Mental Models and Learning: The Case of Base-Rate Neglect

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Motivation

A large literature documents biases in decision-making.

- Base-rate neglect (Kahneman Tversky 73), overconfidence (Mobius et al. 10), sunk-cost effect (Thaler 80), Gamber's fallacy (Rabin 02), correlation neglect (Enke Zimmermann 19), failures of contingent reasoning (Esponda Vespa 14 19, Martinez-Marquina et al. 19), etc.

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Less is known on whether such biases persist when there is feedback.

- Biases may be corrected with experience if agents accumulate evidence indicative of optimal behavior.
- Learning requires agents to be receptive to new information and be attentive in how they record, process, and incorporate this information.

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Initial misconceptions can impact learning from feedback.

- An incorrect understanding of the value of this information or a desire to on to certain types of beliefs might prevent learning.
- Refer to such failures in incorporating relevant information as resulting from incorrect 'mental models'.

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Goal: Study whether suboptimal behavior can persist in the presence of feedback and asses the role mental models play in their persistence.

Proof of concept: Base-Rate Neglect

(Kahneman Tversky 73)

- Disease prevalence in the population: $p = 0.15$.
- Reliability of the test: $q = 0.80$,

  i.e. test is positive (negative) with 80% chance if the person has the disease (does not have the disease).
- What is the likelihood that the person has the disease conditional on a positive test?

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  Bayesian Benchmark:
  $\Pr(\text{sick}|\text{positive}) = \frac{pq}{pq+(1-p)(1-q)} = 0.41.$

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  Base-Rate Neglect (BRN):
  (result of incorrect mental model) $= 0.80$.

Preview of Results

- Are people Bayesian in the long run?

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No, convergence to the benchmark is slow and partial.

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- Can BRN be corrected?

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- Is learning transferable to new settings?

  Yes, but partially.

Why Base-Rate Neglect?

Well documented in the laboratory (Benjamin 19) and with professionals (e.g. Eddy 82, Kennedy Willis Faust 97, Gigerenzer Hoffrage Ebert 98).

Feedback is simple and natural.

- No need to make inferences about others.
- Feedback is informative, easy to process.
- Feedback is exogenous to decisions.

  Problems with learning from endogenous feedback: Esponda Vespa 18, Fudenberg Vespa 19.

Experimental Design

1st goal: Compare initial beliefs to long run beliefs.

Each session consists of 5 parts:

- Part 1 **Introduces the main updating task.**
- Part 2 **Repetition of the task for 200 rounds.**
- Part 3
- Part 4
- Part 5

Experimental Design

Part 1 **Introducing the main updating task.**

Subjects update beliefs on a binary state using a binary signal.

(Kahneman Tversky 73)

- There are 100 projects.
- 15 projects are successes; 85 are failures.
- Task: assess the chance that a randomly selected project is a success conditional on a test result.
- The test result is either positive or negative and has a reliability of 80%.

Experimental Design

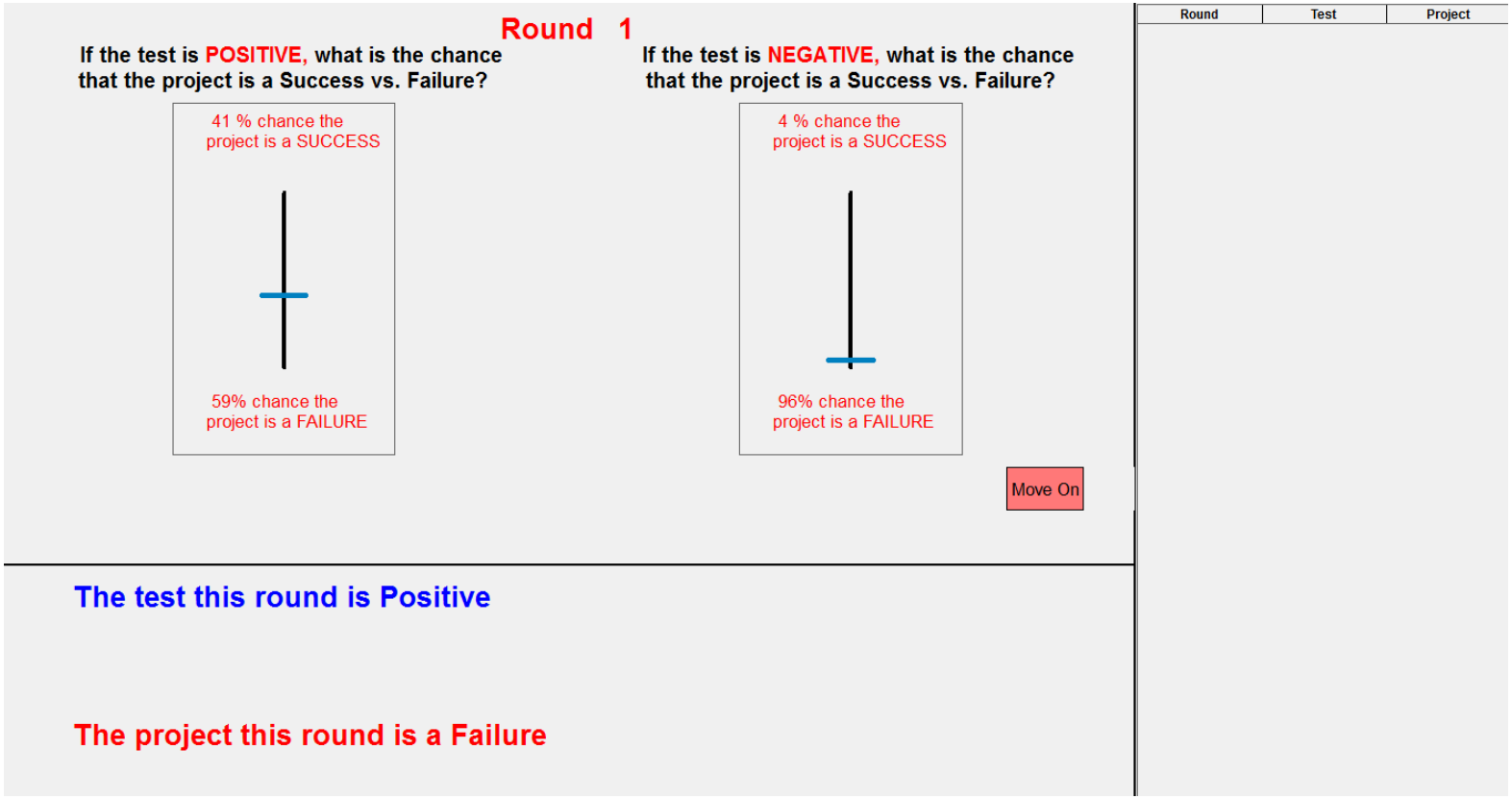
Part 2 **Repetition of the task for 200 rounds.**

- 100 rounds of feedback; beliefs reported every round.

  p and q are constant; state drawn independently in every round.

- 100 rounds of feedback; beliefs reported every 10 rounds.

Round 1 Feedback



ROUND 5 FEEDBACK

Round 5

If the test is POSITIVE, what is the chance that the project is a Success vs. Failure?

80 % chance the project is a SUCCESS

20% chance the project is a FAILURE

If the test is NEGATIVE, what is the chance that the project is a Success vs. Failure?

20 % chance the project is a SUCCESS

80% chance the project is a FAILURE

The test this round is Negative

The project this round is a Failure

Round	Test	Project
1	Positive	Failure
2	Negative	Failure
3	Positive	Failure
4	Positive	Success

Experimental Design

2nd goal: Study how initial misconceptions can impact learning.

2 between-subjects treatments that *only* differ in how the main updating task is introduced.

> **Primitives (P)**: Primitives (p and q) provided.
>
> **NoPrimitives (NoP)**: Primitives (p and q) not provided.

Experimental Design

- There are 100 projects.

  Primitives: 15 projects are successes; 85 are failures.

  NoPrimitives: A certain number of them successes and the remaining ones are failures.

- The computer runs a test on the selected project.

  Primitives: The test result either positive or negative and has a reliability of 80%.

  NoPrimitives: The test result either positive or negative and has a reliability of R%.

Feedback is structurally the same in both treatments, but BRN is possible only in Primitives.

Experimental Design

Other details:

- Participants: 128 UCSB students.
- Subjects paid for one choice in one part (using BDM).
- ~90 min sessions (subjects move at own pace).
- Average payoffs: $22.5 (either $10 or $35).

RESULTS

Round 1: Primitives

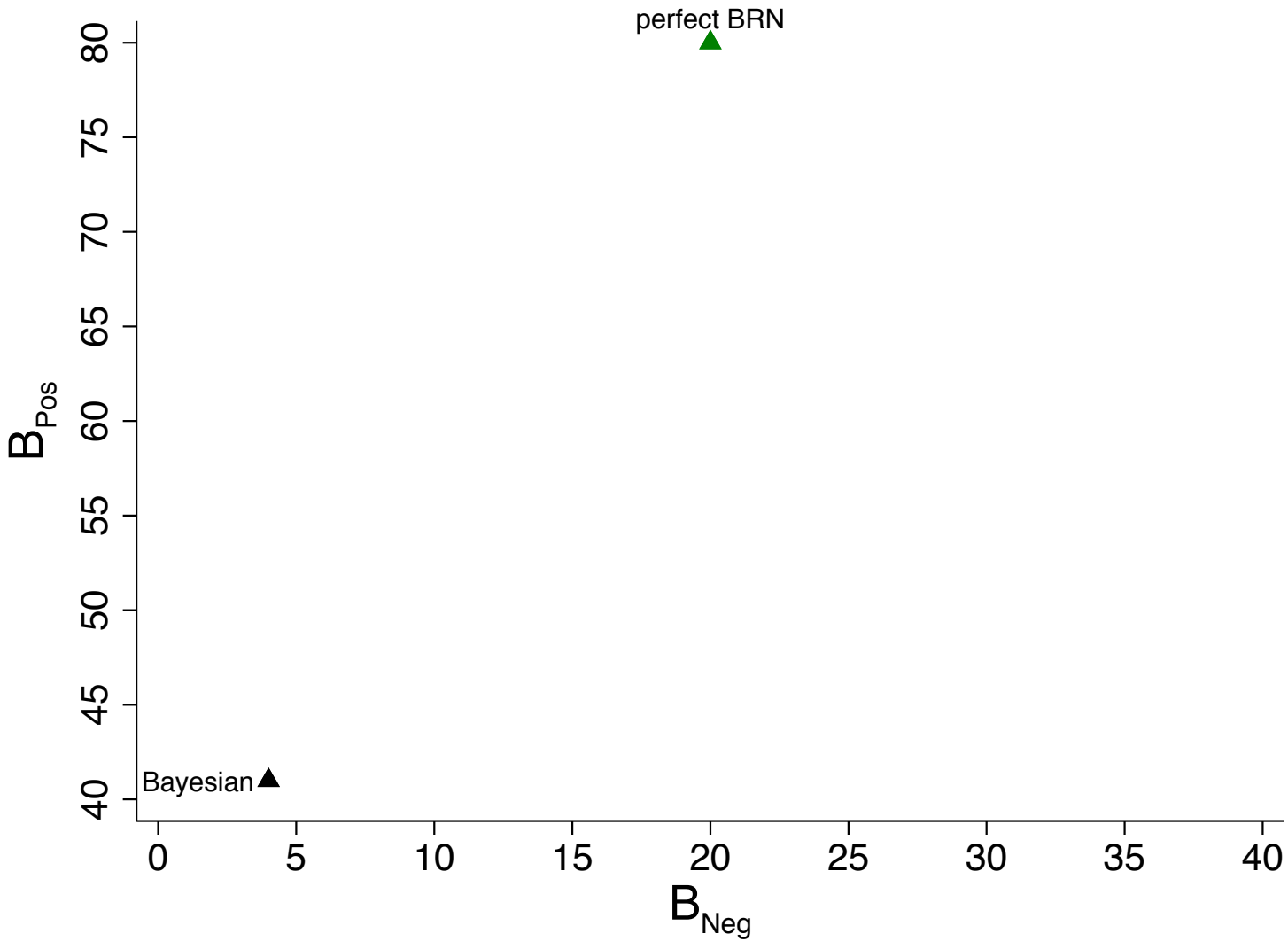
Replication of literature on BRN in Round 1 of Primitives:

- Majority of subjects are consistent with perfect BRN.

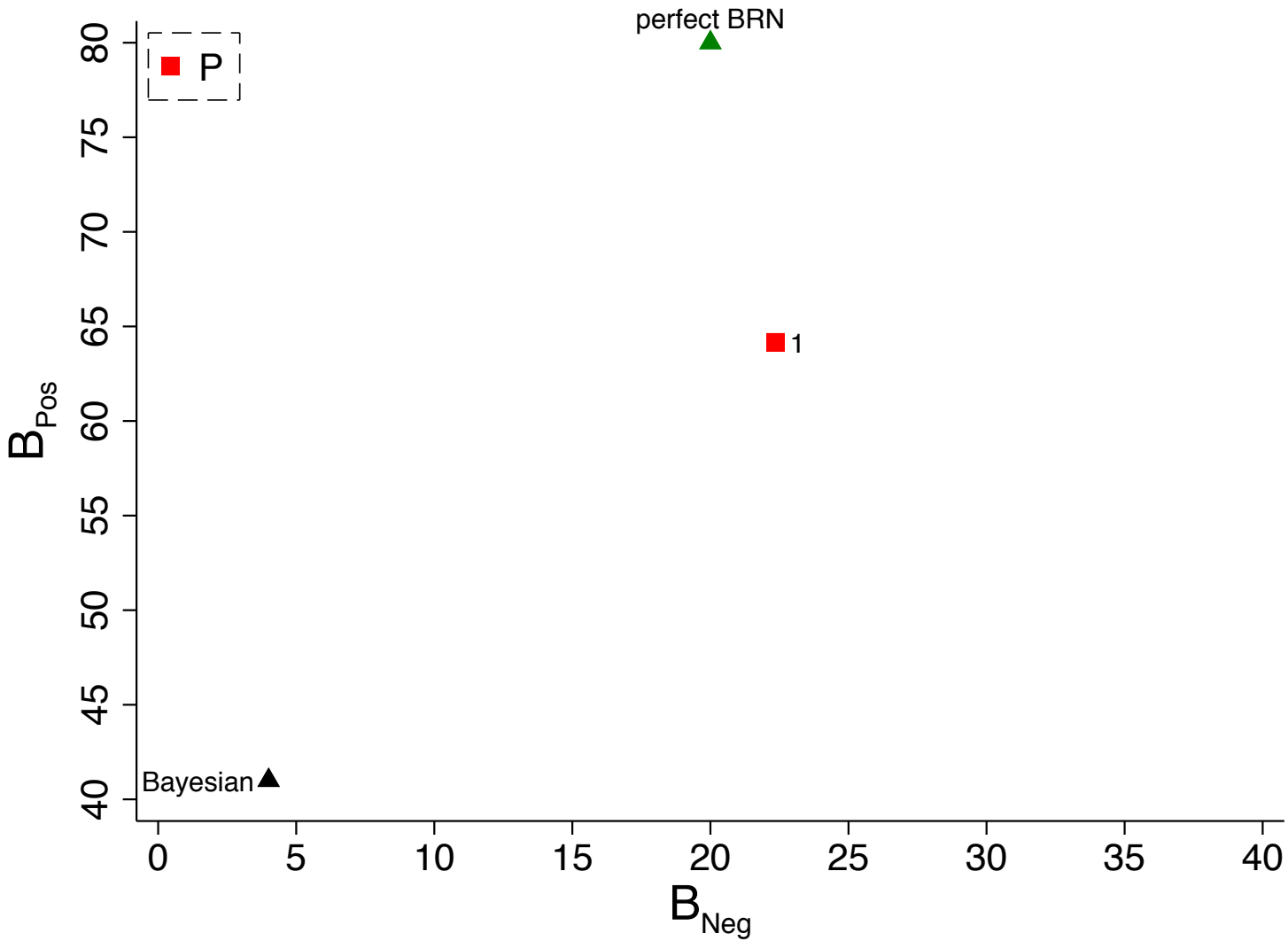
	Round 1 Primitives
perfect Base-Rate Neglect (pBRN)	56.3
Bayesian	4.7

As % of Subjects in each treatment.

Analysis at the aggregate level



Average Beliefs: Primitives



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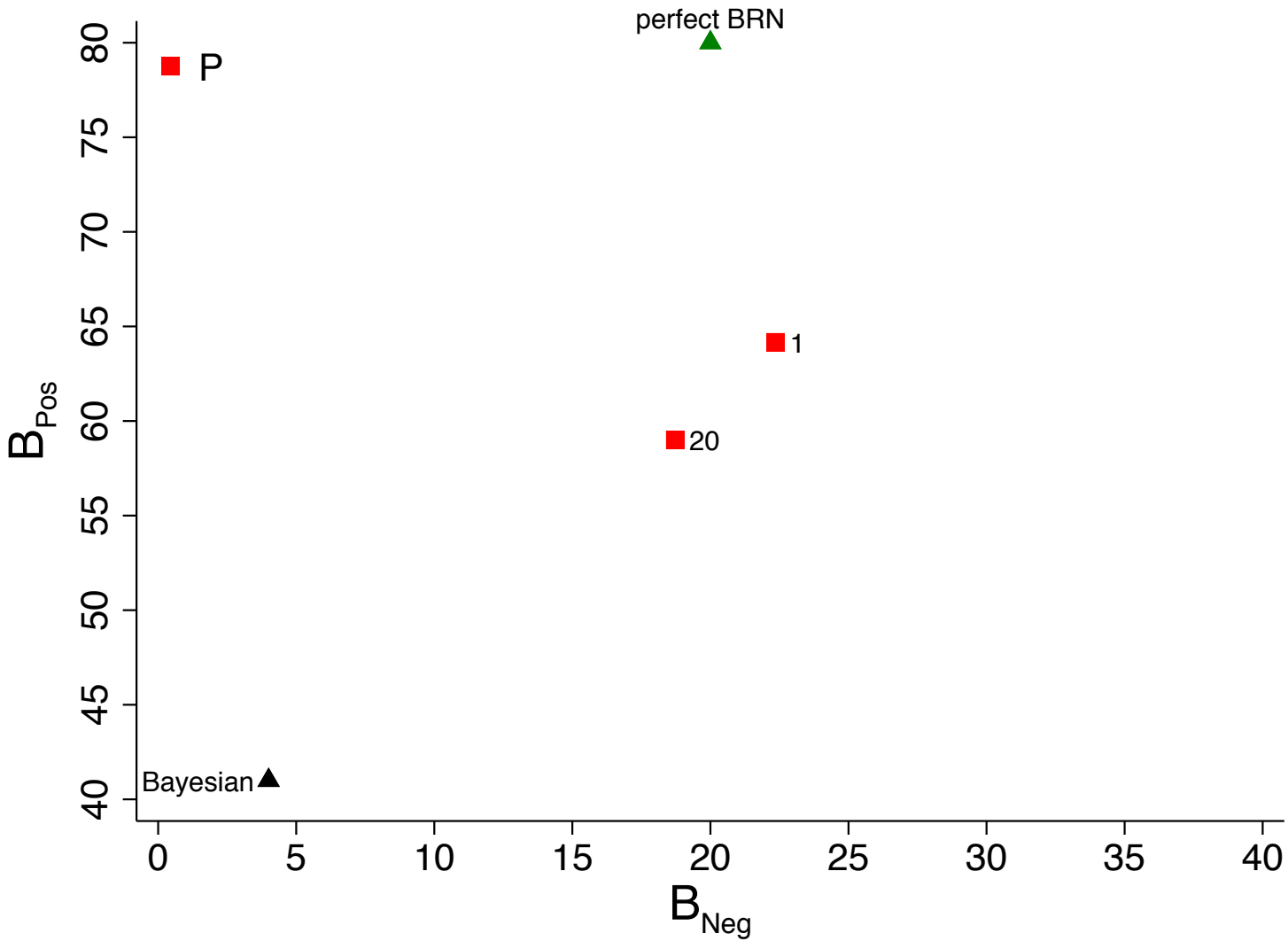

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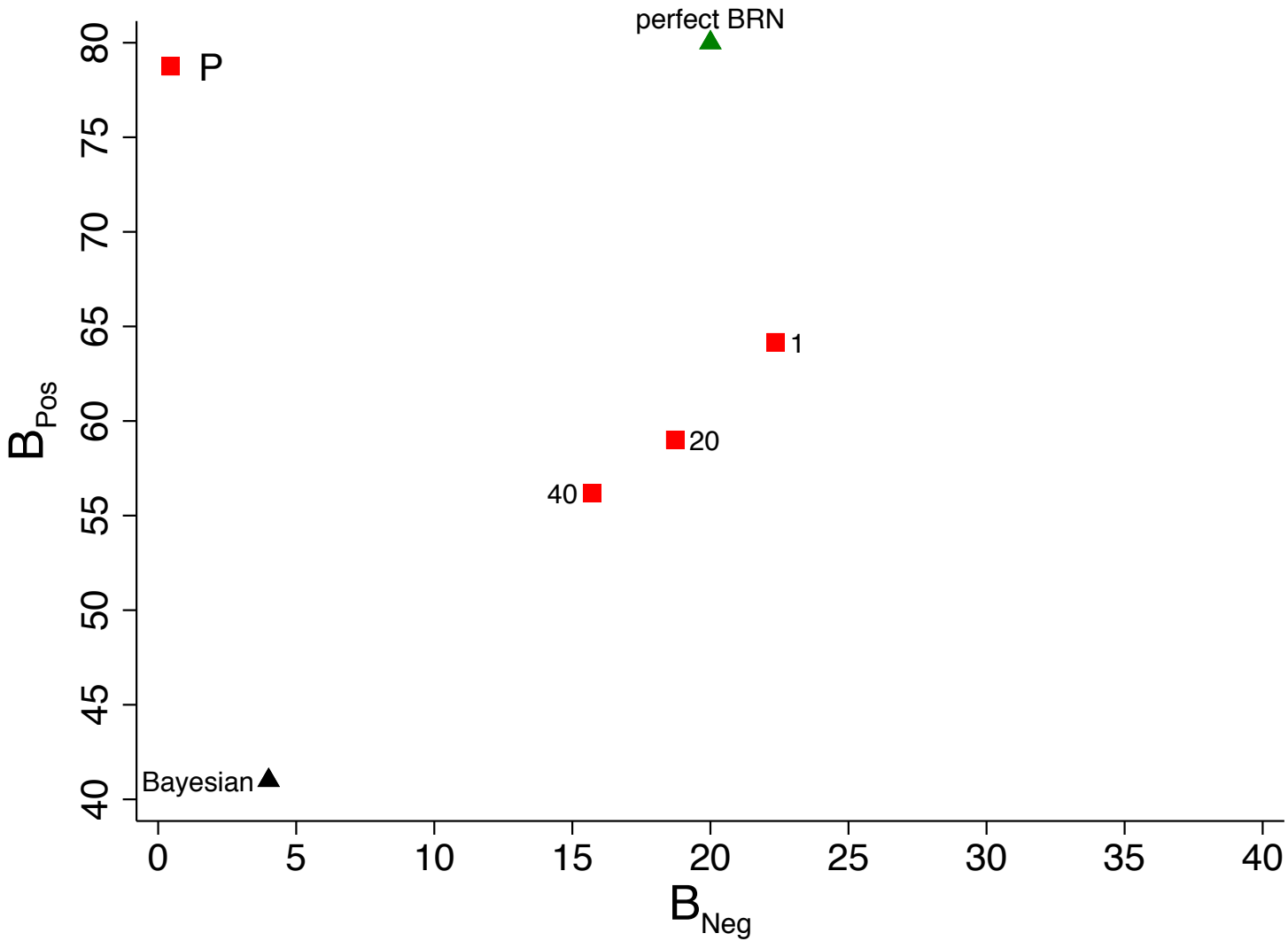

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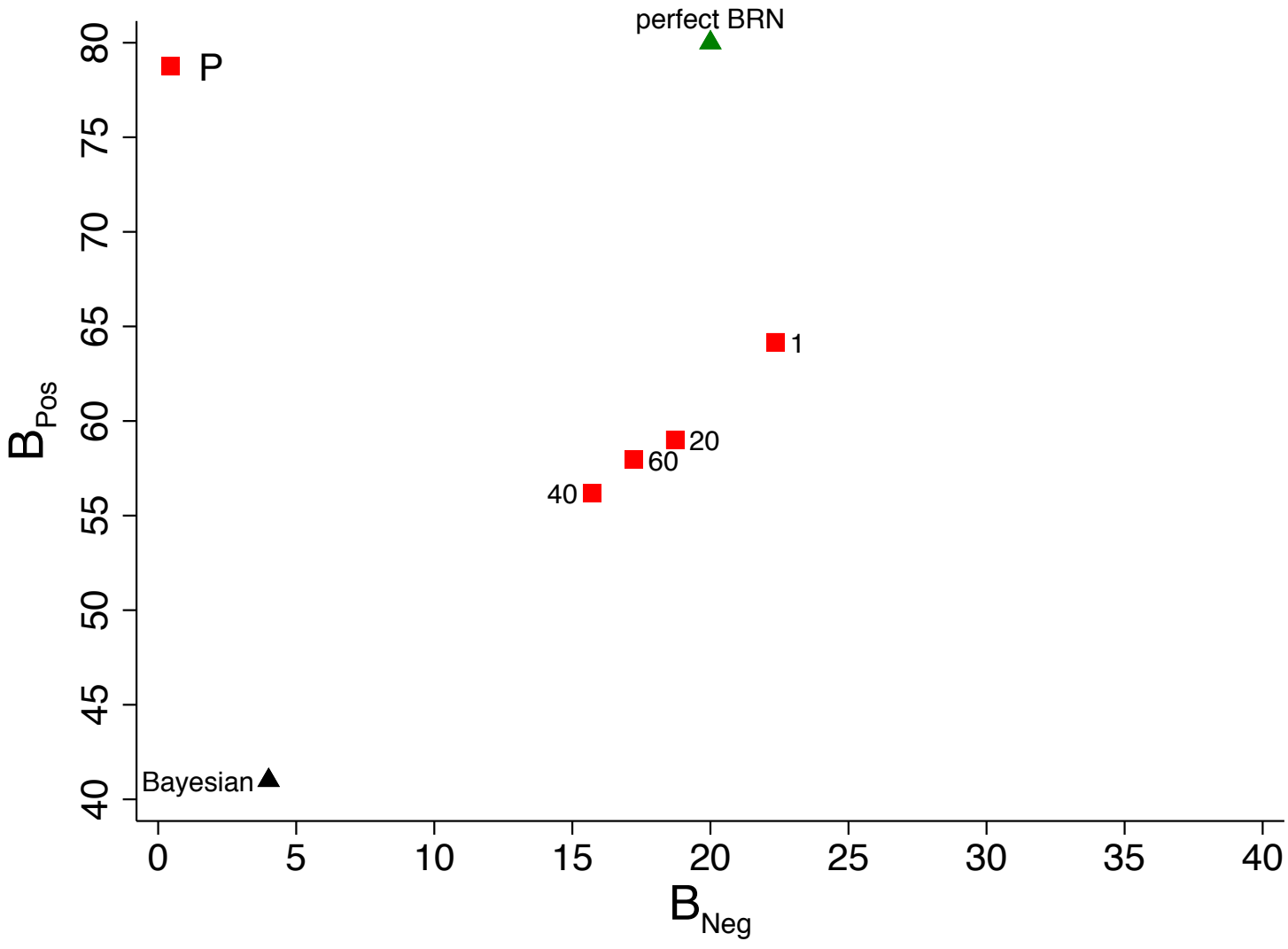

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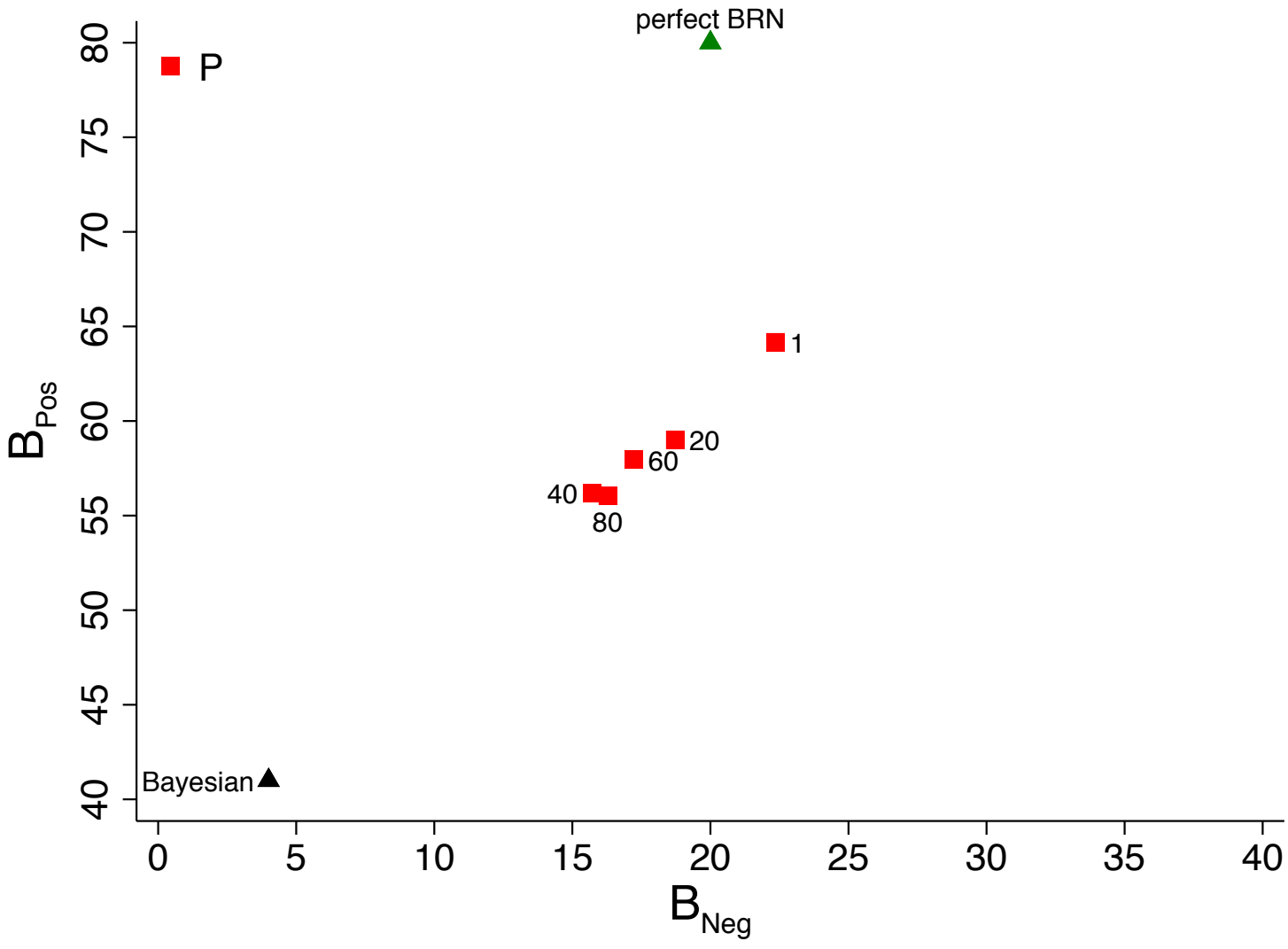

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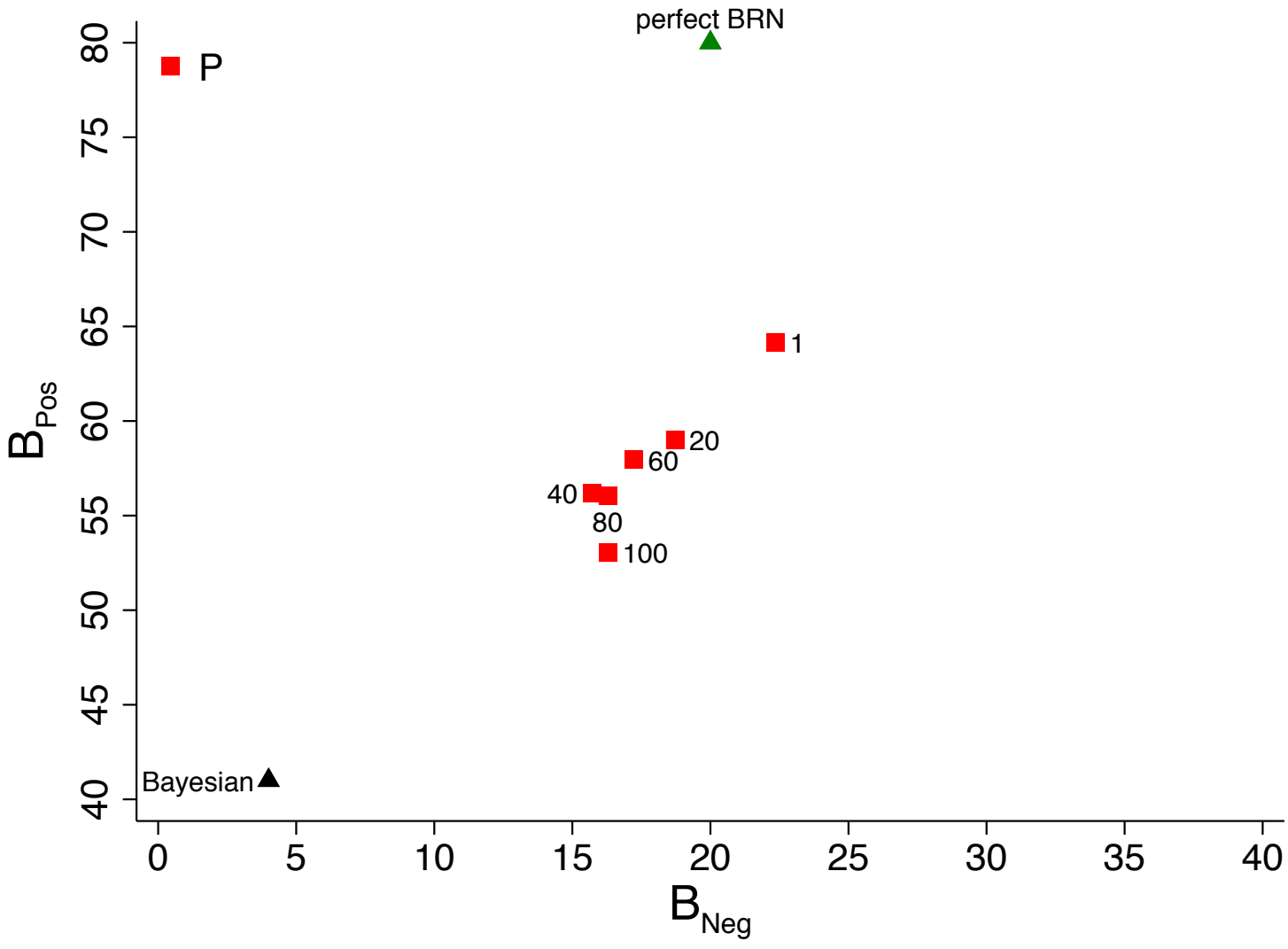
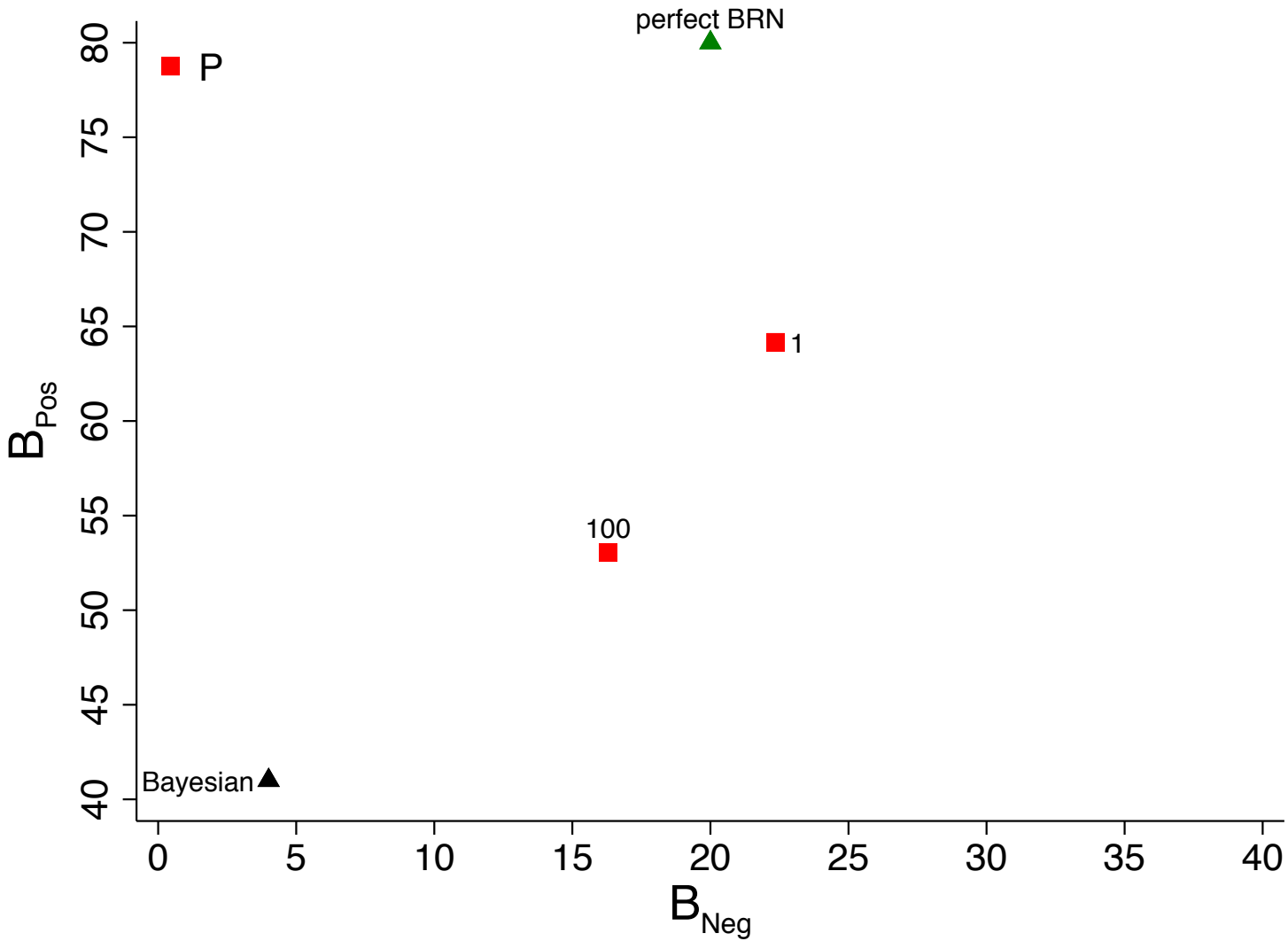
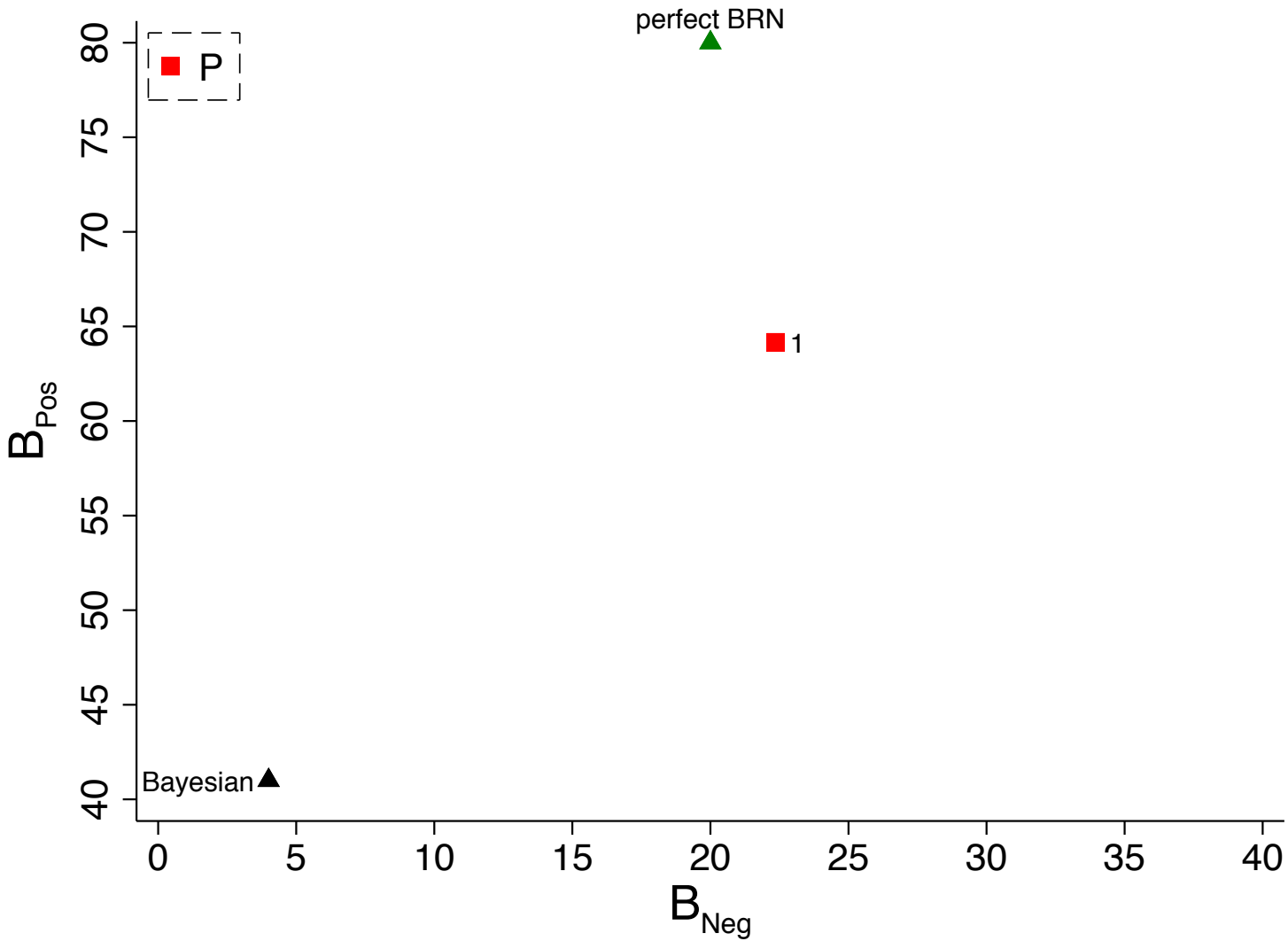


Beliefs in Primitives: Rounds 1 & 100



Average Beliefs: Round 1



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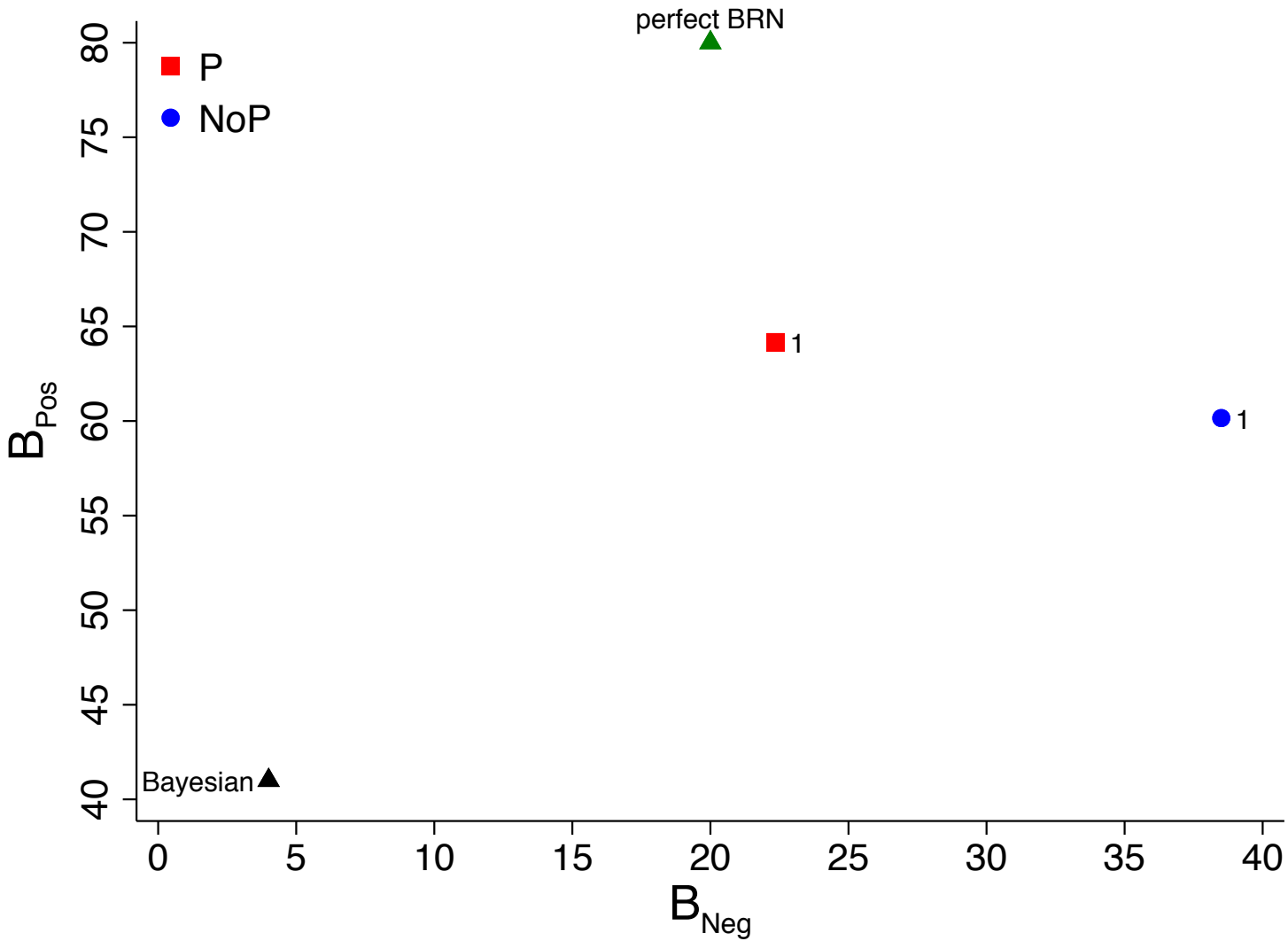
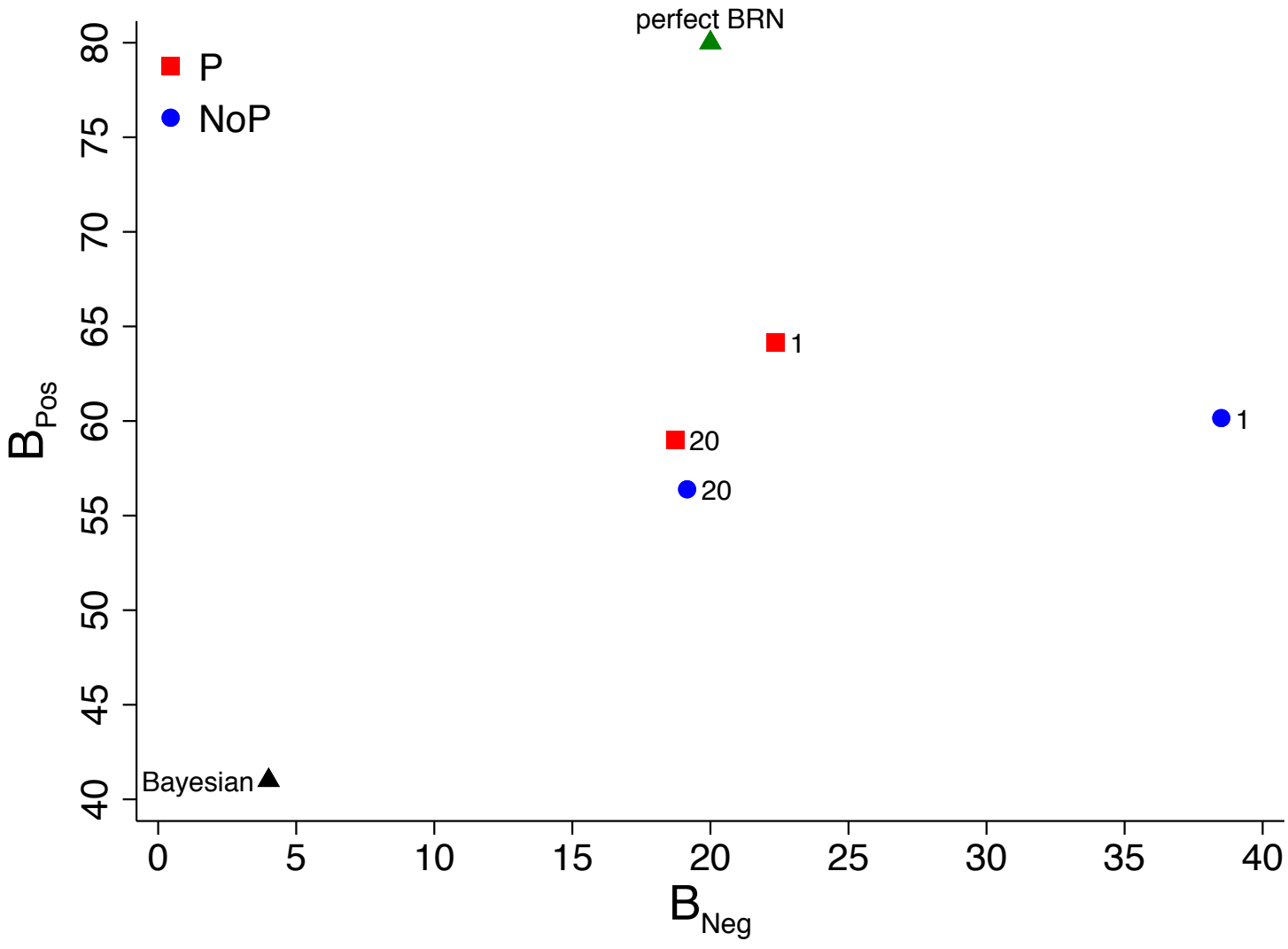


Evolution of Average Beliefs



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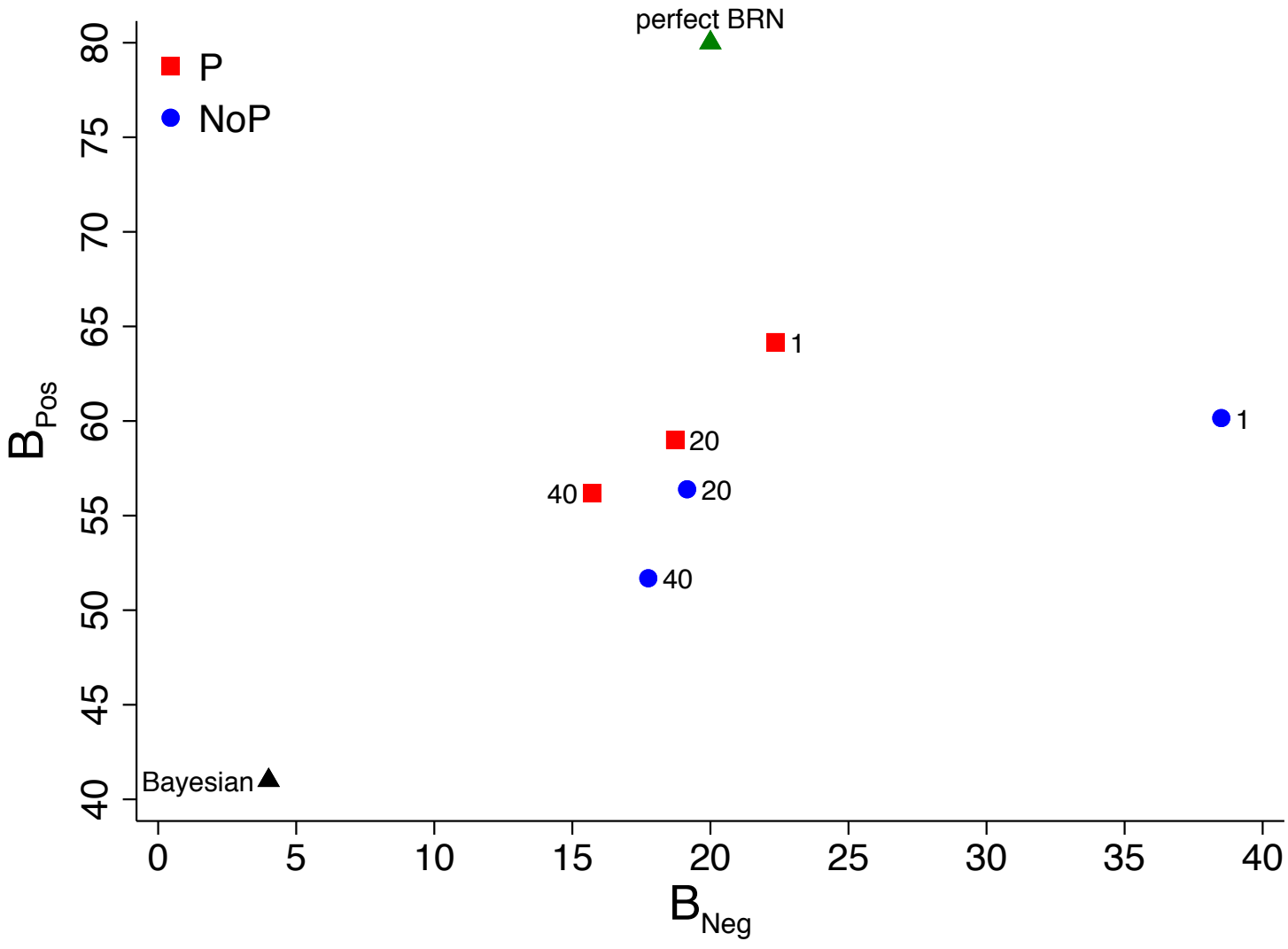

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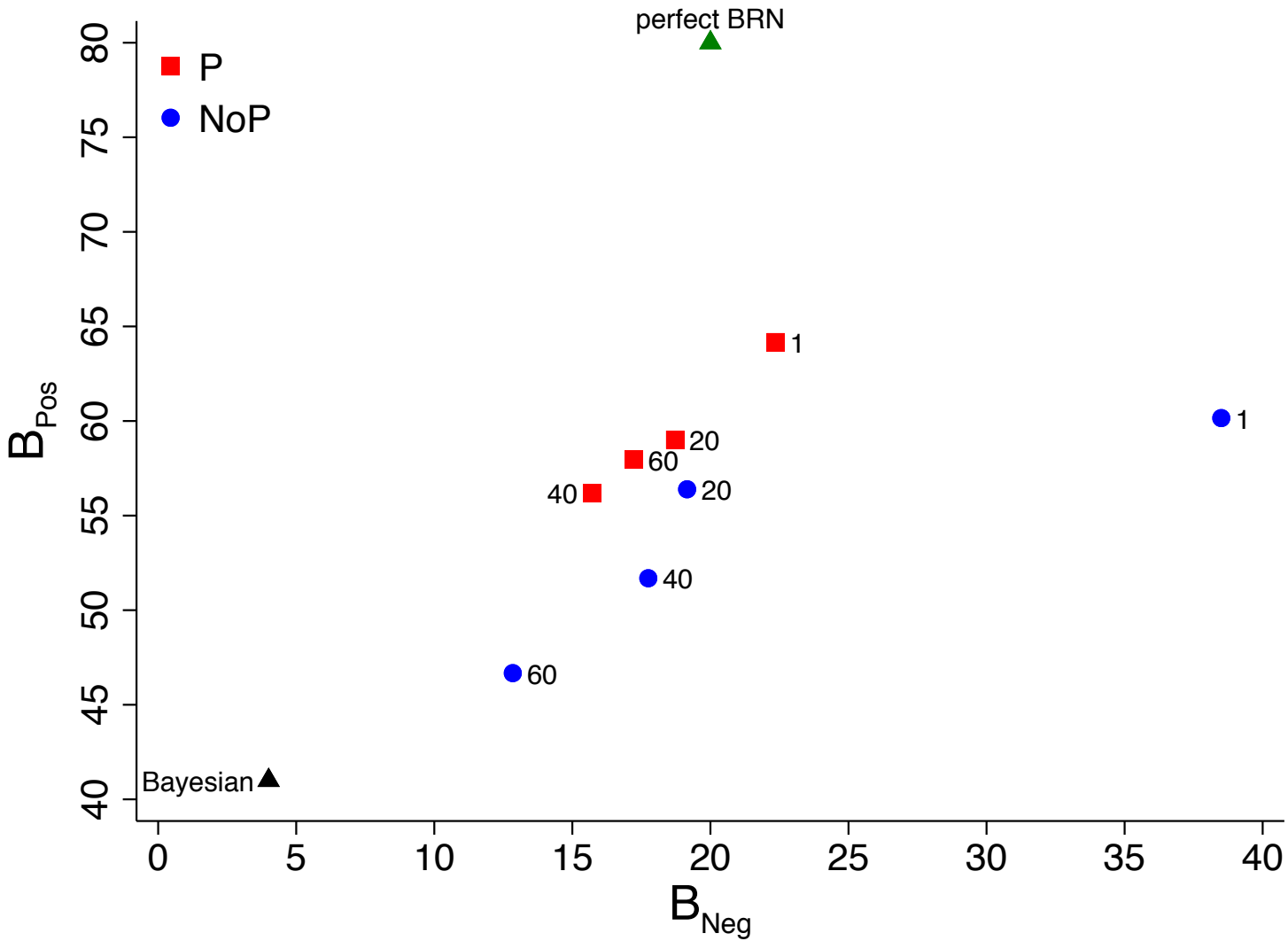

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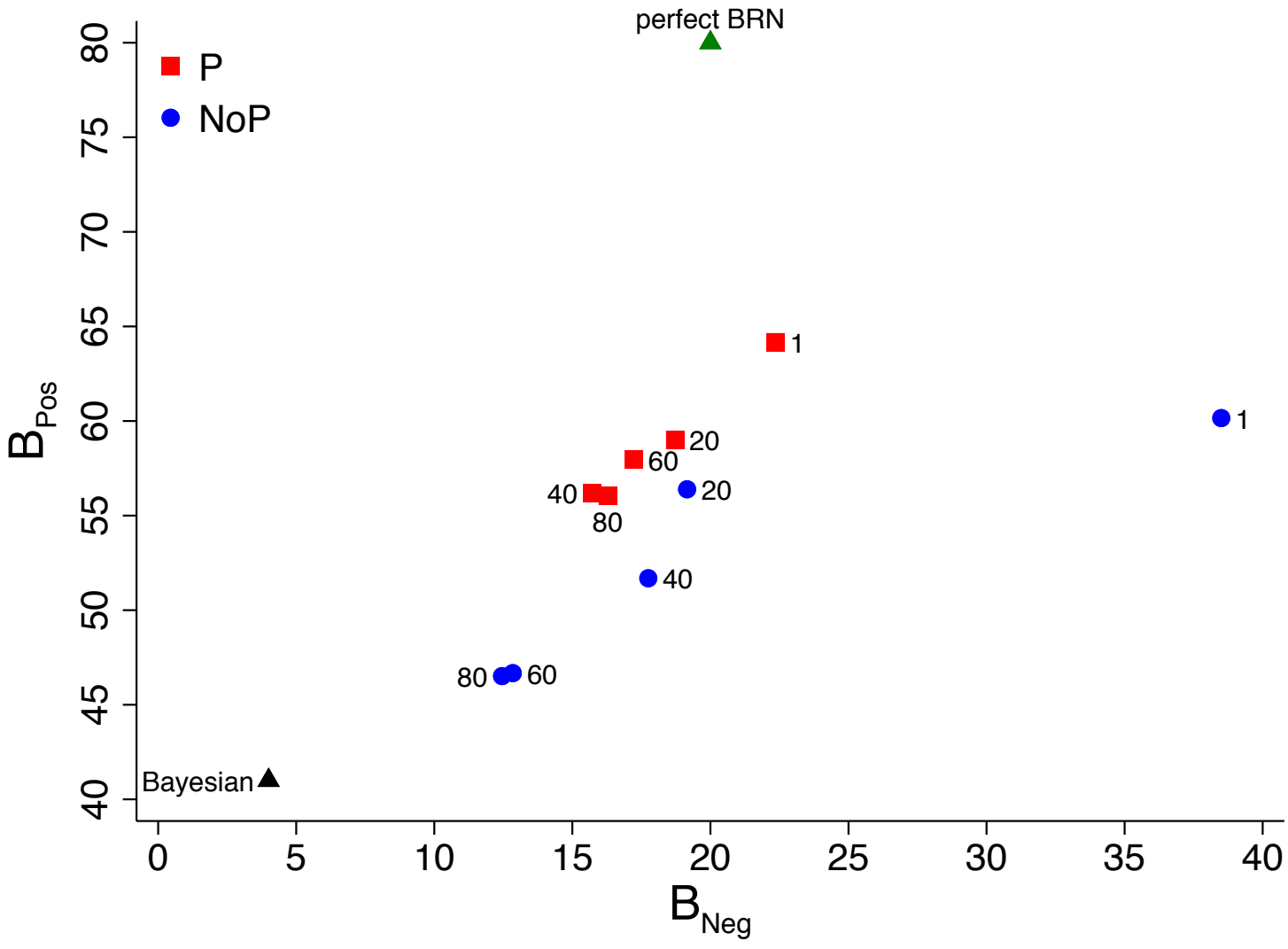

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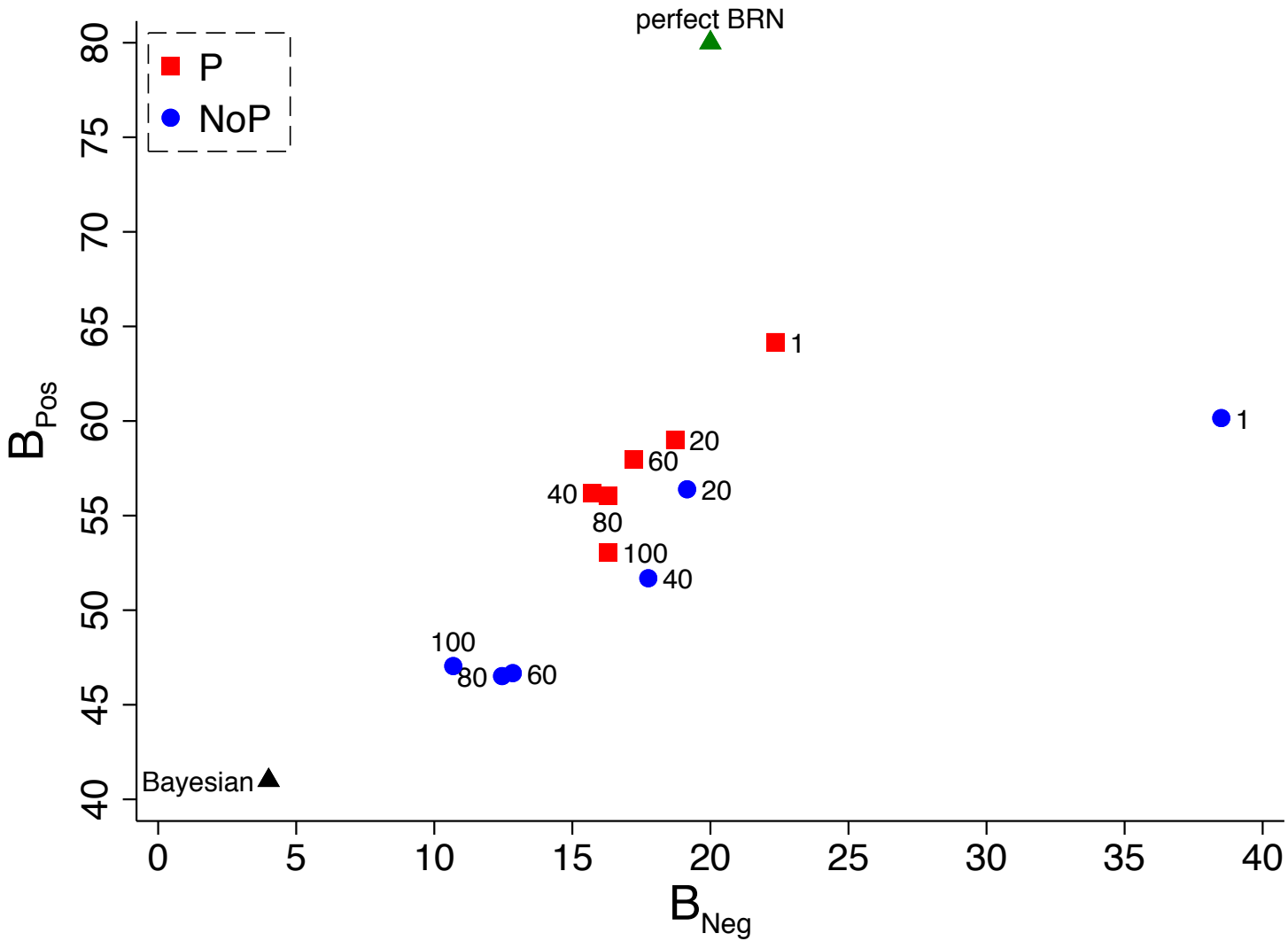
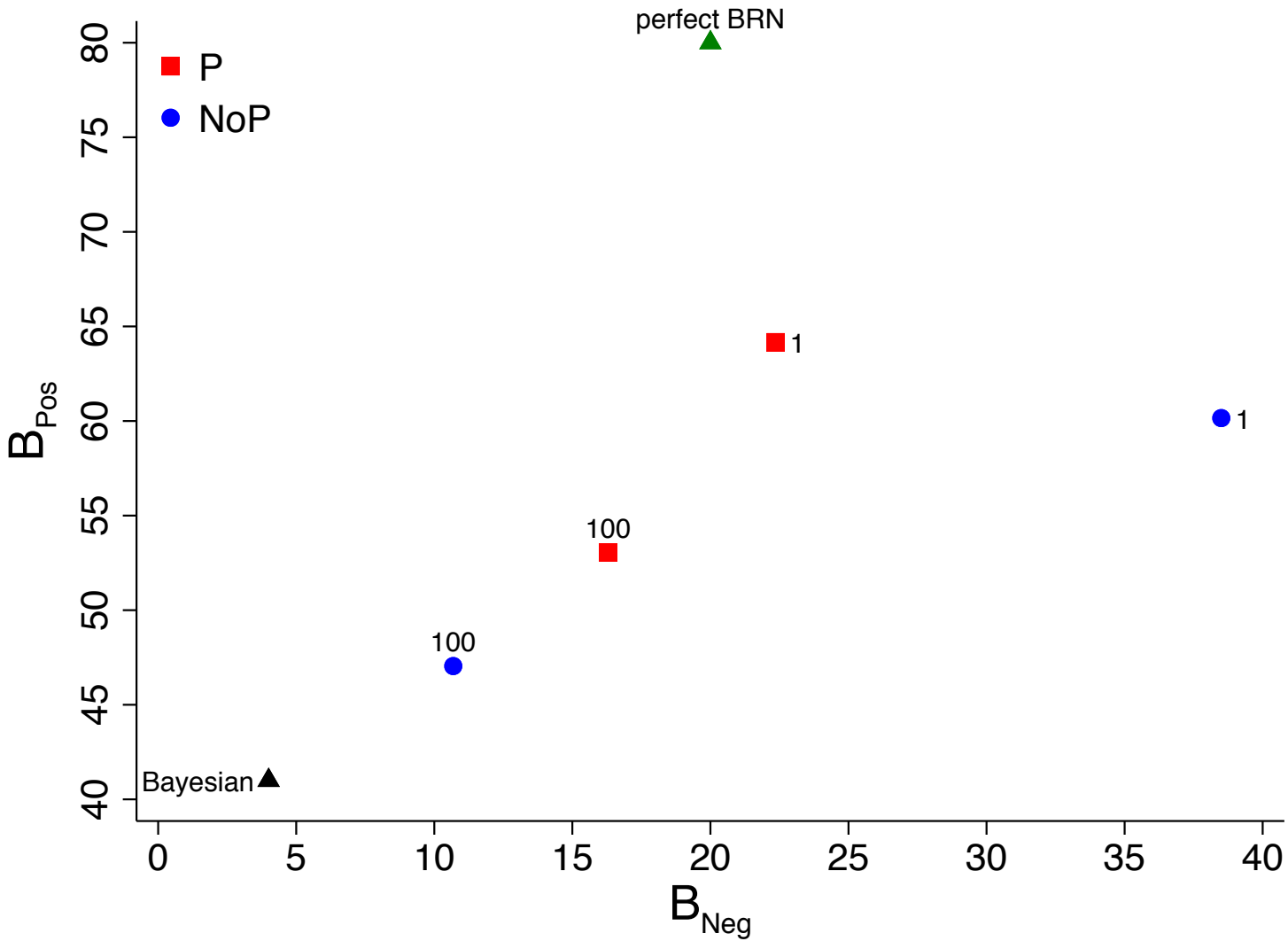
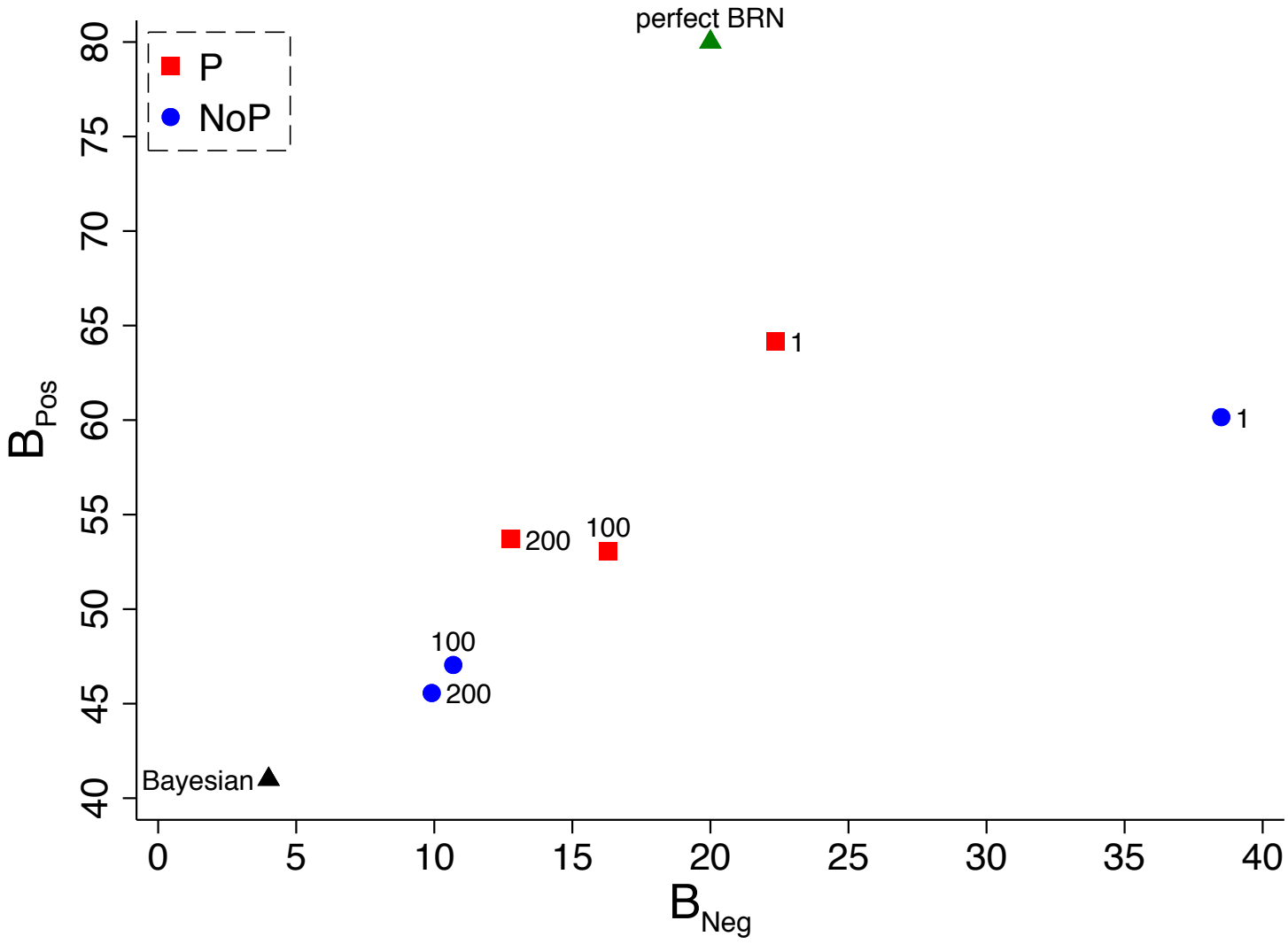


Evolution of Beliefs: Rounds 1 & 100



Evolution of Average Beliefs: Round 200

Statistical analysis



Responsiveness to Prior

Follow approach by Grether (1980):

Bayes' rule for each test:

$$\frac{B_{Pos}}{1 - B_{Pos}} = \frac{p}{1 - p} \times \frac{q}{1 - q}$$

$$\frac{B_{Neg}}{1 - B_{Neg}} = \frac{p}{1 - p} \times \frac{1 - q}{q}$$

Responsiveness to Prior

Follow approach by Grether (1980):

Bayes' rule in logs:

$$\ln\left(\frac{B_{Pos}}{1-B_{Pos}}\right) = \ln\left(\frac{p}{1-p}\right) + \ln\left(\frac{q}{1-q}\right)$$

$$\ln\left(\frac{B_{Neg}}{1-B_{Neg}}\right) = \ln\left(\frac{p}{1-p}\right) + \ln\left(\frac{1-q}{q}\right)$$

Responsiveness to Prior

Follow approach by Grether (1980):

Use B_{Pos} and B_{Neg} to obtain α and β:

$$\ln\left(\frac{B_{Pos}}{1-B_{Pos}}\right) = \alpha\ln\left(\frac{p}{1-p}\right) + \beta\ln\left(\frac{q}{1-q}\right)$$

$$\ln\left(\frac{B_{Neg}}{1-B_{Neg}}\right) = \alpha\ln\left(\frac{p}{1-p}\right) + \beta\ln\left(\frac{1-q}{q}\right)$$

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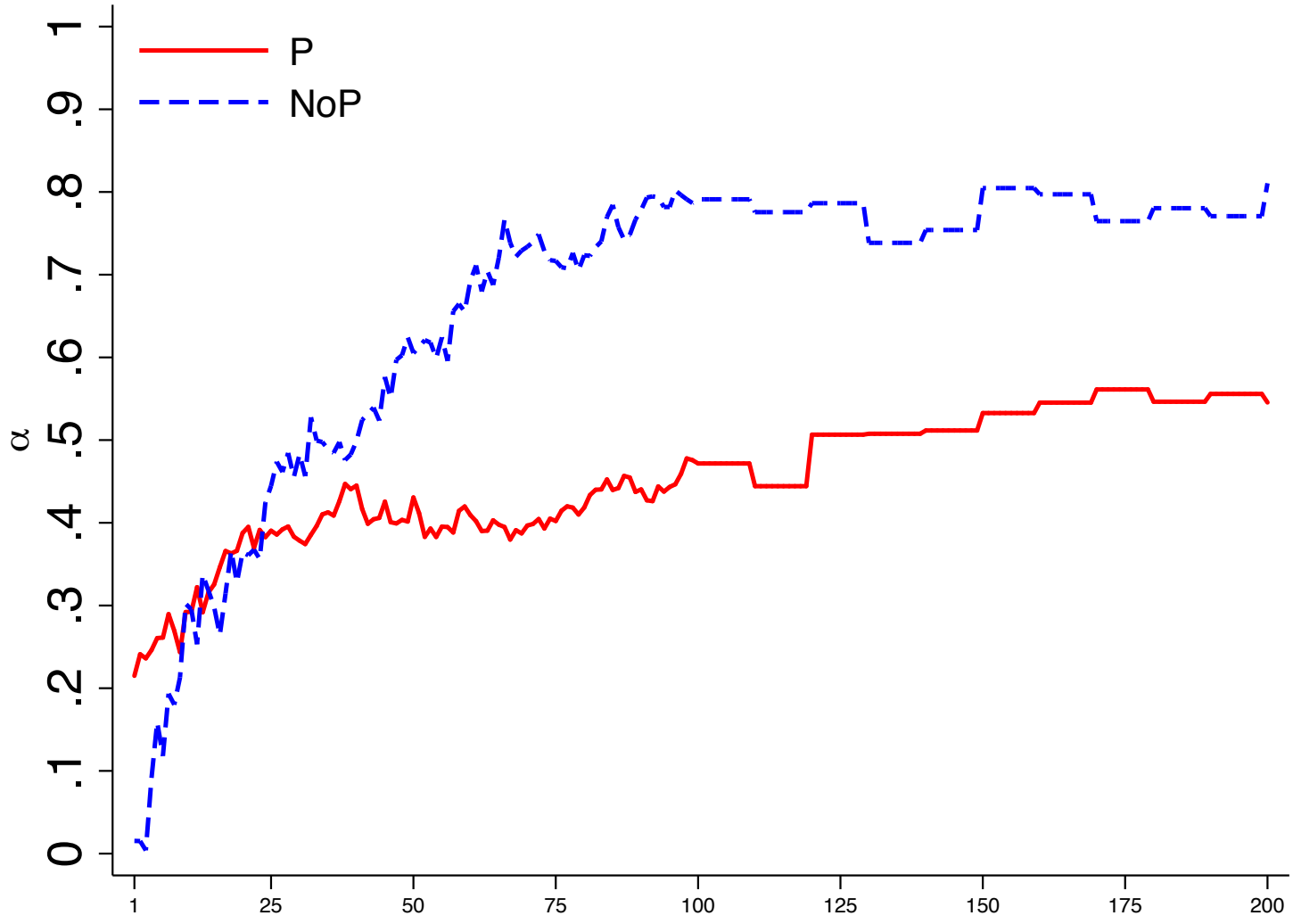
Bayesian benchmark: $\alpha = 1, \beta = 1$.
perfect BRN: $\alpha = 0, \beta = 1$.

Responsiveness to Prior (α)



Result #1

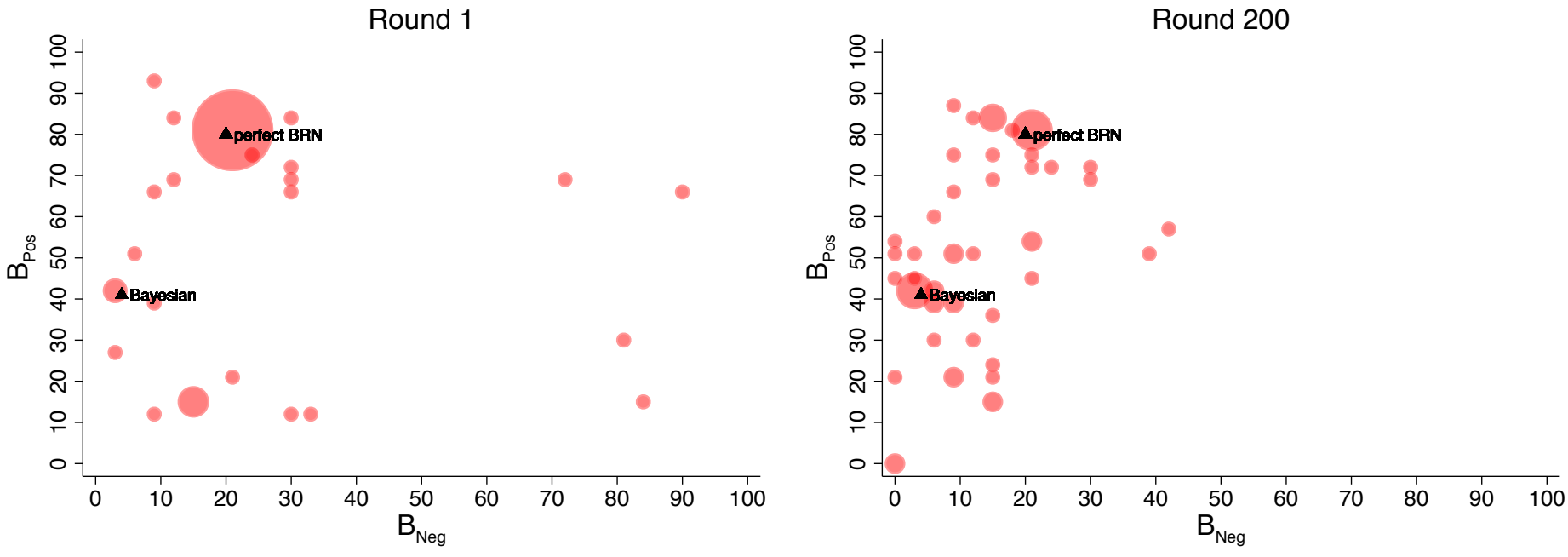
Beliefs in both treatments move closer to the Bayesian benchmark from round 1 to 200.

By round 200,

- Beliefs are significantly different between Primitives and NoPrimitives.
- Beliefs closer to the Bayesian benchmark in NoPrimitives.

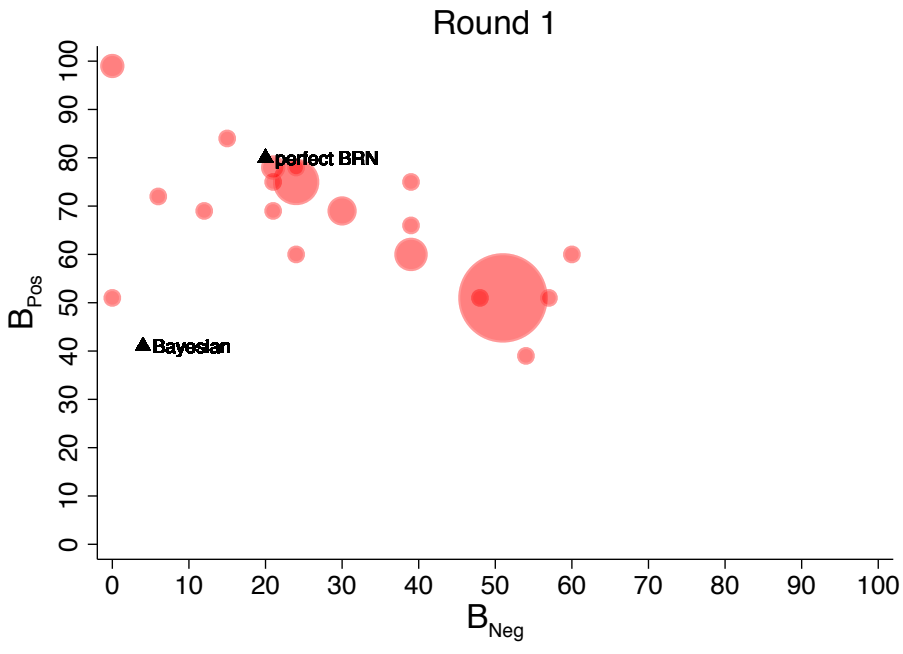
Distribution of Beliefs: Primitives

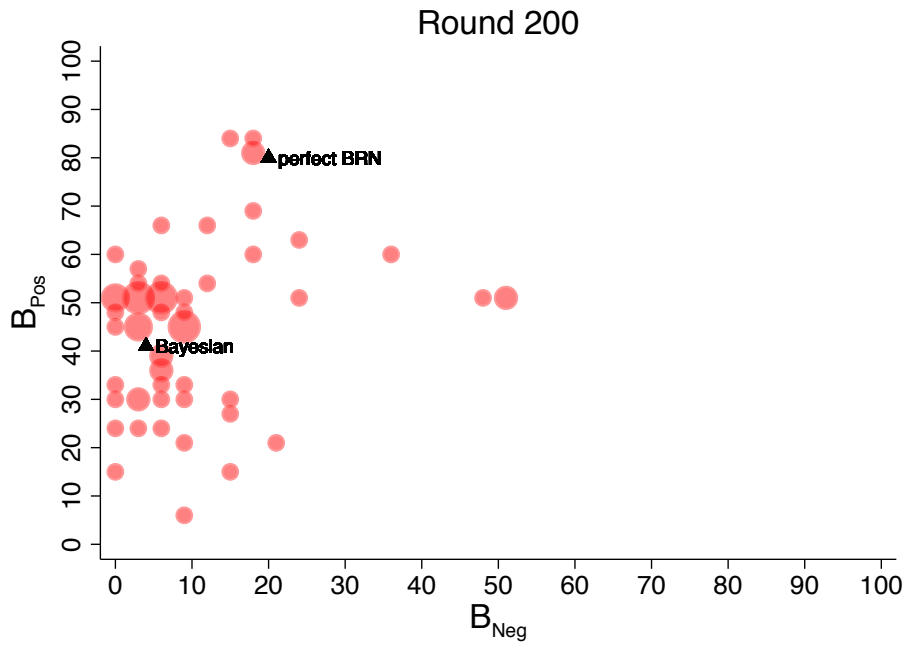
Primitives Treatment



Distribution of Beliefs: NoPrimitives

NoPrimitives Treatment





Heterogeneity in Learning

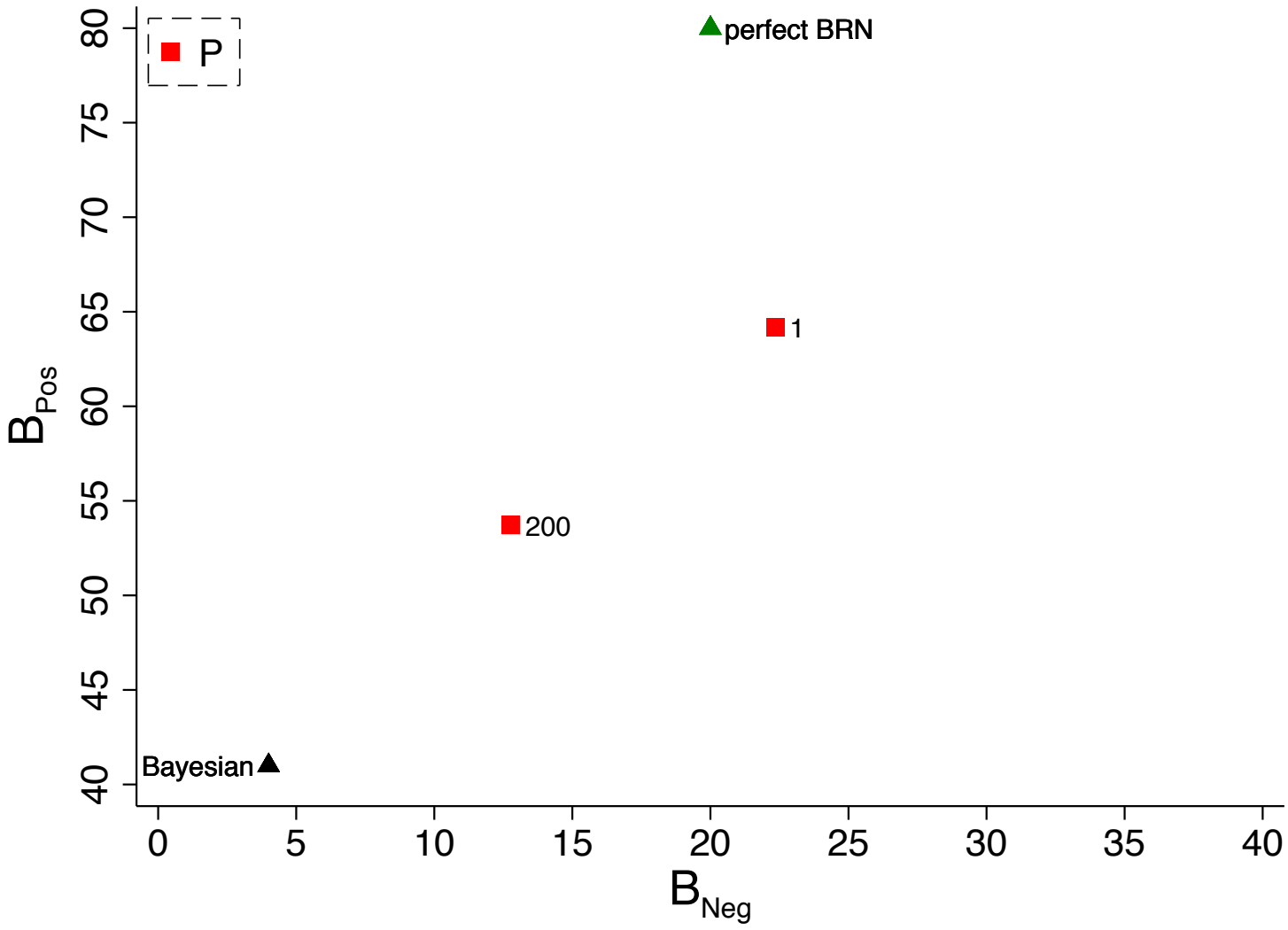
Are subjects with initial BRN responses driving the treatment effect?

- Separate subjects in Primitives into two groups:

  R1 pBRN: Subjects with perfect BRN responses in round 1.

  R1 Others: All other subjects in Primitives treatment.

Average Beliefs: Round 1



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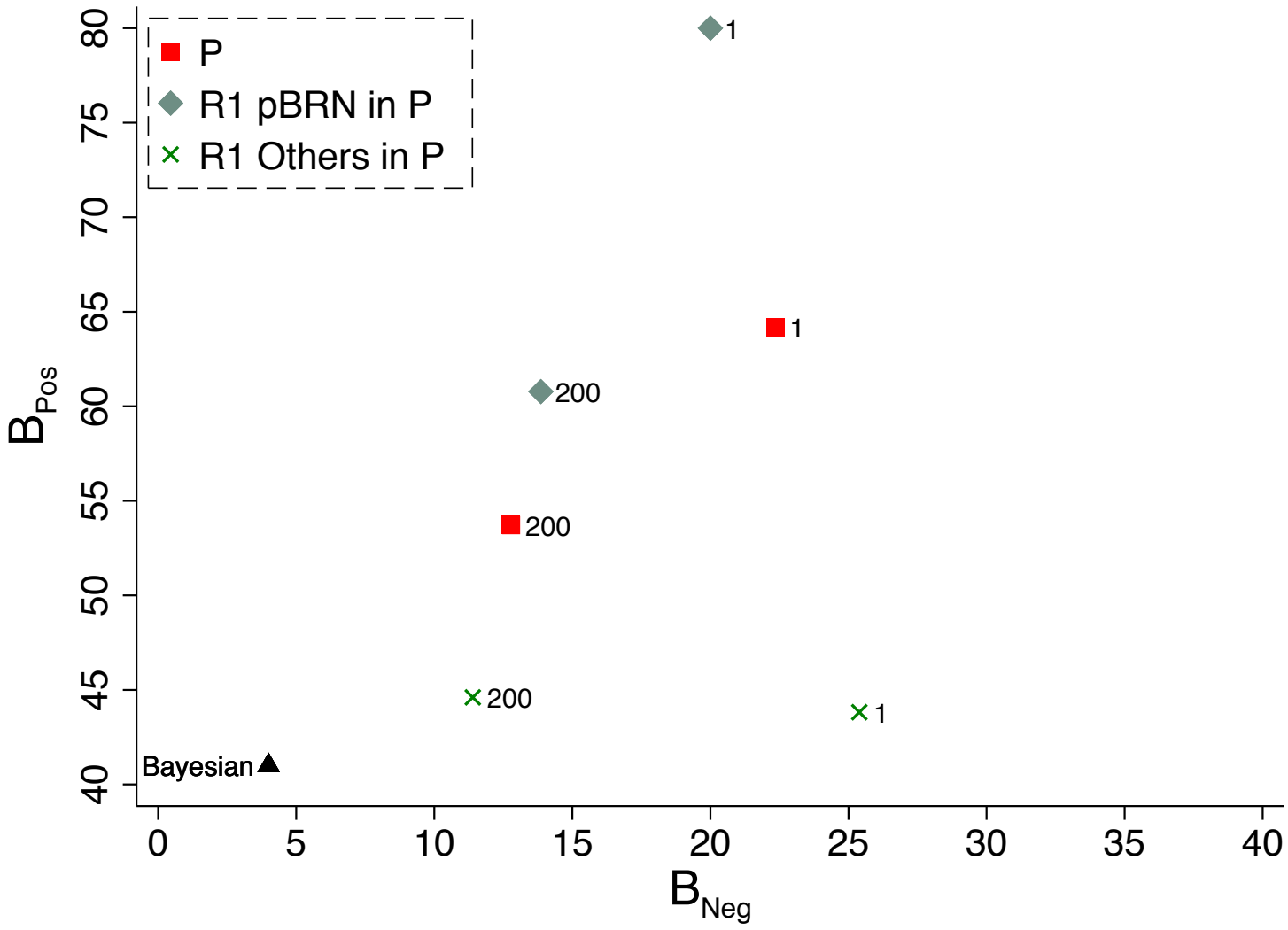
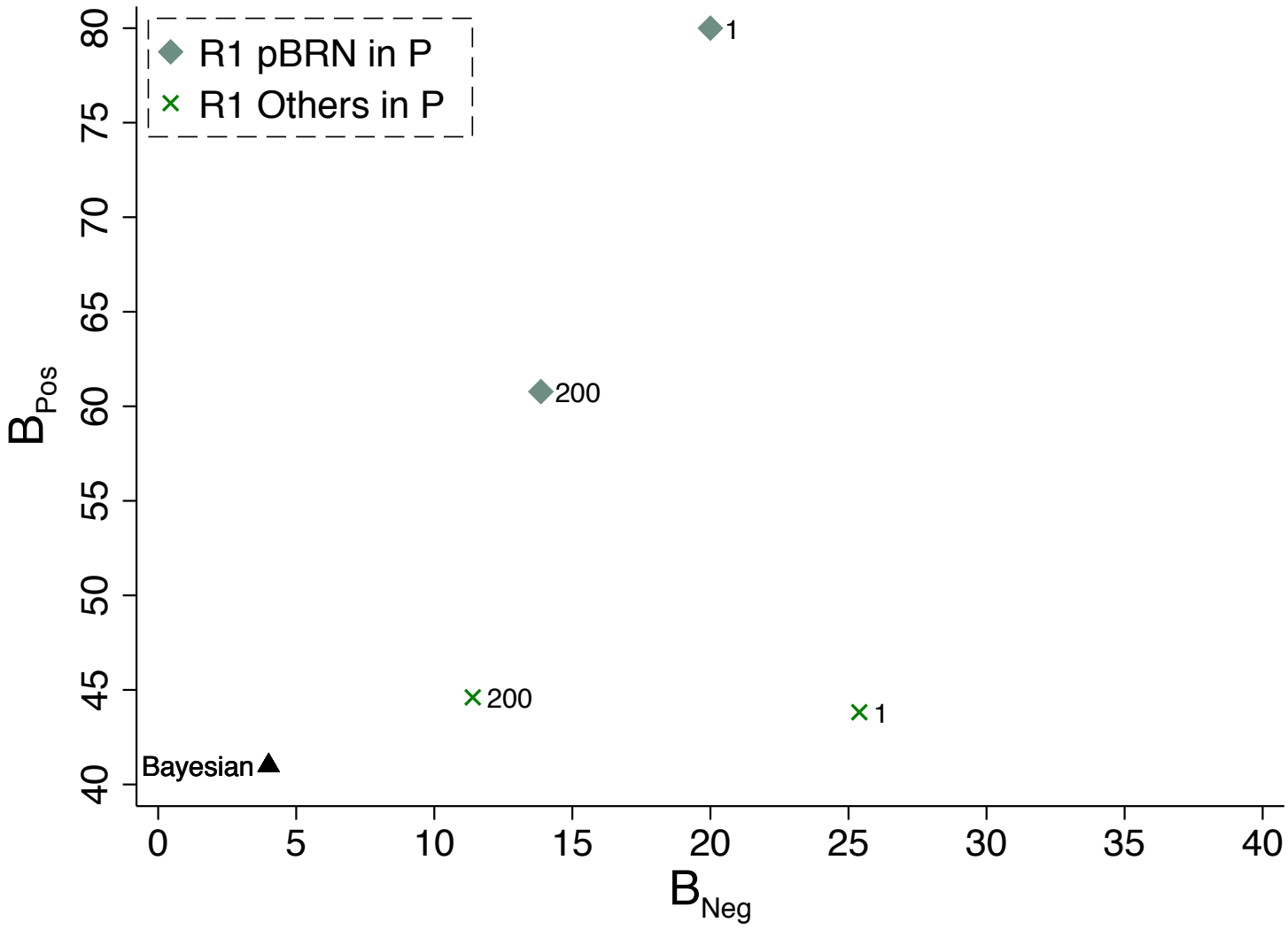


Average Beliefs: Rounds 1 & 200



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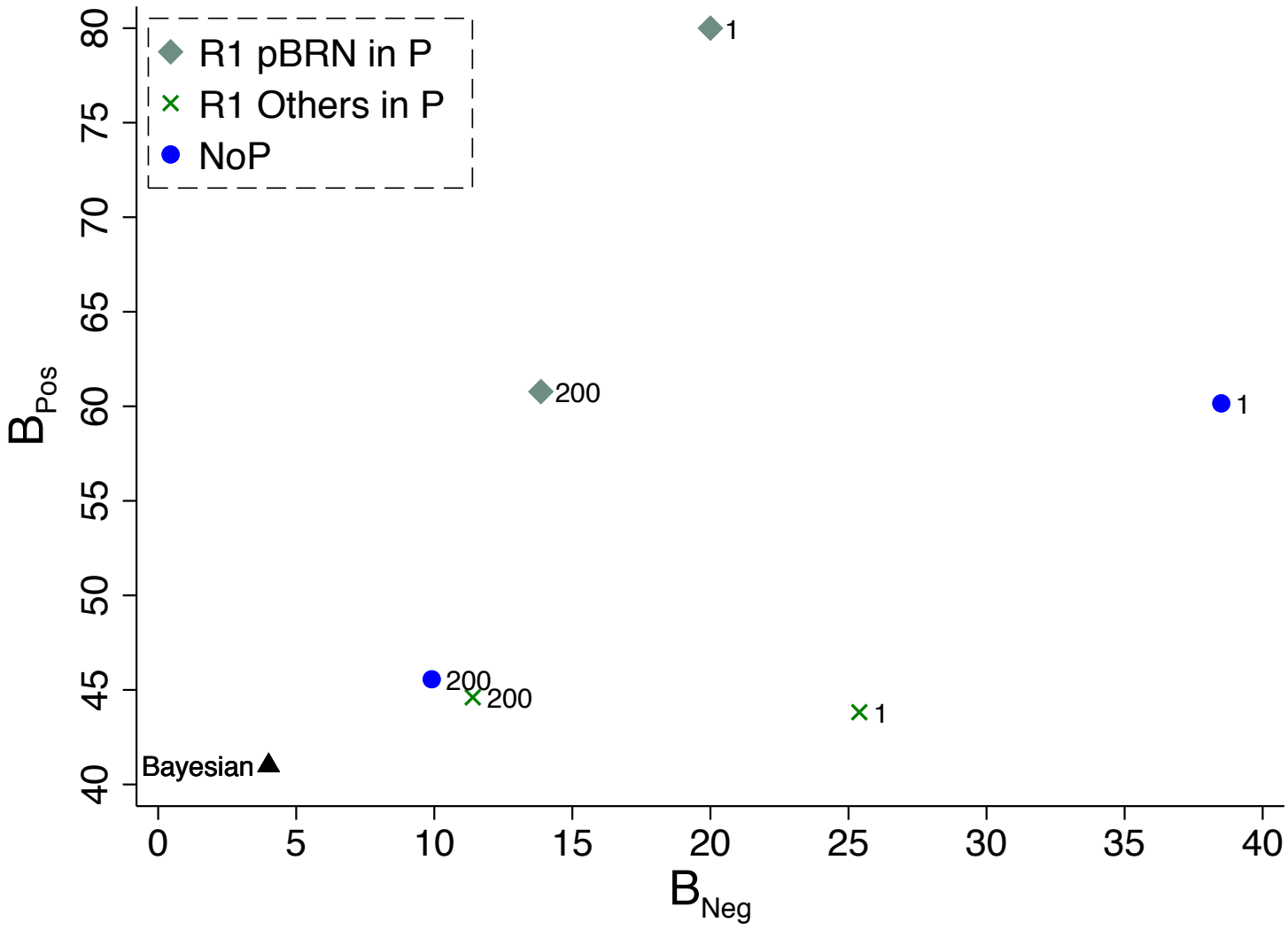

Result #2

Subjects in the Primitives treatment who display BRN in round 1 are driving the treatment effect in round 200.

R1 pBRN subjects and learning from feedback

Result #2 suggests that starting with an incorrect mental model can hinder learning from feedback.

Why is learning slower for R1 pBRN subjects?

- Less attentive to feedback?
- Reluctant to change their beliefs?

Evidence on R1 pBRN subject being less responsive to feedback

R1 pBRN subjects are:

1. Less responsive to immediate/cumulative feedback.
2. More likely to show convergence in beliefs.
3. Spend less time per decision.

Studying attentiveness

R1 pBRN subjects are less responsive to data.

Are they less attentive or choose not to be responsive?

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Each session consists of 5 sections:

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Studying attentiveness

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Are they less attentive or choose not to be responsive?

Each session consists of 5 sections:

- Part 1 Introduces the main updating task.
- Part 2 Repetition of the task for 200 rounds.
- Part 3 **Recollection of feedback.**
  - » Number of rounds each signal-state realization was observed?
- Part 4
- Part 5

REPORT FEEDBACK

Part 5

Enter the number of rounds where the feedback was:

A POSITIVE test result and the project was a SUCCESS ____

A POSITIVE test result and the project was a FAILURE ____

A NEGATIVE test result and the project was a SUCCESS ____

A NEGATIVE test result and the project was a FAILURE ____

	Test was POSITIVE	Test was NEGATIVE
Project was a SUCCESS		
Project was a FAILURE		

Check if your answers sum up to 200

Beliefs, Observed and Recalled Frequencies

1. Are recalled frequencies consistent with observed frequencies?
   R1 pBRN subjects have a noisier recollection of feedback.

2. Are beliefs consistent with recalled frequencies?
   No difference between R1 pBRN subjects and others.

▸ Statistical analysis

Result #3

The evidence suggests that subjects who initially display BRN are less attentive to feedback.

Summary of results so far

By round 200, beliefs in NoPrimitives are closer to the Bayesian benchmark.

The treatment effect driven by subjects with initial BRN responses.

These subjects are less attentive to feedback.

SUMMARY OF RESULTS SO FAR

By round 200, beliefs in NoPrimitives are closer to the Bayesian benchmark.

The treatment effect driven by subjects with initial BRN responses.

These subjects are less attentive to feedback.

Can initial misconceptions (such as BRN) be corrected?

Feedback in summary form

Up to now, feedback is presented round-by-round, but subjects determine whether and how to make use of it.

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Part 1 Introduces the main updating task.

Part 2 Repetition of the task for 200 rounds.

Part 3 Recollection of feedback.

Part 4 **Feedback summarized in table form.**

Part 5

Feedback as Summary Table

Part 6

Summary of the 200 rounds that you actually observed:

A POSITIVE test result and the project was a SUCCESS	23
A POSITIVE test result and the project was a FAILURE	43
A NEGATIVE test result and the project was a SUCCESS	5
A NEGATIVE test result and the project was a FAILURE	129

The two-by-two table on the right also summarizes this information and will be available when you make your choices.

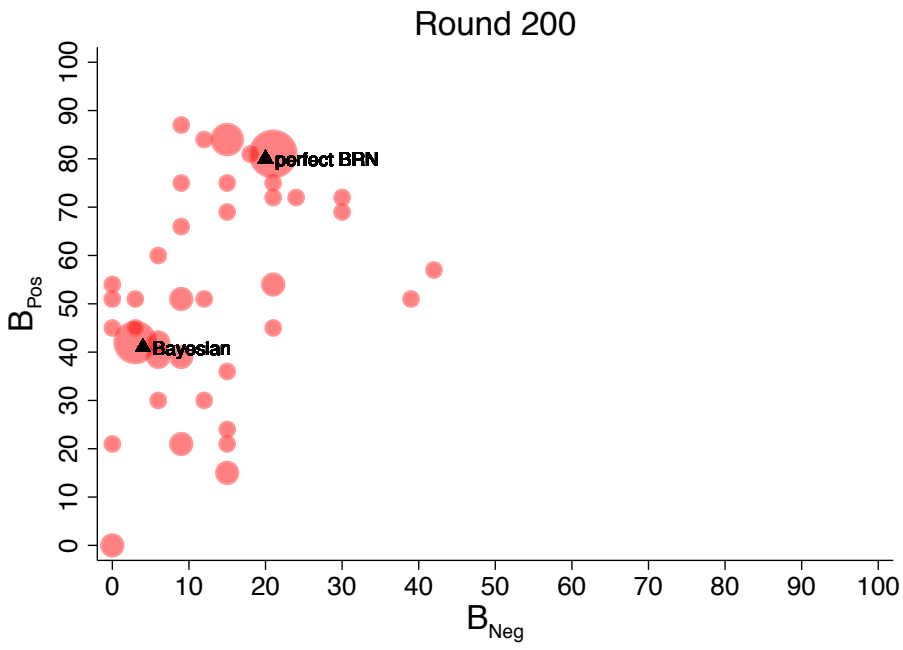
Summary of the 200 rounds that you actually observed:

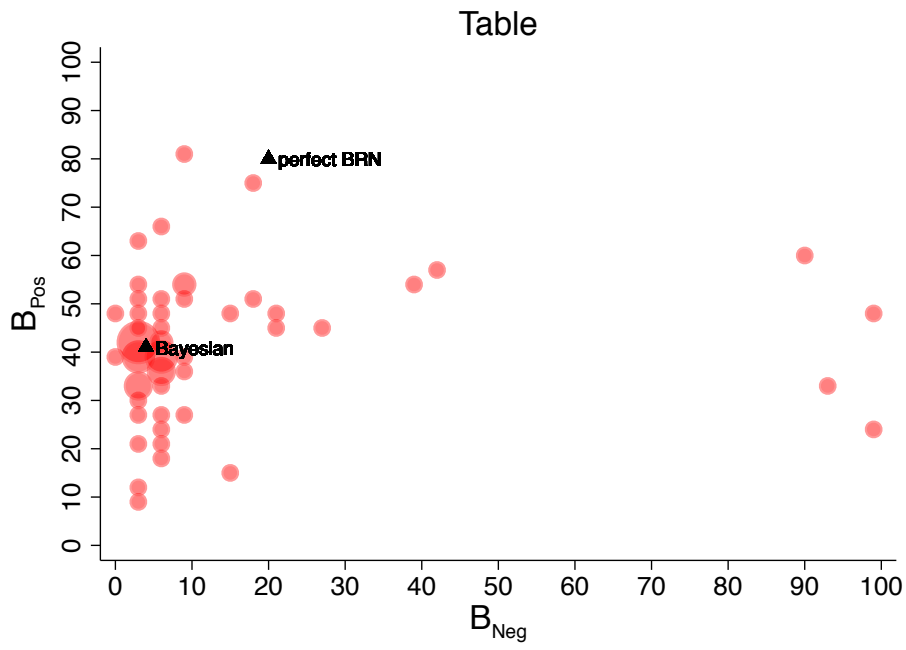
	Test was POSITIVE	Test was NEGATIVE
Project was a SUCCESS	23	5
Project was a FAILURE	43	129

Proceed to Make Choices

Distribution of Beliefs: Primitives

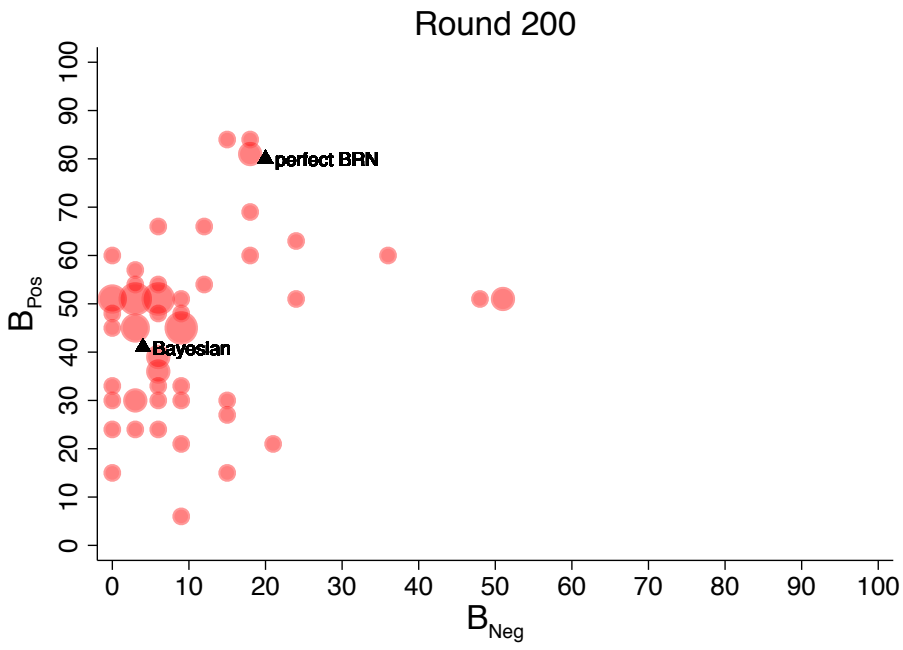
Primitives Treatment

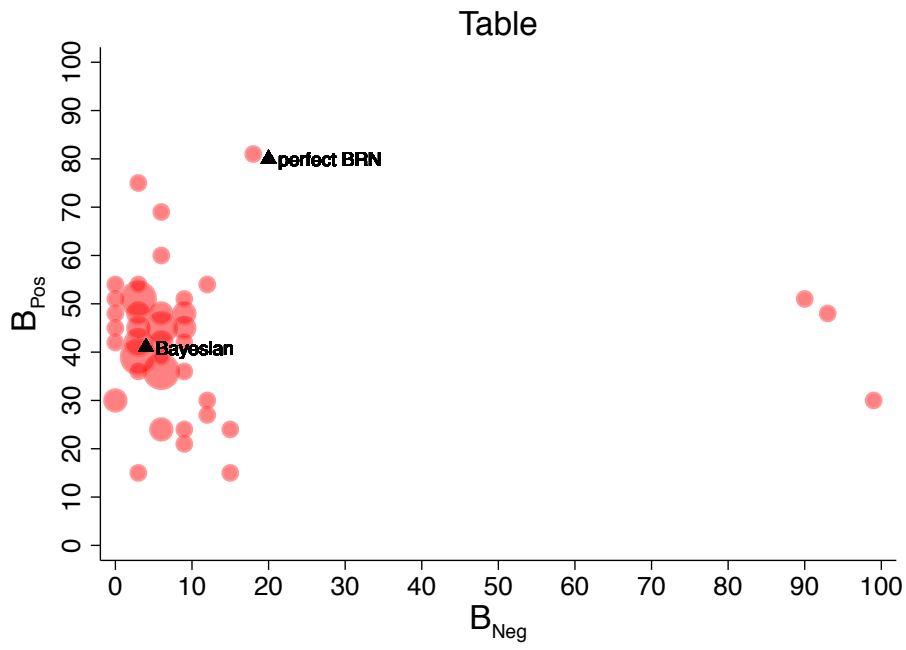




Distribution of Beliefs: NoPrimitives

NoPrimitives Treatment





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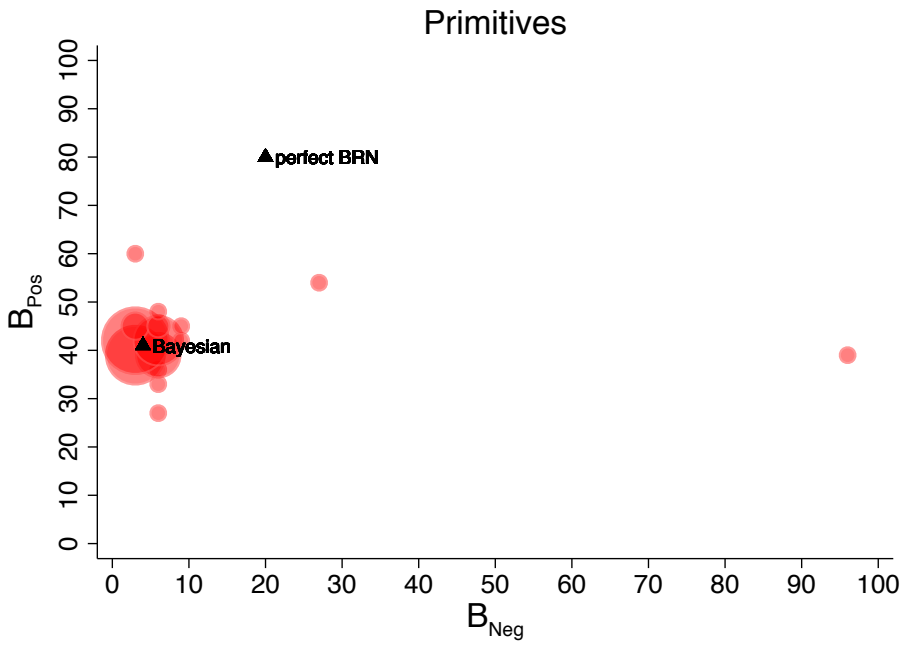
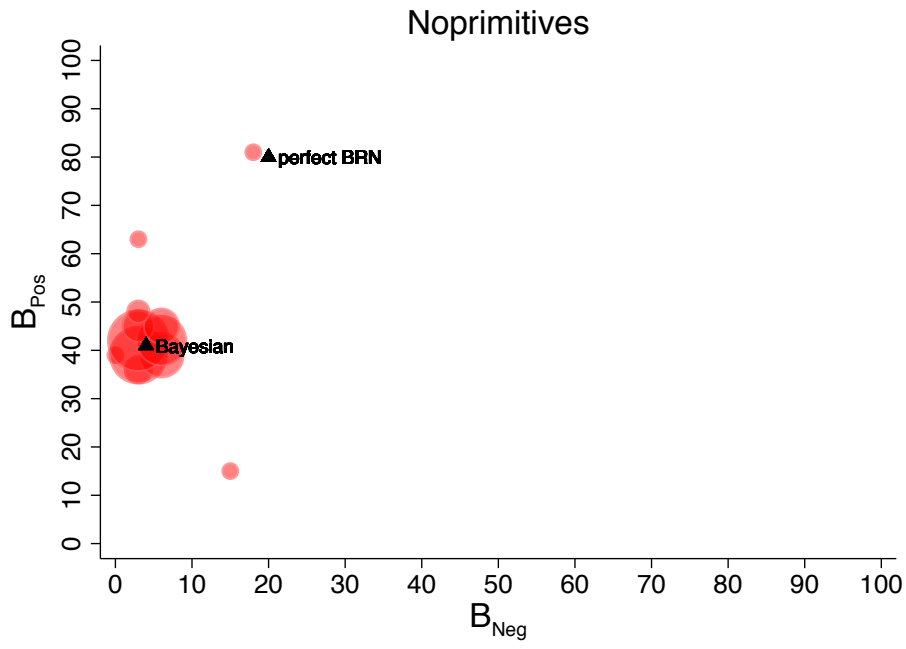
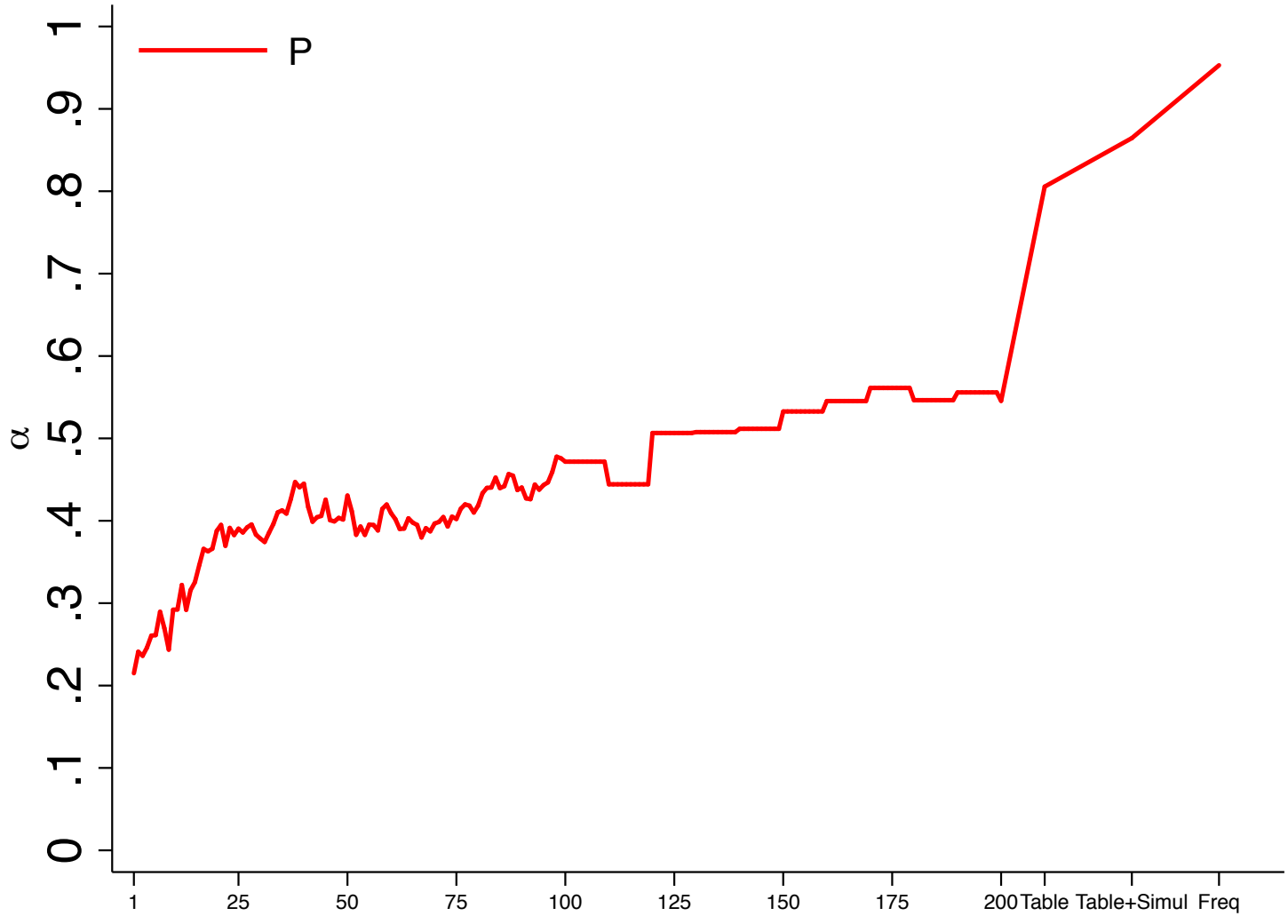




Responsiveness to the prior (α)



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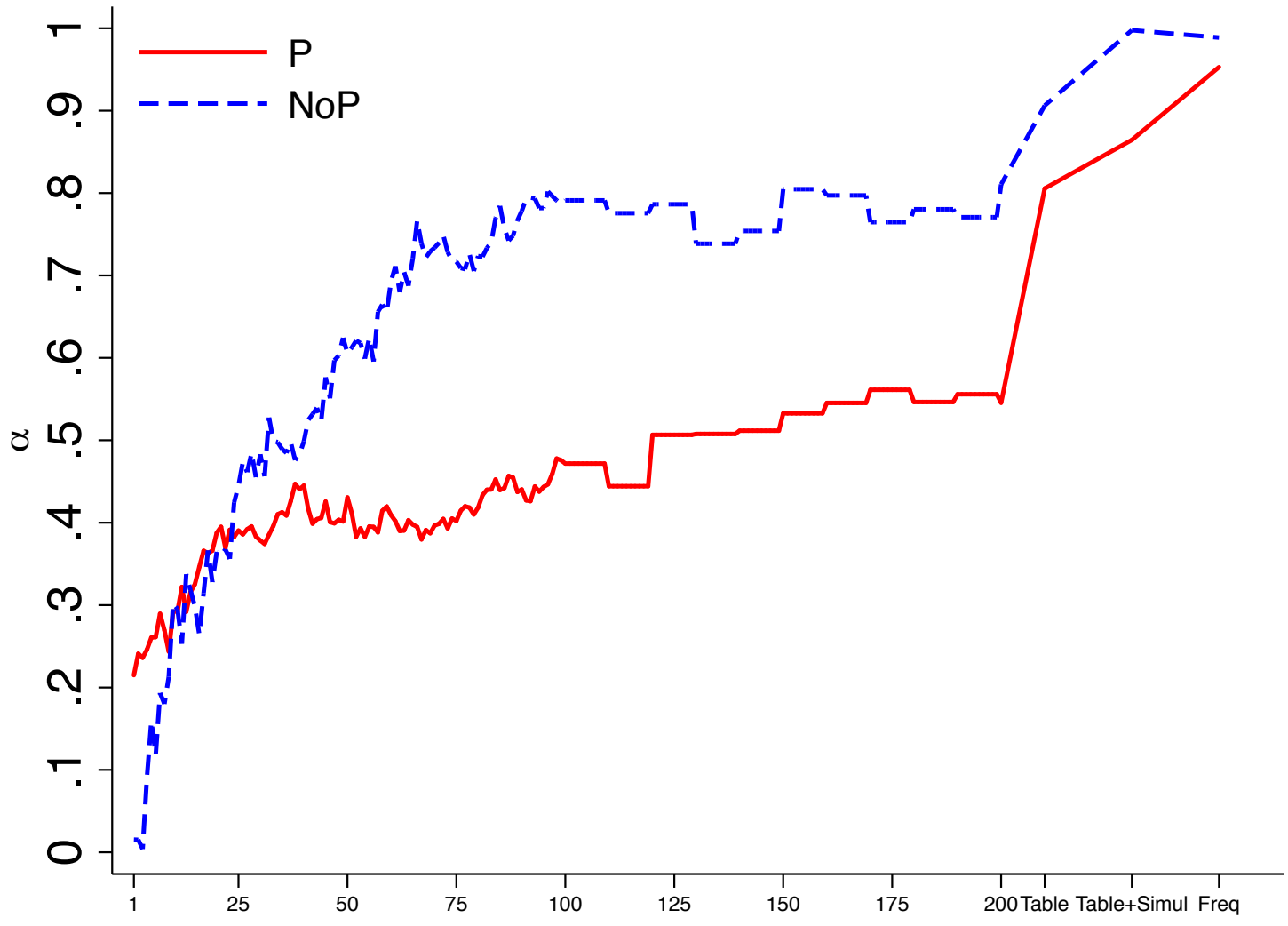

Result #4

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Old vs. New Parameters (Round 1): Primitives

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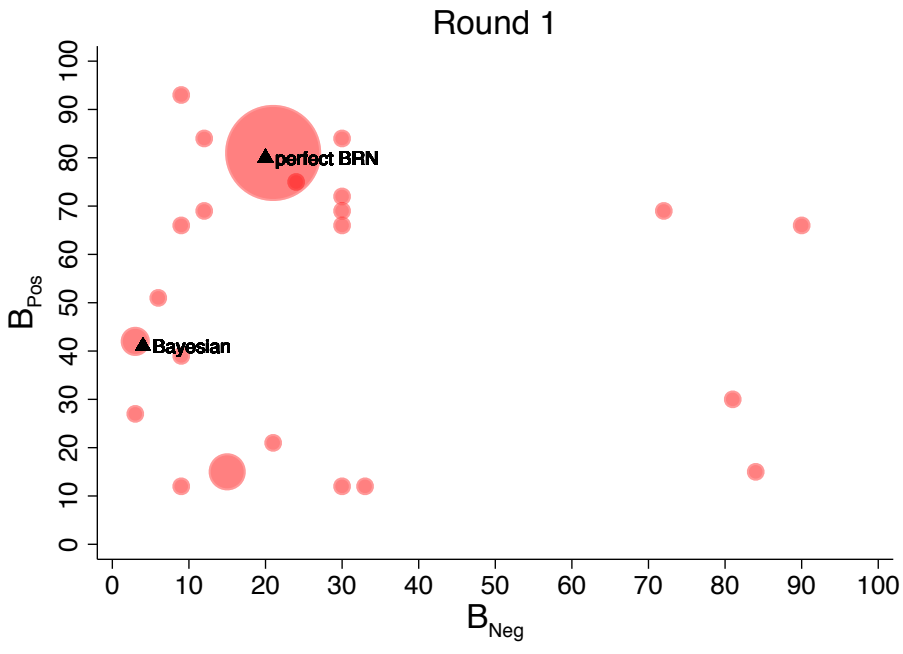
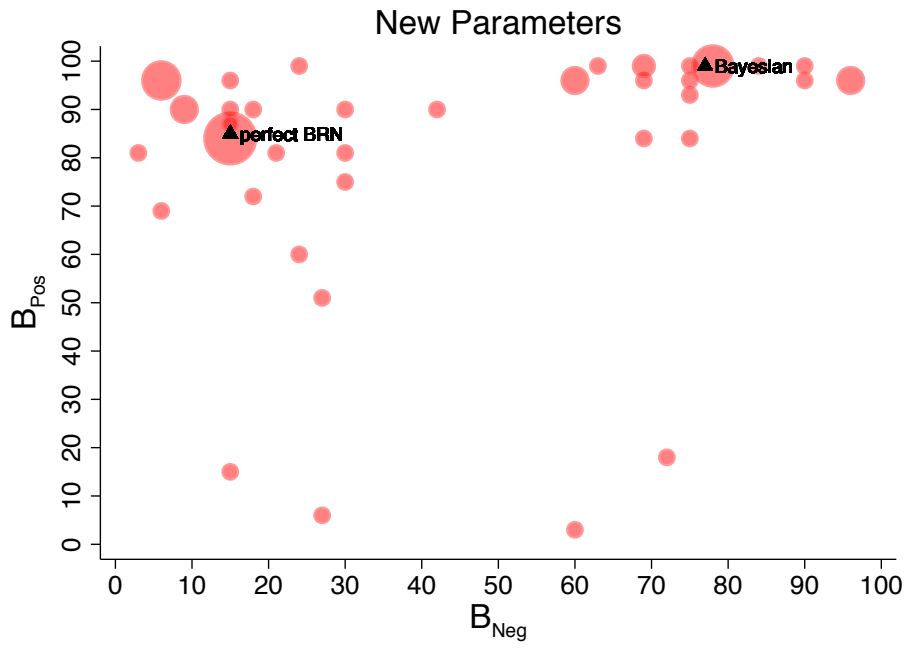
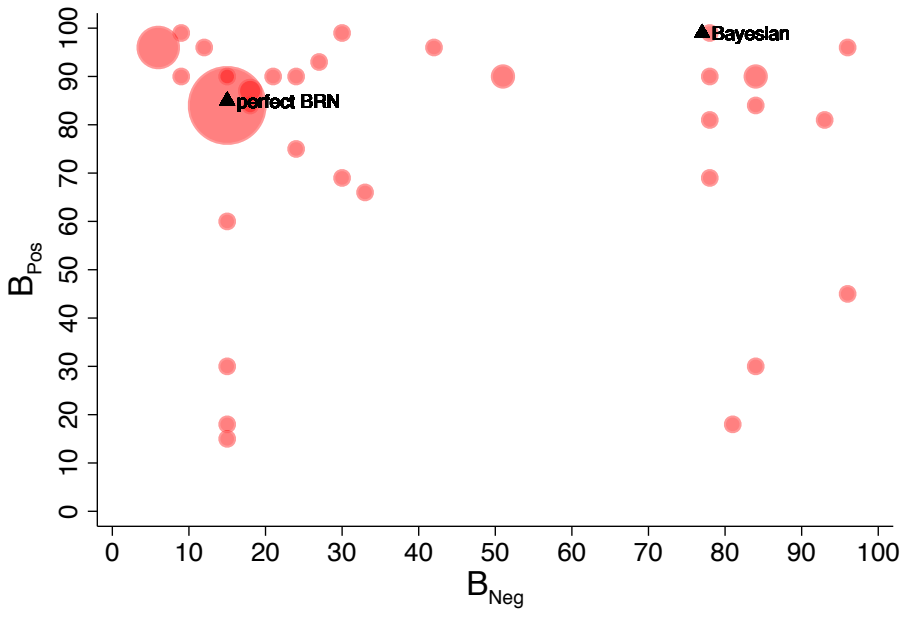


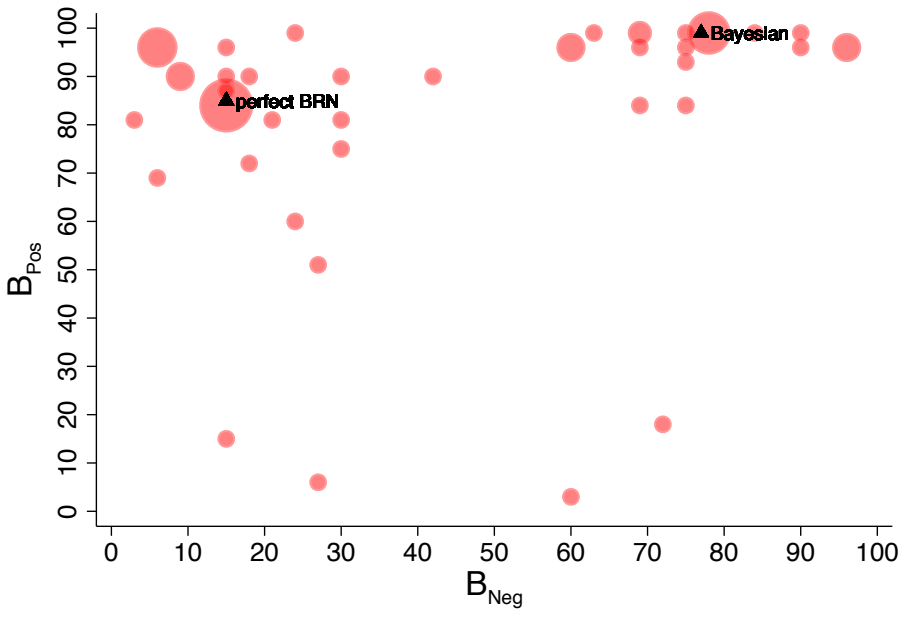


Distribution of Beliefs with New Parameters

NoPrimitives



Primitives



Results: Cross-environment learning

	Old parameters	New parameters	
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perfect Base-Rate Neglect	56.3	**17.2**	**37.5**
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Table reports % initial responses consistent with each answer.

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perfect Base-Rate Neglect	56.3	**17.2**	**37.5**
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Table reports % initial responses consistent with each answer.

With new parameters, BRN is more prevalent in NoPrimitives treatment.

Main Findings

Are beliefs Bayesian in the long run?

- **Adjustment is slow and partial.**

Does starting with BRN hinder learning?

- **Beliefs closer to the Bayesian benchmark in NoPrimitives.**

Why is learning slower for those who initially display BRN?

- **Results suggest they are less attentive to feedback.**

Can BRN be corrected?

- **If feedback is provided in summary form to challenge BRN.**

Is learning transferable to new settings?

- **Yes, but partially.**

Some Implications

Biases can be persistent because they impact how agents learn from feedback.

- Literature studying implications of misspecified models. Esponda Pouzo 16, Fudenberg Romanyuk Strack 17, Bohren Hauser 17, Heidhues Koszegi Strack 18.
- Literature on endogenous attention. Sims 03, Caplin Dean 15, Schwartzstein 14, Gagnon-Bartsch Rabin Schwartzstein 18.

Some Implications

Policy implications:

- Biases can be persistent even in information rich environments.
- Interventions need to influence how agents engage with information.
- Withholding payoff relevant information can improve long run behavior.

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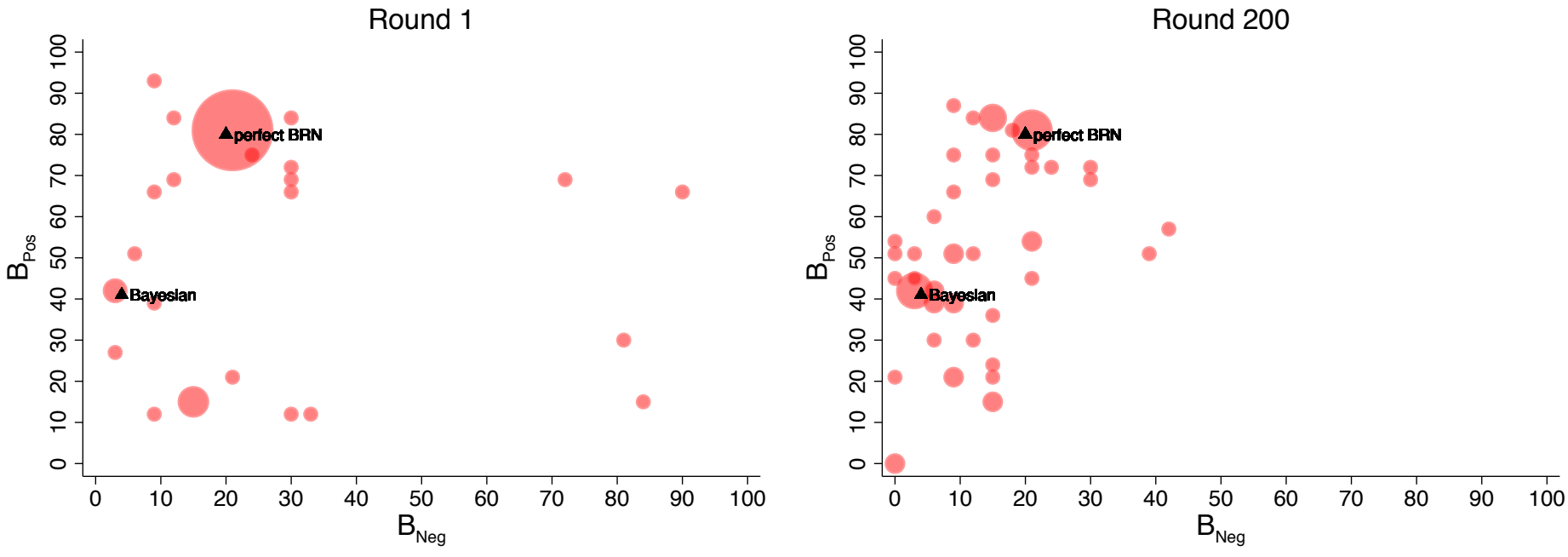
Proof of concept: design can be used to study persistence of biases in other settings.

Thank you

Distribution of Beliefs: Primitives

Primitives Treatment



Distribution of Beliefs: NoPrimitives

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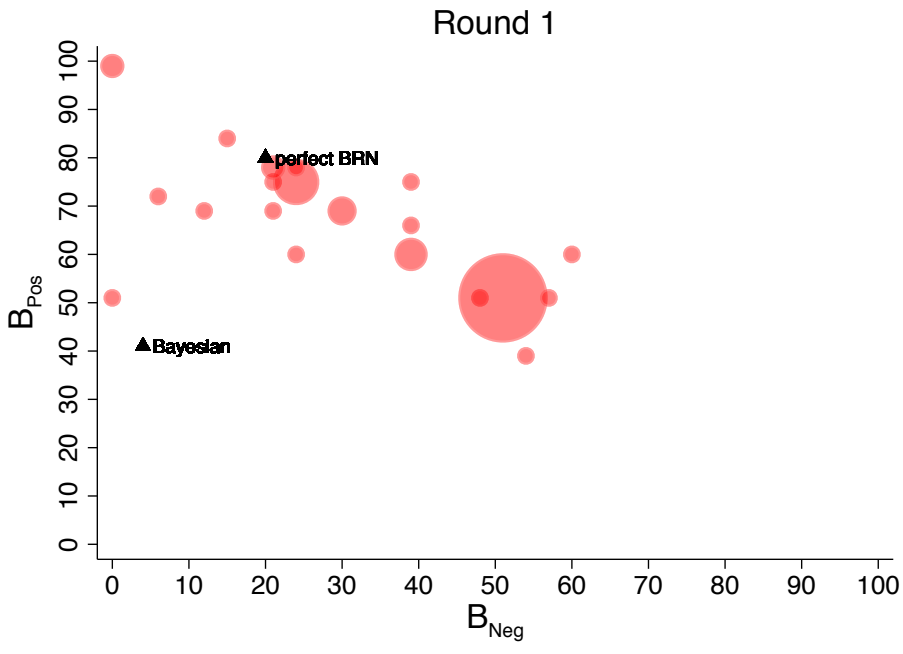
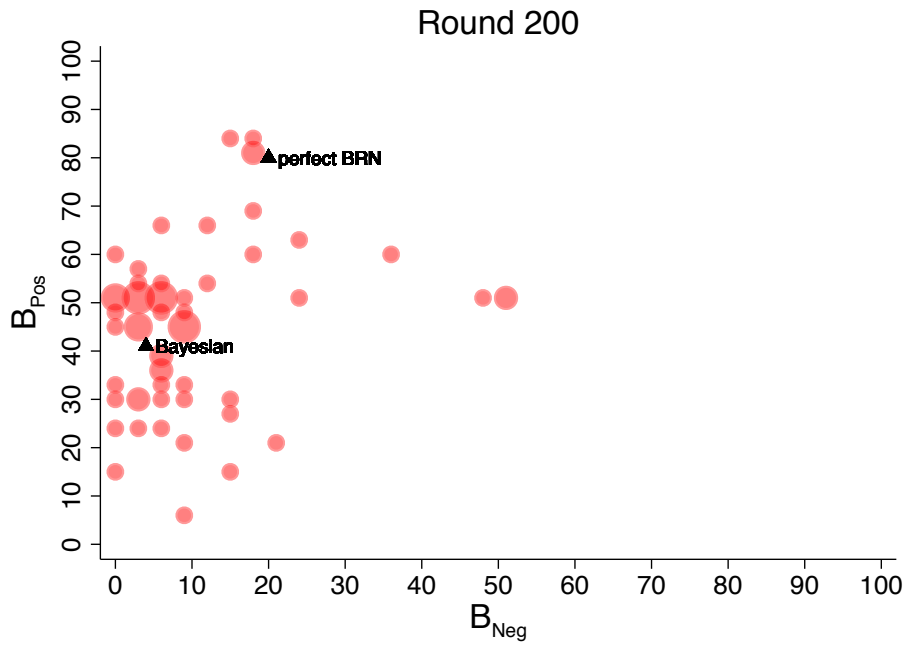




Heterogeneity in Learning

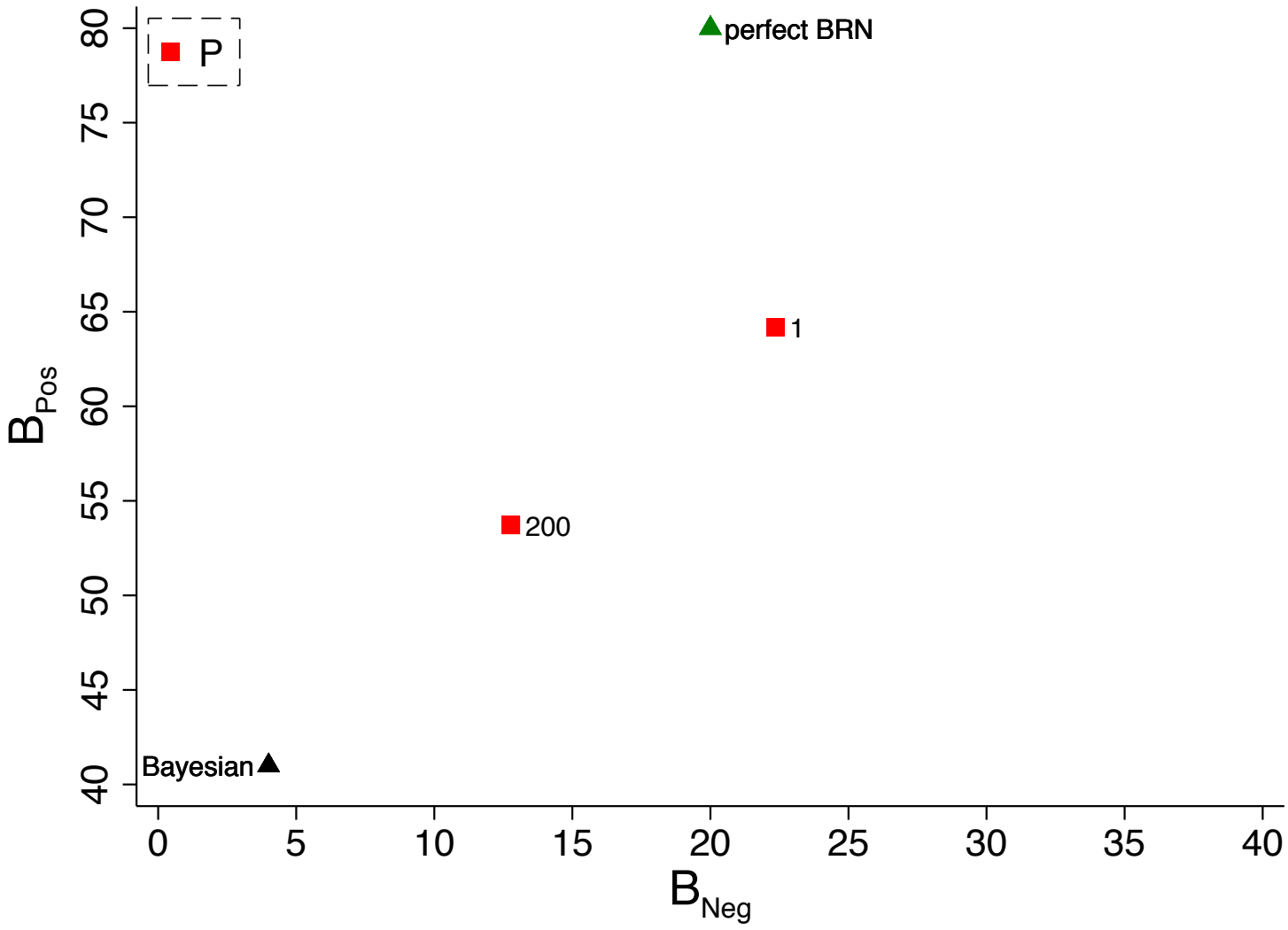
Are subjects with initial BRN responses driving the treatment effect?

- Separate subjects in Primitives into two groups:

  R1 pBRN: Subjects with perfect BRN responses in round 1.

  R1 Others: All other subjects in Primitives treatment.

Average Beliefs: Round 1



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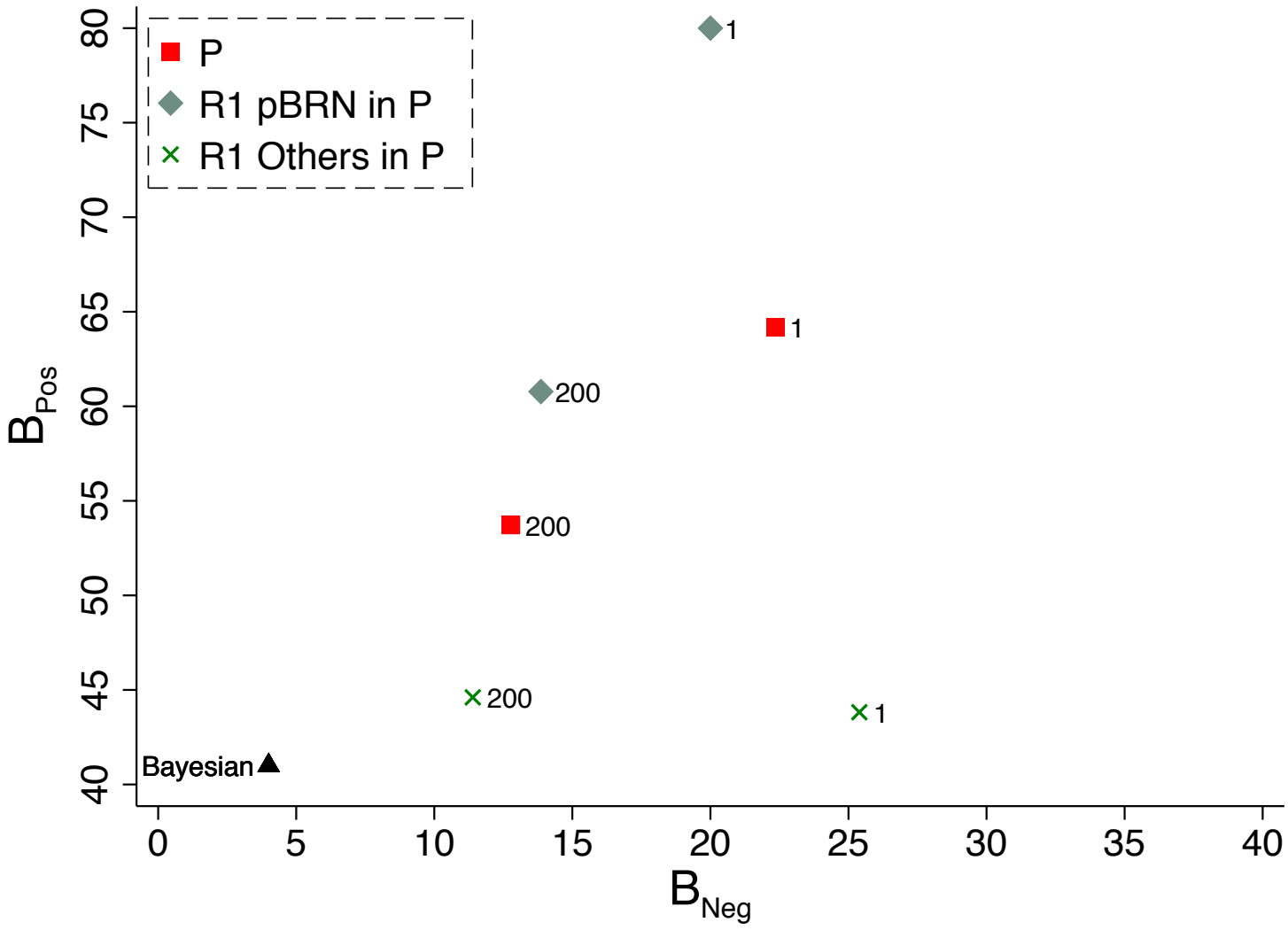
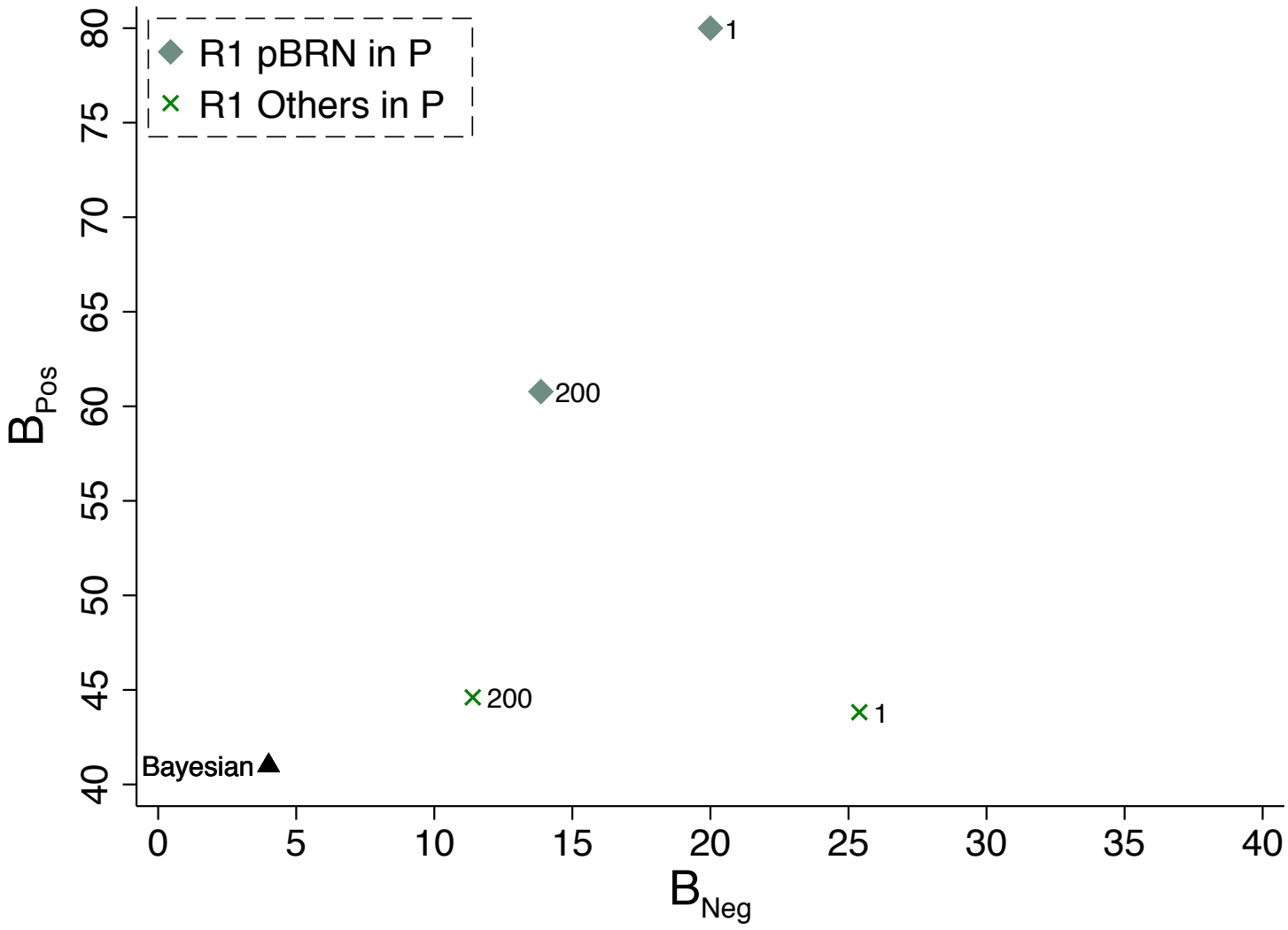


Average Beliefs: Rounds 1 & 200



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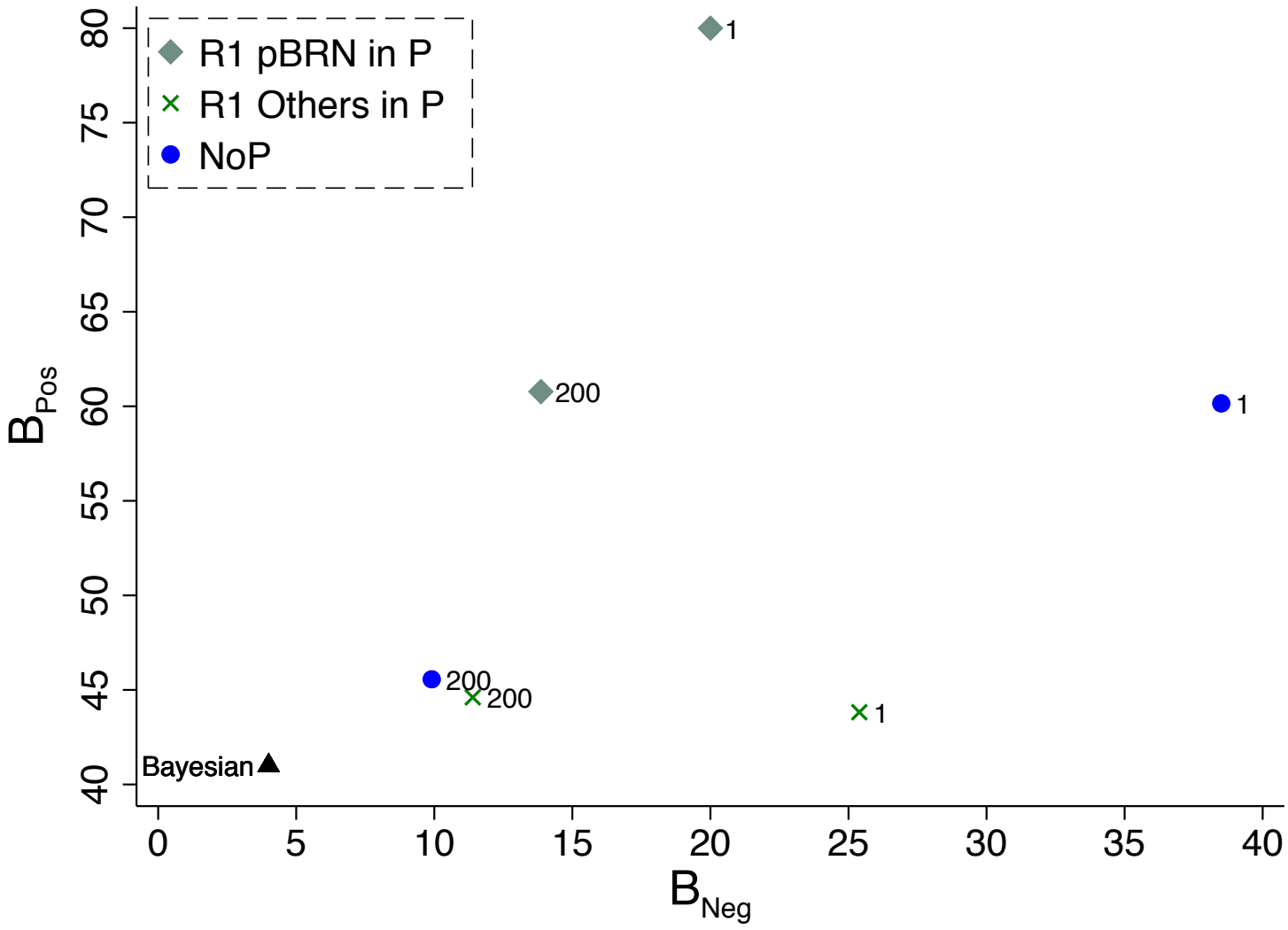

Result #2

Subjects in the Primitives treatment who display BRN in round 1 are driving the treatment effect in round 200.

R1 pBRN subjects and learning from feedback

Result #2 suggests that starting with an incorrect mental model can hinder learning from feedback.

Why is learning slower for R1 pBRN subjects?

- Less attentive to feedback?
- Reluctant to change their beliefs?

Evidence on R1 pBRN subject being less responsive to feedback

R1 pBRN subjects are:

1. Less responsive to immediate/cumulative feedback.
2. More likely to show convergence in beliefs.
3. Spend less time per decision.
4. Have a noisier recollection of the data.

Result #3

The evidence suggests that subjects who initially display BRN are less attentive to feedback.

Summary of results so far

By round 200, beliefs in NoPrimitives are closer to the Bayesian benchmark.

The treatment effect driven by subjects with initial BRN responses.

These subjects are less attentive to feedback.

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By round 200, beliefs in NoPrimitives are closer to the Bayesian benchmark.

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Can initial misconceptions (such as BRN) be corrected?

Feedback in summary form

Up to now, feedback is presented round-by-round, but subjects determine whether and how to make use of it.

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Part 3 Recollection of feedback.

Part 4 **Feedback summarized in table form.**

Part 5

Feedback as Summary Table

Part 6

Summary of the 200 rounds that you actually observed:

A POSITIVE test result and the project was a SUCCESS	23
A POSITIVE test result and the project was a FAILURE	43
A NEGATIVE test result and the project was a SUCCESS	5
A NEGATIVE test result and the project was a FAILURE	129

The two-by-two table on the right also summarizes this information and will be available when you make your choices.

Summary of the 200 rounds that you actually observed:

	Test was POSITIVE	Test was NEGATIVE
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Proceed to Make Choices

Distribution of Beliefs: Primitives

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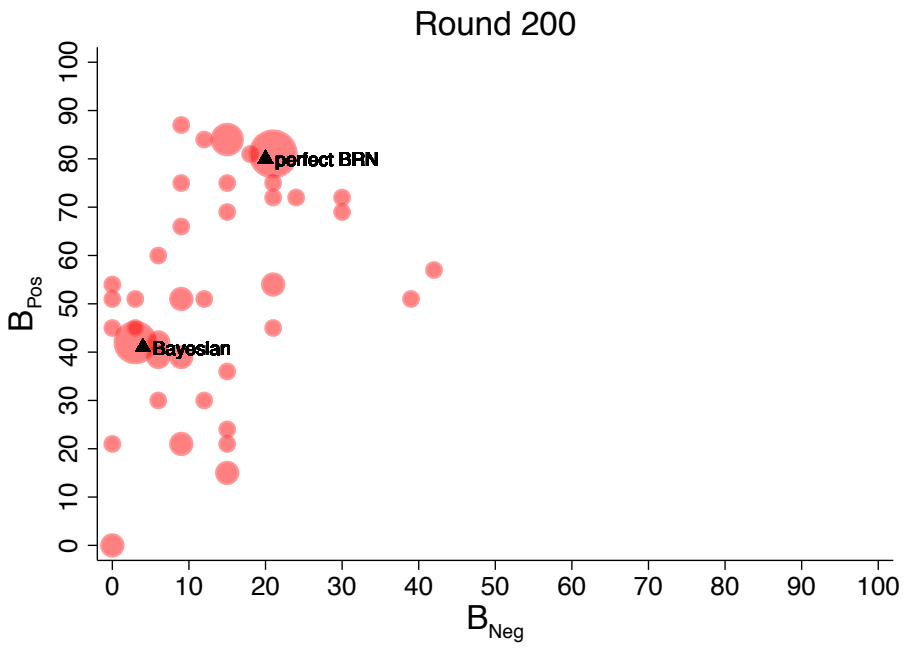

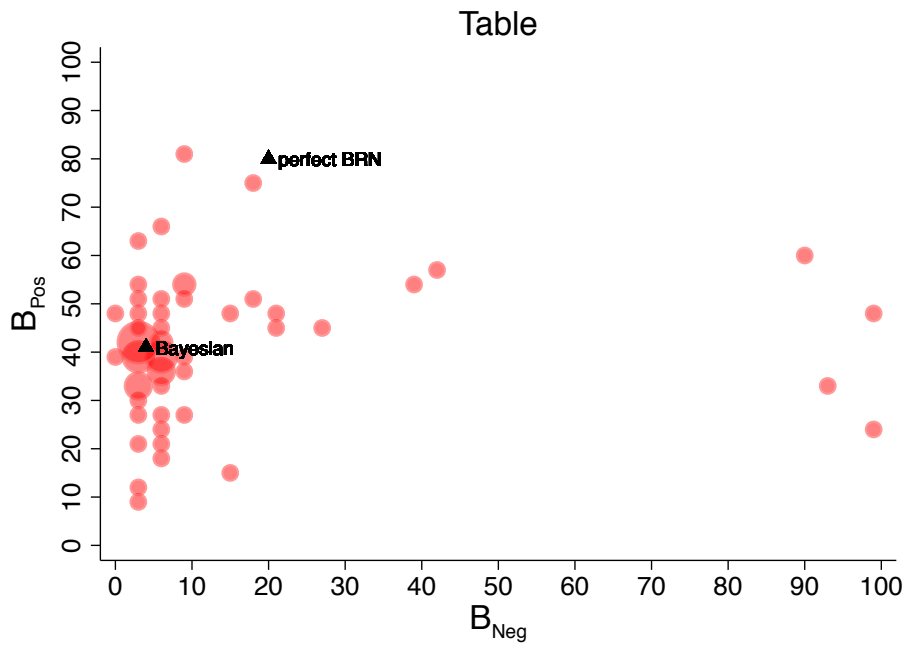

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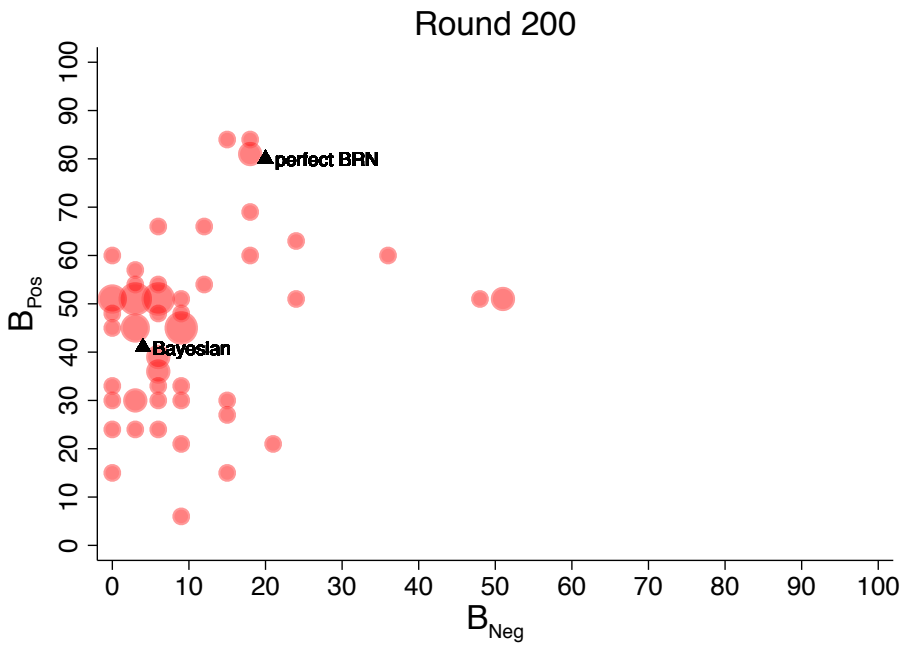

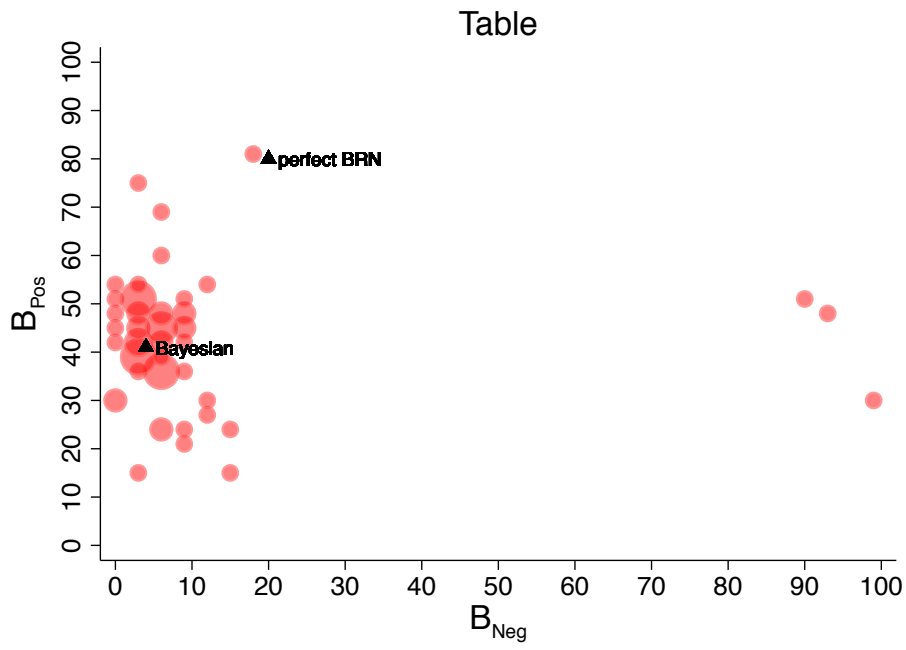

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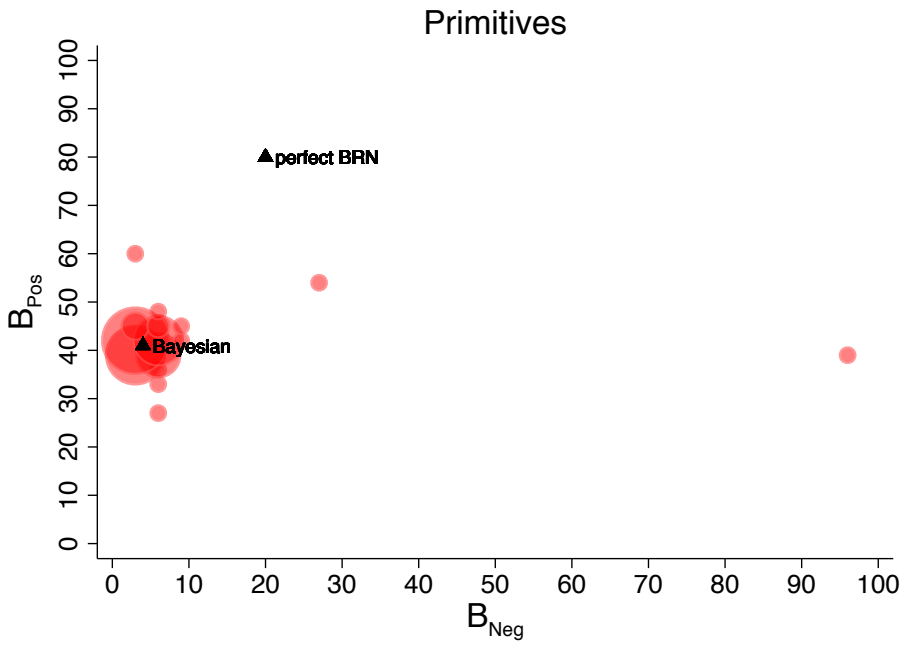

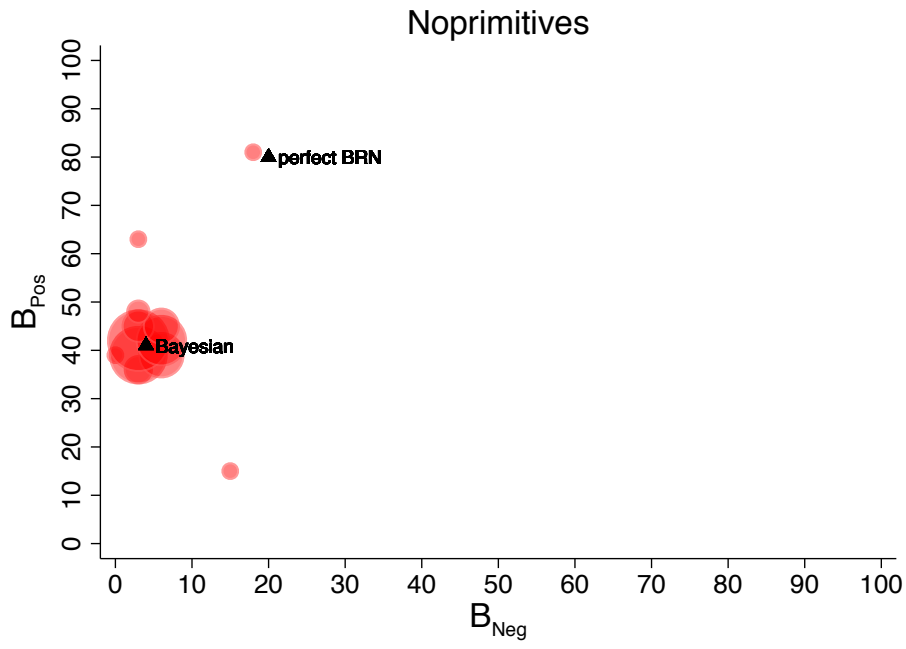

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Old vs. New Parameters (Round 1): Primitives

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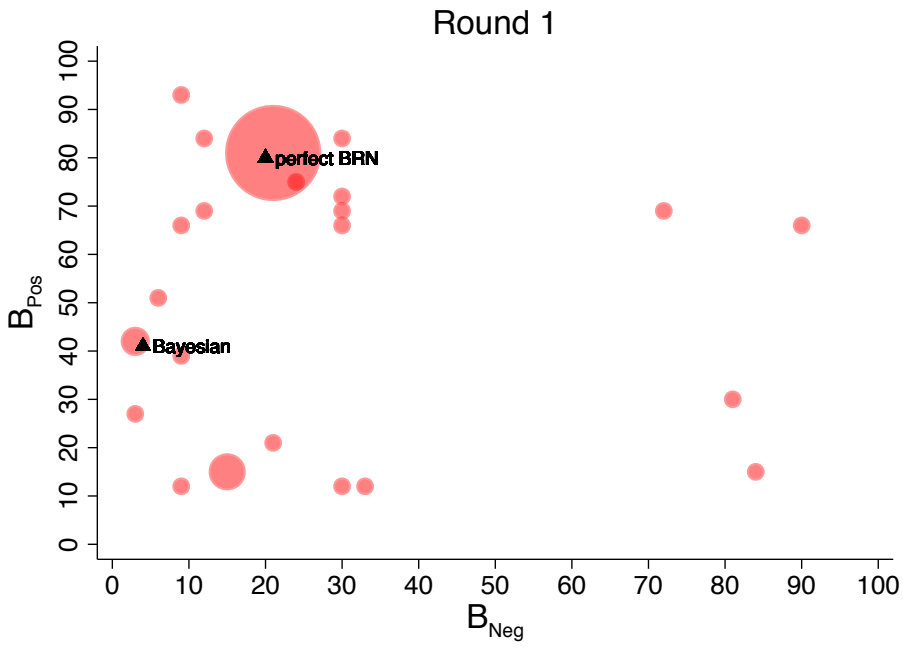

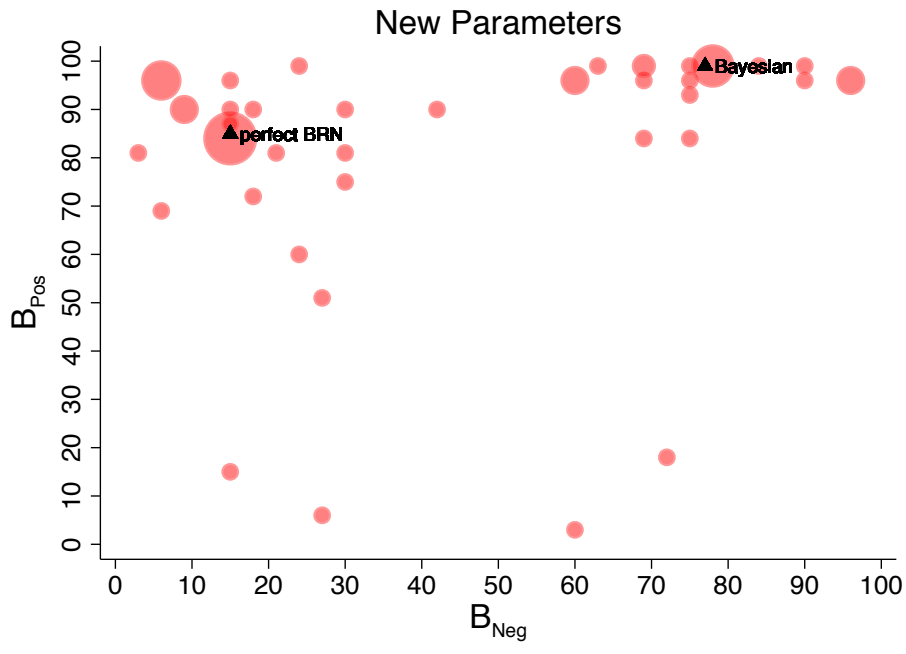

Distribution of Beliefs with New Parameters

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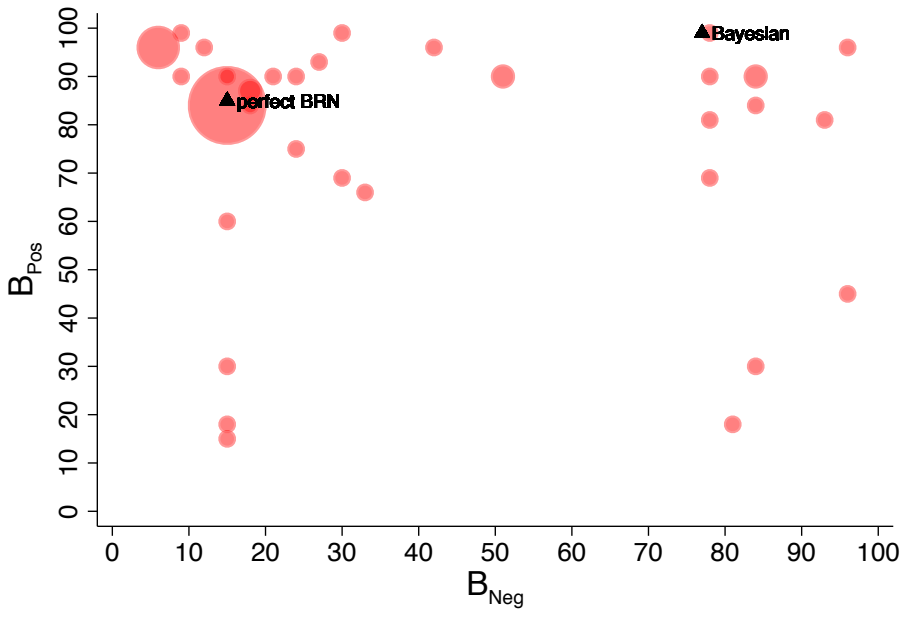
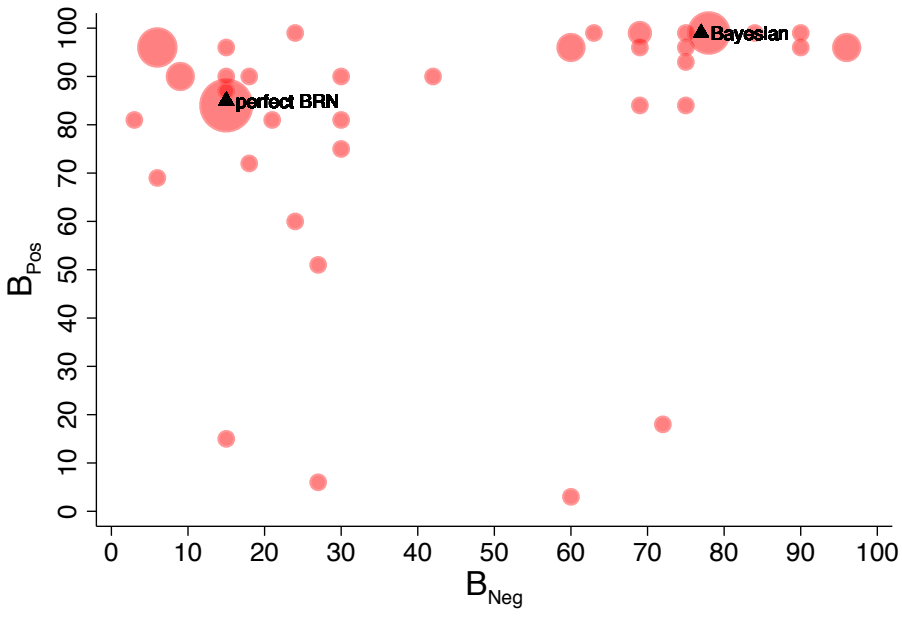


Primitives



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With new parameters, BRN is more prevalent in NoPrimitives treatment.

SUMMARY

	1. Average distance to Bayesian benchmark			2. Distance in Beliefs between P and NoP		
	P	NoP	H_0 : P=NoP (p-value)	b_{Neg}	b_{Pos}	$H_0 : b_{Neg} = b_{Pos} = 0$ (p-value)
R1	25.3	27.0	.203	-16.1	4.0	< .001
R20	21.3	20.9	.843	-0.4	2.6	.829
R100	18.6	13.0	.003	5.6	6.0	.056
R200	15.3	10.5	.004	2.9	8.1	.049

Back to R200

Two ways to study treatment differences

1. Are beliefs closer to the Bayesian benchmark in P or NoP?

   Look at the average distance between beliefs and the benchmark.

2. Are beliefs different between P and NoP?

   Look at the average distance between beliefs in P and NoP.

1. Distance Beliefs & Bayesian benchmark

» Average absolute value:

$$\Delta = \frac{\left|B_{Neg} - 4\%\right| + \left|B_{Pos} - 41\%\right|}{2}.$$

» Euclidean distance gives same qualitative results.

▸ Back to table

2. Distance between beliefs across treatments

» System of equations:

$$B_{Pos} = a_{Pos} + b_{Pos}\mathrm{P} + \epsilon_{Pos}$$
$$B_{Neg} = a_{Pos} + b_{Neg}\mathrm{P} + \epsilon_{Neg}$$

» $\mathrm{P} = 1$ if treatment P.

Back to table

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Table	**9.8**	**8.0**	**.394**	**3.5**	**-1.3**	**.523**

Back to Table

1. Responses to immediate feedback (B_{Pos})

	$[B_{Pos,t} - B_{Pos,t-1}]$
	Primitives
	All
(Positive, Success)$_{t-1}$	0.7***
(Positive, Failure)$_{t-1}$	-1.1***
(Negative, Success)$_{t-1}$	-0.2
(Negative, Failure)$_{t-1}$	0

All Rounds All Subjects, SE clustered by subject.

$(*, **, ***)$: Significant at 10, 5, 1 percent level.

1. Responses to immediate feedback (B_{Pos})

	$[B_{Pos,t} - B_{Pos,t-1}]$	
	Primitives	NoPrimitives
	All	All
(Positive, Success)$_{t-1}$	0.7***	2.4***
(Positive, Failure)$_{t-1}$	-1.1***	-2.8***
(Negative, Success)$_{t-1}$	-0.2	-0.9*
(Negative, Failure)$_{t-1}$	0	.1

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1. Responses to immediate feedback (B_{Pos})

	$[B_{Pos,t} - B_{Pos,t-1}]$		
	Primitives		NoPrimitives
	All	pBRN	All
(Positive, Success)$_{t-1}$	0.7***	0.5	2.4***
(Positive, Failure)$_{t-1}$	-1.1***	-0.9	-2.8***
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	$[B_{Pos,t} - B_{Pos,t-1}]$			
	Primitives			NoPrimitives
	All	pBRN	Others	All
(Positive, Success)$_{t-1}$	0.7***	0.5	1.1*	2.4***
(Positive, Failure)$_{t-1}$	-1.1***	-0.9	-1.5***	-2.8***
(Negative, Success)$_{t-1}$	-0.2	-0.6	0.2	-0.9*
(Negative, Failure)$_{t-1}$	0	-0.2	0.2**	.1

All Rounds All Subjects, SE clustered by subject.

$(*, **, ***)$: Significant at 10, 5, 1 percent level.

1. Responses to immediate feedback (B_{Neg})

	$[B_{Neg,t} - B_{Neg,t-1}]$			
	Primitives			NoPrimitives
	All	pBRN	Others	All
(Positive, Success)$_{t-1}$	0.5	0.2	-0.1	0
(Positive, Failure)$_{t-1}$	0.6	-0.1	0.1	0.4
(Negative, Success)$_{t-1}$	0.9^{***}	0.6	1.3^{**}	2.0^{***}
(Negative, Failure)$_{t-1}$	-0.2^{**}	0.1	-0.3^{***}	-0.6^{***}

All Rounds All Subjects, SE clustered by subject.

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Back

1. Response to cumulative feedback

Are beliefs in round 200 consistent with observed frequencies?

Measure of Distance: $\Delta_{B,F} = \frac{|B_{Neg} - F_{Neg}| + |B_{Pos} - F_{Pos}|}{2}$.

- B_{Neg}/B_{Pos}: Belief conditional on a neg./pos. signal.
- F_{Neg}/F_{Pos}: Observed frequency conditional on a neg./pos. signal.

1. Response to cumulative feedback

	$\Delta_{B,F}$
R1 pBRN in Primitives	17.9
R1 Others in Primitives	11.4
NoPrimitives	9.8
Hypotheses:	
R1 pBRN = R1 Others	.006
R1 pBRN = NoP	.000
R1 Others = NoP	.454

Back

2. Convergence more likely in Primitives

	Primitives	NoPrimitives
Same choices in...	All	All
Round 91-100	77	36
Round 96-100	93	47

As % of Subjects in each treatment.

2. Convergence more likely in Primitives

	Primitives			NoPrimitives
Same choices in...	All	R1 pBRN	R1 Others	All
Round 91-100	77	83	68	36
Round 96-100	93	94	93	47

As % of Subjects in each treatment.

Back

3. Subject take longer to form beliefs in P

The mean (median) number of minutes to complete the first 100 rounds:

- Primitives: 11 (9) vs. NoPrimitives: 15 (13).
- No significant difference between R1 pBRN and R1 Others in Primitives.

Back

Responsiveness to Prior

Follow approach by Grether (1980):

Bayes' rule for each test:

$$\frac{B_{Pos}}{1 - B_{Pos}} = \frac{p}{1 - p} \times \frac{q}{1 - q}$$

$$\frac{B_{Neg}}{1 - B_{Neg}} = \frac{p}{1 - p} \times \frac{1 - q}{q}$$

Responsiveness to Prior

Follow approach by Grether (1980):

Bayes' rule in logs:

$$\ln\left(\frac{B_{Pos}}{1-B_{Pos}}\right) = \ln\left(\frac{p}{1-p}\right) + \ln\left(\frac{q}{1-q}\right)$$

$$\ln\left(\frac{B_{Neg}}{1-B_{Neg}}\right) = \ln\left(\frac{p}{1-p}\right) + \ln\left(\frac{1-q}{q}\right)$$

Responsiveness to Prior

Follow approach by Grether (1980):

Use B_{Pos} and B_{Neg} to obtain α and β:

$$\ln\left(\frac{B_{Pos}}{1-B_{Pos}}\right) = \alpha\ln\left(\frac{p}{1-p}\right) + \beta\ln\left(\frac{q}{1-q}\right)$$

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$$\ln\left(\frac{B_{Pos}}{1-B_{Pos}}\right) = \alpha \ln\left(\frac{p}{1-p}\right) + \beta \ln\left(\frac{q}{1-q}\right)$$

$$\ln\left(\frac{B_{Neg}}{1-B_{Neg}}\right) = \alpha \ln\left(\frac{p}{1-p}\right) + \beta \ln\left(\frac{1-q}{q}\right)$$

Bayesian benchmark: $\alpha = 1, \beta = 1$.
perfect BRN: $\alpha = 0, \beta = 1$.

Responsiveness to Prior (α)

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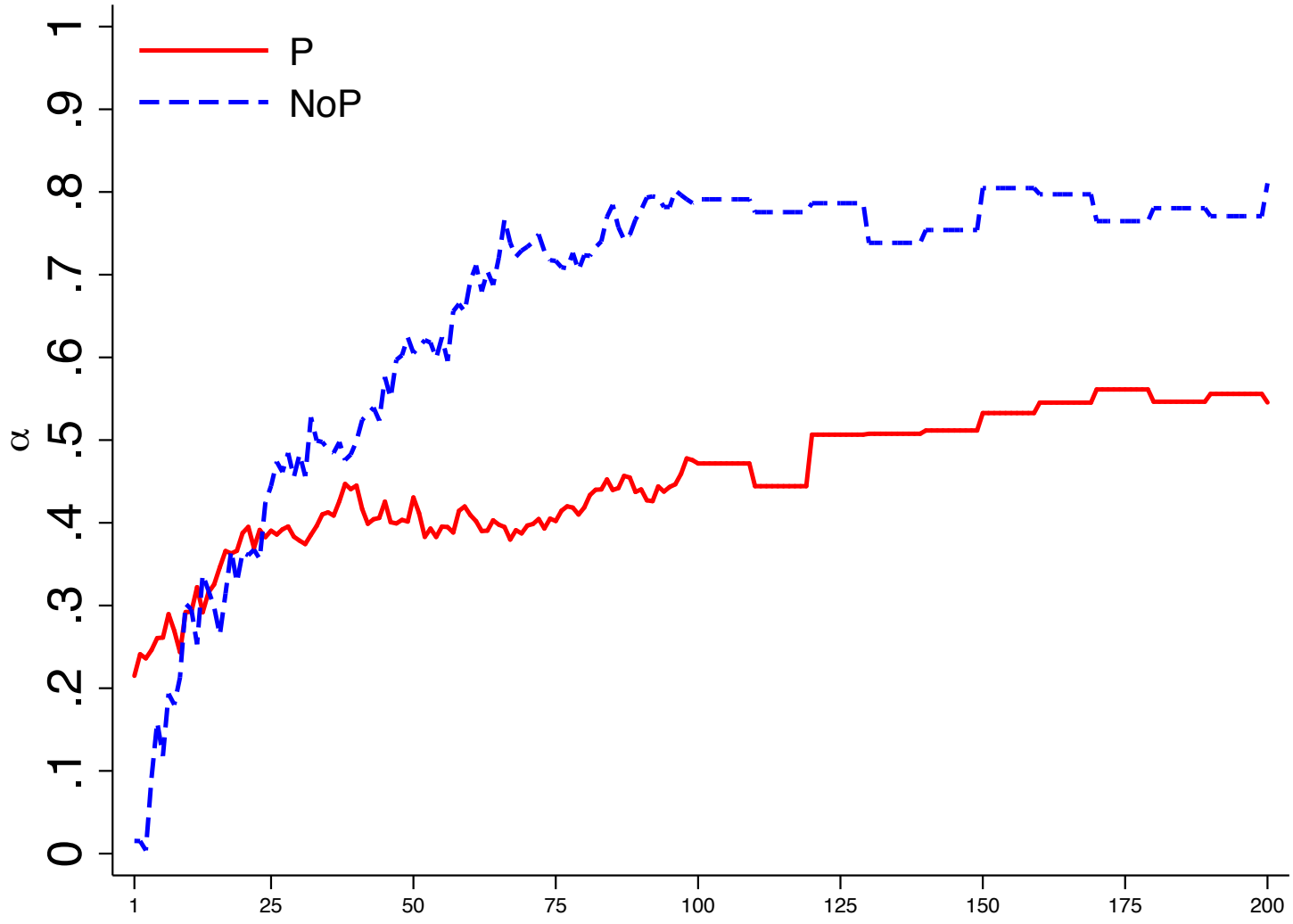

Studying attentiveness

R1 pBRN subjects are less responsive to data.

Are they less attentive or choose not to be responsive?

Each session consists of 5 sections:

- Part 1 Introduces the main updating task.
- Part 2 Repetition of the task for 200 rounds.
- Part 3 **Recollection of feedback.**
- Part 4
- Part 5

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  - » Number of rounds each signal-state realization was observed?
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Beliefs, Observed and Recalled Frequencies

Three questions:

1. Are beliefs consistent with observed frequencies?
2. Are recalled frequencies consistent with observed frequencies?
3. Are beliefs consistent with recalled frequencies?

Beliefs, Observed and Recalled Frequencies

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$$\Delta_{B,R} = \frac{|B_{Neg} - R_{Neg}| + |B_{Pos} - R_{Pos}|}{2}$$

$$\Delta_{R,F} = \frac{|R_{Neg} - F_{Neg}| + |R_{Pos} - F_{Pos}|}{2}$$

- B_{Neg}/B_{Pos}: Belief conditional on a neg./pos. signal.
- F_{Neg}/F_{Pos}: Observed frequency conditional on a neg./pos. signal.
- R_{Neg}/R_{Pos}: Recalled frequency conditional on a pos./neg. signal.

Beliefs, Observed and Recalled Frequencies

1. Are beliefs consistent with observed frequencies?

	$\Delta_{B,F}$	$\Delta_{B,R}$	$\Delta_{R,F}$
R1 pBRN in Primitives	**17.9**	12.3	14.3
R1 Others in Primitives	**11.4**	9.4	8.1
NoPrimitives	**9.8**	10.3	9.6
Hypotheses:			
R1 pBRN = R1 Others	.006	.262	.021
R1 pBRN = NoP	.000	.333	.033
R1 Others = NoP	.454	.719	.542

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Beliefs of R1 pBRN subjects are farther from observed frequencies.

Beliefs, Observed and Recalled Frequencies

3. Are recalled frequencies consistent with observed frequencies?

	$\Delta_{B,F}$	$\Delta_{B,R}$	$\Delta_{R,F}$
R1 pBRN in Primitives	17.9	12.3	**14.3**
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R1 pBRN subjects have a noisier recollection of feedback.

Beliefs, Observed and Recalled Frequencies

2. Are beliefs consistent with recalled frequencies?

	$\Delta_{B,F}$	$\Delta_{B,R}$	$\Delta_{R,F}$
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No difference between groups.

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Learning Model

Beliefs (on B_{Neg} and B_{Pos}) described by Beta distribution updated using outcomes from a Bernoulli process.

- Prior described by the Beta distribution (α_k, β_k), for $k \in \{Neg, Pos\}$.
- $\sigma \in [0, 1]$ describes the attentiveness to data in rounds 1-200.
- Posterior in round r described by (α_k^r, β_k^r) for $k \in \{Neg, Pos\}$.

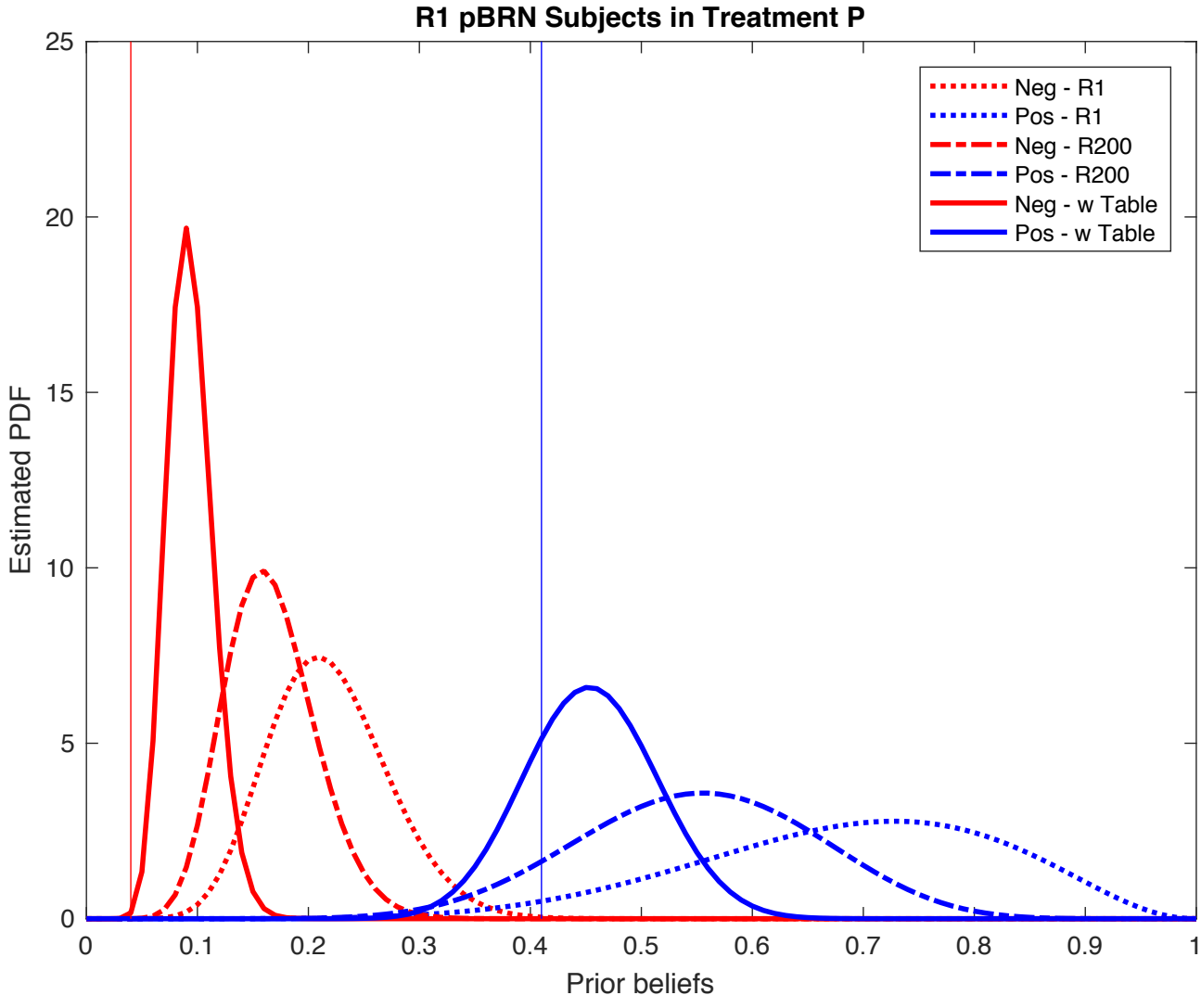
$$\alpha_k^r = \alpha_k + \sigma S_k^r \text{ and } \beta_k^r = \beta_k + \sigma F_k^r,$$

  where S_k^r and F_k^r are observed # successes and failures.

- Agents report $\mathbb{E}(B_k|\alpha_k^r, \beta_k^r) = \frac{\alpha_k^r}{\alpha_k^r + \beta_k^r}$.
- Using ML, find σ and (α_k, β_k), for $k \in \{Neg, Pos\}$.

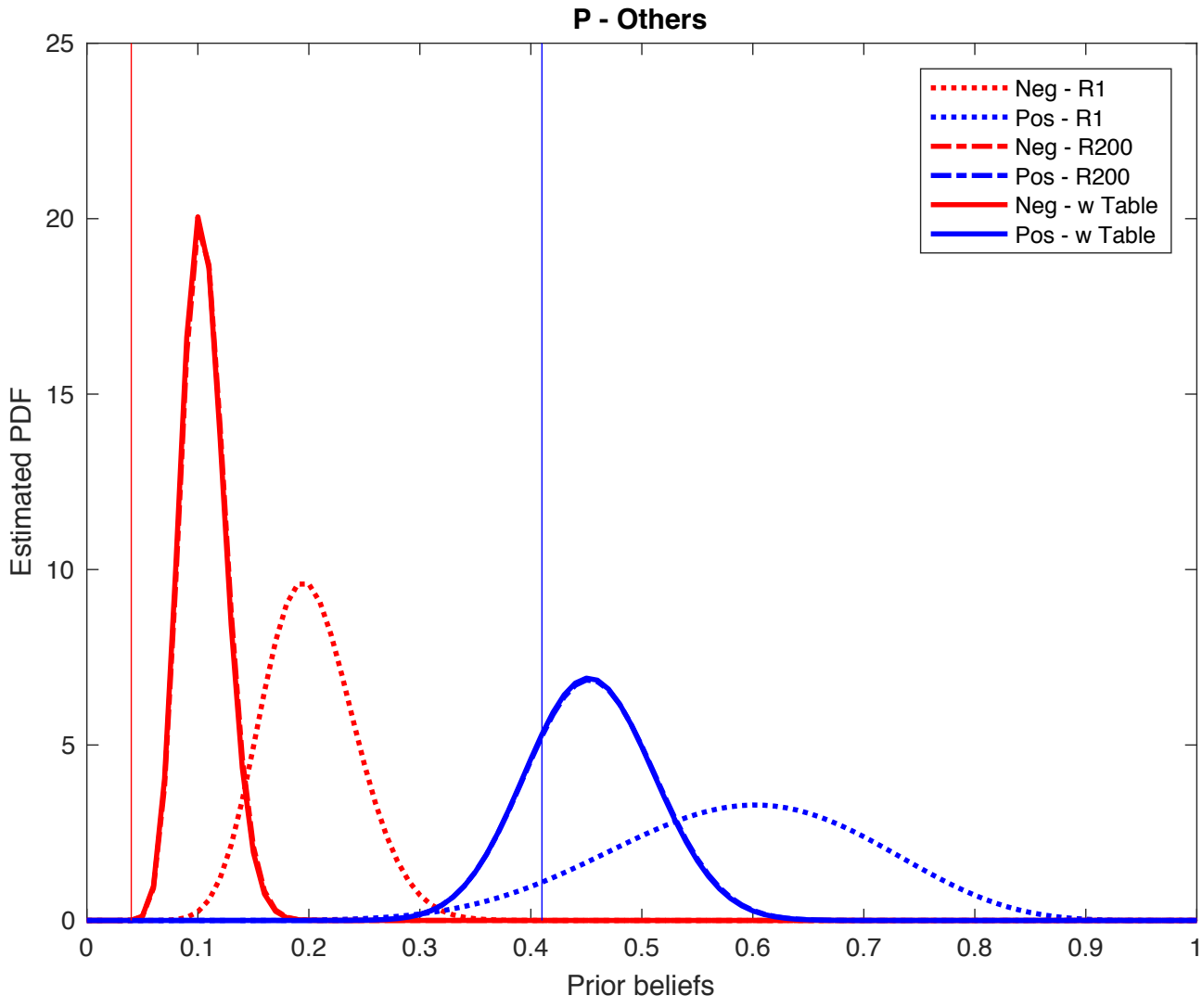
R1 pBRN $\sigma = 0.17$

back



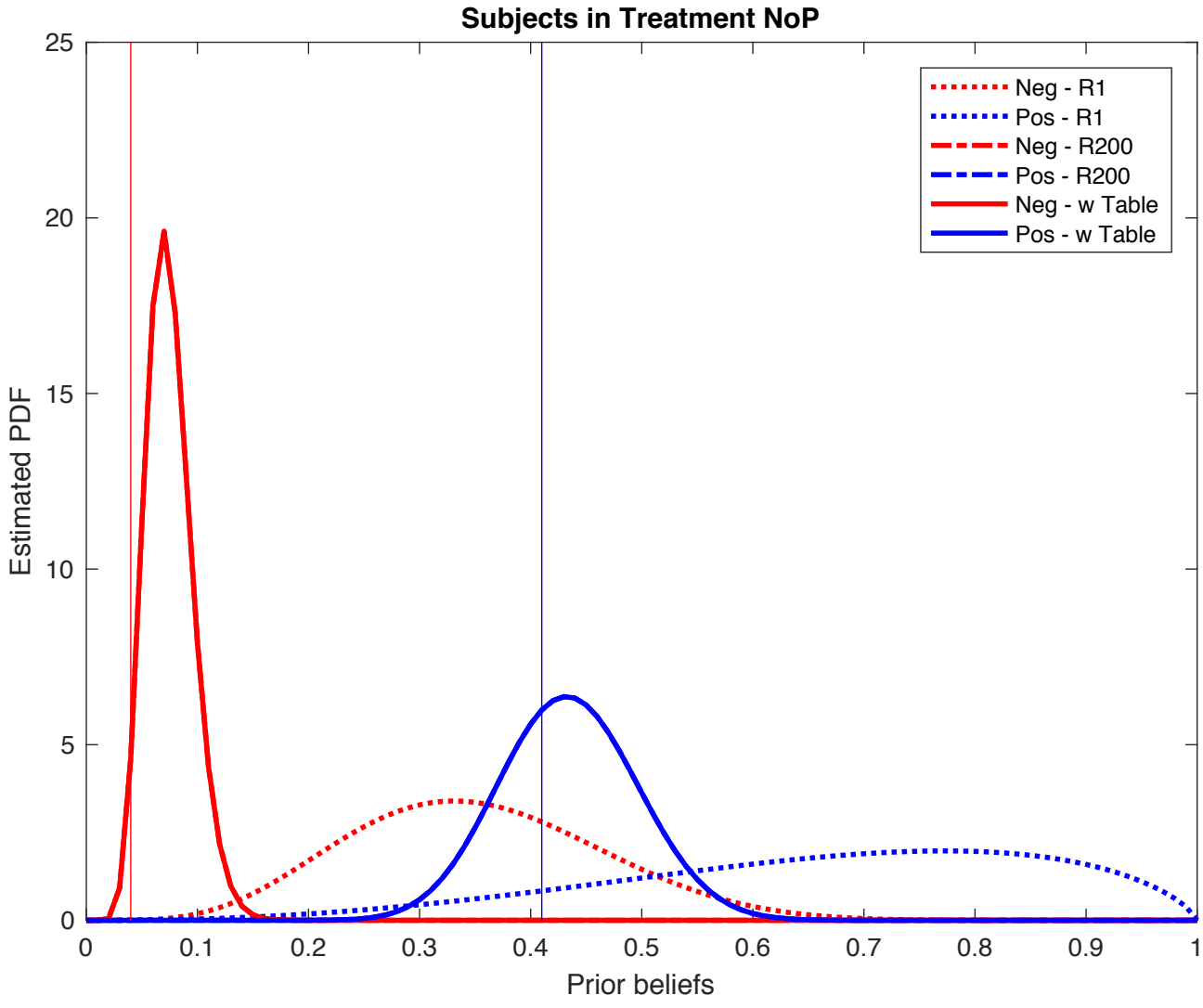
R1 Others $\sigma = 0.98$

back

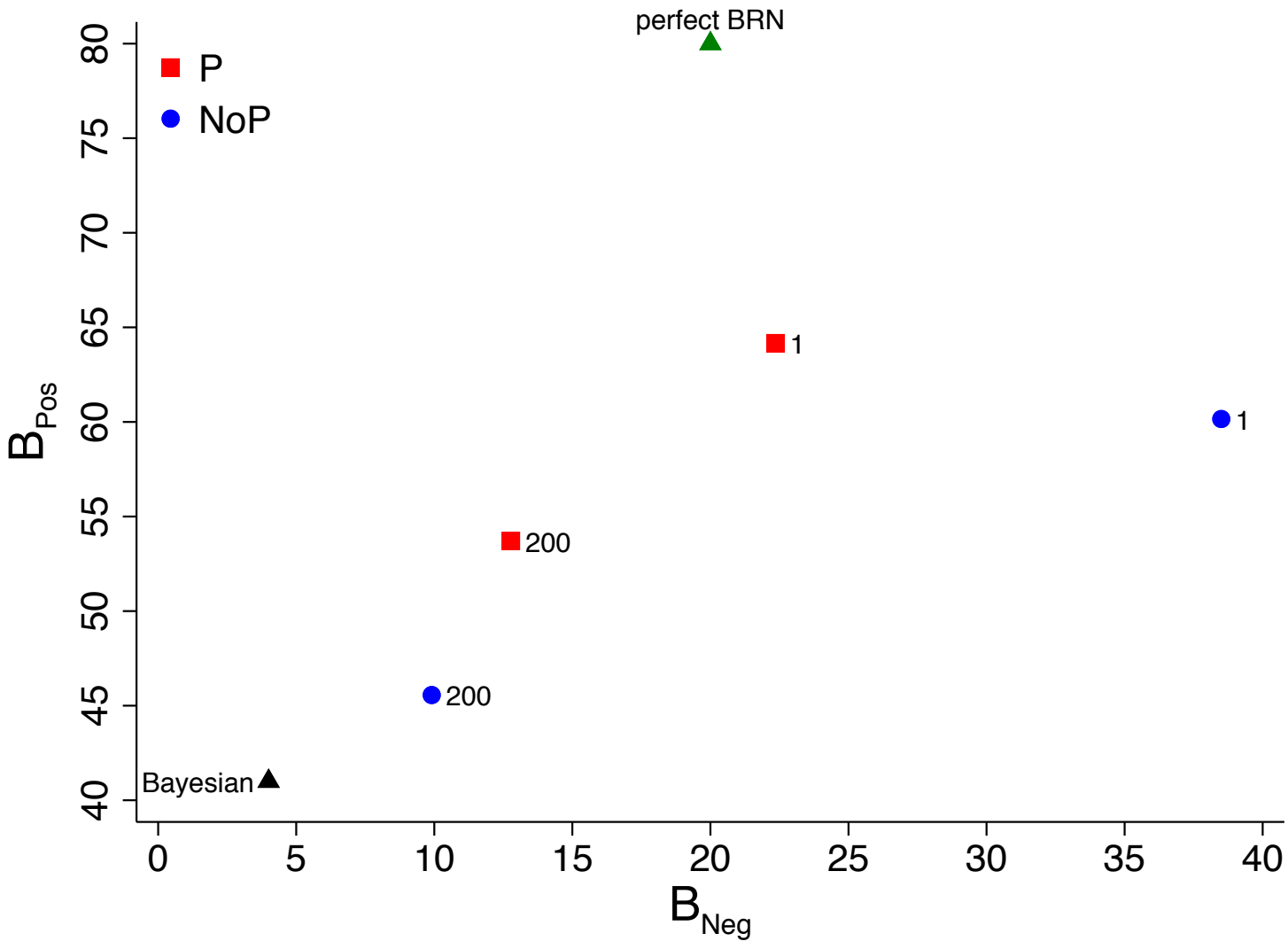


NoP $\sigma = 1.0$

back



Evolution of Average Beliefs: Round 200



Evolution of Average Beliefs: Table