Aggregate Dynamics in Mirrlees Economies:
The Case of Persistent Shocks

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Abstract: I consider a neoclassical growth model with endogenous labor supply in which agents have private information about their idiosyncratic value of leisure. A key assumption is that these shocks follow a persistent stochastic process over time. For this economy I solve the problem of a social planner that seeks to maximize the welfare of agents, subject to incentive compatibility, promise-keeping, threat-keeping, and aggregate feasibility constraints. When preferences over consumption and leisure are logarithmic, I obtain a strong analytical result: All macroeconomic variables are exactly the same under private and full information. However, when the stochastic shocks follow a stochastic process that closely resembles a random walk and there is a constant Frisch elasticity of labor supply I find large quantitative effects of the information frictions in a calibrated version of the model: output, investment, consumption, capital, and labor are all 9.5% lower in the steady-state of the private information economy compared to the full information case.

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1 Introduction

This paper addresses an old question in macroeconomics: What are the implications of imperfect insurance for aggregate dynamics? A common approach in the literature has been to impose exogenous forms of market incompleteness, such as restricting the set of securities available and imposing exogenous borrowing constraints (e.g. Krusell et al. (1998)). In this paper I follow a recent literature that takes a more primitive approach: the restrictions to risk sharing are assumed to arise only because of private information. Conditional on the information structure of the economy, contractual arrangements are optimal.

The model considered is a Mirrleesean neoclassical growth model with endogenous labor supply and perpetual youth. Agents receive idiosyncratic shocks to their value of leisure, which are private information. A key feature of the model is that these shocks are persistent over time. For this economy I solve the problem of a social planner that faces a non-trivial trade-off between providing insurance and incentives to the agents. Importantly, the social planner also faces the aggregate feasibility constraints of the economy.

The main purpose of this paper is to compare the optimal aggregate allocation of this economy with that of exactly the same economy but with full information. In order to gain insight about what factors will determine their differences I derive two equations that represent necessary optimality conditions. These equations, which I term the intratemporal and the intertemporal cross-sectional inverse Euler conditions, hold both under private and full information. Given these equations it is then straightforward to show a striking irrelevance result: When the utility functions for consumption and leisure are both logarithmic, the aggregate allocations for the private and full information economies are exactly the same. The reason is that when the utility functions are logarithmic, the cross-sectional inverse Euler conditions become linear with respect to the individual hours worked and the consumption levels of agents. As a result, they become relations between aggregate consumption and aggregate hours worked.[2] Since these equations hold both for the private and full information economies, this imply that the aggregate allocations of both economies must be the same.

While the cross-sectional inverse Euler conditions deliver an interesting benchmark case in which the information frictions are completely irrelevant for aggregate dynamics, they also provide important insights about what is likely to break the irrelevance result. A simple inspection of the cross-sectional inverse Euler equations indicates that what will preclude them from delivering a

[2] In fact, they become the intratemporal and intertemporal Euler equations of a representative agent economy.

relation between aggregate variables is a Jensen's inequality correction term. For a given amount of heterogeneity, the curvature of the utility functions will affect the correction term. In addition, for given utility functions (other than logarithmic), the amount of heterogeneity will also affect it.

The key question to address is then: To what extent will the information frictions affect aggregate dynamics when preferences and idiosyncratic uncertainty are assumed to take realistic forms? To this end, I make the following assumptions for the rest of the paper. Because they are consistent with a balanced growth path, I retain the assumption of logarithmic preferences with respect to consumption. However, for labor supply I assume preferences with a constant Frisch elasticity. In turn, for the idiosyncratic shocks I assume a stochastic process that approximates a random walk with Pareto innovations.

Under these preferences and idiosyncratic shocks analytical results are no longer available and I must resort to numerical simulations. Since computations are complex I simplify the analysis by focusing on steady-state solutions and by using the first-order approach introduced by Kapicka (2013) ansd Pavan et al. (2014). Calibrating the Frisch elasticity of labor supply to micro estimates, and the variance of the idiosyncratic shocks innovations to reproduce their empirical counterpart for wages, I then compare the steady-state of the model under private information with that of the model under full information. I find large effects of the information frictions on aggregate steady-state dynamics: output, consumption, investment and hours worked are all 9.5% lower in the economy with private information. Thus, considering realistic preferences and idiosyncratic shocks goes a long way in turning around the irrelevance result obtained under logarithmic preferences.

There is a long literature introducing information frictions into macro models. Examples include Atkeson and Lucas (1992), da Costa and Luz (2018), Green (1987), Phelan (1994), Scheuer (2013), and Werning (2007) (see Veracierto (2021) for a review). The most closely related papers to this one are Farhi and Werning (2012), Kapicka (2013) and Veracierto (2021). Farhi and Werning (2012) analyze the transitionary dynamics of a Mirrleesean neoclassical growth model with persistent idiosyncratic shocks. While the social planner is allowed to optimize with respect to the consumption allocations, the labor allocations are taken to be beyond the planner's control. For this economy, Farhi and Werning (2012) show that when the utility of consumption is logarithmic, that all aggregate variables are exactly the same as in the representative agent economy under full information. That is, they show that the information frictions are irrelevant for aggregate dynamics. The current paper extends the irrelevance result in Farhi and Werning (2012) to the case in which not only consumption allocations are optimal, but in which labor supply decisions are optimally chosen as well. Also, while Farhi and Werning (2012) only consider the case of logarithmic preferences, this paper provides numerical results for more realistic preferences.

This paper is also closely related to Kapicka (2013). Since I use exactly the same information structure, preferences and idiosyncratic shock process, it can be shown that the solution to the economy-wide social planner problem in this paper can be decomposed into a series of principal-agent planning problems that are identical to the one considered in that paper. In these principal-agent problems the economy-wide shadow prices for labor and consumption are taken as given. These shadow prices turn out to correspond to the economy-wide solution if the aggregate feasibility constraints of the economy are satisfied. While this aggregation step is missing in Kapicka (2013), it constitutes the main focus of this paper: It allows me to explore the consequences of the information frictions for aggregate dynamics.

Finally this paper is closely related to Veracierto (2021). In that paper I also considered a Mirrleesean neoclassical growth model with idiosyncratic shocks to the value of leisure. The main difference is that while the idiosyncratic shocks were i.i.d. in that paper, here they are persistent. Thus, the irrelevance result under logarithmic preferences presented in this paper can be considered as extending the similar result in Veracierto (2021) to the case of persistent shocks. An important difference between both papers is that when a constant Frisch elasticity of labor supply was considered in Veracierto (2021) the irrelevance result of private information for aggregate dynamics was approximately obtained, while in this paper that result is dramatically changed. The reason why both papers obtain such different results under a constant Frisch elasticity of labor supply is that the i.i.d. shocks generate very little cross-sectional heterogeneity, while the random-walk shocks in this paper generate a large amount. As a consequence, while the Jensen's correction terms (which preclude the cross-sectional inverse Euler equation to aggregate into standard Euler equations) are negligible in Veracierto (2021), they become quite significant in this paper.

The paper is organized as follows. Section 2 uses a simple static economy to provide intuition for the irrelevance result under logarithmic preferences. Section 3 presents the full-blown infinite horizon model and derives the corresponding irrelevance result. Section 4 explains how the first-order approach developed by Kapicka (2013) can be applied to the economy-wide social planning problem when preferences are different from logarithmic, and presents the corresponding quantitative results. Finally, Section 5 concludes the paper.

2 A simple model

In this section I develop intuition for the main analytical results in the paper by considering a two-periods model. This is the simplest framework to study that allows for persistent shocks. The economy is populated by a measure one of agents who value consumption and leisure, and who are subject to idiosyncratic shocks to their value of leisure. These idiosyncratic shocks take values in a finite set. Hereon, the realized value at date t will be denoted by θ_t and the history of shocks between period 0 and 1 will be denoted by $\theta^1 = (\theta_0, \theta_1)$. The initial distribution over θ_0 is denoted by μ_0. Realizations of θ_1, conditional on θ_0, are determined by a transition matrix Q. Thus, the probability distribution over histories θ^1 is given by $\mu_1\left(\theta^1\right) = \mu_0\left(\theta_0\right) Q\left(\theta_0, \theta_1\right)$. Of major interest in this paper is the case in which the realizations of the value of leisure θ_t are private information of the individuals.

Agents' preferences are described by

$$\sum_{t=0}^{1} \sum_{\theta^t} \beta^t \left[u\left(c_t\left(\theta^t\right)\right) + \theta_t n\left(1 - h_t\left(\theta^t\right)\right)\right] \mu_t\left(\theta^t\right),$$

where c_t and h_t are consumption and hours worked, respectively. The utility functions u and n are assumed to satisfy the usual regularity conditions, and $0 < \beta < 1$.

The consumption good is produced using the following production function:

$$Y_t = F\left(K_t, H_t\right), \tag{2.1}$$

where K_t is capital and H_t is aggregate hours worked. Capital depreciates at the rate δ and K_0 is given.

For the rest of the paper it will be useful to follow the literature in specifying the social planner's problem in terms of directly choosing utilities of consumption u_t and of leisure n_t. By the Revelation Principle it suffices to specify u_t and n_t as depending on the history of reports θ^t made by the individuals. The social planner's problem is then given by

$$\max \left\{ \sum_{t=0}^{1} \sum_{\theta^t} \beta^t \left[u_t\left(\theta^t\right) + \theta_t n_t\left(\theta^t\right)\right] \mu_t\left(\theta^t\right) \right\}, \tag{2.2}$$

subject to

$$C_t + K_{t+1} - (1-\delta) K_t \leq F\left(K_t, H_t\right), \tag{2.3}$$

$$C_t = \sum_{\theta^t} c\left(u_t\left(\theta^t\right)\right) \mu_t\left(\theta^t\right), \tag{2.4}$$

$$H_t = \sum_{\theta^t} h\left(n_t\left(\theta^t\right)\right)\mu_t\left(\theta^t\right), \tag{2.5}$$

$$\begin{aligned} & u_0\left(\theta_0\right) + \theta_0 n_0\left(\theta_0\right) + \beta\sum_{\theta_1}\left[u_1\left(\theta_0,\theta_1\right) + \theta_1 n_1\left(\theta_0,\theta_1\right)\right]Q\left(\theta_0,\theta_1\right) \\ \geq\ & u_0\left(\theta_0^c\right) + \theta_0 n_0\left(\theta_0^c\right) + \beta\sum_{\theta_1}\left[u_1\left(\theta_0^c,\theta_1\right) + \theta_1 n_1\left(\theta_0^c,\theta_1\right)\right]Q\left(\theta_0,\theta_1\right), \end{aligned} \tag{2.6}$$

$$u_1\left(\theta_0,\theta_1\right) + \theta_1 n_1\left(\theta_0,\theta_1\right) \geq u_1\left(\theta_0,\theta_1^c\right) + \theta_1 n_1\left(\theta_0,\theta_1^c\right), \tag{2.7}$$

where $c\left(u_t\right) = u^{-1}\left(u_t\right)$ and $h\left(n_t\right) = 1 - n^{-1}\left(n_t\right)$ are the consumption and hours worked implied by the utility levels u_t and n_t, respectively, equation (2.6) holds for every θ_0 and θ_0^c, and equation (2.7) holds for every θ_0, θ_1 and θ_1^c.[3] Equation (2.3) is the aggregate feasibility constraint. Equation (2.4) defines aggregate consumption in terms of the utility levels u_t. Equation (2.5) is similar to equation (2.4) but for aggregate hours worked. Equation (2.6) is the incentive compatibility constraint at $t = 0$. It states that truthfully reporting the realized value θ_0 weakly dominates making any other report θ_0^c. Observe that the true value of leisure θ_0 is used even when deviating from truth-telling (the reports affect the allocations received but not the preferences). Also, observe that the conditional expectations in the continuation values are always taken with respect to the true transition probabilities, independently of the reported type. Equation (2.7) is the incentive compatibility constraint at $t = 1$. It is similar to equation (2.6) except that, being $t = 1$ the last period, there are no continuation values. The incentive compatibility constraints (2.6) and (2.7) only consider one time deviations because it can be shown that if an agent has lied in the past it always dominates to tell the truth in the future (see Fernandes and Phelan (2000)).[4]

Under full information the social planner is restricted to equations (2.2)-(2.5). That is, the incentive compatibility constraints (2.6)-(2.7) are dropped from the problem.[5]

2.1 The cross-sectional inverse Euler conditions

In this subsection I derive necessary conditions to the social planner problem that will play a central role in determining the importance of private information for aggregate dynamics.

[3] θ_t^c can be interpreted as representing an element of the set $\{\theta_t\}^c$.

[4] That is, there is no need to consider deviations from truth-telling both at $t = 0$ and $t = 1$.

[5] Observe that any solution to the social planner's problem, either under private or public information, will have $K_2 = 0$ since $t = 1$ is the last period of the economy.

2.1.1 The intratemporal condition

I first show that if $\{u_t, n_t, K_{t+1}\}_{t=0}^1$ is a solution to the social planner's problem (2.2)-(2.7), then it satisfies the following cross-sectional intratemporal inverse Euler condition:

$$F_H(K_t, H_t) \sum_{\theta^t} \frac{1}{n'(1-h(n_t(\theta^t)))} \mu_t(\theta^t) = \bar{\theta}_t \sum_{\theta^t} \frac{1}{u'(c(u_t(\theta^t)))} \mu_t(\theta^t), \tag{2.8}$$

for every t, where

$$\bar{\theta}_t = \sum_{\theta^t} \theta_t \mu_t(\theta^t) \tag{2.9}$$

is the average value of θ_t.

To see this, suppose that the left-hand side of equation (2.8) is strictly larger than the right-hand side at some $t = t^*$. Consider an alternative allocation $\{\hat{u}_t, \hat{n}_t, K_{t+1}\}_{t=0}^1$, where $\{\hat{u}_t, \hat{n}_t\}_{t=0}^1$ is identical to $\{u_t, n_t\}_{t=0}^1$ except at t^*, which is given by

$$\begin{aligned} \hat{u}_{t^*}\left(\theta^{t^*}\right) &= u_{t^*}\left(\theta^{t^*}\right) + \bar{\theta}_{t^*}\varepsilon, \\ \hat{n}_t\left(\theta^{t^*}\right) &= n_{t^*}\left(\theta^{t^*}\right) - \varepsilon, \end{aligned}$$

for every θ^{t^*}, where ε is a small positive number.[6] Observe that the objective function (2.2) attains the same value under $\{\hat{u}_t, \hat{n}_t, K_{t+1}\}_{t=0}^1$ as under $\{u_t, n_t, K_{t+1}\}_{t=0}^1$. Also, observe that the incentive compatibility constraints (2.6)-(2.7) can be written as

$$\begin{aligned} &0 \le (u_0(\theta_0) - u_0(\theta_0^c)) + \theta_0 (n_0(\theta_0) - n_0(\theta_0^c)) \\ &+\beta \sum_{\theta_1} \left[(u_1(\theta_0,\theta_1) - u_1(\theta_0^c,\theta_1)) + \theta_1 (n_1(\theta_0,\theta_1) - n_1(\theta_0^c,\theta_1))\right] Q(\theta_0,\theta_1), \end{aligned} \tag{2.10}$$

$$0 \le (u_1(\theta_0,\theta_1) - u_1(\theta_0,\theta_1^c)) + \theta_1 (n_1(\theta_0,\theta_1) - n_1(\theta_0,\theta_1^c)). \tag{2.11}$$

Since $\{\hat{u}_t, \hat{n}_t, K_{t+1}\}_{t=0}^1$ preserves all the same differences in equations (2.10)-(2.11) as $\{u_t, n_t, K_{t+1}\}_{t=0}^1$, it is incentive compatible.

Next, I verify that $\{\hat{u}_t, \hat{n}_t, K_{t+1}\}_{t=0}^1$ satisfies the aggregate feasibility condition (2.3) at $t = t^*$

[6] Hereon, $\hat{C}_t$ and $\hat{H}_t$ will denote the aggregate consumption and hours worked that satisfy equations (2.4) and (2.5) under $\{\hat{u}_t, \hat{n}_t\}_{t=0}^1$, respectively.

(which is the only date at which the allocations are affected by the ε deviations). Observe that

$$\begin{aligned}
& F\left(K_{t^*}, \hat{H}_{t^*}\right) - \hat{C}_{t^*} - K_{t^*+1} + (1-\delta) K_{t^*} \\
\simeq\; & F\left(K_{t^*}, H_{t^*}\right) + F_H\left(K_{t^*}, H_{t^*}\right) \sum_{\theta^{t^*}} \frac{1}{n'\left(1 - h\left(n_{t^*}\left(\theta^{t^*}\right)\right)\right)} \mu_{t^*}\left(\theta^{t^*}\right) \varepsilon \\
& - \left(C_t + \sum_{\theta^{t^*}} \frac{1}{u'\left(c\left(u_{t^*}\left(\theta^{t^*}\right)\right)\right)} \mu_{t^*}\left(\theta^{t^*}\right) \bar{\theta}_{t^*} \varepsilon\right) - K_{t^*+1} + (1-\delta) K_{t^*} \\
>\; & F\left(K_{t^*}, H_{t^*}\right) - C_t - K_{t^*+1} + (1-\delta) K_{t^*}
\end{aligned}$$

where the approximate equality in the first line becomes arbitrarily close to an equality when ε is sufficiently small, and the strict inequality follows from the assumption that the left-hand side of equation (2.8) is strictly larger than the right-hand side. Since $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$ satisfies equation (2.3) at t^* it follows that $\{\hat{u}_t, \hat{n}_t, K_{t+1}\}_{t=0}^{1}$ satisfies it with strict inequality. This surplus of consumption goods at t^* can then be used to increase $\hat{u}_{t^*}\left(\theta^{t^*}\right)$ by an additional η (sufficiently small amount) at every θ^{t^*}. The resulting allocation satisfies the aggregate feasibility constraint (2.3) (provided that η is sufficiently small) and the incentive compatibility constraints (2.10)-(2.11) (since $\hat{u}_{t^*}\left(\theta^{t^*}\right)$ increases a uniform additional amount across all θ^{t^*}). Moreover, the objective function (2.2) evaluates at a strictly larger number (since η is positive) than $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$. This contradicts the assumption that $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$ solves the social planner's problem (2.2)-(2.7).

2.1.2 The intertemporal condition

I now show that if $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$ is a solution to the social planner's problem (2.2)-(2.7), then it satisfies the following cross-sectional intertemporal inverse Euler condition:

$$\sum_{\theta^1} \frac{1}{u'\left(c\left(u_1\left(\theta^1\right)\right)\right)} \mu_1\left(\theta^1\right) = \beta\left(F_K\left(K_1, H_1\right) + 1 - \delta\right) \sum_{\theta^0} \frac{1}{u'\left(c\left(u_0\left(\theta^0\right)\right)\right)} \mu_0\left(\theta^0\right). \tag{2.12}$$

To see this, suppose that the right-hand side of equation (2.12) is strictly larger than the left-hand side. Consider an alternative allocation $\left\{\hat{u}_t, n_t, \hat{K}_{t+1}\right\}_{t=0}^{1}$ given by

$$\begin{aligned}
\hat{u}_0\left(\theta^0\right) &= u_0\left(\theta^0\right) - \beta\varepsilon \\
\hat{u}_1\left(\theta^1\right) &= u_1\left(\theta^1\right) + \varepsilon
\end{aligned}$$

for every θ^0 and θ^1, where ε is a small positive number, and $\hat{K}_t$ identical to K_t except at $t = 1$, which is given by

$$\hat{K}_1 = K_1 + C_0 - \hat{C}_0. \tag{2.13}$$

Observe that the objective function (2.2) attains the same value under $\left\{\hat{u}_t, n_t, \hat{K}_{t+1}\right\}_{t=0}^{1}$ as under $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$. Also, since $\left\{\hat{u}_t, n_t, \hat{K}_{t+1}\right\}_{t=0}^{1}$ preserves all the same differences in equations (2.10)-(2.11) as $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$, it is incentive compatible.

Next, I verify that $\left\{\hat{u}_t, n_t, \hat{K}_{t+1}\right\}_{t=0}^{1}$ satisfies the aggregate feasibility condition (2.3) at both time periods. Observe that equation (2.13) leaves the sum of aggregate consumption and investment unchanged at $t = 0$. Thus, equation (2.3) holds at $t = 0$. Also, observe that

$$C_0 - \hat{C}_0 \simeq \sum_{\theta^0} \frac{1}{u'\left(c\left(u_0\left(\theta^0\right)\right)\right)} \mu_0\left(\theta^0\right) \beta\varepsilon.$$

Then,

$$\begin{aligned}
& F\left(\hat{K}_1, H_1\right) - \hat{C}_1 - \hat{K}_2 + (1-\delta)\hat{K}_1 \\
\simeq\ & F\left(K_1, H_1\right) + F_K\left(K_1, H_1\right) \sum_{\theta^0} \frac{1}{u'\left(c_0\left(\theta^0\right)\right)} \mu_0\left(\theta^0\right) \beta\varepsilon - C_1 - \sum_{\theta^1} \frac{1}{u'\left(c\left(u_1\left(\theta^1\right)\right)\right)} \mu_1\left(\theta^1\right) \varepsilon \\
& -K_2 + (1-\delta) K_1 + (1-\delta) \sum_{\theta^0} \frac{1}{u'\left(c_0\left(\theta^0\right)\right)} \mu_0\left(\theta^0\right) \beta\varepsilon \\
>\ & F\left(K_1, H_1\right) - C_1 - K_2 + (1-\delta) K_1
\end{aligned}$$

where the approximate equality becomes arbitrarily close to an equality when ε is sufficiently small, and the strict inequality follows from the assumption that the right-hand side of equation (2.12) is larger than the left-hand side. Since $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$ satisfies equation (2.3) at $t = 1$, it follows that $\left\{\hat{u}_t, n_t, \hat{K}_{t+1}\right\}_{t=0}^{1}$ satisfies it with strict inequality. This surplus of consumption goods at $t = 1$ can then be used to increase $\hat{u}_1\left(\theta^1\right)$ by an additional η (sufficiently small amount) at every θ^1. The resulting allocation satisfies the aggregate feasibility constraint (2.3) (provided that η is sufficiently small) and the incentive compatibility constraints (2.10)-(2.11) (since $\hat{u}_1\left(\theta^1\right)$ increases a uniform additional amount across all θ^1). Moreover, the objective function (2.2) evaluates at a strictly larger number (since η is positive) than $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$. This contradicts the assumption that $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$ solves the social planner's problem (2.2)-(2.7).

2.2 An irrelevance result

Observe that if $\{u_t, n_t, K_{t+1}\}_{t=0}^{1}$ is a solution to the social planner's problem (2.2)-(2.5) under full information, then it must also satisfy the cross-sectional intratemporal and intertemporal inverse Euler conditions (2.8) and (2.12). Otherwise, we could consider exactly the same deviations as above to reach exactly the same contradictions. The only difference is that now there would be no need to verify that the incentive compatibility constraints (2.10)-(2.11) hold under the proposed deviations, since there are no such constraints under full information.

A key question in this paper is what are the effects of private information on the evolution of the macroeconomic variables $\{C_t, H_t, K_{t+1}\}_{t=0}^{1}$? Are these macroeconomic variables the same under private information as under full information? In this subsection I provide an interesting benchmark case in which these macroeconomic variables are exactly the same in the private and full information economies. This is the case in which the utility functions u and n both take logarithmic forms.

To see this observe that when u and n are both logarithmic, equation (2.8) becomes

$$(1 - H_t) F_H (K_t, H_t) = \bar{\theta}_t C_t, \tag{2.14}$$

for every t.

Also, when u is logarithmic equation (2.12) becomes

$$C_{t+1} = \beta (F_K (K_{t+1}, H_{t+1}) + 1 - \delta) C_t, \tag{2.15}$$

for $t = 0$.

Since the aggregate feasibility constraint (2.3) as well as equations (2.14)-(2.15) must be satisfied both in the private and full information economies, we conclude that $\{C_t, H_t, K_{t+1}\}_{t=0}^{1}$ are exactly the same in both economies. That is, under logarithmic preferences the information frictions are completely irrelevant for macroeconomic dynamics.

Observe that the solution $\{C_t, H_t, K_{t+1}\}_{t=0}^{1}$ in both economies coincides with that of a representative agent economy with preferences given by.

$$\sum_{t=0}^{1} \beta^t \left[\ln C_t + \bar{\theta}_t \ln (1 - H_t)\right],$$

where $\bar{\theta}_t$ satisfies equation (2.9).

3 An infinite horizon model

In this section I extend the results from the previous section to an infinite horizon perpetual youth model. Following Phelan (1994), I introduce stochastic lifetimes in order to generate a stationary distribution of agents across promised and threat values, which will play an important role in the recursive formulation of the following section.

The economy is always populated by a measure one of agents. With probability $1 - \sigma$ an agent dies from one period to the next and is replaced by a newborn. Agents do not care about the utility levels of their descendants. The preferences of an agent born at date a are given by

$$\sum_{t=a}^{\infty}\sum_{\theta^{t-a}}(\sigma\beta)^{t-a}\left[u\left(c_t^a\left(\theta^{t-a}\right)\right)+\theta_{t-a}n\left(1-h_t^a\left(\theta^{t-a}\right)\right)\right]\mu_{t-a}\left(\theta^{t-a}\right),$$

where c_t^a and h_t^a are consumption and hours worked, respectively. The initial distribution over θ_0 is denoted by μ_0. Realizations of θ_{t-a}, conditional on θ_{t-a-1}, are determined by a transition matrix Q. Thus, $\mu_{t-a}\left(\theta^{t-a}\right)=\mu_0\left(\theta_0\right)Q\left(\theta_0,\theta_1\right)...Q(\theta_{t-a-1},\theta_{t-a})$.

Similarly to the static economy, the consumption good is produced using the production function in equation (2.1). Also, capital depreciates at the rate δ and K_0 is given.

The social planner chooses utilities of consumption u_t^a and leisure n_t^a as a function of the history of reports θ^{t-a} made by the individuals. Their problem is the following:

$$\max\left\{\sum_{t=0}^{\infty}\sum_{\theta^t}(\sigma\beta)^t\left[u_t^0\left(\theta^t\right)+\theta_t n_t^0\left(\theta^t\right)\right]\mu_t\left(\theta^t\right)\right.$$

$$\left.+\sum_{a=1}^{\infty}\pi^a(1-\sigma)\sum_{t=a}^{\infty}\sum_{\theta^{t-a}}(\sigma\beta)^{t-a}\left[u_t^a\left(\theta^{t-a}\right)+\theta_{t-a}n_t^a\left(\theta^{t-a}\right)\right]\mu_{t-a}\left(\theta^{t-a}\right)\right\} \tag{3.1}$$

subject to

$$C_t+K_{t+1}-(1-\delta)K_t\leq F\left(K_t,H_t\right), \tag{3.2}$$

$$C_t=\sigma^t\sum_{\theta^t}c\left(u_t^0\left(\theta^t\right)\right)\mu_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\sigma^{t-a}\sum_{\theta^{t-a}}c\left(u_t^a\left(\theta^{t-a}\right)\right)\mu_{t-a}\left(\theta^{t-a}\right) \tag{3.3}$$

$$H_t=\sigma^t\sum_{\theta^t}h\left(n_t^0\left(\theta^t\right)\right)\mu_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\sigma^{t-a}\sum_{\theta^{t-a}}h\left(n_t^a\left(\theta^{t-a}\right)\right)\mu_{t-a}\left(\theta^{t-a}\right) \tag{3.4}$$

for every $t\geq 0$, and

$$u_t^a\left(\theta^{t-a-1},\theta_{t-a}\right)+\theta_{t-a}n_t^a\left(\theta^{t-a-1},\theta_{t-a}\right)$$

$$+\sum_{j=1}^{\infty}\sum_{\theta_{t-a+1}^{t-a+j}}(\sigma\beta)^j\left[u_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a},\theta_{t-a+1}^{t-a+j}\right)+\theta_{t-a+j}n_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a},\theta_{t-a+1}^{t-a+j}\right)\right]\left(\prod_{k=1}^{j}Q\left(\theta_{t-a+k-1},\theta_{t-a+k}\right)\right)$$

$$\geq u_t^a\left(\theta^{t-a-1},\theta_{t-a}^c\right)+\theta_{t-a}n_t^a\left(\theta^{t-a-1},\theta_{t-a}^c\right)$$

$$+\sum_{j=1}^{\infty}\sum_{\theta_{t-a+1}^{t-a+j}}(\sigma\beta)^j\left[u_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a}^c,\theta_{t-a+1}^{t-a+j}\right)+\theta_{t-a+j}n_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a}^c,\theta_{t-a+1}^{t-a+j}\right)\right]\left(\prod_{k=1}^{j}Q\left(\theta_{t-a+k-1},\theta_{t-a+k}\right)\right), \tag{3.5}$$

for every $a\geq 0$, $t\geq a$, θ^{t-a-1}, θ_{t-a} and θ_{t-a}^c , and K_0 given. Observe that the objective function (3.1) is the weighted lifetime utility of all agents (current and future), where agents of generation a receive a Pareto weight π^a, with $0<\pi<1$. Equation (3.2) is the aggregate feasibility constraint at date t. Equation (3.3) defines aggregate consumption at date t. The first term is the total consumption of the measure one of agents initially alive at date 0, of which a fraction σ^t survives to date t. The second term sums the total consumption of the measure $1-\sigma$ of agents born at each other date $1 \leq a \leq t$, of which a fraction σ^{t-a} survives to date t. Equation (3.4) is similar to equation (3.3) but for aggregate hours worked. Equation (3.5) is the incentive compatibility constraint at date t for agents of generation a, after the history of reports θ^{t-a-1}.[7] It states that truthfully reporting θ_{t-a} weakly dominates making any other report θ^c_{t-a}. Similarly to the static case, the true value of leisure θ_{t-a} is used to evaluate deviations from truth-telling, the conditional expectations are always taken with respect to the true transition probabilities, and only one-time deviations from truth-telling need to be considered. Also observe that, similarly to the static case, the social planner's problem under full information reduces to (3.1)-(3.4).

In order to simplify the arguments that follow, I hereon assume that $\pi = \beta$. That is, that the social planner discounts future utilities at the same rate as individual agents.

3.1 The cross-sectional inverse Euler conditions

In this subsection I derive the necessary cross-sectional inverse Euler conditions for this infinite horizon economy. Because of its perpetual youth structure the deviations considered will be somewhat more complex that in the simple economy of Section 2. However, the arguments will parallel those already used.

3.1.1 The intratemporal condition

I first show that if $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ is a solution to the social planner's problem (3.1)-(3.5), then it satisfies the following condition:

$$F_H(K_t,H_t)\left(\sigma^t\sum_{\theta^t}\frac{1}{n'\left(1-h\left(n^0_t\left(\theta^t\right)\right)\right)}\mu_t\left(\theta^t\right)+\sum_{a=1}^t(1-\sigma)\,\sigma^{t-a}\sum_{\theta^{t-a}}\frac{1}{n'\left(1-h\left(n^a_t\left(\theta^{t-a}\right)\right)\right)}\mu_{t-a}\left(\theta^{t-a}\right)\right)$$

$$=\bar{\theta}_t\left(\sigma^t\sum_{\theta^t}\frac{1}{u'\left(c\left(u^0_t\left(\theta^t\right)\right)\right)}\mu_t\left(\theta^t\right)+\sum_{a=1}^t(1-\sigma)\,\sigma^{t-a}\sum_{\theta^{t-a}}\frac{1}{u'\left(c\left(u^a_t\left(\theta^{t-a}\right)\right)\right)}\mu_{t-a}\left(\theta^{t-a}\right)\right), \quad (3.6)$$

for every t, where

$$\bar{\theta}_t=\sigma^t\sum_{\theta^t}\theta_t\mu_t\left(\theta^t\right)+\sum_{a=1}^t(1-\sigma)\,\sigma^{t-a}\sum_{\theta^{t-a}}\theta_{t-a}\mu_{t-a}\left(\theta^{t-a}\right) \quad (3.7)$$

is the average value of θ at date t.

[7] Observe that $\theta^{t-a+j}_{t-a+1} = (\theta_{t-a+1}, \theta_{t-a+2}, ..., \theta_{t-a+j})$ denotes a history of shocks between ages $t-a+1$ and $t-a+j$ for an agent born at date $a \leq t$.

To see this, suppose that the left-hand side of equation (3.6) is strictly larger than the right-hand side at some $t = t^*$. Consider an alternative allocation $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, where $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t\right\}_{t=0}^\infty$ is identical to $\left\{\{u_t^a, n_t^a\}_{a=0}^t\right\}_{t=0}^\infty$ except at t^*, which is given by

$$\begin{aligned}
\hat{u}_{t^*}^a\left(\theta^{t^*-a}\right) &= u_{t^*}^a\left(\theta^{t^*-a}\right) + \bar{\theta}_{t^*}\varepsilon, \\
\hat{n}_t^a\left(\theta^{t^*-a}\right) &= n_{t^*}^a\left(\theta^{t^*-a}\right) - \varepsilon,
\end{aligned}$$

for every $0 \leq a \leq t^*$ and θ^{t^*-a}, where ε is a small positive number. Observe that in going from $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ to $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, the objective function (3.1) changes by

$$\begin{aligned}
&\sum_{\theta^{t^*}} (\sigma\beta)^{t^*}\left[\bar{\theta}_{t^*}\varepsilon - \theta_{t^*}\varepsilon\right]\mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\sum_{\theta^{t^*-a}}\sigma^{t^*-a}\beta^{t^*}\left[\bar{\theta}_{t^*}\varepsilon - \theta_{t^*-a}\varepsilon\right]\mu_{t^*-a}\left(\theta^{t^*-a}\right) \\
= \ &\bar{\theta}_{t^*}\varepsilon\beta^{t^*}\left[\sum_{\theta^{t^*}}\sigma^{t^*}\mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\sum_{\theta^{t^*-a}}\sigma^{t^*-a}\mu_{t^*-a}\left(\theta^{t^*-a}\right)\right] \\
&-\varepsilon\beta^{t^*}\left[\sum_{\theta^{t^*}}\sigma^{t^*}\theta_{t^*}\mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\sum_{\theta^{t^*-a}}\sigma^{t^*-a}\theta_{t^*-a}\mu_{t^*-a}\left(\theta^{t^*-a}\right)\right] \\
= \ &\bar{\theta}_{t^*}\varepsilon\beta^{t^*} - \varepsilon\beta^{t^*}\bar{\theta}_{t^*}.
\end{aligned} \tag{3.8}$$

That is, the objective function remains unchanged.[8]. Observe that the incentive compatibility constraint (3.5) can be written as

$$\begin{aligned}
&0 \leq \left(u_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) - u_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right)\right) + \theta_{t-a}\left(n_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) - n_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right)\right) \\
&+\sum_{j=1}^\infty\sum_{\theta_{t-a+1}^{t-a+j}}(\sigma\beta)^j\left(u_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}, \theta_{t-a+1}^{t-a+j}\right) - u_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}^c, \theta_{t-a+1}^{t-a+j}\right)\right)\left(\prod_{k=1}^j Q\left(\theta_{t-a+k-1}, \theta_{t-a+k}\right)\right) \\
&+\sum_{j=1}^\infty\sum_{\theta_{t-a+1}^{t-a+j}}(\sigma\beta)^j\theta_{t-a+j}\left(n_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}, \theta_{t-a+1}^{t-a+j}\right) - n_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}^c, \theta_{t-a+1}^{t-a+j}\right)\right)\left(\prod_{k=1}^j Q\left(\theta_{t-a+k-1}, \theta_{t-a+k}\right)\right)
\end{aligned} \tag{3.9}$$

for every $a \geq 0$, $t \geq a$, θ^{t-a-1}, θ_{t-a} and θ_{t-a}^c. Since $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, preserves all the same differences in equation (3.9) as $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, it is incentive compatible.

Next, I verify that $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ satisfies the aggregate feasibility condition (3.2) at $t = t^*$. Observe that

$$F\left(K_{t^*}, \hat{H}_{t^*}\right) - \hat{C}_{t^*} - K_{t^*+1} + (1-\delta)K_{t^*}$$

[8] Observe that the last line of equation (3.8) uses the facts that $\sigma^t + \sum_{a=1}^t (1-\sigma)\sigma^{t-a} = 1$, that μ_{t-a} is a probability distribution, and that $\bar{\theta}_{t^*}$ satisfies equation (3.7).

$$\simeq F\left(K_{t^*}, H_{t^*}\right) + F_H\left(K_{t^*}, H_{t^*}\right) \times$$

$$\left(\sigma^{t^*} \sum_{\theta^{t^*}} \frac{1}{n'\left(1 - h\left(n^0_{t^*}\left(\theta^{t^*}\right)\right)\right)} \mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*} (1-\sigma)\,\sigma^{t^*-a} \sum_{\theta^{t^*-a}} \frac{1}{n'\left(1 - h\left(n^a_{t^*}\left(\theta^{t^*-a}\right)\right)\right)} \mu_{t^*-a}\left(\theta^{t^*-a}\right)\right) \varepsilon$$

$$- \left(\sigma^{t^*} \sum_{\theta^{t^*}} \frac{1}{u'\left(c\left(u^0_{t^*}\left(\theta^{t^*}\right)\right)\right)} \mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*} (1-\sigma)\,\sigma^{t^*-a} \sum_{\theta^{t^*-a}} \frac{1}{u'\left(c\left(u^a_{t^*}\left(\theta^{t^*-a}\right)\right)\right)} \mu_{t^*-a}\left(\theta^{t^*-a}\right)\right) \bar{\theta}_{t^*} \varepsilon$$

$$-C_{t^*} - K_{t^*+1} + (1-\delta) K_{t^*}$$

$$> F\left(K_{t^*}, H_{t^*}\right) - C_t - K_{t^*+1} + (1-\delta) K_{t^*}$$

where the approximate equality in the first line becomes arbitrarily close to an equality when ε is sufficiently small, and the strict inequality follows from the assumption that the left-hand side of equation (3.6) is strictly larger than the right-hand side. Since $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ satisfies equation (3.2) at t^*, it follows that $\left\{\{\hat{u}^a_t, \hat{n}^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ satisfies it with strict inequality. This surplus of consumption goods at t^* can then be used to increase $\hat{u}^a_{t^*}\left(\theta^{t^*-a}\right)$ by an additional η (sufficiently small amount) for every $0 \leq a \leq t^*$ and θ^{t^*-a}. The resulting allocation satisfies the aggregate feasibility constraint (3.2) at t^* (provided that η is sufficiently small) and the incentive compatibility constraint (3.9) (since $\hat{u}^a_{t^*}\left(\theta^{t^*-a}\right)$ increases a uniform additional amount across all θ^{t^*-a}). Moreover, the objective function (3.1) evaluates at a strictly larger number (since η is positive) than $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$. This contradicts the assumption that $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ solves the social planner's problem (3.1)-(3.5).

3.1.2 The intertemporal condition

I now show that if $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ is a solution to the social planner's problem (3.1)-(3.5), then it satisfies the following cross-sectional intertemporal inverse Euler condition:

$$\sigma^{t+1} \sum_{\theta^{t+1}} \mu_t\left(\theta^{t+1}\right) \frac{1}{u'\left(c^0_{t+1}\left(\theta^{t+1}\right)\right)} + \sum_{a=1}^{t+1} (1-\sigma)\,\sigma^{t+1-a} \sum_{\theta^{t+1-a}} \frac{1}{u'\left(c^a_{t+1}\left(\theta^{t+1-a}\right)\right)} \mu_{t+1-a}\left(\theta^{t+1-a}\right)$$

$$= \beta\left(F_K\left(K_{t+1}, H_{t+1}\right) + 1 - \delta\right) \times$$

$$\left(\sigma^t \sum_{\theta^t} \mu_t\left(\theta^t\right) \frac{1}{u'\left(c^0_t\left(\theta^t\right)\right)} + \sum_{a=1}^{t} (1-\sigma)\,\sigma^{t-a} \sum_{\theta^{t-a}} \frac{1}{u'\left(c^a_t\left(\theta^{t-a}\right)\right)} \mu_{t-a}\left(\theta^{t-a}\right)\right) \tag{3.10}$$

for every $t \geq 0$.

To see this, suppose that the right-hand side of equation (3.10) is strictly larger than the left-hand side at some t^*. Consider an alternative allocation $\left\{\{\hat{u}^a_t, \hat{n}^a_t\}^t_{a=0}, \hat{K}_{t+1}\right\}^\infty_{t=0}$, given by $\hat{n}^a_t = n^a_t$

for every t and $0 \leq a \leq t$, $\hat{u}_t^a = u_t^a$ for every t and $0 \leq a \leq t$, except at t^* and t^*+1, which is given by

$$\hat{u}_{t^*}^a \left(\theta^{t^*-a}\right) = u_{t^*}^a \left(\theta^{t^*-a}\right) - \beta\varepsilon,$$

for every $0 \leq a \leq t^*$ and θ^{t^*-a}, and

$$\hat{u}_{t^*+1}^a \left(\theta^{t^*+1-a}\right) = u_{t^*+1}^a \left(\theta^{t^*+1-a}\right) + \varepsilon,$$

for every $0 \leq a \leq t^*+1$ and every θ^{t^*+1-a}, where ε is a small positive number, and $\hat{K}_t$ identical to K_t except at t^*+1, which is given by

$$\hat{K}_{t^*+1} = K_{t^*+1} + C_t - \hat{C}_t \tag{3.11}$$

Observe that in going from $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ to $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$, the Pareto-weighted welfare of the agents alive at date 0 changes by

$$\sum_{\theta^{t^*}} (\sigma\beta)^{t^*} (-\beta\varepsilon) \mu_t \left(\theta^t\right) + \sum_{\theta^{t^*+1}} (\sigma\beta)^{t^*+1} \varepsilon \mu_{t^*+1} \left(\theta^{t^*+1}\right), \tag{3.12}$$

the sum of Pareto-weighted welfare of the agents born between $1 \leq a \leq t^*$ changes by

$$\sum_{a=1}^{t^*} \pi^a (1-\sigma) \left(\sum_{\theta^{t^*-a}} (\sigma\beta)^{t^*-a} (-\beta\varepsilon) \mu_{t^*-a} \left(\theta^{t^*-a}\right) + \sum_{\theta^{t^*+1-a}} (\sigma\beta)^{t^*+1-a} \varepsilon \mu_{t^*+1-a} \left(\theta^{t^*+1-a}\right) \right) \tag{3.13}$$

and the Pareto-weighted welfare of the agents born at date t^*+1 changes by

$$\pi^{t^*+1} (1-\sigma) \sum_{\theta^0} \varepsilon \mu_0 \left(\theta^0\right) \tag{3.14}$$

Adding the changes in equations (3.12)-(3.14), using the assumption that $\pi = \beta$, that the μ's are probability distributions, and the fact that

$$\sigma^{t^*} + \sum_{a=1}^{t^*} (1-\sigma) \sigma^{t^*-a} = 1,$$

we get that the objective function (3.1) remains unchanged.

Also, since $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$, preserves all the same differences in equation (3.9) as $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, it is incentive compatible.

Next, I verify that $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$ satisfies the aggregate feasibility condition (3.2) at $t = t^*$ and $t = t^*+1$ (which are the only two dates at which the allocations are affected by the ε deviations). Using that $\hat{n}_{t^*}^a = n_{t^*}^a$ for every $0 \leq a \leq t^*$ and equation (3.11), which implies that aggregate investment plus aggregate consumption does not change at $t = t^*$, we have that

$\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$ satisfies the aggregate feasibility condition (3.2) at $t = t^*$ simply from the fact that $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ does.

Observe that

$$C_t - \hat{C}_t \simeq \left(\sigma^{t^*}\sum_{\theta^{t^*}}\frac{1}{u'\left(c_{t^*}^0\left(\theta^{t^*}\right)\right)}\mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\,\sigma^{t^*-a}\sum_{\theta^{t^*-a}}\frac{1}{u'\left(c_{t^*}^a\left(\theta^{t^*-a}\right)\right)}\mu_{t^*-a}\left(\theta^{t^*-a}\right)\right)\beta\varepsilon.$$

Then,

$$\begin{aligned}
&F\left(\hat{K}_{t^*+1}, H_{t^*+1}\right) - \hat{C}_{t^*+1} - \hat{K}_{t^*+2} + (1-\delta)\,\hat{K}_{t^*+1} \\
&\simeq F\left(K_{t^*+1}, H_{t^*+1}\right) + F_K\left(K_{t^*+1}, H_{t^*+1}\right) \times \\
&\left(\sigma^{t}\sum_{\theta^{t}}\frac{1}{u'\left(c_{t^*}^0\left(\theta^{t^*}\right)\right)}\mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\,\sigma^{t^*-a}\sum_{\theta^{t^*-a}}\frac{1}{u'\left(c_{t^*}^a\left(\theta^{t^*-a}\right)\right)}\mu_{t^*-a}\left(\theta^{t^*-a}\right)\right)\beta\varepsilon \\
&-\sigma^{t^*+1}\sum_{\theta^{t^*+1}}\frac{1}{u'\left(c_{t^*+1}^0\left(\theta^{t^*+1}\right)\right)}\mu_{t^*}\left(\theta^{t^*+1}\right)\varepsilon \\
&-\sum_{a=1}^{t^*+1}(1-\sigma)\,\sigma^{t^*+1-a}\sum_{\theta^{t^*+1-a}}\frac{1}{u'\left(c_{t^*+1}^a\left(\theta^{t^*+1-a}\right)\right)}\mu_{t^*+1-a}\left(\theta^{t^*+1-a}\right)\varepsilon \\
&-C_{t^*+1} - K_{t^*+2} + (1-\delta)\,K_{t^*+1} \\
&+(1-\delta)\left(\sigma^{t}\sum_{\theta^{t}}\frac{1}{u'\left(c_{t^*}^0\left(\theta^{t^*}\right)\right)}\mu_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\,\sigma^{t^*-a}\sum_{\theta^{t^*-a}}\frac{1}{u'\left(c_{t^*}^a\left(\theta^{t^*-a}\right)\right)}\mu_{t^*-a}\left(\theta^{t^*-a}\right)\right)\beta\varepsilon \\
&> F\left(K_{t^*+1}, H_{t^*+1}\right) - C_{t^*+1} - K_{t^*+2} + (1-\delta)\,K_{t^*+1}
\end{aligned}$$

where the approximate equality becomes arbitrarily close to an equality when ε is sufficiently small, and the strict inequality follows from the assumption that the right-hand side of equation (3.10) is smaller than the left-hand side. Since $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ satisfies equation (3.2) at $t = t^* + 1$, it follows that $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$ satisfies it with strict inequality. This surplus of consumption goods at $t = t^* + 1$ can then be used to increase $\hat{u}_{t^*+1}^a\left(\theta^{t^*+1-a}\right)$ by an additional η (sufficiently small amount) at every $0 \leq a \leq t^* + 1$ and every θ^{t^*+1-a}. The resulting allocation satisfies the aggregate feasibility constraint (3.2) (provided that η is sufficiently small) and the incentive compatibility constraints (3.9) (since $\hat{u}_{t^*+1}^a\left(\theta^{t^*+1-a}\right)$ increases by a uniform additional amount across all θ^{t^*+1-a}). Moreover, the objective function (3.1) evaluates at a strictly larger number (since η is positive) than $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$. This contradicts the assumption that $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ solves the social planner's problem (3.1)-(3.5).

3.2 An irrelevance result

Observe that, for the same reasons given in Section 2.2, the solution to the social planner's problem (3.1)-(3.4) under full information must also satisfy the cross-sectional intratemporal and intertemporal inverse Euler conditions (3.6) and (3.10). Now assume that the utility functions u and n are both logarithmic. Using equations (3.3) and (3.4) we then get that the inverse Euler conditions (3.6) and (3.10) reduce to equations (2.14) and (2.15), respectively, for every $t \geq 0$. Since the aggregate feasibility constraint (3.2) must be satisfied both in the private and full information economies, we conclude that the aggregate allocation $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ is exactly the same in both economies. That is, once again, under logarithmic preferences the private information becomes completely irrelevant for aggregate dynamics.

4 A first-order approach

When preferences are different from logarithmic, the cross-sectional inverse Euler equations (3.6) and (3.10) do not represent relations between aggregate variables. Jensen's inequality precludes this from happening. It is an open question whether the corrections due to Jensen's inequality will be important enough under realistic preferences and cross-sectional heterogeneity, that the irrelevance result shown above would brake down completely or hold approximately. This section addresses that question. Since solving the social planner's problem (3.1)-(3.5) under realistic preferences and idiosyncratic uncertainty is extremely costly, I will simplify the analysis by focusing on the steady-state of the economy and following the first-order approach described in Kapicka (2013) and Pavan et al. (2014).

The flow utility that I will consider is the following

$$u(c_t) + \hat{\theta}_t n(1 - h_t) = \ln(c_t) - \hat{\theta}_t \frac{\varsigma}{1+\eta} h_t^{1+\eta}, \tag{4.1}$$

where $\varsigma > 0$ and $\eta > 0$. I choose u to be logarithmic so that the preferences are consistent with a balanced growth path (a standard assumption in the macroeconomics literature). For n I assume a constant Frisch elasticity of labor supply, which will allow me to calibrate η to an empirically reasonable magnitude. Hereon, I will define

$$\theta_t = \hat{\theta}_t^{-\frac{1}{1+\eta}}.$$

Thus, h_t can be interpreted as the effective hours worked, θ_t as the individual's idiosyncratic labor productivity, and the disutility term is equation (4.1) as being determined by the amount of effort

h_t/θ_t exerted. In this interpretation, h_t is publicly observable but θ_t is private information. Alternatively, we can retain the interpretation of Section 3 in which h_t is publicly observable total hours worked, and agents receive a privately observed idiosyncratic shock $\hat{\theta}_t$ to their disutility of labor. Under preferences with constant Frisch elasticity of labor supply, the idiosyncratic productivity and preference shocks interpretations are equivalent. Hereon, I will interpret θ_t as an idiosyncratic productivity shock in order to follow Kapicka (2013) most closely.

The idiosyncratic shock θ_t takes values in the interval $(0, \infty)$ and the probability $\Phi(\varepsilon, \theta)$ that $\theta_t < \varepsilon$ conditional on $\theta_{t-1} = \theta_{-1}$ is given by

$$\Phi(\varepsilon, \theta_{-1}) = \begin{cases} \frac{1-\left(\frac{\varepsilon}{\kappa\theta_{-1}}\right)^{-\lambda}}{1-\left(\frac{\zeta}{\kappa}\right)^{-\lambda}}, \text{ if } \kappa\theta_{-1} < \varepsilon < \zeta\theta_{-1}, \\ 0, \text{ if } \varepsilon \leq \kappa\theta_{-1} \\ 1, \text{ if } \zeta\theta_{-1} \leq \varepsilon, \end{cases} \tag{4.2}$$

where $\lambda > 1$, $\kappa = \frac{\lambda-1}{\lambda}$, and $\zeta > \kappa$. Observe that under this distribution function we have that

$$E[\theta|\theta_{-1}] = \frac{1-\left(\frac{\zeta}{\kappa}\right)^{-\lambda+1}}{1-\left(\frac{\zeta}{\kappa}\right)^{-\lambda}}\theta_{-1}.$$

Thus, when ζ becomes large $E[\theta|\theta_{-1}] \simeq \theta_{-1}$, i.e. θ_t approximately follows a random walk.[9]

Instead of working with the economy-wide social planner problem (3.1)-(3.5) it will useful to specify the problem in terms of a series of component planning problems that solve principal-agent problems taking aggregate shadow prices as given. The solution to these component planning problems will correspond to the solution of the economy-wide social planner problem (3.1)-(3.5) if certain side-conditions are satisfied. The advantage of working with the component planning problems is that the recursive first-order approach in Kapicka (2013), which is designed for principal-agent problems, can be directly applied.

In order to represent individual contracts recursively, the key insight in Fernandes and Phelan (2000) was to specify the incentive compatibility constraint (3.5) in terms of present values, i.e. as

$$\begin{aligned} & u_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) + \theta_{t-a}n_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) + v_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) \\ \geq\ & u_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right) + \theta_{t-a}n_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right) + v_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right), \end{aligned}$$

and to have the present values $v_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right)$ (for truth-telling) and $v_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right)$ (for every possible deviation θ_{t-a}^c) represent the state of the recursive contract for the following period. While

[9]Kapicka (2013) directly considers the case of $\zeta = \infty$. In computations, however, ζ is finite.

theoretically this is a valid representation, carrying as a state the present values obtainable under every possible report makes the problem intractable from a computational point of view. For this reason Kapicka (2013) and Pavan et al. (2014) took the much simpler approach of using only two numbers as state variables: the promised value $v_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right)$ under truth-telling, and the partial derivative of $v_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right)$ with respect to θ_{t-a}. That is, under this representation only local deviations from truth-telling are accurately specified. The great advantage of this formulation is that an envelope theorem in Milgrom and Segal (2002) can be used to replace the incentive compatibility constraints (3.5) with a series of envelope conditions. This relaxed problem is what will be analyzed below.[10]

In what follows it will be useful to group agents in two sets: the young and the old. The young are those agents born at the beginning of the current period. The old are the agents born in some previous period. The key distinction between the two groups is that, while the old agents start the current period in an ongoing contract, the young start completely anew. Thus, while the component social planners are committed to deliver the promises made to the old agents in their ongoing contracts, they can start the young on any new contract that they decide.

4.1 The component social planner's problem for old agents

The state of an old agent at the beginning of the period, before the current idiosyncratic productivity shock θ is realized, is given by a triplet (θ_{-1}, w, s), where θ_{-1} is the type reported in the previous period, w is the expected promised value obtained from having truthfully reported θ_{-1} in the previous period, and s is the derivative of the expected promised value with respect to the reported type in the previous period (evaluated at the reported type θ_{-1}).

Define

$$x = \varsigma \frac{h^{1+\eta}}{1+\eta} \tag{4.3}$$

and

$$\gamma = \left(\frac{(1+\eta)}{\varsigma}\right)^{\frac{1}{1+\eta}} \frac{1}{1+\eta}. \tag{4.4}$$

Then, the component social planner's problem for old agents is the following:

$$P\left(\theta_{-1}, w, s\right) = \max_{\{u,x,v,r\}} \left\{ \int_{\kappa\theta_{-1}}^{\zeta\theta_{-1}} \left\{ q\gamma\left(1+\eta\right) x\left(\theta\right)^{\frac{1}{1+\eta}} - e^{u(\theta)} + \pi\sigma P\left[\theta, v\left(\theta\right), r\left(\theta\right)\right] \right\} \phi\left(\theta, \theta_{-1}\right) d\theta \right\} \tag{4.5}$$

[10]The envelope conditions are necessary conditions for the incentive compatibility constraints. This is why the problem considered is a "relaxed" problem. Kapicka (2013) provides sufficient conditions as well, which can be verified after a solution to the relaxed problem is obtained.

subject to

$$\begin{aligned} & u(\theta) - \theta^{-(1+\eta)} x(\theta) + \beta\sigma v(\theta) \\ = & \int_{\kappa\theta_{-1}}^{\theta} \left[(1+\eta)\,\varepsilon^{-(2+\eta)} x(\varepsilon) + \beta\sigma r(\varepsilon) \right] d\varepsilon \\ & + u(\kappa\theta_{-1}) - (\kappa\theta_{-1})^{-(1+\eta)}\, x(\kappa\theta_{-1}) + \beta\sigma v(\kappa\theta_{-1}) \end{aligned} \tag{4.6}$$

$$w = \int_{\kappa\theta_{-1}}^{\zeta\theta_{-1}} \left[u(\theta) - \theta^{-(1+\eta)} x(\theta) + \beta\sigma v(\theta) \right] \phi(\theta, \theta_{-1})\, d\theta \tag{4.7}$$

$$\begin{aligned} s = & \int_{\kappa\theta_{-1}}^{\zeta\theta_{-1}} \left[u(\theta) - \theta^{-(1+\eta)} x(\theta) + \beta\sigma v(\theta) \right] \phi_2(\theta, \theta_{-1})\, d\theta \\ & - \kappa \left[u(\kappa\theta_{-1}) - (\kappa\theta_{-1})^{-(1+\eta)}\, x(\kappa\theta_{-1}) + \beta\sigma v(\kappa\theta_{-1})] \phi(\kappa\theta_{-1}, \theta_{-1}) \right] \\ & + \zeta \left[u(\zeta\theta_{-1}) - (\zeta\theta_{-1})^{-(1+\eta)}\, x(\zeta\theta_{-1}) + \beta\sigma v(\zeta\theta_{-1})] \phi(\zeta\theta_{-1}, \theta_{-1}) \right] \end{aligned} \tag{4.8}$$

where q is the shadow value of labor (which is taken as given), ϕ is the density function of the conditional distribution described in equation (4.2), and

$$\phi_2(\theta, \theta_{-1}) = \frac{d\phi(\theta, \theta_{-1})}{d\theta_{-1}}.$$

From equations (4.3) and (4.4) we have that $h = \gamma(1+\eta)\, x^{\frac{1}{1+\eta}}$. Thus, the objective in equation (4.5) is to maximize the expected discounted contribution to social welfare made by the agent, which is given by the social value of their labor qh net of the consumption goods e^u that they receive (since their labor supply eases the aggregate feasibility constraint 3.4, but their consumption tightens the aggregate feasibility constraint 3.3). Equation (4.6) is the envelope condition. It shows how the lifetime utility must vary with $\theta \in [\kappa\theta_{-1}, \zeta\theta_{-1}]$ if it is to be incentive compatible. Intuitively, each incentive compatibility constraint states a maximization problem: to maximize the agent's lifetime utility with respect to the reported type. These maximization problems are indexed by the agent's true type. If the optimized lifetime utility happens to be differentiable with respect to the agent's true type (i.e. if the the value function is differentiable), its derivative will be given by the derivative of the objective function with respect to the true type (evaluated at the optimal choice). In this case, the value function can be written as the integral of these derivatives, which is what equation (4.6) describes.[11] Equation (4.7) is the promise-keeping constraint: w is the expected lifetime utility obtained under truth-telling. Equation (4.8) is the marginal threat-keeping constraint: s describes how the expected lifetime utility marginally varies with the type θ_{-1} (truthfully) reported by the agent in the previous period.

[11] Kapicka (2013) shows that the regularity conditions for equation (4.6) to be valid are satisfied for the functional forms used here (the functional forms are exactly the same as those in Section 7 in his paper).

4.2 The component social planner's problem for young agents

I assume that the initial idiosyncratic productivity levels of young agents are drawn from the same distribution Φ described in equation (4.2), but conditional on $\theta_{-1} = 1$. Since young agents are born without an ongoing recursive contract, there are no state variables to describe for them. The component social planner's problem for young agents is given by

$$\max_{\{u,x,v,r\}} \int_{\kappa}^{\zeta} \left[\frac{u(\theta) - \theta^{-(1+\eta)} x(\theta) + \beta\sigma v(\theta)}{\psi} + q\gamma(1+\eta) x(\theta)^{\frac{1}{1+\eta}} - e^{u(\theta)} + \pi\sigma P[\theta, v(\theta), r(\theta)] \right] \phi(\theta, 1)\, d\theta \tag{4.9}$$

subject to equation (4.6), where ψ is the shadow value of consumption (which is taken as given) and P is the solution to the Bellman equation (4.5) for old agents (also taken as given). The objective function includes, similarly to the old agents problem, the social value of the labor supplied qh net of the consumption goods e^u received. However, now there is an extra term: the lifetime utility of the young agents $u - \theta^{-(1+\eta)}x + \beta\sigma v$ (divided by the shadow value of consumption ψ, to transform the utiles into consumption units). The reason why the lifetime utility of young agents enters the objective function (4.9) but the lifetime utility of old agents does not enter the objective function (4.5) is that while the choice of lifetime utility for the current young agents directly contributes to the economy-wide social welfare function (3.1), the lifetime utilities of old agents have already been decided in some previous period and the social planner is simply committed to deliver those promised values (given by w in equation 4.7). Also, observe that while the component social planner's problem for young agents is subject to the envelope conditions (4.6) (which, again, substitutes for the incentive compatibility constraints 3.5), there are no promise-keeping (4.7) or marginal threat-keeping constraints (4.8) since, again, young agents are born without ongoing recursive contracts.

4.3 Side conditions

Let (u_o, x_o, v_o, r_o) and (u_y, x_y, v_y, r_y) be the optimal decision rules for old and young agents, respectively. The invariant distribution α of old agents across individual states (θ_{-1}, w, s) satisfies, for all Borel sets $\mathit{\Theta}$, W and S, that

$$\alpha(\mathit{\Theta} \times W \times S) = \sigma \int_A \Phi(\mathit{\Theta}, \theta_{-1}) \alpha(d\theta_{-1} \times dw \times ds) + \sigma\Phi(B, 1) \tag{4.10}$$

where

$$A = \{(\theta_{-1}, w, s) : \; v_o(\theta_{-1}, w, s, \theta) \in W \text{ and } r_o(\theta_{-1}, w, s, \theta) \in S, \text{ for some } \theta \in \mathit{\Theta}\},$$

$$B = \{\theta \in \Theta : v_y(\theta) \in W \text{ and } r_y(\theta) \in S\}.$$

Observe that $\Phi(B, 1)$ is the measure of young agents born with an idiosyncratic productivity in the set B.

The shadow prices q and ψ for labor and consumption, that the component social planners take as given, correspond to the solution of the economy-wide social planner's problem if there exists an aggregate stock of capital K such that following three side conditions are satisfied:

$$\frac{1}{\beta} - 1 + \delta = F_K(K, H), \tag{4.11}$$

$$q = F_H(K, H), \tag{4.12}$$

$$C + \delta K = F(K, H), \tag{4.13}$$

where

$$C = \int c[u_o(\theta_{-1}, w, s, \theta)]\,\Phi(d\theta, \theta_{-1})\alpha(d\theta_{-1} \times dw \times ds) + \int c[u_y(\theta)]\,\Phi(d\theta, 1), \tag{4.14}$$

and

$$H = \int \gamma(1+\eta)\,x_o(\theta_{-1}, w, s, \theta)^{\frac{1}{1+\eta}}\,\Phi(d\theta, \theta_{-1})\alpha(d\theta_{-1} \times dw \times ds) + \int \gamma(1+\eta)\,x_y(\theta)^{\frac{1}{1+\eta}}\,\Phi(d\theta, 1). \tag{4.15}$$

Equation (4.11) is the standard Euler equation for the aggregate stock of capital. Equation (4.12) states that the shadow value of labor is equal to the marginal product of labor. Equation (4.13) is the aggregate feasibility constraint.

4.4 Computations

While the component social planner problems and the first-order approach greatly simplify the problem of computing a steady-state solution, it is still a daunting task. The reason is that the state vector to the component social planner's problem for old agents is constituted by three real numbers (θ_{-1}, w, s), and that for each value of the state vector the planner must decide allocation rules (u, x, v, r) over a continuum of values $\theta \in [\kappa\theta_{-1}, \zeta\theta_{-1}]$ subject to constraints that involve integral forms. Moreover, all these maximizations must be solved many times as the Bellman equation (4.5) is iterated until convergence. In addition, this process must be repeated for different shadow prices q and ψ (and the corresponding invariant distribution α solved for) until the side conditions are satisfied. Fortunately, certain model properties and computational strategies make the problem tractable.

First, observe that as long as the production function F is constant returns to scale, equation (4.11) determines the K/H ratio. Equation (4.12) then determines the shadow value of labor q. This is important because the only shadow price that the component social planner's problem for old agents (4.5)-(4.8) depends on is q (which is already known). Thus, this problem must be solved for only once.

Second, given the functional forms chosen for the utility functions and the distribution of idiosyncratic shocks, it can be shown that the solution P to the Bellman equation (4.5) satisfies the following homogeneity property:

$$P(\theta_{-1}, w, s) = \theta_{-1} P\left(1, w - \frac{\ln \theta_{-1}}{1-\beta}, s\theta_{-1}\right), \tag{4.16}$$

for every (θ_{-1}, w, s).[12] This property is important because it implies that the component social planner's problem for old agents (4.5)-(4.8) has a block structure. In particular, we can first solve the problem for $\theta_{-1} = 1$ (and every (w, s)). Only after this solution is obtained it is that we need to solve the problem for other values of θ_{-1} (and every (w, s)).

Third, for any given duple (w, s), the allocation rules (x_o, v_o, r_o) for the component social planner's problem for old agents are restricted to be continuous piecewise linear functions over the interval $[\kappa\theta_{-1}, \zeta\theta_{-1}]$.[13] The maximization in (4.5)-(4.8) is then performed with respect to the values of these functions at $\kappa\theta_{-1}$ and their slopes at the different θ-segments. The piecewise linearity of (x_o, v_o, r_o) allows to solve the integrals in equation (4.6) analytically and, therefore, get an analytical expression for $u(\theta)$.[14] This expression can then be used in equations (4.7) and (4.8) to also solve their corresponding integrals analytically. It turns out that this procedure converts equations (4.7) and (4.8) into linear relations between (w, s) and the choice variables (constituted by the intercepts and piecewise-slopes of (x_o, v_o, r_o)), which greatly simplifies computations. Since $u(\theta)$ enters exponentially in the objective function (4.5) and since ϕ is the density for the distribution function in equation (4.2), the integral in the objective function cannot be solved for analytically. For this reason, each θ-segment is divided into many subsegments and a trapezoidal rule is used to approximate the corresponding integral. The resulting sum of analytical expressions for the trapezoidal rule is then differentiated with respect to the choice variables (i.e. the intercepts and

[12] Kapicka (2013) showed this property for the case of $\zeta = \infty$. However it can be shown that this property also holds for $\zeta < \infty$.

[13] Kapicka (2013) instead solved his principal-agent problem assuming that the allocation rules are (discontinuous) step functions.

[14] This requires splitting the integrals into the different θ-segments over which the allocation rules are linear.

piecewise-slopes over (x_o, v_o, r_o)) to get their corresponding first order conditions. I then focus on $\theta_{-1} = 1$ and iterate with these first order conditions and the derivatives of $P(1, w, s)$ at a finite rectangular mesh of grid points for (w, s), until convergence is obtained. Since the allocation rules (v_o, r_o) take values outside the grid points, the derivatives of $P(1, v_o(\theta), r_o(\theta))$ are obtained in each iteration using bilinear interpolation/extrapolation over its values at the grid points. Once the derivatives of $P(1, w, s)$ have converged I then solve the problem (4.5)-(4.8) for other values of θ_{-1} and grid points (w, s). These problems can be solved in one step (no iterations are needed).

Fourth, given the derivatives of $P(1, w, s)$ already found, I fix the shadow value of consumption ψ and solve the component planner's problem for young agents (4.9). The allocation rules (x_y, v_y, r_y) are also assumed to be piecewise linear, and I solve the first order conditions for their intercepts and piecewise-slopes. For a given value of ψ, this problem must be solved only once.

Fifth, given the allocation rules (x_o, v_o, r_o) over $(\theta_{-1}, w, s, \theta)$ and (x_y, v_y, r_y) over θ, I simulate a large panel of agents (and their descendants) over a long period of time following those allocation rules. At the end of the simulation period, their consumption levels and effective hours worked are averaged out to obtain aggregate consumption C and hours worked H. Given the K/H ratio already known, these values of C and H are plugged into equation (4.13) to evaluate its equality. Different values for ψ must be considered (and (x_y, v_y, r_y), C and H recomputed) until an equality is obtained.

4.5 Quantitative results

Before turning to the quantitative results the model must by calibrated. In what follows I describe how parameter values are selected using a model time period equal to one year.

The survival probability σ is chosen to be 0.975 so that the expected lifetime of a young agent is equal to 40 years. As is standard in the macro literature, the discount factor β is set to 0.96.

The production function is chosen to have the following Cobb-Douglas form:

$$F(K, H) = AK^{1-\rho}H^{\rho}.$$

Following standard calibrations of the neoclassical growth model the labor share ρ is selected to be 0.64. Similarly, the depreciation rate of capital δ is set to 0.10. The productivity parameter A is chosen so that the steady state shadow value of leisure q is equal to one (a normalization).

Following Kapicka (2013) the disutility of work parameter ς is set to one, which provides a normalization for aggregate hours worked H. In turn η is set to 5.0 to generate a Frisch elasticity of labor supply equal to 0.2, which is consistent with micro estimates.

The parameters λ and ζ for the distribution of idiosyncratic shocks are selected so that

$$E[\theta \mid \theta_{-1}] = 0.999 \times \theta_{-1},$$

i.e. the idiosyncratic shock process approximates a random walk, and so that the variance of the log innovations to θ equals 0.007, which is consistent with estimates of the variance of annual innovations to the wage rate in Heathcote et al. (2005). This delivers values of $\lambda = 11.95$ and $\zeta = 1.63$.[15]

In computations I use a grid of 1,000 points for θ_{-1}. For each θ_{-1} I divide the support $[\kappa\theta_{-1}, \zeta\theta_{-1}]$ for the idiosyncratic shock θ into 16 segments, over which the allocation rules are piecewise linear. The trapezoidal rule used in the integration of $u(\theta)$ over each θ-segment uses 10 subsegments. For each grid point θ_{-1} I use a grid of 40 values for w and 40 values for s. The grids for (w, s) are allowed to depend on θ_{-1}. In practice, they are chosen to cover an area just large enough so that the bilinear interpolations/extrapolations of the derivatives of the value function P do not evaluate outside that area (i.e. only interpolations of the derivatives of P are used). For the Monte Carlo simulations used for computing aggregate consumption and hours worked, I use a panel of about 130,000 agents.

TABLE 1

Information	Y	C	I	H	K
Public	100.0	100.0	100.0	100.0	100.0
Private	90.5	90.5	90.5	90.5	90.5

The basic experiment of interest is to compare the aggregate steady state variables of the economies with private and full information, when these economies have exactly the same parameter values selected above. The results are presented in Table 1. We see that the aggregate effects of the information frictions are very large: output, consumption, investment, hours worked and capital are all 9.5% lower in the economy with private information than in the economy with full information.[16] This is in stark contrast with the irrelevance of private information under logarithmic preferences presented in Section 3. The reason is that the curvature introduced by the constant

[15] Recall that κ has already been restricted to be equal to $\frac{\lambda-1}{\lambda}$.

[16] Since equation (4.11) holds under private and full information, both economies have identical K/H ratios. Thus, the effects on all the aggregate variables shown in Table 1 must be the same.

Frisch elasticity of labor supply and the large cross-sectional heterogeneity introduced by the random walk idiosyncratic shocks generate large enough corrections (due to Jensen's inequality) to the cross-sectional intratemporal inverse Euler equation that the aggregate allocations become significantly different.[17] Under realistic preferences and idiosyncratic shocks the information frictions have large effects on aggregate dynamics.

5 Conclusions

In this paper I explored to what extent information frictions could affect aggregate dynamics. To that end I considered a neoclassical growth model in which agents receive idiosyncratic shocks to their value of leisure, and in which these shocks are private information of the agents. Considering the social planner's problem for this economy I derived two necessary conditions for an optimal allocation: the cross-sectional intratemporal and intertemporal inverse Euler equations. When preferences are logarithmic, these conditions become relations between aggregate variables. Since the cross-sectional Inverse Euler equations apply both to the private and full information economies, this implies that all aggregate variables are exactly the same in both economies. That is, under logarithmic preferences I show that the information frictions are completely irrelevant for aggregate dynamics.

For other preferences, the cross-sectional inverse Euler equations fail to deliver relations between aggregate variables because of a Jensen inequality correction term. Moreover, this correction term is affected by the amount of heterogeneity in the economy. For these reasons it is important to assess the importance of the information frictions for aggregate dynamics using realistic preferences and idiosyncratic shocks. With this in mind, I considered a version of the model with a constant Frisch elasticity of labor supply and idiosyncratic shocks that follow a random walk process. Calibrating parameter values to U.S. observations I then solved the model numerically and compared the steady-state of the model with and without private information. I now found that under this realistic structure the information frictions have large effects on aggregate variables: Steady-state output, consumption investment and hours worked are all 9.5% lower in the economy with private information.

The numerical experiment in the paper focused on a steady-state analysis. It would be extremely interesting to consider off steady-state analysis as well. In particular, it would be important to

[17]Since the utility of consumption u is logarithmic, the cross-sectional intertemporal Euler equation still reduces to equation (2.15) in both economies.

study how the information frictions affect the business cycle dynamics of the economy. In Veracierto (2021) I considered business cycle fluctuations but in a version of the model with i.i.d. idiosyncratic shocks. Computing business cycles when the idiosyncratic shocks are persistent is a much harder task and is left for future research.

Also, the paper focused on comparing the aggregate allocations of the model with private information with those of the same economy but with full information. It would be extremely interesting to compare them with other benchmarks. For instance, similar to Farhi and Werning (2012) they could be compared to a Bewley-type equilibrium. In addition, if transitionary dynamics could be computed, it would be extremely interesting to assess the welfare gains of going from a Bewley-type steady-state to an optimal allocation. This would extend Farhi and Werning (2012) analysis to having an endogenous labor supply and more realistic preferences. This is also left for future work.

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