

How Competition Shapes Information in Auctions*

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Abstract

We consider auctions where buyers can acquire costly information about their valuations and those of others, and investigate how competition between buyers shapes their learning incentives. In equilibrium, buyers find it cost-efficient to acquire some information about their competitors so as to only learn their valuations when they have a fair chance of winning. We show that such learning incentives make competition between buyers less effective: losing buyers often fail to learn their valuations precisely and, as a result, compete less aggressively for the good. This depresses revenue, which remains bounded away from what the standard model with exogenous information predicts, even when information costs are negligible. Finally, we examine the implications for auction design. First, setting an optimal reserve price is more valuable than attracting an extra buyer, which contrasts with the seminal result of Bulow and Klemperer (1996). Second, the seller can incentivize buyers to learn their valuations, hence restoring effective competition, by maintaining uncertainty over the set of auction participants.

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1 INTRODUCTION

In many auctions, participants spend significant time and resources learning about the goods for sale before submitting a bid. Relevant examples are the sales of complex, high-value assets such as companies, broadband licenses, or procurement contracts, during which interested buyers conduct extensive due diligence. For instance, bidders in takeover auctions get access to extensive information about the target company's operations and finances, allowing them to assess synergies and estimate how much they value its acquisition.

In practice, accessing and processing such information is costly, and buyers only want to undertake this investment if they have a fair chance of winning the auction.¹ They then have an incentive to investigate how much other participants value the good for sale: doing so reduces the strategic uncertainty faced in the auction and allows them to assess whether due diligence costs are worth paying. In [Gleyze and Pernoud \(2022\)](#), we show that such an incentive is prevalent and arises under most auction formats. We now argue that it significantly affects the properties of auctions and address the following questions: How does competition between buyers shape the kinds of information they acquire? How does that, in turn, affect the value of competition in auctions?

Answering these questions is key to understanding the performance of some real-life auctions. For example, consider the 1994-1995 spectrum auction run by the Federal Communications Commission. The ascending auction resulted in a low price of \$26 per capita for the Los Angeles license, while less profitable licenses were sold at higher prices.² Some argue that the presence of Pacific Telesis, a company that already operated in California and was presumed to win, scared away its competitors. Indeed, other participants came to learn PacTel's intention to bid, which may explain why they failed to conduct proper investigations and cautiously submitted weak bids.

This paper proposes a simple model of a second-price auction in which buyers can acquire costly information before bidding. Buyers' valuations are drawn i.i.d. from some common knowledge distribution, but are unknown to buyers ex-ante. The main innovation of our model is that it gives buyers flexibility in *what* information they can seek. Specifically, buyers can acquire information about both their own valuations for

¹Due diligence for the acquisition of a company can take several months and involves important legal and advising fees.

²For instance, the Chicago license was sold at \$31 per capita. See <https://www.fcc.gov/auctions-summary> for the auction outcome and [Klemperer \(2002\)](#) for a more detailed account.

the good as well as those of their competitors.³

We model information acquisition as a two-step process. Buyers first have the opportunity to assess the potential competition by acquiring a signal about other bidders' valuations. Then, after having observed that signal's realization, they decide what signal to acquire about their own values. Buyers have some flexibility in choosing a signal and, in particular, can choose how the signal partitions the set of possible valuations. Information is costly, and we require that the cost satisfies an appropriate notion of convexity.

Our first main result is that buyers never converge to becoming fully informed of their valuations in equilibrium, even as information costs become arbitrarily small relative to the value of the good. Instead, they find it cost-efficient to first assess the valuation of their toughest competitor, and only then learn about their own, which they do only when they have a chance of winning. As a result, their private information when entering the auction (i.e., their types) are interdependent: not only do buyers have information that is relevant to others, but their own expected valuations may depend on what they learned about the competitors. We characterize equilibrium information structures in high-stake auctions (i.e., auctions where information costs are small relative to the value of the good) and show that buyers only learn their valuations if it falls in a similar range as that of their toughest competitor. The information buyers acquire is then deeply shaped by the competitive pressure they impose on each other, as depicted in Figure 1.

The rest of the paper examines how buyers' learning incentives, in turn, affect the performance of the second-price auction. We show that expected revenue remains bounded away from what the standard model predicts, even when the cost of information is small. Indeed, losing buyers often fail to learn their valuations precisely, and since they bid their expected valuations for the good in equilibrium, this leads to a regression to the mean of bids. Losing bids are then less dispersed than in the standard model, which depresses the expected second-highest bid and hence expected revenue whenever the number of buyers $N \geq 3$.⁴

³Learning about the competitors can be interpreted as undertaking market research. For instance, bidders in broadband subsidies auctions often start by investigating how much the subsidy is worth to a company with a different technology than their own—e.g., how much it is worth to a company providing broadband via satellite, whereas they use cable. The sole purpose of such information is to predict others' bids and assess their chances of winning the auction.

⁴This effect persists even as information costs become arbitrarily small. This highlights a discontinuity between the standard model, where buyers know their valuations ex-ante (the cost of information is

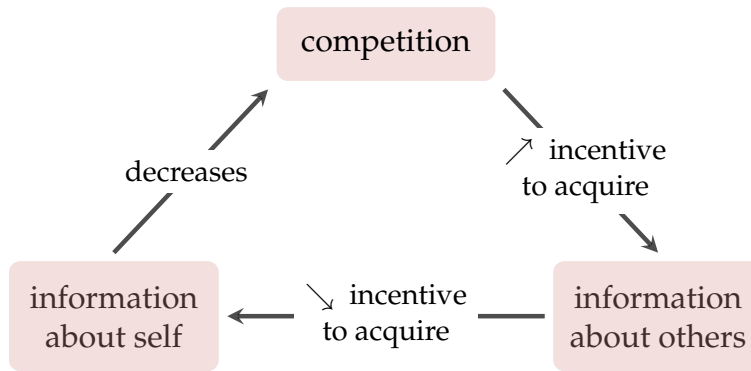


Figure 1: *The interplay between competition and information.*

Our first results highlight a new adverse effect of competition on revenue, and we investigate its implications for auction design. We show that, whenever the number of bidders N is not too small, attracting an additional bidder is less valuable than setting an optimal reserve price,⁵ suggesting that the seminal result of [Bulow and Klemperer \(1996\)](#) relies on buyers knowing their valuations fully. There are several forces at play. First, the presence of an additional bidder does not raise revenue as much as in the standard model, as it negatively impacts others' learning incentives. Second, a carefully chosen reserve price is more valuable as losing bidders oftentimes fail to learn their valuations for the good, leaving a larger expected gap between the highest and second-highest bids. Furthermore, since the optimal reserve price seeks to address this perverse effect of competition on revenue, it varies with the number of buyers N and converges to the highest possible valuation as N grows large. This contrasts with the standard model in which the optimal reserve price is independent of N .

We then show how the seller can mitigate the revenue loss by randomizing access to the auction. Indeed, by only allowing a randomly-chosen subset of buyers to participate in the sale, the seller maintains uncertainty over the extent of competition. Buyers can no longer predict whose bids they will be facing in the auction, which reduces their incentives to learn about their competitors: even if a strong buyer is present, he might not be granted access; others then still have a chance of winning, and hence an incentive to learn about their own valuations. In high-stake auctions, this unambiguously improves expected revenue. This result might explain why bidders in takeover

then effectively zero), and ours.

⁵A reserve price is a minimum acceptable bid set by the seller.

auctions are often required to sign non-disclosure agreements preventing them from revealing, among other things, their participation in the sale. Such agreements are all the more important as buyers' incentives very much conflict with the seller's on that point: high-valuation buyers benefit from disclosing their participation and bids to others, so as to deter them from conducting due diligence and reduce the expected price.⁶

Finally, we close the paper with a discussion of the model's key assumptions, which are the ones imposed on the process of information acquisition.

1.1 Related Literature

First, we build on a previous paper ([Gleyze and Pernoud \(2022\)](#)), which considers a general mechanism design setting with independent private values in which participants can acquire costly information on their preferences as well as others'. We show that most selling mechanisms incentive participants to learn about others' preferences. Hence, such an incentive cannot be designed away by the seller. In the present paper, we investigate how that affects the value of competition in auctions. We focus on the second-price auction, not only because it is a widely-used auction format but also because it is *strategy-proof*: if buyers knew their valuations, they would have a dominant strategy and would have no incentive to inquire about the competition. We can thus isolate the detrimental effect of competition on learning incentives.

Second, our paper contributes to the literature on entry and learning costs in auctions. [Levin and Smith \(1994\)](#) characterize the symmetric (mixed) equilibrium under a second-price auction when buyers pay a fixed cost to learn their values before bidding. They show that equilibrium entry decisions are efficient and revenue-maximizing in the IPV setting.⁷ In a similar setting, [Compte and Jehiel \(2007\)](#) show that dynamic auction formats tend to dominate static ones, as dynamic formats reveal more information on the toughness of competition. Another strand of the literature on learning costs allows buyers to flexibly choose *how much* information to acquire ([Hausch and Li \(1993\)](#); [Persico \(2000\)](#); [Bergemann and Välimäki \(2002\)](#); [Shi \(2012\)](#); [Kim and Koh \(2022\)](#)). Im-

⁶Even if buyers are legally bound not to disclose their bidding intentions, they might still be able to signal them—e.g., toeholds can sometimes signal an intention to bid aggressively in takeover auctions.

⁷A related literature assumes that buyers have some private information when making entry decisions ([Ye \(2007\)](#); [Quint and Hendricks \(2018\)](#); [Lu et al. \(2021\)](#), etc.). Entry then serves as a screening mechanism that the seller can leverage. Such considerations are absent in our paper.

portantly, these papers consider information acquisition about a one-dimensional random variable—usually buyers’ own valuations or a component that is common to all buyers—and so do not allow buyers to separately choose how much to learn about self and about others. Instead, our paper seeks to understand how competition affects buyers’ learning incentives. To that end, we allow buyers to also learn about their competitors’ valuations, leading to new insights and different predictions. One contribution of our paper is then to develop a tractable model of *multidimensional* learning in auctions.

A relatively small literature studies buyers’ incentives to learn about their competitors’ types in first-price auctions (Tian and Xiao (2007)) and auctions with interdependent values (Bobkova (2019); Kim and Koh (2020)).⁸ Information about opponents’ types is valuable as it allows buyers to either alleviate the winner’s curse (in interdependent-value auctions) or shade their bids more aggressively (in first-price auctions). Such incentives are absent in our setting as buyers compete in a second-price auction and their valuations are independent and private.

Our paper also speaks to the literature highlighting the value of competition in selling mechanisms, which often compares auctions to “negotiations.” Bulow and Klemperer (1996) argue that auctions have the benefit of attracting more buyers at the cost of less negotiating power for the seller,⁹ but show that attracting just one more buyer has more value than being able to commit to the optimal reserve price. Relatedly, Bulow and Klemperer (2009) show that with costly entry, actual competition in an auction dominates potential competition from a sequential entry mechanism.¹⁰ These results, however, take buyers’ information as fixed. Instead, our paper asks how competition affects the information buyers acquire and reaches opposite conclusions. Gershkov et al. (2021) also qualify the value of competition and show that it can even hurt revenue if buyers can invest to increase their values before bidding.

A growing empirical literature investigates why some sellers favor negotiations over auctions. Most of it focuses on the market for corporate control, in which both types of selling mechanisms are commonly observed (Boone and Mulherin (2007); Ak-

⁸In these papers, learning is one-dimensional: buyers usually know their own types and acquire costly information about their opponents’. In Bobkova (2019), buyers choose whether to learn about a private versus a common component in their valuations, but they cannot do both.

⁹E.g., the seller might no longer be able to withdraw the object for sale if bids are too low.

¹⁰Roberts and Sweeting (2013) extend their model with ex-ante noisy signals and asymmetries and find that the sequential entry mechanism can dominate the auction.

tas et al. (2010); Gentry and Stroup (2019)). Takeover auctions are particularly relevant applications for our analysis as they involve high due diligence costs. The same is true for procurement auctions, and Bajari et al. (2009) find that auctions tend to perform poorly when procurement projects are complex, which is consistent with our results.

Finally, several papers study the performance of auctions when buyers are ex-ante asymmetric (Maskin and Riley (2000); Compte and Jehiel (2002); Kim and Che (2004); Cantillon (2008); Jehiel and Lamy (2015); Marquez (2021)). In particular, Marquez (2021) shows that the presence of an asymmetrically strong buyer hurts revenue if it deters entry of remaining “regular” buyers. In our paper, buyers are ex-ante identical, but we show that asymmetries in private information arise endogenously, even in symmetric equilibria.

2 THE MODEL

A seller puts a unique, indivisible good for sale through a second-price auction.¹¹ There are N buyers, and buyer i 's valuation for the good is denoted by ν_i . A buyer's valuation is the sum of two components $\nu_i = v_i + u_i$, where $v_i \in V$ should be interpreted as the main component—we sometimes abuse language and refer to v_i as a buyer's valuation—and $u_i \in U$ as small mean-zero noise. Both components are identically and independently distributed across buyers. Main components $(v_i)_i$ are drawn i.i.d. from a finite set $V \subset \mathbb{R}_+$ according to a probability distribution $p \in \Delta V$. Noise terms $(u_i)_i$ are drawn from a compact interval $U \equiv [\underline{u}, \bar{u}] \subset \mathbb{R}$ according to a strictly positive and continuous density, with $\mathbb{E}[u_i] = 0$. They are small, in the sense that $\min_{v'_i \neq v''_i} |v'_i - v''_i| > \bar{u} - \underline{u}$. Hence if a buyer has a strictly greater v_i than another, then he must necessarily have a strictly greater overall valuation ν_i . The noise terms are only included in the model to address technical issues arising from the discreteness of V , but serve no other purpose.¹² We take them to be sufficiently small so as not to interfere with the rest of our analysis.

¹¹The intuition underlying our results seems more general and should extend to other environments—e.g., R&D races and other types of contests, etc.

¹²To understand how the noise terms matter, consider what happens when there are none, and buyers fully learn their valuations. Then equilibrium bids take values in V , and no bid is ever made *strictly in between* two neighboring values. A buyer has then no incentive to learn to distinguish such values. In equilibrium, buyers would have to randomize between bundling and not bundling neighboring values. The noise terms can be interpreted as perturbations à la Harsanyi (1973): they allow us to dispense from such randomization and guarantee that buyers cannot predict others' preferences *perfectly*.

Buyers have quasilinear utility functions. Buyer i 's gross payoff from the auction in state $(\nu_j)_j$ at bid profile $(b_j)_j$ equals

$$U(\nu_i, b_i, b_{-i}) \equiv \begin{cases} \frac{\nu_i - \max_{j \neq i} b_j}{|\{j = 1, \dots, N \text{ s.t. } b_j = b_i\}|} & \text{if } b_i = \max_j b_j \\ 0 & \text{otherwise} \end{cases}.$$

Note that we are considering a setting in which buyers' valuations are independent and private. Hence if buyers knew their own valuations ν_i , it would be a dominant strategy for them to bid truthfully in the auction, and the seller's expected revenue would be the expected second-highest value.

Information Structures. Buyers start with no private information, but they can learn about the realization of $(\tilde{v}_i)_i$ at some cost before competing in the auction. They also learn their own (and only their own) noise terms \tilde{v}_i for free at the end of the information acquisition process.

We assume that buyers can acquire two signals, one about their own valuations \tilde{v}_i and one about others' \tilde{v}_{-i} . Without loss of generality, buyers first acquire information about others' valuations and, conditional on the realization of this signal, acquire information on their own. Indeed, since we are looking at a (strategy-proof) second-price auction, information about others is only valuable insofar as it helps buyers decide how much they should learn about themselves. To reduce the dimension of the problem, we furthermore assume that buyers can only learn about $\max_{j \neq i} \tilde{v}_j$, and not the full vector \tilde{v}_{-i} .¹³

We model information acquisition about any random variable as the choice of a partition $\Pi = \{\pi_1, \dots, \pi_L\}$ of the set of possible realizations V . That is, if a buyer chooses information partition Π , then the buyer learns to which element of the partition the realization of the random variable belongs. If the chosen partition is $\Pi = \{V\}$, then no information is acquired. If $\Pi = \{\{v\}_{v \in V}\}$, then the partition is fully revealing. We furthermore require that buyers choose *monotone* partitions, meaning that if $v', v'' \in \pi_l$ with $v' < v''$, then all $v \in (v', v'')$ also belong to the element π_l of the partition.

¹³This is a sufficient statistic for the highest bid faced by i in any symmetric equilibrium, which is what matters for i 's gross payoff as that determines his allocation and the price he pays if he wins. That said, it is not without loss, and we investigate the robustness of our results to alternative formulations in Section 6.

Information is costly. Letting \mathcal{P} denote the set of all possible monotone partitions of V , the cost of a signal $c : \mathcal{P} \times \Delta(V) \rightarrow \mathbb{R}_+$ is a function of both the chosen partition and the prior belief about the random variable of interest. Indeed, even though both \tilde{v}_i and $\max_{j \neq i} \tilde{v}_j$ take realizations in the same set V , they have different prior probabilities. Thus we want to allow the same partition of V to have different costs depending on whether it provides information about v_i or $\max_{j \neq i} v_j$.

Strategies and Solution Concept. Buyers have two decisions to make. First, they decide what information to acquire. Then, conditional on their information set, they submit a bid to the seller.

As described above, information acquisition is sequential, and an information strategy consists of two parts. Each buyer i first chooses an information partition about others $\Pi_i^{other} \in \mathcal{P}$. Then, conditional on his information set about others $\pi_i^{other} \in \Pi_i^{other}$, he chooses an information partition about his own valuation $\Pi_i^{self} : 2^V \rightarrow \mathcal{P}$. Finally, each buyer chooses a measurable bidding strategy $\beta_i : 2^V \times 2^V \times U \rightarrow \mathbb{R}_+$, which outputs a bid given the buyer's overall information $\pi_i = (\pi_i^{other}, \pi_i^{self}, u_i)$.

Let $\Sigma \equiv \left\{ (\Pi_i^{other}, \Pi_i^{self}, \beta_i) \in \mathcal{P} \times \mathcal{P}^{2^V} \times \mathbb{R}_+^{2^V + 2^V + U} \right\}$ be the overall set of pure strategies,¹⁴ and $\sigma_i \in \Delta \Sigma$ a strategy for buyer i .

Buyer i 's ex-ante expected utility under strategy profile $(\sigma_i)_i$ writes

$$\mathbb{E}_{\sigma_i, \sigma_{-i}} \left[U(v_i, \beta_i(\pi_i), \beta_{-i}(\pi_{-i})) - \lambda \left(c(\Pi_i^{other}, p_{1:N-1}) + c(\Pi_i^{self}(\pi_i^{other}), p) \right) \right],$$

where $\lambda > 0$ is a parameter that scales the cost of information, and $p_{1:N-1}$ is the prior distribution of $\max_j \tilde{v}_j$.

A Nash equilibrium is a strategy profile such that each buyer's equilibrium strategy σ_i maximizes his ex-ante expected utility given that all others follow theirs. As usual in a second-price auction, there exist many unappealing Nash equilibria, and even more so now that the information structure is endogenous. In particular, the concept of Nash equilibrium imposes no discipline on which (losing) bid a buyer submits if, at some information set, he can predict his toughest opponent's bid and knows that he does not want to outbid it. To rule out unrealistic equilibria, we use the following trembling-hand-like refinement.

¹⁴Technically, a buyer could condition his bid not only on what he *learned* π_i but also on the partitions he *chose* $(\Pi_i^{other}, \Pi_i^{self})$ —i.e., not only the the signal *realizations* but on the signals themselves. However, that would never be strictly optimal as π_i is a sufficient statistic for a buyer's posterior belief.

Definition 1. A tremble-robust symmetric equilibrium σ^* is a Nash equilibrium in which,

- (i) all buyers choose the same strategy $\sigma_i = \sigma^*$,
- (ii) there exists a sequence of strictly positive numbers $\{\varepsilon^k\}_{k=1}^\infty$ converging to zero such that, for each buyer i , σ^* is a best-response to all other buyers j playing a perturbed strategy $\sigma_j = \hat{\sigma}^k$, where $\hat{\sigma}^k = \sigma^*$ with probability $1 - \varepsilon^k$ and, with complementary probability, buyer j 's bid is drawn from some continuous distribution F with support $[\min_{\nu_i \in V \times U} \nu_i, \max_{\nu_i \in V \times U} \nu_i]$ (independent of his information set).

Intuitively, we require that each buyer's equilibrium strategy remains optimal when his opponents might tremble with vanishing probability and make a bid drawn from a full-support distribution.

Assumptions on the Cost of Information. First, we assume that the cost of a signal only depends on the chosen partition through its effect on the buyer's belief. Any partition $\Pi = \{\pi_l\}_{l=1, \dots, L}$ induces a distribution over posterior beliefs, which puts weight on as many posteriors as there are elements in the partition $\{\mu_l\}_{l=1, \dots, L}$ with¹⁵

$$\mu_l(v) = \begin{cases} \frac{\Pr(\tilde{v}=v)}{\sum_{v' \in \pi_l} \Pr(\tilde{v}=v')} & \text{if } v \in \pi_l \\ 0 & \text{otherwise} \end{cases}.$$

We suppose that the cost of information only depends on the chosen partition through the extent to which it reduces the amount of uncertainty in the buyer's belief. Formally, there exists a measure of uncertainty $H : \Delta V \rightarrow \mathbb{R}_+$, which is a concave function of a belief, such that

$$c(\Pi, \text{prior}) = H(\text{prior}) - \mathbb{E}(H(\text{posterior}) \mid \Pi).$$

We furthermore assume that H is bounded and continuous. This formulation precludes that the cost of a signal about others be greater (or lower) than the cost of a signal about self per se. We could relax this assumption, and say that the cost of a signal about others is scaled by a different parameter λ^{other} than one about self λ^{self} . All our results would go through for $\lambda^{other} / \lambda^{self}$ not too large. More importantly, the cost

¹⁵In what follows, the random variable \tilde{v} is either a buyer's own valuation \tilde{v}_i or the valuation of his toughest opponent $\max_{j \neq i} \tilde{v}_j$.

of leaning about the competition might scale with the number of competitors $N - 1$, and we discuss how that would affect our results in Section 6.

A notable example of a cost function that has such a form is the entropic cost.

Example 1. Let H be the (extended) entropy function $H(p) = -\sum_v p(v) \log[p(v)]$.¹⁶ The cost of an information partition equals the expected reduction in the entropy of the buyer's belief that it induces:

$$c(\Pi, \Pr(\cdot)) = -\sum_v \Pr(\tilde{v} = v) \log[\Pr(\tilde{v} = v)] + \sum_{\pi^l \in \Pi} \Pr(\tilde{v} \in \pi^l) \sum_v \frac{\Pr(\tilde{v} = v)}{\Pr(\tilde{v} \in \pi^l)} \log\left(\frac{\Pr(\tilde{v} = v)}{\Pr(\tilde{v} \in \pi^l)}\right).$$

The entropic cost has been widely used in the applied literature, in particular in models of rational inattention. It has the advantage of being tractable and having solid information-theoretic foundations.

The key assumption underlying our results is an assumption on the convexity of the cost, or equivalently on the concavity of the measure of uncertainty H .

Assumption 1. The measure of uncertainty H is strongly concave: there exists $m > 0$ such that, for all $q, q' \in \Delta V$ and all $t \in [0, 1]$,

$$tH(q) + (1 - t)H(q') - H(tq + (1 - t)q') \leq -\frac{1}{2}mt(1 - t)\|q - q'\|^2.$$

Since H is concave, the left-hand side is always weakly negative. For H to be *strongly* concave, it must be sufficiently negative: the growth rate of H must have a quadratic upper bound. This assumption ensures that the cost of a partition is sufficiently convex in the fineness of the partition. In particular, it implies that it is cost-efficient for buyers to acquire *some* information about the competitors (i.e., choose an informative, though fairly coarse, partition Π^{other}) to avoid having to become *fully* informed about their own valuations, whenever the prior p is sufficiently uncertain, and m is sufficiently large.

¹⁶The entropy is usually only defined for full-support beliefs. When a buyer learns $v \in \pi^l \neq V$, he however puts zero probability on all realizations not in π^l : his posterior does not have full support. We extend the domain of the entropy function in the following way. For any belief p on the boundary of the simplex, we define $H(p)$ to be the limit of $-\sum_v \hat{p}(v) \log[\hat{p}(v)]$ as $\hat{p} \rightarrow p$ for some full-support \hat{p} .

To formalize this, let $\Pi_{v^*}^{other} \equiv \{\{v : v \leq v^*\}, \{v : v > v^*\}\}$ be the partition that divides the set of valuations V into two subsets: valuations that are below some threshold v^* and those that are above. Arguably, this is a fairly coarse partition whenever the set of valuations V is rich. If the equilibrium is efficient, a buyer i who learns $\max_j v_j > v^*$ has little to no incentive to learn to distinguish all valuations $v_i \leq v^*$ since he loses the auction at all of these. Let $\Pi_{>v^*}^{self} \equiv \{\{v : v \leq v^*\}, \{v\}_{v>v^*}\}$ be the partition that bundles all these lower valuations together. This partition is less costly than becoming fully informed of v_i , and potentially significantly so. Similarly, when $\max_j v_j \leq v^*$, buyer i might not want to distinguish all values $v_i > v^*$, and let $\Pi_{\leq v^*}^{self} \equiv \{\{v\}_{v \leq v^*}, \{v : v > v^*\}\}$. Hence even a coarse signal about others can significantly reduce how finely a buyer should learn about his own valuation.

Lemma 1. *There exists \bar{T} and \bar{m} such that if $\sum_v [p(v)]^2 \leq \bar{T}$ and $m \geq \bar{m}$, then it is less costly for buyers to acquire some information about others instead of becoming fully informed about themselves. Formally,*

$$c(\Pi_{v^*}^{other}, p_{1:N-1}) + (\Pr(v_i \leq v^*))^{N-1} c(\Pi_{\leq v^*}^{self}, p) + \left[1 - (\Pr(v_i \leq v^*))^{N-1}\right] c(\Pi_{>v^*}^{self}, p) < c(\{\{v_i\}_{v_i \in V}\}, p),$$

for some $v^* \in V$.

Proofs of all the results are in Appendix B. In Lemma 1, $\sum_v [p(v)]^2$ captures the precision of the prior belief. Indeed, this sum always lies weakly below one, is equal to one only if the prior is deterministic (i.e., $p(v) = 1$ for some v), and is lowest under a uniform prior. To see why the condition on the prior is necessary, take the extreme case in which only two valuations have strictly positive prior probability: $V = \{\underline{v}, \bar{v}\}$. Then any information acquired about others must fully reveal $\max_j v_j$: the only non-trivial information partition is the fully revealing one $\{\{\underline{v}\}, \{\bar{v}\}\}$. There is then no scope for buyers to save on information costs about their own valuations by learning a bit about others.

For the rest of the paper, we assume that the prior p is sufficiently uncertain and H is sufficiently concave that Lemma 1 holds.

Example 1 (continued). *Consider again the entropy function $H(p) = -\sum_v p(v) \log[p(v)]$. Let $N = 2$ and $V = \{\frac{1}{K}, \frac{2}{K}, \dots, \frac{K}{K}\}$ with $p(v_i) = \frac{1}{K}$ for all $v_i \in V$. The fully revealing*

partition $\Pi^{self} = \{v_i\}_{v_i \in V}$ costs

$$c(\{v_i\}_{v_i \in V}, p) = H(p) = \log(K).$$

Suppose K is even. Learning whether the competitor's value v_j is above or below $v^* = \frac{1}{2}$ costs

$$c(\Pi_{v^*}^{other}, p_{1:N-1}) = \log(K) - \frac{v^*}{K} \log(v^*) - \frac{K - v^*}{K} \log(K - v^*) = \log(2).$$

When buyer i learns v_j is greater (respectively, lower) than $v^* = 0.5$, he only needs to learn his valuation precisely when it is also greater (resp., lower) than $v^* = 0.5$, which costs

$$c(\Pi_{\leq v^*}^{self}, p) = c(\Pi_{> v^*}^{self}, p) = \log(K) - \frac{1}{2}[\log(K) - \log(2)].$$

Hence that allows him to reduce learning costs about his value by $\frac{1}{2}[\log(K) - \log(2)]$, and so learning about the competition is cost-efficient whenever $K > 8$.

3 HOW COMPETITION SHAPES BUYERS' INFORMATION

This section investigates how the competitive pressure between buyers affects what information they seek and the resulting equilibrium information structure. We first consider two benchmark cases in which buyers are either exogenously informed of their valuations or can only acquire costly information about their own valuations. We show that these two benchmarks yield the same predictions when information costs are small relative to the value of the good. This is, however, not the case when buyers can also learn about their competitors.

3.1 Two Benchmark Cases

In the first benchmark we consider, buyers are exogenously informed of their valuations. This case is well understood, and bidding truthfully is a dominant strategy for buyers. Whether or not they know others' valuations, or can acquire information about them, is then irrelevant. We report the properties of the equilibrium for completeness.

Proposition 0. *Suppose buyers know their valuations ex-ante. Then there exists a symmetric equilibrium in which expected revenue equals the expected second-highest valuation $\mathbb{E}[\nu_{(2)}]$.*

Now suppose buyers have no private information ex-ante and can only acquire information on their own valuations. Most papers on information acquisition in auctions focus on this case.

Proposition 1. *Suppose buyers can only learn about themselves. Then, for λ small enough, there exists a symmetric equilibrium in which they all become fully informed about their own valuations, and expected revenue equals the expected second-highest valuation $\mathbb{E}[v_{(2)}]$.*

Hence, the two benchmarks yield similar predictions for small information costs. The intuition is direct: the gains associated with distinguishing two realizations of \tilde{v}_i are always strictly positive, as a buyer might face a price (i.e., a highest bid) that falls precisely between these two realizations. If information costs are small enough, buyers must choose the fully revealing partition $\Pi^{self} = \{\{v\}_{v \in V}\}$.

3.2 The General Case

We now consider our main model specification, in which buyers can acquire information on their valuations as well as others'. We show that buyers have an incentive to learn a bit about their competitors, so as not to waste resources learning about their own valuations when such information makes no difference.

We start by establishing equilibrium existence.

Proposition 2. *There exists a symmetric equilibrium that is robust to trembles for any cost parameter λ .*

In what follows, we use the term “equilibrium” to refer to a tremble-robust symmetric equilibrium.

We now show that, contrary to our benchmarks, buyers cannot all become fully informed of their valuations in equilibrium. This is true even as the cost parameter λ becomes arbitrarily small.

Proposition 3. *There exists $\varepsilon > 0$ such that, for any sequence of equilibria $\{\sigma_\lambda\}_\lambda$,*

$$\lim_{\lambda \rightarrow 0} \Pr(\Pi^{self} = \{\{v_i\}_{v_i \in V}\} \mid \sigma_\lambda) \leq 1 - \varepsilon.$$

The intuition is the following. If buyers learn their valuations fully, they simply bid truthfully in equilibrium, and the good goes to the highest-valuation buyer. It is then

cost-efficient for buyers to first assess how much competitive pressure they will face in the auction (i.e., what is the highest valuation among their competitors) and then only learn their own valuations when it is worth, as this leads to strictly lower overall information costs (Lemma 1). In the proof, we show that doing so does not harm their gross payoff from the auction and must hence be a profitable deviation.

More generally, this highlights buyers' incentive to learn about their competitors. If they do so, then their private information (i.e., their types) when entering the auction will be interdependent. Indeed, not only will buyers have information relevant to others, but their beliefs about their own valuations will depend on what they learned about others. In other words, the equilibrium information structure will fail to satisfy the standard assumption of independent private types, despite buyers' valuations being statistically independent. This significantly complicates the analysis of equilibrium behavior, as buyers no longer have a dominant strategy when deciding how to bid.

To illustrate this, consider a buyer i who acquired no information whatsoever and let $N = 2$. If buyers were not able to acquire information about others, then bidding his expected valuation would be a dominant strategy for i . This is no longer the case in our model. Indeed, buyer j might have learned something about i 's valuation, in which case j 's bid might carry information that is relevant to i . For instance, suppose buyer j becomes full informed about his competitor's valuation (i.e., $\Pi_j^{other} = \{\{v\}_{v \in V}\}$) and always bids just below it (i.e., $\beta_j(\pi_j) = v_i + \underline{u}$). Given j 's strategy, buyer i *always* wants to win the auction since the price he pays always lies strictly below his valuation: a best response is for i to bid sufficiently high so as to be guaranteed a win. Bidding his expected valuation given *his own information set* results in a strictly lower payoff for i , and so it is no longer a dominant strategy.

Overall, buyers' ability to learn about each other can create rich interdependencies between their information and bids, potentially expanding widely the types of behavior sustainable in equilibrium. Yet, focusing on high-stake auctions (that is, auctions with small cost parameter λ) and symmetric equilibria enables us to characterize equilibrium information structures.¹⁷

Theorem 1. *There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, there exists $\varepsilon(\lambda) > 0$ with $\lim_{\lambda \rightarrow 0} \varepsilon(\lambda) = 0$ such that, if an information structure has probability $\Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda) \geq$*

¹⁷Focusing on high-stake auctions ensures that any information that has non-trivial value must be acquired in equilibrium, thus reducing the noise in buyers' behaviors.

$\varepsilon(\lambda)$ in some equilibrium σ_λ , then it solves

$$\min_{\hat{\Pi}^{other}, \hat{\Pi}^{self}} c\left(\hat{\Pi}^{other}, p_{1:N-1}\right) + \mathbb{E}_{\hat{\pi}^{other}} \left[c\left(\hat{\Pi}^{self}(\hat{\pi}^{other}), p\right) \right]$$

s.t.

(\star)

$$\hat{\Pi}^{self}(\hat{\pi}^{other}) = \left\{ \left\{ v_i \mid v_i < \min_{v \in \hat{\pi}^{other}} v \right\}, \left\{ v_i \right\}_{v_i \in \hat{\pi}^{other}}, \left\{ v_i \mid v_i > \max_{v \in \hat{\pi}^{other}} v \right\} \right\} \forall \hat{\pi}^{other} \in \hat{\Pi}^{other}.$$

In words, condition (\star) requires that if buyer i knows his toughest competitor's valuation belongs to some interval π^{other} , then buyer i learns his own valuation fully if and only if it falls in the same interval. Acquiring more information that this does not affect the outcome of the auction in equilibrium, and thus serves no purpose. Figure 2 illustrate condition (\star).

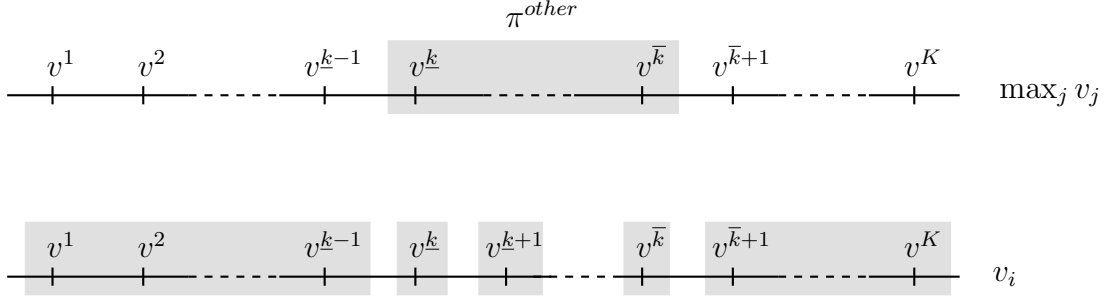


Figure 2: Let $V = \{v^1, v^2, \dots, v^K\}$ and order the valuations in increasing order, i.e., $v^k < v^{k+1}$. If buyer i learns that $\max_j v_j \in \pi^{other} \equiv \{v : v^k \leq v \leq v^{\bar{k}}\}$ (top line), he chooses to fully learn his valuation if it belongs to the set π^{other} , but fails to distinguish all valuations that are for sure lower or higher than $\max_j v_j$ (bottom line).

There exist many information structures satisfying (\star),¹⁸ but an equilibrium information structure must furthermore minimize total information costs. Intuitively, there is a certain amount of information that guarantees buyers make no mistake at the bidding stage (condition (\star)) and buyers choose the cheapest way to achieve it. Lemma 1 then implies $\Pi^{other} \neq \{V\}$, as fully learning one's own valuation is not cost-efficient, and buyers acquire *some* information about others in equilibrium. Overall, the com-

¹⁸For instance, not acquiring any information about the competition $\Pi^{other} = \{V\}$ and fully learning one's own value $\Pi^{self}(\{V\}) = \{\{v\}_{v \in V}\}$.

petitive pressure that buyers impose on each other shapes the information that buyers acquire in significant ways. The rest of the paper examines how that, in turn, affects the value of competition.

4 REVENUE AND ENTRY DISTORTIONS

In this section, we analyze the impact of learning incentives on revenue and entry. We show that the equilibrium information structure leads buyers to compete less aggressively for the good, which depresses revenue and distorts entry.

4.1 Revenue Loss

Since buyers do not learn their valuations fully in equilibrium, expected revenue is likely to be different than in our benchmark cases. Our first main result states that revenue remains strictly lower and bounded away from the expected second-highest valuation, even for small information costs λ .

Theorem 2. *Let $N \geq 3$. There exist $L > 0$ and $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, the revenue generated in any equilibrium σ_λ of the second-price auction is bounded away by L from the expected second-highest valuation:*

$$\mathbb{E} [\text{equilibrium revenue} \mid \sigma_\lambda] < \mathbb{E} [\nu_{(2)}] - L.$$

Note that the constant L is independent of the cost parameter λ . Hence revenue is bounded below the expected second-highest valuation, and does not converge to it as information costs vanish. This contrasts with our above benchmarks, where revenue converges to the expected second-highest valuation as λ goes to zero.

The intuition is simple. In equilibrium, buyers first assess the competition and only learn their own valuations if they fall in a similar range as that of their toughest competitor. As a result, losing bidders often only learn that their valuations are below some threshold (and, in particular, below that of their toughest competitor) but fail to learn it exactly. Our equilibrium refinement guarantees that if losing bidders fail to learn their valuations, they bid their expected valuations given their information sets. This reduces the variance in losing bids and distorts the expected second-highest bid downwards whenever $N \geq 3$. Indeed, since the \max is a convex function, the expected

highest bid among losing bids is greater when losing bids are more dispersed. (With only $N = 2$ buyers, there is only one losing bid, and dispersion plays no role.)

We emphasize that for sufficiently small information costs λ , the equilibrium allocation of the good remains efficient in our model, namely the highest valuation bidder wins the good. Indeed, a buyer only fails to learn his valuation in equilibrium if he is sure of losing (or winning) given what he learned about others. Hence a direct corollary of Theorem 2 is that endogenous information acquisition does not affect total surplus for small information costs λ , but redistributes surplus from the seller to the buyers. It furthermore suggests that, if possible, high-valuation buyers have a strong incentive to signal that they have a high valuation, so as to discourage others from learning about their own and competing aggressively. This is often seen in practice. For instance, jump bidding and toeholds are sometimes seen as signaling devices that aim at deterring competition (Bulow et al. (1999); Betton and Eckbo (2000); Hörner and Sahuguet (2007)).

A Uniform Example. Let $V = \{\frac{1}{K}, \frac{2}{K}, \dots, \frac{K-1}{K}, \frac{K}{K}\}$ be the set of possible valuations, and $\Pr(\tilde{v}_i = v) = \frac{1}{K}$ be the prior probability of each $v \in V$. For K large enough, this approximates a uniform distribution on $[0, 1]$. We set H to be the entropy function as in Example 1, such that $H(p) = -\sum_v p(v) \log(p(v))$ for any belief $p \in \Delta V$.

We know from our analysis that for small enough cost parameter λ , buyers only put non-trivial weight on cost-minimizing information structures satisfying (\star) . Under such information structures, buyers must acquire some information about others $\Pi^{other} \neq \{V\}$, and only learn to distinguish their own valuations when they fall in the same range as that of their toughest competitor. We find the cheapest such information structure numerically,¹⁹ and depict the equilibrium information partition about others Π^{other} for several values of N in Figure 8 of Appendix A.1. We then simulate equilibrium bids and compute expected revenue for vanishing λ .²⁰ We also compute expected revenue when buyers are fully informed of their valuations, which equals the expected second-highest valuation.

Figure 3 shows how expected revenue remains bounded away from its full infor-

¹⁹Though there could technically exist several cost-minimizing information structures, this never happens in our numerical example.

²⁰In Figure 3, we fit a flexible (6th-degree) polynomial on expected revenue to smooth small irregularities arising from the discreteness of the set of valuations V . Indeed, its discreteness induces slight non-monotonicities in N that disappear as the grid of possible valuations gets finer (i.e., as K increases).

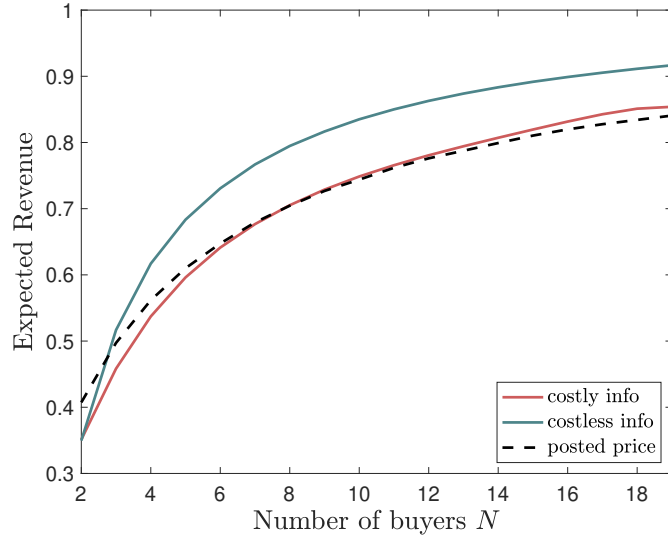


Figure 3: The top (blue) line plots expected revenue in the standard model (i.e., the expected second-highest value). The bottom (red) line plots expected revenue in our model for small λ . For comparison, the dashed (black) line plots expected revenue in the standard model when the seller uses a posted-price mechanism (i.e., commits to a price and, if more than one buyer is interested, the winner is chosen uniformly at random). The difference between the blue and red lines is the revenue loss from Theorem 1. Parameter $K = 30$.

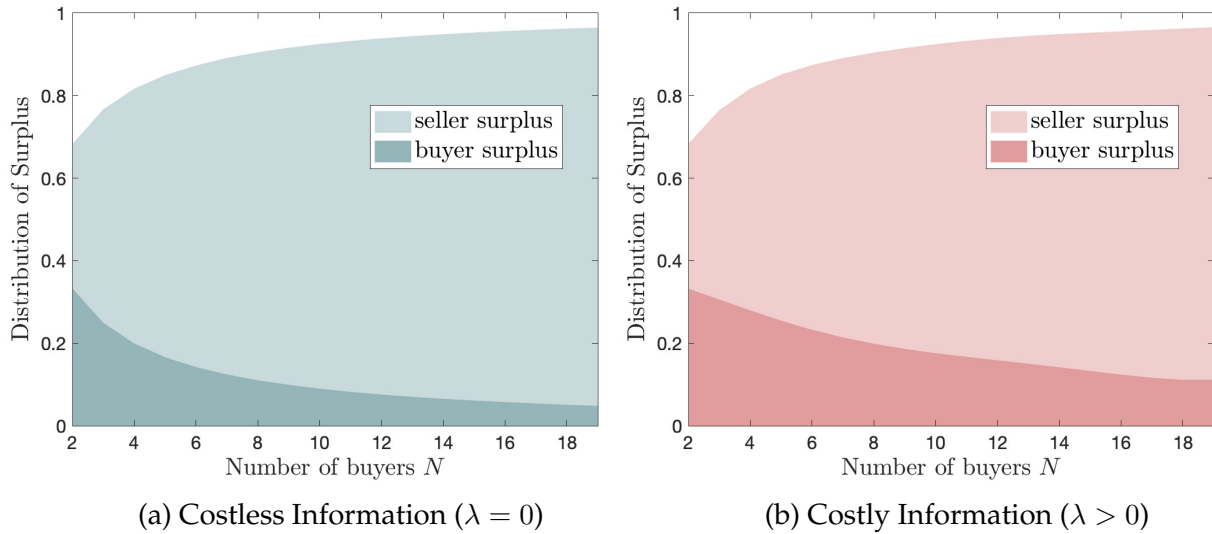


Figure 4: Distribution of surplus between the seller and buyers in our uniform example. As before, parameter $K = 30$.

mation benchmark even as information costs become arbitrarily small. The revenue loss due to endogenous information acquisition is captured by the difference between the top (blue) and the bottom (red) lines. To assess the magnitude of this loss, we compare it to the loss in revenue associated with using a posted-price mechanism instead of an auction in the standard model.²¹ In this example, the revenue loss due to endogenous information acquisition is similar in magnitude to the loss associated with using a suboptimal posted-price mechanism (difference between blue and dashed black lines).

Since the equilibrium allocation remains efficient when λ is small, the revenue loss means that buyers get a higher surplus than in the standard model with exogenous information. This is illustrated in Figure 4.

4.2 Entry Distortion

We now extend our baseline model to include entry decisions. After the information acquisition stage, buyers decide whether or not to participate in the auction. If they do, they incur an entry cost $\kappa > 0$. Formally, a buyer enters the auction and pays the entry cost κ at information set π_i if he makes a non-zero bid, that is, if $\beta_i(\pi_i) > 0$.

Entry costs are common in practice. For instance, to participate in the 1991 auction for television franchises in the UK, TV channels had to provide a detailed listing of what they would air. This resembles more a fixed entry fee than an information acquisition cost. More generally, participating in an auction always entails some fixed (e.g., legal) costs.

Consider first what happens in the standard model where buyers know their valuations ex-ante. In any equilibrium satisfying the tremble-hand-like refinement, buyers who enter the auction bid truthfully. Buyers with higher valuations have a greater incentive to enter, and in equilibrium, a buyer enters if and only if his valuation is above a threshold $\nu_i \geq \nu^*$. A buyer with a valuation precisely at the threshold must be indifferent between entering or not. He knows that he will win the auction only if no one else enters, in which case he pays a price of zero. The indifference condition is then:

$$\Pr \left(\max_{j \neq i} \nu_j < \nu^* \right) \nu^* = \kappa.$$

²¹Under a posted-price mechanism, the seller chooses a (unique) price and, if several buyers express interest in buying the good at that price, allocates the good randomly between them.

Let $\tilde{N}_0 = |\{i \mid \tilde{v}_i \geq \nu^*\}|$ be the (random) number of buyers who enter the auction in that equilibrium.

If buyers can only learn about themselves, then for λ small enough, there exists an equilibrium in which buyers learn to distinguish all valuations they can have above ν^* , and only enter when their valuations are above ν^* . Hence the equilibrium yields the same allocation and revenue as what the standard model predicts. On the contrary, entry decisions are much different when buyers can also learn about their competitors' valuations.

Proposition 4. *Let $N \geq 3$. There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, there exists $\varepsilon(\lambda) > 0$ with $\lim_{\lambda \rightarrow 0} \varepsilon(\lambda) = 0$ such that, in any equilibrium σ_λ , the probability that at least two buyers enter the auction is bounded above by*

$$\Pr\left(\tilde{N}_\lambda > 1 \mid \sigma_\lambda\right) \leq \Pr(v_{(1)} = v_{(2)}) + \varepsilon(\lambda),$$

where $\tilde{N}_\lambda = |\{i \mid \beta_\lambda(\tilde{\pi}_i) > 0\}|$ is the (random) number of buyers who enter.

Proposition 4 states that several buyers enter the auction only if their valuations fall in a similar range. In the proof, we show that two buyers i, j , with values $v_i < v_j$ cannot *both* enter the auction with non-vanishing probability in equilibrium. If they were to both enter, buyer i would never win the auction as his overall valuation ν_i must lie strictly below j 's. He would then want to learn j 's valuation so as not to enter in those states and save on the entry cost. Entry costs then reinforce the revenue loss described in Theorem 2, as losing buyers not only fail to learn their valuations for the good but stay out of the auction altogether. This is precisely what happened in the 1991 U.K. auction for television franchises in two regions (the Midlands and Scotland). In each region, the incumbent firm was expected to win and ended up being the only one putting together a complete programming plan and bidding for the license.²²

A direct corollary of Proposition 4 is that, for sufficiently small entry cost κ , there is more entry in the standard model than in ours. Indeed, for κ small enough, the entry threshold when buyers know their valuations ν^* is very close to the lowest possible valuation. With probability close to one, all buyers then enter the auction. On the contrary, the probability that two or more buyers enter the auction remains bounded below one in our model (Proposition 4), irrespective of the size of the entry cost κ .

²²See [Klemperer \(2002\)](#) for more details.

Observe that, once entry decisions are introduced, the standard model (in which buyers know their own valuations ex-ante, but not that of others) yields different predictions from a model with costless information ($\lambda = 0$, such that buyers know both their own and their toughest competitor's valuations). Indeed, in the former, entry decisions are characterized by the above threshold ν^* . In the latter, two buyers with valuations $v_i \neq v_j$ cannot both enter. Unlike for revenue, there is then no discontinuity in entry decisions between a model in which information is costly ($\lambda > 0$ but small) and one in which it is free ($\lambda = 0$).

5 MARKET DESIGN IMPLICATIONS

So far, we have shown that losing buyers often fail to learn their valuations precisely, which leads them to bid less aggressively and depresses expected revenue. This has implications for market design. First, the value of an optimal reserve price often dominates the value of an additional bidder. This contrasts with common wisdom inherited from [Bulow and Klemperer \(1996\)](#) (thereafter, "BK"). Second, the seller gains by randomizing access to the auction. Doing so maintains uncertainty over the extent of competition and incentivizes buyers to learn their valuations. Overall, this suggests that competition is most effective if it is carefully *designed* by the seller.

5.1 Additional Buyer vs. Reserve Price

In a seminal paper, BK show that the value of an additional bidder in an auction always dominates the value of optimizing the reserve price. This is an important result as it gives market designers a very simple and actionable insight: attract as many bidders as possible.

Two facts bring some nuance to this common wisdom. First, many high-value sales operate via negotiations with just a few buyers instead of open auctions. Indeed, about half of firm acquisitions occur through negotiations instead of auctions ([Boone and Mulherin \(2007\)](#)). Second, and as mentioned in the introduction, auctions with many bidders sometimes fail.

We revisit this result in our setting with information acquisition. To do so, we tweak the model slightly and let the support of the noise terms $[\underline{u}_N, \bar{u}_N]$ depend on the number of buyers N . This will be important for the following result and we discuss why

afterwards.

Theorem 3. *Suppose $(\bar{u}_N - \underline{u}_N) = o(N \exp(-N))$ as $N \rightarrow \infty$. There exists \bar{N} such that, for all $N \geq \bar{N}$ and for λ small enough, revenue with $N + 1$ bidders in the second-price auction without reserve is **lower** than revenue with N bidders in the second-price auction with optimal reserve.*

That are several forces underlying Theorem 3. First, competition is less valuable than in the standard model, as additional buyers not only fail to learn their valuations fully but also exert a negative externality on others' learning incentives. For some values of N , it can be that adding a buyer to the auction changes the equilibrium information structure, leading buyers to learn more about their competitors—a finer Π^{other} —and less about themselves—a coarser Π^{self} in expectation. This has an adverse effect on expected revenue. Second, a reserve price is more valuable in our setting. Indeed, losing buyers often fail to learn their valuations precisely, which leaves a larger expected gap between the highest and second-highest bid, and hence more room for a carefully-designed reserve price to intervene. In the proof, we show that these forces favor a reserve price whenever the number of buyers N is sufficiently large. Note that this result is not driven by higher N having a negative impact on revenue—our numerical examples suggest revenue is still increasing in N ,—but instead shows that auction design has a *relatively* larger value when information is endogenous.

Why is the condition on the support of the noise terms needed? Think of what happens when the number of buyers gets large. With high probability, at least two buyers will have the highest valuation possible $\max_{v \in V} v \equiv v^K$. All these buyers will learn their valuations fully and bid $\nu_i = v^K + u_i$. In such cases, revenue is effectively the same as in the standard auction model with $|\{i : v_i = v^K\}|$ bidders and valuations drawn from $[v^K + \underline{u}, v^K + \bar{u}]$. In that world, the BK result holds as an additional bidder increases the expected highest draw of u_i . This force goes against our result but does not overturn it whenever the support of the u_i s shrinks fast enough with N .

Note that Theorem 3 requires the number of buyers N to be large enough. Our uniform example however suggests that N does not have to be very large for Theorem 3 to apply.

Uniform Example Continued. We revisit the uniform example from Section 3.3 to illustrate Theorem 3. The left panel of Figure 5 plots the value of an additional bidder

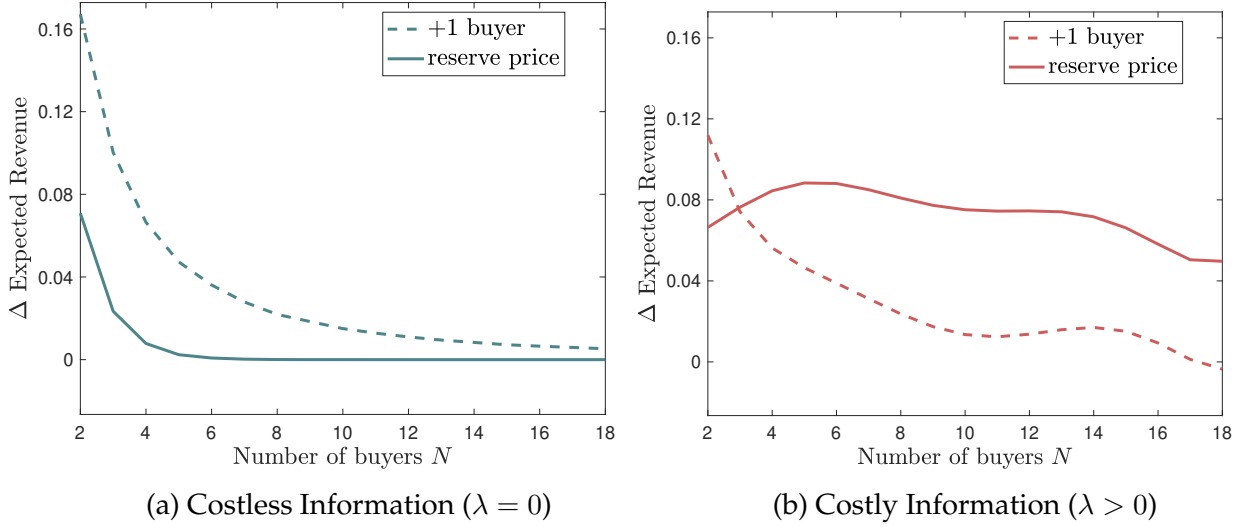


Figure 5: The dashed lines plot the value of an additional bidder—i.e., the difference between expected revenue under $N + 1$ bidders and expected revenue under N bidders—as a function of N . The solid lines plot the value of a reserve price—i.e., the difference between expected revenue under N bidders with an optimal reserve price and without a reserve price—as a function of N . All curves are fitted with a flexible (6th-degree) polynomial to smooth small irregularities arising from the discreteness of the set of valuations V .

and the value of a reserve price in the standard model, in which buyers know their valuations and bid truthfully. The value of an additional bidder is then always greater than that of a reserve price (BK’s result). The opposite is true in our model (right panel).

Not only is the optimal reserve price more valuable in our framework than in standard theory, but it also has different properties. When buyers know their valuations ex-ante, the optimal reserve price is known to be independent of the number of participants N in the auction (see, e.g., Myerson (1981)).²³ In our framework, the optimal reserve price converges to the highest possible valuation as the number of buyers N grows large and, more generally, seems to be increasing in N . See Figure 9 in Appendix A.1 for an illustration.

²³This result technically requires buyers’ set of possible valuations to be an interval and the distribution to satisfy an appropriate regulatory condition. It holds approximately when our finite set of valuations is sufficiently fine.

5.2 Maintaining Uncertainty over Competition

To mitigate the revenue loss, the seller needs to incentivize buyers to learn their valuations for the good. In this section, we show that by inducing uncertainty on the set of buyers allowed into the auction, the seller can induce higher information acquisition and increase revenue.

The set of potential buyers is still exogenous and equal to $N = \{1, \dots, N\}$, but the seller can now commit to only letting some (random) subset of buyers compete in the auction. That is, the seller can commit to only considering some of the submitted bids. Let \tilde{M} be the random set of buyers who get access to the auction—that is, buyers whose bids are taken into consideration,—whose distribution is chosen by the seller. As before, buyers acquire information before bidding and, in particular, before knowing the realization of \tilde{M} .

Consider the following way to randomize access to the auction. With probability $1 - q < 1$, all buyers get access $M = \{1, \dots, N\}$. All bids are then taken into consideration: the good goes to the highest bidder who pays the second-highest bid. With probability q , one buyer chosen uniformly at random is denied access to the auction: $M = \{1, \dots, N\} \setminus i$ for some i . In such an event, the seller acts as if buyer i had not submitted a bid. Hence, even if one learns that another buyer has a greater valuation, there is still some strictly positive probability q/N that the other buyer's bid will not be accounted for. Information about one's own valuation is then strictly beneficial: for λ sufficiently small, buyers become fully informed.

Proposition 5. *Take any $\varepsilon > 0$. There exists an access rule \tilde{M} such that, for λ small enough,*

$$\mathbb{E} [\text{equilibrium revenue} \mid \sigma_\lambda] \geq \mathbb{E} [\nu_{(2)}] - \varepsilon$$

in any equilibrium σ_λ . In particular, the access rule described above with $q = 1 - \frac{\varepsilon}{\mathbb{E}[\nu_{(2)]]}$ yields such revenue.

Proposition 5 suggests that randomizing access to the auction is a powerful tool to incentivize information acquisition. By maintaining uncertainty over the competition that a buyer will face in the auction, the seller reduces the negative effect of competition on learning incentives. In the proof of Proposition 5, we show that if a buyer's toughest competitor has a non-zero chance of being excluded from the auction, then the buyer has a strict incentive to learn his valuation for the good. For λ sufficiently small, he

will then do so.

Our results can explain why sellers sometimes try to keep secret the identity of participants in an auction. For instance, potential bidders in takeover auctions sign a confidentiality agreement that prevents them from revealing, among other things, their participation in the auction and the value of their bids (see [Gentry and Stroup \(2019\)](#) for a description of a typical takeover auction).

Finally, we compare the value of randomizing access to that of setting an optimal reserve price—which is sometimes a complicated endeavor when the seller has little information about the value of the object.

Theorem 4. *Suppose $(\bar{u}_N - \underline{u}_N) = o(N \exp(-N))$ as $N \rightarrow \infty$. There exists \bar{N} such that, for all $N \geq \bar{N}$ and for λ small enough, randomizing access to the auction leads to **higher** revenue than setting an optimal reserve price.*

Hence, when information is endogenous and shaped by competition, randomizing access—and thus sometimes allocating the good to the *wrong* buyer—improves revenue more than setting an optimal reserve price. These two allocative distortions serve different purposes: the former incentivizes buyers to acquire more information about their valuations for the good, while the latter reduces the rent they get from said information. [Theorem 4](#) then says that in our framework, there is more value in incentivizing buyers to acquire information than in reducing their information rent. We revisit our uniform example in [Figure 6](#) to illustrate this.

6 DISCUSSION AND ROBUSTNESS OF THE MODEL

The key assumptions in our analysis are the ones imposed on the process of information acquisition. Some are required for tractability, while others can be relaxed to some extent. We now discuss these assumptions in more detail.

The structure of the Learning Process. We model information acquisition as a two-step process, in which buyers first acquire a signal about their competitors' valuations and then one about their own. The following proposition says that such *ordering* is without loss within the class of two-step learning processes.

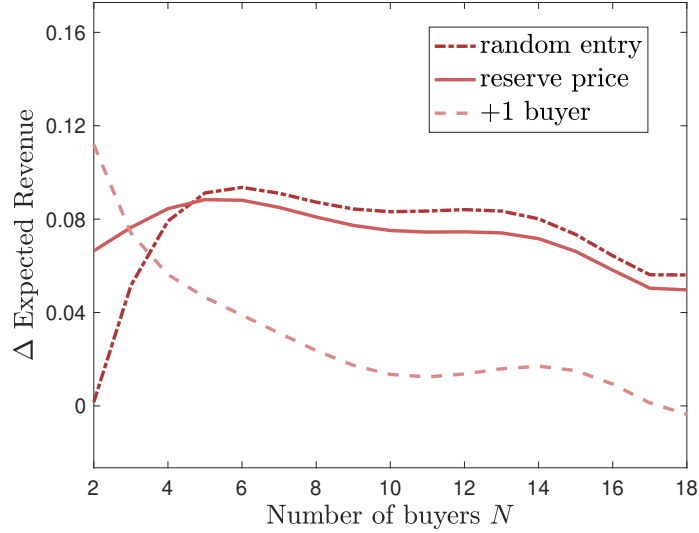


Figure 6: The dash-dotted line plots the value of randomizing access to the auction—i.e., the difference between expected revenue when access is randomized optimally and expected revenue when all buyers are allowed into the auction—for λ arbitrarily small. The other two lines are the same as in Figure 4, and represent the value of a reserve price and of an additional buyer. All curves are fitted with a flexible (6th-degree) polynomial to smooth small irregularities arising from the discreteness of the set of valuations V .

Proposition 6 (order of signals). Consider our main model specification, but suppose that buyers can now choose in which order to acquire the two signals.²⁴ There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$ and in any equilibrium σ_λ , buyers choose to acquire information on others first, and all results are unchanged.

The proof of Proposition 6 consists in showing that, in any equilibrium, buyers choose to acquire information about the competition first. The only alternative ordering would have buyers first choose how much to learn about their own values before learning about others'. For sufficiently small information costs λ , they would then find it optimal to fully learn their values and learn nothing about their competitors.²⁵ This, however, cannot be part of an equilibrium (Proposition 3). In that sense, the ordering of signals imposed in our model is without loss.

²⁴That is, they can choose whether to first acquire a signal about their own value v_i and then one about others $\max_j v_j$, or vice versa.

²⁵Indeed, information about others is only valuable insofar as it helps buyers determine what information to acquire about their own values.

Signals as Partitions. Throughout the paper, we model signals as information partitions. Partitions are a particular type of signals in that they are “deterministic:” when a buyer chooses partition Π^{self} , he learns *with probability one* to which element of the partition his value belongs. In other words, he never gets the “wrong” signal. Naturally, this is a simplifying assumption, but one that is needed for tractability. With more noise in the learning process, characterizing equilibrium bidding becomes much more challenging. Recall that in our setting, the second-price auction is no longer strategy-proof: that is, it might not be optimal for a buyer i to bid his expected valuation given his information set. Indeed, other buyers might have acquired information about their competitors—which include i himself—and so their bids might carry information relevant to i . If i does not know his valuation fully upon bidding, he must then account for the fact that winning at a particular bid from his competitors carries information about his own value. There is no nice structure on how a buyer should update about his value upon tying at a particular bid—e.g., he could update positively at a particular bid but negatively at another. Characterizing equilibrium bidding is then much more challenging. By imposing this particular structure on signals, we reduce the noise in the learning process and recover tractability: if a buyer ties, he must have a valuation close to that of his toughest opponent, and so must have learned his valuation fully. This is also why we focus on small information cost λ .

Learning about the max. A perhaps more economically substantive assumption concerns buyers’ ability to learn about their toughest competitor’s valuation $\max_j v_j$ without having to learn about each competitor $(v_j)_j$ individually. This assumption is mainly made for tractability, as it significantly reduces the dimensionality of the problem. Combined with the assumption on the cost of information, it however does imply that the cost of learning about the competition does not scale with the number of competitors. This seems a good approximation in settings where buyers learn about the competition in aggregate—e.g., by undertaking market research or by hiring a consulting company to inform them of the overall competition. In other settings, however, buyers might only be able to learn about their competitors one at a time. That is, buyers might only be able to learn about $\max_j v_j$ by learning about each v_j independently.

To investigate how this would affect our results, suppose that the cost of information partition Π^{other} about $\max_j v_j$ now equals $\gamma(N - 1)c(\Pi^{other}, p)$, where γ is a scaling function that depends on the number of competitors $N - 1$. A notable example is

$\gamma(N - 1) = N - 1$, such that to learn something about $\max_j v_j$, a buyer must acquire that same partition about each one of its $N - 1$ competitors, and the cost of doing so adds up linearly. More generally, γ could be concave or convex depending on the learning technology.

We revisit our uniform example one last time and derive equilibrium revenue for three different parametrizations of γ (one linear, one concave, and one convex). The results are depicted in Figure 7. Focus first on the top graphs. If the number of com-

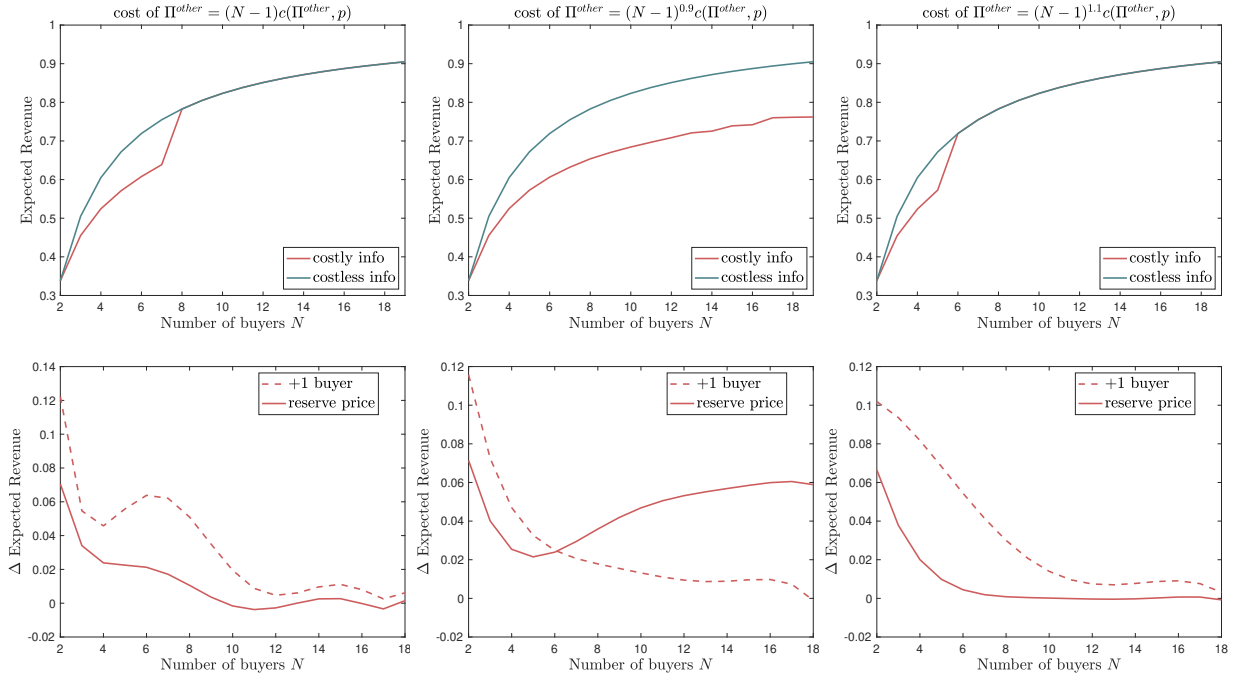


Figure 7: The top figures plot expected revenue in the standard model (blue) and in our model for small λ (red). The bottom figures plot the value of an additional bidder (dashed line) and the value of a reserve price (solid line) in our model. Both are shown for three different parametrizations of how the cost of learning about the competition scales with the number of competitors. The left figures consider a linear scaling function ($\gamma(N - 1) = N - 1$), the middle figures a slightly concave scaling function ($\gamma(N - 1) = (N - 1)^{0.9}$), and the right figures a slightly convex scaling function ($\gamma(N - 1)^{1.1} = N - 1$). All curves are fitted with a flexible (6th-degree) polynomial to smooth small irregularities arising from the discreteness of the set of valuations V . Parameter $K = 100$.

petitors is not too large, then it remains cost-efficient for buyers to acquire some information about the competition before learning their own values, and the revenue loss persists. As N increases, inquiring about the competition becomes increasingly costly,

and above some threshold it is no longer cost-efficient to do so. Hence, under such alternative modeling, Theorem 1 and 2 persist for auctions of intermediate size but not necessarily for large auctions.

Theorem 3 however seems less robust to alternative assumptions on the cost of learning about the competition. Recall that Theorem 3 compares the value of a reserve price to the value of an additional bidder. Now that the cost of learning about others scales with the number of competitors, having an additional bidder in the auction might discourage bidders from learning about the competition. This force goes against our result and was absent in our baseline model. Whether or not it overturns the result depends on how exactly this cost scales with $N - 1$. For instance, when the scaling function is slightly concave (middle panel), then the value of a reserve price is still greater than that of an additional bidder whenever $N \geq 6$.

7 CONCLUSION

This paper develops a tractable model of multidimensional information acquisition in auctions, in which buyers can first learn about the strength of competition before learning about their own valuations. We first characterize how the competitive pressure between buyers shapes the information that they seek. In our framework, buyers find it cost-efficient to first acquire some information about their competitors so as to only learn their valuations when they have a chance to win. Second, we show that competition between buyers is made less effective by learning incentives. Losing buyers often fail to learn their valuations precisely and, as a result, compete less aggressively for the good. This depresses expected revenue. We then propose market design solutions to mitigate these effects. Overall, we show that the seller benefits from carefully designing the competition that buyers face—either via a reserve price or by maintaining uncertainty over the set of auction participants.

Our results suggest that the interactions between information and competition can have large, previously unknown implications for auction design. We believe it provides yet another justification for robust mechanism design, or at least a careful consideration of informational incentives in the practice of market design.

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APPENDIX A ADDITIONAL MATERIAL

A.1 Additional Details on the Uniform Example

Throughout the paper, we use a uniform example in which $V = \{\frac{1}{K}, \frac{2}{K}, \dots, \frac{K-1}{K}, \frac{K}{K}\}$ and $\Pr(\tilde{v}_i = v) = \frac{1}{K}$ for all $v \in V$ to illustrate our results. We know from Theorem 1 that, in equilibrium, all buyers choose the cost-minimizing information structure $(\Pi^{other}, \Pi^{self})$ that satisfies

$$\Pi^{self}(\pi^{other}) = \left\{ \left\{ v_i : v_i < \min_{v \in \pi^{other}} v \right\}, \left\{ v_i \right\}_{v_i \in \pi^{other}}, \left\{ v_i : v_i > \max_{v \in \pi^{other}} v \right\} \right\},$$

for all $\pi^{other} \in \Pi^{other}$. We find this cost-minimizing information structure numerically, and depict Π^{other} for several values of N in Figure 8. For instance, when $N = 3$, Π^{other} partitions the set of valuations into four intervals: buyers learn whether their toughest competitor has a valuation below .28, between .28 and .5, between .5 and .72, or above .72.

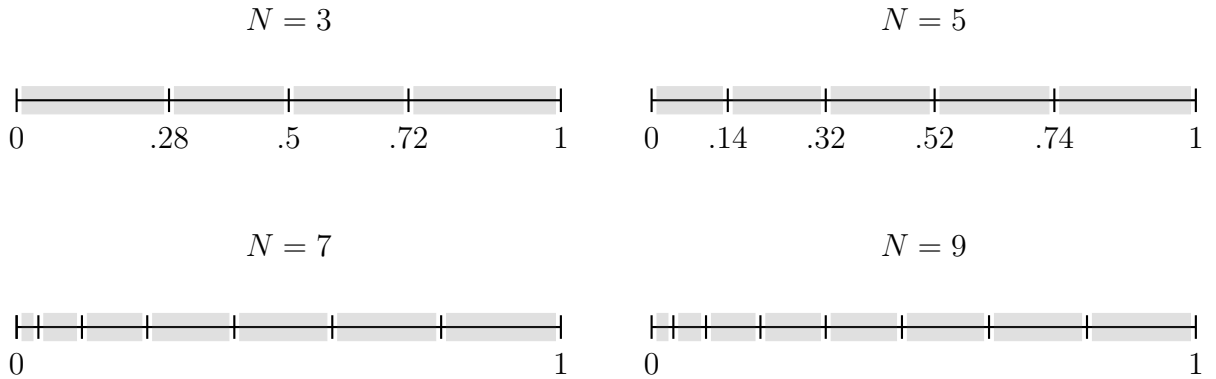


Figure 8: Parameter $K = 50$.

We furthermore compute the optimal reserve price, both in our framework and when buyers are exogenously informed of their valuations, and plot them in Figure 9. In our framework, the optimal reserve price seems increasing in the number of buyers N and we can show that it converges to the highest possible valuation as N goes to infinity. In the standard model, the optimal reserve price is approximately constant (the slight variations are solely driven by the finiteness of the set of valuations).

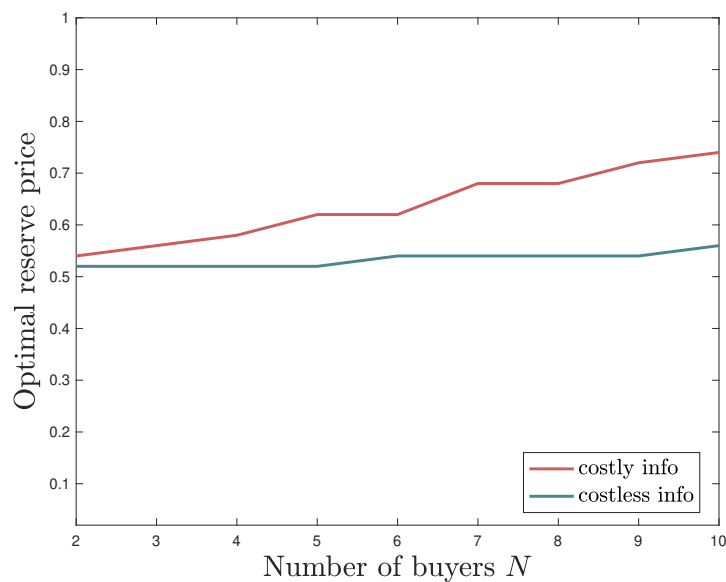


Figure 9: The optimal reserve price under exogenous information (blue line) is close to being independent of N . The fact that it varies slightly with N comes from the discreteness of V , and disappears for K large enough. On the contrary, the optimal reserve price when information is endogenous and shaped by competition is increasing in N . Parameter $K = 50$.

APPENDIX B PROOFS

B.1 Preliminary Analysis

Proof of Lemma 1. We want to show that, for $\sum_v [p(v)]^2$ low enough and m large enough, there exists $v^* \in V$ such that

$$c(\Pi_{v^*}^{other}, p_{1:N-1}) + (\Pr(v_i \leq v^*))^{N-1} c(\Pi_{\leq v^*}^{self}, p) + \left[1 - (\Pr(v_i \leq v^*))^{N-1}\right] c(\Pi_{> v^*}^{self}, p) < c(\{\{v_i\}_{v_i \in V}\}, p).$$

Acquiring partition $\Pi_{v^*}^{other}$ leads agent i to hold one of two possible posterior beliefs about $\max_j v_j$. Let $\delta_{v_{1:N-1} \leq v^*}$ denote i 's posterior about $\max_j v_j$ after learning $\max_j v_j \leq v^*$, that is

$$\delta_{v_{1:N-1} \leq v^*} \equiv \Pr(\cdot \mid \max_j v_j \leq v^*) = \mathbb{1}\{\cdot \leq v^*\} \frac{p_{1:N-1}(\cdot)}{\Pr(\max_j v_j \leq v^*)}.$$

Similarly, let $\delta_{v_{1:N-1} > v^*}$ denote i 's posterior about $\max_j v_j$ after learning $\max_j v_j > v^*$.

Using the fact that the cost of a partition equals the expected reduction in uncertainty in a buyer's beliefs, the above inequality rewrites as

$$\begin{aligned} & H(p_{1:N-1}) - \Pr(\max_j v_j \leq v^*) H(\delta_{v_{1:N-1} \leq v^*}) - \Pr(\max_j v_j > v^*) H(\delta_{v_{1:N-1} > v^*}) \\ & + \Pr(\max_j v_j \leq v^*) \left[H(p) - \Pr(v_i > v^*) H(p(\cdot \mid v_i > v^*)) - \sum_{v \leq v^*} p(v) H(\delta_v) \right] \\ & + \Pr(\max_j v_j > v^*) \left[H(p) - \Pr(v_i \leq v^*) H(p(\cdot \mid v_i \leq v^*)) - \sum_{v > v^*} p(v) H(\delta_v) \right] \\ & < H(p) - \sum_v p(v) H(\delta_v). \end{aligned}$$

Simplifying terms, this becomes

(1)

$$H(p_{1:N-1}) - \Pr(\max_j v_j \leq v^*) H(\delta_{v_{1:N-1} \leq v^*}) - \Pr(\max_j v_j > v^*) H(\delta_{v_{1:N-1} > v^*})$$

$$\begin{aligned}
&< \Pr\left(\max_j v_j \leq v^*\right) \Pr(v_i > v^*) \underbrace{\left[H(p(\cdot | v_i > v^*)) - \sum_{v > v^*} \frac{p(v)}{\Pr(v_i > v^*)} H(\delta_v) \right]}_{\text{gains from not learning } v_i \text{ fully when } v_i > v^* \text{ but } \max_j v_j \leq v^*} \\
&+ \Pr\left(\max_j v_j > v^*\right) \Pr(v_i \leq v^*) \underbrace{\left[H(p(\cdot | v_i \leq v^*)) - \sum_{v \leq v^*} \frac{p(v)}{\Pr(v_i \leq v^*)} H(\delta_v) \right]}_{\text{gains from not learning } v_i \text{ fully when } v_i \leq v^* \text{ but } \max_j v_j > v^*}.
\end{aligned}$$

Intuitively, the LHS is the cost of $\Pi_{v^*}^{other}$ whereas the RHS is the reduction in information costs on self that buyer i achieves after acquiring signal $\Pi_{v^*}^{other}$ about others. The former is a fairly coarse partition—it only has two elements—and so does not grow as the prior gets uncertain. From the assumption of strong concavity, it just have to be above

$$\begin{aligned}
&H(p_{1:N-1}) - \Pr(\max_j v_j \leq v^*) H(\delta_{v_{1:N-1} \leq v^*}) - \Pr(\max_j v_j > v^*) H(\delta_{v_{1:N-1} > v^*}) \\
&\geq \frac{m}{2} \Pr(\max_j v_j \leq v^*) \Pr(\max_j v_j > v^*) \left[\frac{\sum_{v \leq v^*} (p_{1:N-1}(v))^2}{\Pr(\max_j v_j \leq v^*)^2} + \frac{\sum_{v > v^*} (p_{1:N-1}(v))^2}{\Pr(\max_j v_j > v^*)^2} \right],
\end{aligned}$$

but this lower bound goes to zero as $\sum_v (p(v))^2$ goes to zero.

On the contrary, the reduction in information costs on self grows as the prior becomes uncertain. To show this, we find a lower bound for the RHS of (1) using the fact that H is strongly concave. To apply the notion of strong concavity, we need to consider mixtures between two beliefs only. This can be done iteratively in the following way:

$$\begin{aligned}
p(\cdot | v_i \leq v^*) &= \frac{p(v^1)}{\Pr(v_i \leq v^*)} \delta_{v^1} + \left[1 - \frac{p(v^1)}{\Pr(v_i \leq v^*)} \right] \delta_{v^1 < v_i \leq v^*}, \\
\text{where } \delta_{v^1 < v_i \leq v^*} &= \frac{p(v^2)}{\Pr(v^1 < v_i \leq v^*)} \delta_{v^2} + \left[1 - \frac{p(v^2)}{\Pr(v^1 < v_i \leq v^*)} \right] \delta_{v^2 < v_i \leq v^*} \\
\text{and } \delta_{v^2 < v_i \leq v^*} &= \frac{p(v^3)}{\Pr(v^2 < v_i \leq v^*)} \delta_{v^3} + \left[1 - \frac{p(v^3)}{\Pr(v^2 < v_i \leq v^*)} \right] \delta_{v^3 < v_i \leq v^*}, \text{ etc.}
\end{aligned}$$

Using the strong concavity of H iteratively, we then get

$$H(p(\cdot | v_j \leq v^*)) - \sum_{v \leq v^*} \frac{p(v)}{\Pr(v_i \leq v^*)} H(\delta_v)$$

$$\begin{aligned}
&\geq \frac{m}{2} \frac{p(v^1)}{\Pr(v_i \leq v^*)} \left[1 - \frac{p(v^1)}{\Pr(v_i \leq v^*)} \right] \|\delta_{v^1} - \delta_{v^1 < v_i \leq v^*}\|^2 \\
&+ \frac{m}{2} \left[1 - \frac{p(v^1)}{\Pr(v_i \leq v^*)} \right] \frac{p(v^2)}{\Pr(v^1 < v_i \leq v^*)} \left[1 - \frac{p(v^2)}{\Pr(v^1 < v_i \leq v^*)} \right] \|\delta_{v^2} - \delta_{v^2 < v_i \leq v^*}\|^2 \\
&+ \dots
\end{aligned}$$

where

$$\|\delta_{v^k} - \delta_{v^k < v_i \leq v^*}\|^2 = 1 + \sum_{v^k < v_i \leq v^*} \left[\frac{p(v_i)}{\Pr(v^k < v_i \leq v^*)} \right]^2.$$

We hence get

$$\begin{aligned}
H(p(\cdot | v_j \leq v^*)) - \sum_{v \leq v^*} \frac{p(v)}{\Pr(v_i \leq v^*)} H(\delta_v) &\geq \frac{m}{2} \sum_{v \leq v^*} \frac{p(v)}{\Pr(v_i \leq v^*)} \frac{\Pr(v < v_i \leq v^*)}{\Pr(v \leq v_i \leq v^*)} \\
&+ \frac{m}{2} \sum_{v \leq v^*} \frac{p(v)}{\Pr(v_i \leq v^*) \Pr(v < v_i \leq v^*) \Pr(v \leq v_i \leq v^*)} \sum_{v < v_i \leq v^*} [p(v_i)]^2.
\end{aligned}$$

Using the fact that $\frac{\Pr(v < v_i \leq v^*)}{\Pr(v \leq v_i \leq v^*)} = \frac{\Pr(v \leq v_i \leq v^*) - p(v)}{\Pr(v \leq v_i \leq v^*)}$, and collecting all $[p(v_i)]^2$ terms together, we get

$$(2) \quad H(p(\cdot | v_j \leq v^*)) - \sum_{v \leq v^*} \frac{p(v)}{\Pr(v_i \leq v^*)} H(\delta_v) \geq \frac{m}{2} - \frac{m}{2} \frac{1}{\Pr(v_i \leq v^*)} \sum_{v \leq v^*} [p(v)]^2.$$

Similarly,

$$(2') \quad H(p(\cdot | v_j > v^*)) - \sum_{v > v^*} \frac{p(v)}{\Pr(v_i > v^*)} H(\delta_v) \geq \frac{m}{2} - \frac{m}{2} \frac{1}{\Pr(v_i > v^*)} \sum_{v > v^*} [p(v)]^2.$$

Note that these two lower bounds increase and tend to $m/2$ as the prior gets more uncertain.

We now find an upper bound for the cost of $\Pi_{v^*}^{other}$, that is for the LHS of (1). Note that

$$H(p_{1:N-1}) - \Pr(\max_j v_j \leq v^*) H(\delta_{v_{1:N-1} \leq v^*}) - \Pr(\max_j v_j > v^*) H(\delta_{v_{1:N-1} > v^*})$$

$$\begin{aligned}
&= H(p_{1:N-1}) - \sum_v p_{1:N-1}(v) H(\delta_{v_{1:N-1}=v}) \\
&\quad - \Pr(\max_j v_j \leq v^*) \left[H(\delta_{v_{1:N-1} \leq v^*}) - \sum_{v \leq v^*} \frac{p_{1:N-1}(v)}{\Pr(\max_j v_j \leq v^*)} H(\delta_{v_{1:N-1}=v}) \right] \\
&\quad - \Pr(\max_j v_j > v^*) \left[H(\delta_{v_{1:N-1} > v^*}) - \sum_{v > v^*} \frac{p_{1:N-1}(v)}{\Pr(\max_j v_j > v^*)} H(\delta_{v_{1:N-1}=v}) \right].
\end{aligned}$$

Using the same steps as above, we can find bounds on the last two terms, such that

$$\begin{aligned}
&H(p_{1:N-1}) - \Pr(\max_j v_j \leq v^*) H(\delta_{v_{1:N-1} \leq v^*}) - \Pr(\max_j v_j > v^*) H(\delta_{v_{1:N-1} > v^*}) \\
&\leq H(p_{1:N-1}) - \sum_v p_{1:N-1}(v) H(\delta_{v_{1:N-1}=v}) - \frac{m}{2} \left[1 - \sum_v [p_{1:N-1}(v)]^2 \right].
\end{aligned}$$

Combining this bound on the LHS with the bounds (2) and (2') on the RHS, condition (1) holds if

$$\begin{aligned}
&H(p_{1:N-1}) - \sum_v p_{1:N-1}(v) H(\delta_{v_{1:N-1}=v}) - \frac{m}{2} \left[1 - \sum_v [p_{1:N-1}(v)]^2 \right] \\
&< \frac{m}{2} \left[\Pr(\max_j v_j \leq v^*) \Pr(v_i > v^*) + \Pr(\max_j v_j > v^*) \Pr(v_i \leq v^*) \right] \\
&\quad - \frac{m}{2} \left[\Pr(\max_j v_j > v^*) \sum_{v \leq v^*} (p(v))^2 + \Pr(\max_j v_j \leq v^*) \sum_{v > v^*} (p(v))^2 \right].
\end{aligned}$$

Let $T \equiv \sum_v (p(v))^2$, which lies weakly below one. No value $v \in V$ can have prior probability greater than \sqrt{T} . Hence, for small T , it is possible to find v^* such that $\Pr(\max_j v_j \leq v^*)$ is close to 0.5. The RHS can then be arbitrarily close to $m/4$ for sufficiently small T . Hence, for T small enough, the RHS is strictly positive and increasing in m . Similarly, the LHS is positive but decreasing in m . For m high enough, the inequality must then hold strictly, establishing the claim. \square

Lemma 2. *There exists \bar{T} and \bar{m} such that if $\sum_v [p(v)]^2 \leq \bar{T}$ and $m \geq \bar{m}$, then the following is true:*

$$c(\{\{v^1\}, \{v^2, \dots, v^*\}, \{v : v > v^*\}\}, p_{1:N-1}) - c(\{\{v^1\}, \{v : v > v^1\}\}, p_{1:N-1})$$

$$\begin{aligned}
&< \Pr \left(\max_j v_j > v^1 \right) c(\{\{v\}_{v \in V}\}, p) \\
&\quad - \Pr \left(v^2 \leq \max_j v_j \leq v^* \right) c(\{\{v\}_{v \leq v^*}, \{v : v > v^*\}\}, p) \\
&\quad - \Pr \left(v^* < \max_j v_j \right) c(\{v : v \leq v^*\}, \{\{v\}_{v > v^*}\}, p),
\end{aligned}$$

for some $v^* > v^1$.

Lemma 2 says that the cost-efficient information structure has $\Pi^{other} \neq \{\{v^1\}, \{v : v > v^1\}\}$, such that buyers do not just learn whether their toughest opponent has the smallest possible valuation v^1 or not. This is important as such information structure would not lead to a revenue loss, as losing buyers would always learn their valuations fully.²⁶

Proof. We rewrite the condition in terms of the measure of uncertainty H :

(3)

$$\begin{aligned}
&\Pr(\max_j v_j > v^1)H(\delta_{1:N-1 > v^1}) - \Pr(v^1 < \max_j v_j \leq v^*)H(\delta_{v^1 < 1:N-1 \leq v^*}) \\
&\quad - \Pr(\max_j v_j > v^*)H(\delta_{1:N-1 > v^*}) \\
&< \Pr \left(v^2 \leq \max_j v_j \leq v^* \right) \left[\Pr(v_i > v^*)H(\delta_{> v^*}) + \sum_{v \leq v^*} \Pr(v_i = v)H(\delta_v) \right] \\
&\quad + \Pr \left(\max_j v_j > v^* \right) \left[\Pr(v_i \leq v^*)H(\delta_{\leq v^*}) + \sum_{v > v^*} \Pr(v_i = v)H(\delta_v) \right] \\
&\quad - \Pr \left(\max_j v_j > v^1 \right) \sum_v \Pr(v_i = v)H(\delta_v).
\end{aligned}$$

Dividing everything by $\Pr(\max_j v_j > v^1)$, and using the fact that $H(\delta_{\leq v^*}) > \Pr(v_i = v^1)H(\delta_{v^1}) + \Pr(v^1 < v_i \leq v^*)H(\delta_{v^1 < v_i \leq v^*})$, condition (3) holds if

$$H(\delta_{1:N-1 > v^1}) - \Pr \left(\max_j v_j \in (v^1, v^*] \mid \max_j v_j > v^1 \right) H(\delta_{v^1 < 1:N-1 \leq v^*})$$

²⁶Indeed, after learning $\max_j v_j > v^1$, they would still choose the fully-revealing partition $\Pi^{self} = \{\{v\}_{v \in V}\}$.

$$\begin{aligned}
& - \Pr \left(\max_j v_j > v^* \mid \max_j v_j > v^1 \right) H(\delta_{1:N-1 > v^*}) \\
& < \Pr \left(\max_j v_j \in (v^1, v^*] \mid \max_j v_j > v^1 \right) \left[\Pr(v_i > v^*) H(\delta_{> v^*}) - \sum_{v > v^*} \Pr(v_i = v) H(\delta_v) \right] \\
& + \Pr \left(\max_j v_j > v^* \mid \max_j v_j > v^1 \right) \left[\Pr(v_i \leq v^*) H(\delta_{\leq v^*}) - \sum_{v \leq v^*} \Pr(v_i = v) H(\delta_v) \right]
\end{aligned}$$

for some $v^* > v^1$. This is exactly the same condition as in the proof of Lemma 1, but for a redefined set of valuations $\widehat{V} = V \setminus v^1$. We then know from Lemma 1 that for $\sum_{v > v^1} \left(\frac{p(v)}{1-p(v^1)} \right)^2$ small enough and m large enough, the inequality holds for some $v^* \in \widehat{V}$, hence proving the claim. \square

B.2 Proofs of Results of Section 3

B.2.1 Proofs of Proposition 1

Proof of Proposition 1. For Proposition 1, suppose that each buyer can only learn about himself. We look for a symmetric equilibrium in which, for λ small enough, buyers become fully informed. For this to be the case, buyers must choose the partition $\Pi_0 \equiv \{\{v_i\}_{v_i \in V}\}$ with a probability that tends to one as λ goes to zero. We construct a symmetric equilibrium σ that has such property.

Consider the following symmetric pure strategy profile:

- Each buyer chooses the finest partition $\Pi^{self} = \Pi_0$;
- Each buyer bids his valuation $\beta(\{v_i\}, u_i) = v_i + u_i$.

It is a dominant strategy for a buyer who knows his value to bid his valuation for the good $\beta(\{v_i\}, u_i) = v_i + u_i$. Hence we only need to check that buyers do not want to deviate to another information partition.

Let $K \equiv |V|$ and order the possible valuations for the good in increasing order: $V \equiv \{v^1, v^2, \dots, v^K\}$ with $v^1 < v^2 < \dots < v^K$. Any other information partition $\Pi^{self} \neq \Pi_0$ must bundle at least two possible valuations together. That is, there exist v^k and $v^{k'}$ that belong to the same element of the partition Π^{self} . Such bundling reduces a buyer's information costs by

$$\lambda (c(\Pi_0, p) - c(\Pi^{self}, p)) > 0.$$

We however show that such bundling must make buyer i strictly worse off in the auction, and so cannot be optimal for λ small enough.

First, we argue that Π^{self} cannot bundle two non-neighboring values v^k and $v^{k'}$ with $k' > k + 1$. Recall that, by assumption, others follow their equilibrium strategy: they become fully informed of their values and bid truthfully. Hence, with probability $p_{1:N-1}(v^{k+1})$, the highest bid among i 's competitors lies in $[v^{k+1} + \underline{u}, v^{k+1} + \bar{u}]$. At that bid, i wants to lose if his value is v^k and wants to win if his value is $v^{k'}$. Hence there is a strictly positive gain for him in distinguishing $v_i = v^k$ from $v_i = v^{k'}$, and for λ small enough, he must be doing so.

Second, we argue that Π^{self} cannot bundle two neighboring values v^k and v^{k+1} either. This is a bit more subtle, and would not be true absent the noise terms $(u_i)_i$ in buyers' valuations. Suppose a buyer bundles $\{v^k, v^{k+1}\}$. Upon learning that $v_i \in \{v^k, v^{k+1}\}$, it is (weakly) optimal for i to bid truthfully $\mathbb{E}[v_i | v_i \in \{v^k, v^{k+1}\}] + u_i$. Had buyer i chosen the fully revealing partition Π_0 , he would have bid $v^k + u_i$ when $v_i = v^k$, and $v^{k+1} + u_i$ when $v_i = v^{k+1}$. If the highest bid among i 's competitors lies above $v^{k+1} + u_i$ or below $v^k + u_i$, then this bundling does not change anything. Hence any difference in gross payoff between these two partitions must occur when i 's toughest competitor, call him j^* , has a value $\max_j v_j \in (v^k + u_i, v^{k+1} + u_i)$. This requires either $v_{j^*} = v^k$ and $u_{j^*} > u_i$, or $v_{j^*} = v^{k+1}$ and $u_{j^*} < u_i$.

Focus on the first case, where $v_{j^*} = v^k$. Fix a realization of u_i , and consider what happens when $u_{j^*} > u_i$. There are two possible scenarios: either $u_{j^*} > u_i + \mathbb{E}[v_i | v_i \in \{v^k, v^{k+1}\}] - v^k$, in which case buyer i does not win when he bids $\mathbb{E}[v_i | v_i \in \{v^k, v^{k+1}\}] + u_i$. Learning to distinguish v^k from v^{k+1} allows the buyer to win the auction at the latter value, and hence induce a gain in gross payoff of

$$p(v^{k+1}) (v^{k+1} + u_i - v^k - u_{j^*}) > 0.$$

If $u_{j^*} \leq u_i + \mathbb{E}[v_i | v_i \in \{v^k, v^{k+1}\}] - v^k$, then i wins when he bids $\mathbb{E}[v_i | v_i \in \{v^k, v^{k+1}\}]$. Learning to distinguish v^k from v^{k+1} allows the buyer not to win the auction at the former value, and hence induces a gain in gross payoff of

$$p(v^k) (u_{j^*} - u_i) > 0.$$

Hence, as soon as $\Pr(u_{j^*} > u_i) > 0$, the expected gains from distinguishing these two values are strictly positive. This is the case whenever $\bar{u} > \underline{u}$, i.e. whenever the

noise is not degenerate at zero.²⁷ For λ small enough, the cost of distinguishing these values must be strictly below the gains, and so the above strategy profile forms an equilibrium. □

B.2.2 Proofs of Proposition 2 and 3

Proof of Proposition 2. We first prove the existence of a symmetric equilibrium building on Corollary 5.3 in Reny (1999) (Step 1). We then argue that the proof extends for the existence of symmetric equilibria that are robust to trembles (Step 2).

Step 1. Without loss, we can restrict agents' possible bids to belong to $[0, \bar{v}]$ for some \bar{v} that is greater than any possible realized valuation for the good. Agents' pure strategy sets are then compact Hausdorff spaces. Furthermore, their utility functions are bounded, measurable, and symmetric. Corollary 5.3 in Reny (1999) then states that there exists a symmetric mixed strategy equilibrium if (the mixed extension of) our game is better-reply secure along the diagonal. The crux of the proof is then to show that this condition of better-reply security holds in our setting.

Let $w_i(\sigma_i, \sigma_{-i})$ be buyer i 's ex-ante expected utility given the strategy profile, that is

$$w_i(\sigma_i, \sigma_{-i}) \equiv \mathbb{E}_{\sigma_i, \sigma_{-i}} \left[U(\nu_i, \beta_i(\pi_i), \beta_{-i}(\pi_{-i})) - \lambda \left(c(\Pi_i^{other}, p_{1:N-1}) + c(\Pi_i^{self}(\pi_i^{other}), p) \right) \right].$$

Let $\sigma^* \in \Delta\Sigma$ denote a (potentially mixed) strategy. By symmetry, $w_i(\sigma^*, \dots, \sigma^*)$ is independent of i . Let $w(\sigma^*) \equiv w_i(\sigma^*, \dots, \sigma^*)$ be the diagonal payoff function, i.e., the payoff of an agent when all agents play the same strategy $\sigma^* \in \Delta\Sigma$. The game is diagonally better-reply secure if, whenever $(\sigma^*, w^*) \in \Delta\Sigma \times \mathbb{R}$ is in the closure of the graph of its diagonal payoff function and $(\sigma^*, \dots, \sigma^*)$ is *not* an equilibrium, then some buyer i can secure a payoff strictly above w^* along the diagonal at $(\sigma^*, \dots, \sigma^*)$.²⁸

We show that our game satisfies this condition. Suppose that $(\sigma^*, \dots, \sigma^*)$ is not an

²⁷To see what goes wrong with the above argument when there is no noise—i.e. $\Pr(u_i = 0) = 1$ —recall that buyers fully learn their values in the proposed equilibrium. Hence absent noise, other buyers' bids only take values in V . In particular, that means a buyer never faces a bid strictly in between v^k and v^{k+1} . There are thus no opportunity cost in bundling these values together, but only a strictly reduction in information costs. The equilibrium would then require buyers to mix between bundling neighboring values and not bundling them.

²⁸Buyer i can secure a payoff strictly above w^* along the diagonal at $(\sigma^*, \dots, \sigma^*)$ if there exists $\delta > 0$ and $\bar{\sigma}_i$, such that $w_i(\sigma', \dots, \bar{\sigma}_i, \dots, \sigma') \geq w^* + \delta$ for all σ' in some open neighborhood of σ^* .

equilibrium and let (σ^*, w^*) be an element of the closure of the graph of its diagonal payoff function. By definition, there exists a sequence of strategies $\sigma^{(n)}$ converging to σ^* such that $\lim w(\sigma^{(n)}) = w^*$. There are two cases: either w is continuous at σ^* or it is not. Discontinuities in the payoff can only arise from ties at the bidding stage. Hence the first case arises when, under σ^* , no tie occurs with strictly positive probability. In that case, $w(\sigma^*) = w^*$. Since σ^* is not an equilibrium, a buyer i has a deviation σ'_i that yields $w_i(\sigma'_i, \sigma^*_{-i}) > w(\sigma^*) = w^*$. Furthermore, at this deviation $(\sigma'_i, \sigma^*_{-i})$, buyer i 's payoff is continuous in others' strategy σ_{-i} . (If not, that means i ties with strictly positive probability under σ'_i and a marginally close strategy would make him strictly better off.) Hence i can secure a payoff within ε of $w_i(\sigma'_i, \sigma^*_{-i}) > w^*$, as required by better-reply security.

Now consider the second case in which w is discontinuous at σ^* . This means relevant ties must occur with strictly positive probability at σ^* . Some of the buyers who tie with positive probability under σ^* must lose with positive probability at $\sigma^{(n)}$ for large enough n while they would be strictly better off winning. (Indeed, they cannot all be indifferent between winning and losing since a positive mass of them tie and they cannot all have the same expected value conditional on winning, since their u_i s differ.) A buyer i can thus deviate to some σ'_i that breaks all ties in his favor, such that w_i is now continuous in σ_{-i} at $w_i(\sigma'_i, \sigma^{(n)}_{-i}) > w_i(\sigma^{(n)})$ for large n . And so i can secure a payoff strictly above $\lim w_i(\sigma^{(n)}) = w^*$. Overall, the game is then better-reply secure and must admit a symmetric mixed-strategy equilibrium.

Step 2. We now argue that it must also admit a symmetric equilibrium that is robust to trembles. Consider a perturbed version of our game $G^{(k)}$ in which payoffs are perturbed with the $\varepsilon^{(k)}$, with $\lim \varepsilon^{(k)} = 0$.²⁹ For each k , the existence of a symmetric mixed-strategy equilibrium $(\sigma^{(k)})$ follows from the above argument.

We now need to show that the equilibrium of the perturbed game $\sigma^{(k)}$ converges to some feasible strategy $\sigma^* \in \Delta\Sigma$ as k grows large. This is true (at least for some subsequence) since the set of strategies $\Delta\Sigma$ to which $\sigma^{(k)}$ belongs is (sequentially) compact. So there exists a feasible strategy σ^* to which the equilibrium strategy $\sigma^{(k)}$ converges to as perturbations vanish.

²⁹That is, agents have the same strategy space but the payoff associated with a strategy profile equals the expected payoffs if the agent's bid coincides with the one specified by his strategy with probability $1 - \varepsilon^{(k)}$ and is drawn from some exogenous continuous distribution F otherwise.

Finally, we have left to show that the strategy it converges to σ^* forms a symmetric equilibrium of the unperturbed game. By way of contradiction, suppose $(\sigma^*, \dots, \sigma^*)$ is not an equilibrium. Then, by better-reply security, an agent i can secure a payoff strictly above $\lim w(\sigma^{(k)}) \equiv w^*$ using some strategy σ'_i . However σ'_i would then also be a profitable deviation against $\sigma_{-i} = \sigma^{(k)}$ for k large enough since payoffs are continuous in the perturbation $\varepsilon^{(k)}$, and so $\sigma^{(k)}$ cannot be an equilibrium of the perturbed game for k large enough. \square

Proof of Proposition 3. By contradiction, suppose there exists a sequence of equilibria $\{\sigma_\lambda\}_\lambda$ such that $\lim_{\lambda \rightarrow 0} \Pr(\Pi^{self} = \{\{v_i\}_{v_i \in V} \mid \sigma_\lambda\}) = 1$. That is, each buyer i might be mixing over partitions in equilibrium, but he must put a probability that tends to one as λ goes to zero on partition $\Pi^{self} = \Pi_0$.

We construct a profitable deviation. Consider an alternative strategy for buyer i , in which he first acquires information as to whether the maximum valuation among other bidders is above some threshold $v^{k^*} < v^K$, before learning about his own. That is, he chooses $\Pi_{k^*}^{other} = \{\{v^1, \dots, v^{k^*}\}, \{v^{k^*+1}, \dots, v^K\}\}$. Then, when he learns that his toughest competitor has a value above the threshold $\max_j v_j > v^{k^*}$, buyer i chooses to partition his set of valuations into $\Pi_{>k^*}^{self} \equiv \{\{v_i\}_{v_i > v^{k^*}}, \{v_i : v_i \leq v^{k^*}\}\}$. Intuitively, he does not learn to distinguish all the valuations below the threshold, as he most likely would not win at any of these. On the contrary, when he learns $\max_j v_j \leq v^{k^*}$ buyer i chooses the partition $\Pi_{\leq k^*}^{self} \equiv \{\{v_i\}_{v_i \leq v^{k^*}}, \{v_i : v_i > v^{k^*}\}\}$.

By Lemma 1, there exists k^* such that this alternative information strategy leads to strictly lower information cost than becoming fully informed about oneself:

$$\begin{aligned} c(\Pi_0, p) - \Pr(\max_j v_j > v^{k^*})c(\Pi_{>k^*}^{self}, p) \\ - \Pr(\max_j v_j \leq v^{k^*})c(\Pi_{\leq k^*}^{self}, p) - c(\Pi_{k^*}^{other}, \bar{p}_{N-1}) \equiv \Delta c > 0. \end{aligned}$$

However, there is a potential opportunity cost of doing so if partitioning partially his set of valuations yields a lower gross payoff to i . (It has to yield a weakly lower payoff to i as information is valuable.) We now show that this opportunity cost is zero, and hence smaller than $\lambda \Delta c$.

Consider first what happens when i learns $\max_j v_j \leq v^{k^*}$. Since all agents converge to becoming fully informed about themselves by assumption, with a probability that tends to one they all bid at most $\max_j v_j \leq v^{k^*} + \bar{u} < v^{k^*+1} + \underline{u}$.

How does partition $\Pi_{\leq k^*}^{self}$ compare to Π_0 ? If buyer i has a value below the threshold v^{k^*} , then under both partitions he perfectly learns it and gets the same expected payoff. So these partitions can yield different payoff only when i has a value above the threshold. Under the former partition, i fails to distinguish his potential values, only learns $v_i \in \{v'_i \in V \mid v'_i > v^{k^*}\}$, and makes a bid in $[v^{k^*+1} + \underline{u}, v^K + \bar{u}]$. Under the latter partition, i learns his value v_i , and bids $v_i + u_i$. These partitions can then only yield different payoffs if one of i 's competitors sometimes bids strictly above $v^{k^*+1} + \underline{u}$ despite his realized value being lower. Call that buyer j and that particular bid b^* . This can only be optimal for j if he failed to learn his value and chose a partition about himself that bundles his realized value $v_j \leq v^{k^*}$ with some other value(s) strictly above v^{k^*} . That is, buyer j bids b^* at an information set π_j such that $\min_{v_j \in \pi_j^{self}} v_j \leq v^{k^*}$ and $\max_{v_j \in \pi_j^{self}} v_j > v^{k^*}$.³⁰ Such bundling can be optimal for j only if he expects no non-vanishing cost from doing so, given others' equilibrium strategy. In particular, buyer j must believe that with a probability very close to one he will not face a bid in the interval of values he deems possible $[\min_{v_j \in \pi_j^{self}} v_j + \underline{u}, \max_{v_j \in \pi_j^{self}} v_j + \bar{u}]$. By assumption, buyers almost always learn and bid their valuations, and so buyer j must have learned that i 's valuation is even higher than $\max_{v_j \in \pi_j^{self}} v_j$. If buyer i only learns that $v'_i > v^{k^*}$, he thus wants to make a bid that is weakly higher than any other bid he might face given equilibrium strategies, including b^* . He then gets the same gross payoff as if he had learned to distinguish these values, but at a lower information cost.

Overall, when i learns $\max_j v_j \leq v^{k^*}$, he knows for sure that if one of his competitors bids above $v^{k^*+1} + \underline{u}$, then he wants to match that bid. Doing so does not require learning the possible valuations he has that lie about v^{k^*} . Hence the two partitions about self, $\Pi_{\leq k^*}^{self}$ and Π_0 , yield the exact same gross expected payoff.

Now consider what happens when i learns $\max_j v_j > v^{k^*}$. Buyer i then knows with certainty that $v_j > v^{k^*}$ for some j . Call him j^* . Given equilibrium strategies, that agent j^* converges to becoming fully informed about himself, so with a probability that tends to one makes a bid in $[v_{j^*} + \underline{u}, v_{j^*} + \bar{u}]$. How does partition $\Pi_{> k^*}^{self}$ compare to Π_0 ? Again, these partitions only differ when i has a value below the threshold. Under $\Pi_{> k^*}^{self}$, buyer i bundles all such values, and suppose that, in such a case, he bids sufficiently low so as to never win given others' equilibrium strategies. Under Π_0 he learns his value fully and bids $v_i + u_i$. Hence, for these two partitions to lead to different gross

³⁰Note that our starting assumption simply requires buyers to become fully informed *with a probability close to one*. So this is possible, albeit with small probability.

expected payoffs, it has to be that with non-zero probability, the highest bid among i 's competitors lies strictly below $v^{k^*} + \bar{u}$, and that by choosing such finer partition i sometimes wins.

Under what conditions can the highest-value buyer j^* , with value $v_{j^*} > v^{k^*} + \bar{u}$, bid strictly below $v^{k^*} + \bar{u}$ in equilibrium? For j^* to find such low bid optimal, it has to be that buyer j^* fails to learn his value: that is, he sometimes chooses a partition $\Pi_{j^*}^{self}$ that bundles $v_{j^*} > v^{k^*}$ with values below v^{k^*} . Denote that bundle by $\pi_{j^*}^{self}$. For such bundle to be optimal for j^* , it must be that j^* expects no non-vanishing loss from doing so. Since all buyers fully learn and bid their valuation with a probability close to one by assumption, this is only the case when j^* learned that $\max_{j \neq j^*} v_j < \min_{v'_{j^*} \in \pi_{j^*}^{self}} v'_{j^*}$. Hence j^* 's bid must lie above the valuations of all the other buyers, and none of them has any incentive to match his bid. In particular, buyer i never wants to win against j^* 's bid in such scenario, and choosing the finer partition cannot lead to strict gains. \square

B.2.3 Proof of Theorem 1

To prove Theorem 1, we find necessary conditions that must be satisfied by any information structure $(\Pi^{other}, \Pi^{self}) \in \mathcal{P} \times \mathcal{P}^{2^V}$ that has non-vanishing weight in some equilibrium σ_λ . Lemmas 3 to 5 show, in a succession of steps, that an equilibrium information structure $(\Pi^{other}, \Pi^{self})$ must satisfy

$$(\star) \quad \Pi^{self}(\pi^{other}) = \{ \{v_i \mid v_i < \min_{v' \in \pi^{other}} v'\}, \{v_i\}_{v_i \in \pi^{other}}, \{v_i \mid v_i > \max_{v' \in \pi^{other}} v'\} \}$$

for all $\pi^{other} \in \Pi^{other}$. In words, after learning $\max_j v_j \in \pi^{other} \in \Pi^{other}$, the agent bundles all the values he can have that are below (resp. above) his toughest competitor's valuation for sure, as illustrated in Figure 2. Lemma 6 shows that an equilibrium information structure must also minimize total information costs. We know from Lemma 1 that this precludes $\Pi^{other} = \{V\}$, which completes the proof of Theorem 1.

Lemma 3. *There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, there exists $\varepsilon(\lambda) > 0$ with $\lim_{\lambda \rightarrow 0} \varepsilon(\lambda) = 0$ such that if an information structure has probability $\Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda) \geq \varepsilon(\lambda)$ in some equilibrium σ_λ , then it must have the following form: for all $\pi^{other} \in \Pi^{other}$,*

$$\{v_i\} \in \Pi^{self}(\pi^{other}) \quad \forall v_i \in \pi^{other}, v_i \neq \max_{v'_i \in \pi^{other}} v'_i.$$

In words, any information structure that has non-vanishing weight in equilibrium

must satisfy the following condition: if agent i learns that his toughest competitor has value in some interval $\pi^{other} \equiv [v^k, v^{\bar{k}}]$, then i at least learns to distinguish all the values he can have that lie in $[v^k, v^{\bar{k}}]$ as he knows competition will fall into that range.³¹

Proof of Lemma 3. Fix λ , and let $(\Pi^{other}, \Pi^{self})$ be an information structure that is chosen with probability at least $\varepsilon > 0$ in some equilibrium. Take any $\pi^{other} \in \Pi^{other}$, and let $v^k \equiv \min_{v'_i \in \pi^{other}} v'_i$ and $v^{\bar{k}} \equiv \max_{v'_i \in \pi^{other}} v'_i$. That is, upon learning $\max_j v_j \in \pi^{other}$, agent i knows that his toughest competitor has $v_j \in [v^k, v^{\bar{k}}]$.

We prove that, after learning $\max_j v_j \in \pi^{other}$, the equilibrium partition that an agent chooses about himself $\Pi^{self}(\pi^{other})$ cannot bundle a value $v^* \in \pi^{other}$ with some other value weakly below $v^{\bar{k}}$.³² By contradiction, suppose that this is not true: after learning that $\max_j v_j \in \pi^{other}$, an agent chooses a partition that bundles some $v^* \in \pi^{other}$ with another that is weakly below $v^{\bar{k}}$. That bundle can be composed of only these two values, or can have other values in it too. Let \underline{v} (resp, \bar{v}) be the lowest (resp, highest) element in that bundle, and denote that bundle by $\pi_{\underline{v}, \bar{v}}^{self}$. Note that, without loss, $v^* > \underline{v}$. There are indeed two cases: either the bundle is only composed of values in π^{other} , in which case v^* can be any value in the bundle but the smallest one, or it isn't, in which case $\underline{v} < \min_{v'_i \in \pi^{other}} v'_i \leq v^*$.

Let $j^* \in \arg \max_j v_j$ be (any one of) agent i 's toughest competitor(s).³³ At information set $(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i)$ agent i knows that his toughest competitor has $\max_j v_j \in \pi^{other}$ and that his own value v_i might also belong to that set, since $\pi^{other} \cap \pi_{\underline{v}, \bar{v}}^{self} \neq \emptyset$. By the symmetry of the equilibrium, he hence knows that, with strictly positive probability, j^* 's information set is also $\pi_{j^*} = (\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_{j^*})$ for some u_{j^*} . With non-trivial probability, this is the highest bid that agent i faces. Indeed, it is for instance the case when all other agents choose the same information structure as i (with happens with probability ε^{N-1}) and have all value $v_j = v^*$. For his equilibrium bid to be optimal, agent i must then be indifferent between losing the auction and winning *at his equilibrium bid*

³¹Showing that buyer i also wants to distinguish valuation $v_i = v^{\bar{k}}$ is a bit more subtle, and we handle that case in Lemma 4.

³²This is enough to prove the result. The other alternative is that v^* is bundled with values weakly above $v^{\bar{k}}$. If $v^* = v^{\bar{k}}$, this is irrelevant as Lemma 3 makes no claim about $v^{\bar{k}}$. The only relevant case is then $v^* < v^{\bar{k}}$. But if v^* is bundled with values above $v^{\bar{k}}$ then it must be bundled with $v^{\bar{k}}$ as well. However that implies $v^{\bar{k}}$ is bundled with a value v^* weakly below $v^{\bar{k}}$ itself, and this is precisely what we are proving cannot happen.

³³Note that j^* might not be the highest-value bidder if there exists another agent j with $v_j = v_{j^*}$ and $u_j > u_{j^*}$, but this is irrelevant for the following argument.

(i.e., winning at a tie, such that he pays his equilibrium bid):

$$\begin{aligned}\beta\left(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i\right) &= \mathbb{E}\left[v_i \mid \pi_i = \left(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i\right), \max_j \beta(\pi_j) = \beta(\pi_i)\right] + u_i \\ &= \sum_{v_i = \underline{v}}^{\bar{v}} v_i \Pr\left[v_i \mid \pi_i = \left(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i\right), \max_j \beta(\pi_j) = \beta(\pi_i)\right] + u_i.\end{aligned}$$

If not, then agent i would have an incentive to marginally increase or decrease his bid, depending on whether or not he wants to win the auction at that price. Note that $\beta\left(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i\right) \in [\underline{v} + u_i, \bar{v} + u_i]$ since \underline{v} (resp. \bar{v}) is the lowest (resp. highest) value of v_i that agent i deems possible at information set $\left(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i\right)$.

We now show that agent i has a strict, non-vanishing incentive to learn to distinguish values $v_i = \underline{v}$ and $v_i = \bar{v}$, and will hence do so for small enough information costs. We know that, with a strictly positive probability that is increasing in ε and independent of λ , the highest bid made by i 's competitors when i 's information set is $\pi_i = \left(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i\right)$ lies in $[\underline{v} + u_j, \bar{v} + u_j]$ for some u_j . There are then strictly positive gains from unbundling values $v_i = \bar{v}$ and $v_i = \underline{v}$ since i wants to win against such bid when $v_i = \bar{v}$ and $u_i > u_{j^*}$ but wants to lose when $v_i = \underline{v}$ and $u_i < u_{j^*}$. (This is the same argument as in the proof of Proposition 1.)

There is a strictly positive cost $\lambda\Delta c$ associated with unbundling these values, as it requires choosing a finer partition and splitting $\pi_{\underline{v}, \bar{v}}^{self}$ into at least two elements. However, for λ small enough, there exists $\varepsilon(\lambda)$ such that, if $\varepsilon \geq \varepsilon(\lambda)$ then the value of distinguishing these values more than compensates the cost. Hence there exists $\varepsilon(\lambda)$ such that, if an information structure has probability $\Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) \geq \varepsilon(\lambda)$, then it cannot make such a bundle. Furthermore, since the cost of unbundling these values goes to zero when λ goes to zero, then $\varepsilon(\lambda)$ must go to zero as well. \square

Lemma 4. *There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, there exists $\varepsilon(\lambda) > 0$ with $\lim_{\lambda \rightarrow 0} \varepsilon(\lambda) = 0$ such that if an information structure has probability $\Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) \geq \varepsilon(\lambda)$ in some equilibrium σ_λ , then it must have the following form:*

$$\left\{ \max_{v'_i \in \pi^{other}} v'_i \right\} \in \Pi^{self}\left(\pi^{other}\right) \quad \text{for all } \pi^{other} \in \Pi^{other}.$$

Proof of Lemma 4. Suppose not: there exists $\varepsilon > 0$ and a sequence of equilibria $\{\sigma_\lambda\}_\lambda$

such that $\Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda) \geq \varepsilon$ for all λ and $\{\max_{v'_i \in \pi^{other}} v'_i\} \notin \Pi^{self}(\pi^{other})$ for some $\pi^{other} \in \Pi^{other}$. Let $\bar{v} = \max_{v'_i \in \pi^{other}} v'_i$ and $\underline{v} = \min_{v'_i \in \pi^{other}} v'_i$. In words, after learning that $\max_j v_j \in [\underline{v}, \bar{v}]$, agent i does not learn to distinguish $v_i = \bar{v}$.

We know from Lemma 4 that $\Pi^{self}(\pi^{other})$ cannot bundle $v_i = \bar{v}$ with lower values since an agent must learn to distinguish all values $v_i \in \pi^{other} \setminus \bar{v}$. Hence \bar{v} must be bundled with even greater values, and denote this bundle by $\pi_{\geq \bar{v}}^{self}$. Let \hat{v} denote the maximum valuation in $\pi_{\geq \bar{v}}^{self}$, which by definition satisfies $\hat{v} > \bar{v}$.

Step 1. We first show that, at information set $\pi_i = (\pi^{other}, \pi_{\geq \bar{v}}^{self}, u_i)$, an agent must bid arbitrarily close to $\bar{v} + u_i$ for sufficiently small λ . That is, for any $\eta > 0$ there exists $\bar{\lambda}$ such that, for all $\lambda \leq \bar{\lambda}$, $|\beta(\pi^{other}, \pi_{\geq \bar{v}}^{self}, u_i) - \bar{v} - u_i| \leq \eta$. Note that, at this information set, agent i knows that $\max_j v_j \in [\underline{v}, \bar{v}]$ and $v_i \in [\bar{v}, \hat{v}]$. Since the equilibrium is symmetric, he also knows that, with non-vanishing probability, his toughest opponent has the same information set and makes the same equilibrium bid as i . Indeed, this happens whenever $v_i = \bar{v}$, $\max_j v_j = \bar{v}$, and i 's toughest opponent(s) chooses the same information structure as i . Note first that an equilibrium bid at $\pi_i = (\pi^{other}, \pi_{\geq \bar{v}}^{self}, u_i)$ cannot be bounded below the lowest possible valuation at that information set $\bar{v} + u_i$. Indeed, since an agent sometimes ties at that bid, he could slightly increase his bid and make a strict gain. More importantly, $\beta(\pi^{other}, \pi_{\geq \bar{v}}^{self}, u_i)$ cannot be bounded above $\bar{v} + u_i$ either. If it were the case, then the agent would have an incentive to learn whether $v_i = \bar{v}$ or $v_i > \bar{v}$, since he would not want to win against $\beta(\pi^{other}, \pi_{\geq \bar{v}}^{self}, u_i)$ in the former case. For sufficiently small information cost λ , he would do so.

Step 2. We then show that such bid at information set $\pi_i = (\pi^{other}, \pi_{\geq \bar{v}}^{self}, u_i)$ cannot be part of an equilibrium. If it were, then an agent i would make such a bid with probability at least ε in all states of the world consistent with information set π_i . In particular, he would make such a bid in states where his toughest opponent has value $\max_j v_j = \bar{v}$ and he has value $v_i = \hat{v}$. But then his toughest opponent would have a strictly positive, non-vanishing incentive to learn that his valuation is $v_j = \bar{v}$, as he then wants to outbid i if $u_j > u_i$ and to lose if $u_j < u_i$. He would do so in equilibrium, and agent i would have a strict incentive to deviate and learn that his value is $v_i = \hat{v}$. \square

Lemma 5. *There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, there exists $\varepsilon(\lambda) > 0$ with $\lim_{\lambda \rightarrow 0} \varepsilon(\lambda) = 0$ such that if an information structure has probability $\Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda) \geq \varepsilon(\lambda)$ in some*

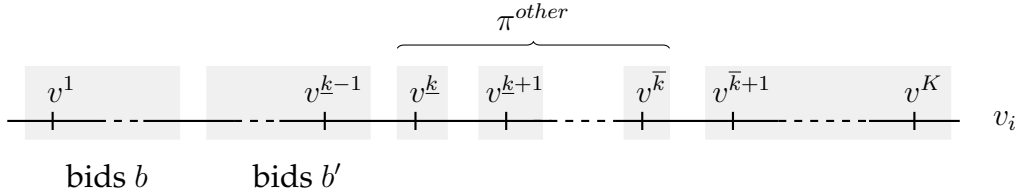
equilibrium σ_λ , then it must have the following form: for all $\pi^{other} \in \Pi^{other}$,

$$\left\{ v_i \mid v_i < \min_{v'_i \in \pi^{other}} v'_i \right\} \in \Pi^{self}(\pi^{other}) \quad \text{and} \quad \left\{ v_i \mid v_i > \max_{v'_i \in \pi^{other}} v'_i \right\} \in \Pi^{self}(\pi^{other}).$$

Proof of Lemma 5. Fix λ and let $(\Pi^{other}, \Pi^{self})$ be an information partition that is chosen with probability at least ε in equilibrium σ_λ . Take any $\pi^{other} \in \Pi^{other}$, and let $v^{\underline{k}} \equiv \min_{v'_i \in \pi^{other}} v'_i$ and $v^{\bar{k}} \equiv \max_{v'_i \in \pi^{other}} v'_i$. That is, upon learning $\max_j v_j \in \pi^{other}$, buyer i knows that his toughest competitor has $v_j \in [v^{\underline{k}}, v^{\bar{k}}]$.

Step 1. We prove that if i learns $\max_j v_j \in \pi^{other}$, then i chooses a signal about himself that bundles all the values he might have that lie for sure below his toughest competitor's valuation: $\{v_i \mid v_i < v^{\underline{k}}\} \in \Pi^{self}(\pi^{other})$. Denote by $j^* \in \arg \max_j v_j$ (one of) i 's toughest competitors.

By contradiction, suppose that the claim is not true, and that in equilibrium buyer i partitions all these values into at least two elements:



Since choosing such finer partition is costly, it must lead him to make better decision-making: so he must make (at least two) different bids b and b' depending on what he learned, and these two bids lead to different outcomes. Hence it must be that, with some strictly positive probability, the highest bid faced by buyer i lies in between these two bids $\Pr(b \leq \max_j \beta(\pi_j) \leq b' \mid \max_j v_j \in \pi^{other}, v_i < v^{\underline{k}}) > 0$, as otherwise these two bids would be completely equivalent.

This implies that buyer j^* , whom we know has a value $v_{j^*} \in \pi^{other}$, must sometimes make a bid below $b' \leq v^{\underline{k}-1} + u_i < v^{\underline{k}} + u_i$.³⁴ For j^* to make such a low bid in equilibrium, he must fail to learn that he has a high value and bundle his high value with lower ones. Let π_{j^*} be the information set at which j^* acts in such a way, with $\beta(\pi_{j^*}) \leq b'$.

³⁴The first inequality comes from the fact that in a tremble-robust equilibrium a buyer can never make a bid that lies *outside* the set of valuations he deems feasible. When i makes bid b' , he knows that his value is at most $v_i \leq v^{\underline{k}-1} + u_i$. The second inequality holds by construction since the noise terms are smaller than the size of the grid defined by V .

We furthermore know that j^* sometimes has this information set when he is a highest-valuation buyer and his value $v_{j^*} \in \pi^{other}$. Hence $\max_{v \in \pi_{j^*}^{self}} v \geq v^{\bar{k}}$.

Failing to learn his valuation is however costly for j^* as it leads him to sometimes lose the auction against i 's bid, despite his value being higher than i 's winning bid. Note that j^* cannot lose against i 's bid with non-vanishing probability, as otherwise it would be profitable for him to learn his valuation and bid it for λ small enough. For such bundle to be optimal for j^* , he must then expect to face a bid weakly above $v_{j^*} + \bar{u}$ with a probability that tends to one as λ goes to zero. If not, then learning to distinguish his high value from lower ones would lead to strictly positive, non-vanishing gains, and for λ small enough he would do so. Hence there must be another buyer, call him k , who bids weakly above $v_{j^*} + \bar{u}$ with non-vanishing probability in those states. Since j^* is a highest-valuation buyer, it must be that $v_k \leq v_{j^*} < v_{j^*} + \bar{u}$ almost surely, and hence that buyer k bids strictly above his valuation. By the symmetry of the equilibrium, buyer k must however tie at his equilibrium bid with non-vanishing probability.³⁵ He then incurs a strict, non-vanishing loss when that occurs, and so such a high bid by k cannot be sustained in equilibrium.

Step 2. Finally, we prove that if i learns $\max_j v_j \in \pi^{other}$, then i chooses a signal about himself that bundles all the values he might have that he knows for sure lie above his toughest competitor's valuation: $\{v_i \mid v_i > v^{\bar{k}}\} \in \Pi^{self}(\pi^{other})$.

By contradiction, suppose this is not the case. Following a similar logic as for Step 1, agent i can only find it worthwhile to learn to distinguish some of the values $v_i > v^{\bar{k}}$ if he sometimes faces a bid in that interval, and sometimes loses at that bid. That means that with positive probability, one of i 's competitors, all of whom have a value at most $v^{\bar{k}} + \bar{u}$, makes a bid strictly above this and wins with non-vanishing probability at that bid. However, if they win they must be paying a price weakly higher than i 's bid, and i 's bid must lie above $v^{\bar{k}} + \bar{u}$. Hence making such a bid leads that agent to have a strictly negative payoff, while he could ensure himself zero by making a lower bid. This cannot be optimal, and hence cannot be part of an equilibrium. \square

Lemma 6. *There exists $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, there exists $\varepsilon(\lambda) > 0$ with $\lim_{\lambda \rightarrow 0} \varepsilon(\lambda) = 0$ such that if an information structure has probability $\Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda) \geq \varepsilon(\lambda)$ in some*

³⁵If buyer k makes such a bid with non-vanishing probability then others must do as well.

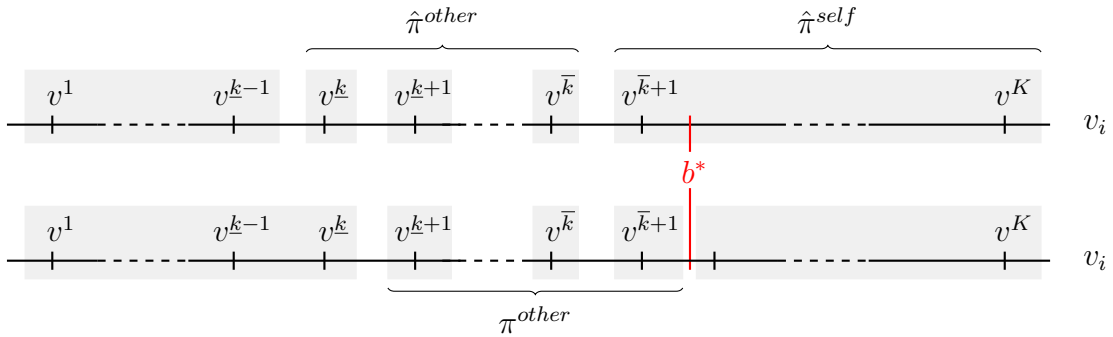
equilibrium σ_λ , then it must be cost-minimizing, that is:

$$\begin{aligned} (\Pi^{other}, \Pi^{self}) \in \arg \min_{\hat{\Pi}^{other}, \hat{\Pi}^{self}} & c\left(\hat{\Pi}^{other}, p_{1:N-1}\right) + \mathbb{E}_{\hat{\pi}^{other}} \left[c\left(\hat{\Pi}^{self}(\hat{\pi}^{other}), p\right) \right] \\ \text{s.t. } & \left(\hat{\Pi}^{other}, \hat{\Pi}^{self}\right) \text{ satisfies } (\star). \end{aligned}$$

Proof of Lemma 6. Suppose not, such that an equilibrium puts non-vanishing probability $\varepsilon > 0$ on some information structure $(\Pi^{other}, \Pi^{self})$ that is not cost-minimizing. That is, there exists another information structure $(\hat{\Pi}^{other}, \hat{\Pi}^{self})$ satisfying (\star) that is strictly cheaper than $(\Pi^{other}, \Pi^{self})$. Let $\lambda\Delta c$ be the difference in information costs between these two information structures.

Take the point of view of some agent i . For $(\hat{\Pi}^{other}, \hat{\Pi}^{self})$ not to be a profitable deviation from $(\Pi^{other}, \Pi^{self})$, it has to be that, under the former, agent i sometimes gets a strictly lower gross payoff at the auction stage. Since $(\hat{\Pi}^{other}, \hat{\Pi}^{self})$ satisfies (\star) , this can only happen when i fails to learn his valuation fully under $(\hat{\Pi}^{other}, \hat{\Pi}^{self})$. That is, it can only happen when either $\hat{\pi}_i^{self} = \{v_i \mid v_i > \max_{v'_j \in \hat{\pi}_i^{other}} v'_j\}$ or $\hat{\pi}_i^{self} = \{v_i \mid v_i < \min_{v'_j \in \hat{\pi}_i^{other}} v'_j\}$.

Consider the first case, which we illustrate in the figure below. For agent i not to get his full-information optimal payoff at $(\hat{\pi}_i^{other}, \hat{\pi}_i^{self} = \{v_i \mid v_i > \max_{v'_j \in \hat{\pi}_i^{other}} v'_j\})$, it has to be that the highest bid i faces at this information set sometimes falls strictly within $[\min_{v_i \in \hat{\pi}_i^{self}} v_i + \underline{u}, \max_{v_i \in \hat{\pi}_i^{self}} v_i + \bar{u}]$. Call b^* such a bid and let j^* be an opponent



that submits it. For agent i to be strictly better off under $(\Pi^{other}, \Pi^{self})$ because of it, it has to be that, under $(\Pi^{other}, \Pi^{self})$, he better discriminates whether he should win against b^* . That is, for some values of $v_i \in \hat{\pi}_i^{self}$ with $v_i < b^*$, i learns his value and bids below b^* . (In the figure, this corresponds to i learning to distinguish $v_i = v^{\bar{k}+1}$

from $v_i > v^{\bar{k}+1}$ under $(\Pi^{other}, \Pi^{self})$, as illustrated in the bottom partition.) However, that means agent j^* is making a strict loss winning against that bid, since we know $v_{j^*} \leq \max_{v_j \in \hat{\pi}_i^{other}} v_j < v_i$. Since information structure $(\Pi^{other}, \Pi^{self})$ has non-vanishing probability, agent j^* is making a non-vanishing loss, and for small enough λ , he must find it profitable to learn enough so as to avoid this costly mistake.

The reasoning for the second case is very similar. \square

Wrapping up, if an information structure has non-vanishing weight in some equilibrium, then it must satisfy (\star) and be cost-minimizing. The information structure under which agents acquire no information about others and fully learn their own valuation does satisfy (\star) . However, it is not cost-minimizing (Lemma 1). Hence if an information structure has non-vanishing weight, it must involve acquire some information about $\max_j v_j$.

B.3 Proof of Results of Section 4

B.3.1 Proof of Theorem 2

We know from Theorem 1 that, for λ small enough, the only information structures that have non-trivial probability must satisfy:

$$\Pi^{self}(\pi^{other}) = \left\{ \left\{ v_i \mid v_i < \min_{v \in \pi^{other}} v \right\}, \left\{ v_i \right\}_{v_i \in \pi^{other}}, \left\{ v_i \mid v_i > \max_{v \in \pi^{other}} v \right\} \right\} \forall \pi^{other} \in \Pi^{other},$$

and must minimize total information costs. A direct implication is that buyers do acquire some information about the competition $\Pi^{other} \neq \{V\}$ (Lemma 1), and as a result sometimes fail to learn their valuations precisely. The proof of Theorem 2 leverages this to show that, in any equilibrium, expected revenue remains bounded away from the expected second-highest valuation even as the cost parameter λ goes to zero. We first show that, when losing buyers fail to learn their valuations, they bid their expected valuations given their information sets (Step 1). We then show that such behavior reduces the second-highest bid in expectation (Step 2).

Step 1. We show that, when a buyer fails to learn his valuations fully but only learns that he does not have the highest one—i.e., when $\pi_i^{self} = \{v_i \mid v_i < \min_{v'_i \in \pi_i^{other}} v'_i\}$,—then he bids his expected valuations given his information set. Denote that bundle by

$\pi_{<}^{self}$. We already know that, in any tremble-robust equilibrium, a buyer cannot make a bid that lies outside the interval of values he deems possible:

$$\beta(\pi^{other}, \pi_{<}^{self}, u_i) \in \left[\min_{v'_i \in \pi_{<}^{self}} v'_i + u_i, \max_{v'_i \in \pi_{<}^{self}} v'_i + u_i \right].$$

Furthermore, at that information set, a buyer must lose the auction with probability one in equilibrium. Suppose not, and let $j^* \in \arg \max_j v_j$ be (one of) i 's toughest competitors. We know buyer i 's bid must lie below j^* 's value since $\beta(\pi^{other}, \pi_{<}^{self}, u_i) < \min_{v'_i \in \pi^{other}} v'_i + u \leq \nu_{j^*}$. We also know that buyer i chooses this information structure with non-vanishing probability. For i to win the auction at that information set, buyer j^* must fail to learn his valuation and bundle his high value in π^{other} with some lower ones. However such information structure cannot be optimal for j^* when λ is small enough as it leads him to lose against i 's bid with non-vanishing probability. Hence it must be that at information set $\pi_i = (\pi^{other}, \pi_{<}^{self}, u_i)$, i always loses the auction in equilibrium.

Buyer i 's equilibrium bid is then disciplined by the trembling-hand-like refinement that we impose. In particular, buyer i 's bid must be optimal given that each of his competitors trembles with vanishing probability. Hence i can only win the auction when j^* trembles. In that scenario, none of i 's competitors' bids reveal any information about i 's value: all buyers $j \neq j^*$ learned about their toughest competitors, which is j^* , and j^* is trembling so his bid is drawn at random. Given that bidding truthfully is a weakly dominant strategy in a SPA, and that buyer i faces a distribution of bids that has full support given j^* 's tremble, he has a strict incentive to bid his expected valuation for the good given his information set:

$$\beta \left(\pi^{other}, \pi_{<}^{self}, u_i \right) = \mathbb{E} \left[v_i \mid v_i < \min_{v'_i \in \pi^{other}} v'_i \right] + u_i.$$

Step 2. We show that there exist $L > 0$ and $\bar{\lambda} > 0$ such that, for all $\lambda \leq \bar{\lambda}$, the expected second-highest bid is lower than $\mathbb{E}[\nu_{(2)}] - L$ in any equilibrium.³⁶ Take any equilibrium, and denote by $q(\lambda)$ the probability that a buyer chooses an information structure satisfying (\star) . We know from Theorem 1 that $\lim_{\lambda \rightarrow 0} q(\lambda) = 1$. We focus on the case where all buyers choose such an information structure—the other case has vanishing

³⁶ $\nu_{(2)}$ denotes the second highest value for every realization of $(\nu_i)_i$.

probability, and induces a revenue that is bounded above by the highest possible valuation. We first show that, *given any realized second-highest bid* $b_{(2)}$, revenue (i.e., the realized second-highest bid) must lie weakly below $\mathbb{E}[\nu_{(2)} \mid b_{(2)}]$. We then show that, with strictly positive probability, it is bounded strictly below $\mathbb{E}[\nu_{(2)} \mid b_{(2)}] - L$ for some $L > 0$.

Note that the highest-valuation buyer wins with probability one if all buyers choose an information structure satisfying (\star) . Let i_1 be the highest-valuation buyer and $\nu_{(1)}$ his realized valuation. (Ties have probability zero as the distribution of u_i is continuous.) There are two cases: either the price is set by a buyer who learned his valuation fully, or it isn't. The first case is direct: since the second-highest bidder (call him j^*) learned his value, he must have bid truthfully, and revenue then equals $b_{(2)} = \nu_{j^*} \leq \nu_{(2)}$. In the second case, the second-highest bidder failed to learn his value, which means that $\pi_{j^*}^{self} = \{v_j \mid v_j \leq \bar{v}\}$ for some $\bar{v} < v_{i_1}$. If j^* wins at such bid, then all other bidders $j \neq j^*, i_1$ must also have had values $v_j \leq \bar{v}$.³⁷ We know from Step 1 that j^* must have bid $\beta(\pi_{j^*}) = \mathbb{E}[\nu_{j^*} \mid v_{j^*} \leq \bar{v}]$, which always lies weakly below $\mathbb{E}[\nu_{(2)} \mid b_{(2)} = \beta(\pi_{j^*})] = \mathbb{E}[\nu_{(2)} \mid v_{i_1} > \bar{v}, v_j \leq \bar{v} \forall j \neq i_1]$.³⁸ Thus, when all agents choose information structures satisfying (\star) , $b_{(2)} \leq \mathbb{E}[\nu_{(2)} \mid b_{(2)}]$, and expected revenue is weakly below the expected second-highest valuation.

We now prove that, with strictly positive non-vanishing probability, the second-highest bid $b_{(2)}$ is bounded strictly below the expected second-highest valuation given $b_{(2)}$. In particular, we show that this is the case when all buyers choose the same information structure and the gap between the highest and second-highest valuations is large enough. Let $V = \{v^1, v^2, \dots, v^K\}$ with $v^{k+1} > v^k$. Take any information structure $(\Pi^{other}, \Pi^{self})$ that has non-vanishing weight in equilibrium. Let $\underline{v} = \min\{v \mid \exists \pi^{other} \in \Pi^{other} \text{ s.t. } v \in \pi^{other}, v^K \in \pi^{other}\}$ denote the smallest valuation that Π^{other} bundles with v^K . We know from Theorem 1 and Lemma 1 that $\Pi^{other} \neq \{V\}$. Similarly, we know from 2 that $\Pi^{other} \neq \{\{v^1\}, \{v^2, \dots, v^K\}\}$. Hence $\underline{v} > v^2$. Consider what happens when the highest-valuation bidder has value $v_{i_1} \geq \underline{v}$ while all others $j \neq i_1$ have value $v_j < \underline{v}$. When all buyers choose information structure $(\Pi^{other}, \Pi^{self})$, all buyers $j \neq i_1$ must learn $\max_{j \neq i} v_j \in [\underline{v}, v^K]$ since $v_{i_1} \geq \underline{v}$. Furthermore, all buyers but i_1 must fail to learn their valuations precisely: $\pi_j^{self} = \{v_j \mid v_j < \underline{v}\}$ with $|\pi_j^{self}| \geq 2$. The second-highest bid

³⁷If not, they would have either learned their values and outbid j^* , or just learned that $v_j \leq \bar{v}'$ for some $\bar{v}' > \bar{v}$, which would also have led them to outbid j^* .

³⁸Indeed, the latter is the expectation of the highest value among the $N - 1 \geq 2$ losing bidders, which is weakly above the expected value of one specific losing bidder j^* .

then equals $\mathbb{E}[v_j \mid v_j < \underline{v}] + \max_{j \neq i_1} u_j$.

Overall, we get

$$\begin{aligned} \mathbb{E}[\text{equilibrium revenue} \mid \sigma_\lambda] - \mathbb{E}[\nu_{(2)}] &\leq (1 - [q(\lambda)]^N) (v^K - \mathbb{E}[\nu_{(2)}]) \\ &+ [q(\lambda)]^N \Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda)^N \Pr(v_{i_1} \geq \underline{v}, v_j < \underline{v} \forall j \neq i_1) \times \\ &\quad \left(\mathbb{E}[v_i \mid v_i < \underline{v}] + \max_{j \neq i_1} u_j - \mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}] - u_{i_2} \right). \end{aligned}$$

We give an upper bound for the second term. Note that $\mathbb{E}[v_i \mid v_i < \underline{v}] - \mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}]$ is strictly negative. Indeed, it compares the expected value of a buyer conditional on it being lower than some bound $\mathbb{E}[v_i \mid v_i < \underline{v}]$ to the expected *second-highest* value conditional on it being lower than that same bound and the highest-value being higher than this bound $\mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}]$. Hence the latter is just the expected value of the best of these $N - 1$ draws, simply truncating the distribution at the bound as we know that all these $N - 1$ draws lie below it. Since there are $N \geq 3$ buyers, the latter is strictly positive whenever there is some variance in the distribution of $v_i < \underline{v}$. This is the case as $|\{v_i \mid v_i < \underline{v}\}| \geq 2$. Hence

$$\mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}] - \mathbb{E}[v_i \mid v_i < \underline{v}] \equiv l > 0.$$

Furthermore, $\max_{j \neq i_1} u_j \geq u_{i_2}$. Then

$$\begin{aligned} \mathbb{E}[\text{equilibrium revenue} \mid (\Pi^{other}, \Pi^{self})] - \mathbb{E}[\nu_{(2)}] &\leq (1 - [q(\lambda)]^N) (v^K - \mathbb{E}[\nu_{(2)}]) \\ &- [q(\lambda)]^N \Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda)^N \Pr(v_{(1)} \geq \underline{v}, v_{(2)} < \underline{v}) \Pr(v_{(1)} \geq \underline{v}, v_{(2)} < \underline{v}) l. \end{aligned}$$

Since $\lim_{\lambda \rightarrow 0} q(\lambda) = 1$, the first RHS term goes to zero as information costs vanish. Since $\lim_{\lambda \rightarrow 0} \Pr(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda) > 0$, the second does not. There exists $\bar{\lambda}$ such that for all $\lambda \leq \bar{\lambda}$, expected revenue is bounded away from the expected second-highest valuation. \square

B.3.2 Proof of Proposition 4

Take any equilibrium σ_λ , and fix some realization of $(v_i)_i$. We show that two bidders i_1, i_2 with values $v_{i_2} < v_{i_1}$ cannot both enter the auction with probability greater than $\varepsilon > 0$ for all λ . Hence if two buyers enter with non-vanishing probability, they must

have the same value. We then argue that they must be the highest-valuation buyers, i.e., $v_{i_1} = v_{i_2} = \max_j v_j$.

Given an information structure $(\Pi^{other}, \Pi^{self})$, let $\pi(v) \in \Pi^{other} \times \Pi^{self}$ denote the information set at which a buyer must be in state $v = (v_i)_i$.³⁹ Formally, we show that

$$\sum_{(\Pi^{other}, \Pi^{self})} \Pr [(\Pi^{other}, \Pi^{self}) \mid \sigma_\lambda] \Pr (\beta(\pi(v)) > 0 \mid \sigma_\lambda)$$

cannot be greater than ε irrespective of λ for both i_1 and i_2 if $v_{i_2} < v_{i_1}$. By contradiction, suppose that it is. It must be that at some information sets π_{i_1}, π_{i_2} that have non-vanishing weight given $(v_i)_i$, these two buyers make non-zero bids $\beta(\pi_i) > 0$ for $i = i_1, i_2$.

Step 1. We first show that i_2 must know and bid his valuation at this information set: $\pi_{i_2}^{self} = \{v_{i_2}\}$ and $\beta(\pi_{i_2}) = v_{i_2} + u_{i_2}$. Suppose not: at this information set i_2 does not know his valuation fully $|\pi_{i_2}^{self}| > 1$, and let \underline{v} and \bar{v} denote the lowest and highest values in π_{i_2} , respectively. For i_2 to find it optimal to enter the auction and bid $\beta(\pi_{i_2}) > 0$, it must be that he wins with strictly positive probability at such bid. In particular, this implies that i_1 is sometimes at an information set at which he bids lower than i_2 . At information set π_{i_2} , buyer i_2 cannot rule out the possibility that some other buyer $j \neq i_1, i_2$ has the same value as him $v_j = v_{i_2}$. Indeed, $\pi_{i_2}^{other}$ is only informative of $\max_{j \neq i_2} v_j \geq v_{i_1} > v_{i_2}$. Hence with non-vanishing probability, one of i_2 's opponents has the same information set as him $\pi_j = \pi_{i_2}$ and makes the same equilibrium bid. With non-vanishing probability, buyer i_2 then ties to win the auction, and must then be indifferent between winning and losing at his equilibrium bid. That equilibrium bid must lie strictly in between $\underline{v} + u_i$ and $\bar{v} + u_i$, which means that there are non-vanishing losses associated with failing to distinguish values $v_i = \underline{v}$ and $v_i = \bar{v}$. Indeed, at the former he does not want to win at his equilibrium bid while he does want to win at the latter. For λ small enough, it must then be optimal to learn to distinguish these values, and it cannot be that $|\pi_{i_2}^{self}| > 1$.

Step 2. We now show that in state $v = (v_i)_i$, agent i_1 must be entering the auction and outbidding i_2 with probability one. Take any information set $\pi(v_{i_1}, v_{-i_1})$ that i_1 might have in equilibrium given the realized $(v_i)_i$. By assumption, one of these information

³⁹That is, $\pi(v) = (\{\pi^{other} \in \Pi^{other} : \max_j v_j \in \pi^{other}\}, \{\pi^{self} \in \Pi^{self}(\pi^{other}) : v_i \in \pi^{self}\})$

sets is the one we started with π_{i_1} at which he enters. At this information set, he must be outbidding i_2 . Indeed we know from Step 1 that i_2 is bidding $v_{i_2} + u_{i_2} < v_{i_1}$, and that i_2 wins sufficiently often at that bid to justify its entry. So there are strictly positive, non-vanishing gains from outbidding i_2 , and for λ small enough, i_1 must learn sufficiently about his valuation to do so.

Suppose that at some other information set $\hat{\pi}(v_{i_1}, v_{-i_1})$ that has positive probability in equilibrium, buyer i_1 stays out of the auction in that state of the world, and gets zero gross payoff. By choosing a finer information structure, buyer i_1 could have entered and gotten a strictly positive, non-vanishing gross payoff. Indeed, since i_1 has a strictly greater value than i_2 , i_1 must have greater, and hence strictly positive, gains from entering the auction. For λ small enough, that other information structure must yield an overall strictly greater payoff, and hence represents a profitable deviation.

Step 3. We now argue that i_2 cannot find it profitable to enter the auction at information set π_{i_2} . There are two cases: either $\pi_{i_2}^{other}$ is sufficiently fine that i_2 can predict that i_1 will enter with probability one and bid higher than v_{i_2} , or not. In the first case, i_2 cannot find it optimal to pay the entry fee κ . In the second, i_2 could deviate to a finer $\pi_{i_2}^{other}$ so as to not enter in this state of the world in which there are no gains from doing so. For λ small enough, this strict increase in gross payoff must be lower than the cost of the finer $\pi_{i_2}^{other}$, and this deviation leads to a strictly higher overall payoff.

Taking stocks, given some realization of $(v_i)_i$, if two buyers enter with non-vanishing probability then they must have the same value v_i . We have left to argue that they must have the highest realized value $\max_i v_i$. By contradiction, suppose not: two buyers i_1 and i_2 enter with non-trivial probability while some other buyers j^* with $v_{j^*} > v_{i_1} = v_{i_2}$ does not. First note that bidders i_1 and i_2 must know their valuation fully for λ small enough and bid it, as they tie against each other with non-vanishing probability. Yet if they find it profitable to enter and bid their values then buyer j^* must find it strictly profitable since he has a strictly higher valuation than them. Hence j^* cannot stay away from the auction in equilibrium, and if two buyers enter with non-vanishing probability then they must have $v_i = \max_j v_j$. \square

B.4 Proof of Results of Section 5

B.4.1 Proof of Theorem 3

Let $\Pi^{other} = \{\{\pi_l^{other}\}_{l=1}^L\}$ be an information partition about $\max_j v_j$ that has non-vanishing probability in equilibrium. Let $v^{\underline{k}_l}$ and $v^{\bar{k}_l}$ denote the lowest and highest values in π_l^{other} , respectively. The equilibrium partition may depend on the number of bidders N but we do not make that dependence explicit so as to keep the notation uncluttered. The proof of Theorem 3 has two steps. We first show that, for N large enough and λ small enough, buyers learn to distinguish whether or not their toughest competitor has a value equal to v^K (i.e., $\{v^K\} \in \Pi^{other}$) under any information structure that has non-vanishing weight. Second, we show that setting a reserve price just below that highest possible valuation yields more revenue than having an additional buyer participate in the auction.

Step 1. *Assume no reserve price. There exists \bar{N}_1 such that, for all $N \geq \bar{N}_1$, there exists $\bar{\lambda}$ such that, for all $\lambda \leq \bar{\lambda}$, buyers learn whether their toughest competitor has the highest possible value: $\{v^K\} \in \Pi^{other}$.*

By contradiction, suppose not: for arbitrarily large N , there exists an information structure that has non-vanishing weight in equilibrium such that $\{v^K\} \notin \Pi^{other}$. That is, with non-trivial probability, buyers choose a partition about others that bundles v^K with some other possible values: $\{v^{\underline{k}_L}, \dots, v^K\} \in \Pi^{other}$ for some $v^{\underline{k}_L} < v^K$.

We know from Theorem 1 that an information structure that has non-vanishing weight in equilibrium must satisfy (\star) and minimize cost. We give an alternative information structure under which $\{v^K\} \in \hat{\Pi}^{other}$ that is strictly cheaper than $(\Pi^{other}, \Pi^{self})$, which hence proves the latter cannot be sustained in equilibrium.

The cost of information partition Π^{other} equals:

$$c(\Pi^{other}, p_{1:N-1}) = H(p_{1:N-1}) - \sum_{l=1}^L \Pr\left(\max_j v_j \in \pi_l^{other}\right) H(\delta_{\pi_l^{other}}),$$

where $\delta_{\pi_l^{other}} \in \Delta V$ is the agent's posterior about $\max_j v_j$ given that he knows $\max_j v_j \in \pi_l^{other}$:

$$\delta_{\pi_l^{other}}(\cdot) = \Pr\left(\cdot \mid \max_j v_j \in \pi_l^{other}\right) = \begin{cases} \frac{p_{1:N-1}(v)}{\sum_{v' \in \pi_l^{other}} p_{1:N-1}(v')} & \text{if } v \in \pi_l^{other} \\ 0 & \text{otherwise} \end{cases}.$$

Consider an alternative information partition about others that unbundles v^K from $\{v^{\underline{k}_L}, \dots, v^{K-1}\}$, but keeps the rest of the partition the same:

$$\widehat{\Pi}^{other} = \{\{v^{\underline{k}_l}, \dots, v^{\bar{k}_l}\}_{l=1, \dots, L-1}, \{v^{\underline{k}_L}, \dots, v^{K-1}\}, \{v^K\}\}.$$

Naturally this partition is finer, and hence costlier than the one we started with. Using the same formula as above, the extra cost equals

$$\begin{aligned} c(\widehat{\Pi}^{other}, p_{1:N-1}) - c(\Pi_\lambda^{other}, p_{1:N-1}) &= \Pr\left(\max_j v_j \geq v^{\underline{k}_L}\right) H(\delta_{\{v^{\underline{k}_L}, \dots, v^K\}}) \\ &\quad - \Pr\left(v^{\underline{k}_L} \leq \max_j v_j < v^K\right) H(\delta_{\{v^{\underline{k}_L}, \dots, v^{K-1}\}}) - \Pr\left(\max_j v_j = v^K\right) H(\delta_{\{v^K\}}). \end{aligned}$$

Note that as N goes to infinity, the second term goes to zero, as $\Pr(\max_j v_j < v^K)$ goes to zero. Furthermore, a buyer's posterior conditional on learning that $\max_j v_j \in \{v^{\underline{k}_L}, \dots, v^K\}$ converges to the degenerate belief that puts probability one on the toughest competitor having the highest possible value $\delta_{\{v^K\}}$. Indeed, as N goes to infinity, the probability of such event goes to one. Hence, $\lim_{N \rightarrow \infty} \delta_{\{v^{\underline{k}_L}, \dots, v^K\}} = \delta_{\{v^K\}}$. Since H is continuous, this implies that the first and last term cancel out in the limit, and so the overall expression goes to zero: the cost of learning whether one of the competitors has the highest valuation possible becomes negligible.⁴⁰

There is however a first-order gain in choosing this alternative partition, as it allows the buyer to save on information cost about his own valuation. If the buyer learns that $\max_j v_j \in \{v^{\underline{k}_L}, \dots, v^K\} = \pi^{other}$, then we know from Theorem 1 that $\Pi^{self}(\pi^{other}) = \{\{v^1, \dots, v^{\bar{k}_{L-1}}\}, \{v^k\}_{k=\underline{k}_L, \dots, K}\}$. If the buyer learns that $\max_j v_j \in \{v^{\underline{k}_L}, \dots, v^{K-1}\}$, then the optimal way to partition his set of valuations is the same one. If however the buyer learns that $\max_j v_j = v^K$, then he optimally chooses a coarser partition for himself: $\widehat{\Pi}^{self}(\{v^K\}) = \{\{v^1, \dots, v^{K-1}\}, \{v^K\}\}$. Hence the gain in information cost on self is

$$\begin{aligned} &\Pr\left(\max_j v_j = v^K\right) \\ &\quad \times \left[c(\{\{v^1, \dots, v^{K-1}\}, \{v^K\}\}, p) - c(\{\{v^1, \dots, v^{\tau_L}\}, \{v^k\}_{k=\tau_L+1, \dots, K}\}, p) \right]. \end{aligned}$$

Since $\Pr(\max_j v_j = v^K)$ tends to one as N tends to infinity, this tends to the strictly

⁴⁰Intuitively, this is due to the fact that such event is so likely that learning about its realization does not move the buyer's belief much.

positive expression in parenthesis. For N large enough, the information structure $(\hat{\Pi}^{other}, \hat{\Pi}^{self})$ is strictly cheaper, and so the information structure we started with cannot have non-trivial weight in equilibrium.

Step 2. *There exists \bar{N}_2 such that, for all $N \geq \bar{N}_2$, there exists $\bar{\lambda}$ such that, for all $\lambda \leq \bar{\lambda}$, setting a reserve price $r \in (v^{K-1}, v^K)$ yields more revenue than having one more bidder in the auction.*

Consider a (somewhat extreme) reserve price that lies just below the highest possible valuation $r = v^K - \eta$ for η small ($|\underline{u}_N| < \eta < v^K - v^{K-1}$). Under such reserve price, and for λ small enough, all buyers find it optimal to acquire no information about others, and to only learn whether their valuations lie above or below the reserve price. Hence they all choose

$$\Pi^{self} = \{\{v^1, v^2, \dots, v^{K-1}\}, \{v^K\}\}.$$

Given a realization of $(v_i)_i$, let $v_{1:N}$ and $v_{2:N}$ be the highest and second-highest valuations, respectively. As λ goes to zero, imposing a reserve price $r = v^K - \eta$ then yields an expected revenue of

$$\text{Revenue}(r, N) = \Pr(v_{1:N} = v^K, v_{2:N} < v^K) r + \Pr(v_{2:N} = v^K) \mathbb{E}(v_{2:N} \mid v_{2:N} \geq v^K + \underline{u}_N).$$

Now consider what happens in equilibrium if no reserve price is imposed, but there are $N + 1$ bidders participating in the auction. By Step 1, it has to be that buyers choose to learn sufficiently finely about the competition, and in particular that, for N large enough, they come to learn whether their toughest competitor has a value of v^K . If they learn that this is the case, they then partition their own set of valuations into $\{\{v^1, v^2, \dots, v^{K-1}\}, \{v^K\}\}$. That is, they learn whether they should compete with their toughest competitor (which only yields a non zero payoff when they also have $v_i = v^K$) and bundle together all valuations below v^K . Hence, in any equilibrium, all buyers who learn $\max_j v_j = v^K$ and $v_i < v^K$ bid $\mathbb{E}[v_i \mid v_i < v^K] + u_i$ (as argued in Step 1 of proof of Theorem 2).

Revenue with $N + 1$ bidders but no reserve price then equals

$$\begin{aligned} \text{Revenue}(0, N + 1) &= \Pr(v_{2:N+1} = v^K) \mathbb{E}(v_{2:N+1} \mid v_{2:N+1} \geq v^K + \underline{u}_{N+1}) \\ &\quad + \Pr(v_{1:N+1} = v^K, v_{2:N+1} < v^K) (\mathbb{E}[v_i \mid v_i < v^K] + \mathbb{E}(u_{1:N})) \end{aligned}$$

$$+ \Pr(v_{1:N+1} < v^K) \mathbb{E}[\text{eq. revenue} | v_i < v^K \forall i = 1, \dots, N+1].$$

The first line captures expected revenue when both the highest- and second-highest-valuation buyers have $v_i = v^K$. In such case they both learn their valuations fully, and revenue simply equals the expected second-highest valuation. The second line captures expected revenue when the highest-valuation buyer has $v_i = v^K$ while the second-highest valuation buyer has $v_j < v^K$. In such case, all losing buyers j fail to learn their valuations and bid $\mathbb{E}[v_i | v_i < v^K] + u_j$. Finally, the third line captures expected revenue when none of the buyers has $v_i = v^K$. The probability of such event is vanishing at a faster rate than the others as N grows. We will thus be able to overlook it and do not need to derive an explicit expression for revenue.

We show that, for N high enough, the above reserve price yields greater expected revenue than having an additional bidder:

$$\Delta \equiv \text{Revenue}(r, N) - \text{Revenue}(0, N+1) > 0.$$

This difference is at least

$$\begin{aligned} \Delta &\geq \Pr(v_{1:N} = v^K, v_{2:N} < v^K) (r - \mathbb{E}[v_i | v_i < v^K] - \bar{u}_{N+1}) \\ &\quad + (\Pr(v_{2:N} = v^K) - \Pr(v_{2:N+1} = v^K)) (v^K + \bar{u}_N) \\ &\quad + (\Pr(v_{1:N} = v^K, v_{2:N} < v^K) - \Pr(v_{1:N+1} = v^K, v_{2:N+1} < v^K)) (\mathbb{E}[v_i | v_i < v^K] + \bar{u}_{N+1}) \\ &\quad - \Pr(v_{1:N+1} < v^K) \mathbb{E}[\text{eq. revenue} | v_i < v^K \forall i = 1, \dots, N+1] \\ &\quad + \Pr(v_{2:N} = v^K) \left[\mathbb{E}(u_{2:M} | M = |\{i = 1, \dots, N | v_i = v^K\}|, M \geq 2) \right. \\ &\quad \quad \left. - \mathbb{E}(u_{2:M} | M = |\{i = 1, \dots, N+1 | v_i = v^K\}|, M \geq 2) \right] \\ &= Np^K (1 - p^K)^{N-1} (r - \mathbb{E}[v_i | v_i < v^K] - \bar{u}_{N+1}) \\ &\quad + (v^K + \bar{u}_N) \left[1 - (1 - p^K)^N - Np^K (1 - p^K)^{N-1} - 1 + (1 - p^K)^{N+1} + (N+1)p^K (1 - p^K)^N \right] \\ &\quad + \left[Np^K (1 - p^K)^{N-1} - (N+1)p^K (1 - p^K)^N \right] (\mathbb{E}[v_i | v_i < v^K] + \bar{u}_{N+1}) \\ &\quad - (1 - p^K)^{N+1} \mathbb{E}[\text{eq. revenue} | v_i < v^K \forall i = 1, \dots, N+1] \\ &\quad + \Pr(v_{2:N} = v^K) \Delta_u^N, \end{aligned}$$

where

$$\begin{aligned}\Delta_u^N &= \mathbb{E}(u_{2:M} | M = |\{i = 1, \dots, N | v_i = v^K\}|, M \geq 2) \\ &\quad - \mathbb{E}(u_{2:M} | M = |\{i = 1, \dots, N + 1 | v_i = v^K\}|, M \geq 2).\end{aligned}$$

Factorizing by $Np^K (1 - p^K)^{N-1}$ this simplifies to

$$\begin{aligned}\frac{\Delta}{Np^K (1 - p^K)^{N-1}} &\geq r - \mathbb{E}[v_i | v_i < v^K] - \bar{u}_{N+1} - p^K(v^K + \bar{u}_N) \\ &\quad + \left[p^K - \frac{1 - p^K}{N} \right] [\mathbb{E}[v_i | v_i < v^K] + \bar{u}_{N+1}] \\ &\quad - \frac{(1 - p^K)^2}{Np^K} \mathbb{E}[\text{eq. revenue} | v_i < v^K \forall i = 1, \dots, N + 1] \\ &\quad + \Pr(v_{2:N} = v^K) \frac{\Delta_u^N}{Np^K (1 - p^K)^{N-1}}.\end{aligned}$$

Using the fact that $r = v^K - \eta$ for some small η , this rewrites as

$$\begin{aligned}\frac{\Delta}{Np^K (1 - p^K)^{N-1}} &\geq (1 - p^K) (v^K - \mathbb{E}[v_i | v_i < v^K]) - \eta - p^K \bar{u}_N - (1 - p^K) \bar{u}_{N+1} \\ &\quad - \frac{1 - p^K}{N} (\mathbb{E}[v_i | v_i < v^K] + \bar{u}_{N+1}) - \frac{(1 - p^K)^2}{Np^K} \mathbb{E}[\text{eq. revenue} | v_i < v^K \forall i] \\ &\quad + \Pr(v_{2:N} = v^K) \frac{\Delta_u^N}{Np^K (1 - p^K)^{N-1}}.\end{aligned}$$

First note that $(1 - p^K) (v^K - \mathbb{E}[v_i | v_i < v^K]) > 0$ and so must be greater than η for η small enough. Furthermore, all terms on the second line go to zero as N goes to infinity. The same is true of the bounds on the noise terms \bar{u}_N . Finally, the last term is bounded above by $\bar{u}_N - \underline{u}_N$ and is arbitrarily small if the noise terms have small support. Hence, there must exist \bar{N}_2 such that, for all $N \geq \bar{N}_2$, the reserve price yields greater revenue than the extra bidder: $\Delta > 0$.

Intuitively, for large N , the first-order difference between a reserve price and an additional bidder occurs when only one bidder has value $v_i = v^K$ and all others have

values $v_i < v^K$.⁴¹ In such a case, an additional bidder only yields revenue $\mathbb{E}[v_i | v_i < v^K] + \max_j u_j$ whereas a reserve price yields revenue arbitrarily close to $v_K + \underline{u}_N$, which is higher.

Combining Steps 1 and 2, for all $N \geq \max\{\bar{N}_1, \bar{N}_2\}$ the claim holds, and the above reserve price outperforms an auction with no reserve but one more bidder. \square

B.4.2 Proof of Proposition 5 and Theorem 4

Proof of Proposition 5. Consider the following way to randomize access to the auction:

$$\Pr(M = N) = 1 - q \quad \text{and} \quad \Pr(M = N \setminus i) = \frac{q}{N} \quad \text{for all } i,$$

for some $q \in (0, 1)$. That is, with probability $1 - q$ all buyers get access to the auction. With remaining probability, one buyer chosen uniformly at random is excluded. Take any information partition about self Π^{self} that has non-vanishing weight in equilibrium. We show that, for any $q > 0$, there exists $\bar{\lambda}$ such that, for all $\lambda \leq \bar{\lambda}$, buyers must always become fully informed of their valuations: $\Pi^{self} = \{\{v\}_{v \in V}\}$.

By contradiction, suppose not: Π^{self} bundles some valuations together. Take any such bundle and let \underline{v} and \bar{v} be the lowest and highest element in that bundle, respectively. Denote that bundle by $\pi_{\underline{v}, \bar{v}}^{self}$ and let $\beta(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i)$ a buyer's equilibrium bid at that bundle. We know that $\beta(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_i) \in [\underline{v} + u_i, \bar{v} + u_i]$.

Consider deviating to $\hat{\Pi}^{self} = \{\{v\}_{v \in V}\}$. Since that partition is finer, it is costlier and increases buyer i 's information costs by

$$\lambda \left[c(\hat{\Pi}^{self}, p) - c(\Pi^{self}, p) \right] > 0.$$

Such a finer partition must (weakly) increase buyer i 's gross payoff. We show that it strictly increases i 's gross payoff. Whatever information i acquired about his toughest opponent's valuation is irrelevant with probability q/N . Indeed, with probability q/N , i 's toughest opponent will be excluded from the auction. In such event, buyer i is left competing with buyers about whose valuations i has no information. In particular, with non-trivial probability, all these buyers chose the same signal about self Π^{self} , all

⁴¹When more than one bidders have values $v_i = v^K$, they learn their valuations fully and the second-highest bid is $v_{2:N} > r$. The only difference in revenue between a reserve price and an additional bidder then comes from the fact that, under the latter, the second-highest bidder has a larger draw of u_i in expectation.

have values $v_j \in \pi_{\underline{v}, \bar{v}}^{self}$, and all bid $\beta(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, u_j)$ for some u_j . But buyer i then has a strict incentive to learn to distinguish $v_i = \bar{v}$ from $v_i = \underline{v}$. Indeed, he wants to win against $\beta(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, \max_j u_j)$ (at least) when $v_i = \bar{v}$ and $u_i > \max_j u_j$, and lose against $\beta(\pi^{other}, \pi_{\underline{v}, \bar{v}}^{self}, \max_j u_j)$ (at least) when $v_i = \underline{v}$ and $u_i < \max_j u_j$.

These gains from distinguishing $v_i = \bar{v}$ and $v_i = \underline{v}$ are strictly positive and independent of λ . Hence, for λ small enough, the increase in information cost must be strictly smaller than the gains, and buyers must become fully informed about their valuations in equilibrium: $\Pi^{self} = \{\{v\}_{v \in V}\}$.

In any equilibrium, fully informed buyers must bid their valuations for the good: $\beta((\pi^{other}, \{v_i\}, u_i)) = v_i + u_i = \nu_i$. Expected revenue is then at least $q\mathbb{E}[\nu_{(2)}]$. Given any $\varepsilon > 0$, set $q = 1 - \frac{\varepsilon}{\mathbb{E}[\nu_{(2)}]}$. For λ small enough, expected revenue from such randomized access is at least $\mathbb{E}[\nu_{(2)}] - \varepsilon$, which completes the proof. \square

Proof of Theorem 4. Consider the randomized access from Proposition 5. For any $\varepsilon > 0$, the seller can ensure himself a revenue of

$$\mathbb{E}[\nu_{(2)}] - \varepsilon$$

for λ small enough. We compare this revenue to the one obtained when the seller sets the optimal reserve price.

Step 1. We first show that, for N large enough and λ small enough, the optimal reserve price r^* must target buyers with the highest possible realization of v_i , that is, $r^* = v^K + u^*$ for some $u^* \in (\underline{u}_N, \bar{u}_N)$. Suppose not, and $r^* < v^K + \underline{u}_N$ for all N and λ . Using the same argument as in the proof of Theorem 3, we show that, for N large enough, buyers only put non-vanishing weight on information structures such that $\{v^K\} \in \Pi^{other}$. In words, they find it cost-efficient to learn whether they face a competitor with the highest possible valuation, as this event has very high probability for large N . (See proof of Theorem 3 for more details.)

After learning that $\max_j v_j = v^K$, a buyer chooses $\Pi^{self}(\{v^K\}) = \{\{v_i : v_i < v^K\}, \{v^K\}\}$ and bids $\mathbb{E}[v_i \mid v_i < v^K] + u_i$ whenever $v_i < v^K$. A reserve price of $r < v^K + \underline{u}_N$ then yields revenue strictly bounded above by

$$\left(1 - (1 - p^K)^N - Np^K (1 - p^K)^{N-1}\right) \mathbb{E}[\nu_{(2)} \mid \nu_{(2)} \geq v^K + \underline{u}_N]$$

$$+ Np^K (1 - p^K)^{N-1} \mathbb{E} [\max\{r, \mathbb{E}[v_i | v_i < v^K] + u_i\}] + (1 - p^K)^N (v^K + \underline{u}_N).$$

In comparison, a reserve price to $r' = v^K + \underline{u}_N$ yields an expected revenue of

$$\begin{aligned} & \left(1 - (1 - p^K)^N - Np^K (1 - p^K)^{N-1}\right) \mathbb{E}[\nu_{(2)} | \nu_{(2)} \geq v^K + \underline{u}_N] \\ & + Np^K (1 - p^K)^{N-1} (v^K + \underline{u}_N). \end{aligned}$$

The latter is strictly greater whenever

$$\begin{aligned} & Np^K (1 - p^K)^{N-1} (v^K + \underline{u}_N - \mathbb{E} [\max\{r, \mathbb{E}[v_i | v_i < v^K] + u_i\}]) \\ & - (1 - p^K)^N (v^K + \underline{u}_N) > 0 \\ \iff & Np^K (v^K + \underline{u}_N - \mathbb{E} [\max\{r, \mathbb{E}[v_i | v_i < v^K] + u_i\}]) - (1 - p^K) (v^K + \underline{u}_N) > 0. \end{aligned}$$

Since $[\max\{r, \mathbb{E}[v_i | v_i < v^K] + u_i\}] \leq r < v^K + \underline{u}_N$, the first term is strictly positive, and for N large enough the inequality must hold. The optimal reserve price must then lie weakly above $v^K + \underline{u}_N$.

Step 2. We now show that revenue under the optimal reserve price is lower than under the above randomized access. Under the optimal reserve price, expected revenue equals

$$\begin{aligned} & r^* N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1} \\ & + \left[1 - (1 - p^K \Pr(u_i \geq u^*))^N - N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1}\right] \\ & \quad \times \mathbb{E}[\nu_{(2)} | \nu_{(2)} \geq r^*]. \end{aligned}$$

The first term arises whenever the reserve price binds, which is the case whenever only one buyer has a valuation greater than $r^* = v^K + u^*$. The second term captures expected revenue when the reserve price does not bind, that is when two or more buyers have valuations $\nu_i = v_i + u_i \geq v^K + u^*$.

Randomizing access yields revenue $\mathbb{E}[\nu_{(2)}] - \varepsilon$. This is greater whenever

$$\begin{aligned} & \left[(1 - p^K \Pr(u_i \geq u^*))^N + N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1}\right] \mathbb{E}[\nu_{(2)} | \nu_{(2)} \leq r^*] \\ & - r^* N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1} - \varepsilon > 0. \end{aligned}$$

This rewrites as

$$(1 - p^K \Pr(u_i \geq u^*))^N \mathbb{E} [\nu_{(2)} \mid \nu_{(1)} \leq r^*] - \varepsilon \\ - N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1} (r^* - \mathbb{E}[\nu_{(2)} \mid \nu_{(2)} \leq r^*, \nu_{(1)} > r^*]) > 0.$$

In words, the gain from randomizing access comes from the fact that buyers fully learn their valuations, and that whenever none of them has a value above the reserve price, then the seller gets a revenue equal to the second-highest value (first line). The loss comes from states of the world in which only one buyer has a value above the reserve price (so only one buyer has $\nu_i \geq v^K + u^*$), in which case the reserve price yields a revenue of $r^* = v^K + u^*$ while the randomization into entry yields a revenue of $\mathbb{E}[\nu_{2:N} \mid \nu_{2:N} \leq v^K + u^*, \nu_{1:N} > v^K + u^*] - \varepsilon = \mathbb{E}[\nu_{1:N-1} \mid \nu_{1:N-1} \leq v^K + u^*] - \varepsilon$ (second line). However this loss goes to zero with N at a faster rate than the gains, as the expected highest-value below the reserve price converges to the reserve price quickly. In particular, with probability $1 - [(1 - p^K)/(1 - p^K \Pr(u_i \geq u^*))]^{N-1}$, the second-highest value is $\nu_{2:n} = v^K + u_i$ for some $u_i < u^*$. Hence the condition rewrites as

$$(1 - p^K \Pr(u_i \geq u^*))^N \mathbb{E} [\nu_{(2)} \mid \nu_{(1)} \leq r^*] - \varepsilon \\ - N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1} \left[1 - \left(\frac{1 - p^K}{1 - p^K \Pr(u_i \geq u^*)} \right)^{N-1} \right] \\ \times (u^* - \mathbb{E}[u_{1:M} \mid M = |\{i : v_i = v^K, u_i < u^*\}|]) \\ - N (p^K \Pr(u_i \geq u^*)) (1 - p^K \Pr(u_i \geq u^*))^{N-1} \left(\frac{1 - p^K}{1 - p^K \Pr(u_i \geq u^*)} \right)^{N-1} \\ \times (v^K + u^* - \mathbb{E}[\nu_{1:N-1} \mid \nu_{1:N-1} \leq v^K + \underline{u}_N]) > 0.$$

Dividing everything by $(1 - p^K \Pr(u_i \geq u^*))^{N-1}$ yields

$$(1 - p^K \Pr(u_i \geq u^*)) \mathbb{E} [\nu_{(2)} \mid \nu_{(1)} \leq r^*] - \varepsilon \\ - N (p^K \Pr(u_i \geq u^*)) \left[1 - \left(\frac{1 - p^K}{1 - p^K \Pr(u_i \geq u^*)} \right)^{N-1} \right] \\ \times (u^* - \mathbb{E}[u_{1:M} \mid M = |\{i : v_i = v^K, u_i < u^*\}|])$$

$$\begin{aligned}
& - N (p^K \Pr(u_i \geq u^*)) \left(\frac{1 - p^K}{1 - p^K \Pr(u_i \geq u^*)} \right)^{N-1} \\
& \quad \times (v^K + u^* - \mathbb{E}[\nu_{1:N-1} \mid \nu_{1:N-1} \leq v^K + \underline{u}_N]) > 0.
\end{aligned}$$

For ε small enough, the first term is strictly positive and remains bounded away from zero as N grows large. The last term goes to zero as N grows large. So does the second term if the noise terms are sufficiently small given N since it scales with $u^* - \underline{u}_N \leq \bar{u}_N - \underline{u}_N$. Hence there exists \bar{N} such that, for all $N \geq \bar{N}$, the claim holds: the above randomization intro entry outperforms an auction where all bids are considered but with an optimal reserve price. \square

B.5 Proof of Results in Section 6

Proof of Proposition 6. We show that there cannot exist an equilibrium in which buyers first choose to learn about their own values and then about others.

By contradiction, suppose such an equilibrium exists, and let Π^{self} be an information partition about self that has non-vanishing probability in equilibrium. We first argue that, for sufficiently small information costs λ , the signal about self Π^{self} that buyers acquire must fully reveal v_i . Suppose not, such that Π^{self} bundles two possible values (i.e., there exists $\pi^{self} \in \Pi^{self}$ such that $|\pi^{self}| > 1$) and let \underline{v} and \bar{v} be the smallest and highest values in π^{self} , respectively. In states of the world where $v_j \in \pi^{self}$ for all buyers j , all buyers are at the same information set π^{self} with non-vanishing probability. Following a similar argument as in the proof of Lemma 3, they then tie for the good with non-vanishing probability, and must be indifferent between losing and winning at their equilibrium bid. Their equilibrium bid then lies strictly in between $\underline{v} + u_i$ and $\bar{v} + u_i$, and given that they face such bid with non-vanishing probability, they have a strict incentive to learn to distinguish value $v_i = \underline{v}$ from value $v_i = \bar{v}$.

Therefore a buyer learns his value fully $\Pi^{self} = \{\{v_i\}_{v_i \in V}\}$. We however know from Proposition 3 that such an equilibrium, in which buyers converge to becoming fully informed of their valuations, cannot exist. \square