THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

August 2024

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section (precise instructions are below)—50 points in each section—for a total of 100 points. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are five (5) pages, including this one.

SECTION I

Please answer any **two** of the three questions from this section.

I.1 (25 points) Robinson Crusoe is stranded on a small deserted island where the only available food consists of coconuts. These can be harvested according to a production function $y = f(h)$ where y is the amount of food produced and h is the number of hours spent harvesting. There are T hours in the day. Crusoe has a utility function $u(x, \ell)$ defined over the amount of food x consumed and the hours of leisure ℓ.

(a) Write down the optimization problem Crusoe faces to optimally allocate his time T between leisure ℓ and time spent harvesting h?

(b) Now suppose that Crusoe starts a "firm" called RC Inc. that produces food via the production function f. The firm seeks to maximize profits, pays Crusoe a wage for his time and charges Crusoe a price for food. Crusoe is the sole owner of RC Inc. and receives all the profits of the firm. Write down what is meant by a Walrasian equilibrium of this one-consumer, one firm economy $\mathcal{E}$.

(c) Under what conditions does a Walrasian equilibrium also solve the optimization problem in part (a)? Conversely, under what conditions does a solution to part (a) also constitute a Walrasian equilibrium allocation of $\mathcal{E}$?

I.2 (25 points) Consider an exchange economy $\mathcal{E}$ with two consumers, three (3) states of nature and one consumption good. The states are equally likely and the two consumers have the same utility function

$$u_i(x_{i1}, x_{i2}, x_{i3}) = \frac{1}{3}\sqrt{x_{i1}} + \frac{1}{3}\sqrt{x_{i2}} + \frac{1}{3}\sqrt{x_{i3}}$$

where x_{is} denotes consumer i's consumption in state s. The endowments are

$$\begin{aligned} w_1 &= (4, 0, 3) \\ w_2 &= (0, 4, 3) \end{aligned}$$

where again w_{is} denotes the endowment of consumer i in state s.

Markets are *incomplete*.

(a) Suppose there are state contingent assets for states 1 and 2 but not for state 3. Find a Radner equilibrium.

(b) Now suppose a state contingent asset for state 3 is introduced. How does this affect the utilities of the two consumers?

I.3 (25 points) Suppose there are two buyers, $i = 1, 2$, and two sellers, $j = 1, 2$. Each seller owns a house and each buyer wants to buy at most one house. Buyer i values the house owned by j at h_{ij} (in dollars). Seller j values the house she owns at c_j (also measured in dollars).

Suppose that $c_1 = c_2 = 1$ whereas $h_{11} = 2$, $h_{12} = 4$ and $h_{21} = 8$, $h_{22} = 8$.

(a) Find a Pareto efficient allocation.

(b) Show that there is a Walrasian equilibrium such that the prices of the two houses $(p_1, p_2) = (1, 3)$.

(c) Now suppose that the value that buyer 1 attaches to house 2 changes to $h_{12} = 5$, while all other values remain unchanged. Is there a Walrasian equilibrium?

Section II is on the next two pages.

Please answer any two of the three questions from this section.

1. Consider the following normal-form game with N players. Each has three strategies, A, B, and C. Given a profile of (pure) strategies, denote by α the number of players who choose A, by β the number of players who choose B, and by γ the number of players who choose C. The payoff of a player who chooses A is γ, that of a player who chooses B is 2α, and that of a player who chooses C is 3β.

   (a) (20) Suppose that $N = 2$. Represent this game as a matrix, identify the rationalizable strategy profiles, and solve for all the Nash equilibria (pure and mixed).

   (b) (30) Find a value of N such that the game has a pure-strategy Nash equilibrium. For this N, how many pure-strategy Nash equilibria are there? Prove this.

2. Consider a normal-form game and two different extensive-form representations of the game.

   (a) (10) Can the set of rationalizable strategies differ across the extensive-form representations? Explain. If the answer is "yes" give an example in which this is true.

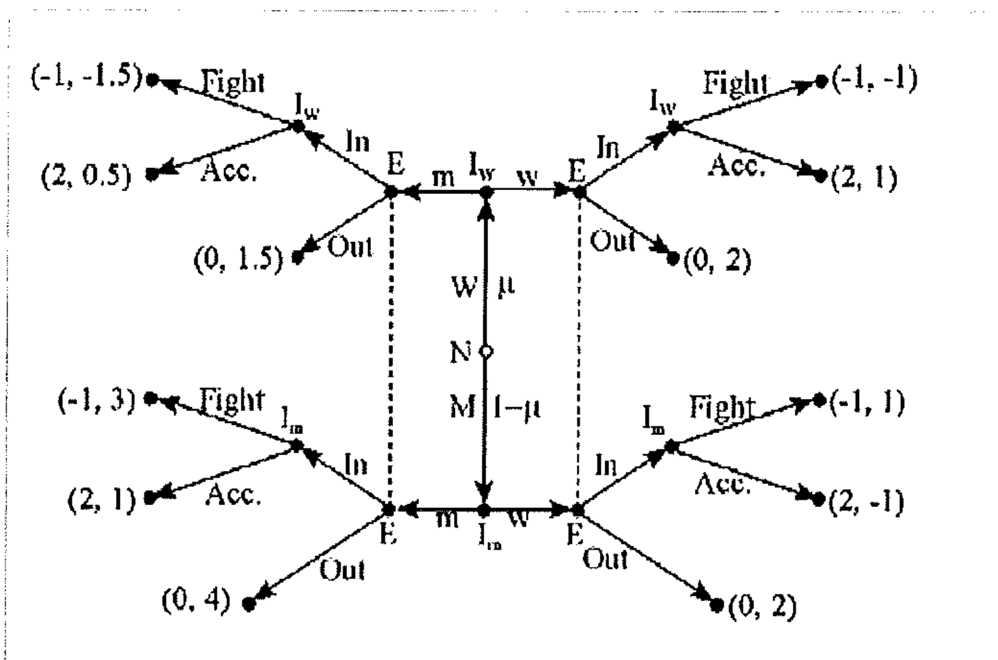
   (b) (10) Can the set of Nash equilibria (pure and mixed) differ across the extensive-form representations? If the answer is "no," explain. If the answer is "yes," give an example in which this is true.

   (c) (10) Can the set of subgame perfect equilibria differ across the extensive-form representations? If the answer is "no," explain. If the answer is "yes," give an example in which this is true.

   (d) (10) Can the set of weak perfect Bayesian equilibria differ across the extensive-form representations? If the answer is "no," explain. If the answer is "yes," give an example in which this is true.

   (e) (10) Can the set of sequential equilibria differ across the extensive-form representations? If the answer is "no," explain. If the answer is "yes," give an example in which this is true.

3. Consider the following version of an entry game with an entrant (player 1) and an incumbent (player 2). The incumbent is either weak (denoted by W in the figure below), with probability $\mu \in (0,1)$, or strong (denoted by M in the figure below), with probability $1-\mu$. The incumbent knows whether he is weak or strong but the entrant does not. A strong incumbent likes to fight and a weak incumbent does not. If the entrant stays out, his payoff is 0 and that of the incumbent is 2 (regardless of his type). If the entrant goes in and the incumbent accommodates him, then the entrant's payoff is 2 and the incumbent's payoff is 1 if he is weak and -1 if he is strong. If the entrant goes in and the incumbent fights him, then the entrant's payoff is -1 and the incumbent's payoff is -1 if he is weak and 1 if he is strong. Before the entrant makes his decision whether to enter, the incumbent has an opportunity to "build reputation" by raiding the market of another firm. A strong incumbent who raids gets a payoff of 2 for raiding and a weak incumbent gets a payoff of $-1/2$ from raiding. The incumbent's payoff from not raiding is 0 (regardless of his type). The payoffs across the raid and the entry stage are additive. This gives rise to the following extensive-form game:



Solve for the sequential equilibria of this game. (Hint: consider $\mu > 1/3$, $\mu < 1/3$, and $\mu = 1/3$.)