Technological Rivalry and Optimal Dynamic Policy in an Open Economy

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Abstract

What are a country's policy options in the face of emerging technologies development in a global economy? To answer this question, we examine optimal dynamic policies in an open economy where technology is endogenously accumulated through R&D innovation. Our key insight is that a country has incentives to influence foreign innovation efforts across sectors and over time—giving rise to optimal policies even when the private innovation allocations are (Pareto) efficient. We derive explicit expressions for optimal taxes linked to both an intratemporal and an intertemporal motive to manipulate foreign technology. A country would want to levy higher tariffs in sectors in which it has a comparative advantage, at the same time invoking domestic innovation subsidies during transition. By contrast, optimal policies under exogenous technology call for uniform tariffs across sectors and no innovation policies.

Keywords: Endogenous Technology, Innovation, Trade, Optimal Dynamic Policies

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1 Introduction

Trade disputes are often not about trade, but about technological rivalry. When Japan took over as the leader of the semiconductors industry in the mid-1980s, it caused alarm in the U.S., where the microchip was invented. The U.S. then started to impose a set of protectionist measures, along with significant subsidies in the domestic semiconductor sectors. The outcome was a renewal of its competitiveness (Langlois and Steinmueller 2000).[1]

Throughout history, we have seen how development in leading-edge sectors has spurred national competition, protectionism, sometimes under the guise of trade wars. One of the core disputes in the 2018 U.S-China trade war was over technology, an event that led to the U.S. banning Huawei and other Chinese companies from accessing critical supplies in the U.S. Along with increasing U.S. restrictions and controls, domestic policies inside China to boost demand and procurement and incentivize research in these goods have soared. At the same time, in the U.S., the passing of the America Competes Act of 2022 earmarks 52 billion dollars of funding for the semiconductors industry. In a world where there is rising demand for technologies such as semiconductors, 5G, a race to gain an edge in emerging technologies such as renewables and AI, a country may have a desire to react.

This paper provides a framework to explore the nexus between trade policies and technological development in the context of globalization and competition. The spirit of contest is captivatingly explored in Samuelson (2004), which argues that developing countries' technological advancement can sometimes harm the interests of advanced economies, by becoming more productive in sectors in which rich countries have a comparative advantage (or conversely, benefit them if rapid productivity growth happens in developing countries' comparative advantage sectors). This simple but powerful argument is based on comparative static analysis over exogenous technology, and does not consider the option that one country may attempt to influence the innovation efforts of another, nor does it consider the costs of innovation. To re-evaluate Samuelson's thesis when technology endogenously evolves, and when countries have a set of instruments to influence trade and technology

[1] Strategy and Circumstance: The Response of American Firms to Japanese Competition in Semiconductors, 1980-1995. Strategic Management Journal , Oct. - Nov., 2000, Vol. 21, No. 10/11 pp. 1163-1173.

dynamics, we theoretically characterize optimal dynamic taxation in a workhorse Ricardian model with endogenous technology.

We show that a government has an incentive to manipulate innovation efforts in *other* countries across sectors and over time. Our main takeaway and novel finding is that even when markets are efficient and the government has access to a full set of trade policies, time-consistent optimal policy will involve domestic innovation policies. This motivation goes beyond conventional industrial policies that justify subsidizing or protecting some sectors at the expense of others on the basis of externalities and spillovers.

The framework we propose to study optimal innovation and trade policy is a dynamic multi-country, multi-sector model with comparative advantage based trade and endogenous technology accumulation through R&D and innovation. Our model builds on the one-sector framework of Eaton and Kortum (2001) and examines optimal taxation. The economy's government is benevolent and can choose a set of sector-specific domestic taxes/subsidies on R&D, as well as differential trade policies across sectors and trading partners. When choosing these policies, the government internalizes its choices on trade and technology development in its own country as well as in others. Other economies' government is taken to be passive in the benchmark scenario. We first consider the case where Home cannot commit to a sequence of taxes, then compare it to the Ramsey optimal policy.

We carefully select this rich dynamic model as it meets three criteria. First, it nests an efficient baseline Ricardian economy. This can help isolate the new mechanism underpinning optimal policy as it obviates the need for any other classic industry policy intervention. Second, the innovation process is such that both sectoral and aggregate trade patterns–and hence optimal policy– can be explicitly characterized. This allows us to compare the results with the standard workhorse Eaton and Kortum (2002) Ricardian model with exogenous technology. Third, the enriched version of the model can accommodate many additional features of interest, for example, economies of scale, knowledge spillovers, or externalities. The complexity of the problem lies in the fact that these additional elements give rise to policy interventions in a closed-economy environment, but added to the fact is that they are intertwined with trade policies. Thus, the sharp predictions stemming from our analytically convenient results are particularly appealing given the richness of these mechanisms.

The baseline economy is efficient. Productivity (innovation efficiency) differences across sectors shape comparative advantage, which determines trade across countries. There are constant returns to scale in production and innovation, and a free flow of labor between research and production. The model features Bertrand competition between producers for each goods, where each producer competes with all other producers in the world. Although the realized markup of each firm follows a distribution, the aggregate endogenous distribution of markups is Pareto, and the expected profit of a firm is a constant share of sales in the industry. Thus, there are constant and identical aggregate markups in each sector. In addition, there are no other taxes, distortions, or externalities in the baseline case that underlie conventional industrial policy motives.

In our baseline closed-economy setting, the planner would choose the same allocation as the market. Openness itself does not affect the level of private innovation intensity: the increased foreign competition that spurs innovation is exactly offset by the larger foreign market effect that tends to reduce innovation effort. However, in the open economy environment, there is scope for policy corresponding to both an intertemporal and intratemporal motive to manipulate technology. Home's planner understands that it can undertake actions to influence foreign profits and research returns, and hence curb foreign innovation efforts in some sectors or channel them towards those that would benefit Home.

To bring clarity to the disparate set of mechanisms, we build up our results by first zeroing in on a dynamic technology manipulation motive in a one-sector model that is efficient— absent any externalities or intratemporal relative prices. Consider a case when Home is innovating and transitioning to a higher level of technology. We show that Home country would want to suppress foreign innovation efforts by curtailing its research returns while raising its cost of innovation. If it could commit to a schedule of trade policies (a path of export taxes), it would suffice to implement an optimal foreign allocation and relative prices over time. Under these Ramsey policies, Home's export tax rises but becomes flatter over time. This generates a higher Foreign price index today and a relatively lower future price, reducing expected profits while pushing up the interest rate. As a result, Foreign reduces innovation investment and produces more, which in turn makes innovation cheaper for Home.

The first key result to underscore is that the time-consistent Markov policies would invoke innovation policies —even in the absence of any domestic externalities and even though they distort domestic R&D efforts. The reason is that without commitment, the government always has an incentive to deviate from the promised trade policy in the following period: it imposes a higher export tax to increase Foreign's prices today, but cannot commit to lowering it tomorrow. Hence, trade policies can only be used to manipulate contemporaneous prices. To influence intertemporal prices, Home subsidizes its own innovation and alters the path of interest rate and foreign wages (innovation costs) by changing its own level of technology. This induces Foreign to innovate less and produce more.

Second, when comparative advantage is introduced, there is a rich set of interweaving intratemporal mechanisms to alter relative prices and innovation. To uncover these different forces, we turn to a multi-sector model in the long run and derive an explicit formula that links a country's optimal tariff in a given sector to the country's comparative advantage: Home would generally want to improve its terms of trade by inducing Foreign to do more research and enhance technology in Foreign's comparative advantage sector (as opposed to Home's). To do so, it imposes a higher tariff in sectors that see larger net exports (relative to the foreign production) and raises that tariff when net exports increase in that sector–for example, due to openness or a rise in global demand. By reducing the demand for foreign goods in those sectors, these tariffs can curb foreigners' research efforts in those sectors.

The heterogenous import tariff schedule looks very different from the uniform optimal tariff under the fixed technology case, as analyzed in Costinot, Donaldson, Vogel, and Werning (2015). They show that a country can exercise monopoly power to tilt relative prices in its favor and improve welfare. A country would opt for a higher export tax in sectors with greater comparative advantage (or a higher subsidy in the comparative disadvantage sectors), combined with a uniform (or zero) import tariff. The reason is that there is more room to manipulate world prices in the comparative advantage sector—achieved through a heterogenous export tax schedule. There is no need to levy differential import tariffs, as a uniform tax is sufficient to bring about a reduction in foreign wages, obviating the need to distort domestic consumption prices further.[2] While this force is also present here, the novel mechanisms in our model give rise to starkly different results: heterogeneous-sector level import tariffs serve the purpose of reducing demand for corresponding foreign goods and hence Foreign's incentives to innovate in these sectors.

Third, our framework provides additional insights when there are more than two countries. Take the example of the US and China competition in a sector such as semiconductors. In the two-country case, our model predicts that the U.S. would want to discourage innovation in China's semiconductor industry as it is the U.S.' comparative advantage. Suppose there is a rise in the global demand for semiconductors, then the U.S. would consequently raise its tariffs on Chinese semiconductors, whereas China would lower its tariff on the U.S. semiconductors. However, the two-country case precludes the realistic scenario in which the U.S. and China are both net exporters of semiconductor products to the rest of the world. In the multi-region scenario, both China and the U.S. would raise their tariffs levied on each other in the semiconductor industry. But both would also impose a higher tariff on the rest of the world's other sector–textiles, for instance— to induce the other competitor to shift innovation efforts into textiles and away from semiconductors.

We explore the long-run optimal policies under a number of scenarios, including a greater demand for certain goods, the developing nation's catching up, a fall in trade costs, etc., in a multi-region long-run model. For illustrative purposes and to contrast with the fixed technology case, we compute optimal policies for a multi-country model with a cross-section of 20 two-digit level manufacturing sectors in the steady state. Our results point to significant heterogeneity across countries/sectors in both tariffs and export taxes.

The main mechanism that optimal trade and industrial policies are employed to influence a foreign nation's innovation efforts is robust to a range of extensions, for example, allowing for varying returns to scale in innovation and intertemporal spillovers. In Sec-

[2] Bagwell and Staiger (1999) emphasize terms-of-trade manipulation and its implication for the WTO. Trade policy in a partial equilibrium setting is explored in Gros (1987), and Broda, Limao, and Weinstein (2008), which show that industry tariff is related to the foreign export supply elasticity. Demidova and Rodríguez-Clare (2009) characterize optimal tariffs in a small economy, single industry, Melitz-Pareto setting. Trade policy analyzed in quantitative or new trade theories include Caliendo and Parro (2021); Costinot and Rodríguez-Clare (2014); Costinot et al. (2015); Demidova (2017); Lashkaripour and Lugovskyy (2016). Costinot, Rodríguez-Clare, and Werning (2020) characterizes optimal firm-level trade policy in a single-sector two-country Melitz model. Ossa (2014) computes optimal tariff across sectors considering traditional, new trade, and political economy motives for protection.

tion 5, we explore a case with decreasing return to scale in innovation. The gap between the private and social return for R&D justifies industrial policies that subsidize innovation when there are externalities. There are also nontrivial interactions between industrial policies and heterogeneous trade policies. For example, if all sectors and countries have the same decreasing returns to scale in innovation, and both Home and Foreign use innovation tax to correct these externalities at the steady state, time-consistent policy will still employ additional innovation policies. Ramsey policy can simply use a constant innovation tax to target the externality and use a path of trade policies. But without commitment, Home resorts to innovation policies to change future marginal innovation costs across sectors in Foreign.

Our Markov results contrast with optimal policy in Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2019) and Lashkaripour and Lugovskyy (2016),[3] whereby industrial policies are used solely to correct domestic wedges/inefficiency. In both papers, the model is static. By contrast, we focus on optimal dynamic policies with individual investment decisions that can influence technology. Under our general setup, optimal policy depends on the dynamic features, in addition to the gap between the private and social choice of R&D generated in the domestic market.

Our contribution is primarily theoretical, as a first attempt to understand the basic contours of optimal policy when there is technological evolution and competition. As such, our technical contribution is to theoretically characterize optimal dynamic policy and derive general results in a framework with elemental features, while providing explicit formulas that give rise to sharp predictions about the structure of optimal policy in special cases. The emphasis of dynamic terms of trade manipulation is closely related to Costinot, Lorenzoni, and Werning (2014), which proposes a theory of capital controls as dynamic terms-of-trade manipulation in an endowment economy. By contrast, in our dynamic economy with endogenous technology, industrial policies serve as intertemporal trade policies. In general, our approach differs from the numerical approaches to computing optimal policy in a particular environment at a moment in time.

[3] Bartelme et al. (2019) characterize optimal policy for a small open economy in a multi-sector Ricardian model with Marshallian externalities. Lashkaripour and Lugovskyy (2016) study optimal industry and trade policy with scale economies.

Our paper sidesteps from issues like international technology diffusion or policy competition. First, there is already a large and expansive literature on the topic of international technology diffusion in the global economy, but few consider optimal policy in these settings.[4] Our paper provides a general setup and solution method that can nest cross-sector and cross-country innovation diffusion. Second, we emphasize a country's incentives on trade and domestic policies and thus assume foreign countries are passive in their policies. With foreign retaliation against Home's actions, the main expression for optimal policy will include an additional term that considers the impact of Home's tariff on foreign export tax revenue and the feedback of foreign income on Home's optimal policies. Furthermore, the policy will internalize the impact of Foreign import tariffs on Home export tax revenue. While the tariff response is more subdued, the main mechanism remains unchanged. Both Home and Foreign experience a welfare loss, but Foreign's welfare loss is smaller than the baseline when Foreign is not allowed to retaliate.

The paper is related to, but has little overlap with, the growth literature emphasizing the importance of R&D on long-run growth. Optimal policies in these contexts depend on assumptions of each theory—featuring either imperfect competition pricing, knowledge spillovers, congestion externalities, or creative destruction. Akcigit, Ates, and Impullitti (2019) explore policies with these features in an open economy with a one-sector model that does not have comparative advantage aspects to trade (and hence heterogeneous tariffs). Liu and Ma (2021) examine optimal R&D policy for a small open economy when there are cross-sector spillovers and externalities without dynamic considerations. In both of these papers, it is the presence of externalities, spillovers, or distortions that justify interventionist policies, whereas optimal dynamic policies in our setting arise from both static and dynamic terms of trade consideration and comparative advantage for an open economy. Furthermore, our model features a dynamic economy with endogenous technology, industrial policies serve as intertemporal trade policies.

Our paper proceeds as follows. Section 2 sets up the multi-country, multi-sector dynamic

[4]Innovation or international technology diffusion in the global economy include works such as Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2018), Atkeson and Burstein (2010), Bloom, Draca, and Van Reenen (2016), Buera and Oberfield (2020), Cai, Li, and Santacreu (2022), Eaton and Kortum (1999), Grossman and Helpman (1990), Grossman and Helpman (1993), Hsieh, Klenow, and Nath (2019), Perla, Tonetti, and Waugh (2021), and Somale (2021).

theoretical framework, while Section 3 zooms in on the dynamic technology manipulation motive in a one-sector model. Section 4 focuses on the intratemporal motive to manipulate technology, and Section 5 concludes with a comparison of various policies in the full-fledged dynamic model with additional rich features.

2 Theoretical Framework

2.1 Model

The model extends the endogenous technology model in Eaton and Kortum (2001), henceforward EK2001, to one that features multiple sectors and countries, and derives optimal R&D and trade policies therein. We then extend the model to allow for various externalities and international spillover.

The world has many countries and sectors. Within a sector, there is a continuum of varieties of consumption goods. All consumers' discount factor is β. Country $n \in N$ has a measure L_n of labor, which can freely flow into the production sector as a worker or the research sector as an innovator.

Consumer preference in each country n is $\sum_{t=0}^{\infty} \beta^t \frac{C_{nt}^{1-\sigma}}{1-\sigma}$, where final goods is a Cobb-Douglas function across the consumption of different sector j goods $C_n = \Pi_{j \in N_s} \left(C_n^j\right)^{\beta_j}$, where β_j is constant and reflects the share of sector j. Within each sector, consumption is also aggregated with a Cobb-Douglas function across individual varieties $C_n^j = \exp \int_0^1 \ln c_n^j(\omega) d\omega$. All goods are tradable with an iceberg trade cost d_{nm} between country n and m.

Innovation incentive and research decision. We start by explaining innovation efforts within each sector, as in the one-sector economy model of Eaton and Kortum (2001). All countries n are capable of producing any variety ω of good with technology $q_n(\omega)$ (where industry j is suppressed for notational convenience), the distribution of which is endogenous and depends on the number of researchers and research productivity.

Researchers draw ideas about how to produce goods. At a Poisson rate α_n, a researcher in country n draws an idea, which consists of the realization of two random variables. One is the good ω to which the idea applies, drawn from the uniform distribution over $[0,1]$. The other is the efficiency $q(\omega)$, drawn from a Pareto distribution with a parameter θ.

Let the measure of researchers in country n at t be L_{nrt}, and the cumulative stock of ideas be T_{nt}. Under a unit interval of varieties, the number of ideas for producing a specific good is Poisson distributed with parameter T_{nt}. Ideas retire with probability δ and hence the evolution of the stock of ideas T_{nt} is:

$$T_{nt} = (1-\delta)T_{n,t-1} + \alpha_{nt}L_{nrt}. \tag{1}$$

Kortum (1997) proves that when the quality of each idea is Pareto distributed, the distribution of technology efficiency frontier is a Frechet distribution with parameter T_{nt} and θ.

Firms engage in Bertrand competition: the lowest-cost producer of each good in each market claims the entire market for that good, charging a markup just enough to keep the second-lowest-cost producer out of the market. In equilibrium, the distribution of the markup is Pareto with the parameter θ. Since all firms selling in the market charge a markup drawn from the same distribution, total profits $prof_{nt}$ at period t earned by firms in the market are a constant share of total sales. Let x_{mt} denote market m's total spending at t (also expenditure per variety in country m given Cobb-Douglas preferences). Thus profits earned by either domestic or foreign firms who sell in that market is $x_{mt}/(1+\theta)$. The probability that a researcher in n draws a q that is the lowest price in market m at t is π_{mnt}/T_{nt}: a firm innovates and surpasses the current set of ideas in its own country with probability $1/T_{nt}$ at time t, but then needs to be the cheapest source of a particular good in country m, with probability π_{mnt}. For example, the expected profit earned by a firm in country 1 (Home country) at time t is:

$$prof_{1t} = \frac{1}{1+\theta}\frac{1}{T_{1t}}\left[\pi_{11t}x_{1t} + \sum_{m\neq 1}\frac{1}{1+\tau_{xmt}}\pi_{m1t}x_{mt}\right] = \frac{1}{\theta}\frac{w_{1t}L_{1pt}}{T_{1t}}, \tag{2}$$

which is obtained using the fact that on expectation a constant fraction of sales goes to profit while the remaining goes to labor income paid to production workers, and shows that export tax of Home would affect Home firms' profit. In addition, the expected profit earned by a firm in country n is

$$prof_{nt} = \frac{1}{1+\theta}\frac{1}{T_{nt}}\left[\frac{1}{1+t^{j}_{nt}}\pi^{j}_{1nt}x_{1t} + \sum_{m\neq 1}\pi^{j}_{mnt}x_{mt}\right] = \frac{1}{\theta}\frac{w_{nt}L_{npt}}{T_{nt}}, \tag{3}$$

which shows that import tariff on foreign country would affect their firms profit and labor in the sector.

We can write the expected discounted value of an idea as

$$v_{nt} = \sum_{s=t}^{\infty}[\beta(1-\delta)]^{s-t}\frac{u_{ns}}{u_{nt}}\frac{P_{nt}}{P_{ns}}prof_{ns}. \tag{4}$$

where u_{ns} is country n's marginal utility of consumption at period s and P_{ns} is its consumer price.

A researcher is motivated by the possibility of coming up with an idea with value. Free mobility across sectors ensures that the present value of the expected profits of being a researcher is equal to the wage of being a worker in the production sector w, i.e., $\alpha_{nt}v_{nt} = w_{nt}$. This determines the level of R&D conducted. Workers engaged in research do not know how good their ideas will be ex-ante. Since each idea is worth v_{nt} in expectations, the total value of research output at time t is $\alpha_{nt}L_{nrt}v_{nt}$. The average value of a researcher is $\alpha_{nt}v_{nt}$. Total number of research workers is $L_{nrt} = r_{nt}L_{nt}$, where r_{nt} is the equilibrium share of research workers—or, research intensity. Thus,

$$\begin{aligned} \alpha_{nt}v_{nt} &= w_{nt} \quad r_{nt} \in [0,1] \\ \alpha_{nt}v_{nt} &< w_{nt} \quad r_{nt} = 0 \\ \alpha_{nt}v_{nt} &> w_{nt} \quad r_{nt} = 1. \end{aligned} \tag{5}$$

The poisson rate α_{nt} reflects how effective the researchers are in country n's innovation process—-or, innovation efficiency. Innovation can exhibit CRS, that is, $\alpha_{nt} = \alpha_n$, or have domestic externality, where $\alpha_{nt} = \alpha_n(L_{nrt})^{\varepsilon-1}(T_{n,t-1})^{\eta}$, including potential DRS ($\varepsilon < 1$), and intertemporal diffusion $\eta \neq 0$; or foreign externality/diffusion, where $\alpha_{nt} = \alpha_n(L_{nrt})^{\varepsilon-1}(T_{m,t-1})^{\eta}$. It can be used to consider policies with input-output innovation structure across sectors, indicated by j and k, where $\alpha^{j}_{nt} = \alpha^{j}_{n}\prod_k(T^{k}_{n,t-1})^{\omega_{jk}}$. To highlight the role of policy, we consider a one-sector model and a multi-sector steady-state model with CRS innovation in Sections 3 and 4 as our baseline. This yields an efficient world private equilibrium, where Home has an incentive to use optimal policies for static and dynamic terms of trade consideration. In the event that there are direct spillovers or externalities, additional classic industry policy motives interact with the highlighted new force.

We now define the world private equilibrium. Variables with prime denote variables in the next period.

Definition 1 (World Private Equilibrium)**.** *The world private equilibrium consists of an allocation of labor and consumption* $\left\{L_{nr}^j, L_{np}^j, C_n^j\right\}$*, technology* $\left\{T_n^j\right\}$*, expenditures* $\{x_n\}$*, prices* $\left\{P_n^j\right\}$*, and wages* $\{w_n\}$ *such that consumers maximize expected discounted utility, firms maximize profits, and the following free entry and market clearing conditions hold:*

1. *Free entry conditions for researchers*

$$w_n = \alpha_n^j\left(L_{nr}^j, T_{n,-1}^j\right)\left(\frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \tilde{\beta}_n v_n^{j\prime}\right), \quad (\forall n \in N, \forall j \in N_s)$$

   where $\tilde{\beta}_n = \beta(1-\delta)\frac{u_n' P_n}{u_n P_n'}$.

2. *Evolution of technology*

$$T_n^j = \alpha_n^j\left(L_{nr}^j, T_{n,-1}^j\right)L_{nr}^j + (1-\delta)T_{n,-1}^j, \quad (\forall n \in N, \forall j \in N_s), \tag{6}$$

3. *Goods market clearing conditions*

$$\frac{1+\theta}{\theta}w_1 L_{1p}^j = \beta_j\left[\pi_{11}^j x_1 + \sum_{m\neq 1}\frac{1}{1+\tau_{xm}^j}\pi_{m1}^j x_m\right], \tag{7}$$

$$\frac{1+\theta}{\theta}w_n L_{np}^j = \beta_j\left[\frac{1}{1+t_n^j}\pi_{1n}^j x_1 + \sum_{m\neq 1}\pi_{mn}^j x_m\right], \tag{8}$$

where the expenditures are given by[5]

$$x_1 = \frac{1+\theta}{\theta} w_1 \sum_j L_{1p}^j + \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \pi_{m1}^j x_m + \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t_m^j}{1+t_m^j} \pi_{1m}^j x_1 \tag{9}$$

$$x_m = \frac{1+\theta}{\theta} w_m \sum_j L_{mp}^j. \tag{10}$$

4. *The labor market clearing conditions for each country n*

$$\sum_j \left(L_{nr}^j + L_{np}^j \right) = L_n. \tag{11}$$

Proposition 1. *At the steady state of the multiple-sector open economy, the private research intensity r_n^j is the same as in the closed economy for all sectors j in country n. Openness reallocates more labor into the comparative-advantage sectors and increases the endogenous level of technology in these sectors.*

Proof. See Appendix A. The optimal research intensity r_n^j in each sector j does not depend on the country's size, research productivity, or trade openness. The reason is that accessing foreign markets increases the potential profits, but competition from foreign inventions decreases them. These two effects exactly cancel out, and the level of openness does not affect research intensity. However, the research level would depend on size, research productivity, and openness. Thus, given the same level of research intensity r_n^j, more labor reallocated to the comparative advantage sector increases the total number of researchers in that sector and hence its technology T_n^j.

2.2 Optimal trade and innovation policies

We consider two types of government policies: Markov and Ramsey policies. Under Markov, time-consistent optimal policies, the Home government chooses current-period policies, which are constrained to depend only on the value of the current period's state.

[5] The trade shares satisfy $\pi_{11}^j = \frac{T_1^j (w_1)^{-\theta}}{T_1^j (w_1)^{-\theta} + \sum_{n\neq 1} T_n^j (w_n(1+t_n^j) d_{1n})^{-\theta}}$, $\pi_{m1}^j = \frac{T_1^j (w_1(1+\tau_{xm}^j) d_{m1})^{-\theta}}{T_1^j (w_1(1+\tau_{xm}^j) d_{m1})^{-\theta} + \sum_{n\neq 1} T_n^j (w_n d_{mn})^{-\theta}}$, $\pi_{mn}^j = \frac{T_n^j (w_n d_{mn})^{-\theta}}{T_1^j (w_1(1+\tau_{xm}^j) d_{m1})^{-\theta} + \sum_{i\neq 1} T_i^j (w_i d_{mi})^{-\theta}}$, $\pi_{1m}^j = \frac{T_m^j (w_m(1+t_m^j) d_{1m})^{-\theta}}{T_1^j (w_1)^{-\theta} + \sum_{n\neq 1} T_n^j (w_n(1+t_n^j) d_{1n})^{-\theta}}$. Note that $\sum_n \pi_{m,n}^j = 1$ for any m.

Private individuals react by taking current and future policies as given. The government cannot commit, but correctly anticipates how future policies will depend on current ones through the state of the economy. For comparison, we also consider the Ramsey problem, whereby the government has the ability to commit to all its future policies at the beginning of time, and chooses a sequence of taxes to maximize utility— taking into consideration the private responses to the policies.

Markov policy The Home government (country 1) chooses optimal unilateral trade policies and domestic $R\&D$ policies by maximizing the aggregate of individuals' instantaneous utilities discounted by β. Foreigners are taken to be passive. Trade policy instruments are restricted to the country-sector level, comprising country-sector-specific import tariffs t_n^j and export taxes τ_{xn}^j directed at country $n \neq 1$. Domestic $R\&D$ policies are sector-specific innovation profit tax/subsidies. We derive optimal domestic innovation summarized by L_{1r}^j and then show how to implement it with taxes on innovation profit. The government rebates the tax income to households in a lump-sum fashion.

The Home government determines researchers in each sector j, taking into account foreign private innovation decisions and equilibrium production and trade. There are number $N \times N_s$ state variables, i.e., the technologies $\left\{T_{n,-1}^j\right\}$ for all country n and sector j. Specifically, the government chooses L_{1r}^j with $j \in \{1, 2, .., N_s\}$, country-sector-specific import tariff t_n^j and export taxes τ_{xn}^j toward country $n > 1$ to solve the following problem:

$$V\left(\left\{T_{n,-1}^j\right\}\right) = \max_{\left\{L_{1r}^j, \tau_{xn}^j, t_n^j\right\}} \frac{x_1^{1-\sigma}}{1-\sigma} + \beta\left[V\left(\left\{T_n^j\right\}\right)\right]$$

subject to the world private equilibrium characterized by

$$\frac{w_n}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)} = \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \tilde{\beta}_n v_n^{j\prime}, \qquad (\gamma_{rn}^j, \quad N_s \times (N-1)) \tag{12}$$

$$\frac{1+\theta}{\theta} w_1 L_{1p}^j = \beta_j\left[\pi_{11}^j x_1 + \sum_{m\neq 1}\frac{1}{1+\tau_{xm}^j}\pi_{m1}^j x_m\right], \quad (\gamma_{L1}^j, \quad N_s) \tag{13}$$

$$\frac{1+\theta}{\theta} w_n L_{np}^j = \beta_j\left[\frac{1}{1+t_n^j}\pi_{1n}^j x_1 + \sum_{m\neq 1}\pi_{mn}^j x_m\right], \quad (\gamma_{Ln}^j \quad N_s \times (N-1)) \tag{14}$$

and the evolution of technology (6), the definition of expenditures (9)-(10), the labor market clearing conditions (11), the normalization of home price $P_1 = 1$.[6]

We consider unilateral policies, without loss of generality, we assume that countries other than 1 (Home) do not invoke trade policies on one another, as reflected in equation (14), nor innovation policies as in equation (12). Optimal conditions for the Markov policy are derived in Appendix B.

Ramsey policy Under Ramsey optimal policies, the Home government can commit to the path of policies. In period 0, Home government chooses a sequence of $\left\{L^j_{1rt}, \tau^j_{xnt}, t^j_{nt}\right\}_{t=0}^{\infty}$ to maximize the present value of utility subject to the world private equilibrium, in particular the worker-researcher choices, in each period t:

$$\frac{w_{nt}}{\alpha^j_n}\frac{u_{nt}}{P_{nt}} = \sum_{s=t}^{\infty}[\beta(1-\delta)]^{s-t}\frac{u_{ns}}{P_{ns}}\left(\frac{1}{\theta}\frac{w_{ns}L^j_{nps}}{T^j_{ns}}\right), \tag{15}$$

along with equations (6) - (11). The government decides on the entire path of policies honored in the future, particularly as future policies affects the foreign individual's expected value of innovation. Let $\beta^t\gamma_{vnt}$ be the multiplier on the worker-research constraint. Following Marcet and Marimon (2019), the state variable now includes the cumulative promise of past multipliers $\Gamma^j_{vn,t-1}$ with

$$\Gamma^j_{vnt} = \sum_{s=0}^{t}(1-\delta)^{t-s}\gamma^j_{vns} = (1-\delta)\Gamma^j_{vn,t-1} + \gamma^j_{vnt}, \qquad \text{for any } n \neq 1 \text{ and } j.$$

Hence, the problem can be written recursively as

$$L\left(\left\{\Gamma^j_{vn,-1}, T^j_{n,-1}\right\}\right) = \inf_{\gamma^j_{vn}}\sup \frac{x_1^{1-\sigma}}{1-\sigma} + \sum_{n\neq 1}\sum_{j}^{N_s}\left[\gamma^j_{vn}\left(\frac{1}{\theta}\frac{L^j_{np}}{T^j_n} - \frac{1}{\alpha^j_n}\right) + (1-\delta)\Gamma^j_{vn,-1}\frac{1}{\theta}\frac{L^j_{np}}{T^j_n}\right] w_n x_n^{-\sigma} P_n^{\sigma-1}$$
$$+ ....\beta L\left(\left\{\Gamma^j_{vn}, T^j_n\right\}\right), \tag{16}$$

[6] Home consumer price is given by $P_1 = \Pi_j\left[T^j_1 w_1^{-\theta} + \sum_{n\neq 1} T^j_n(w_n(1+t^j_n)d_{1n})^{-\theta}\right]^{-\beta_j/\theta}$. The consumer price of country n is $P_n = \Pi_j\left[T^j_1(w_1(1+\tau^j_{xn})d_{n1})^{-\theta} + \sum_{m\neq 1} T^j_m(w_m d_{nm})^{-\theta}\right]^{-\beta_j/\theta}$.

where we omit other constraints (6)-(11), which are the same as in the Markov problem. See Appendix C for the full recursive problem (16), the proof of the mapping between the original Ramsey problem and this recursive problem, and the derivations of the optimal conditions for the Ramsey policy.

3 Dynamic Technology Manipulation

We first study an intertemporal motive to influence foreign innovation in one's favor. To isolate this force, we first consider a two-country, one sector case without domestic externalities. The key takeaway is that the Markov government, unable to commit to its future trade policies, has to deploy domestic innovation policies to manipulate foreign innovation efforts. This contrasts with Ramsey policies.

3.1 Markov policies

As Lerner symmetry holds in the one-sector, two-country model, Home's import tariff can be normalized to zero. The government optimises over export taxes while directly choosing domestic innovation and allocations, respecting the world private equilibrium. Recall γ_{r2} is the multiplier on country 2's worker-researcher choice condition (12). The following proposition characterizes the optimal Markov policies.

Proposition 2. *Under the one-sector, two-country model, the optimal Markov policy is characterized by zero tariffs (Lerner symmetry), export tax, and domestic innovation satisfying the following:*

$$1+\tau_x^M = \frac{1+\theta\pi_{22}}{\theta\pi_{22}+\gamma_{r2}(\sigma-1)\beta(1-\delta)\frac{u'_{c_2}/P'_2}{u_{c_2}/P_2}\frac{w'_2}{\alpha_2}\frac{1}{u_{c_1}x_2}}, \tag{17}$$

$$\frac{w_1}{\alpha_1} = \frac{1}{\theta}\frac{w_1 L_{1p}}{T_1} + \tilde{\beta}_1 v_1 + \underbrace{\frac{\theta}{1+\theta}\frac{\beta(1-\delta)}{u_{c_1}}\frac{1}{u_{c_2}/P_2}\gamma_{r2}\frac{\partial\left(u'_{c_2}\frac{w'_2}{P'_2}\right)}{\partial T_1}}_{\text{innovation wedge}}, \tag{18}$$

Furthermore, $\gamma_{r2} = 0$ at the steady state.

Proof. See Appendix D.

Export tax Compare the export tax (17) with the one under exogenous technology: $\tau_x = 1/(\theta\pi_{22})$, which captures the static terms of trade consideration. The extra term in the denominator $\gamma_{r2}(\sigma-1)\beta(1-\delta)\frac{u'_{c_2}/P'_2}{u_{c_2}/P_2}\frac{w'_2}{\alpha_2}\frac{1}{u_{c_1}x_2}$ captures a dynamic technology consideration. Although the home government cannot use the current export tax to directly affect foreign's future marginal consumption u'_{c2}, future prices w'_2 and P'_2, the current export tax directly enters into current period foreign price index P_2. Thus, it influences foreign researchers' choices through equilibrium prices and the tightness of foreign worker-researcher constraint (12).

Specifically, through its first-order impact on P_2, home export tax influences foreign research incentives through two countervailing forces. First, higher export tax can lower foreign research incentives by increasing the foreign interest rate: all else equal, a higher foreign consumer price P_2 reduces foreign consumption x_2/P_2 and raises its interest rate $\frac{u_{c_2}}{\beta u'_{c_2}}$. This lowers foreign innovation incentives. Moreover, if Foreign shifts towards production away from innovation and produces more goods, it also lowers Home's price index and makes Home's cost of innovation cheaper. Second, higher export tax, which raises P_2, can boost foreign research incentives by lowering foreign real wages, w_2/P_2.

The strength of these two countervailing forces depends on the intertemporal elasticity of substitution. When $\sigma = 0$, the foreign interest rate $\frac{u_{c_2}}{\beta u'_{c_2}}$ is constant at $1/\beta$, and the incentive to manipulate foreign interest rate disappears. But the government still wants to affect the foreign real wage path through export taxes. Note that in this linear consumption case, innovation cost consideration is at work rather than a consumption smoothing motive. When $\sigma = 1$, the interest rate and wage effect cancel out, and τ_x is the same as in the exogenous technology case. When $\sigma > 1$, the government would want to raise the export tax to be above that of the exogenous technology case to depress foreign innovation incentives, while it would want to reduce it below that level if $\sigma < 1$.

Innovation policy The inability of the Markov government to directly influence foreign future wages and prices gives room for domestic innovation policies. In the 'Euler' equation of innovation choices (18), there is an extra wedge term compared to the private equilibrium. The key elements in this wedge term are the multiplier γ_{r2} on foreign worker-researcher constraint (12) and the derivative, denoted as $der \equiv \partial \left(u'_{c_2} \frac{w'_2}{P'_2} \right) / \partial T_1$, for the purpose of discussion.

To unpack these terms, first note that the wedge essentially captures the impact of Home's next period technology state T_1 on Foreign's discounted innovation value, i.e., $\tilde{\beta} v'_2$. To see this, we rewrite this discounted value as

$$\tilde{\beta} v'_2 = \beta(1-\delta) \frac{u'_{c_2}/P'_2}{u_{c_2}/P_2} v'_2 = \beta(1-\delta) \frac{u'_{c_2}/P'_2}{u_{c_2}/P_2} \frac{w'_2}{\alpha_2}, \tag{19}$$

where the second equality comes from researchers' free-entry condition (5). Hence, the derivative *der* captures how the future terms in country 2's discounted future value vary with home T_1.

Although Foreign's current consumption c_2 and price P_2 are jointly determined along with Home's current-period innovation choice and technology T_1, the Home government can use its T_1 to affect foreign's future marginal consumption and prices. This effect is captured by the derivative *der* in the wedge. The changes in future marginal consumption and prices lead to variations in foreign interest rates and real wages, influencing foreign innovation incentives. This affects the tightness of the foreign worker-researcher constraint (12)—the reason why the multiplier of this constraint γ_{r2} shows up in the wedge.

When the interest rate effect dominates, the derivative *der* is negative, meaning that Foreign's marginal utility from increasing real wage falls with T_1. A higher level of technology at Home lowers its future export price and P'_2, raises Foreign consumption c'_2 and lowers its future marginal utility of consumption–hence increasing the interest rate. The real wage, however, would rise with T_1 as a result of the decline in P'_2. When the marginal consumption– or the interest rate effect dominates— $der \leq 0$ in the wedge term.

The sign of the multiplier γ_{r2} depends on whether the home government aims to reduce or raise foreign innovation. When the home government wants to reduce foreign innovation, the multiplier γ_{r2} is negative. From the perspective of Home, Foreign's innovation benefit is too large relative to its innovation cost—it conducts too much innovation. For instance, when home T_1 is low relative to innovation efficiency α_1, it would want to develop its own technology and curb foreign innovation. In this case, γ_{r2} tends to be negative, and along with a negative *der* term, produces a positive wedge.

The higher the wedge, the larger the Home innovation incentive (the right-hand side of the equation (18) reflects the benefit as a researcher). It behaves like a subsidy to Home's innovation efforts. We can define the innovation subsidy τ_d relative to Home firms' current profit as

$$\tau_d = \frac{1}{\frac{1}{\theta}\frac{w_1 L_{1p}}{T_1}} \frac{\theta}{1+\theta} \frac{\beta(1-\delta)}{u_{c_1}} \frac{1}{u_{c_2}/P_2} \gamma_{r2} \frac{\partial(u'_{c_2}\frac{w'_2}{P'_2})}{\partial T_1}.$$

Hence, when home T_1 is low and transits to a higher steady state, Home government subsidizes industry innovation.

Note that $\gamma_{r2} = 0$ at the steady state. In this case, the optimal export tax is the same as in the exogenous case, and there are no innovation subsidies. The intuition is straightforward: when the technology reaches the steady state, the government can no longer use policies to affect the interest rate and wage path. In addition, there are no domestic externality or comparative advantage to create policy incentives.

Numerical example To understand the Markov innovation subsidy and export tax, we consider an example where the Home government's technology T starts below the steady state, while country 2's technology is at its steady state. Figure 1 shows optimal policies employed relative to their steady-state levels during a transition path.[7]

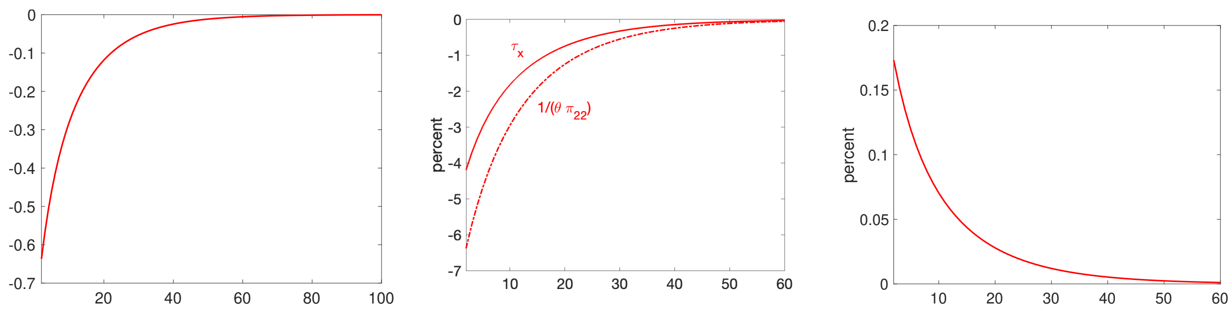
The Markov government utilizes both export taxes and innovation subsidies, as depicted in Panel (b) and (c). The export tax has two salient features. First, it increases when T_1 increases over time. Second, it consistently exceeds the level in the static model of $1/(\theta\pi_{22})$. Moreover, the gap between export tax and $1/(\theta\pi_{22})$ shrinks and eventually goes to zero when T_1 increases over time.

These two features reinforce our previous discussions in Proposition 2. When T_1 increases over time, the home country gains monopoly power and is able to impose a higher export tax. This motive is clear from the rising $1/(\theta\pi_{22})$, capturing the static terms of trade effect. A higher export tax than $1/(\theta\pi_{22})$ reflects the home government's motives to lower foreign innovation efforts by raising foreign interest rates.

[7] In this example, $\theta = 2, \sigma = 2.5, \beta = 0.9, \delta = 0.02, \alpha_1 = 1, \alpha_2 = 0.9, d = 1.1$.

In addition, Home increases subsidies on domestic research efforts to further deter Foreign's. As technology progresses towards its long run value, the need to manipulate foreign innovation incentives diminishes and eventually disappears. As a result, the export tax equals $1/(\theta\pi_{22})$ and innovation subsidies become zero, in the long run.

Figure 1: Optimal Markov Polices: One Sector, Two Countries



(a) T_1 (b) Export tax (c) Innovation subsidy

3.2 Ramsey policies

The Ramsey government can commit to a sequence of export taxes and does not need to undertake any innovation policies. Recall that γ_{v2} is the multiplier on the worker-research constraint, and Γ_{v2} the multiplier recording the past commitment of policies with $\Gamma_{v2} = (1-\delta)\Gamma_{v2,-1} + \gamma_{v2}$. The following proposition characterizes the optimal Ramsey policies.

Proposition 3. *Under the one-sector, two-country model, the optimal Ramsey policy is characterized by zero tariff, and export tax and domestic innovation satisfying the followings:*

$$1+\tau_x^R = \frac{1+\theta\pi_{22}}{\theta\pi_{22} + (\sigma-1)\left(\gamma_{v2}\frac{w_2}{\alpha_2} - \Gamma_{v2}\frac{w_2 L_{2p}}{\theta T_2}\right)\frac{u_{c_2}}{P_2}/(u_{c_1}x_2)}, \tag{20}$$

$$\frac{w_1}{\alpha_1} = \frac{1}{\theta}\frac{w_1 L_{1p}}{T_1} + \tilde{\beta}_1 v_1. \tag{21}$$

Furthermore, $\gamma_{v2} = \Gamma_{v2} = 0$ *at the steady state.*

Proof. See Appendix D.

It is easy to see that the Ramsey innovation condition (21) is the same as that in the private equilibrium (12). The reason is that with commitment, the path of export taxes is sufficient to implement the intertemporal allocation. The formula for the optimal export tax in (20) looks similar to equation (17) in the Markov case. To compare, we plug in the worker-researcher constraint (12) to equation (17), and the Markov export tax becomes

$$1+\tau_x^M = \frac{1+\theta\pi_{22}}{\theta\pi_{22}+(\sigma-1)\left(\gamma_{r2}\frac{w_2}{\alpha_2}-\gamma_{r2}\frac{w_2}{\theta}\frac{L_{2p}}{T_2}\right)/(u_{c_1}x_2)}. \tag{22}$$

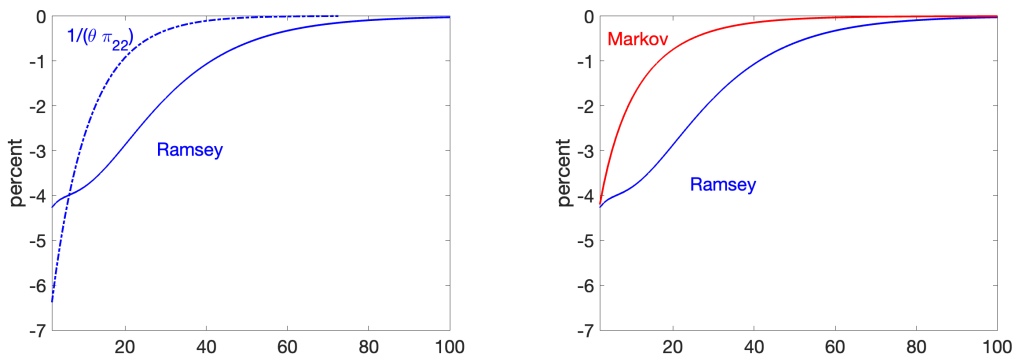
As is clear, the multiplier $\gamma_{v2}u_{c2}/P_2$ in Ramsey corresponds to the multiplier γ_{r2} in the Markov case.

However, taxes under Markov and Ramsey contain an important difference. Markov export taxes (22) depend only on the current multiplier γ_{r2} associated with the foreign innovation incentive, while Ramsey export taxes τ_x^R depend not only on the current multiplier γ_{v2}, but also on cumulative past commitments $\Gamma_{v2,-1}$. Past commitments $\Gamma_{v2,-1}$ dampen the impact of the current multiplier γ_{v2} on the export tax, showing up as the negative term in the denominator of equation (20).

To see why all past multipliers Γ_{v2} enter the Ramsey tax equation (20), consider the fact that the Ramsey government, with similar incentives as a Markov one, also uses export taxes to manipulate terms of trade and foreign innovation incentives. But the difference is that the Ramsey government can commit to a lifetime policy path at time 0, with full knowledge that the period t's export tax τ_{xt} directly affects foreign prices P_{2t}, which enters all past worker-researcher choice constraints (15) for periods $s \le t$. When the interest rate effect dominates, higher P_{2t} lowers period t's innovation incentive but increases period $s < t$'s research incentives. This also explains why the past multipliers counteract the impact of the current multiplier γ_{v2}.

Numerical example Figure 2 plots Ramsey policies against static and Markov policies in the same example as in Figure 1. Panel (a) exhibits an export tax that is higher under Ramsey for low levels of T_1 than under the static case $(1/(\theta\pi_{22}))$, and lower in later periods. Panel (b) shows that the path of export taxes tends to be flatter under Ramsey than under Markov. This discrepancy arises from the ability of a Ramsey government to manipulate not only the current foreign price P_2 through the current τ_x, but also future prices P_2' through commitments to future τ_x'. Consequently, the government is able to avoid excessive increases in the current τ_x, which could lead to undesirable distortions. In contrast, the Markov government cannot commit to a sequence of export taxes to influence the terms of trade, and it is obliged to implement distortionary domestic industry policies such as innovation policies.

Figure 2: Optimal Ramsey Polices: One Sector, Two Countries



(a) Export tax   (b) Export tax

In the long run, neither Markov nor Ramsey policies have any additional incentive to alter the terms of trade. Both end up with the same steady state of technology and export taxes. Note that the steady-state innovation and trade policy becomes the same as in a static model only in the special case with one sector and no externalities. We proceed to discuss this in detail in the following section and explore why multiple-sector and multiple-country model illuminate additional insights.

4 Comparative Advantage: Multi-Sector, Multi-Country Model

When comparative advantage is introduced, intratemporal considerations to manipulate foreign technology become more intricate. To make these forces transparent, we derive optimal taxes in a long-run model with multiple sectors, countries, before incorporating potential externalities and considering transition paths. In contrast to a uniform tariff applied across all sectors under exogenous technology, conventionally emphasized, optimal policies with endogenous technology consist of heterogenous tariffs and export taxes across sectors.

Our starting point is a baseline model characterized by an efficient private equilibrium with no distortions, externalities, or international spillovers. Specifically, innovation has constant returns to scale (CRS), $\alpha^j_{nt} = \alpha^j_n$, and the allocation between researchers and workers are efficient. Under Bertrand competition, the endogenous markup of each firm follows a distribution that is invariant over time and does not depend on the destination to which the firm sells. Hence, there are identical and constant aggregate markups across all sectors.

Proposition 4 (Exogenous Technology). *When technology is exogenous,*

1. *optimal trade policies consist of a country-specific import tariff and country-sector-specific export taxes that rise with a sector's trade share in country n, π^j_{n1}. Specifically,*

$$t^j_n = \bar{t}_n, \qquad 1 + \tau^j_{xn} = \frac{1 + \theta(1 - \pi^j_{n1})}{\theta \sum_{m \neq 1}(1 + \bar{t}_m)\pi^j_{nm}}; \tag{23}$$

2. *with lower trade cost, Home charges a higher export tax in sectors with higher net exports (comparative advantage sectors). That is, Home allows for greater differentiation of taxes across sectors.*

When T is exogenous, optimal tariffs are uniform across sectors, and their overall level, denoted by $\bar{t}$, is not uniquely pinned down (tax neutrality holds as in Lerner (1936) and Costinot and Werning (2019)). Thus, we can set zero tariffs in all sectors in one country. Export taxes are used to exploit the country's monopoly power, and they increase with π^j_{n1}, the share of good j that country n imports from Home. In other words, the export tax for a specific country-sector increases as the market power of that sector in the destination country increases. This schedule of trade policies is consistent with the one proposed by Costinot et al. (2015), where the government can manipulate relative prices in its favor by limiting the supply of its export goods.

Proposition 5 (Two-Country Endogenous Technology). *In the two-country steady state case with endogenous technology and no domestic externalities,*

1. *optimal policies do not distort domestic innovation (zero innovation taxes), but consist of heterogenous import tariffs and export taxes across countries and sectors;*

2. *Home optimal tariffs satisfy*

$$t^j - t^{j'} = \frac{1}{\theta}\left(\frac{\frac{1+\theta}{\theta}w_1 L^j_{1p} - \beta_j x_1}{\frac{1+\theta}{\theta}w_2 L^j_{2p}} - \frac{\frac{1+\theta}{\theta}w_1 L^{j'}_{1p} - \beta_{j'} x_1}{\frac{1+\theta}{\theta}w_2 L^{j'}_{2p}}\right); \tag{24}$$

   Home's import tariff imposed on good j increases with its net exports normalized by production in country 2, relative to sector j' (tax neutrality holds so that one of the sector's tariffs can be normalized to zero).

3. *Home optimal export taxes satisfy* $(1+\tau^j_x)(1+t^j) = \frac{1+\theta\pi^j_{22}}{\theta\pi^j_{22}}$;
4. *Openness or technological change affects optimal policies: tariffs rise by more in sectors that see a greater increase in net exports.*

PROOF in Appendix E. Proposition 5 shows that tariffs are now sector specific rather than uniform in the exogenous technology case. Home's tariff in sector j is $(\frac{1+\theta}{\theta}w_1 L^j_{1p} - \beta_j x_1)$ – in other words, the excess supply of labor in sector j, which relates to its net exports of good j, nomalized by foreign sector size. This reflects the motive to reduce foreign innovation efforts in sectors where Home has a comparative advantage. If Home exports more of the sector's goods, it would want to charge a higher tariff; if Home imports relatively more of a good from country n, it would want to charge a relatively lower tariff, all else equal.

The optimal tariff in equation (24) satisfies

$$t^j_n = -\frac{\gamma^j_{Ln}}{u_c} = -\frac{\gamma^j_{rn}}{(1+\theta)T^j_n u_c} + Const_n, \tag{25}$$

where γ^j_{Ln} is the multiplier on the goods market clearing condition (14) and γ^j_{rn} is the multiplier on the free-entry condition of researchers (12). The multiplier γ^j_{rn} reflects the impact of a foreign country's excess incentive of innovation on Home welfare, and γ^j_{Ln} reflects the impact of excess demand of a foreign good on Home welfare. This equation demonstrates that Home would like to use tariffs to affect foreign's labor and production across sectors, hence profit and incentive for innovation.

Appendix E.1 proves the optimal innovation policy at the steady state of the baseline model. The Home government chooses to not distort its own R&D, but would like to use heterogenous tariffs to influence Foreign's innovation, as shown in equation (25). The sign of γ_{rn}^{j} determines whether Home will impose a tariff on country n in sector j. As for the level of optimal tariffs, multiple considerations are taken into account: the impact of its tariffs on Foreign and Home's own labor and production, Foreign profits that affect its technology, Home's own tariff revenue, consumption price, and terms of trade through changes in wages. Despite the complexity, sectoral optimal tariff can be summarized by Home's net export normalized by foreign sector output.

Home's optimal export taxes show that the government would still want to use export taxes to exploit the country's monopoly power at the steady state, as in the case with exogenous technology—but in conjunction with heterogenous import tariffs across sectors aimed at influencing Foreign's innovation.

Proposition 6 (**Multi-Country Endogenous Technology**). *In the multi-country steady-state case with endogenous technology and no domestic externalities,*

1. *optimal policies do not distort domestic innovation (zero innovation taxes), but consist of heterogenous import tariffs and export taxes across countries and sectors;*
2. *On average, tariffs are higher for sectors with relatively higher net exports, for any sector j:*

$$\sum_{n\neq 1}(t_n^j+\xi_n)\frac{1+\theta}{\theta}w_n L_{np}^j=\frac{1+\theta}{\theta}w_1 L_{1p}^j-\beta_j x_1, \tag{26}$$

where ξ_n are country-specific.[8] *In the case where only Home and one foreign country (country 2) produce sector j and j′ goods, e.g., rest of the world have $\alpha_{ROW}^{j}=0$ and $\alpha_{ROW}^{j'}=0$,*

$$t_2^j-t_2^{j'}=\frac{1}{\theta}\left(\frac{\frac{1+\theta}{\theta}w_1L_{1p}^j-\beta_j x_1}{\frac{1+\theta}{\theta}w_2L_{2p}^j}-\frac{\frac{1+\theta}{\theta}w_1L_{1p}^{j'}-\beta_{j'} x_1}{\frac{1+\theta}{\theta}w_2L_{2p}^{j'}}\right);$$

[8] $\xi_n=\sum_{i\neq 1}^{N}\sum_k^{N_s}t_i^k\beta_k\pi_{n,i}^k-\sum_k^{N_s}\beta_k\frac{\tau_{xn}^k}{1+\tau_{xn}^k}\pi_{n1}^k$.

3. *Within a country*

$$\frac{t_n^j}{1+t_n^j}\frac{\beta^j \pi_{1n}^j x_1}{\frac{1+\theta}{\theta} w_n L_{np}^j} = \frac{\sum_{m\neq 1} \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \beta^j \pi_{mn}^j x_m}{\frac{1+\theta}{\theta} w_n L_{np}^j} + \delta_n, \tag{27}$$

where δ_n are country specific.[9]

4. *Home optimal export taxes satisfy*

$$1+\tau_{xn}^j = \frac{1+\theta(1-\pi_{n1}^j)}{\theta \sum_{m\neq 1}(1+t_m^j)\pi_{nm}^j}; \tag{28}$$

5. *Openness or technological change affects optimal policies: tariffs rise by more in sectors that see a greater increase in net exports.*

PROOF: see Appendix F.

Different from the two-country case, tariffs on one foreign country now affect *other* foreign countries' sectoral profits and incentives to do innovation—and hence technology. The levels of optimal tariffs and export taxes are jointly determined across countries and sectors.

With multiple countries, Home's tariffs are on average higher for sectors with relatively higher net exports ($\frac{1+\theta}{\theta} w_1 L_{1p}^j - \beta_j x_1$): equation (26) shows that for that sector, the weighted average of Home's tariff across countries is higher. For a specific country/sector, the optimal tariff on country n is increasing in the ratio of country n's exports to other countries relative to its exports to Home, as in equation (27). That is, if Home imports relatively more from country n, it would want to charge a relatively lower tariff, all else equal. This tariff also depends on other countries' export taxes in sector j and other general equilibrium effects, such as the impact on other countries' labor reallocation across sectors, as captured in equation (27). The optimal tariffs still satisfy

$$t_n^j = -\frac{\gamma_{Ln}^j}{u_c} = -\frac{\gamma_{rn}^j}{(1+\theta)T_n^j u_{c_1}} + Const_n,$$

which demonstrates that Home would like to use tariffs to affect foreign's labor and pro-

[9] $\delta_n = \frac{1}{1+\theta}\{\theta \sum_j^{N_s} t_n^j \frac{\frac{1+\theta}{\theta} w_n L_{np}^j}{x_n} + \frac{\mu_n L_{np}}{u_c x_n} - 1\}$

duction across sectors and the profit and incentive for innovation. The differences from the two-country case will be further illustrated in numerical examples below.

Moreover, the same motive of not distorting one's own R&D also holds in the multi-country steady state case. The Home government would want to use export taxes to exploit the country's monopoly power, in conjunction with heterogenous import tariffs across sectors to influence Foreign's innovation. Moreover, in the multi-country case, these taxes depend on how much other countries export to country n (π^j_{nm}), reflecting the degree of monopoly power Home has on country n in sector j.

From the expressions above, the tariff Home imposes on sector j is related to j's excess supply of labor, which is by definition ($\frac{1+\theta}{\theta} w_1 L^j_{1p} - \beta_j x_1$), hence net exports in the two-country case, and analogously in the multi-country case. An important difference arises between the two-country and multi-country cases. With two countries, one country's net exports in a sector are necessarily the other country's net imports. With multiple countries, there could exist sectors in which both Home and Foreign are net exporters to the rest of the world. This difference has important implications for optimal policies. Unlike in the two-country case whereby only one country has the incentive to levy a tariff in a given sector j, it is possible in the multi-country case that Home and Foreign are net exporters of certain goods and *both* countries' unilateral polices would want to impose high tariffs in those sectors. Whether the tariff they impose in that sector towards another country is higher or lower than tariffs in other sectors depends on that sector's net exports (normalized by another country's production, reflecting the fact that tariffs are related to the elasticity, not an absolute level)—as is shown in the above proposition. Note that net exports of a good j are total net exports to the world, rather than to one economy alone, gauging a country's overall comparative advantage. The above result also shows that an event that raises both Home and Foreign's net exports of good j (for instance, a fall in trade costs or a rise in global demand in sector j) will also raise the tariffs they levy on each other in that sector.

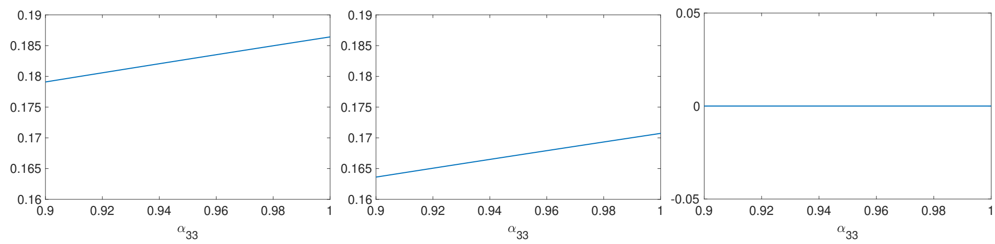
Proposition 7. *With endogenous technology accumulation and no domestic externalities, in the steady state, Markov and Ramsey governments choose the same trade policies. Moreover, neither type of government uses domestic innovation policies.*

PROOF in Appendix G.

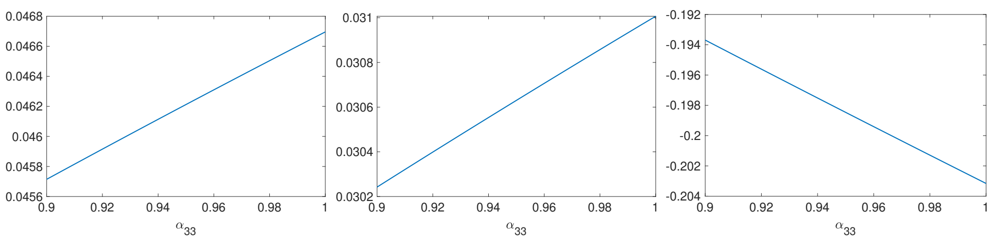
Numerical Example The interlinking relationships and spillover of tariffs across countries and sectors may appear to be complex but follow the basic principle outlined in the above proposition. That is— the within-country tariff ranking has to do with the country's comparative advantage. That strict ranking disappears when we deviate from the special case, but still follows some basic trade patterns—further explored in a simple quantitative analysis in Appendix K. Still, to gain some intuition, and to investigate predictions of theory when cast in realistic scenarios, we consider an example where a third country becomes larger. Appendix H also considers a few other cases.

There are three sectors (S1, S2, and S3) and three economies: the United States (US), China (CN), and the rest of the world (ROW). The innovation efficiencies in the three sectors accord with US $(1, 0.9, 0.9)$, CN $(0.9, 0.9, 1)$, ROW $(0, 0, \alpha_{33})$ with α_{33} varying from 0.9 to 1. Since ROW's innovation efficiency is zero and cannot produce goods from S1 and S2, it imports them from the United States and China. Furthermore, S2 is China's comparative advantage (CA) sector, and S1 is U.S.'s comparative advantage sector.

Figure 3: Optimal Policies for the United States when Global Demand Rises



(a) Tariff on CN: sector 1 (b) Tariff on CN: sector 2 (c) Tariff on CN: sector 3

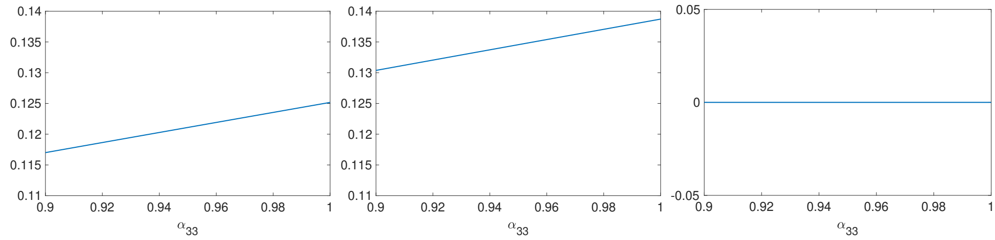


(d) US net export: sector 1 (e) US net export: sector 2 (f) US net export: sector 3

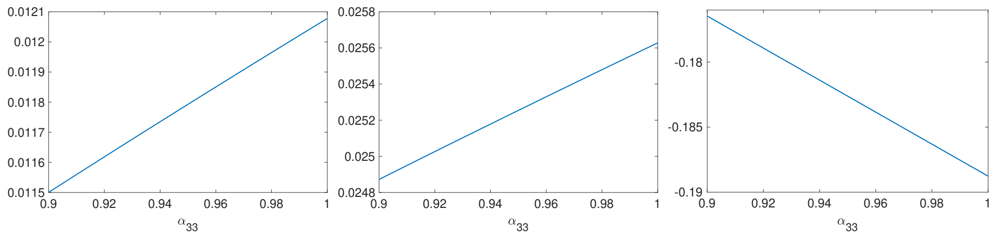
Note: The upper panel plots U.S. optimal tariff on imports from China in sector 1, sector 2, and sector 3 when sector 3 in ROW becomes more efficient, i.e. higher α_{33}. The lower panel plots the corresponding US's net export in sector 1, 2, and 3.

To start with, we increase α_{33} in the first example to make ROW larger. This led to a surge in the global demand for goods in S1 and S2. Note that the U.S. always imposes

Figure 4: Optimal Policies for China when Global Demand Rises



(a) Tariff on US: sector 1 (b) Tariff on US: sector 2 (c) Tariff on US: sector 3



(d) CN net export: sector 1 (e) CN net export: sector 2 (f) CN net export: sector 3

Note: The upper panel plots China's optimal tariff on imports from the U.S. in sectors S1, S2, and S3 when S3 in ROW becomes more efficient, i.e. higher α_{33}. The lower panel plots China's net export in the three sectors.

tariffs on S1, which is its CA sector. Larger global demand for S1 and S2 boosts exports in these sectors, and thus the U.S. increases tariffs on both sectors. Moreover, the tariff on S2 increases by more as the net export in this sector rises faster.

Figure 3 shows that the U.S. increases tariffs on China's sector 2 goods more than tariffs on sector 1 goods. In addition, the U.S. imposes higher tariffs on ROW's sector 3 goods, aiming to induce Chinese labor to shift from sector 1 and 2 to sector 3. However, the impact of this policy on the rise of Chinese labor in sector 3 is not significant. The same optimal policy applies symmetrically to China, which levies a higher tariff on the US goods in sector 1 and 2, but higher in sector 2, see Figure 4. The larger the rise in global demand, the larger the tariffs levied by both China and the U.S. to endogenously improve each country's own technology in that sector.

We also explore optimal policies under several alternative scenarios in Appendix H. Specifically, we consider a case when there is an increase in China S1's innovation efficiency α_{11}. In this case, China levies a relatively higher tariff on imports from U.S. sector 1 to discourage innovation in the U.S., see Figure A-3. We also study a case when there is a rise in the importance of sector 1 goods. Figure A-4 demonstrates that both the US and China would raise tariffs on importing sector 1 goods from the other country.

For illustrative purposes and to contrast with the fixed technology case, we also compute optimal policies for a multi-country model with a cross-section of 20 two-digit level manufacturing sectors in the steady state in Appendix K. Our results point to significant heterogeneity across countries/sectors in both tariffs and export taxes. By implementing heterogeneous tariffs, there is an additional 20% gain in welfare at the steady state compared to the fixed technology case.

5 Optimal Policies during Transition

Having established separately the two key motives of government policies—to manipulate dynamic technology accumulation and intra-period terms of trade, we proceed to examine policies during transition when these two effects are simultaneously present. We complete the analysis of the full-fledged multi-country-multi-sector model by introducing an externality in technology accumulation. The key result is that the two main motives are still at play, and a conventional tax to target the externality is insufficient to implement optimal policies. For a Markov government, there would need to be additional industrial policies in the long run when there are externalities. Three motives underlie innovation tax or subsidies: to correct for externalities, to manipulate dynamic technology accumulation, and to account for the intertwining effects of externality and technology accumulation across sectors.

The domestic externality is hereby modeled with returns to scale in the innovation effort. Specifically, the innovation efficiency in country n sector j depends on a constant α_n^j and the research effort in this sector,

$$\alpha_{nt}^j = \alpha_n^j (L_{nrt}^j)^{\varepsilon - 1},$$

where $\varepsilon \neq 1$ captures the externality. When $\varepsilon = 1$, the economy goes back to the efficient case with constant α_n^j.

We assume that all countries can use the Pigouvian innovation tax/subsidy to correct for the domestic externality. In addition, foreign countries are still passive and do not use other policies. In this case, the worker-researcher choice in country $n > 1$ becomes

$$\frac{w_n}{\alpha_n^j (L_{nrt}^j)^{\varepsilon-1}} = \varepsilon \frac{w_n L_{np}^j}{\theta T_n^j} + \tilde{\beta}_n v_n^{j\prime}, \qquad (\gamma_{rn}^j), \tag{29}$$

where the ε in front of $\frac{w_n L_{np}^j}{\theta T_n^j}$ reflects foreign country n's Pigouvian innovation tax/subsidy.

Proposition 8 (Markov Policy). *In the general case, the Markov policy is characterized by the following export tax, import tariff, and innovation subsidy τ_d^{jM}, for $n > 1$ any sector j*

$$1+\tau_{xn}^{jM} = \frac{1+\theta(1-\pi_{n1}^j)}{\theta \sum_{m\neq 1}(1+t_m^{jM})\pi_{nm}^j + \left(\sum_k \gamma_{rn}^k (\sigma-1)\tilde{\beta}_n v_n^k\right)/(u_{c_1} x_n)}, \tag{30}$$

$$t_n^{jM} = -\frac{\gamma_{Ln}^j}{u_c} = -\frac{\varepsilon \gamma_{rn}^j}{(1+\theta) T_n^j u_{c_1}} + Const_n, \tag{31}$$

$$\tau_d^{jM} = (\varepsilon - 1) + \underbrace{\frac{\varepsilon}{\frac{1}{\theta}\frac{w_1 L_{1p}^j}{T_1^j}} \frac{\theta}{1+\theta} \frac{\beta(1-\delta)}{u_{c_1}} \sum_{n\neq 1}^{N} \sum_{k}^{N_s} \frac{1}{u_{c_n}/P_n} \gamma_{rn}^k \frac{\partial \left(u_{c_n}' \frac{w_n'/P_n'}{\alpha_n^k (L_{nr}^{k\prime})^{\varepsilon-1}}\right)}{\partial T_1^j}}_{\text{additional subsidies}}. \tag{32}$$

where γ_{rn}^j and γ_{Ln}^j are the multipliers for worker-researcher condition (12) and goods clearing condition (14), respectively.

Proof. See Appendix I.

The optimal export tax τ_{xn}^j in this full model encapsulates comparative advantage—the static terms of trade effect—as in the long-run case (28). In addition, it captures the home government's incentive to manipulate country n's technology accumulation in all sectors, similar to the one-sector case in (17). This dynamic incentive is captured by the multiplier γ_{rn}^k on the worker-researcher constraints, for all sector k in country n.

In our numerical example shown below, we find that in the long run, the second term $\left(\sum_k \gamma_{rn}^k(\sigma-1)\tilde{\beta}_n v_n^k\right)/(u_{c_1} x_n)$ in the denominator of (30) is zero even with externalities. This implies that export taxes have the same formula as ones without externalities in equation (28). The reason is that the government uses export taxes to manipulate dynamic terms of trade, directly through P_n. In the steady state, the relative prices across countries remain constant. Hence, the dynamic aspect of the export tax disappears, and the second term in the denominator becomes zero, even with externalities.

The optimal import tariff t_n^j has a similar formula to the one in the long run. However, the externality shows up directly in (31) simply because we allow foreign countries to correct the externalities through innovation taxes, as shown in equation (29). Home government chooses country-sector-specific tariffs to affect both innovation effort and innovation efficiency across sectors. First, as in the case without externality, the government uses these tariffs to influence the relative prices to induce the desired innovation efforts across different sectors in a foreign country. Second, with decreasing returns to scale in innovation, the change in innovation effort further influences innovation efficiency $\alpha_n^j (L_{nr}^j)^{\varepsilon-1}$ directly.

The optimal innovation effort satisfies the Euler equation with an extra wedge relative to the private equilibrium. To implement this desired innovation effort, the government can choose an innovation subsidy (32), the ratio of the wedge to current profit. This innovation subsidy consists of two terms. The first term $(\varepsilon - 1)$ is constant and reflects the standard incentive of industrial policies, i.e., correcting the externality. When $\varepsilon = 1$, the first term disappears, and the formula returns to the efficient case. The second term is related to dynamic manipulation, as shown in the multipliers γ_{rn}^k and the derivatives of future discounted value to T_1^j. Hence, the government internalizes the impact of its technology in sector j on the technology accumulation in any sector k in other countries.

Notably, when $\varepsilon \neq 1$ for some k, the wedge and the innovation subsidy are not zero even in the long run. In contrast, when there are no externalities, the wedge equals zero, as shown in Proposition 5 and 6. The reason is that the Markov government cannot commit to using future tariffs to adjust the next period's $L_{nr}^{k\prime}$, which affects the technology of country n even when all T reaches the long-run steady state. It is obliged to implement extra distortionary innovation policies even at the SS, reflected by $\frac{\partial \left(u'_{c_n} \frac{w'_n / P'_n}{\alpha_n^k (L_{nr}^{k\prime})^{\varepsilon-1}} \right)}{\partial T_1^j}$ in the extra innovation policy term. This becomes clear as the result is contrasted with the Ramsey policy below.

Proposition 9 (**Ramsey Policy**). *The Ramsey government chooses export tax, import tariff, and* *innovation subsidy* τ_d^{jR} *as*

$$1+\tau_{xn}^{jR} = \frac{1+\theta(1-\pi_{n1}^{j})}{\theta\sum_{m\neq 1}(1+t_m^{jR})\pi_{nm}^{j} + (\sigma-1)\sum_k^{N_s}\left(\gamma_{vn}^{k}\frac{w_n}{\alpha_n^{j}} - \varepsilon\Gamma_{vn}^{k}\frac{w_n L_{np}^{k}}{\theta T_n^{k}}\right)\frac{u_{c_n}}{P_n}/(u_{c_1}x_n)} \tag{33}$$

$$t_n^{jR} = -\frac{\gamma_{L_n}^{j}}{u_c} = -\frac{\varepsilon\Gamma_{vn}^{j}}{(1+\theta)T_n^{j}u_{c_1}P_n/u_{c_n}} + Const_n, \tag{34}$$

$$\tau_d^{jR} = \varepsilon - 1. \tag{35}$$

In addition, in the steady state, $\sum_k^{N_s}\gamma_{vn}^{k}\frac{w_n}{\alpha_n^{j}} = 0, \sum_k \varepsilon\Gamma_{vn}^{k}\frac{w_n L_{np}^{k}}{\theta T_n^{k}} = 0$, *for any foreign country n, even with externality* $\varepsilon \neq 1$.

Proof. See Appendix J.

For the Ramsey government, a constant subsidy/tax rate $\varepsilon - 1$ on innovation is sufficient to correct the externalities.[10] While industrial policies fix externalities, optimal trade policies address terms of trade/technological competition as before, for both static and dynamic considerations. The Ramsey government also considers its past promises on trade policies with Γ_{vn}^{k} showing up in the export tax formula.

The optimal tariffs exhibit a similar structure to that of the Markov case, related to the multipliers of foreign worker-researcher constraints. While Markov tariffs are pinned down by current multipliers γ_{rn}^{j}, Ramsey tariffs depend on all past multipliers Γ_{vn}^{j}. Clearly–tariffs determine a country's cross-sector innovation benefit: in the dynamic setting, innovation benefits in period s are also part of innovation benefits for periods $m < s$. The Ramsey government internalizes the impact of period s tariffs on all the past worker-researcher tradeoffs, and thus Γ_{vn}^{j} shows up in the tariff formula. In contrast, the Markov government cannot commit in which case it only considers the current multiplier γ_{r2}^{j}.

With externalities, labor allocation also affects innovation efficiency $\alpha_n^{j}(L_{nr}^{j})^{\varepsilon-1}$. For instance, $\varepsilon > 1$ means that subsidies endogenously strengthen the comparative advantage sectors and hence raise the incentives to conduct trade policies.

Most importantly, through labor allocations, tariffs determine a country's cross-sector

[10] If there is foreign externality/diffusion, then $\alpha_{nt}^{j} = \alpha_n^{j}(L_{nrt}^{j})^{\varepsilon_n^{j}-1}(T_{m,t-1}^{j})^{\eta}$ where $m \neq 1$. The expression can be modified to account for international spillovers, but the wedge would no longer be a constant.

average cost of innovation $\frac{w_n}{\alpha_n^j (L_{nr}^j)^{\varepsilon-1}}$, which affects the tightness of the constraint γ_{vn}^j. In sum, the Ramsey government does not use additional domestic innovation policies other than the constant Pigouvian tax, relying solely on a path of trade policies. At the steady state, the Ramsey government can use committed tariffs to affect future innovation efficiency, and thus does not impose the same policies as Markov in the case of externalities. The Markov government is incentivized to use tariffs to affect labor allocation and its own comparative advantage even in the steady state, showing up as an extra wedge apart from the Pigouviann tax.

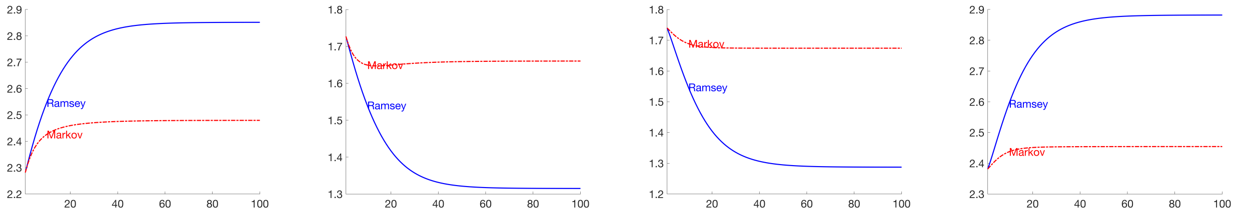
Numerical Example It is worth noting the computational challenges involved in solving this multi-country and multi-sector model. The Markov problem includes a large number of state variables, specifically, $N \times N_s$. In a two-country, two-sector model, for instance, there would be four state variables. But then the Ramsey planner needs to consider all past cumulative multipliers Γ_{vn}^j, which raises the total number of state variables to $N \times N_s \times (N-1) \times N_s$. Another problem is that without externalities, the equilibrium tends to run into corner solutions, where L_{nr}^j might be binding at zero for some sectors and countries. The equilibrium under decreasing return to scale circumvents the issue of corner solutions but still requires guessing the derivatives of future allocations and prices with respect to current technology choices. To check the robustness of our computations, we solve the models with both global methods under sparse grids and perturbation methods. The two methods give identical results, and they are consistent with our theory.

Consider an example with two countries, Home (country 1) and Foreign (country 2), and two sectors. Home has the comparative advantage in sector 1 in that its α_1^1/α_1^2 is higher than foreign α_2^1/α_2^2. In period 1, α_1^1 increases and stays at this high level.[11]

Figures 5 - 7 compare Markov versus Ramsey during the transition path and at the steady state. Both cases start with the same technology levels, i.e., the steady state of Markov technologies before the change of α_1^1. Figure 5 plots the evolution of technologies $\{T_1^1, T_1^2, T_2^1, T_2^2\}$. The Markov and Ramsey equilibrium share some salient features. When

[11] In this example, $\theta = 2$, $\sigma = 2.5$, $\beta = 0.9$, $\delta = 0.02$, $d = 1.4$, and $\varepsilon = 0.99$ for all countries and sectors. In addition, $\alpha_2^2 = 1$ and $\alpha_1^2 = \alpha_2^1 = 0.9$ at all the time. At period 0, $\alpha_1^1 = 0.95$, and it increases to 1 at period 1 and stays at this value.

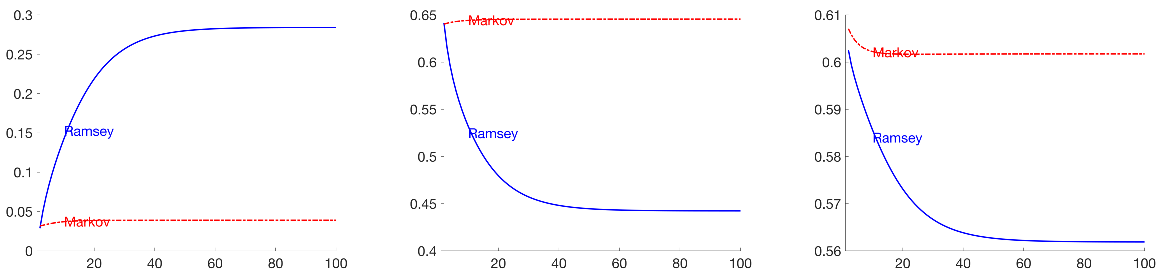
Figure 5: Technology under Ramsey vs Markov: multi-sector with externality



(a) T_1^1 (b) T_1^2 (c) T_2^1 (d) T_2^2

α_1^1 increases, on impact, the return of researchers in sector 1 of Home country becomes higher, which draws more researchers into sector 1. As a result, T_1^1 increase over time, and T_1^2 falls at the Home country. The technology in Country 2 moves in the opposite direction, as the demand for sector 1 goods decreases. Markov differs from Ramsey in that the increase in T_1^1 in Markov is smaller. The technology gap between sectors within a country is also smaller in Markov in the long run. These are due to the different policies Ramsey and Markov use in the long run when there are externalities.

Figure 6: Trade policies under Ramsey vs Markov: multi-sector with externality



(a) t_2^1 (b) τ_{x2}^1 (c) τ_{x2}^2

Figure 7: Ramsey multiplier: multi-sector with externality

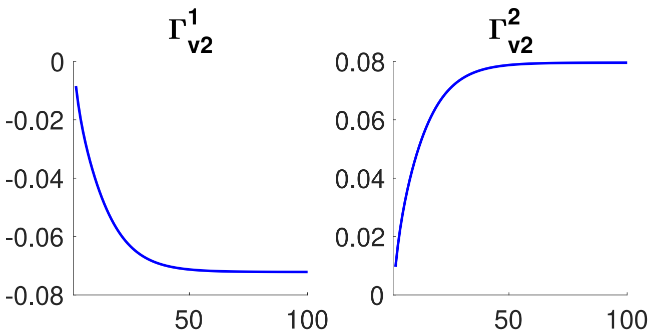
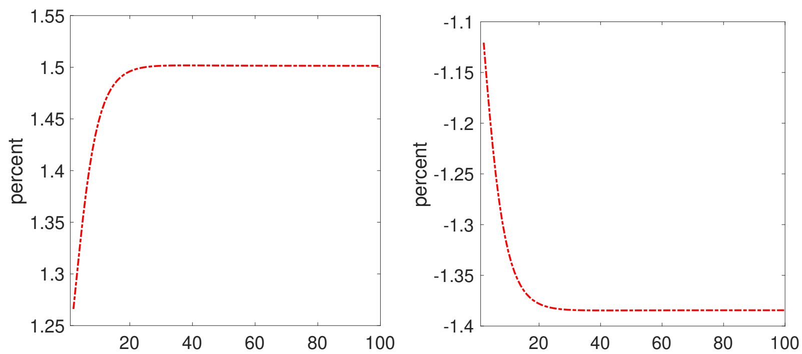


Figure 6 compares trade policies under Ramsey and Markov. We normalize the tariff on sector 2 imports to zero. Given that Home has the comparative advantage in Sector 1, Home would like to impose a tariff on this sector, see Figure 6 (a). Over time, Sector 1 becomes more productive and experiences larger exports, leading to higher import tariffs in both Markov and Ramsey. However, the increase in tariff in Ramsey is larger and more persistent than that in Markov. With this large and persistent increase in import tariff, Ramsey government changes the export taxes according to equation (33).

To understand Figure 6, let us revisit the impact of tariffs on innovating incentives. With or without externality, higher tariffs in sector 1 reduce the sector's profit and discourage innovation or research L^1_{r2}. When $\varepsilon < 1$, an opposing force is also present. Decreasing return to scale ($\varepsilon < 1$) together with lower L^1_{r2} boosts the innovation efficiency $\alpha^1_2(L^1_{r2})^{\varepsilon-1}$. As a result, the opportunity cost of innovating $w_2/[\alpha^1_2(L^1_{r2})^{\varepsilon-1}]$ falls, encouraging innovation in sector 1. These two effects, the lower benefit versus the lower opportunity cost, work against each other and dampen the government's incentive for innovation when $\varepsilon < 1$. This explains why the tariff increase in Markov is small.

The two effects are also present in Ramsey. However, the Ramsey government considers the impact of the current tariff on all past benefits of foreign innovation, while the opportunity cost only shows up in one period. As a result, the larger benefit induces higher tariffs in Ramsey. Figure 7 shows that the Ramsey policy wants Foreign to do less innovation in sector 1 ($\Gamma^1_{v2} < 0$) and more in sector 2, and for this reason, levies a tariff on sector 1 imports.

Figure 8: Additional innovation subsidies under Markov: multi-sector with externality



(a) Innovation Subsidy$_1$ (b) Innovation Subsidy$_2$

Unable to commit to raising tariffs in the future, the Markov government resorts to innovation policies. Figure 8 exhibits the additional innovation subsidies deployed in Markov, which is the second term in Equation (32). The classic industry policy would prescribe an innovation tax, independent across sectors and from trade, when the externality parameter $\varepsilon < 1$ is constant and the same across sectors. In contrast, even with constant ε, the innovation subsidies in Markov problem vary across sectors and time. The first reason is that the Home government subsidizes sector 1 during the transition. Second, even if the Foreign government has used industry policies to correct the externality, Home still uses extra innovation policies at the steady state. The reason is that Home can affect foreign's future marginal innovation cost $\frac{w_n'}{\alpha_n'^k (L_{nr}^{k\prime})^{\varepsilon-1}}$ by affecting $L_{nr}^{k\prime}$ through T_1.

In summary, when externalities are present, both Ramsey and Markov utilize domestic innovation policies in conjunction with heterogeneous export taxes and import tariffs across sectors. However, their policies diverge at the steady state. Ramsey, after imposing the conventional externality tax, relies solely on trade policies. In contrast, Markov requires both trade policies and additional innovation policies to achieve the desired outcomes.

6 Conclusion

Our intention in this paper has been to examine optimal policies for countries when new technologies emerge or when the rest of the world grows. The question can be fully answered only by examining a dynamic model, where there are multiple goods in a multiple-region world economy. Two important motives for governments—a dynamic manipulation of foreign technology and an intra-period terms of trade effect–underlie optimal policies. Time-consistent optimal policies of a country invoke innovation and trade policies, even when private innovation allocations are efficient. Innovation policies can be used to manipulate the benefit and cost of foreign innovation investment in the absence of commitment. In contrast to the Markov government, Ramsey optimal policies are able to avoid deploying innovation policies that distort its own innovation investment by invoking trade policies (heterogenous export tax and tariff across sectors and over time). The full-fledged model is important but entails non-trivial technical challenges. And yet, our model yields explicit expressions for optimal policies and more general results for various specifications, all of which make the mechanisms transparent. These results stand in sharp contrast to the standard models with exogenous technology, where optimal policies call for uniform tariffs across sectors. Future work can take up an analysis of a more quantitative nature, engage with other models of technology competition, or with the various state-of-the-art developments in trade theory.

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Online Appendix to "Technological Rivalry and Optimal Dynamic Policy in an Open Economy"

by Yan Bai, Keyu Jin, and Dan Lu

This appendix is organized as follows.

A Proof for Proposition 1

Note the proposition holds for general case of α_n^j, even when there are externality or intertemporal diffusions. At the steady state, the evolution of technology implies,

$$\delta T_n^j = \alpha_n^j L_{nr}^j = \alpha_n^j r_n^j L_n^j, \tag{A.1}$$

where the second equality uses the definition of research intensity, $r_n^j = L_{nr}^j / L_n^j$, i.e., the share of research labor in sector j of country n. The free-entry condition implies that private research efforts satisfy

$$\frac{w_n}{\alpha_n^j} = \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + E\tilde{\beta}_n v_n^{j\prime}. \tag{A.2}$$

Combining the equation (A.1), (A.2), the definition of $\tilde{\beta} = \beta(1-\delta)\frac{u_n' P_n}{u_n P_n'}$, and $u_n' = u_n$, $P_n' = P_n$ at the steady state, we find that the steady-state research intensity for any country n and sector j is given by

$$r_{n,ss}^j = \frac{\delta}{\theta\left[1-\beta(1-\delta)\right]+\delta}.$$

Hence, the research intensity is constant across sectors and countries.

B Characterization of Markov policy

In this appendix, we first lay out the Home government's problem under the assumption that it cannot commit to future policies. We then characterize the Markov policy by taking first-order conditions. Note that the characterization of optimal Markov policy applies to the general form of innovation efficiency, $\alpha_n^j(L_{nr}^j, T_{n,-1}^j)$.

B.1 Markov problem

Home government (country 1) chooses optimal unilateral trade and domestic R&D policies and solves

$$V\left(\left\{T_{n,-1}^j\right\}\right) = \max_{\left\{L_{nr}^j, L_{np}^j T_n^j, w_n, x_n, \tau_{xn}^j, t_n^j\right\}} \frac{x_1^{1-\sigma}}{1-\sigma} + \beta\left[V\left(\left\{T_n^j\right\}\right)\right] \tag{A.3}$$

Subject to

$$\frac{w_n}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)} = \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \tilde{\beta}_n v_n^{j\prime}, \qquad (\gamma_{rn}^j, \quad N_s \times (N-1)) \tag{A.4}$$

$$T_n^j = \alpha_n^j(L_{nr}^j, T_{n,-1}^j)L_{nr}^j + (1-\delta)T_{n,-1}^j, \quad (\gamma_{Tn}^j, \quad N_s \times N) \tag{A.5}$$

$$P_1 = \Pi_j\left[T_1^j w_1^{-\theta} + \sum_{n\neq 1} T_n^j (w_n(1+t_n^j)d_{1n})^{-\theta}\right]^{-\beta_j/\theta} = 1, \quad (\gamma_P) \tag{A.6}$$

$$\frac{1+\theta}{\theta} w_1 L^j_{1p} = \beta_j \left[\pi^j_{11} x_1 + \sum_{m \neq 1} \frac{1}{1+\tau^j_{xm}} \pi^j_{m1} x_m \right], \quad (\gamma^j_{L1}, \quad N_s) \tag{A.7}$$

$$\frac{1+\theta}{\theta} w_n L^j_{np} = \beta_j \left[\frac{1}{1+t^j_n} \pi^j_{1n} x_1 + \sum_{m \neq 1} \pi^j_{m,n} x_m \right] \quad (\gamma^j_{Ln}, \quad N_s \times (N-1)) \tag{A.8}$$

$$\sum_j \left(L^j_{nr} + L^j_{np} \right) = L_n, \quad (\mu_n, \quad N) \tag{A.9}$$

$$x_1 = \frac{1+\theta}{\theta} w_1 \sum_j L^j_{1p} + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau^j_{xm}}{1+\tau^j_{xm}} \pi^j_{m1} x_m + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t^j_m}{1+t^j_m} \pi^j_{1m} x_1, \quad (\gamma_4) \tag{A.10}$$

where

$$x_m = \frac{1+\theta}{\theta} w_m \sum_j L^j_{mp} \tag{A.11}$$

$$P_m = \Pi_j \left[T^j_1 (w_1 (1+\tau^j_{xm}) d_{m1})^{-\theta} + \sum_{n \neq 1} T^j_n (w_n d_{mn})^{-\theta} \right]^{-\beta_j/\theta} \tag{A.12}$$

$$\pi^j_{11} = \frac{T^j_1 (w_1)^{-\theta}}{T^j_1 (w_1)^{-\theta} + \sum_{n \neq 1} T^j_n (w_n (1+t^j_n) d_{1n})^{-\theta}} \tag{A.13}$$

$$\pi^j_{m1} = \frac{T^j_1 (w_1 (1+\tau^j_{xm}) d_{m1})^{-\theta}}{T^j_1 (w_1 (1+\tau^j_{xm}) d_{m1})^{-\theta} + \sum_{n \neq 1} T^j_n (w_n d_{mn})^{-\theta}} \tag{A.14}$$

$$\pi^j_{mn} = \frac{T^j_n (w_n d_{mn})^{-\theta}}{T^j_1 (w_1 (1+\tau^j_{xm}) d_{m1})^{-\theta} + T^j_n (w_n d_{mn})^{-\theta} + \sum_{i \neq \{m,n\}} T^j_i (w_i d_{mi})^{-\theta}} \tag{A.15}$$

$$\pi^j_{1m} = \frac{T^j_m (w_m (1+t^j_m) d_{1m})^{-\theta}}{T^j_1 (w_1)^{-\theta} + T^j_m (w_m (1+t^j_m) d_{1m})^{-\theta} + \sum_{n \neq \{1,m\}} T^j_n (w_n (1+t^j_n) d_{1n})^{-\theta}} \tag{A.16}$$

Note that $\sum_n \pi^j_{mn} = 1$ for any m.

Note that sum up (A.7) we get the balanced trade condition for country 1

$$\sum_{m=1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{1}{1+t^j_m} \pi^j_{1m} x_1 = \sum_{m=1}^{N} \sum_{j=1}^{N_s} \beta_j \pi^j_{m1} x_m,$$

and sum up (A.8) we get the balanced trade condition for country $n \neq 1$,

$$\sum_{m=1}^{N} \sum_{j=1}^{N_s} \beta_j \pi^j_{nm} x_n = \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \pi^j_{mn} x_m + \sum_{j=1}^{N_s} \beta_j \frac{1}{1+t^j_n} \pi^j_{1n} x_1.$$

As one of them is redundant, we can drop one of the (A.8), so we end up with number $(N-1)N_s - 1$ for γ^j_{Ln} with $n \neq 1$.

B.2 Optimal conditions for Markov policy

Here we first take the first-order conditions over the Markov problem (A.3) assuming interior solution. We then derive the optimal conditions for Markov policy from these first-order conditions.

First order conditions For ease of notation, we make the following definitions:

$$G^j_n = \left\{ \left(x'_n\right)^{-\sigma} \left(P'_n\right)^{\sigma-1} \frac{w'_n}{\alpha^j_n(L^{j\prime}_{nr}, T^j_n)} \right\}, \qquad M^j_n = G^j_n x^\sigma_n P^{1-\sigma}_n, \tag{A.17}$$

where G^j_n captures the future value of a researcher in future prices, and M^j_n is the future value of the researcher in current prices.

FOC over x_1:

$$u_c + \sum_{n=1}^{N} \sum_j \gamma^j_{Ln} \beta_j \frac{1}{1+t^j_n} \pi^j_{1n} - \gamma_4 \left(1 - \sum_{n\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t^j_n}{1+t^j_n} \pi^j_{1n}\right) = 0. \tag{A.18}$$

FOC over L^j_{1r}:

$$\mu_1 = \gamma^j_{T1} \left[\alpha^j_1(L^j_{1r}, T^j_{1,-1}) + \frac{\partial \alpha^j_1}{\partial L^j_{1r}} L^j_{1r} \right], \tag{A.19}$$

FOC over L^j_{1p}:

$$\mu_1 = \left(\gamma_4 - \gamma^j_{L1}\right) \frac{1+\theta}{\theta} w_1. \tag{A.20}$$

FOC over L^j_{nr} for $n > 1$:

$$\mu_n = \gamma^j_{Tn} \left[\alpha^j_n(L^j_{nr}, T^j_{n,-1}) + \frac{\partial \alpha^j_n}{\partial L^j_{nr}} L^j_{nr} \right] + \gamma^j_{rn} \frac{w_n}{\left[\alpha^j_n(L^j_{nr}, T^j_{n,-1})\right]^2} \frac{\partial \alpha^j_n}{\partial L^j_{nr}}, \tag{A.21}$$

FOC over L^j_{np} for $n > 1$:

$$\mu_n = \gamma^j_{rn} \frac{1}{\theta} \frac{w_n}{T^j_n} - \gamma^j_{Ln} \frac{1+\theta}{\theta} w_n + \sum_{k=1}^{N_s} \gamma^k_{L1} \beta_k \frac{1}{1+\tau^k_{xn}} \pi^k_{n1} \frac{1+\theta}{\theta} w_n \tag{A.22}$$
$$+ \sum_{i\neq 1}^{N} \sum_k^{N_s} \gamma^k_{Li} \beta_k \pi_{ni} \frac{1+\theta}{\theta} w_n + \gamma_4 \sum_k^{N_s} \beta_k \frac{\tau^k_{xn}}{1+\tau^k_{xn}} \pi^k_{n1} \frac{1+\theta}{\theta} w_n + \sum_k^{N_s} \gamma^k_{rn} \beta (1-\delta) \sigma M^k_n \frac{1+\theta}{\theta} \frac{w_n}{x_n}.$$

FOC over τ_{xn}^j for $n > 1$:

$$\left(\sum_k \gamma_{rn}^k \beta(1-\delta)(1-\sigma)M_n^k\right)\beta_j\pi_{n1}^j - \gamma_{L1}^j\frac{1}{(1+\tau_{xn}^j)}\beta_j\pi_{n1}^j x_n - \gamma_{L1}^j\frac{1}{(1+\tau_{xn}^j)}\theta(1-\pi_{n1}^j)\beta_j\pi_{n1}^j x_n$$
$$+\sum_{i\neq 1}^N \gamma_{Li}^j\beta_j\left[\theta\pi_{ni}^j\pi_{n1}^j x_n\right] + \gamma_4\beta_j\frac{1}{(1+\tau_{xn}^j)}\pi_{n1}^j x_n - \gamma_4\beta_j\frac{\tau_{xn}^j}{1+\tau_{xn}^j}\theta\pi_{n1}^j(1-\pi_{n1}^j)x_n = 0 \tag{A.23}$$

FOC over import tariff t_n^j for $n > 1$:

$$-\gamma_P\beta_j\frac{\pi_{1n}^j}{1+t_n^j} + \gamma_{L1}^j\beta_j\frac{\partial\pi_{11}^j}{\partial t_n^j}x_1 - \gamma_{Ln}^j\beta_j\frac{\pi_{1n}^j}{\left(1+t_n^j\right)^2}x_1 + \gamma_{Ln}^j\frac{1}{1+t_n^j}\frac{\partial\pi_{1n}^j}{\partial t_n^j}x_1 + \sum_{m\neq\{1,n\}}\gamma_{Lm}^j\frac{1}{1+t_m^j}\frac{\partial\pi_{1m}^j}{\partial t_n^j}x_1$$
$$+\gamma_4\beta_j\frac{1}{\left(1+t_n^j\right)^2}\pi_{1n}^j x_1 + \gamma_4\sum_{m\neq 1}^N\beta_j\frac{t_m^j}{1+t_m^j}\frac{\partial\pi_{1m}^j}{\partial t_n^j}x_1 = 0 \tag{A.24}$$

FOC over T_1^j:

$$-\gamma_{T1}^j + \beta\frac{\partial V}{\partial T_1^j} + \gamma_P\frac{\beta_j}{\theta}\frac{\pi_{11}^j}{T_1^j} - \left(\sum_{i\neq 1}^N\sum_k^{N_s}\gamma_{ri}^k\beta(1-\delta)(1-\sigma)M_i^k\right)\left(\frac{\beta_j}{\theta}\frac{\pi_{i1}^j}{T_1^j}\right) \tag{A.25}$$
$$+\gamma_{L1}^j\beta_j\left[\frac{\partial\pi_{11}^j}{\partial T_1^j}x_1 + \sum_{m\neq 1}^N\frac{1}{1+\tau_{xm}^j}\frac{\partial\pi_{m1}^j}{\partial T_1^j}x_2\right] + \sum_{n\neq 1}^N\gamma_{Ln}^j\beta_j\left[\frac{1}{1+t_n^j}\frac{\partial\pi_{1n}^j}{\partial T_1^j}x_1 + \sum_{m\neq 1}\frac{\partial\pi_{mn}^j}{\partial T_1^j}x_m\right]$$
$$+\gamma_4\sum_{m\neq 1}^N\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\frac{\partial\pi_{m1}^j}{\partial T_1^j}x_m + \gamma_4\sum_{m\neq 1}^N\beta_j\frac{t_m^j}{1+t_m^j}\frac{\partial\pi_{1m}^j}{\partial T_1^j}x_1 + \beta(1-\delta)\sum_{n\neq 1}^{N_s}\sum_k\gamma_{r,n}^k\frac{\partial G_n^k}{\partial T_1^j}x_n^\sigma P_n^{1-\sigma} = 0$$

where $\frac{\partial V}{\partial T_{1,-1}^j} = \gamma_{T1}^j\left(1-\delta+\frac{\partial\alpha_1^j}{\partial T_{1,-1}^j}\right)$ by the envelop theorem.

FOC over T_n^j for $n > 1$:

$$\gamma_{Tn}^j = \beta\left[\gamma_{Tn}^{j\prime}\left(1-\delta+\frac{\partial\alpha_n^{j\prime}}{\partial T_n^j}L_{nr}^j + \gamma_{rn}^j\frac{w_n'}{\left(\alpha_n^{j\prime}\right)^2}\frac{\partial\alpha_n^{j\prime}}{\partial T_n^j}\right)\right] + \gamma_P\frac{\beta_j}{\theta}\frac{\pi_{1n}^j}{T_n^j}$$
$$+\gamma_{L1}^j\beta_j\left[\frac{\partial\pi_{11}^j}{\partial T_n^j}x_1 + \sum_{m\neq 1}\frac{1}{1+\tau_{xm}^j}\frac{\partial\pi_{m1}^j}{\partial T_n^j}x_m\right] + \sum_{i\neq 1}^N\gamma_{Li}^j\beta_j\left[\frac{1}{1+t_i^j}\frac{\partial\pi_{1i}^j}{\partial T_n^j}x_1 + \sum_{m\neq 1}\frac{\partial\pi_{mi}^j}{\partial T_n^j}x_m\right]$$
$$+\gamma_4\sum_{m\neq 1}^N\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\frac{\partial\pi_{m1}^j}{\partial T_n^j}x_m + \gamma_4\sum_{m\neq 1}^N\beta_j\frac{t_m^j}{1+t_m^j}\frac{\partial\pi_{1m}^j}{\partial T_n^j}x_1$$
$$-\gamma_{rn}^j\frac{1}{\theta}\frac{w_nL_{np}^j}{\left(T_n^j\right)^2} - \sum_{i\neq 1}^N\left(\sum_k^{N_s}\gamma_{ri}^k\beta(1-\delta)(1-\sigma)M_i^k\right)\frac{\beta_j}{\theta}\frac{\pi_{in}^j}{T_n^j} + \sum_{i\neq 1}^N\sum_k^{N_s}\gamma_{ri}^k\beta(1-\delta)\frac{\partial G_i^k}{\partial T_n^j}x_i^\sigma P_i^{1-\sigma} \tag{A.26}$$

FOC over w_1:

$$-\gamma_P \sum_j^{N_s} \beta_j \frac{\pi_{11}^j}{w_1} - \sum_j^{N_s} \gamma_{L1}^j \frac{1+\theta}{\theta} L_{1p}^j + \sum_j^{N_s} \gamma_{L1}^j \beta_j \frac{\partial \pi_{11}^j}{\partial w_1} x_1 + \sum_j^{N_s} \gamma_{L1}^j \beta_j \sum_{m\neq 1}^{N} \frac{1}{1+\tau_{xm}^j} \frac{\partial \pi_{m1}^j}{\partial w_1} x_m$$
$$+ \sum_{n\neq 1}^{N} \sum_j^{N_s} \gamma_{Ln}^j \beta_j \frac{1}{1+t_n^j} \frac{\partial \pi_{1n}^j}{\partial w_1} x_1 + \sum_{n\neq 1}^{N} \sum_j^{N_s} \gamma_{Ln}^j \beta_j \sum_{m\neq 1}^{N} \frac{\partial \pi_{mn}^j}{\partial w_1} x_m$$
$$+ \gamma_4 \frac{1+\theta}{\theta} \sum_j L_{1p}^j + \gamma_4 \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \frac{\partial \pi_{m1}^j}{\partial w_1} x_m + \gamma_4 \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t_m^j}{1+t_m^j} \frac{\partial \pi_{1m}^j}{\partial w_1} x_1$$
$$+ \sum_{n\neq 1}^{N} \sum_j^{N_s} \gamma_{rn}^j \beta (1-\delta)(1-\sigma) M_n^j \left(\sum_k^{N_s} \beta_k \pi_{n1}^k \right) w_1^{-1} = 0 \tag{A.27}$$

FOC over w_n for $n > 1$:

$$-\gamma_P \sum_j^{N_s} \beta_j \frac{\pi_{1n}^j}{w_n} + \sum_j^{N_s} \gamma_{L1}^j \beta_j \frac{\partial \pi_{11}^j}{\partial w_n} x_1 + \sum_j^{N_s} \gamma_{L1}^j \beta_j \sum_{m\neq 1}^{N} \frac{1}{1+\tau_{xm}^j} \frac{\partial \pi_{m1}^j}{\partial w_n} x_m + \sum_j^{N_s} \gamma_{L1}^j \beta_j \frac{1}{1+\tau_{xn}^j} \pi_{n1}^j \frac{x_n}{w_n}$$
$$- \sum_j^{N_s} \gamma_{Ln}^j \frac{1+\theta}{\theta} L_{np}^j + \sum_{i\neq 1}^{N} \sum_j^{N_s} \gamma_{Li}^j \beta_j \frac{1}{1+t_i^j} \frac{\partial \pi_{1i}^j}{\partial w_n} x_1 + \sum_{i\neq 1}^{N} \sum_j^{N_s} \gamma_{Li}^j \beta_j \sum_{m\neq 1} \frac{\partial \pi_{mi}^j}{\partial w_n} x_m + \sum_{i\neq 1}^{N} \sum_j^{N_s} \gamma_{Li}^j \beta_j \pi_{ni}^j \frac{x_n}{w_n}$$
$$+ \gamma_4 \sum_{m\neq}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \frac{\partial \pi_{m1}^j}{\partial w_n} x_m + \gamma_4 \sum_{j=1}^{N_s} \beta_j \frac{\tau_{xn}^j}{1+\tau_{xn}^j} \pi_{n1}^j \frac{x_n}{w_n} + \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t_m^j}{1+t_m^j} \frac{\partial \pi_{1m}^j}{\partial w_n} x_1$$
$$+ \sum_j^{N_s} \gamma_{rn}^j \left[\frac{1}{\theta} \frac{L_{np}^j}{T_n^j} - \frac{1}{\alpha_n^j (L_{nr}^j, T_{n,-1}^j)} + \beta(1-\delta)\sigma M_n^j \frac{1}{w_n} \right]$$
$$+ \sum_{m\neq 1}^{N} \sum_j^{N_s} \gamma_{rm}^j \left[\beta(1-\delta)(1-\sigma) M_m^j \left(\sum_k \beta_k \frac{\pi_{mn}^k}{w_n} \right) \right] = 0 \tag{A.28}$$

Simplifying conditions First, according to FOC (A.20) on L_{1p}^j, it has to be the case that $\gamma_{L1}^j = \gamma_{L1}$ for any sector j at Home country. This implies that the marginal return of production labor has to be equalized across sectors. We can further prove that $\gamma_P / x_1 = u_c = \gamma_4 - \gamma_{L1}$.

With the condition $\gamma_{L1}^j = \gamma_{L1}$ and $u_c = \gamma_4 - \gamma_{L1}$, we can further simplify the optimal export tax equation (A.23) as, for $n > 1$ and $j \in [1, N_s]$

$$1 + \tau_{xn}^j = \frac{(\gamma_4 - \gamma_{L1}) \left[1 + \theta(1 - \pi_{n1}^j) \right]}{\gamma_4 \theta (1 - \pi_{n1}^j) - \theta \left(\sum_{i\neq 1}^{N} \gamma_{Li}^j \pi_{ni}^j \right) + \left(\sum_k \gamma_{rn}^k \beta (1-\delta)(\sigma - 1) M_n^k \right) x_n^{-1}}. \tag{A.29}$$

For the import tariff, we can plug in the expenditure derivatives and further simplify the equation

(A.24) to

$$\frac{1}{1+t_n^j} = \frac{u_c - \sum_{m\neq\{1,n\}}^N \left(\gamma_4 - \gamma_{Lm}^j\right) \frac{t_m^j}{1+t_m^j}\theta\pi_{1m}^j + \gamma_4\theta(1-\pi_{1n}^j) - \gamma_{L1}\theta\pi_{11}^j - \sum_{m\neq\{1,n\}} \gamma_{Lm}^j\theta\pi_{1m}^j}{\left(\gamma_4 - \gamma_{Ln}^j\right)\left[1+\theta(1-\pi_{1n}^j)\right]}$$

or equivalently,

$$1+t_n^j = \frac{\left(\gamma_4 - \gamma_{Ln}^j\right)\left[1+\theta(1-\pi_{1n}^j)\right]}{(\gamma_4 - \gamma_{L1})\,\theta\pi_{11}^j + \frac{\gamma_P}{x_1} + \sum_{m\neq\{1,n\}}^N \left(\gamma_4 - \gamma_{Lm}^j\right)\frac{1}{1+t_m^j}\theta\pi_{1m}^j}$$

Using the equilibrium condition $\gamma_P/x_1 = u_c = \gamma_4 - \gamma_{L1}$, we arrive at for each foreign country $n > 1$ and $j \in [1, N_s]$

$$1+t_n^j = \frac{\gamma_4 - \gamma_{Ln}^j}{u_c}. \tag{A.30}$$

We combine the FOC for L_{1r}^j, L_{1p}^j (A.20) and T_1^j (A.25). Specifically, Putting in enevelop and derivatives

$$\begin{aligned}
\gamma_{T1}^j = \beta\left[\gamma_{T1}^{j\prime}\left(1-\delta+\frac{\partial\alpha_1^{j\prime}}{\partial T_{n,}^j}\right)\right] + \frac{1}{\theta}\frac{1}{T_1^j}[\gamma_P\beta_j\pi_{11}^j - \left(\sum_{n\neq 1}^N\sum_k^{N_s}\gamma_{rn}^k\beta(1-\delta)(1-\sigma)M_n^k\right)\left(\beta_j\pi_{n1}^j\right) \\
+ \gamma_{L1}\theta\beta_j\left[\pi_{11}^j(1-\pi_{11}^j)x_1 + \sum_{m\neq 1}^N \frac{1}{1+\tau_{xm}^j}\pi_{m1}^j(1-\pi_{m1}^j)x_m\right] \\
- \sum_{n\neq 1}^N \gamma_{Ln}^j\theta\beta_j\left[\frac{1}{1+t_n^j}\pi_{11}^j\pi_{1n}^j x_1 + \sum_{m\neq 1}^N \pi_{m1}^j\pi_{mn}^j x_m\right] + \theta\gamma_4\sum_{n\neq 1}^N\beta_j\frac{\tau_{xn}^j}{1+\tau_{xn}^j}\pi_{n1}^j(1-\pi_{n1}^j)x_n \\
- \theta\gamma_4\sum_{n\neq 1}^N \beta_j\frac{t_n^j}{1+t_n^j}\pi_{11}^j\pi_{1m}^j x_1 + \theta\sum_{n\neq 1}^N\sum_k^{N_s}\gamma_{r,n}^k\beta(1-\delta)\frac{\partial G_n^k}{\partial T_1^j}x_n^\sigma P_n^{1-\sigma}T_1^j] = 0,
\end{aligned}$$

FOC on τ_{xn}^j are

$$\begin{aligned}
\left(\sum_k \gamma_{rn}^k\beta(1-\delta)(1-\sigma)M_n^k\right)\beta_j\pi_{n1}^j - \gamma_{L1}^j\frac{1}{(1+\tau_{xn}^j)}\beta_j\pi_{n1}^j x_n - \gamma_{L1}^j\frac{1}{(1+\tau_{xn}^j)}\theta(1-\pi_{n1}^j)\beta_j\pi_{n1}^j x_n \\
+ \sum_{i\neq 1}^N \gamma_{Li}^j\beta_j\left[\theta\pi_{ni}^j\pi_{n1}^j x_n\right] + \gamma_4\beta_j\frac{1}{(1+\tau_{xn}^j)}\pi_{n1}^j x_n - \gamma_4\beta_j\frac{\tau_{xn}^j}{1+\tau_{xn}^j}\theta\pi_{n1}^j(1-\pi_{n1}^j)x_n = 0,
\end{aligned}$$

sum over $n \neq 1$ and substitute it into the FOC on T_1^j, we can further write it as:

$$\gamma_{T1}^j = \beta\left[\gamma_{T1}^{j\prime}\left(1-\delta+\frac{\partial\alpha_1^{j\prime}}{\partial T_{n,}^j}\right)\right]+\frac{1}{\theta}\frac{1}{T_1^j}\{\gamma_P\beta_j\pi_{11}^j$$
$$+\gamma_{L1}\theta\beta_j\left[\pi_{11}^j(1-\pi_{11}^j)x_1\right]+\sum_{n\neq 1}(\gamma_4-\gamma_{L1})\frac{1}{(1+\tau_{xn}^j)}\beta_j\pi_{n1}^j x_n$$
$$-\theta\pi_{11}^j x_1\sum_{n\neq 1}^N\frac{1}{1+t_n^j}\gamma_{Ln}^j\beta_j\pi_{1n}^j-\theta\gamma_4\sum_{n\neq 1}^N\beta_j\frac{t_n^j}{1+t_n^j}\pi_{11}^j\pi_{1m}^j x_1\}+\sum_{n\neq 1}^N\sum_k^{N_s}\gamma_{r,n}^k\beta(1-\delta)\frac{\partial G_n^k}{\partial T_1^j}x_n^\sigma P_n^{1-\sigma}.$$

Furthermore, because optimal tariffs satisfy

$$\gamma_P\beta_j\pi_{11}^j+\theta\beta_j\pi_{11}^j x_1\left\{\gamma_{L1}(1-\pi_{11}^j)-\sum_{n\neq 1}^N\frac{1}{1+t_n^j}\gamma_{Ln}^j\pi_{1n}^j-\gamma_4\sum_{n\neq 1}^N\frac{t_n^j}{1+t_n^j}\pi_{1m}^j\right\}+(\gamma_4-\gamma_{L1})\sum_{n\neq 1}\frac{1}{(1+\tau_{xn}^j)}\beta_j\pi_{n1}^j x_n$$
$$=\gamma_P\beta_j\pi_{11}^j+(\gamma_4-\gamma_{L1})\frac{1+\theta}{\theta}w_1L_{1p}^j-(\gamma_4-\gamma_{L1})\beta_j\pi_{11}^j x_1$$
$$=(\gamma_4-\gamma_{L1})\frac{1+\theta}{\theta}w_1L_{1p}^j,$$

hence

$$\gamma_{T1}^j=\beta\left[\gamma_{T1}^{j\prime}(1-\delta)\right]+\frac{1}{\theta}\frac{1}{T_1^j}(\gamma_4-\gamma_{L1})\frac{1+\theta}{\theta}w_1L_{1p}^j+\sum_{n\neq 1}^N\sum_k^{N_s}\gamma_{r,n}^k\beta(1-\delta)\frac{\partial G_n^k}{\partial T_1^j}x_n^\sigma P_n^{1-\sigma}.$$

The FOC of L_{1r}^j implies

$$\gamma_{T1}^j=(\gamma_4-\gamma_{L1})\frac{1+\theta}{\theta}\frac{w_1}{\alpha_1^j}$$

Plug back into the above equation, we have the optimal conditions for Home innovation for each sector $j\in[1,N_s]$:

$$\frac{w_1}{\alpha_1^j(L_{1r}^j,T_{1,-1}^j)+\frac{\partial\alpha_1^j}{\partial L_{1r}^j}L_{1r}^j}=\frac{1}{\theta}\frac{w_1L_{1p}^j}{T_1^j}+\beta\left[\frac{1}{u_c}\frac{u_c'w_1'}{\alpha_1^j(L_{1r}^{j\prime},T_1^j)+\frac{\partial\alpha_1^{j\prime}}{\partial L_{1r}^{j\prime}}L_{1r}^{j\prime}}\left(1-\delta+\frac{\partial\alpha_1^{j\prime}}{\partial T_{n,}^j}\right)\right]$$
$$+\frac{\theta}{1+\theta}\frac{1}{u_c}\sum_{n\neq 1}^N\sum_k^{N_s}\gamma_{r,n}^k\beta(1-\delta)\frac{\partial G_n^k}{\partial T_1^j}x_n^\sigma P_n^{1-\sigma},\qquad\text{(A.31)}$$

Similarly, plugging in the expenditure derivatives and FOCs on tariffs t_n^j (A.30) to FOC (A.26) of

T_n^j, we have a simplified FOC for T_n^j for each country $n>1$ and $j \in [1, N_s]$

$$\begin{aligned}
\gamma_{Tn}^{j} =& \beta \left[\gamma_{Tn}^{j\prime} \left(1-\delta+\frac{\partial \alpha_n^{j\prime}}{\partial T_n^j} L_{nr}^j + \gamma_{rn}^j \frac{w_n'}{\left(\alpha_n^{j\prime}\right)^2} \frac{\partial \alpha_n^{j\prime}}{\partial T_n^j} \right) \right] + \frac{1}{T_n^j} \{ \gamma_P \frac{\beta_j}{\theta} \pi_{1n}^j \\
& - \sum_{i=1}^{N} \gamma_{Li}^j \beta_j \sum_{m\neq 1} \pi_{mi}^j \pi_{mn}^j x_m + \gamma_{Ln}^j \beta_j \sum_{m\neq 1} \pi_{mn}^j x_m - u_c \sum_{m\neq 1} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \pi_{m1}^j \pi_{mn}^j x_m \\
& - \gamma_{rn}^j \frac{1}{\theta} \frac{w_n L_{np}^j}{\left(T_n^j\right)} - \sum_{i\neq 1}^{N} \left(\sum_k^{N_s} \gamma_{ri}^k \beta (1-\delta)(1-\sigma) M_i^k \right) \frac{\beta_j}{\theta} \pi_{in}^j \} + \sum_{i\neq 1}^{N} \sum_k^{N_s} \gamma_{ri}^k \beta (1-\delta) \frac{\partial G_i^k}{\partial T_n^j} x_i^\sigma P_i^{1-\sigma}.
\end{aligned} \tag{A.32}$$

Using the expenditure derivatives and the optimal tariff conditions (A.30), we can write the FOC (A.27) on w_1

$$\begin{aligned}
&-\gamma_P \sum_j^{N_s} \beta_j \pi_{11}^j - \gamma_{L1} \sum_j^{N_s} \left[\frac{1+\theta}{\theta} w_1 L_{1p}^j + \beta_j \sum_{m\neq 1}^{N} \frac{1}{1+\tau_{xm}^j} \theta \pi_{m1}^j (1-\pi_{m1}^j) x_m + \beta_j \theta \sum_{m\neq 1}^{N} \pi_{m1}^j \pi_{m1}^j x_m \right] \\
&\quad + \sum_n^N \sum_j^{N_s} \gamma_{Ln}^j \beta_j \sum_{m\neq 1}^{N} \theta \pi_{mn}^j \pi_{m1}^j x_m + \gamma_4 \sum_j^{N_s} \left[\frac{1+\theta}{\theta} w_1 L_{1p}^j - \sum_{m\neq 1}^{N} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \theta \pi_{m1}^j (1-\pi_{m1}^j) x_m \right] \\
&\quad + \sum_{n\neq 1}^{N} \sum_j^{N_s} \gamma_{rn}^j \beta (1-\delta)(1-\sigma) M_n^j \left(\sum_k^{N_s} \beta_k \pi_{n1}^k \right) = 0,
\end{aligned}$$

and FOC (A.28) on w_n for $n>1$

$$\begin{aligned}
&-\gamma_P \sum_j^{N_s} \beta_j \pi_{1n}^j + \gamma_{L1} \left[\sum_{m\neq 1}^{N} \sum_j^{N_s} \beta_j \frac{1}{1+\tau_{xm}^j} \theta \pi_{m1}^j \pi_{mn}^j x_m + \sum_j^{N_s} \beta_j \frac{1}{1+\tau_{xn}^j} \pi_{n1}^j x_n \right] \\
&+ \sum_{i\neq\{1\}}^{N} \sum_j^{N_s} \gamma_{Li}^j \beta_j \left(\sum_{m\neq 1} \theta \pi_{mi}^j \pi_{mn}^j x_m + \pi_{n,i}^j x_n \right) - \sum_j^{N_s} \gamma_{Ln}^j \left(\theta \beta_j \sum_{m\neq 1} \pi_{mn}^j x_m + \frac{1+\theta}{\theta} w_n L_{np}^j \right) \\
&+ \gamma_4 \sum_{j=1}^{N_s} \beta_j \left(\sum_{m\neq 1}^{N} \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \theta \pi_{m1}^j \pi_{mn}^j x_m + \frac{\tau_{xn}^j}{1+\tau_{xn}^j} \pi_{n1}^j x_n \right) \\
&+ \sum_j^{N_s} \gamma_{rn}^j \left[\beta (1-\delta)(\sigma-1) M_n^j \right] + \sum_{m\neq 1}^{N} \sum_j^{N_s} \gamma_{rm}^j \left[\beta (1-\delta)(1-\sigma) M_m^j \left(\sum_k^{N_s} \beta_k \pi_{mn}^k \right) \right] = 0.
\end{aligned} \tag{A.33}$$

Summary of optimal Markov conditions The Markov equilibrium includes the variables

$$\Big\{ \underbrace{L_{nr}^j}_{N_s\times N}, \underbrace{L_{np}^j}_{N_s\times N}, \underbrace{T_n^j}_{N_s\times N}, \underbrace{\tau_{xn}^j}_{N_s\times(N-1)}, \underbrace{t_n^j}_{N_s\times(N-1)}, \underbrace{\gamma_{Ln}^j}_{N_s\times(N-1)-1}, \underbrace{\gamma_{rn}^j}_{N_s\times(N-1)}, \underbrace{\gamma_{Tn}^j}_{N_s\times(N-1)}, \underbrace{\mu_n}_{N-1}, \underbrace{w_n}_{N}, \gamma_{L1}, \gamma_4 \Big\}$$

that satisfies the private equilibrium (A.4)-(A.12), the optimal Home innovation (A.31), export tax formula (A.29), import tariff formula (A.30), FOC (A.32) on foreign T_n^j, and FOC (A.22) on foreign production labor L_{np}^j, FOC (A.21) on foreign research effort L_{nr}^j, FOC on foreign wages (A.28), and the condition $u_c = \gamma_4 - \gamma_{L1}$.

B.3 Proof of $\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0$ at the steady state when $\alpha_n^j(L_{nr}^j, T_{n,-1}^j) = \bar{\alpha}_n^j$.

In this appendix, we prove that when $\alpha_n^j(L_{nr}^j, T_{n,-1}^j) = \bar{\alpha}_n^j$, the following equation holds at the steady state

$$\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0. \tag{A.34}$$

1. Use FOC (A.22) on L_{np}^j, we get

$$\gamma_{rn}^j \frac{1}{\theta} \frac{w_n}{T_n^j} - \gamma_{Ln}^j \frac{1+\theta}{\theta} w_n - \mu_n + \sum_{i \neq 1}^N \sum_k^{N_s} \gamma_{Li}^k \beta_k \pi_{ni} \frac{1+\theta}{\theta} w_n$$
$$+ u_c \sum_k^{N_s} \beta_k \frac{\tau_{xn}^k}{1+\tau_{xn}^k} \pi_{n1}^k \frac{1+\theta}{\theta} w_n + \sum_k^{N_s} \gamma_{rn}^k \beta (1-\delta) \sigma M_n^k \frac{1+\theta}{\theta} \frac{w_n}{x_n} = 0$$

multiply by total production labor $\sum_j (L_{np}^j)$ and use the relation $x_n = \frac{1+\theta}{\theta} w_n \sum_j (L_{np}^j)$,

$$\gamma_{rn}^j \frac{1}{1+\theta} \frac{x_n}{T_n^j} - \gamma_{Ln}^j x_n - \mu_n \sum_j (L_{np}^j) + \sum_{i \neq 1}^N \sum_k^{N_s} \gamma_{Li}^k \beta_k \pi_{ni} x_n$$
$$+ u_c \sum_k^{N_s} \beta_k \frac{\tau_{xn}^k}{1+\tau_{xn}^k} \pi_{n1}^k x_n + \sum_k^{N_s} \gamma_{rn}^k \beta (1-\delta) \sigma M_n^k = 0 \tag{A.35}$$

2. Use FOC (A.33) on w_n, $\gamma_P = u_c x_1$, $\gamma_{L1} = 0$, and $\gamma_4 = u_c$, we get

$$- u_c x_1 \sum_j^{N_s} \beta_j \pi_{1n}^j + \sum_{i \neq 1}^N \sum_j^{N_s} \gamma_{Li}^j \beta_j \left(\sum_{m \neq 1} \theta \pi_{mi}^j \pi_{mn}^j x_m \right)$$
$$+ \sum_{i \neq 1}^N \sum_j^{N_s} \gamma_{Li}^j \beta_j \left(\pi_{ni}^j x_n \right) - \sum_j^{N_s} \gamma_{Ln}^j \left(\theta \beta_j \sum_{m \neq 1} \pi_{mn}^j x_m + \frac{1+\theta}{\theta} w_n L_{np}^j \right)$$
$$+ u_c \sum_{j=1}^{N_s} \beta_j \left(\sum_{m \neq 1}^N \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \theta \pi_{m1}^j \pi_{mn}^j x_m + \frac{\tau_{xn}^j}{1+\tau_{xn}^j} \pi_{n1}^j x_n \right)$$
$$+ \sum_j^{N_s} \gamma_{rn}^j \left[\beta(1-\delta)(\sigma-1) M_n^j \right] + \sum_{m \neq 1}^N \sum_j^{N_s} \gamma_{rm}^j \left[\beta(1-\delta)(1-\sigma) M_m^j \left(\sum_k^{N_s} \beta_k \pi_{mn}^k \right) \right] = 0, \tag{A.36}$$

3. Substract equation (A.36) from equation (A.35):

$$\gamma^j_{rn}\frac{1}{1+\theta}\frac{x_n}{T^j_n}-\gamma^j_{Ln}x_n-\mu_n\sum_j^{N_s}(L^j_{np})+u_c x_1\sum_j^{N_s}\beta_j\pi^j_{1n}-\sum_{i\neq 1}^{N}\sum_j^{N_s}\gamma^j_{Li}\beta_j\left(\sum_{m\neq 1}\theta\pi^j_{mi}\pi^j_{mn}x_m\right)$$
$$+\sum_j^{N_s}\gamma^j_{Ln}\theta\beta_j\sum_{m\neq 1}\pi^j_{mn}x_m+\sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np}-u_c\sum_{j=1}^{N_s}\beta_j\left(\sum_{m\neq 1}^{N}\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\theta\pi^j_{m1}\pi^j_{mn}x_m\right)$$
$$+\sum_j^{N_s}\gamma^j_{rn}\left[\beta(1-\delta)M^j_n\right]+\sum_{m\neq 1}^{N}\sum_j^{N_s}\gamma^j_{rm}\left[\beta(1-\delta)(\sigma-1)M^j_m\left(\sum_k^{N_s}\beta_k\pi^k_{mn}\right)\right]=0, \tag{A.37}$$

4. At the steady state $\gamma^j_{T_n}=\gamma^{j\prime}_{T_n}$, the FOC (A.32) of T^j_n for $n>1$ becomes,

$$\theta(1-\beta(1-\delta))\gamma^j_{Tn}T^j_n=-\gamma^j_{rn}\frac{w_n L^j_{np}}{T^j_n}+u_c x_1\beta_j\pi^j_{1n}-\theta u_c\sum_{m\neq 1}^{N}\beta_j\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\pi^j_{m1}\pi^j_{mn}x_m$$
$$-\theta\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma^j_{Li}\beta_j\pi^j_{mi}\pi^j_{mn}x_m+\theta\gamma^j_{Ln}\beta_j\sum_{m\neq 1}\pi^j_{mn}x_m$$
$$-\sum_{i\neq 1}^{N}\left(\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)(1-\sigma)M^k_i\right)\beta_j\pi^j_{in}+\theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial G^k_i}{\partial T^j_n}x^\sigma_i P^{1-\sigma}_i$$

summing over j

$$-\theta(1-\beta(1-\delta))\sum_j^{N_s}\gamma^j_{Tn}T^j_n-\sum_j^{N_s}\gamma^j_{rn}\frac{w_n L^j_{np}}{T^j_n}+u_c x_1\sum_j^{N_s}\beta_j\pi^j_{1n}-\theta u_c\sum_j^{N_s}\sum_{m\neq 1}^{N}\beta_j\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\pi^j_{m1}\pi^j_{mn}x_m$$
$$-\theta\sum_j^{N_s}\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma^j_{Li}\beta_j\pi^j_{mi}\pi^j_{mn}x_m+\theta\sum_j^{N_s}\gamma^j_{Ln}\beta_j\sum_{m\neq 1}\pi^j_{mn}x_m$$
$$-\sum_{i\neq 1}^{N}\left(\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)(1-\sigma)M^k_i\right)\sum_j\beta_j\pi^j_{in}+\sum_j\theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial G^k_i}{\partial T^j_n}x^\sigma_i P^{1-\sigma}_i, \tag{A.38}$$

5. Combine equation (A.37) and (A.38)

$$\gamma^j_{rn}\frac{1}{1+\theta}\frac{x_n}{T^j_n}-\gamma^j_{Ln}x_n-\mu_n\sum_j^{N_s}L^j_{np}+\sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np}+\theta(1-\beta(1-\delta))\sum_j^{N_s}\gamma^j_{Tn}T^j_n+\sum_j^{N_s}\gamma^j_{rn}\frac{w_n L^j_{np}}{T^j_n}$$
$$+\sum_j^{N_s}\gamma^j_{rn}\left[\beta(1-\delta)M^j_n\right]-\sum_j\theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial G^k_i}{\partial T^j_n}x^\sigma_i P^{1-\sigma}_i=0. \tag{A.39}$$

According to FOC (A.21) on L^j_{nr} for $n>1$ and no externalities, we have $\gamma^j_{Tn}=\mu_n/\alpha^j_n$. In addition, at the steady state $T_n^j = \frac{\alpha_n^j L_{nr}^j}{\delta} = \frac{\alpha_n^j}{\delta}\bar{r}L_n^j$. Hence, equation (A.39) becomes

$$\gamma_{rn}^j \frac{1}{1+\theta}\frac{x_n}{T_n^j} - \gamma_{Ln}^j x_n - \mu_n(1-\bar{r})L_n + \sum_j^{N_s}\gamma_{Ln}^j \frac{1+\theta}{\theta} w_n L_{np}^j + \theta(1-\beta(1-\delta))\frac{1}{\delta}\bar{r}\mu_n L_n$$
$$+ \sum_j^{N_s}\gamma_{rn}^j \frac{w_n L_{np}^j}{T_n^j} + \sum_j^{N_s}\gamma_{rn}^j \left[\beta(1-\delta)M_n^j\right] - \sum_j \theta T_n^j \sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma_{ri}^k \beta(1-\delta)\frac{\partial G_i^k}{\partial T_n^j} x_i^\sigma P_i^{1-\sigma} = 0.$$

Using $\bar{r} = \frac{\delta}{\theta(1-\beta(1-\delta))+\delta}$ proved in Proposition 1 and definition of G_i^k from (A.17), we have

$$\gamma_{rn}^j \frac{1}{1+\theta}\frac{x_n}{T_n^j} - \gamma_{Ln}^j x_n + \sum_j^{N_s}\gamma_{Ln}^j \frac{1+\theta}{\theta} w_n L_{np}^j$$
$$+ \sum_j^{N_s}\gamma_{rn}^j \frac{w_n\delta(1-\bar{r})}{\alpha_n^j \bar{r}} + \sum_j^{N_s}\gamma_{rn}^j \left[\beta(1-\delta)\frac{w_n}{\alpha_n^j}\right] - \sum_j \theta T_n^j \sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma_{ri}^k \beta(1-\delta)\frac{\partial(x_i'^{-\sigma}P_i'^{\sigma-1}w_i')}{\partial T_n^j}\frac{1}{\alpha_i^k} x_i^\sigma P_i^{1-\sigma} = 0. \tag{A.40}$$

6. Multiply equation (A.40) with $\frac{\frac{1+\theta}{\theta}w_n L_{np}^j}{x_n}$ on both sides and sum over j

$$\sum_j^{N_s}\gamma_{rn}^j \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \sum_j^{N_s}\gamma_{rn}^j \frac{w_n\delta(1-\bar{r})}{\alpha_n^j\bar{r}} + \sum_j^{N_s}\gamma_{rn}^j\beta(1-\delta)\frac{w_n}{\alpha_n^j}$$
$$- \sum_j \theta T_n^j \sum_{i\neq 1}^{N}\sum_k^{N_s}\frac{\gamma_{ri}^k}{\alpha_i^k}\beta(1-\delta)\frac{\partial(x_i'^{-\sigma}P_i'^{\sigma-1}w_i')}{\partial T_n^j} x_i^\sigma P_i^{1-\sigma} = 0$$

which implies

$$\sum_j^{N_s}\gamma_{rn}^j \frac{w_n(1-\bar{r})L_n^j}{\theta\alpha_n^j\bar{r}L_n^j/\delta} + \sum_j^{N_s}\gamma_{rn}^j \frac{w_n\delta(1-\bar{r})}{\alpha_n^j\bar{r}} + \sum_j^{N_s}\gamma_{rn}^j\beta(1-\delta)\frac{w_n}{\alpha_n^j}$$
$$- \sum_j \theta T_n^j \sum_{i\neq 1}^{N}\sum_k^{N_s}\frac{\gamma_{ri}^k}{\alpha_i^k}\beta(1-\delta)\frac{\partial(x_i'^{-\sigma}P_i'^{\sigma-1}w_i')}{\partial T_n^j} x_i^\sigma P_i^{1-\sigma} = 0.$$

Combining terms, we have

$$\left(\sum_k^{N_s}\frac{\gamma_{rn}^k}{\alpha_n^k}\right)(1+\theta(1-\beta(1-\delta)))\,w_n - \beta(1-\delta)\sum_j \theta T_n^j \sum_{i\neq 1}^{N}\left(\sum_k^{N_s}\frac{\gamma_{ri}^k}{\alpha_i^k}\right)\frac{\partial(x_i'^{-\sigma}P_i'^{\sigma-1}w_i')}{\partial T_n^j} x_i^\sigma P_i^{1-\sigma} = 0.$$

Zero vector is a solution of the homogenous system of linear equations. For every $n \neq 1$, $\sum_k^{N_s}\frac{\gamma_{rn}^k}{\alpha_n^k} = 0$.

C Characterization of Ramsey policy

In this appendix, we first lay out the Home government's Ramsey problem. We then characterize the Ramsey policy by taking first-order conditions. In C.3 we prove the steady-state properties.

C.1 Ramsey problem

At period 0, Home planner chooses a squence of $\left\{L^j_{1rt}, \tau^j_{xnt}, t^j_{nt}\right\}_{t=0}^{\infty}$ to maximize the present value of utility subject to the implementability constraints (private equilibrium every period):

$$V = \max \sum_{t=0}^{\infty} \beta^t \frac{x_{1t}^{1-\sigma}}{1-\sigma},$$

subject to, for each $t \geq 0$

$$T^j_{nt} = \alpha^j_n L^j_{nrt} + (1-\delta) T^j_{n,t-1}, \quad (\gamma^j_{Tnt})$$

$$P_1 = \Pi_j \left[T^j_{1t} (w_{1t} d)^{-\theta} + \sum_{n \neq 1} T^j_{nt} (w_{nt}(1+t^j_{nt}) d)^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = 1, \quad (\gamma_{Pt})$$

$$\frac{w_{nt}}{\alpha^j_n} x_{nt}^{-\sigma} P_{nt}^{\sigma-1} = \sum_{s=t}^{\infty} [\beta(1-\delta)]^{s-t} x_{ns}^{-\sigma} P_{ns}^{\sigma-1} \frac{1}{\theta} \frac{w_{ns} L^j_{nps}}{T^j_{ns}}, \quad (\beta^t \gamma^j_{vnt})$$

$$\frac{1+\theta}{\theta} w_{1t} L^j_{1pt} = \beta_j \left[\pi^j_{11t} x_{1t} + \sum_{m \neq 1} \frac{1}{1+\tau^j_{xmt}} \pi^j_{m1t} x_{mt} \right], \quad (\gamma^j_{L1t})$$

$$\frac{1+\theta}{\theta} w_{nt} L^j_{npt} = \beta_j \left[\frac{1}{1+t^j_{nt}} \pi^j_{1nt} x_{1t} + \sum_{m \neq 1} \pi^j_{mnt} x_{mt} \right], \quad (\gamma^j_{Lnt})$$

$$\sum_j \left(L^j_{nrt} + L^j_{npt} \right) = 1, \quad (\mu_{nt})$$

$$x_{1t} = \frac{1+\theta}{\theta} w_{1t} \sum_j L^j_{1pt} + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau^j_{xmt}}{1+\tau^j_{xmt}} \pi^j_{m1t} x_{mt} + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t^j_{mt}}{1+t^j_{mt}} \pi^j_{1mt} x_{1t}, \quad (\gamma_{4t})$$

where

$$x_{mt} = \frac{1+\theta}{\theta} w_{mt} \sum_j L^j_{mpt}$$

$$P_{mt} = \Pi_j \left[T^j_{1t}(w_{1t}(1+\tau^j_{xmt})d)^{-\theta} + \sum_{n\neq 1} T^j_{nt}(w_{nt}d)^{-\theta} \right]^{-\frac{\beta_j}{\theta}}$$

The government decides the entire path of policies which will be honored in the future, as in particular, future policies would affect the foreign individual expected value of innovation. The Langrangian

$$L = \sum_{t=0}^{\infty} \beta^t \frac{x_{1t}^{1-\sigma}}{1-\sigma} + \sum_{t=0}^{\infty} \sum_{n\neq 1} \sum_{j}^{N_s} \beta^t \gamma^j_{vnt} \left[\sum_{s=t}^{\infty} [\beta(1-\delta)]^{s-t} \frac{1}{\theta} \frac{w_{ns} L^j_{nps}}{T^j_{ns}} x_{ns}^{-\sigma} P_{ns}^{\sigma-1} - \frac{w_{nt}}{\alpha^j_n (L^j_{nr})} x_{nt}^{-\sigma} P_{nt}^{\sigma-1} \right]$$
$$+ \sum_{t=0}^{\infty} \sum_{n} \sum_{j}^{N_s} \beta^t \gamma^j_{Tnt} (\alpha^j_n L^j_{nrt} + (1-\delta) T^j_{n,t-1} - T^j_{nt})$$
$$+ \sum_{t=0}^{\infty} \sum_{j}^{N_s} \beta^t \gamma^j_{L1t} [\beta_j (\pi^j_{11t} x_{1t} + \sum_{m\neq 1} \frac{1}{1+\tau^j_{xmt}} \pi^j_{m1t} x_{mt}) - \frac{1+\theta}{\theta} w_{1t} L^j_{1pt}]$$
$$+ \sum_{t=0}^{\infty} \sum_{n\neq 1} \sum_{j}^{N_s} \beta^t \gamma^j_{Lnt} [\beta_j (\frac{1}{1+t^j_{nt}} \pi^j_{1nt} x_{1t} + \sum_{m\neq 1} \pi^j_{mnt} x_{mt}) - \frac{1+\theta}{\theta} w_{nt} L^j_{npt}]$$
$$+ \sum_{t=0}^{\infty} \beta^t \gamma_{Pt} [1 - \Pi_j \left[T^j_{1t}(w_{1t}d)^{-\theta} + \sum_{n\neq 1} T^j_{nt}(w_{nt}(1+t^j_{nt})d)^{-\theta} \right]^{-\frac{\beta_j}{\theta}}]$$
$$+ \sum_{t=0}^{\infty} \sum_{n} \beta^t \mu_{nt} [1 - \sum_j \left(L^j_{nrt} + L^j_{npt} \right)]$$
$$+ \sum_{t=0}^{\infty} \beta^t \gamma_{4t} (\frac{1+\theta}{\theta} w_{1t} \sum_j L^j_{1pt} + \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau^j_{xmt}}{1+\tau^j_{xmt}} \pi^j_{m1t} x_{mt} + \sum_{m\neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t^j_{mt}}{1+t^j_{mt}} \pi^j_{1mt} x_{1t} - x_{1t})$$

can be rewritten as

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{x_{1t}^{1-\sigma}}{1-\sigma} - \sum_{n\neq 1} \sum_{j}^{N_s} \gamma^j_{vnt} \frac{w_{nt}}{\alpha^j_n (L^j_{nr})} x_{nt}^{-\sigma} P_{nt}^{\sigma-1} + \sum_{n\neq 1} \sum_{j}^{N_s} \left(\sum_{s=0}^{t} \gamma^j_{vns} (1-\delta)^{t-s} \right) \frac{1}{\theta} \frac{w_{nt} L^j_{npt}}{T^j_{nt}} x_{nt}^{-\sigma} P_{nt}^{\sigma-1} + ... \right\}$$

where we omit those constraints that are the same as in the Markov problem.

Let

$$\Gamma^j_{vnt} = \sum_{s=0}^{t} (1-\delta)^{t-s} \gamma^j_{vns}, \qquad \text{for any } n \neq 1 \text{ and } j$$

Recursively

$$\Gamma^j_{vnt} = (1-\delta)\Gamma^j_{vnt-1} + \gamma^j_{vnt}$$

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{x_{1t}^{1-\sigma}}{1-\sigma} - \sum_{n\neq 1} \sum_{j}^{N_s} \gamma^j_{vnt} \frac{w_{nt}}{\alpha^j_n (L^j_{nr})} x_{nt}^{-\sigma} P_{nt}^{\sigma-1} + \sum_{n\neq 1} \sum_{j}^{N_s} \left((1-\delta)\Gamma^j_{vnt-1} + \gamma^j_{vnt} \right) \frac{1}{\theta} \frac{w_{nt} L^j_{npt}}{T^j_{nt}} x_{nt}^{-\sigma} P_{nt}^{\sigma-1} + ... \right\}$$

Hence the problem can be written as recursively,

$$L\left(\left\{\Gamma^j_{n,-1}, T^j_{n,-1}\right\}\right) = \inf_{\gamma^j_{vn}} \sup_{x_1,..} \frac{x_1^{1-\sigma}}{1-\sigma} + \sum_{n\neq 1} \sum_{j}^{N_s} \left[\frac{1}{\theta} \frac{L^j_{np}}{T^j_n} \left(\gamma^j_{vn} + (1-\delta)\Gamma^j_{vn,-1} \right) - \frac{1}{\alpha^j_n (L^j_{nr})} \gamma^j_{vn} \right] w_n x_n^{-\sigma} P_n^{\sigma-1}$$
$$+ ....\beta L\left(\left\{\Gamma^j_n, T^j_n\right\}\right)$$

where

$$\Gamma^j_{vn} = (1-\delta)\Gamma^j_{vn,-1} + \gamma^j_{vn}.$$

Similar to the proof in Section E, the FOCs of export taxes, import tariffs and L_{1r}, combine with T_1's give us no distortion in domestic innovation. The FOCs of T_n, L_{np}, L_{nr} will give us multipliers γ and tariffs.

C.2 Optimal conditions for Ramsey policy

FOC over T^j_1 changes to

$$-\gamma^j_{T1} + \beta E \frac{\partial V}{\partial T^j_1} + \gamma_P \frac{\beta_j}{\theta} \frac{\pi^j_{11}}{T^j_1} + \sum_{n\neq 1}^{N} \sum_{k}^{N_s} \left[\gamma^k_{vn} \left(\frac{1}{\alpha^j_n} - \frac{1}{\theta} \frac{L^k_{np}}{T^k_n} \right) - (1-\delta)\Gamma^k_{vn,-1} \frac{1}{\theta} \frac{L^k_{np}}{T^k_n} \right] w_n x_n^{-\sigma} P_n^{\sigma-1} (\sigma-1) \left(\frac{\beta_j}{\theta} \frac{\pi^j_{n1}}{T^j_1} \right)$$
$$+ \gamma^j_{L1}\beta_j \left[\frac{\partial \pi^j_{11}}{\partial T^j_1} x_1 + \sum_{m\neq 1}^{N} \frac{1}{1+\tau^j_{xm}} \frac{\partial \pi^j_{m1}}{\partial T^j_1} x_m \right] + \sum_{n\neq 1}^{N} \gamma^j_{Ln}\beta_j \left[\frac{1}{1+t^j_n} \frac{\partial \pi^j_{1n}}{\partial T^j_1} x_1 + \sum_{m\neq 1} \frac{\partial \pi^j_{mn}}{\partial T^j_1} x_m \right]$$
$$+ \gamma_4 \sum_{m\neq 1}^{N} \beta_j \frac{\tau^j_{xm}}{1+\tau^j_{xm}} \frac{\partial \pi^j_{m1}}{\partial T^j_1} x_m + \gamma_4 \sum_{m\neq 1}^{N} \beta_j \frac{t^j_m}{1+t^j_m} \frac{\partial \pi^j_{1m}}{\partial T^j_1} x_1 = 0$$

FOC over τ^j_{xn}

$$\sum_k^{N_s}\left[\gamma^k_{vn}\left(\frac{1}{\alpha^j_n}-\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\right)-(1-\delta)\Gamma^k_{vn,-1}\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\right]w_n x_n^{-\sigma}P_n^{\sigma-1}(1-\sigma)\beta_j\frac{\pi^j_{n1}}{1+\tau^j_{xn}}$$
$$+\gamma^j_{L1}\beta_j\frac{\partial\pi^j_{11}}{\partial\tau^j_x}x_1-\gamma^j_{L1}\beta_j\frac{1}{(1+\tau^j_{xn})^2}\pi^j_{n1}x_n+\gamma^j_{L1}\beta_j\frac{1}{1+\tau^j_{xn}}\frac{\partial\pi^j_{n1}}{\partial\tau^j_{xn}}x_n+\sum_{i\neq1}^{N}\gamma^j_{Li}\beta_j\left[\frac{\partial\pi^j_{1i}}{\partial\tau^j_{xn}}x_1+\sum_{m\neq1}\frac{\partial\pi^j_{m,i}}{\partial\tau^j_{xn}}x_m\right]$$
$$+\gamma_4\beta_j\frac{1}{(1+\tau^j_{xn})^2}\pi^j_{n1}x_n+\gamma_4\sum_{m\neq1}^{N}\sum_{j=1}^{N_s}\beta_j\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\frac{\partial\pi^j_{m1}}{\partial\tau^j_{xn}}x_m=0$$

Summing FOC on τ^j_{xn} over $n\neq1$, we can further write FOC on T^j_1

$$\gamma^j_{T1}=\beta E\left[\gamma^{j\prime}_{T1}(1-\delta)\right]+\frac{1}{\theta}\frac{1}{T^j_1}\{\gamma_P\beta_j\pi^j_{11}$$
$$+\gamma_{L1}\theta\beta_j\left[\pi^j_{11}(1-\pi^j_{11})x_1\right]+\sum_{n\neq1}(\gamma_4-\gamma_{L1})\frac{1}{(1+\tau^j_{xn})}\beta_j\pi^j_{n1}x_n$$
$$-\theta\pi^j_{11}x_1\sum_{n\neq1}^{N}\frac{1}{1+t^j_n}\gamma^j_{Ln}\beta_j\pi^j_{1n}-\theta\gamma_4\sum_{n\neq1}^{N}\beta_j\frac{t^j_n}{1+t^j_n}\pi^j_{11}\pi^j_{1m}x_1\}$$

As can be seen, the wedge $\sum_{n\neq1}^{N}\sum_k^{N_s}\left[\gamma^k_{vn}\left(\frac{1}{\alpha^j_n}-\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\right)-(1-\delta)\Gamma^k_{vn,-1}\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\right]w_n x_n^{-\sigma}P_n^{\sigma-1}(\sigma-1)\left(\frac{\beta_j}{\theta}\frac{\pi^j_{n1}}{T^j_1}\right)$ in Home government Euler for T^j_1 reflects dynamic terms of trade consideration. It can be fully implemented by country-sector specific export tax, as the term

$$\sum_k^{N_s}\left[\gamma^k_{vn}\left(\frac{1}{\alpha^j_n}-\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\right)-(1-\delta)\Gamma^k_{vn,-1}\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\right]w_n x_n^{-\sigma}P_n^{\sigma-1}(1-\sigma)\beta_j\frac{\pi^j_{n1}}{1+\tau^j_{xn}}$$

shows up in the FOCs of export taxes. So after substituting FOCs of export taxes, the dynamic wedges in Euler disappear.

Furthermore, sum over the FOCs of optimal tariffs

$$\gamma_P\beta_j\pi^j_{11}+\theta\beta_j\pi^j_{11}x_1\left\{\gamma_{L1}\left[(1-\pi^j_{11})\right]-\sum_{n\neq1}^{N}\frac{1}{1+t^j_n}\gamma^j_{Ln}\pi^j_{1n}-\gamma_4\sum_{n\neq1}^{N}\frac{t^j_n}{1+t^j_n}\pi^j_{1m}\right\}+(\gamma_4-\gamma_{L1})\sum_{n\neq1}\frac{1}{(1+\tau^j_{xn})}\beta_j\pi^j_{n1}x_n$$
$$=\gamma_P\beta_j\pi^j_{11}+(\gamma_4-\gamma_{L1})\frac{1+\theta}{\theta}w_1L^j_{1p}-(\gamma_4-\gamma_{L1})\beta_j\pi^j_{11}x_1$$
$$=(\gamma_4-\gamma_{L1})\frac{1+\theta}{\theta}w_1L^j_{1p},$$

hence

$$\gamma_{T1}^{j}=\beta(1-\delta)E\left[\gamma_{T1}^{j\prime}\right]+\frac{1}{\theta}\frac{1}{T_1^j}\left(\gamma_4-\gamma_{L1}\right)\frac{1+\theta}{\theta}w_1L_{1p}^{j}.$$

Country-sector specific tariff implement the intratemporal allocation across countries sectors.

The FOC of L_{1r}^{j} implies

$$\gamma_{T1}^{j}=\left(\gamma_4-\gamma_{L1}\right)\frac{1+\theta}{\theta}\frac{w_1}{\alpha_1^j},$$

plug back into the above equation, we get

$$\left(\gamma_4-\gamma_{L1}\right)\frac{1+\theta}{\theta}\frac{w_1}{\alpha_1^j}=\beta(1-\delta)\left(\gamma_4'-\gamma_{L1}'\right)\frac{1+\theta}{\theta}\frac{w_1'}{\alpha_1^j}+\frac{1}{\theta}\frac{1}{T_1^j}\left(\gamma_4-\gamma_{L1}\right)\frac{1+\theta}{\theta}w_1L_{1p}^{j}$$

Since $\left(\gamma_4-\gamma_{L1}\right)=u_c$, it further reduces to:

Home government Euler on home research

$$\frac{w_1}{\alpha_1^j}=\frac{1}{\theta}\frac{w_1L_{1p}^{j}}{T_1^j}+\beta(1-\delta)E\left[\frac{u_c'w_1'}{u_c\alpha_1^j}\right]. \tag{A.41}$$

Government do not use industry policies.

Optimal Ramsey export tax

$$1+\tau_{xn}^{jR}=\frac{1+\theta(1-\pi_{n1}^{j})}{\theta\sum_{m\neq1}(1+t_m^{jR})\pi_{nm}^{j}+\sum_k^{N_s}\left(\gamma_{vn}^{k}\frac{1}{\alpha_n^j}-\Gamma_{vn}^{k}\frac{1}{\theta}\frac{L_{np}^{k}}{T_n^k}\right)w_nx_n^{-\sigma}P_n^{\sigma-1}(\sigma-1)/(u_{c_1}x_n)} \tag{A.42}$$

Optimal Ramsey t_n^{jR}

$$t_n^{jR}=-\frac{\gamma_{L_n}^{j}}{u_c} \tag{A.43}$$

C.3 Proof of $\sum_j^{N_s} \frac{\gamma_{Tn}^j}{\alpha_n^j} = 0$ at the steady state when $\alpha_n^j(L_{nr}^j, T_{n,-1}^j) = \bar{\alpha}_n^j$.

FOC over L_{np}^j

$$\frac{1}{\theta}\frac{w_n}{T_n^j}\Gamma_{vn}^j x_n^{-\sigma}P_n^{\sigma-1} - \gamma_{Ln}^j\frac{1+\theta}{\theta}w_n - \mu_n + \sum_{k=1}^{N_s}\gamma_{L1}^k\beta_k\frac{1}{1+\tau_{xn}^k}\pi_{n1}^k\frac{1+\theta}{\theta}w_n + \sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma_{Li}^k\beta_k\pi_{n,i}\frac{1+\theta}{\theta}w_n$$
$$+ \gamma_4\sum_k^{N_s}\beta_k\frac{\tau_{xn}^k}{1+\tau_{xn}^k}\pi_{n1}^k\frac{1+\theta}{\theta}w_n + \sum_k^{N_s}\left[\frac{1}{\theta}\frac{L_{np}^k}{T_n^k}\Gamma_{vn}^k - \frac{1}{\alpha_n^k}\gamma_{vn}^k\right]w_n x_n^{-\sigma}P_n^{\sigma-1}(-\sigma)\frac{1+\theta}{\theta}\frac{w_n}{x_n} = 0 \quad \text{(A.44)}$$

FOC over w_n

$$-\gamma_P\sum_j^{N_s}\beta_j\pi_{1n}^j + \gamma_{L1}\left[\sum_{m\neq 1}^{N}\sum_j^{N_s}\beta_j\frac{1}{1+\tau_{xm}^j}\theta\pi_{m1}^j\pi_{mn}^j x_m + \sum_j^{N_s}\beta_j\frac{1}{1+\tau_{xn}^j}\pi_{n1}^j x_n\right]$$
$$+\sum_{i\neq\{1\}}^{N}\sum_j^{N_s}\gamma_{Li}^j\beta_j\left(\sum_{m\neq 1}\theta\pi_{mi}^j\pi_{m,n}^j x_m + \pi_{n,i}^j x_n\right) - \sum_j^{N_s}\gamma_{Ln}^j\left(\theta\beta_j\sum_{m\neq 1}\pi_{m,n}^j x_m + \frac{1+\theta}{\theta}w_n L_{np}^j\right)$$
$$+\gamma_4\sum_{j=1}^{N_s}\beta_j\left(\sum_{m\neq 1}^{N}\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\theta\pi_{m1}^j\pi_{mn}^j x_m + \frac{\tau_{xn}^j}{1+\tau_{xn}^j}\pi_{n1}^j x_n\right) + (1-\sigma)\sum_j^{N_s}\left[\frac{1}{\theta}\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j - \frac{1}{\alpha_n^j}\gamma_{vn}^j\right]w_n x_n^{-\sigma}P_n^{\sigma-1}$$
$$+\sum_{m\neq 1}^{N}\sum_j^{N_s}\left[\frac{1}{\theta}\frac{L_{mp}^j}{T_m^j}\Gamma_{vm}^j - \frac{1}{\alpha_m^j}\gamma_{vm}^j\right]w_m x_m^{-\sigma}P_m^{\sigma-1}(\sigma-1)\left(\sum_k\beta_k\pi_{mn}^k\right) = 0 \quad \text{(A.45)}$$

FOC over T_n^j

$$\gamma_{Tn}^j = \beta E\left[\gamma_{Tn}^{j\prime}(1-\delta)\right] + \frac{1}{T_n^j}\{\gamma_P\frac{\beta_j}{\theta}\pi_{1n}^j$$
$$-\sum_{i=1}^{N}\gamma_{Li}^j\beta_j\sum_{m\neq 1}\pi_{m,i}^j\pi_{mn}^j x_m + \gamma_{Ln}^j\beta_j\sum_{m\neq 1}\pi_{m,n}^j x_m - u_c\sum_{m\neq 1}^{N}\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m,1}^j\pi_{mn}^j x_m$$
$$-\left[\frac{1}{\theta}\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_{i\neq 1}\sum_k^{N_s}\left[\frac{1}{\theta}\frac{L_{ip}^k}{T_i^k}\Gamma_{vi}^k - \frac{1}{\alpha_i^k}\gamma_{vi}^k\right]w_i x_i^{-\sigma}P_i^{\sigma-1}(\sigma-1)\frac{\beta_j}{\theta}\pi_{in}^j\} \quad \text{(A.46)}$$

1. At the steady state $\gamma_{T_n}^j = \gamma_{T_n}^{j\prime}$, summing over j the FOC over T_n^j:

$$-\theta(1-\beta(1-\delta))\sum_j^{N_s}\gamma_{Tn}^j T_n^j + u_c x_1\sum_j^{N_s}\beta_j\pi_{1n}^j - \theta u_c\sum_j^{N_s}\sum_{m\neq 1}^{N}\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j\pi_{mn}^j x_m$$
$$-\theta\sum_j^{N_s}\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma_{Li}^j\beta_j\pi_{mi}^j\pi_{mn}^j x_m + \theta\sum_j^{N_s}\gamma_{Ln}^j\beta_j\sum_{m\neq 1}\pi_{mn}^j x_m$$
$$-\sum_j^{N_s}\left[\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_{i\neq 1}\sum_k^{N_s}\left[\frac{1}{\theta}\frac{L_{ip}^k}{T_i^k}\Gamma_{vi}^k - \frac{1}{\alpha_i^k}\gamma_{vi}^k\right]w_i x_i^{-\sigma}P_i^{\sigma-1}(\sigma-1)\sum_j^{N_s}\beta_j\pi_{in}^j, \quad \text{(A.47)}$$

2. Substract FOC over L^j_{np} and w_n from the above equation:

$$\frac{1}{\theta}\frac{w_n L_{np}}{T^j_n}\Gamma^j_{vn}x_n^{-\sigma}P_n^{\sigma-1} - \gamma^j_{Ln}x_n - \mu_n\sum_j^{N_s} L^j_{np} + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np} + \theta(1-\beta(1-\delta))\sum_j^{N_s}\gamma^j_{Tn}T^j_n$$
$$+\sum_j^{N_s}\left[\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_k^{N_s}\left[\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\Gamma^k_{vn} - \frac{1}{\alpha^k_n}\gamma^k_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} = 0 \quad \text{(A.48)}$$

3. At the steady state

$$T^j_n = \frac{\alpha^j_n(L^j_{nr})}{\delta}, \quad \mu_n = \gamma^j_{Tn}\alpha^j_n,$$

hence

$$\frac{1}{\theta}\frac{w_n L_{np}}{T^j_n}\Gamma^j_{vn}x_n^{-\sigma}P_n^{\sigma-1} - \gamma^j_{Ln}x_n - \mu_n\sum_j^{N_s} L^j_{np} + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np} + \theta(1-\beta(1-\delta))\sum_j \frac{1}{\delta}\mu_n L^j_{nr}$$
$$+\sum_j^{N_s}\left[\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_k^{N_s}\left[\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\Gamma^k_{vn} - \frac{1}{\alpha^k_n}\gamma^k_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} = 0 \quad \text{(A.49)}$$

At the steady state, the research intensity is given by

$$\bar{r}^j = \frac{\delta}{\theta(1-\beta(1-\delta))+\delta}.$$

Using this research intensity, $L^j_{np} = (1-\bar{r})L^j_n$, and $L^j_{nr} = \bar{r}L^j_n$, we can cancel out the two terms involving μ_n in equation (A.63). Then we have

$$\frac{1}{\theta}\frac{w_n L_{np}}{T^j_n}\Gamma^j_{vn}x_n^{-\sigma}P_n^{\sigma-1} - \gamma^j_{Ln}x_n + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np}$$
$$+\sum_j^{N_s}\left[\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_k^{N_s}\left[\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\Gamma^k_{vn} - \frac{1}{\alpha^k_n}\gamma^k_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} = 0.$$

4. Multiply the above equation $\frac{\frac{1+\theta}{\theta}w_n L^j_{np}}{x_n}$ on both sides and sum over j

$$\sum_j^{N_s}\left[\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} + \sum_k^{N_s}\frac{1}{\alpha^k_n}\gamma^k_{vn}w_n x_n^{-\sigma}P_n^{\sigma-1} = 0.$$

Finally,

$$\sum_k^{N_s}\frac{\gamma^k_{vn}}{\alpha^k_n}(1+\frac{\theta(1-\beta(1-\delta))}{\delta})w_n x_n^{-\sigma}P_n^{\sigma-1} = 0.$$

The equation only holds when for every $n \neq 1$, $\sum_k^{N_s} \frac{\gamma_{vn}^k}{\alpha_n^k} = 0$.

D Proof for Proposition 2 and 3: one-sector two-country

Proof for Proposition 2 Markov optimal policy under one-sector two-country case.

Under the assumption of no externality and one sector, the innovation efficiency is constant $\alpha_1^j(L_{1r}^j, T_{1,-1}^j) = \alpha_1$ and $\frac{\partial \alpha_1^j}{\partial L_{1r}^j} = 0$ for $j = 1$. In this case, Home's optimal condition for innovation (A.31) becomes

$$\frac{w_1}{\alpha_1} = \frac{1}{\theta}\frac{w_1 L_{1p}}{T_1} + \tilde{\beta}_1 v_1 + \underbrace{\frac{\theta}{1+\theta}\frac{\beta(1-\delta)}{u_{c_1}}\frac{1}{u_{c_2}/P_2}\gamma_{r2}\frac{\partial\left(u'_{c_2}\frac{w'_2}{P'_2}\right)}{\partial T_1}}_{\text{wedge}},$$

where we have substituted $G_2 = (x'_2)^{-\sigma}(P'_2)^{\sigma-1}w'_2/\alpha_2$ with $u'_{c_2}\frac{w'_2}{\alpha_2 P'_2}$ since $u_{c_2} = C_2^{-\sigma} = x_2^{-\sigma}P_2^{\sigma}$. We also used the definition of $\tilde{\beta}_1$ and $v_1 = w_1/\alpha_1$. This proved equation (18) in Proposition 2.

The Lerner symmetry holds, we normalize home tariff to be zero. Using the tariff formula (A.30) and $\gamma_4 = u_c$, we can back out the multiplier $\gamma_{L2} = \gamma_4 - u_c(1+t_n^j) = 0$. Using these conditions and the assumption of one sector, equation (A.29) becomes:

$$1 + \tau_x^M = \frac{1+\theta\pi_{22}}{\theta\pi_{22} + \gamma_{r2}(\sigma-1)\beta(1-\delta)\frac{u'_{c_2}/P'_2}{u_{c_2}/P_2}\frac{w'_2}{\alpha_2}\frac{1}{u_{c_1}x_2}},$$

where we used the condition $\pi_{22} = 1 - \pi_{21}$ and definition of M_n from (A.17) under one sector. This proved equation (17) in Proposition 2.

According to Appendix B.3, $\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0$ at the steady state when $\alpha_n^j(L_{nr}^j, T_{n,-1}^j) = \bar{\alpha}_n^j$. This implies under one sector $\gamma_{r2} = 0$ at the SS. □

Proof for Proposition 3 Ramsey optimal policy in one-sector two-country case.

The Euler equation, A.41 in the one sector case becomes:

$$\frac{w_1}{\alpha_1} = \frac{1}{\theta}\frac{w_1 L_{1p}}{T_1} + \beta(1-\delta)E\left[\frac{u'_c w'_1}{u_c \alpha_1}\right].$$

This is the same as private innovation equation.

Lerner symmetry hold, we normalize tariff to zero. The optimal Ramsey export tax, equation

A.42 becomes:

$$1+\tau_x^R = \frac{1+\theta\pi_{22}}{\theta\pi_{22}+(\sigma-1)\left(\gamma_{v2}\frac{w_2}{\alpha_2}-\Gamma_{v2}\frac{w_2}{\theta}\frac{L_{2p}}{T_2}\right)\frac{u_{c_2}}{P_2}/(u_{c_1}x_2)}.$$

Furthermore, According to Appendix C.3, at the SS, $\sum_j^{N_s}\frac{\gamma^j_{vn}}{\alpha^j_n}=0$ when $\alpha^j_n(L^j_{nr},T^j_{n,-1})=\bar{\alpha}^j_n$. This implies under one sector $\gamma_{v2}=0$ and $\Gamma_{v2}=0$ at the SS. □

E Proof for Proposition 5

E.1 Proof for no industry policy at SS

In the baseline, innovation has constant returns to scale (CRS), $\alpha_{nt}=\alpha_n$. There are no other distortions, no externalities nor international spillover. In Proposition 5, we consider a case with multi sectors and two countries.

Under these conditions, the optimal innovation choice of Home country satisfies,

FOC over T_1^j changes to

$$\begin{aligned}
&-\gamma^j_{T1}+\beta\frac{\partial V}{\partial T^j_1}+\gamma_P\frac{\beta_j}{\theta}\frac{\pi^j_{11}}{T^j_1}-\left(\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)(1-\sigma)M^k_i\right)\left(\frac{\beta_j}{\theta}\frac{\pi^j_{i1}}{T^j_1}\right)\\
&+\gamma^j_{L1}\beta_j\left[\frac{\partial\pi^j_{11}}{\partial T^j_1}x_1+\sum_{m\neq 1}^{N}\frac{1}{1+\tau^j_{xm}}\frac{\partial\pi^j_{m1}}{\partial T^j_1}x_2\right]+\sum_{n\neq 1}^{N}\gamma^j_{Ln}\beta_j\left[\frac{1}{1+t^j_n}\frac{\partial\pi^j_{1n}}{\partial T^j_1}x_1+\sum_{m\neq 1}\frac{\partial\pi^j_{mn}}{\partial T^j_1}x_m\right]\\
&+\gamma_4\sum_{m\neq 1}^{N}\beta_j\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\frac{\partial\pi^j_{m1}}{\partial T^j_1}x_m+\gamma_4\sum_{m\neq 1}^{N}\beta_j\frac{t^j_m}{1+t^j_m}\frac{\partial\pi^j_{1m}}{\partial T^j_1}x_1+\beta(1-\delta)\sum_{n\neq 1}^{N_s}\sum_k\gamma^k_{r,n}\frac{\partial G^k_n}{\partial T^j_1}x^\sigma_nP^{1-\sigma}_n=0.
\end{aligned}$$

The envelop condition is:

$$\frac{\partial V}{\partial T^j_{1,-1}}=\gamma^j_{T1}(1-\delta).$$

From the FOC of τ^j_{xn}

$$1+\tau^j_{xn}=\frac{(\gamma_4-\gamma_{L1})\left[1+\theta(1-\pi^j_{n1})\right]}{\gamma_4\theta(1-\pi^j_{n1})-\theta\left(\sum_{i\neq 1}^{N}\gamma^j_{Li}\pi^j_{ni}\right)+\left(\sum_k\gamma^k_{rn}\beta(1-\delta)(\sigma-1)M^k_n\right)x_n^{-1}},$$

From the FOC of t^j_n

$$1+t^j_n=\frac{\gamma_4-\gamma^j_{Ln}}{\gamma_4-\gamma_{L1}},$$

In the Baseline, there are no externality and spillover, so $\frac{\partial \alpha_1^j}{\partial L_{1r}^j} = 0$ and $\frac{\partial \alpha_1^{j'}}{\partial T_n^j} = 0$. At the SS, we have proved $\sum_j \gamma_{rn}^j / \alpha_n^j = 0$ in Appendix B.3 and given α_n^j are constant and do not depend on endogenous variables, we have $\sum_{n\neq 1}^N \sum_k^{N_s} \gamma_{r,n}^k \beta(1-\delta) \frac{\partial G_n^k}{\partial T_1^j} x_n^\sigma P_n^{1-\sigma} = 0$ and $\left(\sum_k \gamma_{rn}^k \beta(1-\delta)(\sigma-1) M_n^k\right) x_n^{-1} = 0$. The optimal policies simplify to:

$$1 + \tau_{xn}^j = \frac{(\gamma_4 - \gamma_{L1})\left[1 + \theta(1 - \pi_{n1}^j)\right]}{\gamma_4 \theta (1 - \pi_{n1}^j) - \theta \left(\sum_{i\neq 1}^N \gamma_{Li}^j \pi_{ni}^j\right)},$$

The innovation:

$$\frac{w_1}{\alpha_1^j} = \frac{1}{\theta} \frac{w_1 L_{1p}^j}{T_1^j} + \beta(1-\delta)\left[\frac{u_c'}{u_c} \frac{w_1'}{\alpha_1^j}\right],$$

which is the same as private free entry condition for researchers. So there are no distortions on Home innovation, Home government uses heterogenous tariff and tax across sectors.

E.2 Proof for optimal tariff in Proposition 5

Optimal innovation is proved in Section E.1. In this section we prove the optimal time-consistent trade policies at the SS in baseline model with two countries.

From the FOC for L_{np}^j

$$\mu_n = \gamma_{rn}^j \frac{1}{\theta} \frac{w_n}{T_n^j} - \gamma_{Ln}^j \frac{1+\theta}{\theta} w_n + \sum_{k=1}^{N_s} \gamma_{L1}^k \beta_k \frac{1}{1+\tau_{xn}^k} \pi_{n1}^k \frac{1+\theta}{\theta} w_n$$
$$+ \sum_{i\neq 1}^N \sum_k^{N_s} \gamma_{Li}^k \beta_k \pi_{ni} \frac{1+\theta}{\theta} w_n + \gamma_4 \sum_k^{N_s} \beta_k \frac{\tau_{xn}^k}{1+\tau_{xn}^k} \pi_{n1}^k \frac{1+\theta}{\theta} w_n + \sum_k^{N_s} \gamma_{rn}^k \beta(1-\delta)\sigma M_n^k \frac{1+\theta}{\theta} \frac{w_n}{x_n},$$

in a two-country case, for any two sectors,

$$\frac{\gamma_{r_2}^j}{T_2^j(1+\theta)} - \gamma_{L_2}^j = \frac{\gamma_{r_2}^{j'}}{T_2^{j'}(1+\theta)} - \gamma_{L_2}^{j'}. \tag{A.50}$$

From the FOC for T_n^j

$$\gamma_{Tn}^j = \beta\left[\gamma_{Tn}^{j\prime}\left(1-\delta+\frac{\partial\alpha_n^{j\prime}}{\partial T_n^j}L_{nr}^j+\gamma_{rn}^j\frac{w_n'}{\left(\alpha_n^{j\prime}\right)^2}\frac{\partial\alpha_n^{j\prime}}{\partial T_n^j}\right)\right]+\frac{1}{T_n^j}\{\gamma_P\frac{\beta_j}{\theta}\pi_{1n}^j$$
$$-\sum_{i=1}^{N}\gamma_{Li}^j\beta_j\sum_{m\neq 1}\pi_{mi}^j\pi_{mn}^j x_m+\gamma_{Ln}^j\beta_j\sum_{m\neq 1}\pi_{mn}^j x_m-u_c\sum_{m\neq 1}^{N}\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j\pi_{mn}^j x_m$$
$$-\gamma_{rn}^j\frac{1}{\theta}\frac{w_nL_{np}^j}{\left(T_n^j\right)}-\sum_{i\neq 1}^{N}\left(\sum_{k}^{N_s}\gamma_{ri}^k\beta(1-\delta)(1-\sigma)M_i^k\right)\frac{\beta_j}{\theta}\pi_{in}^j\}+\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma_{ri}^k\beta(1-\delta)\frac{\partial G_i^k}{\partial T_n^j}x_i^\sigma P_i^{1-\sigma},$$

In the baseline at the SS

$$\theta(1-\beta(1-\delta))\gamma_{Tn}^jT_n^j=u_cx_1\beta_j\pi_{1n}^j-\theta u_c\sum_{m\neq 1}^{N}\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j\pi_{mn}^j x_m$$
$$-\theta\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma_{Li}^j\beta_j\pi_{mi}^j\pi_{mn}^j x_m+\theta\gamma_{Ln}^j\beta_j\sum_{m\neq 1}\pi_{mn}^j x_m-\gamma_{rn}^j\frac{w_nL_{np}^j}{T_n^j}$$

In a two-country case,

$$\theta(1-\beta(1-\delta))\gamma_{T_2}^jT_2^j=u_cx_1\beta_j\pi_{12}^j-\theta u_c\beta_j\frac{\tau_{x2}^j}{1+\tau_{x2}^j}\pi_{21}^j\pi_{22}^j x_2$$
$$-\theta\gamma_{L_2}^j\beta_j\pi_{22}^j\pi_{22}^j x_2+\theta\gamma_{L2}^j\beta_j\pi_{22}^j x_2-\gamma_{r2}^j\frac{w_2L_{2p}^j}{T_2^j} \tag{A.51}$$

From the FOC for L_{nr}^j

$$\mu_n=\gamma_{Tn}^j\alpha_n^j \tag{A.52}$$

Combine A.51 and A.52,

$$\frac{\alpha_2^j}{T_2^j}\left\{u_cx_1\beta_j\pi_{12}^j-\theta u_c\beta_j\frac{\tau_{x2}^j}{1+\tau_{x2}^j}\pi_{21}^j\pi_{22}^j x_2-\theta\gamma_{L_2}^j\beta_j\pi_{22}^j\pi_{22}^j x_2+\theta\gamma_{L2}^j\beta_j\pi_{22}^j x_2-\gamma_{r2}^j\frac{w_2L_{2p}^j}{T_2^j}\right\} \tag{A.53}$$
$$=\frac{\alpha_2^{j\prime}}{T_2^{j\prime}}\left\{u_cx_1\beta_{j\prime}\pi_{12}^{j\prime}-\theta u_c\beta_{j\prime}\frac{\tau_{x2}^{j\prime}}{1+\tau_{x2}^{j\prime}}\pi_{21}^{j\prime}\pi_{22}^{j\prime}x_2-\theta\gamma_{L_2}^{j\prime}\beta_{j\prime}\pi_{22}^{j\prime}\pi_{22}^{j\prime}x_2+\theta\gamma_{L2}^{j\prime}\beta_{j\prime}\pi_{22}^{j\prime}x_2-\gamma_{r2}^{j\prime}\frac{w_2L_{2p}^{j\prime}}{T_2^{j\prime}}\right\}$$

In the two-country case,

$$1+t_2^j=\frac{u_c-\gamma_{L2}^j}{u_c}, \tag{A.54}$$

$$1+\tau_{x2}^{j}=\frac{(\gamma_4-\gamma_{L1})\left[1+\theta(1-\pi_{21}^{j})\right]}{\gamma_4\theta(1-\pi_{21}^{j})-\theta\gamma_{L2}^{j}\pi_{22}^{j}}=\frac{1+\theta\pi_{22}^{j}}{\theta\pi_{22}^{j}(1+t_2^{j})}$$

Equation (A.53) can be further simplified to

$$\frac{\alpha_2^j}{T_2^j}\left\{u_c x_1\beta_j\pi_{12}^j-u_c\frac{1}{1+\tau_x^j}\beta_j\pi_{21}^j x_2-\gamma_{r2}^j\frac{w_2L_{2p}^j}{T_2^j}\right\}=\frac{\alpha_2^{j'}}{T_2^{j'}}\left\{u_c x_1\beta_{j'}\pi_{12}^{j'}-u_c\frac{1}{1+\tau_x^{j'}}\beta_j'\pi_{21}^{j'}x_2-\gamma_{r2}^{j'}\frac{w_2L_{2p}^j}{T_2^j}\right\}$$

Combine with A.50 and A.54, and goods market clearing conditions, we get

$$t^j-t^{j'}=\frac{1}{\theta}\left(\frac{\frac{1+\theta}{\theta}w_1L_{1p}^j-\beta_jx_1}{\frac{1+\theta}{\theta}w_2L_{2p}^j}-\frac{\frac{1+\theta}{\theta}w_1L_{1p}^{j'}-\beta_{j'}x_1}{\frac{1+\theta}{\theta}w_2L_{2p}^{j'}}\right).$$

F Proof for Proposition 6

In this section we prove Proposition 6, i.e., the optimal time-consistent policies at the SS in baseline model with multiple countries.

Let's prove the followings hold

$$\sum_{n\neq 1}(t_n^j+\xi_n)\frac{1+\theta}{\theta}w_nL_{np}^j=\frac{1+\theta}{\theta}w_1L_{1p}^j-\beta_jx_1 \tag{A.55}$$

$$\frac{t_n^j}{1+t_n^j}\frac{\beta^j\pi_{1n}^jx_1}{\frac{1+\theta}{\theta}w_nL_{np}^j}=\delta_n+\frac{\sum_{m\neq 1}\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\beta^j\pi_{mn}^jx_m}{\frac{1+\theta}{\theta}w_nL_{np}^j} \tag{A.56}$$

Proof:

1. Proof of equation (A.55). Similar to the proof of Proposition 5, we use the FOCs for L_{np}^j and T_n^j to derive the optimal policy for tariffs. We also need to use other optimal conditions, e.g., export tax τ_{xn}^j to simplify the fomula.

   Sum over $m\neq 1$ of the FOCs on τ_{xn}^j

$$\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma_{Li}^j\beta_j\left[\theta\pi_{mi}^j\pi_{m1}^jx_m\right]+u_c\sum_{m\neq 1}\beta_j\frac{1}{(1+\tau_{xm}^j)}\pi_{m1}^jx_m-u_c\theta\sum_{m\neq 1}\sum_{n\neq 1}\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j\pi_{mn}^jx_m=0$$

Sum over $n \neq 1$ of the FOCs on T_n^j

$$\sum_{n\neq 1}\theta(1-\beta(1-\delta))\gamma_{Tn}^j T_n^j = -\sum_{n\neq 1}\gamma_{rn}^j \frac{w_n L_{np}^j}{T_n^j} + \sum_{n\neq 1} u_c x_1 \beta_j \pi_{1n}^j - \theta u_c \sum_{n\neq 1}\sum_{m\neq 1}^{N}\beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j \pi_{mn}^j x_m$$
$$-\theta\sum_{n\neq 1}\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma_{Li}^j\beta_j\pi_{mi}^j\pi_{mn}^j x_m + \theta\sum_{n\neq 1}\gamma_{Ln}^j\beta_j\sum_{m\neq 1}\pi_{mn}^j x_m$$

Combine these two:

$$\sum_{n\neq 1}\theta(1-\beta(1-\delta))\gamma_{Tn}^j T_n^j = -\sum_{n\neq 1}\gamma_{rn}^j \frac{w_n L_{np}^j}{T_n^j} + \sum_{n\neq 1} u_c x_1 \beta_j \pi_{1n}^j - u_c \sum_{m\neq 1}\beta_j \frac{1}{(1+\tau_{xm}^j)}\pi_{m1}^j x_m$$

which is

$$\sum_{n\neq 1}\theta(1-\beta(1-\delta))\gamma_{Tn}^j T_n^j = -\sum_{n\neq 1}\frac{\gamma_{rn}^j}{T_n^j} w_n L_{np}^j + u_c\left[\sum_{n\neq 1}\beta_j\pi_{1n}^j x_1 - \sum_{m\neq 1}\beta_j\frac{1}{(1+\tau_{xm}^j)}\pi_{m1}^j x_m\right]$$
$$= -\sum_{n\neq 1}\frac{\gamma_{rn}^j}{T_n^j} w_n L_{np}^j - u_c(\frac{1+\theta}{\theta}w_1 L_{1p}^j - \beta_j x_1)$$

At the steady state $\gamma_{Tn}^j\alpha_n^j - \mu_n = 0$,

$$\frac{\theta(1-\beta(1-\delta))}{\delta}\sum_{n\neq 1}\mu_n L_{nr}^j = -\sum_{n\neq 1}\frac{\gamma_{rn}^j}{T_n^j} w_n L_{np}^j - u_c\underbrace{(\frac{1+\theta}{\theta}w_1 L_{1p}^j - \beta_j x_1)}_{\text{Home net export of sector } j} \tag{A.57}$$

Recall the FOC for L_{np}^j, sum over country n

$$-\gamma_{rn}^j\frac{1}{\theta}\frac{w_n}{T_n^j} = -\gamma_{Ln}^j\frac{1+\theta}{\theta}w_n + \mu_n - \sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma_{Li}^k\beta_k\pi_{ni}\frac{1+\theta}{\theta}w_n - u_c\sum_{k}^{N_s}\beta_k\frac{\tau_{xn}^k}{1+\tau_{xn}^k}\pi_{n1}^k\frac{1+\theta}{\theta}w_n$$

and substitute back to equation A.57, and use $t_n^j = -\frac{\gamma_{Ln}^j}{u_c}$,

$$\frac{\theta(1-\beta(1-\delta))}{\delta}\sum_{n\neq 1}\mu_n L_{nr}^j - \sum_{n\neq 1}\mu_n L_{np}^j + \sum_{n\neq 1}\sum_{i\neq 1}^{N}\left(\sum_{k}^{N_s}\gamma_{Li}^k\beta_k\pi_{ni}^k\right)\frac{1+\theta}{\theta}w_n L_{np}^j +$$
$$u_c\sum_{n\neq 1}\left(\sum_{k}^{N_s}\beta_k\frac{\tau_{xn}^k}{1+\tau_{xn}^k}\pi_{n1}^k\right)\frac{1+\theta}{\theta}w_n L_{np}^j = u_c\sum_{n\neq 1}t_n^j\frac{1+\theta}{\theta}w_n L_{np}^j - u_c(\frac{1+\theta}{\theta}w_1 L_{1p}^j - \beta_j x_1)$$

$$\sum_{n\neq 1} t_n^j \frac{1+\theta}{\theta} w_n L_{np}^j + \sum_{n\neq 1}\sum_{i\neq 1}^{N}\left(\sum_k^{N_s} t_i^k \beta_k \pi_{ni}^k\right)\frac{1+\theta}{\theta} w_n L_{np}^j - \sum_{n\neq 1}\left(\sum_k^{N_s}\beta_k \frac{\tau_{xn}^k}{1+\tau_{xn}^k}\pi_{n1}^k\right)\frac{1+\theta}{\theta} w_n L_{np}^j$$
$$= \frac{1+\theta}{\theta} w_1 L_{1p}^j - \beta_j x_1$$

$$\sum_{n\neq 1}(t_n^j + \sum_{i\neq 1}^{N}\sum_k^{N_s} t_i^k \beta_k \pi_{ni}^k - \sum_k^{N_s}\beta_k \frac{\tau_{xn}^k}{1+\tau_{xn}^k}\pi_{n1}^k)\frac{1+\theta}{\theta} w_n L_{np}^j$$
$$= \frac{1+\theta}{\theta} w_1 L_{1p}^j - \beta_j x_1 \tag{A.58}$$

With multiple countries, Home's tariffs are on average higher for sectors with relatively higher net exports. For that sector, the weighted average of Home's tariff across country is higher. Different from the two-country case, now tariffs on one foreign country would affect other foreign countries sector profit and incentive to do innovation, hence technology, so the levels of optimal tariffs are jointly determined across country and sector.

In the special cases of two countries or multiple countries but only Home and one foreign country (country 2) produce sector j and j' goods, we don't need to consider the supply of other foreign countries that been affected by the tariff imposed on country 2. The left hand side of equation (A.58) only has country 2. Then again

$$t_2^j - t_2^{j'} = \frac{1}{\theta}\left(\frac{\frac{1+\theta}{\theta} w_1 L_{1p}^j - \beta_j x_1}{\frac{1+\theta}{\theta} w_2 L_{2p}^j} - \frac{\frac{1+\theta}{\theta} w_1 L_{1p}^{j'} - \beta_{j'} x_1}{\frac{1+\theta}{\theta} w_2 L_{2p}^{j'}}\right)$$

2. Now we prove

$$\frac{t_n^j}{1+t_n^j}\frac{\beta^j \pi_{1n}^j x_1}{\frac{1+\theta}{\theta} w_n L_{np}^j} = \delta_n + \frac{\sum_{m\neq 1}\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\beta^j \pi_{mn}^j x_m}{\frac{1+\theta}{\theta} w_n L_{np}^j}$$

Go back to the FOC of T_n^j

$$\theta(1-\beta(1-\delta))\gamma_{Tn}^j T_n^j = -\gamma_{rn}^j \frac{w_n L_{np}^j}{T_n^j} + u_c x_1 \beta_j \pi_{1n}^j - \theta u_c \sum_{m\neq 1}^{N}\beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j \pi_{mn}^j x_m$$
$$-\theta \sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma_{Li}^j \beta_j \pi_{mi}^j \pi_{mn}^j x_m + \theta\gamma_{Ln}^j \beta_j \sum_{m\neq 1}\pi_{mn}^j x_m$$

using τ_{xn}

$$\theta \sum_{i \neq 1}^{N} \gamma_{Li}^{j} \beta_j \pi_{ni}^{j} = -u_c \beta_j \frac{1}{(1+\tau_{xn}^{j})} + u_c \beta_j \frac{\tau_{xn}^{j}}{1+\tau_{xn}^{j}} \theta (1 - \pi_{n1}^{j})$$

the FOC becomes

$$\theta(1-\beta(1-\delta))\gamma_{Tn}^{j} T_n^{j} = -\gamma_{rn}^{j} \frac{w_n L_{np}^{j}}{T_n^{j}} + u_c x_1 \beta_j \pi_{1n}^{j} - \theta u_c \sum_{m \neq 1}^{N} \beta_j \frac{\tau_{xm}^{j}}{1+\tau_{xm}^{j}} \pi_{m1}^{j} \pi_{mn}^{j} x_m$$
$$+ \sum_{m \neq 1} \pi_{mn}^{j} x_m \left(u_c \beta_j \frac{1}{(1+\tau_{xm}^{j})} \right) - \sum_{m \neq 1} \pi_{mn}^{j} x_m \left(u_c \beta_j \frac{\tau_{xm}^{j}}{1+\tau_{xm}^{j}} \theta (1 - \pi_{m1}^{j}) \right) + \theta \gamma_{Ln}^{j} \beta_j \sum_{m \neq 1} \pi_{mn}^{j} x_m$$

Combine items,

$$\theta(1-\beta(1-\delta))\frac{1}{\delta}\mu_n L_{nr}^{j} = -\gamma_{rn}^{j} \frac{w_n L_{np}^{j}}{T_n^{j}} + u_c x_1 \beta_j \pi_{1n}^{j}$$
$$+ u_c \sum_{m \neq 1} \beta_j \frac{1}{(1+\tau_{xm}^{j})} \pi_{mn}^{j} x_m - \theta u_c \sum_{m \neq 1} \beta_j \frac{\tau_{xm}^{j}}{1+\tau_{xm}^{j}} \pi_{mn}^{j} x_m + \theta \gamma_{Ln}^{j} \beta_j \sum_{m \neq 1} \pi_{mn}^{j} x_m$$

Replace the last term with $\left(\frac{1+\theta}{\theta} w_n L_{np}^{j} - \beta_j \frac{1}{1+t_n^{j}} \pi_{1n}^{j} x_1 \right)$

$$\theta(1-\beta(1-\delta))\frac{1}{\delta}\mu_n L_{nr}^{j} = -\gamma_{rn}^{j} \frac{w_n L_{np}^{j}}{T_n^{j}} + u_c x_1 \beta_j \pi_{1n}^{j}$$
$$+ u_c \sum_{m \neq 1} \beta_j \frac{1}{(1+\tau_{xm}^{j})} \pi_{mn}^{j} x_m - \theta u_c \sum_{m \neq 1} \beta_j \frac{\tau_{xm}^{j}}{1+\tau_{xm}^{j}} \pi_{mn}^{j} x_m + \gamma_{Ln}^{j} \left((1+\theta) w_n L_{np}^{j} - \theta \beta_j \frac{1}{1+t_n^{j}} \pi_{1n}^{j} x_1 \right)$$

Using the following equation

$$\gamma_{Ln}^{j} = \gamma_{rn}^{j} \frac{1}{1+\theta} \frac{1}{T_n^{j}} + \sum_{j}^{N_s} \gamma_{Ln}^{j} \frac{\frac{1+\theta}{\theta} w_n L_{np}^{j}}{x_n}, \quad \gamma_{Ln}^{j} = -t_n^{j} u_c,$$

we have

$$\theta(1-\beta(1-\delta))\frac{1}{\delta}\mu_n L_{nr}^{j} = (1+\theta) w_n L_{np}^{j} \left[\sum_{j}^{N_s} \gamma_{Ln}^{j} \frac{\frac{1+\theta}{\theta} w_n L_{np}^{j}}{x_n} \right] + u_c x_1 \beta_j \pi_{1n}^{j}$$
$$+ u_c \sum_{m \neq 1} \beta_j \frac{1}{(1+\tau_{xm}^{j})} \pi_{mn}^{j} x_m - \theta u_c \sum_{m \neq 1} \beta_j \frac{\tau_{xm}^{j}}{1+\tau_{xm}^{j}} \pi_{mn}^{j} x_m + \theta t_n^{j} u_c \beta_j \frac{1}{1+t_n^{j}} \pi_{1n}^{j} x_1$$

Using the FOC of τ_x, we get

$$\theta(1-\beta(1-\delta))\frac{1}{\delta}\mu_n L^j_{nr} = (1+\theta)w_n L^j_{np}\left[\sum_j^{N_s}\gamma^j_{Ln}\frac{\frac{1+\theta}{\theta}w_n L^j_{np}}{x_n}\right]$$

$$+(1+\theta)u_c\beta_j\frac{t^j_n}{1+t^j_n}\pi^j_{1n}x_1 + u_c\frac{1+\theta}{\theta}w_n L^j_{np} - (1+\theta)u_c\sum_{m\neq 1}\beta_j\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\pi^j_{mn}x_m$$

or

$$\frac{t^j_n}{1+t^j_n}\frac{\beta^j\pi^j_{1n}x_1}{\frac{1+\theta}{\theta}w_n L^j_{np}} = \delta_n + \frac{\sum_{m\neq 1}\beta_j\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\pi^j_{mn}x_m}{\frac{1+\theta}{\theta}w_n L^j_{np}}$$

where

$$\delta_n = \frac{1}{1+\theta}\{-\theta\sum_j^{N_s}\gamma^j_{Ln}\frac{\frac{1+\theta}{\theta}w_n L^j_{np}}{u_c x_n} + \mu_n\frac{\theta(1-\beta(1-\delta))}{\delta}\frac{\bar{r}}{1-\bar{r}}\frac{1}{u_c w_n}\frac{\theta}{1+\theta} - 1\}$$

$$= \frac{1}{1+\theta}\{-\theta\sum_j^{N_s}\gamma^j_{Ln}\frac{\frac{1+\theta}{\theta}w_n L^j_{np}}{u_c x_n} + \frac{\mu_n L_{np}}{u_c x_n} - 1\}$$

G Proof for Proposition 7

Section E.1 proves that at the steady state Markov doesn't use any industry policy. Equation A.41 in Section C.2 shows Ramsey doesn't use any industry policy.

According to Appendix B.3, in Markov problem, $\sum_j^{N_s}\frac{\gamma^j_{rn}}{\alpha^j_n} = 0$ at the steady state when $\alpha^j_n(L^j_{nr}, T^j_{n,-1}) = \bar{\alpha}^j_n$. Furthermore, According to Appendix C.3, in Ramsey problem at the SS, $\sum_j^{N_s}\frac{\gamma^j_{vn}}{\alpha^j_n} = 0$ when $\alpha^j_n(L^j_{nr}, T^j_{n,-1}) = \bar{\alpha}^j_n$. Then equation A.42 shows Ramsey export taxes have the same formula as in the Markov at the SS.

Ramsey tariff equation A.43 has the same formula as in the Markov. As $\sum_j^{N_s}\frac{\gamma^j_{vn}}{\alpha^j_n} = 0$, the Ramsey FOCs of w_n, L^j_{np} and T^j_n (A.45-A.46) are also the same as Markov, hence γ^j_{Ln} are the same and optimal tariffs are the same.

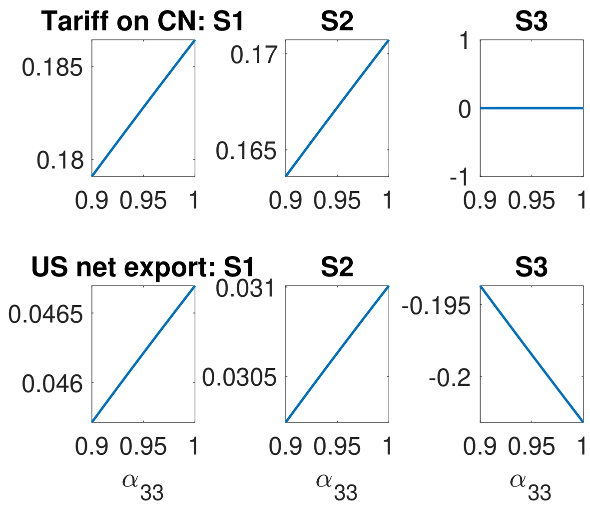
With no domestic externalities, in the steady state, Markov and Ramsey governments choose the same policies and have the same allocations.

H Numerical example

The interlinking relationships and spillover of tariffs across countries and sectors may appear to be complex but follow the basic principle outlined in the proposition. That is— the within-country tariff ranking has to do with the country's comparative advantage. That strict ranking disappears when we deviate from the special case, but still follows some basic trade patterns—further explored in our quantitative analysis. Still, to gain some intuition, and to investigate predictions of theory, we consider a few numerical examples.

Consider the world with three sectors (S1, S2, and S3) and three economies: the United States (US), China (CN), and the rest of the world (ROW). The innovation efficiencies in the three sectors accord with US $(1, 0.9, 0.9)$, CN $(0.9, 0.9, 1)$, ROW $(0, 0, \alpha_{33})$ with α_{33} varying from 0.9 to 1. Since ROW's innovation efficiency are zero and cannot produce goods from S1 and S2, it imports them from the United States and China. Furthermore, S2 is China's comparative advantage (CA) sector, and S1 is U.S.'s comparative advantage sector.

Figure A-1: Optimal Policies for the United States when Global Demand Rises

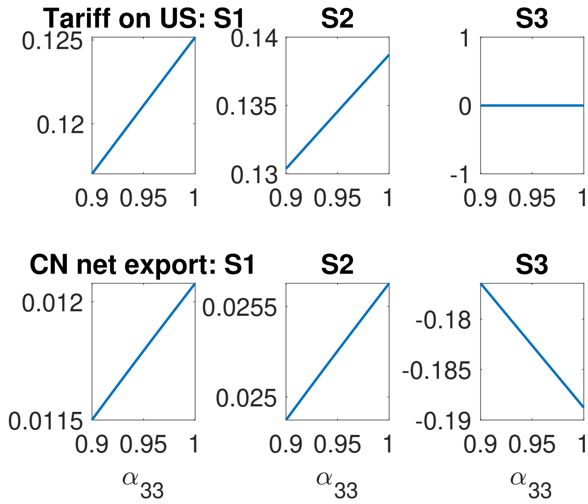


Note: The upper panel plots U.S. optimal tariff on imports from China in sector 1 ($S1$), sector 2 ($S2$), and sector 3 ($S3$) when sector 3 in ROW becomes more efficient, i.e. higher α_{33}. The lower panel plots the corresponding US's net export in sector 1, 2, and 3.

Consider the case in which ROW becomes larger, with a higher α_{33}. This leads to a surge in the global demand for goods in S1 and S2. Note that the U.S. always imposes tariffs on S1, which is its CA sector. Larger global demand for S1 and S2 boosts exports in these sectors, and thus the U.S. increases tariffs on both sectors. Moreover, the tariff on S2 increases by more as the net export in this sector rises faster—the same reasoning for China.

Figure A-1 shows that the U.S. raises tariffs on China's sector 2 goods more than tariffs on sector

Figure A-2: Optimal Policies for China when Global Demand Rises



Note: The upper panel plots China's optimal tariff on imports from the U.S. in sectors S1, S2, and S3 when S3 in ROW becomes more efficient, i.e. higher α_{33}. The lower panel plots China's net export in the three sectors.

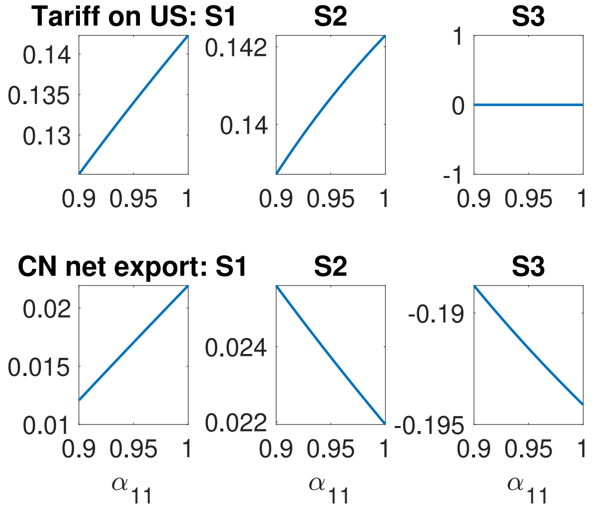
1 goods. In addition, the U.S. imposes higher tariffs on ROW's sector 3 goods, aiming to induce Chinese labor to shift from sector 1 and 2 to sector 3. However, the impact of this policy on the rise of Chinese labor in sector 3 is not significant, although it dampens China's labor flowing into sectors 1 and 2. The same optimal policy applies symmetrically to China, which levies a higher tariff on the US goods in sector 1 and 2, but higher in sector 2, and on ROW in sector 3, see Figure A-2. The larger the rise in global demand, the larger the tariffs levied by both China and the U.S. to endogenously improve each country's own technology in that sector.

Next, consider a case where there is an increase in China S1's innovation efficiency α_{11}, which takes on a value between 0.9 and 1. Figure A-3 shows that China levies a relatively higher tariff on the U.S. in S1 to discourage innovation in the U.S. It would also impose a higher tariff on ROW's sector 3 so as to induce U.S. labor to flow towards sector 3. Overall, this results in U.S. labor (and hence innovation) falling in sector 1.

However, from the perspective of the U.S., it will lower tariffs imposed on China's S1 goods. The reason is that China has become more productive in producing these goods, and thus it is more efficient for the U.S. to increase its imports from China in sector 1. Note, however, that this result may change if there are other forces at play—for instance, an event that causes there to be more profits to be reaped in that sector. This can occur if, along with rising Chinese efficiency, there is also a rise in the US efficiency in this sector, or in the importance for sector 1 goods (see Figure A-4), or if overall trade costs have fallen. In this case, the U.S. would want to raise its tariffs on Chinese sector

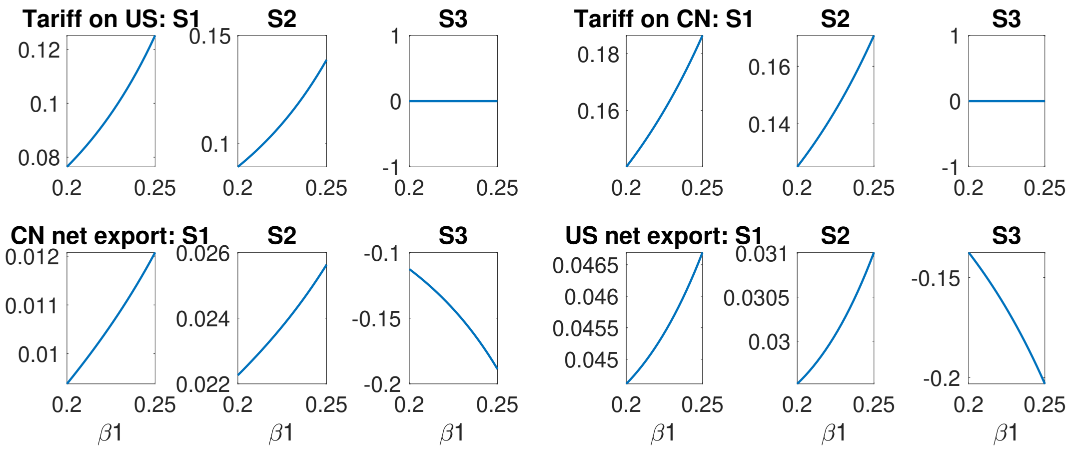
1 goods to stave off competition. These results show that in our framework, competitive policies arise not from a country becoming better at producing another country's comparative advantage goods (in this case more trade is better), but from the incrementally bigger market that is 'up for grabs' and where competition is made more intense when the other country is a closer rival.

Figure A-3: Optimal Policies for China when S1 Efficiency Rises



Note: The upper panel plots China's optimal tariff on imports from the U.S. in sector 1 ($S1$), sector 2 ($S2$), and sector 3 ($S3$) when sector 1 in China becomes more efficient, i.e., higher α_{11}. The lower panel plots the corresponding China's net export in sector 1, 2, and 3.

Figure A-4: Optimal Policies for China and US when S1 becomes more important



(a) China's Tariffs (b) US's Tariffs

Note: The upper panel plots the optimal tariff in sector 1 ($S1$), sector 2 ($S2$), and sector 3 ($S3$) when sector 1 becomes more important, i.e., higher β_1. The lower panel plots the corresponding net export in sector 1, 2, and 3.

We have shown examples whereby a rise in net exports induces an increase in import tariffs—in the case where T is endogenous and optimal policies across sectors and countries are jointly determined. There are other points to be made here. First, the larger the comparative advantage, the larger the motive to impose tariffs on one another. For instance, as the overall trade costs fall, there is more trade across all countries, and hence there is a larger incentive to undertake policies. Second, for each foreign country, there is no simple relationship between optimal tariffs and Home sector's degree of comparative advantage compare to the Foreign country. As is shown from the example above, China will raise its tariffs on the U.S. in sector 1 more if the global demand for good 1 and 2 rises—even though sector 1 is not China's comparative advantage sector. By contrast, if overall trade costs fall, then China will increase its tariff on sector 2. The reason is that China will increase its imports from the U.S. in sector 1 and thus lower its tariffs on U.S.'s sector 1 goods. The key barometer here is the change in *net exports* in any given sector: a country will raise tariffs imposed on the other country if its net exports in a sector rises; conversely, it will lower tariffs in sectors where its net exports falls.

I Proof for Proposition 8

Appendix B lists optimal conditions for Markov policy in general case, including with externalities. Appendix B.3 proved $\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0$ at baseline SS. The prove of $\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0$ with externality is similar, and to avoid repetition, we just highlight the differences here.

All the optimal conditions are the same except that we consider ε in front of the current profit reflecting Foreign country's Pigouvian innovation tax/subsidy

$$\frac{w_n}{\alpha_n^j (L_{nrt}^j)^{\varepsilon-1}} = \varepsilon \frac{w_n L_{np}^j}{\theta T_n^j} + \tilde{\beta}_n v_n^{j\prime}, \qquad (\gamma_{rn}^j),$$

Follow the same steps in Appendix B.3, Equation A.39 now becomes

$$\gamma_{rn}^j \frac{\varepsilon}{1+\theta} \frac{x_n}{T_n^j} - \gamma_{Ln}^j x_n - \mu_n \sum_j^{N_s} L_{np}^j + \sum_j^{N_s} \gamma_{Ln}^j \frac{1+\theta}{\theta} w_n L_{np}^j + \theta(1-\beta(1-\delta)) \sum_j^{N_s} \gamma_{Tn}^j T_n^j + \sum_j^{N_s} \gamma_{rn}^j \frac{\varepsilon w_n L_{np}^j}{T_n^j}$$
$$+ \sum_j^{N_s} \gamma_{rn}^j \left[\beta(1-\delta) M_n^j\right] - \sum_j \theta T_n^j \sum_{i \neq 1}^{N} \sum_k^{N_s} \gamma_{ri}^k \beta(1-\delta) \frac{\partial G_i^k}{\partial T_n^j} x_i^\sigma P_i^{1-\sigma} = 0 \tag{A.59}$$

At the steady state

$$T_n^j = \frac{\alpha_n^j (L_{nr}^j)^{\varepsilon-1} L_{nr}^j}{\delta} = \frac{\alpha_n^j}{\delta} (L_{nr}^j)^{\varepsilon}, \quad \mu_n = \varepsilon \gamma_{Tn}^j \alpha_n^j (L_{nr}^j)^{\varepsilon-1} + \gamma_{rn}^j (\varepsilon - 1) \frac{w_n}{\alpha_n^j (L_{nr}^j)^{\varepsilon}},$$

hence

$$\gamma^j_{rn}\frac{\varepsilon}{1+\theta}\frac{x_n}{T^j_n} - \gamma^j_{Ln}x_n - \mu_n\sum_j^{N_s} L^j_{np} + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np} + \theta(1-\beta(1-\delta))\left(\sum_j \frac{1}{\delta\varepsilon}\mu_n L^j_{nr} - \sum_j^{N_s}\gamma^j_{rn}\frac{(\varepsilon-1)w_n}{\delta\varepsilon\alpha^j_n(L^j_{nr})^{\varepsilon-1}}\right)$$
$$+\sum_j^{N_s}\gamma^j_{rn}\frac{\varepsilon w_n L^j_{np}}{T^j_n} + \sum_j^{N_s}\gamma^j_{rn}\left[\beta(1-\delta)M^j_n\right] - \sum_j \theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial G^k_i}{\partial T^j_n}x_i^\sigma P_i^{1-\sigma} = 0. \tag{A.60}$$

At the steady state, the research intensity is given by

$$\bar{r}^j = \frac{\delta\varepsilon}{\theta(1-\beta(1-\delta))+\delta\varepsilon}.$$

Using this research intensity, $L^j_{np} = (1-\bar{r})L^j_n$, and $L^j_{nr} = \bar{r}L^j_n$, we can cancel out the two terms involving μ_n in equation (A.60). Finally, we have

$$\gamma^j_{rn}\frac{\varepsilon}{1+\theta}\frac{x_n}{T^j_n} - \gamma^j_{Ln}x_n + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np} - \theta(1-\beta(1-\delta))(\sum_j^{N_s}\gamma^j_{rn}\frac{(\varepsilon-1)w_n}{\delta\varepsilon\alpha^j_n(rL^j_n)^{\varepsilon-1}})$$
$$+\sum_j^{N_s}\gamma^j_{rn}\frac{\varepsilon w_n L^j_{np}}{T^j_n} + \sum_j^{N_s}\gamma^j_{rn}\left[\beta(1-\delta)M^j_n\right] - \sum_j \theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial G^k_i}{\partial T^j_n}x_i^\sigma P_i^{1-\sigma} = 0.$$

Multiply the above equation $\frac{\frac{1+\theta}{\theta}w_n L^j_{np}}{x_n}$ on both sides and sum over j

$$\sum_j^{N_s}\gamma^j_{rn}\frac{1}{\theta}\frac{\varepsilon w_n L^j_{np}}{T^j_n} - \theta(1-\beta(1-\delta))(\sum_j^{N_s}\gamma^j_{rn}\frac{(\varepsilon-1)w_n}{\delta\varepsilon\alpha^j_n(rL^j_n)^{\varepsilon-1}})$$
$$+\sum_j^{N_s}\gamma^j_{rn}\frac{\varepsilon w_n L^j_{np}}{T^j_n} + \sum_j^{N_s}\gamma^j_{rn}\beta(1-\delta)\frac{w_n}{\alpha^j_n(rL^j_n)^{\varepsilon-1}} - \sum_j \theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial x_i'^{-\sigma}P_i'^{\sigma-1}\frac{w_i'}{\alpha_i'^k(rL^k_i)^{\varepsilon-1}}}{\partial T^j_n}x_i^\sigma P_i^{1-\sigma} = 0.$$

$$\sum_k^{N_s}\frac{\gamma^k_{rn}}{\alpha^k_n(rL^k_n)^{\varepsilon-1}}(1+\frac{\theta(1-\beta(1-\delta))(1+(\delta-1)\varepsilon)}{\delta\varepsilon})w_n$$
$$-\sum_j \theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial x_i'^{-\sigma}P_i'^{\sigma-1}\frac{w_i'}{\alpha_i'^k(rL^k_i)^{\varepsilon-1}}}{\partial T^j_n}x_i^\sigma P_i^{1-\sigma} = 0$$

The equation holds when for every $n \neq 1$, $\sum_k^{N_s}\frac{\gamma^k_{rn}}{\alpha^k_n(rL^k_n)^{\varepsilon-1}} = 0$, and $\sum_j \theta T^j_n\sum_{i\neq 1}^{N}\sum_k^{N_s}\gamma^k_{ri}\beta(1-\delta)\frac{\partial x_i'^{-\sigma}P_i'^{\sigma-1}\frac{w_i'}{\alpha_i'^k(rL^k_i)^{\varepsilon-1}}}{\partial T^j_n}x_i^\sigma P_i^{1-\sigma} = 0$.

With $\sum_k^{N_s}\frac{\gamma^k_{rn}}{\alpha^k_n(rL^k_n)^{\varepsilon-1}} = 0$, export taxes at the SS have the same formula as without externality;

however, the wedges in each industry at SS will not go to zero, average wedge goes to zero at the SS.

J Proof for Proposition 9

Appendix C lists optimal conditions for Ramsey policy, and Appendix C.3 proved $\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0$ at baseline SS. The prove of $\sum_j^{N_s} \frac{\gamma_{rn}^j}{\alpha_n^j} = 0$ with externality is similar. We use the FOCs of the Ramsey problem.

FOC over L_{np}^j

$$\begin{aligned}
&\varepsilon \frac{1}{\theta}\frac{w_n}{T_n^j}\Gamma_{vn}^j x_n^{-\sigma} P_n^{\sigma-1} - \gamma_{Ln}^j \frac{1+\theta}{\theta} w_n - \mu_n + \sum_{k=1}^{N_s} \gamma_{L1}^k \beta_k \frac{1}{1+\tau_{xn}^k} \pi_{n1}^k \frac{1+\theta}{\theta} w_n + \sum_{i\neq 1}^{N}\sum_k^{N_s} \gamma_{Li}^k \beta_k \pi_{n,i} \frac{1+\theta}{\theta} w_n \\
&+ \gamma_4 \sum_k^{N_s} \beta_k \frac{\tau_{xn}^k}{1+\tau_{xn}^k} \pi_{n1}^k \frac{1+\theta}{\theta} w_n + \sum_k^{N_s} \left[\varepsilon \frac{1}{\theta}\frac{L_{np}^k}{T_n^k}\Gamma_{vn}^k - \frac{1}{\alpha_n^k}\gamma_{vn}^k \right] w_n x_n^{-\sigma} P_n^{\sigma-1} (-\sigma) \frac{1+\theta}{\theta}\frac{w_n}{x_n} = 0
\end{aligned}$$

FOC over w_n

$$\begin{aligned}
&-\gamma_P \sum_j^{N_s} \beta_j \pi_{1n}^j + \gamma_{L1} \left[\sum_{m\neq 1}^{N}\sum_j^{N_s} \beta_j \frac{1}{1+\tau_{xm}^j} \theta \pi_{m1}^j \pi_{mn}^j x_m + \sum_j^{N_s} \beta_j \frac{1}{1+\tau_{xn}^j} \pi_{n1}^j x_n \right] \\
&+ \sum_{i\neq\{1\}}^{N}\sum_j^{N_s} \gamma_{Li}^j \beta_j \left(\sum_{m\neq 1} \theta \pi_{mi}^j \pi_{m,n}^j x_m + \pi_{n,i}^j x_n \right) - \sum_j^{N_s} \gamma_{Ln}^j \left(\theta \beta_j \sum_{m\neq 1} \pi_{m,n}^j x_m + \frac{1+\theta}{\theta} w_n L_{np}^j \right) \\
&+ \gamma_4 \sum_{j=1}^{N_s} \beta_j \left(\sum_{m\neq 1}^{N} \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \theta \pi_{m1}^j \pi_{mn}^j x_m + \frac{\tau_{xn}^j}{1+\tau_{xn}^j} \pi_{n1}^j x_n \right) + (1-\sigma) \sum_j^{N_s} \left[\varepsilon \frac{1}{\theta}\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j - \frac{1}{\alpha_n^j}\gamma_{vn}^j \right] w_n x_n^{-\sigma} P_n^{\sigma-1} \\
&+ \sum_{m\neq 1}^{N}\sum_j^{N_s} \left[\varepsilon \frac{1}{\theta}\frac{L_{mp}^j}{T_m^j}\Gamma_{vm}^j - \frac{1}{\alpha_m^j}\gamma_{vm}^j \right] w_m x_m^{-\sigma} P_m^{\sigma-1} (\sigma-1) \left(\sum_k \beta_k \pi_{mn}^k \right) = 0
\end{aligned}$$

FOC over T_n^j

$$\begin{aligned}
\gamma_{Tn}^j =& \beta E\left[\gamma_{Tn}^{j\prime} (1-\delta) \right] + \frac{1}{T_n^j} \{ \gamma_P \frac{\beta_j}{\theta} \pi_{1n}^j \\
&- \sum_{i=1}^{N} \gamma_{Li}^j \beta_j \sum_{m\neq 1} \pi_{m,i}^j \pi_{mn}^j x_m + \gamma_{Ln}^j \beta_j \sum_{m\neq 1} \pi_{m,n}^j x_m - u_c \sum_{m\neq 1}^{N} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \pi_{m,1}^j \pi_{mn}^j x_m \\
&- \left[\varepsilon \frac{1}{\theta}\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j \right] w_n x_n^{-\sigma} P_n^{\sigma-1} - \sum_{i\neq 1}\sum_k^{N_s} \left[\varepsilon \frac{1}{\theta}\frac{L_{ip}^k}{T_i^k}\Gamma_{vi}^k - \frac{1}{\alpha_i^k}\gamma_{vi}^k \right] w_i x_i^{-\sigma} P_i^{\sigma-1} (\sigma-1) \frac{\beta_j}{\theta} \pi_{in}^j \}
\end{aligned}$$

1. At the steady state $\gamma^j_{T_n} = \gamma^{j\prime}_{T_n}$, summing over j the FOC over T^j_n:

$$
\begin{aligned}
&-\theta(1-\beta(1-\delta))\sum_j^{N_s}\gamma^j_{Tn}T^j_n + u_c x_1 \sum_j^{N_s}\beta_j\pi^j_{1n} - \theta u_c \sum_j^{N_s}\sum_{m\neq 1}^{N}\beta_j \frac{\tau^j_{xm}}{1+\tau^j_{xm}}\pi^j_{m1}\pi^j_{mn}x_m \\
&-\theta\sum_j^{N_s}\sum_{m\neq 1}\sum_{i\neq 1}^{N}\gamma^j_{Li}\beta_j\pi^j_{mi}\pi^j_{mn}x_m + \theta\sum_j^{N_s}\gamma^j_{Ln}\beta_j\sum_{m\neq 1}\pi^j_{mn}x_m \\
&-\sum_j^{N_s}\left[\varepsilon\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_{i\neq 1}\sum_k^{N_s}\left[\varepsilon\frac{1}{\theta}\frac{L^k_{ip}}{T^k_i}\Gamma^k_{vi} - \frac{1}{\alpha^k_i}\gamma^k_{vi}\right]w_i x_i^{-\sigma}P_i^{\sigma-1}(\sigma-1)\sum_j^{N_s}\beta_j\pi^j_{in}, \quad \text{(A.61)}
\end{aligned}
$$

2. Substract FOC over L^j_{np} and w_n from the above equation:

$$
\begin{aligned}
&\varepsilon\frac{1}{\theta}\frac{w_n L_{np}}{T^j_n}\Gamma^j_{vn}x_n^{-\sigma}P_n^{\sigma-1} - \gamma^j_{Ln}x_n - \mu_n\sum_j^{N_s}L^j_{np} + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np} + \theta(1-\beta(1-\delta))\sum_j^{N_s}\gamma^j_{Tn}T^j_n \\
&+\sum_j^{N_s}\left[\varepsilon\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_k^{N_s}\left[\varepsilon\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\Gamma^k_{vn} - \frac{1}{\alpha^k_n}\gamma^k_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} = 0 \quad \text{(A.62)}
\end{aligned}
$$

3. At the steady state

$$
T^j_n = \frac{\alpha^j_n (L^j_{nr})^{\varepsilon-1}L^j_{nr}}{\delta} = \frac{\alpha^j_n}{\delta}(L^j_{nr})^{\varepsilon}, \quad \mu_n = \varepsilon\gamma^j_{Tn}\alpha^j_n (L^j_{nr})^{\varepsilon-1} + \gamma^j_{vn}(\varepsilon-1)\frac{w_n}{\alpha^j_n (L^j_{nr})^{\varepsilon}}x_n^{-\sigma}P_n^{\sigma-1},
$$

hence

$$
\begin{aligned}
&\varepsilon\frac{1}{\theta}\frac{w_n L_{np}}{T^j_n}\Gamma^j_{vn}x_n^{-\sigma}P_n^{\sigma-1} - \gamma^j_{Ln}x_n - \mu_n\sum_j^{N_s}L^j_{np} + \sum_j^{N_s}\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n L^j_{np} \\
&+\theta(1-\beta(1-\delta))\left(\sum_j\frac{1}{\delta\varepsilon}\mu_n L^j_{nr} - \sum_j^{N_s}\gamma^j_{vn}\frac{(\varepsilon-1)w_n}{\delta\varepsilon\alpha^j_n(L^j_{nr})^{\varepsilon-1}}x_n^{-\sigma}P_n^{\sigma-1}\right) \\
&+\sum_j^{N_s}\left[\varepsilon\frac{L^j_{np}}{T^j_n}\Gamma^j_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_k^{N_s}\left[\varepsilon\frac{1}{\theta}\frac{L^k_{np}}{T^k_n}\Gamma^k_{vn} - \frac{1}{\alpha^k_n}\gamma^k_{vn}\right]w_n x_n^{-\sigma}P_n^{\sigma-1} = 0 \quad \text{(A.63)}
\end{aligned}
$$

At the steady state, the research intensity is given by

$$
\bar{r}^j = \frac{\delta\varepsilon}{\theta(1-\beta(1-\delta))+\delta\varepsilon}.
$$

Using this research intensity, $L^j_{np} = (1-\bar{r})L^j_n$, and $L^j_{nr} = \bar{r}L^j_n$, we can cancel out the two terms involving μ_n in equation (A.63). Then we have

$$\varepsilon\frac{1}{\theta}\frac{w_n L_{np}}{T_n^j}\Gamma_{vn}^j x_n^{-\sigma}P_n^{\sigma-1} - \gamma_{Ln}^j x_n + \sum_j^{N_s}\gamma_{Ln}^j\frac{1+\theta}{\theta}w_n L_{np}^j - \theta(1-\beta(1-\delta))(\sum_j^{N_s}\gamma_{vn}^j\frac{(\varepsilon-1)w_n}{\delta\varepsilon\alpha_n^j(rL_n^j)^{\varepsilon-1}})x_n^{-\sigma}P_n^{\sigma-1}$$
$$+\sum_j^{N_s}\left[\varepsilon\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j\right]w_n x_n^{-\sigma}P_n^{\sigma-1} - \sum_k^{N_s}\left[\varepsilon\frac{1}{\theta}\frac{L_{np}^k}{T_n^k}\Gamma_{vn}^k - \frac{1}{\alpha_n^k}\gamma_{vn}^k\right]w_n x_n^{-\sigma}P_n^{\sigma-1} = 0.$$

4. Multiply the above equation $\frac{\frac{1+\theta}{\theta}w_n L_{np}^j}{x_n}$ on both sides and sum over j

$$-\theta(1-\beta(1-\delta))(\sum_j^{N_s}\gamma_{vn}^j\frac{(\varepsilon-1)w_n}{\delta\varepsilon\alpha_n^j(rL_n^j)^{\varepsilon-1}})x_n^{-\sigma}P_n^{\sigma-1} + \sum_j^{N_s}\left[\varepsilon\frac{L_{np}^j}{T_n^j}\Gamma_{vn}^j\right]w_n x_n^{-\sigma}P_n^{\sigma-1} + \sum_k^{N_s}\frac{1}{\alpha_n^k}\gamma_{vn}^k w_n x_n^{-\sigma}P_n^{\sigma-1} = 0.$$

Finally,

$$\sum_k^{N_s}\frac{\gamma_{vn}^k}{\alpha_n^k(rL_n^k)^{\varepsilon-1}}(1+\frac{\theta(1-\beta(1-\delta))}{\delta\varepsilon})w_n x_n^{-\sigma}P_n^{\sigma-1} = 0.$$

The equation only holds when for every $n \neq 1$, $\sum_k^{N_s}\frac{\gamma_{vn}^k}{\alpha_n^k(rL_n^k)^{\varepsilon-1}} = 0$.

K Quantitative optimal policies at baseline SS

In this quantitative analysis, the world consists of five countries with China, the U.S., Germany, Japan, and the rest of the world (ROW). We take China as the home country and compute unilateral optimal policies. To use observables in the data, we extend the exact hat method to compute the counterfactual equilibrium under optimal policies in the long run.

Data and Measurement To conduct the analysis, we need sectoral-level data on gross production and bilateral trade for each country. The data on bilateral trade flows are sourced from the United Nations' Statistical Division Commodity Trade (COMTRADE) database, while the annual gross production data are obtained from the OECD Structural Analysis Database (STAN) and National Accounts and Industrial Statistics Database (UNIDO) compiled by the United Nations. The gross production data are available for 2-digit level ISIC industries, while the 6-digit H.S. trade data is mapped onto two-digit ISIC industries, resulting in 20 two-digit manufacturing sectors for the year 2018.

To compute the total production of a sector in ROW, we aggregate the sectoral production of all countries within the ROW while disregarding bilateral trade between them. However, there is a potential problem with this approach in which the ROW may appear excessively large and productive in this model with endogenous technology. To address this, we conduct a sensitivity analysis by excluding major economies and China's trading partners from the ROW and the results change very little.

Optimal Policies We use the 'exact hat method' to compute the counterfactual equilibrium under optimal policies. This method allows us to calculate optimal policies and equilibrium changes using bilateral trade and sector-level production data without backing out fundamental research efficiency $\{\alpha_n^j\}$ and trade costs $\{d_{nm}^j\}$. We adapt the standard exact hat method but incorporate endogenous technology adoption and optimal policies. In particular, the counterfactuals include the calculation of the multipliers and optimal policies.

Let variables without 'prime' denote the observed variables, which includes the trade matrix $\{\pi_{ni}^j\}$ and sectoral production $\{\frac{1+\theta}{\theta} w_n L_{pn}^j\}$. Variables denoted with 'prime' represent counterfactuals after implementing the optimal policies, and variables with 'hats' denote the ratios of prime variables to the observed ones. The full system of equations that determines the counterfactual changes can be found in Appendix K.1.

In the data, China runs a large surplus, but our exercise features a long-run counterfactual equilibrium with balanced trade. This implies that the welfare gains computed include both the gains from eliminating trade imbalances and those from implementing optimal policies. To separate them, we first run a counterfactual to eliminate the imbalances. We then take the new equilibrium to be our private equilibrium observations which are used to calculate the optimal policies and welfare changes. The results show additional changes brought about by optimal policies.[12]

Table A-1 shows the optimal policies across sectors as a consequence of heterogenous tariffs. We take China to be the home country and consider its optimal unilateral policies. Table A-1 ranks sectors by China's net export as a share of world production in that sector. For example, relative to other sectors, China's textile sector sees the highest share of net exports, whereas China's chemicals are imported on net.

The first finding is that export tax and import tariffs are heterogeneous across sectors/ countries,

[12]When exogenous imbalances are eliminated, tax neutrality holds. Hence, a uniform increase in export tax is equivalent to a uniform increase in tariff. We can also solve the optimal policies with trade imbalances. In this case, tax neutrality no longer holds, and there is an additional valuation effect for the fixed amount of deficit. Home would like to change its optimal policies to increase its domestic prices and inflate away the deficit. To avoid this complication and focus on our main mechanism, we only consider the balanced trade in the long run.

and exhibit a wide range of values. China's optimal import tariffs on the U.S. range from 14% to 35%, for instance. Tariffs also vary across countries. For electrical machinery, the optimal tariff imposed on the U.S. is 17% whereas it is 43% for ROW. Similarly, China imposes a tax of 1.8% for exporting machinery to the U.S. but subsidizes the ROW with an export tax of −1.7%. Of course, all optimal taxes and tariffs are relative, as tax neutrality holds.

Second, what we can surmise from these patterns is that tariffs imposed by China on other countries are generally higher in its comparative advantage sectors (first four sectors compared to others ranked below). However, there is no strict ranking of tariffs that accords with comparative advantage. Since all cross country-and-sector specific taxes and tariffs are jointly determined, no bilateral relationship between tariff/export tax and some revealed country pair-wise comparative advantage exists. Only in the special case, multiple region model examined in Section 4, is there a strict relationship between a sector's net exports/production and its optimal tariff rate. What we can observe, however, is that there tends to be a larger gap in tariffs imposed across countries among comparative advantage sectors. Tariffs imposed are more homogenous among the comparative disadvantage sectors.

Tariff rates are not directly comparable across countries as they reflect individual elasticities associated with a particular trading partner, and take into account that trade costs can be different across sector/countries. Still, there are some general intuition behind these results. Take the example of electrical machinery: tariffs fall heavily on ROW (43%) relative to those in other sectors. But for the U.S. (17%) and Germany (12%), tariffs in this sector are lower than in other sectors. If only consider China's net export to ROW, Electrical machinery is not the highest, this high tariff reflect China particularly import less from ROW than from US and Germany, so that higher tariff in ROW would encourage US and Germany labor move into the sector.

For 'office and computing machinery', the next sector down, China's net export to the US is not the highest in this sector comparing to other sectors. China exports a lot to the rest of the world and therefore imposes a high tariff on the U.S. (35%) and Germany (47%) relative to other sectors. But it levies a relatively lower tariff on the ROW (14%) because it also imports a great deal from the ROW. Similarly, China imports a lot of 'other transport equipment' from the U.S. and hence imposes a lower tariff (14%). Hence one cannot conclude that the country that is imposed the highest tariff in a sector is China's chief rival in that sector. On the contrary, the reason is that a higher tariff imposed on a country is caused by an incentive to induce it to move labor out of this sector because it is not very efficient, but into other sectors where China imports a great deal. Turning to export taxes, we also see significant heterogeneity both across sectors and across countries. For instance, China imposes a tax on the U.S. in the furniture industry, whereas it subsidizes the ROW.

These results stand in stark contrast to the case of exogenous technology, emphasized in past literature. For comparison, we run the counterfactual equilibrium under exogenous technology in Table A-2. As consistent with theory, optimal import tariffs are uniform across sectors –we fixed the level at 25% for the US, which is about the average level under endogenous T case. Optimal export taxes tend to be higher in sectors in which China exports relatively more (on net). For example, textiles, furniture, office, and machinery and equipment face about 2% export taxes, while food, paper, and chemicals face less than 0.5%. Since Home has a comparative advantage and high monopoly power in these high-tax sectors, it can charge a higher markup upon exporting, whilst technology remains immune to policies. Note that there is also no strict relationship between the ordering of sector based on the "overall" comparative advantage and attendant export taxes as there are pairwise comparative advantage, interaction among countries, and trade costs that can differ across sectors and multiple countries.

Our finding of heterogeneous export taxes and import tariffs can perhaps help bridge the gap between the existing theories and the data. As pointed out by Caliendo and Parro (2021), the literature remains highly disconnected from the data: " theoretical result on uniform import tariffs and heterogenous export tax across goods" is in stark contrast to “ the observed wide range of tariff changes across products during the recent trade war...." If the conventional wisdom guiding trade policy relates to manipulating the terms of trade, then it fails to explain the recent trade war between the U.S. and China, for instance. To explain these observed trade policies, the recent literature pursues mechanisms other than terms of trade manipulation, such as externalities or political considerations. In contrast, our model shows that optimal tariffs should differ across products under endogenous technology and technology rivalry, even absent externalities and political considerations.

A natural question is what the suggestive welfare gains amount to from this exercise. Table A-3 reports the gains from the optimal policies. As is evident from Table A-1, under the optimal unilateral policies, China gains by 0.58%, while ROW loses about -1.22%. By contrast, with uniform tariff and heterogenous export tax the gains are 0.5%. Note that even with uniform tariff the gain is about 0.5% and heterogenous export tax generate very little additional gains. Heterogenous tariffs bring about 20% higher gains.

Note that the magnitude of our welfare gains is higher than the standard estimations, and small welfare gains in the literature are commonplace. A higher welfare gain can result from an input-

Table A-1: Optimal Policies with Endogenous Technology

	Export Tax				Import Tariff			
	US	JP	GER	ROW	US	JP	GER	ROW
Leather	0.57	-0.27	-2.34	0.35	25.76	32.41	27.50	34.23
Apparel	-0.11	-0.38	-1.82	-0.59	25.81	27.18	27.11	31.62
Textiles	0.13	0.33	-1.74	-0.40	28.53	28.25	26.58	32.16
Furniture	0.52	0.14	-1.42	-0.58	27.23	27.06	27.42	30.19
Electrical machinery	1.80	0.79	0.88	-1.70	17.71	24.34	12.31	43.41
Office	-2.18	0.70	-4.55	8.33	35.35	28.24	47.48	13.98
Machinery and equip.	1.00	2.39	0.10	-0.38	24.68	22.91	23.53	30.04
Rubber and plastics	0.53	0.81	-0.79	-0.84	25.43	24.44	25.64	28.08
Fabricated metal	0.07	0.06	-0.98	-1.11	25.67	25.57	26.41	27.97
Non-metallic mineral	-0.00	-0.17	-1.47	-1.27	25.83	25.74	27.39	27.89
Printing	0.04	-0.33	-1.52	-1.22	25.39	25.49	27.16	27.11
Other transport	6.51	-0.66	0.25	0.39	14.25	26.79	23.65	29.21
Wood	0.55	-0.39	-1.66	-0.13	24.76	26.36	28.09	25.87
Tobacco	-0.16	-0.40	-1.59	-1.38	25.15	25.16	27.02	26.99
Food	0.02	-0.36	-1.56	-0.62	25.00	25.82	27.48	26.03
Fuel	0.07	-0.26	-1.60	-0.37	24.83	25.44	27.42	26.06
Vehicles	0.96	0.65	0.75	-1.43	22.27	24.29	20.54	28.99
Basic metals	0.02	-0.02	-1.95	-0.34	24.50	25.16	28.51	26.94
Paper	0.47	0.22	-2.22	0.41	24.69	25.04	29.83	24.94
Chemicals	0.29	0.70	-2.69	0.75	24.94	24.76	30.40	24.88

Table A-2: Optimal Policies Under Fixed Technology

	Export Tax				Import Tariff			
	US	JP	GER	ROW	US	JP	GER	ROW
Leather	1.43	3.16	-0.08	3.14	25.00	26.12	26.05	26.08
Apparel	0.94	0.47	-0.13	2.07	25.00	26.12	26.05	26.08
Textiles	2.50	1.70	-0.21	2.29	25.00	26.12	26.05	26.08
Furniture	1.97	0.82	0.04	1.43	25.00	26.12	26.05	26.08
Electrical machinery	2.03	0.21	0.35	1.59	25.00	26.12	26.05	26.08
Office	1.07	0.46	-0.01	5.02	25.00	26.12	26.05	26.08
Machinery and equipment	1.13	0.32	-0.24	0.81	25.00	26.12	26.05	26.08
Rubber and plastics	0.84	-0.40	-0.40	0.20	25.00	26.12	26.05	26.08
Fabricated metal	0.55	-0.35	-0.51	0.04	25.00	26.12	26.05	26.08
Non-metallic mineral	0.60	-0.45	-0.45	-0.09	25.00	26.12	26.05	26.08
Printing	0.31	-0.83	-0.70	-0.50	25.00	26.12	26.05	26.08
Other transport equipment	0.01	-0.61	-0.64	0.34	25.00	26.12	26.05	26.08
Wood	0.29	-0.35	-0.58	-0.34	25.00	26.12	26.05	26.08
Tobacco	-0.02	-0.87	-0.84	-0.70	25.00	26.12	26.05	26.08
Food	-0.00	-0.61	-0.77	-0.66	25.00	26.12	26.05	26.08
Fuel	-0.06	-0.73	-0.81	-0.45	25.00	26.12	26.05	26.08
Vehicles	-0.06	-0.76	-0.72	-0.48	25.00	26.12	26.05	26.08
Basic metals	-0.09	-0.65	-0.70	0.09	25.00	26.12	26.05	26.08
Paper	0.14	-0.65	-0.73	-0.29	25.00	26.12	26.05	26.08
Chemicals	0.08	-0.25	-0.68	0.27	25.00	26.12	26.05	26.08

Table A-3: Welfare Gain Implications

	Optimal policies	If view T exog
CN	0.58	0.50
US	−0.40	−0.36
JP	−0.48	−0.49
Germany	−0.34	−0.06
ROW	−1.22	−1.23

output structure or from assumptions of externality. For example, Bartelme et al. (2019) use a model with externality and find an average gain from optimal industry policies of 0.98%. Caliendo and Parro (2015) evaluate the welfare gains for NAFTA with input-output structure. They find that the gain for the U.S. is 0.08% and 1.3% for Mexico. In contrast, our model emphasizes the role of endogenous technology adoption in trade policy. The welfare gain is about 0.6% for China.

K.1 Counterfactual welfare gains from optimal policies

In this section, we compute the counterfactual equilibrium under optimal policies using exact hat method and our FOCs for the optimal policies. Variables without prime are originals from data (trade matrix π_{ni}^{j}; sectoral output $\frac{1+\theta}{\theta} w_n L_{np}^{j}$), and variables with prime are counterfactuals after implementing optimal policies. Variables with hat are the ratio of prime and original.

$$\hat{T}_n^j = \hat{L}_n^j$$

$$w_n' L_n' = \hat{w}_n \hat{L}_n w_n L_n$$

$$\hat{P}_n = \Pi_j \left[\pi^j_{n1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xn}) \right)^{-\theta} + \sum_{i=2}^{N} \pi^j_{ni} \hat{T}^j_i \hat{w}_i^{-\theta} \right]^{-\beta^j_n/\theta}$$

$$\hat{P}_1 = \Pi_j \left[\pi^j_{11} \hat{T}^j_1 \hat{w}_1^{-\theta} + \sum_{i=2}^{N} \pi^j_{1i} \hat{T}^j_i (\hat{w}_i (1 + t'^j_{xi}))^{-\theta} \right]^{-\beta^j_1/\theta}$$

$$\pi'^j_{11} = \frac{\pi^j_{11} \hat{T}^j_1 \hat{w}_1^{-\theta}}{\pi^j_{11} \hat{T}^j_1 (\hat{w}_1)^{-\theta} + \sum_{n=2}^{N} \pi^j_{1n} \hat{T}^j_n \left(\hat{w}_n (1 + t^{j\prime}_n) \right)^{-\theta}}$$

$$\pi'^j_{m1} = \frac{\pi^j_{m1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xm}) \right)^{-\theta}}{\pi^j_{m1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xm}) \right)^{-\theta} + \sum_{i=2}^{N} \pi^j_{mi} \hat{T}^j_i (\hat{w}_i)^{-\theta}}$$

$$\pi'^j_{mn} = \frac{\pi^j_{mn} \hat{T}^j_n (\hat{w}_n)^{-\theta}}{\pi^j_{m1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xm}) \right)^{-\theta} + \pi^j_{mn} \hat{T}^j_n (\hat{w}_n)^{-\theta} + \sum_{i=\{m,n\}}^{N} \pi^j_{mi} \hat{T}^j_i (\hat{w}_i)^{-\theta}}$$

$$\pi'^j_{1n} = \frac{\pi^j_{1n} \hat{T}^j_n \left(\hat{w}_n (1 + t^j_n) \right)^{-\theta}}{\pi^j_{11} \hat{T}^j_1 (\hat{w}_1)^{-\theta} + \pi^j_{1n} \hat{T}^j_n \left(\hat{w}_n (1 + t^{j\prime}_n) \right)^{-\theta} + \sum_{m \neq \{1,n\}}^{N} \pi^j_{1m} \hat{T}^j_m \left(\hat{w}_m (1 + t^{j\prime}_m) \right)^{-\theta}}$$

$$x'_1 = \frac{1+\theta}{\theta} \hat{w}_1 w_1 L_{1p} + \sum_{n=2}^{N} \sum_j \frac{\tau'^j_{xn}}{1 + \tau'^j_{xn}} \pi'^j_{n1} \beta^j_n x'_n + \sum_{i=2}^{N} \sum_j \frac{t'^j_i}{1 + t'^j_n} \pi'^j_{1i} \beta^j_1 x'_1$$

$$x'_n = \frac{1+\theta}{\theta} \hat{w}_n w_n L_{np}$$

$$\frac{1+\theta}{\theta} \hat{w}_1 \hat{L}^j_1 w_1 L^j_{1p} = \pi'^j_{11} \beta^j_1 x'_1 + \sum_{m \neq 1} \frac{1}{1 + \tau'^j_{xm}} \pi'^j_{m1} \beta^j_m x'_m$$

$$\frac{1+\theta}{\theta} \hat{w}_n \hat{L}^j_{np} w_n L^j_{np} = \frac{1}{1 + t'^j_n} \pi'^j_{1n} \beta^j_1 x'_1 + \sum_{m \neq 1} \pi'^j_{mn} \beta^j_m x'_m$$

$$\sum_j \left(\hat{L}^j_{np} w_n L^j_{np} \right) = w_n L_{np}$$

The optimal trade policies are

$$1 + \tau'^j_{xn} = \frac{\left(1 + \theta(1 - \pi'^j_{n1})\right)}{\theta \sum_{m \neq 1} (1 + t'^j_m) \pi'^j_{nm}},$$

$$1 + t'^j_n = \frac{u_c - \gamma^j_{Ln}}{u_c},$$

For the following FOCs, we use $\gamma^j_{Tn} T^j_n = \mu_n r L'^j_n / \delta$, also solve $\frac{\mu_n}{w'_n}$ and $\frac{\gamma^j_{rn}}{T'^j_n}$ instead of μ_n and γ^j_{rn}, hence we solve counterfactual without backing out foundementals α^j_n, d^j_{nm}.

FOCs become

$$(1-\beta(1-\delta))\frac{\mu_n}{w'_n}\frac{r\hat{w}_n\hat{L}^j_n w_n L^j_n}{\delta} = \gamma_P\frac{\beta^j_1}{\theta}\pi'^j_{1n} - \sum_{i=1}^{N}\gamma^j_{Li}\sum_{m\neq 1}\pi'^j_{mi}\pi'^j_{mn}\beta^j_m x'_m + \gamma^j_{Ln}\sum_{m\neq 1}\pi'^j_{mn}\beta^j_m x'_m$$
$$-u_c\sum_{m\neq 1}^{N}\beta^j_m\frac{\tau'^j_{xm}}{1+\tau'^j_{xm}}\pi'^j_{m1}\pi'^j_{mn}x'_m - \frac{\gamma^j_{rn}}{\left(T'^j_n\right)}\frac{1}{\theta}\hat{w}_n\hat{L}^j_{np}w_n L^j_{np}, \qquad (T^j_n, \quad N_s(N-1))$$

$$\frac{\gamma^j_{rn}}{T'^j_n}\frac{1}{\theta} - \gamma^j_{Ln}\frac{1+\theta}{\theta} - \frac{\mu_n}{w'_n} + \sum_{k=1}^{N_s}\gamma^k_{L1}\beta^k_n\frac{1}{1+\tau'^k_{xn}}\pi'^k_{n1}\frac{1+\theta}{\theta}$$
$$+\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{Li}\beta^k_n\pi'^j_{ni}\frac{1+\theta}{\theta} + \gamma_4\sum_{k}^{N_s}\beta^k_n\frac{\tau'^k_{xn}}{1+\tau'^k_{xn}}\pi'^k_{n1}\frac{1+\theta}{\theta} = 0 \qquad (L^j_{np}, \quad N_s(N-1))$$

$$-u_c x'_1\sum_{j}^{N_s}\beta^j_1\pi'^j_{1n} + \gamma_{L1}\left[\sum_{m\neq 1}^{N}\sum_{j}^{N_s}\beta^j_m\frac{1}{1+\tau'^j_{xm}}\theta\pi'^j_{m1}\pi'^j_{mn}x'_m + \sum_{j}^{N_s}\beta^j_n\frac{1}{1+\tau'^j_{xn}}\pi'^j_{n1}x'_n\right]$$
$$+\sum_{i\neq\{1\}}^{N}\sum_{j}^{N_s}\gamma^j_{Li}\left(\sum_{m\neq 1}\theta\pi'^j_{mi}\pi'^j_{mn}\beta^j_m x'_m + \pi'^j_{ni}\beta^j_n x'_n\right) - \sum_{j}^{N_s}\gamma^j_{Ln}\left(\theta\sum_{m\neq 1}\pi'^j_{mn}\beta^j_m x'_m + \frac{1+\theta}{\theta}\hat{w}_n\hat{L}^j_{np}w_n L^j_{np}\right)$$
$$+\gamma_4\sum_{j=1}^{N_s}\left(\sum_{m\neq 1}^{N}\frac{\tau'^j_{xm}}{1+\tau'^j_{xm}}\theta\pi'^j_{m1}\pi'^j_{mn}\beta^j_m x'_m + \frac{\tau'^j_{xn}}{1+\tau'^j_{xn}}\pi'^j_{n1}\beta^j_n x'_n\right) = 0, \qquad (w_n, N-1)$$

Compute Baseline Steady State Counterfactual

1. We plugged in conditions $\gamma_{L1} = 0$ and $\gamma_4 = u_c$.
2. $$r^j_n = \frac{\delta}{\theta\left[1-\beta(1-\delta)\right]+\delta}$$
3. Guess $N + N_s \times N + 2(N_s-1)N$ unknows $(\hat{w}_n, \hat{L}^j_n, \tau'^j_{xn}, t'^j_n)$
   (a) $$\hat{T}^j_n = \hat{L}^j_n$$
   (b) Evaluate for $(\pi'^j_{m1}, \pi'^j_{mn}, \pi'^j_{11}, \pi'^j_{1m}, \hat{P_m}, x'_n)$
   (c) Solve for labor hence $\hat{L}^j_{np}$

$$\frac{1+\theta}{\theta}\hat{w}_1\hat{L}^j_{1p}w_1 L^j_{1p} = \beta_j\left[\pi'^j_{11}x'_1 + \sum_{m\neq 1}\frac{1}{1+\tau'^j_{xm}}\pi'^j_{m1}x'_m\right]$$

$$\frac{1+\theta}{\theta}\hat{w}_n\hat{L}^j_{np}w_nL^j_{np}=\beta_j\left[\frac{1}{1+t'^j_n}\pi'^j_{1n}x'_1+\sum_{m\neq 1}\pi'^j_{mn}x'_m\right]$$

(d) Solve linear equations for Multipliers $\left\{\underbrace{\gamma^j_{Ln}}_{N_s\times(N-1)-1},\underbrace{\gamma^j_{rn}}_{N_s\times(N-1)},\underbrace{\mu_n}_{N-1}\right\}$. Note that one of γ^j_{Ln} can be set at 0 due to tax neutrality holds.

$$(1-\beta(1-\delta))\frac{\mu_n}{w'_n}\frac{r\hat{w}_n\hat{L}^j_nw_nL^j_n}{\delta}=\gamma_P\frac{\beta^j_1}{\theta}\pi'^j_{1n}-\sum_{i=1}^{N}\gamma^j_{Li}\sum_{m\neq 1}\pi'^j_{mi}\pi'^j_{mn}\beta^j_mx'_m+\gamma^j_{Ln}\sum_{m\neq 1}\pi'^j_{mn}\beta^j_mx'_m$$
$$-u_c\sum_{m\neq 1}^{N}\beta^j_m\frac{\tau'^j_{xm}}{1+\tau'^j_{xm}}\pi'^j_{m1}\pi'^j_{mn}x'_m-\frac{\gamma^j_{rn}}{\left(T'^j_n\right)}\frac{1}{\theta}\hat{w}_n\hat{L}^j_{np}w_nL^j_{np},\qquad (T^j_n,\quad N_s(N-1))$$

$$\frac{\gamma^j_{rn}}{T'^j_n}\frac{1}{\theta}-\gamma^j_{Ln}\frac{1+\theta}{\theta}-\frac{\mu_n}{w'_n}+\sum_{k=1}^{N_s}\gamma^k_{L1}\beta^k_n\frac{1}{1+\tau'^k_{xn}}\pi'^k_{n1}\frac{1+\theta}{\theta}$$
$$+\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{Li}\beta^k_n\pi'^j_{ni}\frac{1+\theta}{\theta}+\gamma_4\sum_{k}^{N_s}\beta^k_n\frac{\tau'^k_{xn}}{1+\tau'^k_{xn}}\pi'^k_{n1}\frac{1+\theta}{\theta}=0\qquad (L^j_{np},\quad N_s(N-1))$$

$$-u_cx'_1\sum_{j}^{N_s}\beta^j_1\pi'^j_{1n}+\gamma_{L1}\left[\sum_{m\neq 1}^{N}\sum_{j}^{N_s}\beta^j_m\frac{1}{1+\tau'^j_{xm}}\theta\pi'^j_{m1}\pi'^j_{mn}x'_m+\sum_{j}^{N_s}\beta^j_n\frac{1}{1+\tau'^j_{xn}}\pi'^j_{n1}x'_n\right]$$
$$+\sum_{i\neq\{1\}}^{N}\sum_{j}^{N_s}\gamma^j_{Li}\left(\sum_{m\neq 1}\theta\pi'^j_{mi}\pi'^j_{mn}\beta^j_mx'_m+\pi'^j_{ni}\beta^j_nx'_n\right)-\sum_{j}^{N_s}\gamma^j_{Ln}\left(\theta\sum_{m\neq 1}\pi'^j_{mn}\beta^j_mx'_m+\frac{1+\theta}{\theta}\hat{w}_n\hat{L}^j_{np}w_nL^j_{np}\right)$$
$$+\gamma_4\sum_{j=1}^{N_s}\left(\sum_{m\neq 1}^{N}\frac{\tau'^j_{xm}}{1+\tau'^j_{xm}}\theta\pi'^j_{m1}\pi'^j_{mn}\beta^j_mx'_m+\frac{\tau'^j_{xn}}{1+\tau'^j_{xn}}\pi'^j_{n1}\beta^j_nx'_n\right)=0,\qquad (w_n,N-1)$$

(e) Equations to solve for $(\hat{w}_n, \hat{L}_n^j, \tau_{xn}'^j, t_n'^j)$

$$\begin{aligned}
\hat{P}_1 = \Pi_j \left[\pi_{11}^j \hat{T}_1^j \hat{w}_1^{-\theta} + \sum_{i=2}^{N} \pi_{1i}^j \hat{T}_i^j (\hat{w}_i (1 + t_{xi}'^j))^{-\theta} \right]^{-\beta_j/\theta} &= 1 \\
\sum_j \left(\hat{L}_{np}^j w_n L_{np}^j \right) &= w_n L_{np} \\
\hat{L}_{1p}^j &= \hat{L}_1^j \\
\hat{L}_{np}^j &= \hat{L}_n^j \\
1 + \tau_{xn}'^j &= \frac{\left(1 + \theta(1 - \pi_{n1}'^j)\right)}{\theta \sum_{m \neq 1} (1 + t_m'^j) \pi_{nm}'^j} \\
1 + t_n'^j &= \frac{u_c - \gamma_{Ln}^j}{u_c}
\end{aligned}$$

L Optimal innovation policy without tariff at baseline SS

In Section E, we prove with optimal country-sector tariffs and export taxes, time-consistent policy do not distort domestic innovation at the SS. In this section, we show without tariffs, Home would like to use innovation policies.

Without tariff, Home government's Euler on T_1^j

$$\begin{aligned}
\gamma_{T1}^j = \beta(1-\delta) \left[\gamma_{T1}^{j\prime} \right] + \frac{1}{\theta} \frac{1}{T_1^j} \{ \gamma_P \beta_j \pi_{11}^j + \gamma_{L1} \theta \beta_j \left[\pi_{11}^j (1 - \pi_{11}^j) x_1 \right] \\
+ \sum_{n \neq 1} (\gamma_4 - \gamma_{L1}) \frac{1}{(1 + \tau_{xn}^j)} \beta_j \pi_{n1}^j x_n - \theta \pi_{11}^j x_1 \sum_{n \neq 1}^{N} \gamma_{Ln}^j \beta_j \pi_{1n}^j \} + \sum_{n \neq 1}^{N} \sum_{k}^{N_s} \gamma_{r,n}^k \rho \frac{\partial G_n^k}{\partial T_1^j} x_n^\sigma P_n^{1-\sigma} = 0
\end{aligned}$$

Inside

$$\begin{aligned}
&\gamma_P \beta_j \pi_{11}^j + \theta \pi_{11}^j x_1 \left\{ \gamma_{L1} \beta_j \left[(1 - \pi_{11}^j) \right] - \sum_{n \neq 1}^{N} \gamma_{Ln}^j \beta_j \pi_{1n}^j \right\} + (\gamma_4 - \gamma_{L1}) \sum_{n \neq 1} \frac{1}{(1 + \tau_{xn}^j)} \beta_j \pi_{n1}^j x_n \\
=&\gamma_P \beta_j \pi_{11}^j + \theta \beta_j \pi_{11}^j x_1 \left\{ \gamma_{L1} (1 - \pi_{11}^j) - \sum_{n \neq 1}^{N} \gamma_{Ln}^j \pi_{1n}^j \right\} + (\gamma_4 - \gamma_{L1}) \frac{1+\theta}{\theta} w_1 L_{1p}^j - (\gamma_4 - \gamma_{L1}) \beta_j \pi_{11}^j x_1
\end{aligned}$$

Recall

$$\gamma_{T_1}^j = (\gamma_4 - \gamma_{L1}) \frac{1+\theta}{\theta} \frac{w_1}{\alpha_1^j}$$

$$u_c + \sum_{n=1}^{N} \sum_j \gamma^j_{Ln} \beta_j \pi^j_{1n} - \gamma_4 = 0$$

Hence government Euler on home research

$$\frac{w_1}{\alpha^j_1} = \frac{1}{\theta} \frac{w_1 L^j_{1p}}{T^j_1} + \beta(1-\delta) \left[\frac{\gamma'_4 - \gamma'_{L1}}{\gamma_4 - \gamma_{L1}} \frac{w'_1}{\alpha^j_1} \right] + \frac{\theta}{1+\theta} \frac{1}{\gamma_4 - \gamma_{L1}} \sum_{n \neq 1}^{N} \sum_k^{N_s} \gamma^k_{r,n} \rho \frac{\partial G^k_n}{\partial T^j_1} x^\sigma_n P^{1-\sigma}_n$$
$$+ \frac{1}{1+\theta} \frac{1}{T^j_1} \frac{\beta_j \pi^j_{11} x_1}{\gamma_4 - \gamma_{L1}} \left\{ \theta \gamma_{L1} (1 - \pi^j_{11}) - \theta \sum_{n \neq 1}^{N} \gamma^j_{Ln} \pi^j_{1n} + \gamma_{L1} - \sum_{n=1}^{N} \sum_j \gamma^j_{Ln} \beta_j \pi^j_{1n} \right\}$$

and at the SS, $\sum_j \gamma^j_{rn} / \alpha^j_n = 0$ at steady state.

$$(1 - \beta(1-\delta)) \frac{w_1}{\alpha^j_1} = \frac{1}{\theta} \frac{w_1 L^j_{1p}}{T^j_1} + \underbrace{\frac{1}{1+\theta} \frac{1}{T^j_1} \frac{\beta_j \pi^j_{11} x_1}{u_c - \gamma_{L1}} \left\{ \theta \gamma_{L1} (1 - \pi^j_{11}) - \theta \sum_{n \neq 1}^{N} \gamma^j_{Ln} \pi^j_{1n} + \gamma_{L1} - \sum_{n=1}^{N} \sum_k \gamma^k_{Ln} \beta_k \pi^k_{1n} \right\}}_{\text{wedge}}$$

1. With two countries, without tariff, the SS optimal innovation satisfies:

$$[1 - \beta(1-\delta)] \frac{w_1}{\alpha^j_1} = \frac{1}{\theta} \frac{w_1 (1 - r^j_1) L^j_1}{T^j_1} + \frac{1}{1+\theta} \frac{\beta_j}{T^j_1} \frac{\pi^j_{11} x_1}{\gamma_4 - \gamma_{L1}} \left[\left(\gamma_{L1} - \gamma^j_{L2} \right) \theta \pi^j_{12} + \sum_k (\gamma_{L1} - \gamma^k_{L2}) \beta_k \pi^k_{12} \right] \tag{A.64}$$

   and the optimal export tax are:

$$1 + \tau^j_x = \frac{(\gamma_4 - \gamma_{L1}) \left(1 + \theta \pi^j_{22} \right)}{\left(\gamma_4 - \gamma^j_{L2} \right) \theta \pi^j_{22}}$$

2. Sum over j of the FOC for L_{1r}, (A.64), we get:

$$(1 - \beta(1-\delta)) \sum_j L^j_{1r} = \frac{\delta}{\theta} \sum_j L^j_{1p}$$

   which hold in market equilibrium, i.e., government distort own innovation across sectors but not overall. The wedges across sectors average to zero. The sign of $\left(\gamma_{L1} - \gamma^j_{L2} \right) \theta \pi^j_{12}$ determines which sector's R&D should be subsidized. Sectors have higher net exports would be subsidized.

3. With multi-countries,

$$\frac{w_1}{\alpha_1^j} = \frac{1}{\theta}\frac{w_1 L_{1p}^j}{T_1^j} + \beta(1-\delta)\left[\frac{w_1'}{\alpha_1^{j\prime}}\right]$$
$$\underbrace{+\frac{1}{1+\theta}\frac{1}{T_1^j}\frac{\beta_j \pi_{11}^j x_1}{u_c - \gamma_{L1}}\left\{\theta\gamma_{L1}(1-\pi_{11}^j) - \theta\sum_{n\neq 1}^{N}\gamma_{Ln}^j \pi_{1n}^j + \gamma_{L1} - \sum_{n=1}^{N}\sum_k \gamma_{Ln}^k \beta_k \pi_{1n}^k\right\}}_{\text{wedge}}$$

Without a full set of country-sector-specific tariff, Home would subsidize/tax some industries' innovation. The wedge on innovation across sectors still averages to zero. The sign of $\theta\gamma_{L1}(1-\pi_{11}^j) - \theta\sum_{n\neq 1}^{N}\gamma_{Ln}^j\pi_{1n}^j = \theta\sum_{n\neq 1}^{N}(\gamma_{L1} - \gamma_{Ln}^j)\pi_{1n}^j$ determines which sector's R&D should be subsidized. Sectors have higher net exports would be subsidized. It is suboptimal as policies distort the country's own innovation in order to affect foreign demand and innovation, it is inferior to first-best policies with tariffs as it cannot target countries differentially. Nevertheless, it is better than no policies at all.

M Nash tariff at baseline SS

Multi-sector, Multi-Country Nash: Let τ_{xmn}^j denote the export tax imposed by country n on the export to country m in sector j, and t_{mn}^j: country m's import tariff on country n in sector j(follow the flow of goods).

Country h planner solves the following problem

$$V_h\left(\left\{T_{n,-1}^j\right\}\right) = \max_{\left\{L_{nr}^j, L_{np}^j T_n^j, w_n, x_n, \tau_{xn}^j, t_n^j\right\}} \frac{(x_h/P_h)^{1-\sigma}}{1-\sigma} + \beta\left[V_h\left(\left\{T_n^j\right\}\right)\right]$$

subject to the private equilibrium

$$T_n^j = \alpha_n^j L_{nr}^j + (1-\delta)T_{n,-1}^j, \quad (\gamma_{Tnh}^j, \quad N_s \times N)$$

$$\frac{w_n}{\alpha_n^j} = \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \beta(1-\delta)G_n^j\left(\left\{T_n^j\right\}\right)x_n^\sigma P_n^{1-\sigma}, \qquad (\gamma_{rnh}^j, \quad N_s \times (N-1) \quad \text{for } n \neq h)$$

$$\frac{1+\theta}{\theta} w_n L_{np}^j = \beta_j\left[\sum_m^N \frac{1}{(1+\tau_{xmn}^j)(1+t_{mn}^j)}\pi_{mn}^j x_m\right], \quad (\gamma_{Lnh}^j, \quad N \times N_s)$$

$$\sum_j \left(L_{nr}^j + L_{np}^j\right) = 1, \quad (\mu_{nh}, \quad N)$$

$$x_n = \frac{1+\theta}{\theta} w_n \sum_j L_{np}^j + \sum_{m \neq n}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau_{xmn}^j}{1+\tau_{xmn}^j} \frac{1}{1+t_{mn}^j} \pi_{mn}^j x_m + \sum_{m \neq n}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t_{nm}^j}{1+t_{nm}^j} \pi_{nm}^j x_n \quad (\gamma_{xnh}, \quad N)$$

where

$$P_n = \Pi_j \left[\sum_{m=1}^{N} T_m^j (w_m d (1+\tau_{xnm}^j)(1+t_{nm}^j))^{-\theta} \right]^{-\frac{\beta_j}{\theta}}$$

$$G_n^j \left(\left\{ T_n^j \right\} \right) = \left[(x_n')^{-\sigma} (P_n')^{\sigma-1} \frac{w_n'}{\alpha_n^j} \right]$$

$$\pi_{mn}^j = \frac{T_n^j (w_n d (1+\tau_{xmn}^j)(1+t_{mn}^j))^{-\theta}}{\sum_i T_i^j (w_i d (1+\tau_{xmi}^j)(1+t_{mi}^j))^{-\theta}}$$

Use the FOCs, optimal tariffs can be simplified to:

$$1+t_{hn}^j = \frac{\gamma_{xhh} - \gamma_{Lnh}^j \frac{1}{(1+\tau_{xhn}^j)} - \gamma_{xnh} \frac{\tau_{xhn}^j}{1+\tau_{xhn}^j}}{\gamma_{xhh} - \gamma_{Lhh}}$$

Optimal export tax:

$$(1+\tau_{xnh}^j)(1+t_{nh}^j) =$$

$$\frac{(\gamma_{xhh} - \gamma_{Lhh}) \left[1 + \theta(1-\pi_{nh}^j) \right]}{\frac{\theta \gamma_{xhh}}{1+t_{nh}^j}(1-\pi_{nh}^j) - \sum_{i \neq h}^{N} \frac{\theta \gamma_{Lih}^j \pi_{ni}^j}{(1+\tau_{xni}^j)(1+t_{ni}^j)} - \sum_{i \neq \{n,h\}}^{N} \frac{\gamma_{xih} \tau_{xni}^j \theta \pi_{ni}^j}{(1+\tau_{xni}^j)(1+t_{ni}^j)} - \gamma_{xnh} \left[\sum_{m \neq \{n,h\}}^{N} \frac{t_{nm}^j \theta \pi_{nm}^j}{1+t_{nm}^j} - \frac{t_{nh}^j \theta (1-\pi_{nh}^j)}{1+t_{nh}^j} \right] + \left(\sum_k^{N_s} \gamma_{rnh}^k \beta (1-\delta)(\sigma-1) \frac{M_n^k}{x_n} \right)}$$

For example, Under the two country case,

$$1+t_{12}^j = \frac{\gamma_{x1} - \gamma_{L2}^j \frac{1}{(1+\tau_{x12}^j)} - \gamma_{x2} \frac{\tau_{x12}^j}{1+\tau_{x12}^j}}{\gamma_{x1} - \gamma_{L1}}$$

$$1+\tau_{x21}^j = \frac{(\gamma_{x1} - \gamma_{L1}) \left(1 + \theta \pi_{22}^j \right)}{\gamma_{x1} \theta \pi_{22}^j - \gamma_{L2}^j (1+t_{21}^j) \theta \pi_{22}^j + \gamma_{x2} t_{21}^j \theta \pi_{22}^j + (1+t_{21}^j) \left(\sum_k^{N_s} \gamma_{r21}^k \beta (1-\delta)(\sigma-1) \frac{M_2^k}{x_2} \right)}$$

Government wouldn't use innovation policies at SS. Nash tariffs have similar feature that each individual countries put higher tariff on own CA sector. Home knows that, first, Foreign will also use optimal tariff and export tax, and second, own tariff would affect foreign's export tax revenue, own export tax would affect foreign's import tariff revenue. First, if Home and Foreign are different, the opposite policies strengthen CA and generate higher incentive of differential tariffs. Second, both use trade policies and countries will trade less, endogenous CA are less and incentive for differential tariff will be smaller in the equilibrium. Overall, countries trade much less with Nash policies, and policies are less differentiated across sectors. Both countries loss from Nash tariffs, though Foreign has a smaller welfare loss than the case where it is not allowed to retaliate (a case in which the Home has a welfare gain).