction of overall dispersion (20.87 percent).

5.1.3 Inefficient Mobility

We now revisit the possibility in this model for workers to make socially suboptimal job-to-job mobility decisions. Recall that this can only occur when an R-firm and an N-firm compete for a given worker, and the condition $\varphi(\theta_N) < \theta_R < \theta_N$ holds. In this case, the worker may either (a) reject an offer from the N-firm with a dominating match while working for an R-firm; or (b) accept an offer from an R-firm with a dominated match while working for the N-firm. Since $\theta_R < \theta_N$, we can see that either decision is suboptimal from the social planner's perspective.

Using the solution for φ implied by our estimates, in Figure 9 we plot the combination of R-firm and N-firm values that result in an inefficient mobility decision. We can see that this set of combinations comprises a sizeable proportion of the match space. However, the true extent of the phenomenon will clearly depend on the steady state distribution of workers across R and N firms, the fraction p_R, and the match distribution F_θ. More precisely, we define the *rate of inefficient mobility* to be the steady state proportion of on-the-job encounters that result in an inefficient job-acceptance decision. We use this statistic as a measure of this inefficiency's severity in the model. It can be computed as:

$$\text{Rate of inefficient mobility} = \int [F_\theta(\varphi^{-1}(x)) - F_\theta(x)] \cdot [(1-p_R)g_R(x) + p_R g_N(x)]dx. \tag{23}$$

Using our point estimates, we find that 10.09 percent of on-the-job encounters result in an inefficient mobility decision. Of course, when $p_R = 0$ or $p_R = 1$, there is no inefficient mobility. Thus, the proportion of type R firms posting vacancies and in the steady state is a key determinant of the extent of inefficient mobility. In the counterfactual policy experiments we report below, the level of the minimum wage will be seen to have important effects on the equilibrium value of p_R, which in turn will have significant impacts on the degree of inefficient mobility in equilibrium.

5.2 A Partial Equilibrium Counterfactual

In order to investigate the quantitative importance of the frequency of bargaining, as measured by p_R, in determining the level of wage inequality, efficiency, and worker welfare in the labor market, we conduct three counterfactual experiments. First, we eliminate bargaining altogether by setting $p_R = 0$. Second, we consider the changes in partial equilibrium when the fraction of bargaining firms is exogenously set at $p_R = 0.5$. Finally, we consider the case in which $p_R = 1$. In Table 6 we document some changes in important aggregate statistics for these three counterfactual scenarios, relative to baseline. Consistent with our previous arguments concerning wage inequality and renegotiating firms, we see that increasing the fraction of R-type firms to 0.5 leads to an increase in the amount of wage dispersion due to search frictions. In fact, increasing the fraction of R-type firms to 0.5 nearly doubles the variance of the log-wage residual.[27] However, we also see that in the case in which all firms renegotiate ($p_R = 1$), the amount of wage dispersion due to search frictions decreases with respect to the $p_R = 0.5$ case. This is due to the fact that when $p_R = 0.5$, so that in the steady state a very large proportion of jobs are at R-type firms, the variance in the wage offers at N-type firms increases accordingly due to the large degree of competition they face from R-type firms. Thus, the amount of variance in log wages accounted for by search frictions is not a monotonic function of p_R.

We see that increasing the fraction of R-type firms to $p_R = 0.5$ leads to a marked increase in the rate of inefficient mobility. In the counterfactual equilibrium more than 20 percent of firm interactions result in an inefficient job choice. This has consequences for average output per worker, which is reduced by 5 cents an hour. Though this may seem small, one should be reminded that this is a flow value for a single worker, which may still aggregate (across workers and over time) to a sizeable loss in output in the overall market. The increase in inefficient mobility is mechanical in the sense that, for lower values of p_R, increasing this fraction serves to increase the rate at which N and R firms compete for workers. On the other hand, we know that if *all* firms were type R there would also be no inefficient mobility, and this is documented in the last column of Table 6. The mapping between p_R and the measure of inefficient moves is not monotonic.

Workers, according to the measure of average worker welfare in the steady state, appear to prefer an environment in which all firms post wages, at least among the four scenarios we consider

[27] Our sample consists only of individuals who have at most a high school education. Since Hall and Krueger find that individuals with higher levels of completed schooling are more likely to bargain over wages, this may be one factor in the much greater degrees of wage and earnings dispersion for college-completers in comparison with those who have lower levels of education.

here. Since N-type firms, which trade off profits for increases in worker retention, are forced through competition to offer higher wages, this is preferable in equilibrium to being at an R-type firm where high wages are achieved primarily through renegotiation. Validation of this result can be found by returning to Figure 6, that shows that for a large range of match values, the implied bargaining power of workers at N-type firms is much higher than it is at R-type firms. In general, the answer to whether workers prefer a greater fraction of R-type firms depends on the values of the primitive parameters. In particular, the worker's bargaining share, α, at R-type firms, and the rate at which OTJ offers can be solicited, λ_E, are crucial determinants of the payoff to being at a renegotiating firm. When α is higher, being able to engage in bargaining is more profitable for the worker. Thus as $\alpha \to 1$, workers will prefer (in partial equilibrium) a greater fraction of R-firms.

6 General Equilibrium

In this section, we move toward understanding the implications of our modeling framework for policy analysis. The results from our partial equilibrium simulations in the previous section established that the presence of R-firms ($p_R > 0$) has significant impacts on the wage distribution and worker welfare. An immediate corollary to this observation is that equilibrium adjustments in p_R provide a new and quantitatively-relevant mechanism through which a given labor market intervention may affect aggregate statistics of interest. Here we will focus on the model's response in general equilibrium to a minimum wage increase, paying particular attention to the role played by bargaining decisions. We view the minimum wage as an application of first-order interest, since it is a prominent instrument in the modern policy landscape, and is already known to have important implications for efficiency, output, wages, and welfare in frictional labor markets (Flinn, 2006; Flinn and Mullins, 2015).

6.1 Determination of Demand-Side Parameters

We have established, using estimates of the model in partial equilibrium, that the bargaining decisions of firms have important cross-sectional and aggregate consequences for wages, worker welfare, and efficiency. Furthermore, the analysis of Section 3 demonstrated binding minimum wages can "interfere" with the bargaining process at R-type firms, while setting the lower bound for wage offers at N-type firms. This results in a shift in profit margins at both firms which, if different, changes the composition of firm types in equilibrium.

In order to investigate quantitatively the effects of policy changes, such as an increase in the minimum wage, on labor market outcomes in general equilibrium requires us to use particular values for the parameters that characterize firms' vacancy posting decisions. As is well known (e.g., Flinn (2006)), not all of the parameters characterizing the demand side of the model are normally estimable. This is even more the case in our application, since there are two vacancy posting problems. We now discuss the manner in which we have chosen values in order to perform the quantitative minimum wage experiment.

Recall that we assumed that the flow cost of posting a vacancy of type j was

$$c_j(v_j) = \frac{c_j}{1+\psi} v_j^{1+\psi}, \; j = R, N.$$

We have already restricted the elasticity $1+\psi$ to be the same for both R-type and N-type postings. It is most often assumed that $\psi = 0$, in which case the marginal cost of posting a type j vacancy is simply c_j. For technical reasons, in performing counterfactual exercises it is useful to generalize this to the case in which $\psi \neq 0$ (see Lise and Robin (2017)). Lise and Robin obtain a point estimate of ψ of 0.084, with a relatively large 95 percent confidence interval: $[0.005, 0.162]$.[28] Given their estimate, we use the value $\psi = 0.05$ to perform our policy experiments. In light of the uncertainty surrounding what constitutes a sensible choice for this parameter, we conduct a sensitivity analysis of our results to the choice of different values of ψ.

We must also fix a value for the elasticity of the Cobb-Douglas match technology with respect to the measure of vacancies, γ. A standard choice for γ is 0.5, as per the review of Petrongolo and Pissarides (2001), and is the value most often used in studies of this type (e.g., Flinn (2006); Flinn and Mullins (2015); Flinn et al. (2017)). After having fixed the values of γ and ψ, the remaining parameters can be backed out as follows. First, given estimates of the equilibrium values of λ_U and λ_E from estimating the partial equilibrium model parameters Ω, we can infer that $\hat{\kappa} = \hat{\lambda}_U^{1/\gamma}$ and $\hat{\mu}_E = \hat{\lambda}_E / \hat{\lambda}_U$. This allows us to determine the measure of vacancies as: $\widehat{v_R + v_N} = \hat{\kappa}[\hat{M}_U + \hat{\mu}_E(1 - \hat{M}_U)]$. Our estimate of the equilibrium value of the parameter p_R allows us to determine a point estimate of v_R, $\hat{v}_R = \hat{p}_R(\widehat{v_R + v_N})$. Turning to equations (17) and (18), we see that all variables in this expression are known from these demand-side parameter values and consistent estimates of $\hat{\Omega}$ with the exceptions of c_R and c_N. Consistent estimates of these values, which we report in Table 10, are obtained by imposing the two FECs, (17) and (18). In the baseline scenario, for which $\psi = 0.05$, the free entry conditions imply that c_N is equal to \$613.16 and c_R is equal to \$2657.45 . While the difference in costs may appear large, the magnitude of the difference is mainly due to the normalization of scale that must be made between these numbers and the rate, q, at which firms meet workers through the matching function. To get a sense of how reasonable the "real" difference in vacancy costs is, recall that they are implicitly determined by the absolute differences in profitability for each firm type under the free entry condition. Thus, we compute the expected value of entering the market with an R type and an N type vacancy, and find that the difference between these two values is \$151.50 , equivalent to

[28] We compute this using the reported standard errors for the estimate in Table 2 of Lise and Robin (2017)

about 75 cents an hour when discounting by ρ. Thus, we require our differential vacancy costs to rationalize a reasonably small discrepancy in the profitability of each firm type.

6.2 Policy Counterfactual

Having obtained values for all of the parameters required to compute the general equilibrium of the model in any environment nested within our modeling framework, we analyze two counterfactual scenarios: setting the minimum wage to \$10 and \$15 per hour (in 2016 dollars). These two scenarios roughly represent the lower and upper bound of recent minimum wage proposals. In order to properly run the counterfactuals, we use the Bureau of Labor Statistics' Consumer Price Index (CPI) to infer that this would amount to minimum wages of \$7.82 and \$11.72 in 2004 dollars, the year in which our data were collected. Table 11 shows the aggregate statistics associated with the new equilibrium generated by both minimum wage increases.[29] A limitation of our analysis is that we only consider the new steady state equilibria, and do not consider transition dynamics.

Table 11 shows that increases in the minimum wage to binding levels leads to an increase in p_R in the new steady state equilibrium. That p_R increases in our simulation implies that the negative impact on profit is more severe for N-type firms. As discussed, imposing the minimum wage transfers surplus to workers through two different, direct mechanisms: (1) for workers at R firms, when the wage that achieves the surplus split α is less than the legal minimum, the worker is able to claim a fraction greater than α; and (2) at N firms, the minimum wage sets the lower bound for φ, thereby shifting up the support of the wage offer distribution at N-type firms. As profit shifts away from firms to workers, two major aggregate effects play out. First, adjusting to the loss in profit, firms post fewer vacancies, and the contact rates for workers (λ_U, λ_E) decrease. This leads to an increase in unemployment and a loss in output per worker. Second, the lower minimum wage ($m = \$10/hr$) leads to welfare gains (on average) for workers, while the higher minimum wage is dominated by the baseline scenario (i.e., no minimum wage). In general, the average welfare of individuals displays a non-monotonic relationship with minimum wages, since increasing unemployment (driven by a reduction in job creation) begins to outweigh the increase in utility conditional on having a job. This is consistent with the analysis of Flinn (2006), which did not include OTJ search and assumed all firms bargained wages.

6.2.1 Aggregate Impacts on Wages and Welfare

An additional effect that is specific to this model is that, according to our results from the partial equilibrium counterfactual (Table 6), an increase in p_R may lead to welfare losses for workers. Given our estimates of Ω, this seems to be the case here.

[29] Tables 15 and 16 show the results from different choices in ψ. The reader can verify that the general patterns of the results discussed below are preserved under these different choices.

In order to further examine the role played by an increase in p_R, we calculate the same statistics under a counterfactual in which the contact rates are allowed to adjust, but p_R is fixed to its baseline value of 0.074 .[30] Table 12 shows a comparison to the general equilibrium response when $m = \$10/hr$, while Table 13 shows an equivalent comparison when $m = \$15/hr$. Both tables confirm that adjustments in p_R play a crucial role in the equilibrium response. When this fraction is fixed to its baseline level, we see larger welfare gains in both cases. In particular, Table 13 shows that although the estimated model predicts an aggregate welfare loss at $\$15/hr$, this is only true when p_R is permitted to adjust.

Tables 12 and 13 also shed light on exactly how renegotiating firms moderate the welfare impact of minimum wages. First, their presence affects the traditional minimum wage policy trade-off between wage inflation (making workers better off) and higher unemployment (making workers worse off). By again comparing the general equilibrium case to that in which p_R is fixed, we see that allowing p_R to upwardly adjust dampens the inflationary effect on wages of a minimum wage increase. Thus, adjustments in p_R are pushing against the "positive" side of this policy trade-off. This occurs because, while N-firms are forced to shift their entire wage offer function up, using m as a lower bound, R-firms are able to pay an increasing proportion of their workers m as m increases.

Second, the model introduces another unique and quantitatively relevant mechanism in the analysis of minimum wage impacts on labor market equilibrium. When p_R increases, this significantly increases the rate at which workers make inefficient job-acceptance decisions. Tables 12 and 13 both suggest, by virtue of the same comparison used previously, that the increase in this inefficiency significantly affects output and therefore contributes to the observed loss in welfare. Thus, while our partial equilibrium experiments indicated that inefficient mobility was not an extremely important problem quantitatively, our general equilibrium analysis suggests that it can easily become an issue of concern if a policy causes p_R to increase from its baseline level.

6.2.2 Impacts on Wage Dispersion

Looking beyond welfare effects, it is known that increases in the minimum wage can, by shifting the lower tail of the wage distribution up, compress the wage distribution. Since this effect is mechanical, it plays out in both wage-posting models (Burdett and Mortensen, 1998)[31] and in wage-bargaining models (Postel-Vinay and Robin, 2002; Flinn, 2006), and therefore should also be expected in this hybrid setting. Table 11 confirms this to be true: the variance of the log-wage residual (the component of wage variance attributable to search frictions) decreases with

[30] While there is no theoretically coherent way to do this, we simulate this outcome by letting λ_U and λ_E retain their new general equilibrium values, re-setting $p_R = 0.074$, and re-computing equilibrium objects assuming these values of the primitives.

[31] See Engbom and Moser (2017) for an empirical study of this effect at play in Brazil.

m, taking a value a little over a third of its baseline value when $m = \$15/hr$. Figure 10 depicts this compression effect graphically, showing a growing mass point that shifts to the right as the minimum wage increases. We see that the minimum wage eliminates wages formerly to the left of m at type R firms, where the surplus division protocol and α would otherwise have produced them. In addition, at type N firms, binding minimum wages shrink the support of the wage offer function φ which, all else being equal, reduces the dispersion of wages at N-type firms.

While removal of the left tail clearly plays an important role in this observed reduction in wage variance, we wish to highlight two further channels that pertain explicitly to the component of wage variance due to search frictions, $\mathbb{V}[\log(\omega)]$. First, since the contact rate λ_E decreases with respect to δ, workers have fewer opportunities to move up the wage ladder relative to their exit probability, which contracts the right tail of the distribution. Second, we established by virtue of a comparative static in partial equilibrium (Table 6) that R-firms tend to increase $\mathbb{V}[\log(\omega)]$, implying that the endogenous increase in p_R might limit the wage-compression effect of increasing m that would otherwise be expected. Indeed, even in the baseline steady state we can see that the variance of $\log(\omega)$ is nearly four times as large at R firms than it is at N firms.

To test the importance of this mechanism, we exploit the same counterfactual used to evaluate the role of p_R in welfare effects, computing wage dispersion when the general equilibrium effects on contact rates are preserved, but with p_R set back to its baseline value of 0.074 . Inspecting Tables 12 and 13 confirms that this compression effect on the distribution of $\log(\omega)$ is indeed significantly more pronounced when p_R is not permitted to adjust. Thus, in the same sense that an adjustment in p_R limits the welfare impacts of a minimum wage increase, it also limits the extent to which minimum wages can diminish wage dispersion.

6.2.3 Heterogeneity in Policy Effects

We now turn to the analysis of welfare effects across ability types. This not only allows us to reinterpret the aggregate results, but also tells a unique story about crucial welfare impacts not revealed in the aggregate statistics. Results are presented in Table 14. We highlight the differential welfare impacts of the minimum wage increase on types, which is non-monotonic in ability level. To assist in exposition, it will help to clarify the three dominant welfare effects at play. First, conditional on being employed, the minimum wage typically increases wages at all levels (for reasons previously discussed in Section 3), as long as the minimum wage increases are not too dramatic.[32] This effect will clearly be more dominant for lower ability workers whose wages

[32] In the simpler environment of Flinn (2006), with no OTJ search and with all firms negotiating wages, too large of a minimum wage increase could lower the value of unemployed search, which was the individual's outside option. This could then decrease wages over a measurable set of productivity draws. Although we are discussing these factors as if they are isolated, in fact the disemployment effect of the minimum wage increase will be reflected in the wage distributions at type R and type N firms.

are most likely to be constrained by m. Second, these less able workers are more likely to draw matches which are unacceptable to the firm (when $a\theta < m$), and so this disemployment effect works against the inflationary effect on wages. Third is the general equilibrium disemployment effect, as firms receive smaller fractions of the match surplus and post fewer vacancies. The consequent reduction in contact rates affects *all* workers, and particularly higher ability workers, whose wages are less likely to be directly affected by the legal minimum. Lower contact rates increase rates of unemployment in the steady state, but also deflate wages, since employed workers have fewer opportunities to find better matches or renegotiate which is also, in equilibrium, internalized by N-type firms' wage offer function, φ. Thus, while minimum wages are *inflationary* for low ability workers, they can in fact be *deflationary* for high ability workers in the same market, through this general equilibrium channel. It is worthwhile pointing out that this negative impact on the welfare of high-ability workers would be mitigated or eliminated altogether if they operated in separate labor markets. However, in our application all sample members have 12 or fewer years of education and it is less likely that they inhabit distinct labor markets.

At $m = \$10/hr$, we see that the lowest three ability types ($k = 1, 2, 3$) benefit from the minimum wage increase, while the two highest types ($k = 4, 5$) suffer under the lower contact rates, which lead to higher steady state unemployment and deflated wages. When the minimum wage is further increased to \$15/hr, we see a minor reversal in effects. The lowest ability type is rendered (essentially) unemployable. The minimum wage has pushed them out of the labor market, as almost no match is profitable for the firm when associated with this type of worker. This has extreme welfare consequences for workers at this ability level. However, workers of type $k = 4$ benefit from the new minimum wage, which is sufficiently high for them to enjoy some inflationary effect on their wages, offsetting the previously discussed disemployment effects.

This exercise demonstrates that there are non-trivial and non-linear impacts on welfare when disaggregated by worker ability. Our results suggest that minimum wages in the range of recent proposals have quite contrasting impacts on workers at different positions in the ability spectrum. A planner with particular interest in the welfare of low-ability workers would opt to use a modest minimum wage increase, but must understand that not only firms, but also higher ability workers in this labor market segment will be required to pay for such a welfare gain. Additionally, minimum wage impacts cannot necessarily be ranked by their degree of progressivity. An "optimal" minimum wage, as measured by average welfare gains,[33] may well involve significant welfare losses for worker types at the top or bottom of the ability distribution.

[33] Crucially, we do not mean optimal in the traditional, Pareto sense, since the minimum wage is unambiguously output reducing and would be trivially sub-optimal if the planner has access to transfers.

7 Conclusion

In this paper we have developed an estimable model in which firms decide not only whether to post a job vacancy but also whether it be one in which a fixed wage is offered given the worker's productivity or whether it will set a wage given the individual's productivity and her best outside option. The proportion of posted vacancies that involve bargaining is given by p_R, and we have seen that this equilibrium outcome is affected by changes in the economic environment, which in our application was the minimum wage.

Previous research has also examined this question theoretically and quantitatively (Postel-Vinay and Robin (2004); Doniger (2015)), but those authors assumed that there existed only worker and firm heterogeneity in productivities, with the total productivity of a match equal to the product of the worker's productivity and the firm's. This led to sorting equilibria in which certain firms bargained and others posted wages, but the implication of these models was that job-taking decisions remained efficient. In our framework, which assumed no firm heterogeneity but instead allowed for idiosyncratic, worker-firm match heterogeneity in productivity, this was not the case. The proportion p_R was determined through the imposition of free entry conditions, and the model generates inefficient job-taking decisions whenever $p_R \in (0, 1)$.

Under our model estimates, we find that a small increase in p_R leads to a loss in worker welfare and an increase in the wage dispersion due to search frictions. Small increases in p_R from our baseline estimate also lead to an increase in the rate at which workers make inefficient job-taking decisions. Thus, we have introduced here a novel and quantitatively relevant mechanism for policy effects in the labor market, which we then evaluated in the context of a minimum wage increase. We saw that increases in the statutory minimum wage led to a significant increase in the proportion of R-type vacancies in equilibrium, and that the positive impacts of a minimum wage increase (higher wages, lower wage dispersion) were markedly dampened by this general equilibrium channel.

We also showed, as have others (Flinn and Mullins, 2015; Flinn et al., 2017), that large increases in the minimum wage can have extremely negative impacts on the welfare of the least able among any group of workers. In our case, where sample members were restricted to have 12 or fewer years of formal schooling, we still found evidence for there being large differences in unobserved, individual-specific productivity, with the most able type being three times more productive than the least productive. In a general equilibrium setting, small but significant increases in the minimum wage tended to increase the welfare of the least able, while reducing the welfare of the most able through a disemployment effect. Large changes in the minimum wage, instead, essentially made the lowest skill types unemployable and led to disastrous decreases in their average welfare. These results indicate that the choice of a minimum wage rate should be made with as much knowledge as possible about the structure of the labor market in which it is imposed. Early evidence from an analysis of the labor market in Seattle after that city's imposition of a $15 minimum wage indicates that low-skill individuals lost jobs and hours in the immediate aftermath of the change (Jardim et al., 2017). Such a finding is consistent with the results reported here.

We also found that local increases in p_R led to increases in earnings inequality. Recent laws such as the one in Massachusetts that seek to limit employers' use of the current labor market status of an applicant in determining the wage offer to an individual would seem to lead to reduced wage inequality given our results. The motivation for these types of laws is the reduction or elimination of a discrimination "multiplier," whereby discrimination against an individual at a current employer does not carry over to the wage offered at the next job. We do not consider discrimination in our search framework (see instead Flabbi (2010)), but we have seen how success in previous job-finding activities carries over to the current job market state for all jobs occupied during the same employment spell. Eliminating this carry-over reduces the transmission of the luck factor across jobs and produces more equitable labor market outcomes. It is also the case that a legislated value of $p_R = 0$ insures efficient job-taking decisions, just as $p_R = 1$ does. From an equity point of view, within our search framework, it would seem that $p_R = 0$ is a desirable outcome.

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A Tables

Table 1: Descriptive Statistics from Hall and Krueger (2012) Survey

	High School, 21-30	All High School	All College
% Bargain	15.52	22.738	40.871
	[7.60, 25.89]	[19.565, 26.387]	[38.298, 44.366]
Mean Wage	12.73	14.367	22.888
	[10.94, 14.61]	[13.668, 15.129]	[22.193, 23.542]
Mean Wage | Bargain	17.36	14.735	25.007
	[8.35, 27.64]	[13.040, 16.602]	[23.658, 26.214]
Mean Wage | No Bargain	11.77	12.323	18.194
	[10.18, 14.13]	[11.640, 13.241]	[17.177, 19.113]
Sample Size	74	691	1822

Notes: This table shows some descriptive statistics from the survey data collected in Hall and Krueger (2012). Bracketed intervals indicate 95% confidence intervals for the statistics calculated. Bargained wages are judged by the answer to the survey question "When you were offered your current/previous job, did your employer make a "take-it-or leave-it" offer or was there some bargaining involved?" The left column shows statistics computed for high school graduates aged 21-30, the middle column shows high school graduates of all ages, the right column is for college graduates. Data is publicly available at `http://www.stanford.edu/~rehall/Hall_Krueger_2011-0071_programs_and_results`

Table 2: Descriptive Statistics from SIPP Sample

Description	Notation	Moment
Average duration of unemployment spells	$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	5.847
		[5.527, 6.216]
Average duration of employment spells	$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	13.897
		[13.540, 14.278]
% Of (non-truncated) employment spells	$P[E_{i,s+1} = 1 \mid E_{i,s} = 1,\ s < S_i]$	31.825
ending in EE transition		[27.382, 35.116]
Average number of spells per worker	$\mathbb{E}[S_i]$	1.936
		[1.889, 2.014]
Average wage at beginning of sample	$\mathbb{E}[W_{i,0,0} \mid E_{i,0} = 1]$	11.239
		[10.986, 11.532]
% Unemployed at beginning of sample	$P[E_{i,0} = 0]$	20.632
		[18.815, 22.447]
Sample size	N	1488

Notes: This table shows descriptive statistics from the SIPP. Durations are reported in months, wages are reported in \$/hour. Bracketed intervals indicate 95% confidence intervals for the statistics calculated.

Table 3: Estimates

Model Parameters								
λ_U	λ_E	δ	p_R	α	b	σ_0	σ_1	π_θ
0.115	0.026	0.020	0.074	0.192	0.213	0.057	0.139	0.502
$[0.104, 0.123]$	$[0.023, 0.030]$	$[0.018, 0.022]$	$[0.058, 0.086]$	$[0.122, 0.221]$	$[0.212, 0.286]$	$[0.016, 0.057]$	$[0.065, 0.238]$	$[0.497, 0.504]$
Measurement Error				Ability Distribution				
$\sigma_{\epsilon,1}$	$\sigma_{\epsilon,2}$	π_ϵ		a_1	a_2	a_3	a_4	a_5
0.025	0.132	0.503		7.58	9.26	11.50	13.96	20.16
$[0.009, 0.038]$	$[0.111, 0.169]$	$[0.500, 0.507]$		$[7.31, 8.06]$	$[8.99, 11.52]$	$[9.56, 14.34]$	$[11.51, 14.51]$	$[19.03, 21.15]$
				π_1	π_2	π_3	π_4	π_5
				0.20	0.20	0.20	0.20	0.20

Notes: This table presents estimates from the SMD procedure of the baseline model, in which minimum wages do not bind. Parameters are as described in the text. $F\theta$, the match distribution, is modeled as a mixture of two normals with standard deviations (σ_1, σ_2) and mixing probability $\pi\theta$. Measurement error, $F\epsilon$ is modeled similarly. Numbers in square brackets show the 95% confidence intervals for each parameter, which have been computed by nonparametric bootstrap, using 100 resamples of the data.

Table 4: Model Fit I: Transitions

Moment	Model	Data
$\mathbb{E}[T_{i,s} \mid E_{i,s} = 0]$	5.86	5.85
$\mathbb{E}[T_{i,s} \mid E_{i,s} = 1]$	13.90	13.90
$P[E_{i,s+1} = 1 \mid E_{i,s} = 1]$	0.31	0.32
$P[\text{Wage Bargained}]$	0.16	0.16

Table 5: Model Fit II: Distributions

	$\Delta \log(w) \mid EE$		$\Delta \log(w) \mid EUE$		$\Delta \log(w) \mid t_i = 24$		$\log(w)$	
	Model	Data	Model	Data	Model	Data	Model	Data
q_{10}	-0.12	-0.24	-0.24	-0.35	-0.09	-0.08	1.88	1.87
q_{20}	-0.05	-0.06	-0.16	-0.21	0.00	0.02	1.98	2.00
q_{30}	-0.01	0.00	-0.11	-0.09	0.03	0.05	2.09	2.08
q_{40}	0.02	0.00	-0.06	-0.04	0.07	0.07	2.19	2.20
q_{50}	0.05	0.06	-0.02	0.00	0.10	0.10	2.30	2.30
q_{60}	0.09	0.12	0.02	0.05	0.13	0.12	2.40	2.40
q_{70}	0.13	0.17	0.06	0.12	0.16	0.16	2.51	2.52
q_{80}	0.17	0.22	0.11	0.20	0.22	0.22	2.68	2.67
q_{90}	0.24	0.36	0.19	0.36	0.31	0.37	2.88	2.88

Table 6: Results for Partial Equilibrium Experiments

	Baseline ($p_R = 0.074$)	$p_R = 0$	$p_R = 0.5$	$p_R = 1$
$\mathbb{E}[W \mid N - \text{firm}]$	9.61	9.89	7.97	-
$\mathbb{E}[W \mid R - \text{firm}]$	8.59	-	8.61	10.26
Worker Welfare	138.11	141.36	123.55	127.53
$\mathbb{V}[\log(\omega)]$	0.0301	0.0170	0.0905	0.0677
$\mathbb{V}[\log(\omega) \mid N - \text{firm}]$	0.0173	0.0170	0.0230	-
$\mathbb{V}[\log(\omega) \mid R - \text{firm}]$	0.0793	-	0.1170	0.0677
Average output per worker ($/hr)	10.96	11.04	10.88	11.11
Rate of inefficient mobility (%)	10.09	0.00	22.08	0.00
Percentage of dispersion from search frictions (%)	20.87	12.96	44.22	37.24

Table 7: Model Estimates for $p_R = 0$ and $p_R = 1$

	Baseline Model	No Renegotiation ($p_R = 0$)	All Renegotiation ($p_R = 1$)
λ_U	0.115	0.118	0.122
λ_E	0.026	0.026	0.021
δ	0.020	0.020	0.020
p_R	0.074	0	1
α	0.192	-	0.097
b	0.213	0.263	0.572
σ_0	0.057	0.182	0.191
σ_1	0.139	0.010	0.005
π_θ	0.502	0.502	0.499
$\sigma_{\epsilon,0}$	0.025	0.045	0.005
$\sigma_{\epsilon,1}$	0.132	0.238	0.151
π_ϵ	0.503	0.506	0.505
θ^*	0.787	0.865	0.838
Q_N	0.061	0.394	0.089

Table 8: Model Fit for $p_R = 0$ and $p_R = 1$: Transitions

Moment	$p = 0$	$p = 1$	Data
$\mathbb{E}[t_{i,s} \mid E_{i,s} = 0]$	5.85	5.83	5.85
$\mathbb{E}[t_{i,s} \mid E_{i,s} = 1]$	13.89	13.82	13.90
$P[E_{i,s+1} = 1 \mid E_{i,s} = 1]$	0.32	0.31	0.32

Table 9: Model Fit for $p_R = 0$ and $p_R = 1$: Distributions

	$\Delta\log(w) \mid EE$			$\Delta\log(w) \mid EUE$			$\Delta\log(w) \mid t_i = 24$		
	$p=0$	$p=1$	Data	$p=0$	$p=1$	Data	$p=0$	$p=1$	Data
q_{10}	-0.25	-0.09	-0.24	-0.33	-0.26	-0.35	0.00	-0.11	-0.08
q_{20}	-0.11	-0.01	-0.06	-0.19	-0.17	-0.21	0.00	-0.01	0.02
q_{30}	-0.03	0.04	0.00	-0.12	-0.12	-0.09	0.00	0.03	0.05
q_{40}	0.01	0.08	0.00	-0.06	-0.07	-0.04	0.00	0.07	0.07
q_{50}	0.05	0.11	0.06	-0.01	-0.02	0.00	0.00	0.10	0.10
q_{60}	0.09	0.13	0.12	0.03	0.00	0.05	0.00	0.13	0.12
q_{70}	0.15	0.16	0.17	0.09	0.02	0.12	0.00	0.15	0.16
q_{80}	0.23	0.21	0.22	0.17	0.07	0.20	0.00	0.21	0.22
q_{90}	0.35	0.29	0.36	0.30	0.15	0.36	0.00	0.30	0.37

Table 10: Implied value of cost parameters in General Equilibrium

Case	c_n	c_r	γ
$\psi = 0.05$	613.16	2657.45	0.5
$\psi = 0.1$	805.96	3963.47	0.5
$\psi = 0.15$	1059.39	5911.36	0.5

Table 11: Aggregate Results for General Equilibrium Experiments

	Baseline	$m = 10$	$m = 15$
λ_U	0.115	0.10	0.06
λ_E	0.026	0.02	0.01
p_R	0.074	0.31	0.48
$\mathbb{E}[W \mid N-\text{firm}]$	9.61	9.71	12.00
$\mathbb{E}[W \mid R-\text{firm}]$	8.59	9.68	12.74
Worker Welfare	138.11	143.85	129.84
$\mathbb{V}[\log(\omega)]$	0.0301	0.0244	0.0117
$\mathbb{V}[\log(\omega) \mid N-\text{firm}]$	0.0173	0.0080	0.0015
$\mathbb{V}[\log(\omega) \mid R-\text{firm}]$	0.0793	0.0396	0.0170
Average output per worker ($/hr)	10.96	10.34	6.56
Rate of inefficient mobility (%)	10.09	20.83	19.70

Table 12: Fixed vs Endogenous p in GE ($m = \$10/hr$)

	Baseline	Endogenous	Fixed
λ_U	0.115	0.10	0.10
λ_E	0.026	0.02	0.02
p_R	0.074	0.31	0.07
$\mathbb{E}[W \mid N-\text{firm}]$	9.61	9.71	10.57
$\mathbb{E}[W \mid R-\text{firm}]$	8.59	9.68	9.90
Worker Welfare	138.11	143.85	153.43
$\mathbb{V}[\log(\omega)]$	0.0301	0.0244	0.0138
$\mathbb{V}[\log(\omega) \mid N-\text{firm}]$	0.0173	0.0080	0.0096
$\mathbb{V}[\log(\omega) \mid R-\text{firm}]$	0.0793	0.0396	0.0301
Average output per worker ($/hr)	10.96	10.34	10.78
Rate of inefficient mobility (%)	10.09	20.83	8.30

Table 13: Fixed vs Endogenous p in GE ($m = \$15/hr$)

	Baseline	Endogenous	Fixed
λ_U	0.115	0.06	0.06
λ_E	0.026	0.01	0.01
p_R	0.074	0.48	0.07
$\mathbb{E}[W \mid N-\text{firm}]$	9.61	12.00	12.94
$\mathbb{E}[W \mid R-\text{firm}]$	8.59	12.74	13.02
Worker Welfare	138.11	129.84	148.51
$\mathbb{V}[\log(\omega)]$	0.0301	0.0117	0.0077
$\mathbb{V}[\log(\omega) \mid N-\text{firm}]$	0.0173	0.0015	0.0063
$\mathbb{V}[\log(\omega) \mid R-\text{firm}]$	0.0793	0.0170	0.0132
Output (\$/hr)	10.96	6.56	8.08
Rate of inefficient mobility (%)	10.09	19.70	7.01

Table 14: Results by Ability for General Equilibrium Experiments

		$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$\mathbb{E}[w \mid N-\text{firm}]$	Baseline	5.84	7.14	8.85	10.77	15.52
	$m=10$	7.91	8.32	8.96	9.74	13.18
	$m=15$	NaN	NaN	11.80	12.08	13.04
$\mathbb{E}[w \mid R-\text{firm}]$	Baseline	5.11	6.25	7.89	9.56	13.72
	$m=10$	7.94	8.31	9.04	9.87	12.80
	$m=15$	NaN	11.78	11.87	12.31	13.89
$\mathbb{V}[\log(\omega)]$	Baseline	0.0307	0.0311	0.0299	0.0304	0.0308
	$m=10$	0.0003	0.0036	0.0141	0.0313	0.0664
	$m=15$	NaN	0.0001	0.0004	0.0037	0.0288
$\mathbb{V}[\log(\omega) \mid N-\text{firm}]$	Baseline	0.0172	0.0174	0.0172	0.0172	0.0172
	$m=10$	0.0001	0.0018	0.0058	0.0126	0.0175
	$m=15$	NaN	NaN	0.0000	0.0005	0.0037
$\mathbb{V}[\log(\omega) \mid R-\text{firm}]$	Baseline	0.0806	0.0803	0.0797	0.0817	0.0808
	$m=10$	0.0005	0.0053	0.0221	0.0497	0.1107
	$m=15$	NaN	0.0001	0.0005	0.0005	0.0037
Welfare	Baseline	138.11	138.11	138.14	138.15	138.08
	$m=10$	178.44	156.10	138.49	126.02	120.22
	$m=15$	47.65	189.36	160.32	140.31	111.56
Output	Baseline	6.66	8.11	10.15	12.27	17.55
	$m=10$	5.38	7.89	9.79	11.78	16.86
	$m=15$	0.00	0.73	6.78	10.39	14.88
Inefficient Mobility	Baseline	10.09	10.09	10.09	10.09	10.09
	$m=10$	8.41	20.57	23.66	24.19	24.33
	$m=15$	NaN	0.58	9.85	23.28	24.81

Table 15: Sensitivity Analysis: Aggregate Results for General Equilibrium, $\psi = 0.10$

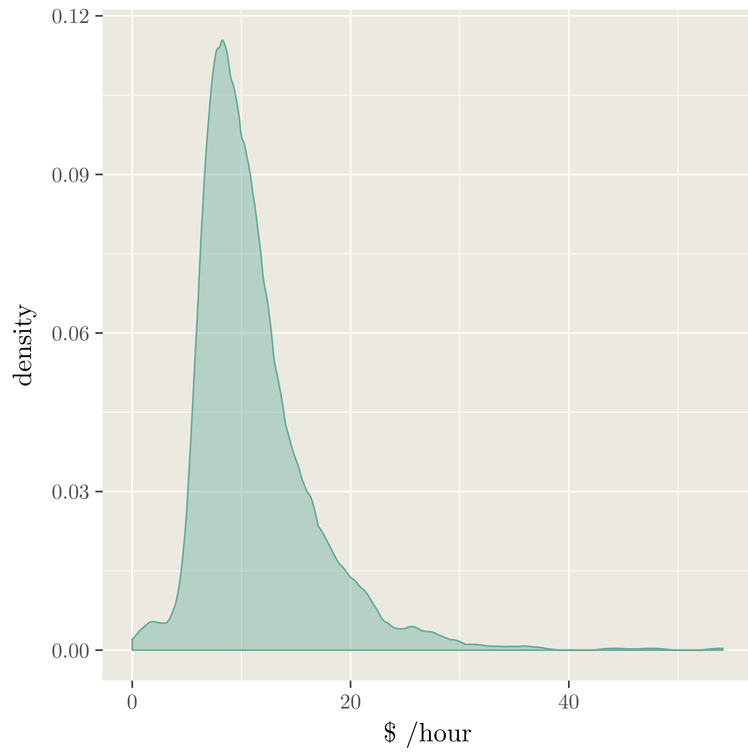
	Baseline	$m = 10$	$m = 15$
λ_U	0.115	0.10	0.06
λ_R	0.026	0.02	0.01
p_R	0.074	0.22	0.36
$\mathbb{E}[W \mid N - \text{firm}]$	9.61	9.96	12.16
$\mathbb{E}[W \mid R - \text{firm}]$	8.59	9.65	12.67
Worker Welfare	138.11	145.53	130.61
$\mathbb{V}[\log(\omega)]$	0.0301	0.0195	0.0087
$\mathbb{V}[\log(\omega) \mid N - \text{firm}]$	0.0173	0.0084	0.0021
$\mathbb{V}[\log(\omega) \mid R - \text{firm}]$	0.0793	0.0351	0.0146
Average output per worker ($/hr)	10.96	10.37	6.62
Rate of inefficient mobility (%)	10.09	18.15	20.13

Table 16: Sensitivity Analysis: Aggregate Results for General Equilibrium, $\psi = 0.15$

	Baseline	$m = 10$	$m = 15$
λ_U	0.115	0.10	0.06
λ_E	0.026	0.02	0.01
p_R	0.074	0.17	0.28
$\mathbb{E}[W \mid N - \text{firm}]$	9.61	10.10	12.28
$\mathbb{E}[W \mid R - \text{firm}]$	8.59	9.65	12.62
Worker Welfare	138.11	146.59	131.30
$\mathbb{V}[\log(\omega)]$	0.0301	0.0176	0.0070
$\mathbb{V}[\log(\omega) \mid N - \text{firm}]$	0.0173	0.0088	0.0026
$\mathbb{V}[\log(\omega) \mid R - \text{firm}]$	0.0793	0.0331	0.0127
Average output per worker ($/hr)	10.96	10.40	6.68
Rate of inefficient mobility (%)	10.09	15.71	18.80

B Figures

Figure 1: Steady State Wage Distribution from the SIPP

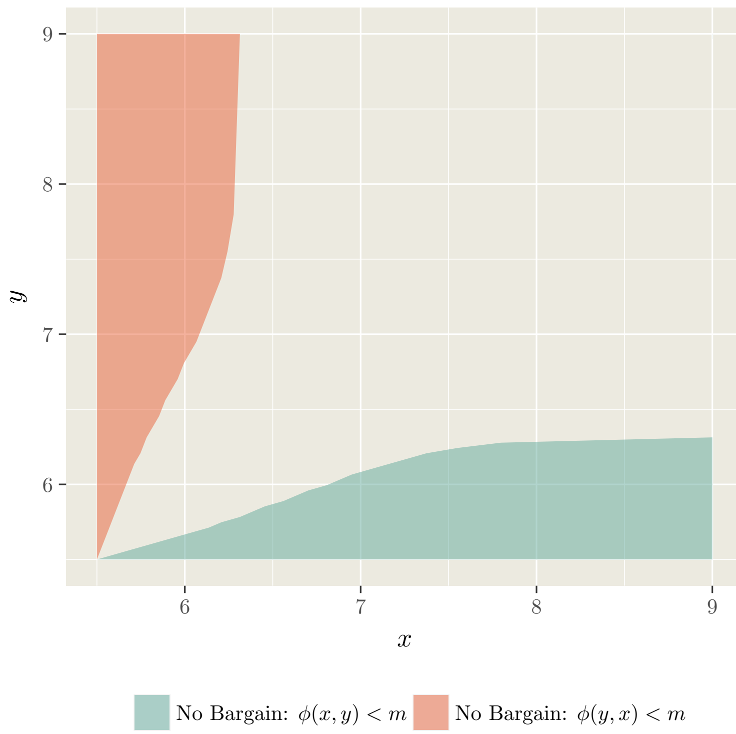


Notes: This figure shows a nonparametric density plot of "steady state" wages, which are taken from workers who are employed at the begnning of our 24-month SIPP sample.

Figure 2: Binding Minimum Wages at R-Firms

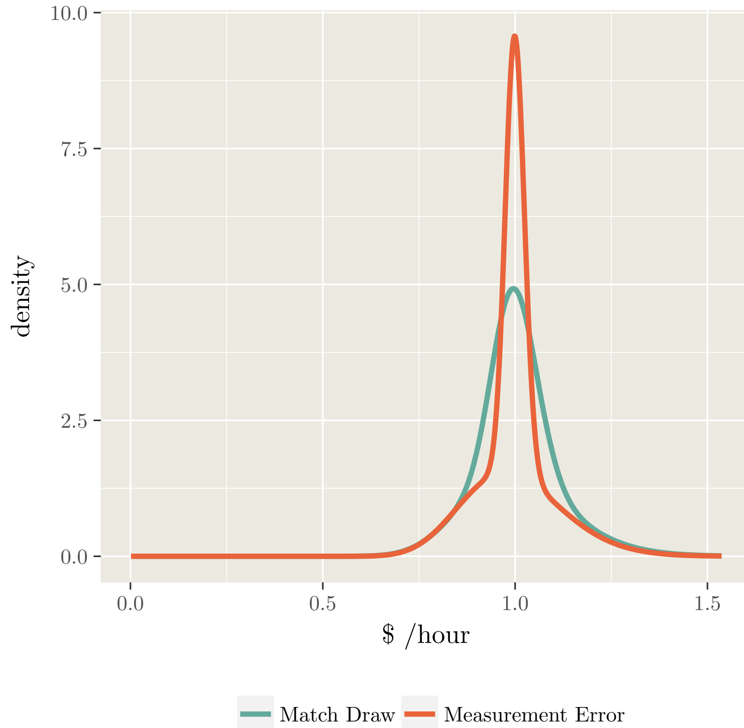
Notes: In this figure we plot bargained wages ϕ. In the left panel, we fix the outside option at a match value of \$6/hr and vary the outside option y. On the right, we fix the outside option at \$6/hr and vary the winning match y.

Figure 3: Region of the match pair space in which the minimum wage binds



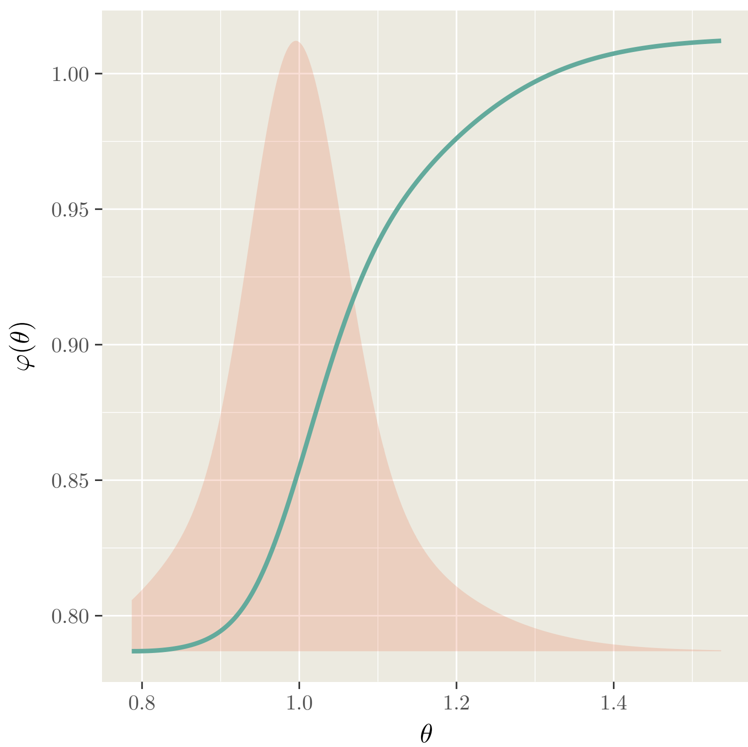
Notes: This figure shows the combinations of x and y for which the minimum wage interferes with the bargaining process.

Figure 4: Densities of the match distribution, F_θ, and measurement error, F_ϵ



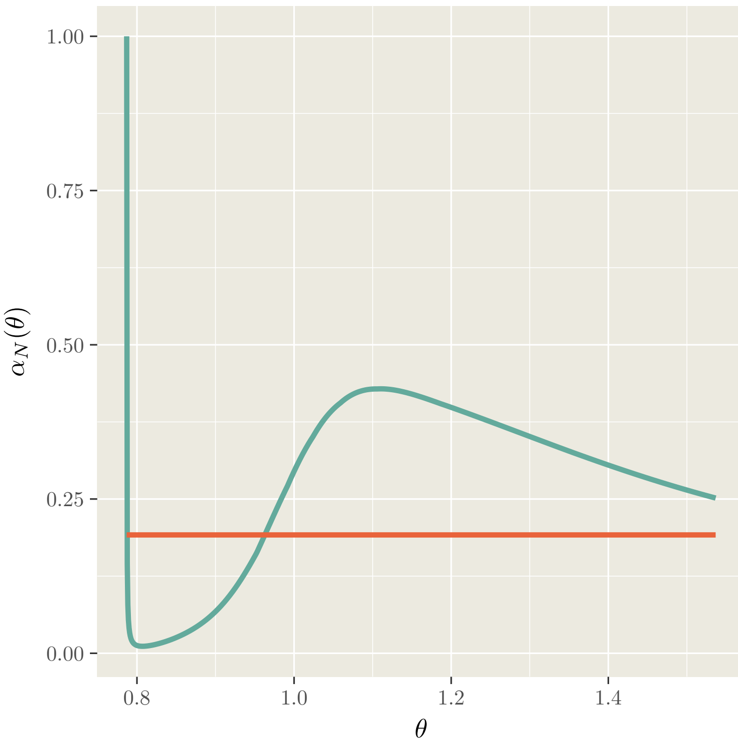
Notes: This figure shows the the density of the distribution of match values, F_θ, and the density of the measurement error distribution, F_ϵ. Realizations of both random variables can be measured in $/hr.

Figure 5: Wage offer function, φ, and density of match values, f_θ



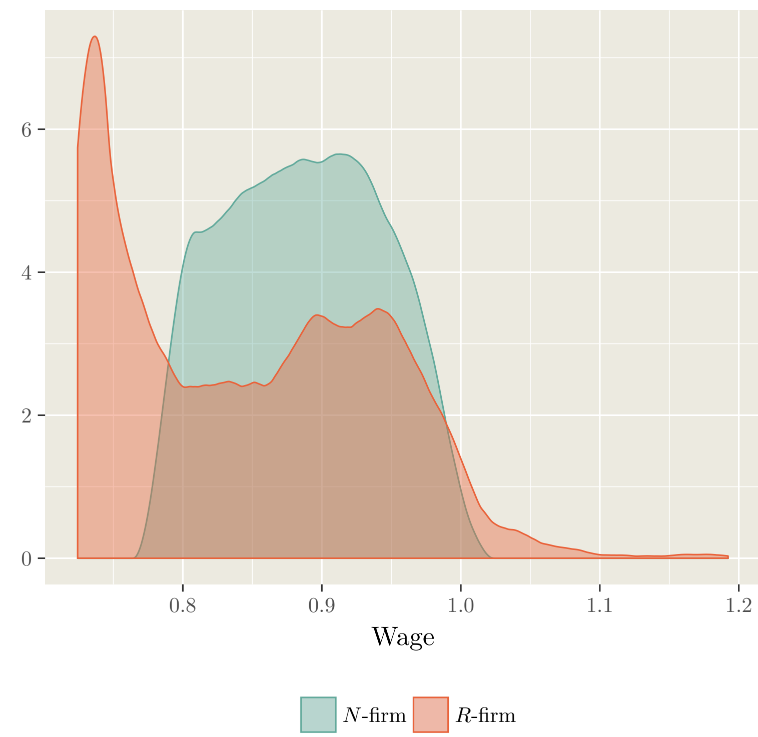
Notes: This figure shows the wage offer function, φ of N-firms. For exposition, the (re-scaled) density f_θ of the match distribution F_θ is shown in the background.

Figure 6: Implied Bargaining Parameters for N-firms, $\alpha_N(\theta)$.



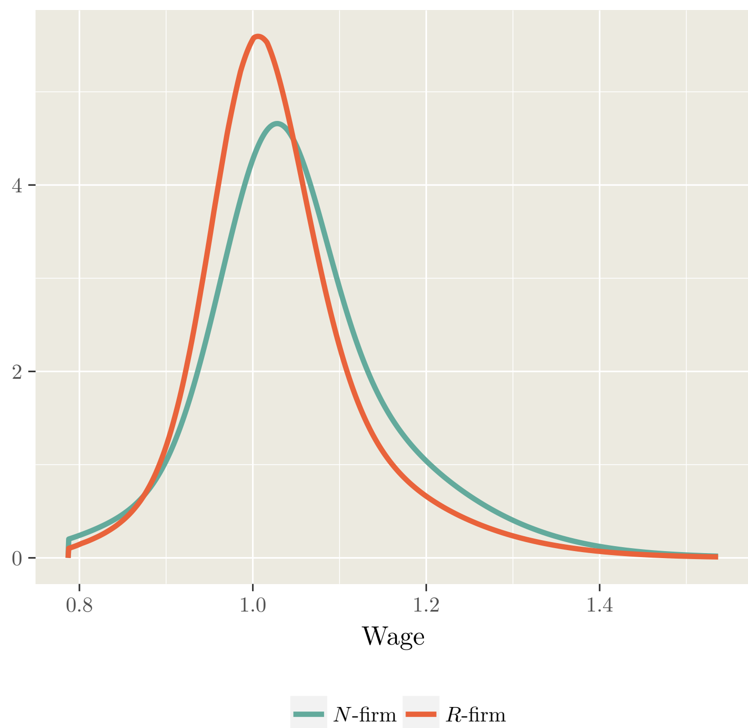
Notes: This figure shows the implied bargaining parameter, defined as the share of surplus to the worker given the wage offer, $\varphi(\theta)$ of N-firms. For reference, the estimated bargaining parameter for R-firms, α, is plotted also. See equation (22) for further explanation.

Figure 7: Wage distribution in steady state at R and N firms



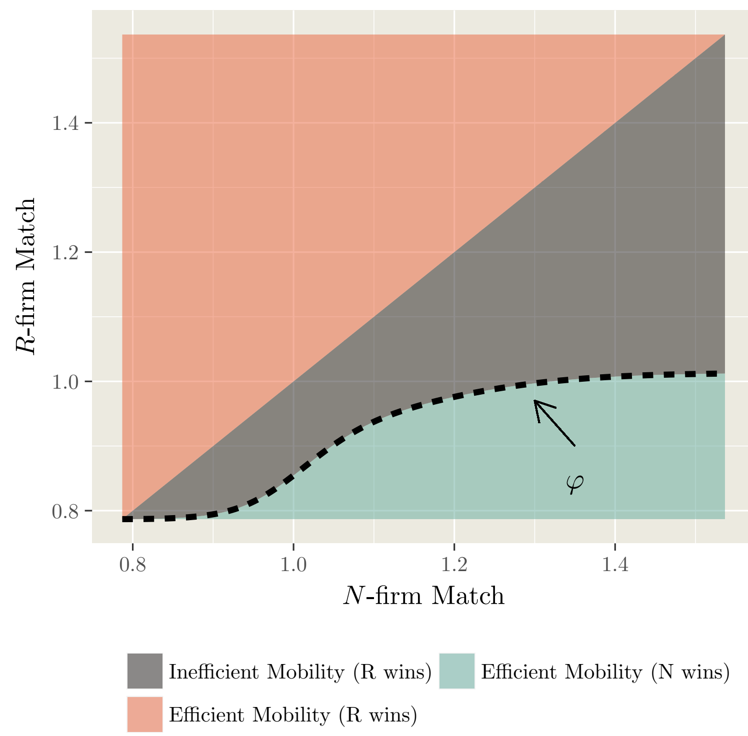
Notes: This figure shows the wage distribution in steady state at N and R firms, using parameter estimates from the baseline model.

Figure 8: Densities of match values in steady state at R and N firms



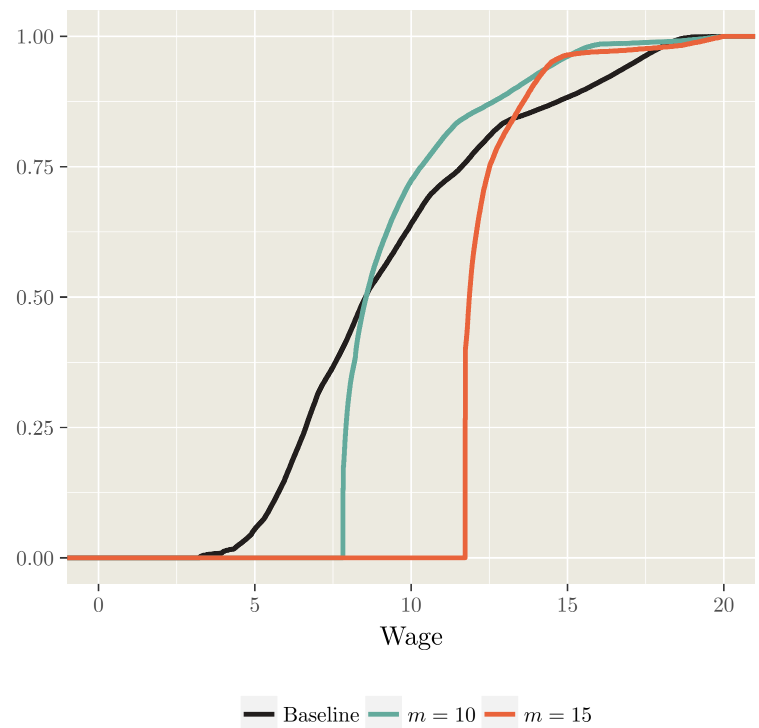
Notes: This figure shows the distribution in steady state of match values at N and R firms, using parameter estimates from the baseline model.

Figure 9: Inefficient Mobility



Notes: This figure shows the combinations of matches at R and N firms that result in efficient and inefficient mobility. An N-firm with match x wins if and only if the wage offer $\varphi(x)$ is greater than the R-firm's match y. When $\varphi(x) < y < x$, the model exhibits inefficient mobility.

Figure 10: Steady State Wage Distributions



Notes: This figure shows the cumulative distribution of wages in steady-state for three general equilibrium scenarios: the baseline, when $m = \$10/hr$, and when $m = \$15/hr$.

C Proofs

In the following set of results, we make extensive use of equations (11) and (13), which express the N-firm's value function as

$$J_N(\theta, w) = \Gamma(\Phi(w), F_\theta(w))(\theta - w) = \Gamma(w)(\theta - w)$$

where we adopt, in the second equality, a convenient redefinition of the function Γ. The second equality technically requires a redefinition of Γ, but we is notationally abusive, but useful, so we adopt it here. The function Γ is an expression that combines the probability that w successfully hires a worker, with the rate at which the worker is lost when w is the non-negotiable wage.

Secondly, we assume the following tie-breaking rule: when two-firms make equally valuable wage offers, the worker moves from the incumbent to the new firm. This is an inconsequential assumption since such ties occur with zero probability. Assuming the alternative tie-breaking rule produces an observationally equivalent equilibrium outcome.

Proof of Lemma 2

Proof. Since all offers $w < \theta^*$ are, by definition, never accepted, we know that $\underline{w} \geq \theta^*$. Now assume that $\underline{w} > \theta^*$, and consider the optimal offer made by a firm when a match $x \in (\theta^*, \underline{w})$ is drawn. Since any offer $w \in (\theta^*, x)$ is both profitable to the firm and acceptable to an unemployed worker (who is met with positive probability), we have a contradiction. ∎

Lemma 4. Γ *is (i) strictly increasing; and (ii) continuous if and only if* Φ *is continuous.*

Proof. **(i)**: This follows directly from our assumption that F_θ is strictly increasing in w (the support of F_θ is a connected set) and Φ is, by definition, non-decreasing. Thus, Γ must be strictly increasing in w, given its form in (13).

(ii): This is immediate, since Γ is a continuous transformation of Φ and F_θ. ∎

Lemma 5. *In equilibrium, the wage offer distribution* Φ *is continuous.*

Proof. Note that a discontinuity in Φ at some w implies a mass point at w and, by Lemma 4, Γ is discontinuous. Given the tie-breaking rule, we have that $\lim^+ \Gamma(w) > \Gamma(w)$. This is caused by a discontinuous increase in the probability of retaining a worker.[34] Hence, $\lim^+ J(\theta, w) > J(\theta, w)$ for any θ, and for any firm offering wage w, an improvement in profit can be made by offering $w + \epsilon$ where ϵ is arbitrarily small. Thus no firm prefers to offer w, a contradiction. ∎

The following corollary is immediate.

Corollary 1. Γ *is continuous.*

Lemma 6. *In equilibrium, wages are given by an almost everywhere deterministic function,* φ*.*

Proof. Suppose otherwise. Then for a firm with match θ, the firm is indifferent over a set $\mathcal{W}$ with positive Lebesgue measure:

$$\Gamma(w)(\theta - w) = c, \ \forall \ w \in \mathcal{W}.$$

Likewise, for a firm with match $\hat{\theta} \neq \theta$, indifference is achieved over a set $\hat{\mathcal{W}}$:

$$\Gamma(w)(\hat{\theta} - w) = \hat{c}, \ \forall \ w \in \hat{\mathcal{W}}.$$

If $\mathcal{W} \cap \hat{\mathcal{W}}$ has positive measure, we must have $\Gamma(w)(\theta - \hat{\theta}) = c - \hat{c}$ for all w in this intersection, which can be true only if $\Gamma(w)$ is everywhere constant, contradicting Lemma 4. Therefore, $\mathcal{W} \cap \hat{\mathcal{W}} = \emptyset$, and so this can only be true for a countable set of matches, which have measure zero under our regularity assumptions on F_θ.

∎

Lemma 7. *The wage offer function,* φ*, is strictly increasing in match values,* θ*.*

[34] Notice that if we had assumed the alternative tie-breaking rule, there would be a discontinuous increase in the probability of hiring the worker, and the result would still follow.

Proof. Let $\varphi(\theta) = w$. This implies that:

$$\Gamma(w)(\theta - w) > \Gamma(\hat{w})(\theta - \hat{w}), \ \forall \hat{w} < w$$

Rearranging this expression we get:

$$(\Gamma(w) - \Gamma(\hat{w}))\theta > \Gamma(w)w - \Gamma(\hat{w})\hat{w}, \ \forall \hat{w} < w$$

By Lemma 4, $\Gamma(w) - \Gamma(\hat{w}) > 0$, which implies that for any $\theta' > \theta$, we have

$$(\Gamma(w) - \Gamma(\hat{w}))\theta' > \Gamma(w)w - \Gamma(\hat{w})\hat{w}, \ \forall \hat{w} < w$$

So when the match value is θ', the above inequality implies that w is also preferred to all $\hat{w} < w$, and so $\varphi(\theta') \geq \varphi(\theta)$, However, if this inequality is not strict, repeated application of the above inequality implies that $\varphi(z) = w$ for all $z \in [\theta, \theta']$. However, this implies a discontinuity in Φ, contradicting Lemma 5. Thus, the inequality must be strict. ∎

To prove differentiability, we make use of the following commonly known result.

Lemma 8. *If a function, $f : \mathbb{R} \mapsto \mathbb{R}$, is bounded, and monotonically increasing, it is almost everywhere (according to Lebesgue measure) differentiable.*

Proof. See, for example, Result 11.42 in Titchmarsh (1932). ∎

Lemma 9. *The wage offer function, φ is almost everywhere differentiable and lower semi-continuous.*

Proof. Consider the function φ on the domain $[\theta^*, \theta]$. Since $\varphi(\theta)$ is bounded above by θ, bounded below by θ^*, and strictly monotonically increasing, it follows from Lemma 8 that φ must be almost everywhere differentiable (and hence almost everywhere continuous).

Consider now a potential discontinuity in φ at θ. Let d^+ and d^- denote the differentiation operation, taking *right* and *left* limits, respectively. Let $\varphi^-(\theta) = w_0$ and $\varphi^+(\theta) = w_1$. We know that $w_0 < w_1$. A discontinuity in φ implies that the distribution Φ is flat over the range $[w_0, w_1]$, and hence: $d^+\Phi(w_0) = d^-\Phi(w_1) = 0$. Suppose, for contradiction, that the function is upper semicontinuous, such that $\varphi(\theta) = w_1$. Optimality of this wage choice implies that the pair of inequalities

$$d^+J(\theta, w_1) \leq 0, \qquad d^-J(\theta, w_1) \geq 0$$

must hold. Taking left and right derivatives at this point gives inequalities

$$\frac{\lambda_E(\rho + 2\Psi(w_1))(p_R f_\theta(w_1) + (1 - p_R)d^+\Phi(w_1))}{(\rho + \Psi(x))\Psi(x)} - 1 \geq 0$$
$$\frac{\lambda_E(\rho + 2\Psi(w_1))(p_R f_\theta(w_1) + 0)}{(\rho + \Psi(x))\Psi(x)} - 1 \leq 0$$

Since $d^+\Phi(w_1) = \phi(w_1) > 0$, one inequality here contradicts the other. Hence, φ must be lower semi continuous (application of the above inequalities at w_0 yields no such contradiction). ∎

D Model Solution

D.1 Solving the Steady State

We first derive the distribution of best attainable offers, G, of employed workers across this state by balancing the flow equation:

$$dG(x) = -(\delta+\lambda_E p_R \tilde{F}_\theta(x)+\lambda_E(1-p_R)\tilde{\Phi}(x))G(x)M_E+\lambda_U[p_R(F_\theta(x)-F_\theta(\theta^*))+(1-p_R)(\Phi(x)-F_\theta(\theta^\star))]M_U$$

Setting $dG(x) = 0$ and rearranging gives the steady state distribution as:

$$G(x) = \frac{p_R(F_\theta(x) - F_\theta(\theta^*)) + (1-p_R)(\Phi(x) - \Phi(\theta^\star))}{\delta + \lambda_E p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)} \frac{\lambda_U M_U}{M_E}$$

It will be helpful to substitute the expression:

$$\Psi(x) = \delta + \lambda_E p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)$$

such that:

$$G(x) = \frac{p(F_\theta(x) - F_\theta(\theta^*)) + (1-p_R)(\Phi(x) - F_\theta(\theta^\star))}{\Psi(x)} \frac{\lambda_U M_U}{M_E}.$$

$\Psi(x)$ is the exit rate at a firm where the maximum attainable wage is x. Next, let $G(x, R)$ indicate the measure of workers at R-firms with match value x, and let $G(x, N)$ indicate the measure of workers at N-firms with wage x, such that $G(x, R) + G(x, N) = G(x)$. We can derive the flow equations:

$$dg(x, R) = -[\delta + \lambda_E p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)]g(x, R)M_E + p_R f_\theta(x)[\lambda_U M_U + \lambda_E M_E G(x)] \quad (24)$$

$$dg(x, N) = -[\delta + \lambda_E p_R \tilde{F}_\theta(x) + \lambda_E(1-p_R)\tilde{\Phi}(x)]g(x, N)M_E + (1-p_R)\phi(x)[\lambda_U M_U + \lambda_E M_E G(x)] \quad (25)$$

Subsituting the derived expression for G and imposing a blanced flow steady state yields:

$$g(x, R) = \frac{\lambda_U p_R f_\theta(x)(\delta + \lambda_E \tilde{F}_\theta(\theta^*)}{\Psi(x)^2} \frac{M_U}{M_E} \quad (26)$$

$$g(x, N) = \frac{\lambda_U (1-p_R)\phi(x)(\delta + \lambda_E \tilde{F}_\theta(\theta^*)}{\Psi(x)^2} \frac{M_U}{M_E} \quad (27)$$

Finally, to derive the distribution of wages at renegotiating firms, we think about the conditional distribution of workers at a firm with match x whose last best offer had value q. The flow equation for the mass of workers of this type is:

$$d(H(q|x)g_r(x)) = -(\delta+\lambda_E p_R \tilde{F}_\theta(x)+\lambda_E(1-p_R)\tilde{\Phi}(x))H(q|x)g(x.R)M_E+\lambda_E p_R f_\theta(x)G(q)M_E+\lambda_U p_R f_\theta(x)M_U$$

Once again, we can substitute our expressions for g_r and G, impose that the steady-state flow is equal to zero, and rearrange to get:

$$H(q|x) = \left(\frac{\Psi(x)}{\Psi(q)}\right)^2$$

D.2 Solving the Wage Equation, φ

In the main text, we derived a condition such that each wage offer, $\varphi(\theta)$, solves the first order condition for a firm at each match value θ, given the local shape of φ at θ. We can rearrange this condition, in equation (15), to get a first order differential equation:

$$\varphi'(\theta) = \frac{\Gamma_1\big(F_\theta(\theta), F_\theta(w)\big) f_\theta(\theta)}{\Gamma\big(F_\theta(\theta), F_\theta(w)\big)/(\theta - w) - \Gamma_2\big(F_\theta(\theta), F_\theta(w)\big) f_\theta(w)} \tag{28}$$

$$\Gamma_1\big(F_\theta(\theta), F_\theta(w)\big) = \frac{\lambda_E(1-p_R)(\rho + 2\text{exit}(\theta, w))}{(\text{exit}(\theta, w)(\rho + \text{exit}(\theta, w))^2} \tag{29}$$

$$\Gamma_2\big(F_\theta(\theta), F_\theta(w)\big) = \frac{\lambda_E p_R(\rho + 2\text{exit}(\theta, w))}{(\text{exit}(\theta, w)(\rho + \text{exit}(\theta, w))^2} \tag{30}$$

$$\text{exit}(\theta, w) = \delta + \lambda_E(p_R\tilde{F}_\theta(w) + (1-p_R)\tilde{F}_\theta(\theta)) \tag{31}$$

The term $\text{exit}(\theta, w)$ is the rate at which a worker at an N-firm with match θ and wage w will leave the firm, in equilibrium. One issue in using the differential equation above is that Proposition 1 does not guarantee that φ is everywhere continuous, and the first-order condition is known only to be necessary and not sufficient. The algorithm we use accounts for potential discontinuities in φ by globally checking for optimality at each step.

To do this, we need to use the following profit function, which gives the profit to the firm under the equilibrium condition that wage offers are ranked according to θ.

$$J^*(\theta, w) = \frac{\theta - w}{\text{exit}(\theta, w)(\rho + \text{exit}(\theta, w))} \tag{32}$$

The algorithm proceeds as follows, given a predetermined grid $\{\theta_0, \theta_1, ..., \theta_J\}$ with $\theta_0 = \theta^*$. To initialize the algorithm, we set $w_0 = \theta^*$:

1. Given θ_{j-1}, $w_{j-1}(= \varphi(\theta_{j-1}))$, use (28) and either Euler's method or a more advanced method such as Runge-Kutta to compute the step $d\varphi_j$.

2. Check for global optimality by solving $w^* = \arg\max_{w \in [w_{j-1}, \theta_j]} J^*(\theta_j, w)$.

3. If $w^* > w_{j-1}$, set $w_{j+1} = w^*$.

4. Otherwise, set $w_j = w_{j-1} + d\varphi_j$.

The idea here is that, if $w^* > w_{j-1}$, then the shape of the match distribution F supports a discontinuity at θ_j, such that no firm offers between w_{j-1} and w^*. In addition, the marginal firm θ_j is indifferent between these wage offers. If, on the other hand, the firm prefers to offer w_j (the lowest wage available) then we must introduce marginal wage competition by way of $\varphi'(\theta)$.

D.3 Solving Surplus Equations and R-Firm wages

In this section we provide further details for solving the total value function, T_R, which defines values at both R-firms and N-firms. For ease of exposition, we provide once more the recursive definition of T:

$$(\rho+\delta)T_R(x) = x+\lambda_E p_R \int_x \alpha(T_R(y)-T_R(x))dF_\theta+\lambda_E(1-p_R)\int_{\varphi^{-1}(x)}(T_R(\varphi(y))-T_R(x))dF_\theta+\delta V_U$$

If we differentiating the surplus equation and rearranging gives:

$$T_R'(x) = \frac{1}{\rho+\delta+\lambda_E(\alpha p_R \tilde{F}_\theta(x)+(1-p_R)\tilde{\Phi}(x)}$$

bearing in mind that $\Phi(x) = F_\theta \varphi^{-1}(x))$. This, in turn, permits us to write:

$$T_R(x) = T_R(\theta^*) + \int_{\theta^*}^x \frac{1}{\rho+\delta+\lambda_E(\alpha p_R \tilde{F}_\theta(z)+(1-p_R)\tilde{\Phi}(z))}dz = T_R(\theta^*) + \hat{T}_R(x)$$

In fact, adapting the method proposed by Cahuc et al. (2006), integration by parts yields the following analytic solution:

$$(\rho+\delta)T_R(x) = x+\lambda_E \int_x \frac{\alpha p_R F_\theta(z)+(1-p)\Phi(z)}{\rho+\delta+\lambda_E(\alpha p_R \tilde{F}_\theta(z)+(1-p_R)\tilde{\Phi}(z)}dz + \delta T_R(\theta^*)$$

In practice, we solve the model by linearly interpolating $\hat{T}_R$ over grid points in the space for θ. In addition, the wage equation ϕ can be written as:

$$\begin{aligned}\phi(x,y) = {}& (\rho+\delta+\lambda_E p_R \tilde{F}_\theta(y)+\lambda_E(1-p_R)\tilde{\Phi}(y))(\alpha\hat{T}_R(x)+(1-\alpha)\hat{T}_R(y)) - \rho T_R(x^*) \\ & - \lambda_E p \left[\int_y^x [(1-\alpha)\hat{T}_R(z)+\alpha\hat{T}_R(x)]dF_\theta(z) + \int_x [(1-\alpha)\hat{T}_R(x)+\alpha\hat{T}_R(z)]dF_\theta(z)\right] \\ & - \lambda_E p_R \left[\int_{\varphi^{-1}(y)}^{\varphi^{-1}(x)} [(1-\alpha)\hat{T}_R(\varphi(z))+\alpha\hat{T}_R(x)]dF_\theta(z) + \int_{\varphi^{-1}(x)} \hat{T}_R(\varphi(z))dF_\theta(z)\right] \end{aligned} \quad (33)$$

Alternatively, using the restriction that $V(x,y) = (1-\alpha)T_R(y)+\alpha T_R(x)$, algebra yields the following expression for wages:

$$\phi(x,y) = (1-\alpha)y+\alpha x - \lambda_E p_R(1-\alpha)^2\left[\int_y^x T_R(z)dF_\theta(z)+\tilde{F}(x)T_R(x)-\tilde{F}(y)T_R(y)\right] \quad (34)$$

The third term in this expression signifies the extent to which a worker is compensated for lower wages today with the promise of future appreciation in wages. This, in turn, depends critically on the proportion, p, of firms that are willing to bargain. Finally, we can also solve for the value of unemployment as:

$$\rho T_R(\theta^\star) = b+\lambda_U \int_{\theta^\star}(p_R\alpha\hat{T}_R(x)+(1-p_R)\hat{T}_R(\varphi(x)))dF_\theta x)$$

Similarly the surplus equation at $\theta^\star$ is

$$\rho T_R(\theta^\star) = \theta^\star+\lambda_E \int_{\theta^\star}(p_R\alpha\hat{T}_R(x)+(1-p_R)\hat{T}_R(\varphi(x)))dF_\theta x)$$

Combining these two expressions is sufficient to pin down $\theta^\star$. This concludes our practical discussion of how to solve the model in equilibrium.

E Data and Sample Construction

Our sample construction works as follows. In each wave of the survey, information is collected on employment spells for the four months prior. This includes the following information:

- An employer index that uniquely identifies jobs across spells.
- For each job held during this time, the beginning and end dates of the employment spell.
- Whether the worker is still currently working for this employer.
- If the respondent is no longer working for this employer, the reason for termination of the job.
- Information on weekly hours for each job, pay rates, and monthly earnings, for each job.

Using this information, we infer employment spells by working backwards from the last chosen wave to the first. When each wave is administered, information from the previous wave is made available on the survey instrument in order to ensure employment spells and employers are reported accurately over time. Because of an error in this feedback mechanism during the administration of wave 3, comparisons to previous employers between wave 3 and wave 2 are not guaranteed to be accurate.[35] For this reason, we use data from the 24 month period beginning in wave 3.

E.1 Sample Selection

We drop all individuals who:

- Are missing from an intermediate wave of the sample; or
- Have employment information imputed in any wave of the sample; or
- Are flagged as contingent (i.e. freelance) workers or business owners; or
- Report working more than 20 hours per week at two separate jobs for a period of more than 3 weeks; or
- Report that they have no job and are not looking for work for the first full month of the sample, and report no subsequent employment spell in the ensuing 24 month period; or
- Held a part-time job during this time period, which we define as a job at which they report working less than 20 hours per week at both the beginning and end of the job spell.

[35] See https://www.census.gov/programs-surveys/sipp/tech-documentation/user-notes/2004w3CoreLaborForceFeedback.html for further details on this survey error

In addition to these restrictions, sometimes jobs reported in previous waves "go missing", in that information is not reported for these jobs in subsequent waves. In this case, we impute the end of the spell date as being the end date of the wave in which information for the job was last provided. If the implied duration of the spell is less than 3 weeks, then this spell is dropped from the sample.

E.2 Measuring Wages

When information on hourly pay is provided, we use this data as the reported wage. When it is missing, we impute the wage as reported average weekly earnings on weekly hours worked. Because this sometimes leads to extremely mismeasured values, we trim the bottom and top 1% of imputed wages.

E.3 Measuring Transitions

Subject to the above restrictions, we assume a job-to-job transition has been made when either:

- One employment spell begins on or before the end date of the previous one; or
- The respondent explicitly states that one spell ended in order for them to take another job. Thus, our assumption is the next reported spell is the "other job" that this worker left to accept.

Finally, in between all employment spells that are not measured as job-to-job transitions, we impute an unemployment spell, with duration equal to the length of time between the end date of the preceding spell and the start date of the one that follows.