THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

August 2023

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section (precise instructions are below)—50 points in each section—for a total of 100 points. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are five (5) pages, including this one.

SECTION I

Please answer any **two** of the three questions from this section.

I.1 (25 points) Let $X = \{x_1, x_2, ..., x_n\}$ be a set of monetary prizes such that $x_1 < x_2 < ... < x_n$. Let $\triangle(X)$ denote the set of lotteries (or probability distributions) over X with typical element $p = (p_1, p_2, ..., p_n)$ such that p_i is the probability of getting prize x_i. An individual has preferences $\succcurlyeq$ over $\triangle(X)$, that is, $p \succcurlyeq p'$ means that the individual weakly prefers p to p' and $p \succ p'$ means that she strictly prefers p to p'. Preferences have an expected utilty representation if there exists a $u : X \to \mathbb{R}$ such that $p \succcurlyeq p'$ if and only if the expectation of u under p is weakly greater than the that under p'.

(a) Suppose the individual is a pessimist and has preferences such that she weakly prefers one lottery to another if and only if the worst possible prize in one is the same or exceeds the worst possible prize in the other. Formally, $p \succcurlyeq p'$ if and only if $\min\{x_i : p_i > 0\} \geq \min\{x_i : p'_i > 0\}$ and $p \succ p'$ if the inequality is strict. Do such preferences have an expected utility representation? Why or why not?

(b) Suppose the individual's preferences are defined as follows. There is a fixed number k such that $1 < k < n$ and one lottery is preferred to another if and only if the probability of getting a prize of at least k in one is the same or exceed the same probability under the other. Formally, $p \succcurlyeq p'$ if and only if $\sum_{i=k}^{n} p_i \geq \sum_{i=k}^{n} p'_i$ and $p \succ p'$ if the inequality is strict. Do such preferences have an expected utility representation? Why or why not?

I.2 (25 points) Consider an economy in which there are two consumers, two states of nature and a single consumption good. The consumers' utility functions are of the expected utility form; that is

$$\begin{aligned} U_1(x_{11}, x_{12}) &= \pi_1 x_{11} + (1 - \pi_1) x_{12} \\ U_2(x_{21}, x_{22}) &= \pi_2 v(x_{21}) + (1 - \pi_2) v(x_{22}) \end{aligned}$$

where x_{is} denotes i's consumption of the good in state s. Consumer 1's subjective probability of state 1 is $\pi_1 > 0$ and consumer 2's subjective probability of state 1 is $\pi_2 > 0$. The function v is strictly concave—that is, consumer 2 is risk averse. Finally, suppose that the two consumers have the same endowment vector.

(a) First, suppose that $\pi_1 = \pi_2$. Show that at any Walrasian equilibrium (with complete markets) with strictly positive consumptions for both consumers in both states (an interior equilibrium), it must be that consumer 2 insures completely, that is, $x_{21} = x_{22}$.

(b) Next, suppose that $\pi_1 < \pi_2$. What can you say about the relative consumptions of the two consumers in the two states, again at an interior Walrasian equilibrium?

I.3 (25 points) Consider a simple economy with four (4) consumers labelled $i = 1, 2, 3, 4$. There are four houses h_1, h_2, h_3, h_4 labelled so that consumer i owns house h_i. Houses are indivisible. The preferences of the consumers are as follows:

1	2	3	4
h_4	h_1	h_1	h_2
h_3	h_4	h_3	h_3
h_1	h_3	h_4	h_4
h_2	h_2	h_2	h_1

so that consumer 1 *strictly* prefers h_4 to h_3 to h_1 to h_2, etc.

(a) Find an allocation (an assignment of houses to consumers) that is in the core of this economy.

(b) Find a set of prices p_1, p_2, p_3, p_4 for the houses so that the allocation you found in part (a) is a Walrasian equilibrium allocation.

SECTION II

Please answer any **two** of the three questions from this section.

II.1 (25 points) Players 1 and 2 are playing the following stage game repeatedly. (The usual conventions about payoffs apply. The players move simultaneously in each play and after each play of the stage game, everyone finds out what has been played.)

	L_2	M_2	R_2
L_1	$1, 5$	$2, 1$	$5, 0$
M_1	$0, 1$	5,2	$2, 2$
R_1	$0, 5$	$1, 2$	$4, 4$

(a) Is there a subgame perfect equilibrium in which (R_1, R_2) is played in the first period, (i) if the game is played twice, (ii) if the game is played three times ?

(b) Is there a subgame perfect equilibrium in which (R_1, R_2) is played in the second period if the game is played three times?

If the answer to any of the questions above is "Yes", please write down the relevant subgame perfect equilibrium *strategies.* (Remember what a strategy is—an action specified after every history.)

II.2 (25 points) There are two players and a public good is to be provided. The value of this public good is 1 to both the players. The good is provided if one of the two individuals provides it. Player $i's$ cost of providing the good is c_i. The c_i's are i.i.d draws from the uniform distribution on $[0, 1]$.

The game is as follows: Nature chooses the cost realization for each player. Each player knows her own cost exactly but not the cost realization of the other player. Once the costs are realized, players i $(i = 1, 2)$ simultaneously choose times $t_i \geq 0$ such that if the good has not been provided until time t_i, player i would provide it. Time is continuous here. The rules of the game are common knowledge.

If the public good is provided at time $t \geq 0$ by player i, then i obtains a payoff of $e^{-rt}(1 - c_i)$ and player $j \neq i$ obtains a payoff of e^{-rt}. Here r is the discount *rate* and e^{-rt} is the discount factor.

(a) (5 points) Does a player with cost 0 have a dominant strategy? What is it?

(b) (20 points) Suppose there is a symmetric equilibrium in which player i uses a continuously differentiable, strictly increasing strategy $t_i = \tau(c_i)$. Use the revelation principle to construct an incentive compatible direct mechanism equivalent to the game above. Then using incentive compatibility, derive the symmetric Bayes Nash equilibrium strategy $\tau(.)$. (Remember the example where we derived a direct mechanism for the first-price auction and obtained the symmetric equilibrium strategy from it.)

II.3 (25 points) Suppose there are two sellers. Each of them has one unit of an *indivisible* good to sell. There is a buyer who demands a unit of this indivisible good. The buyer's maximum willingness to pay for the good is 1. A seller's reservation price (the minimum price she is willing to accept) is c_1 with probability π and c_2 with probability $(1-\pi)$ where,

$$0 < c_1 < c_2 \leq 1$$

Each seller knows her own reservation price but not the other's. The probability distribution and the rules of the game are commonly known.

The payoffs for prices of p_j, $j = 1, 2$ are as follows: $(1 - p_\ell)$ for the buyer, $(p_\ell - c_i)$ for seller ℓ, if her cost is c_i and $1 \geq p_\ell = \min\{p_1, p_2\}$ and 0 for the seller who made no sale. If $p_1 = p_2$, the buyer buys from either seller with equal probability. (Thus, all players are risk-neutral).

The game is as follows: Nature chooses values of c_j for each seller j and seller j is told her value. Then sellers simultaneously and independently choose prices $p_j \geq 0$. The buyer buys from the seller who quotes the lower price, provided this is no greater than 1. If two prices are the same, the buyer randomises between the buyers.

Find a symmetric Bayesian Nash equilibrium of this game (in randomised behavioural strategies if necessary).