# Pareto Weights in Practice: Income Inequality and Tax Reform* 

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#### Abstract

We develop a quantitative, heterogeneous-agents general equilibrium model that reproduces the income inequalities of 31 countries in the Organization for Economic Co-operation and Development. Using this model, we compute the optimal income tax rate for each country under the equal-weight utilitarian social welfare function. We simulate the voting outcome for the utilitarian optimal tax reform for each country. Finally, we uncover the Pareto weights in the social welfare functions of each country that justify the current redistribution policy.


Keywords: Income Inequality, Optimal Tax, Pareto Weights, Political Economy, Redistribution

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## 1. Introduction

The unequal distribution of economic resources and opportunities has always been a primary concern for social scientists and policy makers. According to the 2007 Survey of Consumer Finances, the top 1\% of the population in the United States owns $34 \%$ of country's total wealth, whereas the bottom $20 \%$ of the population owns almost nothingin fact, they are in debt. Income and wealth inequality is widespread across the world, and many countries adopt redistribution policies to alleviate this issue. ${ }^{1}$ Figure 1 plots the Gini coefficients of incomes (for both before- and after-tax/transfers) for 31 countries in the Organization for Economic Co-operation and Development (OECD) based on 201 date from the OECD database. ${ }^{2}$ The Gini coefficient of income before tax and transfers ranges from 0.34 (South Korea) to 0.59 (Ireland). The "improvement rate" in income inequality (measured by the percentage decrease in the Gini coefficients between beforeand after-tax/transfer incomes) ranges from $4 \%$ in Chile to $55 \%$ in Ireland. ${ }^{3}$

Understanding and comparing redistribution policies across countries in a unified framework is not an easy task. Economists' ability to quantitatively evaluate the political outcome of redistribution policies is limited because it requires modeling a complex political process and aggregating individual preferences. However, recent developments in general-equilibrium heterogeneous-agents models enable us to take first steps in addressing these issues. We can compute the optimal income tax rate under various welfare criteria and simulate voting outcomes of alternative policy reforms. We can even uncover

[^1]the welfare weights (so-called Pareto weights) that justify each country's current redistribution policy. To our knowledge, this is the first study that compares how societies (or governments) aggregate individual preferences over the redistribution policies, and does so across a large set of countries. We relate our estimated Pareto weights to the Democracy Index and the social perception about redistribution in the World Values Survey, both of which are often used in the political science and sociology.

More specifically, we ask three questions: (i) What is the optimal proportional income tax rate (and lump-sum transfer) for each country under the equal-weight utilitarian social welfare function? (ii) What would be the outcome of voting on the policy reform needed to adopt this utilitarian optimal tax rate be? (iii) What are the Pareto weights in the social welfare function that justify the current redistribution policy (which is suboptimal according to the equal-weight criteria)?

We examine these questions through the lens of the model seen in Aiyagari (1994), where households face uncertainty about future earnings. As a result of the precautionary savings and labor supply motive to insure against this future uncertainty, the cross-sectional wealth distribution emerges as an equilibrium.

We calibrate the model economy for each of the 31 OECD countries. The stochastic process of individual productivity shocks (which is the source of the cross-sectional income inequality) is chosen to match the before-tax income Gini coefficient in the data. In our model, the government adopts a simple income redistribution policy through a proportional income tax and lump-sum transfer. ${ }^{4}$ We choose the income tax rate to match the after-tax Gini coefficient in the data. As a result, for each country, the model exactly matches the before- and after-tax income Gini coefficients. The tax rates in our model turn out to be remarkably close to those in the data (measured by the tax-to-GDP ratio or the so-called "income tax wedge"), indicating that our model captures important characteristics of income inequality and redistribution in these countries quite well. For example, in the U.S., the implied tax rate in the model is $23.8 \%$, identical to the tax-GDP ratio in year 2010. This value is close to, but slightly lower than the average tax wedge,

[^2]$25.4 \%$, for the household with two earners and two children in 2010.
According to our model, the optimal tax rate under the equal-weight utilitarian social welfare function ranges between $31.8 \%$ (South Korea) and $49.4 \%$ (Chile). For 18 out of the 31 OECD countries we consider, the optimal tax rate is higher than the current average tax rate (measured by the tax-to-GDP ratio). In the other 13 countries, mostly Scandinavian or former communists countries, the optimal tax rate is lower than the current average tax rate. ${ }^{5}$

We then simulate the voting outcome for the reform policy needed to adopt the utilitarian optimal tax rate - i.e., the policy that changes the current tax rate to the optimal level which maximizes the equal-weight utilitarian social welfare function. ${ }^{6}$ Optimal tax reform is favored by the majority of the population in all of the OECD countries we consider with the exception of France and the Netherlands. For example, in Chile, which has a very low average tax rate ( $4.3 \%$ according to our model-implied measure), $90 \%$ of population supports the policy to increase the income tax rate to the socially optimal rate of $49.4 \%$.

If optimal tax reform is supported by the majority of citizens in these countries, why do current tax rates prevail? Exactly what the optimal tax rate is determined to be depends on the specification of the social welfare function. However, it is not obvious whether or not each government is trying to maximize an equal-weight utilitarian welfare function. There could be alternative criteria. For example, one may argue that it is desirable for a society to maximize the welfare of the poorest members of society instead of the average welfare of all (i.e., Rawlsian framework). Moreover, the process under which policies are actually determined is much more complicated than by simple majority rule. For instance, the rich often have more resources to influence the outcome of politics (e.g., lobbies). The political equilibrium under a multi-party system can be different from that under the

[^3]median voter theorem. These questions are immensely important, but beyond the scope of this paper.

Within the utilitarian framework, we instead ask a simple positive (not normative) question. For each country, what are the weights in the social welfare function that justify the current tax rate as an optimum? We interpret these relative weights in the social welfare function as broadly representing each society's preference for redistribution and political arrangement. We can view this as a reduced form representation of the policy determination process. If a society is plutocratic, the social welfare function assigns relatively larger weights to rich households, whereas an egalitarian society is likely to assign larger weights to poor households.

We interpret the persistence of the current suboptimal tax rate (despite overwhelming support for optimal tax reform) as evidence for deviation from equal weighting in the social welfare function. The welfare weights that justify the current tax rate reveal interesting social preferences for each country. ${ }^{7}$ According to our calculations, in Sweden, the Pareto weight on the richest $20 \%$ of the population is only $7.4 \%$, whereas that on the poorest $20 \%$ is $40 \%$. By contrast, in Chile, the Pareto weight on the richest $20 \%$ is $63 \%$, whereas that on the poorest $20 \%$ is a mere $2 \%$.

Our results are closely related to those in the existing literature. Romer (1975) and Roberts (1977) present a model of a median-voter-led income redistribution. Our model enables us to compute the individual welfare under a specific policy and simulate a voting result. Acemoglu, Naidu, Restrepo, and Robinson (2013) find that among 184 countries, there is a significant and robust effect of democracy on tax revenues as a fraction of GDP. Since their measure of democracy is dichotomous, their results are better interpreted as the effect of democratization. Our findings suggest that the degree to which democracy is embraced in each OECD countries affects the adoption of redistribution policies. Chang and Kim (2011) compute the welfare of two extreme socioeconomic systems: laissez-faire

[^4]and egalitarianism. We examine the redistribution policies of 31 OECD countries and uncover the implied Pareto weights of the social welfare function of each country.

The remainder of the paper is organized as follows. Section 2 documents key statistics about equality and welfare across 31 OECD countries and investigates their relationships. Section 3 lays out the benchmark model economy which is calibrated to match the beforeand after-tax income Ginis for each country. In Section 4, we compute the optimal tax rate under the equal-weight utilitarian social welfare function and examine whether the optimal tax reform is supported by the majority of the population. We then uncover the Pareto weights that justify the current tax rate. Section 5 concludes.

## 2. Income Inequality in the OECD Countries

In this section, we document stylized facts about the income inequality and redistribution policies of OECD countries. These facts are summarized in Tables 1 and 2. The first and second columns of Table 1 report the available before- and after-tax income Ginis for OECD countries, which are taken from the OECD database. ${ }^{8}$ The before-tax income Gini ranges from 0.34 (South Korea) to 0.59 (Ireland), with an average of 0.47 and a standard deviation of 0.05 . The after-tax income Gini varies from 0.24 (Iceland) to 0.51 (Chile), with an average of 0.31 and a standard deviation of 0.06 .

Figure 1 plots the before- and after-tax income Ginis for 31 OECD countries. Three countries, Hungary, Mexico and Turkey, are not included because their before-tax income Ginis are not available. All 31 countries are located below the 45-degree line, indicating that in all countries incomes are redistributed from the rich to the poor. The two income Ginis are, however, modestly correlated with the correlation coefficient of 0.4 , indicating varying degrees of redistribution policies across countries.

Based on before- and after-tax income Gini coefficients, we calculate the improvement rate in income inequality-i.e., the percentage decrease of the Gini coefficient after tax and transfers-for each country, which serves as a measure of the strength of the redistribution

[^5]Table 1: Key Statistics for the 34 OECD Countries

|  | Before Gini | After Gini | Improvement (\%) | $\underset{(\%)}{\operatorname{Tax} / Y}$ | Democracy Index | Wealth Gini | per capita $\operatorname{GDP}^{1}(\$)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Australia | 0.469 | 0.334 | 28.8 | 25.6 | 9.22 | 0.636 | 57,535 |
| Austria | 0.479 | 0.267 | 44.3 | 42.2 | 8.49 | 0.693 | 45,171 |
| Belgium | 0.478 | 0.262 | 45.2 | 43.5 | 8.05 | 0.655 | 43,292 |
| Canada | 0.447 | 0.320 | 28.4 | 30.6 | 9.08 | 0.728 | 47,297 |
| Chile | 0.531 | 0.508 | 4.3 | 19.5 | 7.67 | 0.774 | 12,727 |
| Czech Republic | 0.449 | 0.256 | 43.0 | 33.9 | 8.19 | 0.743 | 18,873 |
| Denmark | 0.429 | 0.252 | 41.3 | 47.4 | 9.52 | 0.701 | 56,428 |
| Estonia | 0.487 | 0.319 | 34.5 | 34.0 | 7.68 | 0.660 | 14,212 |
| Finland | 0.479 | 0.260 | 45.7 | 42.5 | 9.19 | 0.662 | 44,134 |
| France | 0.505 | 0.303 | 40.0 | 42.9 | 7.77 | 0.755 | 39,596 |
| Germany | 0.492 | 0.286 | 41.9 | 36.2 | 8.38 | 0.777 | 40,418 |
| Greece | 0.522 | 0.337 | 35.4 | 31.6 | 7.92 | 0.714 | 26,379 |
| Hungary | ... | 0.272 | ... | 38.0 | 7.21 | 0.641 | 12,750 |
| Iceland | 0.393 | 0.244 | 37.9 | 35.2 | 9.65 | 0.663 | 39,511 |
| Ireland | 0.591 | 0.266 | 55.0 | 27.4 | 8.79 | 0.727 | 45,921 |
| Israel | 0.501 | 0.376 | 25.0 | 32.4 | 7.48 | 0.783 | 30,396 |
| Italy | 0.503 | 0.319 | 36.6 | 43.0 | 7.83 | 0.646 | 33,982 |
| Japan | 0.488 | 0.336 | 31.1 | 27.6 | 8.08 | 0.596 | 42,918 |
| Korea | 0.341 | 0.310 | 9.1 | 25.1 | 8.11 | 0.726 | 20,540 |
| Luxembourg | 0.464 | 0.270 | 41.8 | 37.3 | 8.88 | 0.623 | 102,568 |
| Mexico |  | 0.466 | ... | 18.9 | 6.93 | 0.780 | 9,189 |
| Netherlands | 0.424 | 0.288 | 32.1 | 38.9 | 8.99 | 0.812 | 46,783 |
| New Zealand | 0.454 | 0.317 | 30.2 | 31.1 | 9.26 | 0.725 | 32,757 |
| Norway | 0.423 | 0.249 | 41.1 | 42.6 | 9.8 | 0.779 | 86,101 |
| Poland | 0.468 | 0.305 | 34.8 | 31.7 | 7.05 | 0.753 | 12,198 |
| Portugal | 0.522 | 0.344 | 34.1 | 31.2 | 8.02 | 0.725 | 21,512 |
| Slovak Republic | 0.437 | 0.261 | 40.3 | 28.3 | 7.35 | 0.621 | 16,073 |
| Slovenia | 0.453 | 0.246 | 45.7 | 38.1 | 7.69 | 0.639 | 22,938 |
| Spain | 0.507 | 0.338 | 33.3 | 32.5 | 8.16 | 0.662 | 30,058 |
| Sweden | 0.441 | 0.269 | 39.0 | 45.4 | 9.5 | 0.806 | 49,375 |
| Switzerland | 0.372 | 0.298 | 18.7 | 28.1 | 9.09 | 0.806 | 70,523 |
| Turkey | ... | 0.409 | ... | 26.2 | 5.73 | 0.842 | 10,015 |
| United Kingdom | 0.523 | 0.345 | 34.8 | 34.9 | 8.16 | 0.675 | 36,869 |
| United States | 0.499 | 0.380 | 23.8 | 23.8 | 8.18 | 0.852 | 48,287 |
| Average | 0.470 | 0.312 | 34.8 | 33.8 | 8.27 | 0.717 | 37,275 |
| Std. Dev. | 0.050 | 0.061 | 10.5 | 7.3 | 0.90 | 0.069 | 21,270 |

Note: See Appendix A for a detailed description of data.
Source: OECD (2010), Economist Intelligence Unit (2011), and Credit Suisse (2012)

Table 2: Correlations for the 34 OECD Countries

|  | Before <br> Gini | After <br> Gini | Improve- <br> ment (\%) | Tax/Y <br> $(\%)$ | Democracy <br> Index | Wealth <br> Gini |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Before Gini | 1.00 | 0.41 | 0.26 | -0.11 | -0.38 | -0.08 |
| After Gini | 0.41 | 1.00 | -0.76 | -0.71 | -0.51 | 0.37 |
| Improvement (\%) | 0.26 | -0.76 | 1.00 | 0.64 | 0.17 | -0.33 |
| Tax/Y (\%) | -0.11 | -0.71 | 0.64 | 1.00 | 0.39 | -0.22 |
| Demo Index | -0.38 | -0.51 | 0.17 | 0.39 | 1.00 | -0.07 |
| Wealth Gini | -0.08 | 0.37 | -0.33 | -0.22 | -0.07 | 1.00 |

Source: Authors' calculation with data from OECD (2010), Economist Intelligence Unit (2011), and Credit Suisse (2012)

Figure 1: Before-Tax and After-Tax Income Inequality

policies. The improvement rate, shown in the third column of Table 1, varies widely, from $4 \%$ (Chile) to $55 \%$ (Ireland), with an average of $34.8 \%$ and a standard deviation of $10.5 \%$. The improvement rates are only weakly correlated with the before-tax income Gini (the correlation coefficient of 0.26 ), suggesting that a country with high income inequality does not necessarily adopt a stronger redistribution policy.

While redistribution policies take various forms across countries, we will use the average tax rate - measured by the tax revenue to GDP ratio - as a summary statistic of redistribution policies. As Figure 2 illustrates, the improvement rate is fairly strongly correlated with the tax-to-GDP ratios (correlation coefficient of 0.64). This is confirmed by Figure 3, which shows a strong negative correlation, -0.71 , between the tax-to-GDP ratio and the after-tax income Gini. The OECD also reports average the tax wedge (an employer's labor cost minus employee's take-home value) for various household types. We find that the tax-to-GDP ratio is actually quite close to the average tax wedge in the data, confirming that the tax-to-GDP ratio is a good approximation of the average income tax rate for households. The correlation of the average tax wedge with the after-tax income Gini is -0.49 and its correlation with the improvement rate is 0.61 . Broadly speaking, high taxes are likely to be used for income redistribution purposes and thus make the society more equalized. Latin American countries such as Mexico (18.9\%) and Chile (19.5\%) tend to show low values of their tax-to-GDP ratios. Nordic countries such as Denmark (47.4\%) and Sweden (45.4\%) exhibit, on average, high tax-to-GDP ratios. The tax-to-GDP ratio, however, does not show a strong correlation (-0.11), with the before-tax income Gini.

Not all government spending is used for the redistribution of income. Table 3 reports the composition of government spending for the 32 OECD countries with information available from the OECD (all except for New Zealand and Chile). The general government-spending-to-GDP ratio varies from $23 \%$ (Mexico) to $65 \%$ (Ireland), with an average of $46 \%$. Expenditure on social benefits account for the largest share of general government spending in most OECD countries, with the social benefits-to-GDP ratio varying from $2 \%$ in Mexico to $25.6 \%$ in France. The social benefits expenditure is widely considered to be an income-redistribution policy. In fact, Figure 4 shows that the improvement rate is highly positively correlated with the social benefits-to-GDP ratio, with

Figure 2: Tax/GDP and Improvement Rate


Figure 3: Tax/GDP and After-Tax Income Inequality


Figure 4: Social Benefits to GDP Ratio and Improvement Rate

a correlation coefficient of 0.62 , which is similar to the correlation with the tax-to-GDP ratio. The tax-to-GDP ratio also shows a high correlation, 0.6 , with the social benefits-toGDP ratio. We argue that the tax-to-GDP ratio serves as a good proxy for the strength of redistribution policies. It is also consistent with the policy measure (proportional income tax rate) in our model economy.

The adoption of a redistribution policy depends on the political system as well as society's preferences. One may expect that a democratic society is less subject to plutocracy and tends to advocate an equal distribution of resources and opportunities. Thus, it is more likely to adopt an aggressive redistribution policy. To examine this premise, we obtain the Democracy Index from the Economist Intelligent Unit (EIU). The EIU evaluates the development of democracy in a society based on 60 questions in 5 categories: (i) electoral process and pluralism, (ii) functioning of government, (iii) political participation, (iv) political culture, and (v) civil liberties. A country is scored from 0 to 10 in each of the 5 categories. A country's democracy index is its average score across these 5 categories, though not all questions are directly related to redistribution policy, including tax

Table 3: Composition of General Government Expenditure (\% of GDP)

|  | Total | Employee <br> Compensation | Social <br> Benefits | Consumption | Fixed <br> Capital <br> Formation | Other |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| Australia | 36.4 | $\ldots$ | 10.4 | $\ldots$ | 3.8 | $\ldots$ |
| Austria | 52.8 | 9.8 | 25.5 | 4.5 | 1.1 | 12.0 |
| Belgium | 52.6 | 12.6 | 24.9 | 3.7 | 1.6 | 9.8 |
| Canada | 42.3 | 12.4 | 9.7 | 9.9 | 4.7 | 5.6 |
| Chile | $\ldots$ | 8.2 | .. | $\ldots$ | 2.2 | .. |
| Czech Republic | 43.7 | 7.5 | 19.5 | 6.2 | 4.2 | 6.3 |
| Denmark | 57.7 | 19.0 | 18.5 | 9.9 | 2.2 | 8.2 |
| Estonia | 40.5 | 11.9 | 14.8 | 7.5 | 3.9 | 2.4 |
| Finland | 55.8 | 14.5 | 21.0 | 11.5 | 2.5 | 6.3 |
| France | 56.6 | 13.4 | 25.6 | 5.8 | 3.1 | 8.7 |
| Germany | 47.9 | 7.8 | 25.4 | 4.9 | 1.7 | 8.1 |
| Greece | 51.4 | 12.5 | 21.4 | 6.0 | 2.3 | 9.3 |
| Hungary | 50.0 | 11.0 | 18.6 | 7.9 | 3.4 | 9.1 |
| Iceland | 51.6 | 14.8 | 7.9 | 12.2 | 2.9 | 13.8 |
| Ireland | 65.5 | 12.2 | 18.0 | 5.9 | 3.4 | 26.1 |
| Israel | 42.3 | 11.1 | 5.8 | 12.2 | 1.5 | 11.8 |
| Italy | 50.4 | 11.1 | 22.2 | 5.8 | 2.1 | 9.2 |
| Japan | 40.7 | 6.1 | 21.5 | 4.1 | 3.3 | 5.7 |
| Korea | 30.1 | 6.9 | 7.0 | 4.0 | 5.1 | 7.2 |
| Luxembourg | 43.5 | 8.2 | 20.8 | 3.7 | 4.1 | 6.7 |
| Mexico | 23.1 | 8.9 | 2.0 | 2.8 | 2.7 | 6.8 |
| Netherlands | 51.3 | 10.1 | 23.0 | 8.0 | 3.6 | 6.6 |
| New Zealand | .. | .. | .. | .. | .. | .. |
| Norway | 45.2 | 13.6 | 15.8 | 6.7 | 3.2 | 5.8 |
| Poland | 45.4 | 10.2 | 17.0 | 6.2 | 5.6 | 6.4 |
| Portugal | 51.5 | 12.2 | 22.0 | 5.2 | 3.8 | 8.4 |
| Slovak Republic | 40.0 | 7.7 | 19.4 | 4.9 | 2.6 | 5.4 |
| Slovenia | 49.4 | 12.7 | 19.4 | 6.8 | 4.5 | 6.0 |
| Spain | 46.3 | 12.0 | 18.5 | 5.9 | 4.0 | 5.9 |
| Sweden | 52.3 | 14.5 | 18.3 | 9.2 | 3.5 | 6.8 |
| Switzerland | 33.9 | 7.8 | 11.9 | 4.7 | 2.3 | 7.2 |
| Turkey | 40.2 | 8.7 | 11.4 | 5.5 | 2.7 | 1.9 |
| United Kingdom | 49.9 | 11.4 | 14.9 | 13.0 | 2.5 | 8.2 |
| United States | 42.6 | 11.0 | 15.0 | 7.6 | 4.1 | 5.0 |
|  |  |  |  |  |  |  |

Source: National Accounts at a Glance (2010), OECD database.
and transfers. ${ }^{9}$ According to the EIU, among OECD countries, Norway (9.8) is the most democratic, Turkey (5.73) is the least, and the U.S. (8.18) is around the median. The democracy index is modestly positively correlated (0.39), with the tax-to-GDP ratio. It is only weakly correlated (0.17), with the improvement rate. However, this does not necessarily imply that the democracy does not affect income redistribution. The democracy index is modestly negatively correlated (-0.38), with the before-tax income Gini, while it is more negatively correlated $(-0.51)$, with the after-tax income Gini. This pattern may imply that incomes in democratic societies are likely to already equally distributed to the extent that the socio-economic systems of such countries are developed to guarantee equal opportunities. Any redistribution policies captured by the tax-GDP ratio makes the after-tax Gini even smaller, resulting in the after-tax income Gini being more negatively correlated with the democracy index.

Progressive income taxation has been regarded as a powerful redistribution policy. Progressivity is often measured by the marginal tax rate for the highest income group. However, the top statutory tax rate is not systematically correlated with the improvement rate (correlation coefficient of 0.08 ), but is positively correlated ( 0.45 ), with the democracy index (not shown in the table). Moreover, the correlation of the top statutory income tax rate with the social benefits-to-GDP ratio is 0.24 , and its correlation with the tax-to-GDP ratio is 0.39 . In summary, democratic societies are likely to adopt more progressive tax rates at the highest income bracket. However, the marginal tax rate at the top bracket does not necessarily appear to be a good proxy for the strength of overall redistribution of incomes among OECD countries.

We have also collected the wealth Gini coefficients for OECD countries from the 2012 edition of Global Wealth Databook issued by Credit Suisse. Wealth is distributed more unevenly than incomes are. The average wealth Gini coefficient among OECD is 0.72.

[^6]Unfortunately, unlike the data on incomes, wealth data is known to be both less reliable and not standardized across countries. Thus, our analysis will mainly focus on income Ginis. ${ }^{10}$

## 3. Model

The model economy will serve as a laboratory for various quantitative analyses. The benchmark economy extends Aiyagari's (1994) model to endogenous labor supply.

Households: There is a continuum (measure one) of worker-households who have identical preferences and face idiosyncratic productivity shock $x$, which evolves over time according to a Markov process with a transition probability distribution function $\pi_{x}\left(x^{\prime} \mid x\right)=$ $\operatorname{Pr}\left(x_{t+1} \leq x^{\prime} \mid x_{t}=x\right)$. When a household with labor productivity $x_{t}$ chooses to work for $h_{t}$ hours, its labor income is $w_{t} x_{t} h_{t}$, where $w_{t}$ is the wage rate for the efficiency unit of labor. Households hold assets, $a_{t}$, that yield the real rate, $r_{t}$. Both labor and capital incomes are subject to income taxes at the rate $\tau$. Households receive a lump-sump transfer $T_{t}$ from the government. A household maximizes its lifetime utility, shown as:

$$
\max _{\left\{c_{t}, h_{t}\right\}_{t=0}^{\infty}} \mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{c_{t}^{1-\sigma}-1}{1-\sigma}-B \frac{h_{t}^{1+1 / \gamma}}{1+1 / \gamma}
$$

subject to

$$
\begin{gathered}
c_{t}+a_{t+1}=(1-\tau)\left(w_{t} x_{t} h_{t}+r_{t} a_{t}\right)+a_{t}+T_{t} \\
a_{t+1} \geq \underline{a}
\end{gathered}
$$

where $c_{t}$ is consumption. Parameters $\sigma$ and $\gamma$ represent relative risk aversion and laborsupply elasticity, respectively. Capital markets are incomplete in the sense that physical capital is the only available asset for households to insure against idiosyncratic shocks to

[^7]their productivity, and households face a borrowing constraint: $a_{t} \geq \underline{a}$ for all t . Households differ ex post with respect to their productivity $x_{t}$ and asset holdings $a_{t}$, whose cross-sectional joint distribution is characterized by the probability measure $\mu_{t}\left(a_{t}, x_{t}\right)$.

Firms: The representative firm produces output through constant-returns-to-scale CobbDouglas technology using capital, $K_{t}$, and effective units of labor, $L_{t}=\int h_{t} x_{t} d \mu$. The aggregate productivity of the country is $A$. Capital depreciates at the rate $\delta$ each period:

$$
Y_{t}=A F\left(L_{t}, K_{t}\right)=A L_{t}^{\alpha} K_{t}^{1-\alpha}
$$

Government: The government operates a simple fiscal policy characterized by a flat tax rate $(\tau)$ on workers' total income and a lump-sum transfer $(T)$ to all households. It would be more realistic to incorporate progressive taxation. However, as shown in section , the progressivity of a tax rate is only weakly correlated with the overall strength of redistribution (measured by the improvement rate of income Gini coefficients after the redistribution). As the average tax rate (measured by tax-to-GDP ratio) is much more strongly correlated with the improvement rate in the data, we assume that the government adopts a simple proportional income tax rate and lump-sum transfer. We further assume that the government runs a balanced budget, in which all tax revenues are transferred to households:

$$
T_{t}=\int \tau\left\{w_{t} x_{t} h\left(a_{t}, x_{t}\right)+r_{t} a_{t}\right\} d \mu\left(a_{t}, x_{t}\right)
$$

Recursive Representation: It is useful to consider a recursive equilibrium. Let $V(a, x)$ denote the value function of a household with asset holdings $a$ and productivity $x$. Then $V$ can be expressed as follows:

$$
V(a, x)=\max _{c, h}\left\{\frac{c^{1-\sigma}-1}{1-\sigma}-B \frac{h^{1+1 / \gamma}}{1+1 / \gamma}+\beta \mathbb{E}\left[V\left(a^{\prime}, x^{\prime}\right) \mid x\right]\right\}
$$

subject to

$$
\begin{gathered}
c+a^{\prime}=(1-\tau)(w x h+r a)+a+T, \\
a^{\prime} \geq \underline{a} .
\end{gathered}
$$

The intertemporal first-order condition for optimal consumption is:

$$
c(a, x)^{-\sigma}=\beta(1-\tau)(1+r) \mathbb{E}\left[c\left(a^{\prime}, x^{\prime}\right)^{-\sigma}\right] .
$$

The intra-temporal first-order condition for optimal hours worked is:

$$
B h(a, x)^{1 / \gamma} c(a, x)^{\sigma}=(1-\tau) w x .
$$

Equilibrium: A stationary equilibrium consists of a value function, $V(a, x)$; a set of decision rules for consumption, asset holdings, and labor supply, respectively, $c(a, x)$, $a^{\prime}(a, x)$, and $h(a, x)$; aggregate input, $K$ and $L$; and the invariant distribution of households, $\mu(\mathrm{a}, \mathrm{x})$, such that:

1. Individual households optimize: Given $w$ and $r$, the individual decision rules $c(a, x)$, $a^{\prime}(a, x), h(a, x)$, and $V(a, x)$ solve the Bellman equation.
2. The representative firm maximizes profits:

$$
\begin{aligned}
w & =A \alpha(K / L)^{1-\alpha} \\
r+\delta & =A(1-\alpha)(K / L)^{-\alpha} .
\end{aligned}
$$

3. The goods market clears:

$$
\int\left\{a^{\prime}(a, x)+c(a, x)\right\} d \mu=A F(L, K)+(1-\delta) K
$$

4. The factor markets clear:

$$
\begin{aligned}
L & =\int x h(a, x) d \mu \\
K & =\int a d \mu .
\end{aligned}
$$

5. The government balances the budget:

$$
T=\int \tau\{w x h(a, x)+r a\} d \mu
$$

6. Individual and aggregate behaviors are consistent: For all $A^{0} \subset \mathcal{A}$ and $X^{0} \subset \mathcal{X}$,

$$
\mu\left(A^{0}, X^{0}\right)=\int_{A^{0}, X^{0}}\left\{\int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{a^{\prime}=a^{\prime}(a, x)} d \pi_{x}\left(x^{\prime} \mid x\right) d \mu\right\} d a^{\prime} d x^{\prime} .
$$

## 4. Quantitative Analysis

### 4.1. Calibration

Our overall calibration strategy is as follows: (i) We benchmark our model to the U.S. economy. (ii) Preferences are identical across countries. (iii) Three country-specific parameters (aggregate productivity, the magnitude of individual productivity shocks, and income tax rate) are chosen to match three moments (per capita output, before-tax income Gini, and the after-tax income Gini) for each country.

Common Parameters: The time unit is one year. Workers are not allowed to borrow, so $\underline{a}=0$. The labor-income share, $\alpha$, is 0.64 , and the annual depreciation rate of capital, $\delta$, is $10 \%$. The labor supply elasticity, $\gamma$, is set to 1 . This value is larger than the typical micro estimate. However, considering that typical micro estimates do not reflect the extensive margin of labor (i.e., the labor-market participation decision), suing a larger value is desirable. The time discount factor, $\beta$, is set so that the real interest rate is $4 \%$, which is the average real rate of returns to capital in the U.S. for the post-World War II period. The disutility from working, $B$, is chosen so that average hours worked in the steady state is 0.323 , which is the average share of discretionary time devoted to working. ${ }^{11}$ The relative risk aversion, $\sigma=1.4$, is chosen to match the cross-country correlation between hours and labor productivity in the data (-0.68). ${ }^{12}$

Country-Specific Parameters: There are three parameters that are country specific: $A, \sigma_{x}$, and $\tau$. For each country, the aggregate TFP, $A$, is set to match the per capita GDP. Individual productivity $x$ is assumed to follow an $\mathrm{AR}(1)$ process: $\ln x^{\prime}=\rho_{x} \ln x+\varepsilon_{x}$, where $\varepsilon_{x} \sim N\left(0, \sigma_{x}^{2}\right)$. A sizeable literature has estimated this process using wages from panel data, including, Floden and Linde (2001), Chang and Kim (2006), and Heathcote et al. (2008). While there are differences in the estimate on the magnitude of the shocks,

[^8]the consensus is that these shocks are large and persistent. Our benchmark model adopts a persistence value of $\rho_{x}=0.92$, also used in Floden and Linde (2001) and Pijoan-Mas (2006). We assume that this value is common across countries, consistent with many empirical studies that find a highly persistent process in wages in various countries. For the magnitude of shocks, $\sigma_{x}$ is used to match the before-tax income Gini coefficient in each country. The chosen value of $\sigma_{x}$ for the U.S. is 0.32 , somewhat larger than the estimate (0.21) by Floden and Linde (2001) based on the Panel Data of Income Dynamics. We interpret $x$ as a broad measure of households' ability to generate labor income (broader than the pure stochastic components of individual wages). Thus, the model requires a larger value of $\sigma_{x}$ to match the overall cross-sectional distribution of household incomes. The value for other countries ranges between 0.22 (South Korea) and 0.38 (Chile).

Finally, we choose the income tax rate $(\tau)$, the redistribution measure, to match the after-tax income Gini of the country. It turns out that with the proportional income tax rate and lump-sum transfers, the implied tax rate is the same as the improvement rate of income Ginis (before and after tax and transfer). ${ }^{13}$

For the U.S., the model-implied income tax rate, $\tau$, is $23.8 \%$, exactly the same as the tax-to-GDP ratio in the year 2010. Across 31 countries, the model-implied $\tau$ ranges between $4 \%$ (Chile) and $55 \%$ (Ireland), as does the improvement rate in income Ginis.

[^9]Table 4: Parameters of the Benchmark Economy

| Common for All Countries |  |  |
| :--- | :---: | :--- |
| $\beta$ | 0.9408 | Time discount factor |
| $B$ | 7.0090 | Disutility from working |
| $\sigma$ | 1.4 | Relative risk aversion |
| $\gamma$ | 1.0 | Labor supply elasticity |
| $\rho$ | 0.92 | Persistence of idiosyncratic productivity |
| $\alpha$ | 0.64 | Labor share in production function |
| $\delta$ | 0.1 | Depreciation rate of captial |
| Country Specific |  |  |
| $A$ | $0.30-2.14$ | Aggregate productivity |
| $\sigma_{x}$ | $0.22-0.38$ | Std. deviation of idiosyncratic shocks |
| $\tau$ | $0.04-0.55$ | Income tax rate |

Table 4 summarizes the parameter values of the benchmark model economy. Table C. 3 in the Appendix lists the values of $\sigma_{x}$ and $\tau$ for all 31 countries.

### 4.2. Steady State

According to our calibration strategy, the model exactly matches the before- and and after-tax Gini coefficients in the data. The wealth Gini coefficient for the U.S. is 0.79 or 0.82 according to the PSID and SCF, respectively, whereas it is 0.72 in our model. Table 5 compares the quintile groups of the wealth distribution between the model and the data. It shows the wealth share, the ratio of group average to economy-wide average, and the earnings share across quintiles. The top $20 \%$ (the 5th quintile) of households own $83.4 \%$ or $76.2 \%$ of total wealth in the SCF and PSID, respectively; the corresponding share in our model is $73.5 \%$. The PSID found that households in the 1st, 2nd, 3rd, and 4th quintiles own $-0.52 \%$ (in debt), $0.50 \%, 5.06 \%$, and $18.74 \%$, respectively. These shares are also similar in the SCF. The corresponding shares in our model economy are $0 \%, 0.84 \%, 5.93 \%$, and $19.74 \%$. Broadly speaking, the wealth distribution from our benchmark model economy resembles that from the U.S. data quite well, although the model cannot generate an extremely high concentration of the wealth at the top $5 \%$ or

Table 5: Earnings-Wealth Distributions in the U.S.

|  |  | Quintile |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1st | 2nd | 3rd | 4th | 5 th | Total |
| SCF |  |  |  |  |  |  |
| Share of wealth | -.2 | 1.1 | 4.5 | 11.2 | 83.4 | 100 |
| Group avg/pop avg | -.0 | .1 | .2 | .6 | 4.2 | 1 |
| Share of earnings | 6.9 | 10.8 | 14.9 | 19.4 | 48.0 | 100 |
| PSID |  |  |  |  |  |  |
| $\quad$ Share of wealth | -.5 | .5 | 5.1 | 18.7 | 76.2 | 100 |
| Group avg/pop avg | -.0 | .0 | .3 | .9 | 3.8 | 1 |
| Share of earnings | 7.5 | 11.3 | 18.7 | 24.2 | 38.2 | 100 |
|  |  |  |  |  |  |  |
| Model |  |  |  |  |  | 1000 |
| Share of wealth | .0 | 0.8 | 5.9 | 19.7 | 73.5 | 100 |
| Group avg/pop avg | .0 | .0 | .3 | 1.0 | 3.7 | 1 |
| Share of earnings | 3.1 | 7.3 | 12.8 | 20.7 | 56.1 | 100 |

Source: The SCF statistics are based on Díaz-Giménez, Glover, and Ríos-Rull (2011). The PSID statistics are based on Chang and Kim (2011).
$1 \%$. For example, according to the SCF, the top $1 \%$ of the population owns $34 \%$ of the total wealth in the U.S., whereas the top $1 \%$ of households own $8 \%$ of the total wealth in our model.

The model-implied tax rate in the U.S. is $23.8 \%$, exactly the same as the tax-to-GDP ratio in year 2010, but somewhat lower than the average tax wedge of $25.4 \%$ for households with two earners and two children. ${ }^{14}$ Table 6 also compares the tax rates implied by the model to those in the data for 3 countries: U.S., Sweden, and Chile. (We report these statistics for all 31 OECD countries in Appendix Table C.1.) We chose Sweden as an example of a country that adopts an aggressive redistribution policy: the Gini coefficient decreases by $39 \%$ as a result of tax and transfer. We chose Chile as the opposite, an

[^10]Table 6: Results for 3 Countries

|  | U.S. | Sweden | Chile |
| :--- | :---: | :---: | :---: |
| Magnitude of Shock $\left(\sigma_{x}\right)$ | 0.324 | 0.257 | 0.377 |
| Implied Tax Rate $(\tau)$ | 0.238 | 0.390 | 0.043 |
| Tax/Y in 2010 | 0.238 | 0.454 | 0.195 |
| Tax Wedge in 2010 | 0.254 | 0.386 | 0.066 |
|  |  |  |  |
| Optimal Tax Rate $\left(\tau^{*}\right)$ | 0.446 | 0.364 | 0.494 |
| Approval Rate for $\tau^{*}$ | 0.690 | 0.584 | 0.899 |
| Tax Rate by Majority Voting | 0.428 | 0.340 | 0.473 |
|  |  |  |  |
| Weighting Function Parameter $(\eta)$ | 0.378 | -1.592 | 1.826 |
| Pareto Weight on 1st Quintile | 0.148 | 0.397 | 0.016 |
| 2nd Quintile | 0.176 | 0.239 | 0.051 |
| 3rd Quintile | 0.197 | 0.169 | 0.104 |
| 4th Quintile | 0.220 | 0.122 | 0.203 |
| 5th Quintile | 0.260 | 0.074 | 0.625 |

example country that exhibits the lowest improvement rate of the Gini coefficient, with only a $4 \%$ decrease after tax and transfers. The implied tax rates square well with the tax-to-GDP ratio in the data. For Sweden, the model-implied tax rate is $39.0 \%$, somewhat lower than but still close to the tax-to-GDP ratio, $45.4 \%$, and the average tax wedge, $38.6 \%$. For Chile, the implied tax rate is $4.3 \%$, much lower than the tax-to-GDP ratio of $19.5 \%$, but close to the average tax wedge of $6.6 \%$.

Figure 5 plots the implied tax rates and the tax-to-GDP ratios for 31 countries. Countries cluster around the 45-degree line, indicating that the model-implied tax rate approximates the average tax rate in the data fairly well with the exceptions of Chile and Ireland, which exhibit the lowest and the highest Gini improvement rates after tax and transfer. Figure 6 plots the model-implied tax rates and the average tax wedges (for a household with two earners and two children) in the data. The model fits the data even better. In summary, our model provides a successful approximation to the actual redistribution policies. This pattern is consistent with previous research such as Romer (1975) and Meltzer and Richard (1981), in which a proportional income tax and a lump-sum transfer

Figure 5: Implied Tax Rate and Tax/GDP

is commonly used as a simple approximation of redistribution policies.
Since we choose the magnitude of the individual productivity shock ( $\sigma_{x}$ ) and the income tax rate $(\tau)$ to exactly replicate the before- and after-tax income inequality, these parameters affect the equilibrium real interest rate $(r)$ and hours worked $(h) .{ }^{15}$ A larger $\sigma_{x}$ implies a larger uncertainty for individual households. This strengthens precautionary savings motives, which in turn lowers the real interest rate and increases average hours worked (Pijoan-Mas, 2006). A strong redistribution policy in the form of a high tax rate $(\tau)$ reduces the incentive to work and provides insurance (thus raising the real interest rate). The aggregate productivity $(A)$ also affects the hours worked through income effects. Since the relative risk aversion, $\sigma$, is larger than 1 , the income effect dominates the substitution effect in our model. As a result, workers in rich countries tend to work less than those in poor countries. Figure 7 compares hours worked from the model and the data. The model fits the data fairly well except for Chile and Ireland where the improvement rate of income inequality is the lowest (4\% for Chile) and the highest (55\%

[^11]Figure 6: Implied Tax Rate and Tax Wedge

for Ireland) among the 31 OECD countries. (Note that the income tax rate is calibrated to match the improvement rate in our model.) Table C. 1 in the Appendix reports the equilibrium real interest rate and hours worked for all 31 countries.

### 4.3. Robustness: Consumption Tax and Progressivity

The tax and transfer schemes in practice are much more complicated than the simple proportional income tax and lump-sum transfer in our model. Almost all countries adopt progressive taxation to some extent. Consumption or sales taxes are common, and transfers are often targeted at specific groups. In this subsection, we check whether the implied tax rates in the model are still consistent with the actual tax rates with respect to consumption tax or progressivity. With the existence of a consumption tax, a more appropriate tax rate for households' consumption-leisure decisions is $1-\tilde{\tau}=\left(1-\tau_{l}\right) /\left(1+\tau_{c}\right)$, where $\tau_{l}$ is the income tax rate and $\tau_{c}$ is the consumption tax rate. We compute $\tilde{\tau}$ for each of the 31 OECD countries. We use the ratio of the labor-income tax revenue to total labor income for $\tau_{l}$ and the ratio of the consumption-tax revenue to total consumption for $\tau_{c}$. The adjusted income tax rate $(\tilde{\tau})$ is highly correlated with both our benchmark measure of tax rate, the total tax revenue to GDP ratio, as well as with the model-implied

Figure 7: Hours Worked

tax rate. The correlation of $\tilde{\tau}$ with $T / Y$ is 0.89 and that with $\tau$ is 0.66 . Alternatively, we also compute the adjusted tax rate ( $\tilde{\tau}$ ) based on the average tax wedge (two-earner households) and consumption tax rate. This is also highly correlated with the model-implied tax rate (correlation coefficient of 0.66).

While the income tax rate is constant, a lump-sum transfer generates progressivity in our model. To compare the progressivity of income tax in the model to that in the data, we compute the so-called net tax rate-i.e., taxes minus transfers, divided by income-for different levels of income. For the income tax rate at the corresponding income level in the data, we use the average tax wedge of single households from the OECD database. ${ }^{16}$ Since detailed information about the transfer amount at different income levels is not available across countries, we assume that a constant lump-sum transfer is given to all households. For the constant lump-sum transfer amount we use two different measures: (i) the average tax revenue and (ii) the average social-benefit expenditure. The former is a too-generous measure because it assumes that all tax revenue is rebated back to

[^12]households. The latter is a conservative measure of transfer because it assumes that only the social benefit expenditures are given back to households, and that all households are receiving the same amount of transfer from the government regardless of their income level. The reality probably lies between these two measures. Table C. 2 in the Appendix compares these two measures of net tax rates at $67 \%, 100 \%$, and $133 \%$ of the average income level to that in the model for all 31 OECD countries. For the U.S. economy, when the average total tax revenue is used to approximate the lump-sum transfer, the net tax rates are $-7.8 \%, 6.7 \%$, and $15.9 \%$ for the income levels of $66 \%, 100 \%$, and $133 \%$ of the average income. The corresponding net tax rates when the social benefit expenditure is used to approximate the lump-sum transfer are $15.3 \%, 22.1 \%$, and $27.4 \%$, respectively. The net tax rates from the model are $-2.8 \%, 6.1 \%$, and $10.5 \%$, respectively.

## 5. Optimal Tax Reforms and Pareto Weights

In the previous section, we developed a quantitative model that exactly matches the before- and after-tax income Gini coefficients in the data and illustrated that it approximates the redistribution policy of 31 OECD countries fairly well. We now address the following questions with this model economy: (i) What is the optimal income tax rate for each country under the equal-weight utilitarian social welfare function? (ii) Would a majority of population vote for the policy reform that proposes to adopt the utilitarian optimal tax rate? (iii) What are the Pareto weights in the social welfare function that justify the current tax rate, which is suboptimal under the equal weights?

### 5.1. Optimal Tax and Social Welfare

One of the most important goals in public finance is to characterize the optimal tax policy. This task often requires appropriately aggregating individual preferences, which is challenging and controversial. A common practice is to use a social welfare function that averages the utility of the population with equal weights. For example, in the context of our model, the social welfare function can be written as: ${ }^{17}$

[^13]$$
\mathcal{W}(\tau)=\int V\left(a_{0}, x_{0} ; \tau\right) d \mu\left(a_{0}, x_{0} ; \tau\right)
$$
where $V\left(a_{0}, x_{0} ; \tau\right)$ is the discounted sum of the lifetime utility of a household with asset holdings $a_{0}$ and productivity $x_{0}$, and $\mu\left(a_{0}, x_{0} ; \tau\right)$ is the distribution of households over $\left(a_{0}, x_{0}\right)$ in the steady state given the tax rate $\tau$. In other words,
$$
V\left(a_{0}, x_{0} ; \tau\right)=\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t}\left\{\frac{c\left(a_{t}, x_{t} ; \tau\right)^{1-\sigma}-1}{1-\sigma}-B \frac{h\left(a_{t}, x_{t} ; \tau\right)^{1+1 / \gamma}}{1+1 / \gamma}\right\}
$$

First, we look for the tax rate $\tau$ that maximizes the above equal-weight utilitarian social welfare function for each of the 31 OECD countries. We assume that each country is at the steady state under its current income tax rate $\tau$ as reported in Table C.1. We then look for the new tax rate, $\tau^{*}$, that maximizes $\mathcal{W}(\tau)$, including the welfares of households during the transition period to the new steady state. A detailed computational algorithm is provided in Appendix B.2. ${ }^{18}$

Table 6 reports the optimal tax rates of 3 countries: the U.S., Sweden, and Chile. In the U.S., the optimal income tax rate is $44.6 \%$, much higher than the current tax rate of $23.8 \%$. For Chile, the optimal rate is $49.4 \%$, more than 10 times larger than the current tax rate of $4 \%$. This is not surprising because under the equal-weight utilitarian criteria, reallocating the resources from the rich (whose marginal utility is low) to the poor (whose marginal utility is high) would increase the average welfare. For Sweden, however, the current redistribution policy is somewhat excessive from the perspective of the equal-weight utilitarian social welfare, as the optimal rate (36.4\%) is lower than the current rate (39\%).

Table C. 3 reports the optimal tax rates for all countries. The optimal tax rates are higher than the current tax rates in 18 countries, whereas the opposite is true in 13 countries. The latter group includes northern European welfare states (Austria, Belgium,

[^14]Denmark, Finland, Germany, Iceland, Ireland, Luxembourg, Norway, and Sweden) and formerly communist societies (the Czech Republic, the Slovak Republic, and Slovenia).

### 5.2. Voting on Optimal Tax Reform

According to our model, current tax rates are close to the socially optimal level in France and Germany. In most countries, however, the current tax rate is far from optimal. A policy to adopt the optimal tax rate will hardly be Pareto improving-there will be winners and losers as a result of reform. Examining the complicated political process needed to select a policy in each country is beyond our ability and knowledge. Instead we ask a simple question: Would the majority be better off under reform to adopt the optimal tax rate? We assume that voting is binary between the current $(\tau)$ and the optimal $\left(\tau^{*}\right)$ tax rates. An individual household would vote for the optimal tax reform if $V\left(a, x ; \tau^{*}\right)>V(a, x ; \tau)$. In computing the welfare under the optimal tax rate, we include the welfare during the transition to the new steady state.

Table 6 reports this voting outcome in our model. The optimal tax reform is supported by the majority of the population in all three countries. In the U.S., the policy proposal to increase the tax rate to the optimal rate $\left(\tau^{*}=0.45\right)$ is supported by $69 \%$ of the population. In Chile, $90 \%$ of the population is better off under the optimal $\operatorname{tax}\left(\tau^{*}=0.49\right)$. In Sweden, the approval rate to decrease the tax rate to the optimal rate $\left(\tau^{*}=0.36\right)$ is $58 \%$. The approval rates for optimal tax reform for all 31 OECD countries are reported in the Appendix Table C.3. For all of the countries except for France and the Netherlands, optimal tax reform is supported by the majority of the population.

### 5.3. Tax Rate Chosen by the Median Voter

According to the median voter theorem, the tax rate that maximizes the utility of the median household can be a political equilibrium under the simple-majority voting system. However, the median voter theorem may not be easily applicable where households differ in multiple dimensions and stochastic states (such as asset and productivity in our model), because the median voter changes under each tax rate. We address this issue by simulating successive voting on different tax rates. Starting with a tax rate that is $1 \%$ higher (or lower) than the current rate, we keep increasing (or decreasing) towards the optimal tax
rate until the proposed tax rate fails to obtain a majority vote. The results based on successive voting for all 31 countries are summarized in the last column in the Table C.3. The tax rates chosen by the successive voting are actually very close to, but slightly lower than, the optimal tax rates in most countries. For example, in the U.S. the tax rate chosen by the successive majority voting is 0.428 , just a little bit lower than the optimal tax rate of 0.446 . Alternatively, we also pick the median household in the current steady state in terms of consumption. The tax rate that maximizes median-consumption household is 0.396 in the U.S.

### 5.4. Pareto Weights in Practice

We have shown that the current tax rate is far from optimal in most of the 31 OECD countries. We have also shown that for all countries except for France and the Netherlands, the majority of the population would be better off under the optimal tax reform. Why, then, haven't these countries adopted the optimal tax rate suggested by the model? The calculation of the optimal tax rate depends on the specification of the social welfare function. It is not necessarily obvious that the equal-weight utilitarian welfare is the right objective function for a society (or government) to maximize. For example, one may argue that it is desirable to maximize the welfare of the poorest members of a society not the average of all (e.g., Rawlsian criteria). Moreover, the decision-making process by which policies are selected is much more complicated than a simple majority rule. For instance, the rich often have more resources to influence the outcome of policy debates (e.g., lobbies). These questions are immensely important but beyond the scope of this paper.

We ask a rather simple question within the utilitarian framework: For each country, what are the weights in the social welfare function that would justify the current tax rate? We interpret these weights-the so-called Pareto weights-as a reduced form representation of a society's preferences and political decision-making process. If a society is plutocratic, the government assigns relatively larger weights to rich households, whereas an egalitarian society is likely to assign larger weights to poor households. Since democratic societies tend to promote an equal distribution of resources and opportunities, they
are more likely to adopt aggressive redistribution policies (which is interpreted as having larger weights on poor households in the social welfare function).

In order to find the Pareto weights that justify the current tax rates, we allow the weights in the social welfare function to vary with the level of consumption. In principle, individual welfare depends on two individual state variables: asset holdings (a) and productivity $(x)$. For computational simplicity, we specify the Pareto weights in one dimension, the level of consumption. According to the permanent income hypothesis, consumption should reflect the overall welfare of the household. More specifically, we assume that the Pareto weight on household $i, \theta_{i}$, exhibits the following parametric form in consumption:

$$
\mathcal{W}=\int \theta_{i} V\left(a_{i}, x_{i}\right) d \mu(a, x)
$$

where

$$
V\left(a_{i}, x_{i}\right)=\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} U\left(c_{i, t}, h_{i, t}\right), \quad \theta_{i}=\frac{c\left(a_{i, t}, x_{i, t}\right)^{\eta}}{\int c\left(a_{i, t}, x_{i, t}\right)^{\eta} d \mu}
$$

The case with $\eta=0$ corresponds to the equal-weight utilitarian social welfare function. The case with $\eta>0$ can be interpreted as a plutocracy or the political system in which the rich have more influence in the determination of policies. The case with $\eta<0$ can be interpreted as sympathy for the poor, or a strong preference for an egalitarian society. The value of $\eta$ that justifies the current income tax rate, $\tau$, as the socially optimal level is the solution to the following problem:

$$
\tau=\operatorname{argmax} W(\tau)=\int \theta_{i} V\left(a_{i}, x ; \tau\right) d \mu(a, x ; \tau)
$$

With the value of $\eta$ that solves the above problem, we can uncover the Pareto weights on households based on the distribution of households $\mu(a, x)$.

For the U.S., $\eta=0.38$ is required to justify the current tax rate as socially optimal. The Pareto weight increases with the level of consumption, as the social welfare function assigns larger weights to the rich. Table 6 reports the Pareto weights (implied by this value of $\eta$ ) for 5 consumption-quintile groups. The average Pareto weights are $15 \%, 18 \%$, $20 \%, 22 \%$, and $26 \%$, respectively, from the 1st (poorest $20 \%$ ) to the 5 th (richest $20 \%$ )
quintiles. In Sweden, $\eta=-1.59$ justifies the current tax rate. The social welfare function assigns larger weights on the poor. The average weight on the poorest $20 \%$ of households is $40 \%$, whereas that on the richest $20 \%$ is only $7.4 \%$. By contrast, in Chile $\eta=1.83$ justifies the very low current tax rate (4.3\%). The average Pareto weight on the poorest $20 \%$ households is a mere $2 \%$, whereas that on the richest $20 \%$ is almost $63 \%$.

We report the values of $\eta$ and the Pareto weights that justify the current tax rates in Appendix Table C.4. All but 4 countries (the U.S., Israel, South Korea, and Chile) exhibit negative values of $\eta$. Some Northern and Central European countries show especially strongly negative values of $\eta$. Ireland, which showed the biggest improvement rate in income Gini coefficients, yields $\eta=-9.2$, the lowest of all. Slovenia (-3.2), Belgium (-2.5), Norway (-2.4), and Czech Republic (-2.4) are the next lowest- $\eta$ countries. For the United Kingdom, Canada, Australia, and Japan, $\eta$ 's are moderately negative, between 0 and -0.3. Among those 4 countries where $\eta$ is positive (the U.S., Israel, South Korea, and Chile), the size of $\eta$ is moderate (between 0 and 0.5 ) in all countries except Chile. ${ }^{19}$

## 6. Conclusion

Economic inequality is at the heart of policy debates in almost every society. We develop a quantitative general-equilibrium model that can be used for the quantitative analysis of the political economy of redistribution policies. With this model, calibrated to exactly match the before- and after-tax income inequality in data drawn from the OECD database, we ask the following questions for 31 OECD countries: (i) What is the optimal income tax and transfer policy under the equal-weight utilitarian social welfare function? (ii) Is the optimal tax reform supported by the majority of the population? (iii) What

[^15]is the Pareto weight in the social welfare function that justifies the current redistribution policy?

According to our model, the optimal tax rate under the equal-weight utilitarian social welfare is quite high, between $31.8 \%$ (South Korea) and $49.4 \%$ (Chile). For 18 countries, the optimal tax rate is higher than the current average tax rate - measured by the tax-to-GDP ratio. For 13 European countries (Austria, Belgium, Czech Republic, Denmark, Finland, Germany, Iceland, Ireland, Luxembourg, Norway, Slovak Republic, Slovenia, and Sweden), the optimal tax rate is lower than the current average tax rate. In France, the current tax rate is close to the social optimum.

In our model, policy reform to adopt the optimal tax rate is supported by the majority of the population in all countries except for France and the Netherlands. For example, in Chile the policy to increase the income tax rate to the socially optimal rate ( $49.4 \%$ ) is supported by $90 \%$ of the population. We interpret the persistence of the current suboptimal tax rate (despite the population's overwhelming support for the optimal tax reform) as evidence that Pareto weights in the social welfare function are far from equal. For example, in Chile, the Pareto weight - which would justify the currently low tax rates-on the richest $20 \%$ of households is $63 \%$, whereas that on the poorest $20 \%$ is a mere $2 \%$. Designing an appropriate redistribution policy requires aggregating preferences across different households. We argue that the Pareto weights (which map individual welfare into social welfare) uncovered in this analysis will be a useful step for further quantitative analysis in public finance and social choice as well as macroeconomics.

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## Appendix A: Data Appendix

## A.1. Income and Wealth Distribution

The income Gini coefficients used in this paper are obtained from the Organization for Economic Cooperation and Development (OECD) database. The base year is 2010 except for coefficients of Chile, Hungary, Ireland, Japan, New Zealand, Switzerland, and the UK, where the base year is 2009. The OECD database provides two Gini coefficients (beforeand after-taxes and transfers) based on household income per equivalent-household member. The social security contributions and transfers are also included. The statistics about the U.S. earnings and wealth distribution based on the Survey of Consumer Finance (SCF) are from Díaz-Giménez, Glover, and Ríos-Rull (2011), and those based on the Panel Study of Income Dynamics (PSID) are from Chang and Kim (2011). The wealth Gini coefficients for OECD countries are obtained from the 2012 version of the Global Wealth Databook by Credit Suisse. Wealth is defined as the marketable value of financial assets plus non-financial assets less debt.

## A.2. Taxes, Expenditures, and Working Hours

Data on tax revenues, tax rates, and tax wedges are from the OECD tax database. The base year is 2010. The tax wedge is the difference between labor costs to the employer and the corresponding net take-home pay of the employee, which is calculated by expressing the sum of personal income tax and employee plus employer social security contributions together with any payroll tax, minus benefits as a percentage of labor costs (definition by the OECD). The OECD provides the tax wedges for several types of households categorized by the number of members in the household, number of earners, and income level. The composition of general government expenditure is also from the OECD database ("National Accounts at a Glance"). The gap between tax revenues and expenditures reflects the government budget deficit and non-tax revenue. Working hours are calculated using the information on average annual working hours from the OECD database. We divide the OECD's numbers by 5200 hours, the total amount of annual discretionary time. The per capita GDP are also from the OECD database.

## A.3. Democracy Index

The Democracy Index is obtained from the Economist Intelligent Unit (EIU). The EIU evaluates the development of democracy in a society based on 60 questions in 5 categories: (i) electoral process and pluralism, (ii) functioning of government, (iii) political participation, (iv) political culture, and (v) civil liberties. A country is scored from 0 to 10 in each of the 5 categories. The Democracy Index is the average of these 5 scores.

## Appendix B: Computational Procedures

## B.1. Steady-State Equilibrium

The distribution of households, $\mu(a, x)$, is time-invariant in the steady state, as are factor prices. We modify the algorithm suggested by José-Víctor Ríos-Rull (1999) in finding a time-invariant distribution $\mu$. Computing the steady-state equilibrium amounts to finding the value functions, the associated decision rules, and the time-invariant measure of households. The proportional income tax rate $\tau$ is read from the improvement rate of income Gini in the data. We search for (i) the discount factor $\beta$ that clears the capital market at the given quarterly rate of return of $1 \%$; (ii) the standard deviation of idiosyncratic productivity, $\sigma_{x}$, that matches the before-tax Gini coefficient; and (iii) the disutility parameter $B$ to match the average hours worked, 0.323 . The aggregate productivity, $A$, is set to 1 in the U.S. The details are as follows:

1. Choose the grid points for asset holdings ( $a$ ) and idiosyncratic productivity $(x)$. The number of grids is denoted by $N_{a}$ and $N_{x}$, respectively. We use $N_{a}=292$ and $N_{x}=31$. The asset holding $a_{t}$ is in the range of [0, 29.6]. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate the savings decisions of households near the borrowing constraint.
2. Pick initial values of $\beta, B$, and $\sigma_{x}$. For idiosyncratic productivity, we construct a grid vector of length $N_{x}$ of which elements (each denoted by $\ln x_{j}$ ) are equally spaced on the interval $\left[-3 \sigma_{x} / \sqrt{1-\rho_{x}^{2}}\right]$. Then, we approximate the transition matrix of the idiosyncratic productivity using the algorithm from Tauchen (1986).
3. Start with an initial amount of government transfers $T$. Given $\beta, B, \sigma_{x}, \tau$, and $T$, we solve the individual value functions $V$ at each grid point for individual states. In this step, we also obtain the optimal decision rules for asset holdings $a^{\prime}\left(a_{i}, x_{j}\right)$ and labor supply $h\left(a_{i}, x_{j}\right)$. This step involves the following procedure:
(a) Initialize value functions $V_{0}\left(a_{i}, x_{j}\right)$ for all $i=1,2, \cdots, N_{a}$, and $j=1,2, \cdots, N_{x}$.
(b) Update value functions by evaluating the discretized versions:

$$
\begin{aligned}
V_{1}\left(a_{i}, x_{j}\right)=\max \{ & u\left((1-\tau)\left(w h\left(a_{i}, x_{j}\right) x_{j}+r a_{i}\right)+a_{i}+T-a^{\prime}, h\left(a_{i}, x_{j}\right)\right) \\
& \left.\left.+\beta \sum_{j^{\prime}=1}^{N_{x}} V_{0}\left(a^{\prime}, x_{j}^{\prime}\right)\right) \pi_{x}\left(x_{j^{\prime}} \mid x_{j}\right)\right\}
\end{aligned}
$$

where $\pi_{x}\left(x_{j^{\prime}} \mid x_{j}\right)$ is the transition probabilities of $x$, which is approximated using Tauchen's algorithm.
(c) If $V_{1}$ and $V_{0}$ are close enough for all grid points, then we have found the value functions. Otherwise, set $V_{0}=V_{1}$, and go back to step 3(b).
4. Using $a^{\prime}\left(a_{i}, x_{j}\right)$ and $\pi_{x}\left(x_{j^{\prime}}, x_{j}\right)$ obtained from step 3 , we obtain the time-invariant measures $\mu^{*}\left(a_{i}, x_{j}\right)$ as follows
(a) Initialize the measure $\mu_{0}\left(a_{i}, x_{j}\right)$.
(b) Update the measure by evaluating the discretized version of a law of motion:

$$
\mu_{1}\left(a_{i^{\prime}}, x_{j^{\prime}}\right)=\sum_{i=1}^{N_{a}} \sum_{j=1}^{N_{x}} \mathbf{1}_{a_{i^{\prime}}=a^{\prime}\left(a_{i}, x_{j}\right)} \mu_{0}\left(a_{i}, x_{j}\right) \pi_{x}\left(x_{j^{\prime}} \mid x_{j}\right) .
$$

(c) If $\mu_{1}$ and $\mu_{0}$ are close enough in all grid points, then we have found the timeinvariant measure. Otherwise, replace $\mu_{0}$ with $\mu_{1}$ and go back to step 4(b).
5. Using decision rules and invariant measures, check the balance of the government budget. Total tax revenues are:

$$
T R=\int_{a, x} \tau(w x h+r a) d \mu(a, x)
$$

If $T R$ is close enough to $T$, then we have obtained the amount of government transfers. Otherwise, choose a new $T$ and go back to step 3 .
6. We calculate the real interest rate, Gini coefficient, individual hours worked, and other aggregate variables of interest using $\mu^{*}$ and decision rules. If the calculated real interest rate, average hours worked, and before-tax Gini coefficient are close to the assumed ones, we have found the steady state. Otherwise, we choose a new $\beta$, $B$, and $\sigma_{x}$, and go back to step 2 .

The computational procedure for other countries is similar except that we fix $\beta$ and $B$ from the U.S. case. The aggregate productivity $A$ is chosen to match the per capita GDP relative to U.S.

## B.2. Optimal Tax Reform

Individual utilities include those in the transition periods from the initial to the new steady state. We compute the value functions and decision rules backwards and the measure of households forward. Computing the transition equilibrium amounts to finding the value functions, the associated decision rules, and measure of households in each period. The details are as follows:

1. Compute the initial steady state under the current tax rate. Use the algorithm for the steady-state equilibrium.
2. Choose a new tax rate and compute all transition paths as follows:
(a) Compute the final steady state under a new tax rate. Use the algorithm for steady-state equilibrium.
(b) Assume that the transition is completed after $T-1$ periods, and that the economy is in the initial steady state at time 1 and in the final steady state at $T$. Choose a $T$ big enough so that the transition path is unaltered by increasing $T$.
(c) Guess the capital per effective labor $\left\{K_{t} / E_{t}\right\}_{t=2}^{T-1}$ and compute the associated $\left\{r_{t}, w_{t}\right\}_{t=2}^{T-1}$.
(d) Guess the path of government transfers $\{T\}_{t=2}^{T-1}$. Note that the amounts of government transfers are all different in each period, since decision rules and measures are different. Going backward, compute the value functions and policy functions for all transition periods by using $V_{T}(\cdot)$ from the final steady state. Using the initial steady-state distribution $\mu_{1}$ and the decision rules, find the measures of all periods $\left\{\mu_{t}\right\}_{t=2}^{T-1}$.
(e) Based on the decision rules and measures, compute the aggregate variables and total tax revenues. If the total tax revenue is close to the assumed transfers, we obtain the amount of transfers. Otherwise, choose a new path of government transfers and go back to 2(d).
(f) Compute the paths of aggregated capital and effective labor and compare them with the assumed paths. If they are close enough in each period, we find the transition paths. Otherwise, update $\left\{K_{t} / E_{t}\right\}_{t=2}^{T-1}$ and go back to 2(c).
3. Choose the tax rate that yields the highest social welfare. This is the optimal tax rate under the utilitarian criteria. We also compute the voting outcome for this tax reform policy. Voting takes place at the beginning of period 2, after the idiosyncratic productivity shock has been realized. The voting decision of an individual with state $(a, x)$ is determined as follows: if $V\left(a, x, \tau^{\text {new }}, \tilde{r}, \tilde{w}\right)>V\left(a, x, \tau^{\text {current }}, r^{*}, w^{*}\right)$, then this individual votes in favor of the new tax rate.

## B.3. Pareto Weights

We search for the value of $\eta$ so that the current tax rate provides the highest social welfare in the steady state. Note that we compare the steady-state social welfares. Details are as follows:

1. Define a set of tax rates around the current one.
2. Given a tax rate, compute $c(a, x), V(a, x), \mu(a, x)$, and other related variables using the algorithm for the steady-state equilibrium.
3. Assume $\eta$ and compute the social welfare under each tax rate:

$$
\mathcal{W}=\int \theta_{i} V\left(a_{i}, x_{i}\right) d \mu(a, x)
$$

where

$$
\theta_{i}=\frac{c\left(a_{i}, x_{i}\right)^{\eta}}{\int c\left(a_{i}, x_{i}\right)^{\eta} d \mu}
$$

4. Compare the social welfares and choose the highest social welfare and the corresponding tax rate. If the tax rate is close enough to $\tau$, then we obtain $\eta$ and Pareto weights for individuals. Otherwise, we choose a new $\eta$ and go back to step 3.

## Appendix C: Additional Tables

Table C.1: Steady State for OECD

|  | Implied Tax <br> Rates $(\tau)$ | Tax/Y <br> in 2010 | Tax Wedge <br> in 2010 | Interest <br> Rates $(r)$ | Hours <br> Worked $(H)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Australia | 0.288 | 0.256 | 0.237 | 0.051 | 0.299 |
| Austria | 0.443 | 0.422 | 0.400 | 0.078 | 0.261 |
| Belgium | 0.452 | 0.435 | 0.488 | 0.080 | 0.261 |
| Canada | 0.284 | 0.306 | 0.266 | 0.053 | 0.315 |
| Chile | 0.043 | 0.195 | 0.066 | 0.014 | 0.528 |
| Czech Republic | 0.430 | 0.339 | 0.343 | 0.079 | 0.321 |
| Denmark | 0.413 | 0.474 | 0.337 | 0.078 | 0.266 |
| Estonia | 0.345 | 0.380 | 0.358 | 0.058 | 0.368 |
| Finland | 0.457 | 0.425 | 0.369 | 0.081 | 0.258 |
| France | 0.400 | 0.429 | 0.451 | 0.065 | 0.278 |
| Germany | 0.420 | 0.362 | 0.417 | 0.071 | 0.273 |
| Greece | 0.354 | 0.316 | 0.383 | 0.054 | 0.315 |
| Iceland | 0.379 | 0.352 | 0.304 | 0.076 | 0.300 |
| Ireland | 0.550 | 0.274 | 0.162 | 0.091 | 0.207 |
| Israel | 0.250 | 0.324 | 0.133 | 0.041 | 0.350 |
| Italy | 0.366 | 0.430 | 0.424 | 0.059 | 0.299 |
| Japan | 0.311 | 0.276 | 0.254 | 0.052 | 0.306 |
| Korea | 0.091 | 0.251 | 0.179 | 0.045 | 0.454 |
| Luxembourg | 0.418 | 0.373 | 0.213 | 0.075 | 0.230 |
| Netherlands | 0.321 | 0.389 | 0.318 | 0.062 | 0.306 |
| New Zealand | 0.302 | 0.311 | 0.138 | 0.055 | 0.332 |
| Norway | 0.411 | 0.426 | 0.334 | 0.079 | 0.245 |
| Poland | 0.348 | 0.317 | 0.306 | 0.061 | 0.381 |
| Portugal | 0.341 | 0.312 | 0.325 | 0.052 | 0.334 |
| Slovak Republic | 0.403 | 0.283 | 0.315 | 0.075 | 0.344 |
| Slovenia | 0.457 | 0.381 | 0.340 | 0.085 | 0.298 |
| Spain | 0.334 | 0.325 | 0.367 | 0.053 | 0.318 |
| Sweden | 0.390 | 0.454 | 0.386 | 0.072 | 0.279 |
| Switzerland | 0.199 | 0.281 | 0.158 | 0.052 | 0.322 |
| United Kingdom | 0.340 | 0.349 | 0.284 | 0.052 | 0.300 |
| United States | 0.238 | 0.238 | 0.254 | 0.040 | 0.323 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Source: OECD database (2010) and authors' calculation.

Table C.2: Net Tax Rate (\%) at Different Income Levels

|  | $T=$ Total Revenue |  |  | $T=$ Social Benefit |  | Model |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Relative income |  |  |  |  |  |  |  |  |  |
| to the average | $66 \%$ | $100 \%$ | $133 \%$ | $66 \%$ | $100 \%$ | $133 \%$ | $66 \%$ | $100 \%$ | $133 \%$ |
| Australia | -17.6 | 1.2 | 10.5 | 9.9 | 19.5 | 24.3 | -4.1 | 6.9 | 12.4 |
| Austria | -19.9 | 6.0 | 19.6 | 12.9 | 27.8 | 36.0 | -8.7 | 8.9 | 17.8 |
| Belgium | -15.0 | 12.4 | 26.5 | 19.4 | 35.4 | 43.7 | -9.0 | 9.0 | 18.1 |
| Canada | -18.9 | -0.1 | 8.9 | 16.4 | 23.4 | 26.6 | -4.2 | 6.7 | 12.1 |
| Chile | -22.3 | -12.5 | -7.6 | 0.9 | 2.9 | 3.9 | -0.1 | 1.3 | 2.1 |
| Czech Republic | -12.0 | 8.2 | 18.3 | 16.2 | 27.0 | 32.4 | -8.5 | 8.7 | 17.2 |
| Denmark | -33.9 | -8.8 | 6.1 | 14.5 | 23.5 | 30.3 | -8.1 | 8.4 | 16.6 |
| Estonia | -12.3 | 6.1 | 15.3 | 20.0 | 27.6 | 31.4 | -5.4 | 7.9 | 14.5 |
| Finland | -27.0 | -0.2 | 14.1 | 12.7 | 26.3 | 33.9 | -9.3 | 9.1 | 18.2 |
| France | -17.8 | 6.9 | 19.8 | 17.5 | 30.4 | 37.4 | -6.9 | 8.7 | 16.5 |
| Germany | -9.4 | 12.9 | 23.6 | 16.1 | 29.9 | 36.3 | -7.7 | 8.8 | 17.1 |
| Greece | -11.7 | 8.4 | 19.2 | 16.0 | 26.9 | 33.1 | -5.4 | 8.2 | 15.0 |
| Iceland | -24.4 | -1.8 | 9.5 | 20.3 | 28.0 | 31.9 | -7.3 | 7.7 | 15.3 |
| Ireland | -24.3 | -1.6 | 11.0 | 5.5 | 18.3 | 25.9 | -11.9 | 10.4 | 21.5 |
| Israel | -34.6 | -11.7 | 1.2 | 7.3 | 16.3 | 22.2 | -2.9 | 6.4 | 11.1 |
| Italy | -20.5 | 4.2 | 18.0 | 15.6 | 28.3 | 36.0 | -5.9 | 8.3 | 15.4 |
| Japan | -12.5 | 2.6 | 10.9 | 7.0 | 15.7 | 20.6 | -4.6 | 7.3 | 13.3 |
| Korea | -20.2 | -4.9 | 2.7 | 8.7 | 14.4 | 17.1 | -1.1 | 2.3 | 4.0 |
| Luxembourg | -28.3 | -3.0 | 10.9 | 0.9 | 16.5 | 25.5 | -8.0 | 8.6 | 16.9 |
| Netherlands | -24.9 | -0.8 | 10.9 | 7.3 | 20.6 | 27.0 | -5.3 | 7.1 | 13.4 |
| New Zealand | -32.5 | -14.1 | -2.7 | 3.5 | 9.8 | 15.3 | -4.6 | 7.0 | 12.8 |
| Norway | -30.0 | -5.4 | 8.7 | 11.6 | 22.4 | 29.5 | -8.2 | 8.3 | 16.5 |
| Poland | -14.3 | 2.5 | 10.9 | 15.4 | 22.3 | 25.8 | -5.7 | 7.8 | 14.5 |
| Portugal | -14.7 | 5.9 | 16.3 | 12.2 | 23.8 | 29.7 | -4.9 | 8.1 | 14.6 |
| Slovak Republic | -7.8 | 9.6 | 18.3 | 14.0 | 24.2 | 29.2 | -7.7 | 8.3 | 16.3 |
| Slovenia | -18.6 | 4.4 | 16.1 | 16.1 | 27.5 | 33.4 | -9.5 | 8.9 | 18.1 |
| Spain | -12.3 | 7.3 | 17.5 | 17.0 | 26.8 | 32.1 | -4.9 | 7.8 | 14.2 |
| Sweden | -27.5 | -2.7 | 13.3 | 16.8 | 26.9 | 35.4 | -7.3 | 8.2 | 15.9 |
| Switzerland | -22.9 | -6.0 | 3.6 | 4.5 | 12.3 | 17.2 | -2.9 | 4.7 | 8.5 |
| United Kingdom | -22.9 | -2.3 | 8.4 | 13.8 | 22.2 | 26.7 | -5.0 | 8.0 | 14.5 |
| United States | -7.8 | 6.7 | 15.9 | 15.3 | 22.1 | 27.4 | -2.8 | 6.1 | 10.5 |
|  |  |  |  |  |  |  |  |  |  |

Note: The net tax rate is defined by the average tax rate after transfers: (tax - transfer ) / income. For the tax rate at different income levels, we use the average tax wedge of single households from the OECD. For the lump-sum transfer, we use two measures: total tax revenue and social benefit expenditures. Revenues and expenditures are from the OECD, but expenditures for Chile and New Zealand are from the International Monetary Fund.
Source: OECD database (2010) and authors' calculation.

Table C.3: Optimal Tax Reform and Approval Rate

|  | Magnitude <br> Shock $\left(\sigma_{x}\right)$ | Implied Tax Rate ( $\tau$ ) | Optimal Tax <br> Rate ( $\tau^{*}$ ) | Approval Rate | Majority Tax Rate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Australia | 0.294 | 0.288 | 0.411 | 0.595 | 0.388 |
| Austria | 0.278 | 0.443 | 0.388 | 0.645 | 0.373 |
| Belgium | 0.275 | 0.452 | 0.385 | 0.650 | 0.362 |
| Canada | 0.277 | 0.284 | 0.392 | 0.531 | 0.364 |
| Chile | 0.377 | 0.043 | 0.494 | 0.899 | 0.473 |
| Czech Republic | 0.257 | 0.430 | 0.363 | 0.650 | 0.340 |
| Denmark | 0.245 | 0.413 | 0.348 | 0.649 | 0.323 |
| Estonia | 0.299 | 0.345 | 0.421 | 0.519 | 0.395 |
| Finland | 0.275 | 0.457 | 0.385 | 0.652 | 0.367 |
| France | 0.304 | 0.400 | 0.420 | 0.475 | 0.400 |
| Germany | 0.291 | 0.419 | 0.404 | 0.575 | 0.389 |
| Greece | 0.325 | 0.354 | 0.444 | 0.527 | 0.424 |
| Iceland | 0.224 | 0.379 | 0.319 | 0.646 | 0.299 |
| Ireland | 0.348 | 0.550 | 0.458 | 0.662 | 0.450 |
| Israel | 0.324 | 0.250 | 0.446 | 0.686 | 0.430 |
| Italy | 0.308 | 0.366 | 0.425 | 0.515 | 0.406 |
| Japan | 0.305 | 0.311 | 0.424 | 0.586 | 0.401 |
| Korea | 0.221 | 0.091 | 0.318 | 0.653 | 0.281 |
| Luxembourg | 0.270 | 0.418 | 0.382 | 0.594 | 0.358 |
| Netherlands | 0.255 | 0.321 | 0.363 | 0.490 | 0.341 |
| New Zealand | 0.280 | 0.302 | 0.396 | 0.527 | 0.372 |
| Norway | 0.241 | 0.411 | 0.342 | 0.650 | 0.321 |
| Poland | 0.284 | 0.348 | 0.403 | 0.505 | 0.378 |
| Portugal | 0.327 | 0.341 | 0.445 | 0.589 | 0.431 |
| Slovak Republic | 0.253 | 0.403 | 0.358 | 0.642 | 0.333 |
| Slovenia | 0.256 | 0.457 | 0.360 | 0.666 | 0.337 |
| Spain | 0.317 | 0.333 | 0.435 | 0.580 | 0.413 |
| Sweden | 0.257 | 0.390 | 0.364 | 0.584 | 0.340 |
| Switzerland | 0.232 | 0.199 | 0.333 | 0.554 | 0.309 |
| United Kingdom | 0.328 | 0.340 | 0.448 | 0.589 | 0.430 |
| United States | 0.324 | 0.238 | 0.446 | 0.690 | 0.428 |

Table C.4: Pareto Weights across Consumption Quintiles

|  | Pareto Weights |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underline{\text { Parameter }}$ |  | Quintile |  |  |  |
|  | $\eta$ | 1st | 2nd | 3rd | 4th | 5th |
| Australia | -0.125 | 0.217 | 0.206 | 0.199 | 0.193 | 0.184 |
| Austria | -2.186 | 0.459 | 0.246 | 0.151 | 0.096 | 0.048 |
| Belgium | -2.461 | 0.485 | 0.245 | 0.143 | 0.086 | 0.040 |
| Canada | -0.266 | 0.236 | 0.212 | 0.198 | 0.186 | 0.169 |
| Chile | 1.826 | 0.016 | 0.051 | 0.104 | 0.203 | 0.625 |
| Czech | -2.386 | 0.481 | 0.239 | 0.145 | 0.090 | 0.045 |
| Denmark | -2.338 | 0.477 | 0.236 | 0.146 | 0.093 | 0.048 |
| Estonia | -0.474 | 0.263 | 0.221 | 0.196 | 0.174 | 0.145 |
| Finland | -2.609 | 0.499 | 0.244 | 0.139 | 0.081 | 0.036 |
| France | -0.949 | 0.321 | 0.237 | 0.187 | 0.149 | 0.105 |
| Germany | -1.406 | 0.374 | 0.244 | 0.175 | 0.127 | 0.079 |
| Greece | -0.295 | 0.240 | 0.216 | 0.199 | 0.184 | 0.163 |
| Iceland | -2.256 | 0.467 | 0.231 | 0.149 | 0.099 | 0.054 |
| Ireland | -9.172 | 0.804 | 0.171 | 0.022 | 0.003 | 0.000 |
| Israel | 0.331 | 0.154 | 0.179 | 0.198 | 0.217 | 0.252 |
| Italy | -0.551 | 0.273 | 0.225 | 0.195 | 0.170 | 0.137 |
| Japan | -0.185 | 0.225 | 0.209 | 0.199 | 0.190 | 0.176 |
| Korea | 0.252 | 0.170 | 0.189 | 0.200 | 0.211 | 0.229 |
| Luxembourg | -1.791 | 0.418 | 0.243 | 0.163 | 0.112 | 0.064 |
| Netherlands | -0.861 | 0.311 | 0.228 | 0.187 | 0.156 | 0.118 |
| New Zealand | -0.359 | 0.248 | 0.216 | 0.197 | 0.181 | 0.158 |
| Norway | -2.414 | 0.484 | 0.234 | 0.144 | 0.091 | 0.046 |
| Poland | -0.709 | 0.294 | 0.228 | 0.191 | 0.162 | 0.126 |
| Portugal | -0.192 | 0.226 | 0.211 | 0.199 | 0.189 | 0.175 |
| Slovak | -1.948 | 0.436 | 0.239 | 0.158 | 0.107 | 0.060 |
| Slovenia | -3.219 | 0.558 | 0.231 | 0.120 | 0.065 | 0.026 |
| Spain | -0.215 | 0.229 | 0.211 | 0.199 | 0.188 | 0.172 |
| Sweden | -1.592 | 0.397 | 0.239 | 0.169 | 0.122 | 0.074 |
| Switzerland | -0.241 | 0.230 | 0.209 | 0.198 | 0.188 | 0.174 |
| United Kingdom | -0.158 | 0.222 | 0.209 | 0.200 | 0.191 | 0.179 |
| United States | 0.378 | 0.148 | 0.176 | 0.197 | 0.220 | 0.260 |


[^0]:    *We would like to thank Romans Pancs, Mark Bils, Andreas Hornstein, and seminar participants at the University of Rochester, KIPF, and the Federal Reserve Bank of Richmond.

[^1]:    ${ }^{1}$ Alesina and Glaeser (2004) provide a detailed survey of a broad range of sources, their interactions, and the socio-economic consequences of income and wealth inequality.
    ${ }^{2}$ Out of the 34 OECD countries, we exclude Turkey, Mexico, and Hungary from our analysis because before-tax Gini coefficients are not available for these countries.
    ${ }^{3}$ The specific measures and degree of income redistribution caused by individual policies differ considerably across countries. Progressive income taxation and a variety of income transfer programs are typical redistribution policies intended to reduce the inequality of disposable income. There are also indirect transfer programs, which redistribute wealth through providing goods and services that individuals would have otherwise purchased at their own expense. Examples include free education, health care and child care. Different countries have a variety of policy tools. For example, the top statutory personal income-tax rate ranges from $15 \%$ (Czech Republic) to $57 \%$ (Sweden). The property tax share of total tax revenue varies from a mere $1.1 \%$ (Estonia) to $13 \%$ (U.S.).

[^2]:    ${ }^{4}$ In our model, the average tax rate after the transfer is still progressive because of the lump-sum transfer, even though the marginal tax rate is constant.

[^3]:    ${ }^{5}$ These countries are Austria, Belgium, Czech Republic, Denmark, Finland, Germany, Iceland, Ireland, Luxembourg, Norway, Slovak Republic, Slovenia, and Sweden.
    ${ }^{6}$ For this, we compute the changes in the value functions of individual households not only in the new steady state with the optimal tax rate but also en route to the new steady state. If the majority of people benefit from the change in the tax rate, this policy reform can be supported as a political equilibrium.

[^4]:    ${ }^{7}$ Since democratic societies tend to promote an equal distribution of resources and opportunities, they are more likely to adopt aggressive redistribution policies and, as a result, exhibit larger Pareto weights in the social welfare function. In fact, the correlation between the tax-toGDP ratio and the Democracy Index (from Economist Intelligence Unit, 2010) is 0.4 among the 34 OECD countries.

[^5]:    ${ }^{8}$ The OECD database provides the income Gini coefficients that are standardized across sources and measures: http://stats.oecd.org. For Turkey, Mexico, and Hungary, the before-tax income Gini is not available.

[^6]:    ${ }^{9}$ Two alternative - and perhaps more commonly-used-measures of democracy are the those from the Freedom House and Polity IV. However, these measures are not suitable for our analysis, since they do not show much variation across the 34 OECD countries. For instance, according to the Freedom House democracy index, 32 OECD countries are classified as "Free" and only Mexico and Turkey are ranked "Partially Free." According to the Polity IV index, most of the OECD countries score 10 with the exceptions of Estonia (9), France (9), Belgium (8), the Czech Republic (8), South Korea (8), Mexico (8), and Turkey (8).

[^7]:    ${ }^{10}$ The wealth Gini coefficient based on data from Creidt Suisse shows a weak correlation with the before-tax income Gini from the OECD database, while it is positively correlated (0.37), with the after-tax income Gini. Interestingly, unlike income Ginis, the wealth Gini is not highly correlated with any of the redistribution policy measures we consider above, such as the tax-to-GDP ratio, the top statutory income tax rate, the property tax revenue share, or the Democracy Index. The correlation coefficients (with the wealth Gini) for those measures are $-0.22,0.02,0.04$, and -0.07 , respectively. These patterns may arise due to the difficulty in collecting precise wealth data across countries. The wealth measures in the Credit Suisse data are not as standardized across countries as those for income data in the OECD database.

[^8]:    ${ }^{11}$ We normalize the average annual working hours in the OECD data by the total discretionary hours of 5,200 .
    ${ }^{12}$ This is a standard procedure in the literature: The risk aversion parameter is chosen to match the cross-sectional correlation between wages and hours (e.g., Heathcote, Storesletten, and Violante, 2008; and Pijoan-Mas, 2006).

[^9]:    ${ }^{13}$ Let $x$ be a population share from the bottom and $y(x)$ be the income of the marginal agent at $x$. The Lorenze curve $(L(x))$ is the cumulative income share up to $x$ : i.e., $L(x)=\int_{0}^{x} y(x) d x / Y$, where $Y$ is the aggregate output. The Gini coefficient $(G)$ is defined by $1-2 \int_{0}^{x} L(x) d x$. Let $G_{B}$ and $G_{A}$ denote the before- and after-tax/transfers Gini coefficients, respectively. The balanced budget implies $T=\tau Y$.

    $$
    \begin{aligned}
    L_{A}(x) & =\int_{0}^{x}\{(1-\tau) y(x)+T\} d x / Y d x=(1-\tau) L_{B}(x)+\tau x \\
    \text { Then, } G_{A} & =1-2 \int_{0}^{1} L_{A}(x) d x=1-2 \int_{0}^{1}\left((1-\tau) L_{B}(x)+\tau x\right) d x \\
    & =1-2(1-\tau) \int_{0}^{1} L_{B}(x) d x-2 \tau\left[1 / 2 x^{2}\right]_{0}^{1} \\
    & =(1-\tau)\left(1-2 \int_{0}^{1} L_{B}(x) d x\right)=(1-\tau) G_{B}
    \end{aligned}
    $$

[^10]:    ${ }^{14}$ The tax wedge is the difference between gross income and after-tax income. According to the OECD's definition, it is the sum of personal income tax and employee plus employer social security contributions together with any payroll tax less cash transfers, expressed as a percentage of labor costs.

[^11]:    ${ }^{15}$ In the calibration for the benchmark (the U.S. economy), we set the common preference parameters $B$ and $\beta$ so that the real interest rate is $4 \%$ and the average hours worked is 0.323 in the steady state.

[^12]:    ${ }^{16}$ The OECD reports the average tax wedge of a single household at different income levels, including "one-earner married couple" and "two-earner-married couple."

[^13]:    ${ }^{17}$ This utilitarian social welfare function has been commonly used in the literature, for example, in Aiyagari and McGrattan (1998).

[^14]:    ${ }^{18}$ We include the welfares of households during the transition from the current steady state to a new steady state. When a new tax rate is in place at the current steady state, households start re-optimizing their consumption and hours worked. As a result, the corresponding paths of the value functions, $V_{t}\left(a_{t}, x_{t} ; \tilde{\tau}^{*}\right)$, and the distribution, $\mu_{t}\left(a_{t}, x_{t} ; \tilde{\tau}^{*}\right)$, will be different from those in the old steady state. Hence, the computation of the optimal tax needs to take into account changes in value functions and the distribution during transition periods until the economy reaches a new steady state.

[^15]:    ${ }^{19}$ The finding of a negative value of $\eta$, that is, larger weights on the poor, in most OECD countries is not surprising, as many developed countries adopt aggressive redistribution policies. Alesina and Glaeser (2004) explain this as differences in political structure. Almost all European countries have proportional representation systems, but the US does not. Also, socialism does not succeed in the U.S., unlike in Europe. The American federal system as well as the system of checks and balances also makes policies less favorable to the poor. Our results also support the idea that more democratic societies put larger weights on the poor as there is a negative correlation between the democracy index and $\eta$. This relationship becomes stronger when we exclude some formerly communist countries such as the Czech Republic, Slovak Republic, and Slovenia, which have low democracy indicies but emphasize equality.

