Probabilistic Assortative Matching and Search: TU and NTU

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In many contexts, one-to-one matching patterns tend to be assortative. Like marry likes. And more efficient firms recruit workers with higher ability.

But perfect assortative matching (PAM) —two agents match only if of equal rank —is rarely observed.

cf. Abowd-Kramarz-Margoli (1999) on worker-firm matching — Chiappori-Fiorio-Galichon-Verzillo (WP) on marriages-along-income.

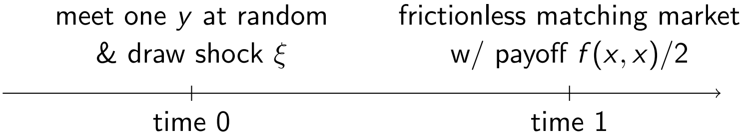
We ask: do there exist fundamental forces that contravene assortative matching? Answer: Search frictions!

A stripped down version. Symmetric populations $x, y \in [0, 1]$

x, y generate complementary output $f(x, y) = xy$ when paired

w/out frictions, matching is assortative ($y = x$) & payoff $f(x, x)/2$

Now consider (non-stationary) search frictions:



Discount the future match payoff $f(x, x)/2$ at rate δ

—the value-of-search (or value of waiting) is $V(x) = \delta f(x, x)/2$

Question: which y is x's likeliest partner type?

TU paradigm. Time $t = 0$ matching is efficient: (x, y) match if

$$f(x,y) + \xi - V(x) - V(y) \geq 0$$

Type x's likeliest partner type is

$$y^\star(x) = \underset{y \in [0,1]}{\operatorname{argmax}}\, xy - \delta x^2/2 - \delta y^2/2$$

This is given by $y^\star(x) = \min\{\frac{x}{\delta}, 1\}$.

As $y^\star(x) > x$, x matches up rather than with complementary types

Note: this tendency rises as frictions increase (smaller δ)

Why? Frictions erode more the bargaining power of high types

(a) no search ($\delta = 1$) (b) no shocks ($\xi = 0$) (c) search and shocks

NTU paradigm. Exogenous payoffs (here: split $f(x, y)$ evenly)

May result in disagreement—(x, y) match at $t = 0$ if both accept:

$$\frac{xy}{2} + \xi \geq \delta \frac{x^2}{2} \qquad \text{and} \qquad \frac{xy}{2} + \xi \geq \delta \frac{y^2}{2}$$

Who is x's likeliest partner type? Surely no less than x

But possibly types $y > x$ accept x with larger probability than x

$$Prob\big[y \text{ accepts } x\big] \propto \frac{xy}{2} - \delta \frac{y^2}{2}$$

When $\delta > \frac{1}{2}$ (small frictions), this is largest for $y < x$

But when $\delta \leq \frac{1}{2}$, this is largest for $y = \min\{\frac{x}{2\delta}, 1\}$

Thus NTU repeats the TU pattern to match up

$$y^\star(x) = \begin{cases} x & \text{if } \delta > \frac{1}{2} \\ \min\{\frac{x}{2\delta}, 1\} & \text{if } \delta \leq \frac{1}{2} \end{cases}$$

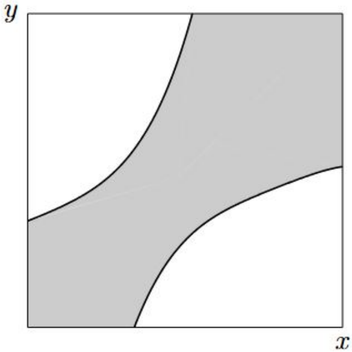
Why? Search frictions have asymmetric impact

Due to complementarities, high y more highly value matching, so with delay high y accept lower shocks ξ

Why shocks?

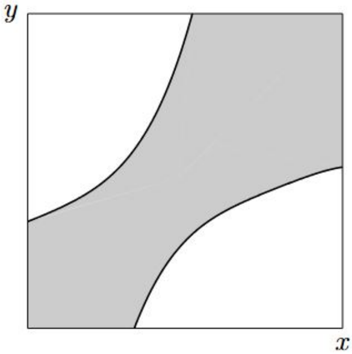
Matching up—not with complementary types—reveals a fundamental failure of PAM

Masked when match probabilities upon meeting are binary, as is the case without shocks: $m_t(x, y) \in \{0, 1\}$



PAM with Search

In Shimer and Smith (2000):



PAM holds if matching sets U_t form a lattice

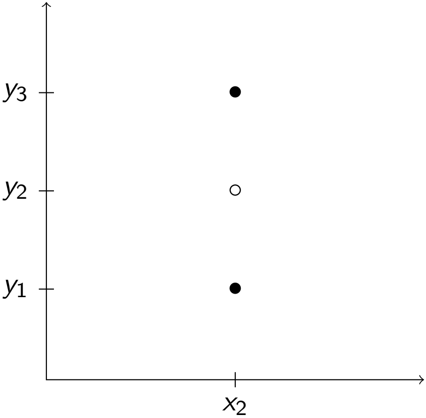
$$(x_1, y_2), (x_2, y_1) \in U_t \quad \Rightarrow \quad (x_2, y_1), (x_1, y_2) \in U_t$$

$\Leftrightarrow$ higher types match with *intervals* of higher types

— But where are intervals centered? Around complementary types?

Shimer and Smith (2000): PAM Implies Convexity

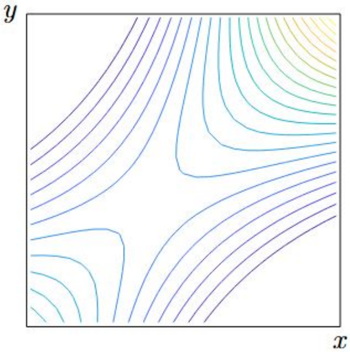
If all $U_t(y) = \{x : m_t(x, y) = 1\}$ is non-empty and U_t is a lattice, then $U_t(x)$ is convex.



$(x_2, y_1), (x_2, y_3) \in U_t$ but $(x_2, y_2) \notin U_t$.

P-PAM with Search

ξ generates probabilistic matching, $m_t(x, y) \in [0, 1]$

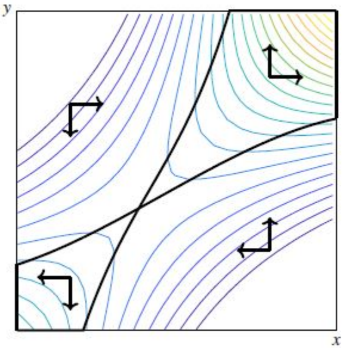


P-PAM holds if $U_t(p) = \{(x, y) : m_t(x, y) \geq p\}$ is a lattice,

$$(x_1, y_2), (x_2, y_1) \in U_t(p) \quad \Rightarrow \quad (x_2, y_1), (x_1, y_2) \in U_t(p),$$

and convex in each dimension.

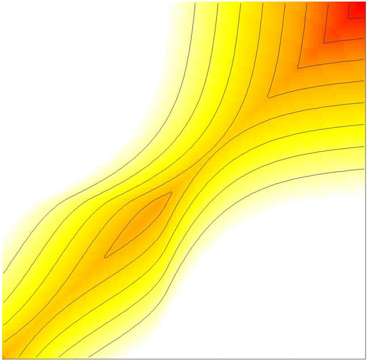
Below P-PAM fails:



Different (equivalent) ways of attesting:

1. Upper contour sets do not form a lattice
2. Likeliest partner types do not 'coincide'
3. Some level lines are decreasing

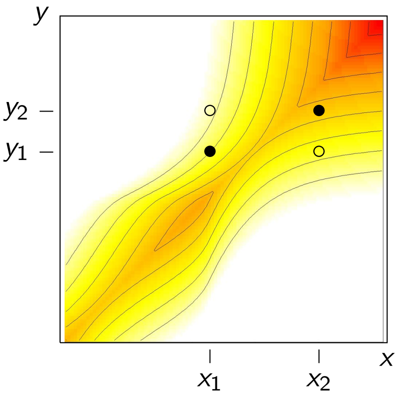
Below P-PAM holds



Different (equivalent) ways of attesting:

1. Upper contour sets form a lattice
2. Likeliest partner types 'coincide'
3. Level lines are non-decreasing

Below P-PAM holds



Different (equivalent) ways of attesting:

1. Upper contour sets form a lattice
2. Likeliest partner types 'coincide'
3. Level lines are non-decreasing

Preview of Results

General model predicts when P-PAM holds

1. Definition + Axiomatization of P-PAM
2. NTU: P-PAM holds w/out complementarity + sorting conditions from previous paper
3. TU: P-PAM always fails with search

   But match probabilities are single-peaked with complementarity
4. existence + uniqueness of a *stochastic* search *equilibrium*

Related Literature

Search: Morgan (1995), Shimer & Smith (2000), Smith (2006), Atakan (2006), Bonneton & Sandmann (2025)

Shocks: Choo & Siow (2006), Galichon & Salanie (2018), Chiappori et al. (2025)

Search + shocks: Gousse, Jacquemet and Robin (2017), Galichon (wP), Borovickova & Shimer (WP)

The Model

Set-up: the agents

Consider two populations with finite types in $X, Y \subset [0,1]$

TU: If (x,y) match, they generate output $f(x,y)+\xi$

NTU: If (x,y) match, they generate payoffs
$\pi^X(y|x)+\alpha^X\xi$ and $\pi^Y(x|y)+\alpha^Y\xi$

In both cases, ξ is an idiosyncratic shock with $\xi \sim \Xi$

Definition (increasing payoffs)

$\pi^X(y_2|x) > \pi^X(y_1|x)$ for $y_2 > y_1$, $\pi^Y(x_2|y) > \pi^Y(x_1|y)$ for $x_2 > x_1$.
Means: *absolute advantage*; types are *vertically differentiated*.

Definition (supermodular output)

$f(x_2,y_2) - f(x_1,y_2) - f(x_2,y_1) + f(x_1,y_1) > 0$ for $x_2 > x_1$, $y_2 > y_1$.
Means *complementarity*; types are *horizontally differentiated*.

Set-up: Non-Stationary Search

Time $t \in [0, \infty)$ is continuous, discounted at rate ρ

Mass of agent types x searching at time t is $\mu_t^X(x)$

Meetings. x meets a type y at time-varying rate $\lambda_t^X(y) \propto \mu_t^Y(y)$

Matching. Agents accept a match if shock ξ is large

Gives rise to meeting-contingent **match probabilities** $m_t(x, y)$:

$$m_t(x,y) = \begin{cases} P\left[\xi : \begin{array}{c} \pi^X(y|x) + \alpha^X \xi \geq V_t^X(x) \\ \pi^Y(x|y) + \alpha^Y \xi \geq V_t^Y(y) \end{array}\right] & \text{(NTU)} \\ \\ P\Big[\xi : \underbrace{S_t(x,y)}_{=f(x,y) - V_t^X(x) - V_t^Y(y)} + \xi \geq 0\Big] & \text{(TU)} \end{cases}$$

Set-up: Search Pool Dynamics

Entry and exit aggregate into a FSDE:

(forward stochastic differential equation)

$$\mu_t^X(x) = \mu_0^X(x) + \int_0^t \Big[-\sum_y \underbrace{\lambda_\tau^X(y)\mu_\tau^X(x)m_\tau(x,y)}_{\text{flow match creation}} + \underbrace{\mu_\tau^X(x)\eta_\tau^X(x)}_{\text{entry}} \Big] d\tau + \int_0^t \underbrace{\mu_\tau^X(x)\sigma_\tau^X(x)dB_\tau(x)}_{\text{noise}}.$$

Set-up: Payoffs

Transferable Utility (TU)

Payoff $\equiv$ value-of-search $V_t^X(x)$ plus α^X share of $S_t(x,y)+\xi$

(Nash bargaining where search-values define threat points)

In expectation:

$$\Pi_t^X(y|x) = V_t^X(x) + \alpha^X\left(S_t(x,y) + \mathbb{E}\left[\xi \middle| S_t(x,y) + \xi \geq 0\right]\right)$$

Non-transferable Utility (NTU)

Payoff $\equiv$ exogenous $\pi^X(y|x)$ plus α^X share of ξ

In expectation:

$$\Pi_t^X(y|x) = \pi^X(y|x) + \alpha^X \mathbb{E}\left[\xi \middle| \min\left\{ \begin{array}{c} \frac{\pi^X(y|x)-V_t^X}{\alpha^X} \\ \frac{\pi^Y(x|y)-V_t^Y}{\alpha^Y} \end{array} \right\} + \xi \geq 0\right]$$

Set-up: Value-of-Search

Expected discounted match payoff is

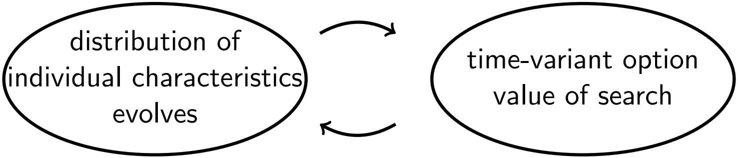
$$V_t^X(x) \equiv \mathbb{E}\Big[\int_t^\infty e^{-\rho(\tau-t)} \sum_{y\in Y} \Pi_\tau^X(y|x) P_t^X(d\tau)(y|x) \Big| \mathcal{F}_t\Big]$$

$P_t^X(\tau)(y|x)$ is the prob. that x matches with some y in $[t,\tau)$.

Equilibrium Definition

An equilibrium is a pair (μ, V):

μ solves population dynamics, V is the value-of-search.



Lemma

$(\mu, V) \equiv (\exp\gamma, V)$ is an equilibrium if and only if (γ, V, Z) solves the following system FBSDEs:

$$\begin{cases} \gamma_t^X(x) = \gamma_0^X(x) + \int_0^t \Big[-\sum_y \lambda_\tau^X(y) m_\tau(x,y) + \eta_\tau^X(x) - \frac{\left(\sigma_\tau^X(x)\right)^2}{2} \Big] d\tau \\ \qquad\qquad + \int_0^t \sigma_\tau^X(x) dB_\tau \\ V_t^X(x) = \int_t^T \Big[\sum_y \left[\Pi_\tau^X(y|x) - V_\tau^X(x)\right] \lambda_\tau^X(y) m_\tau(x,y) - \rho V_\tau^X(x) \Big] d\tau \\ \qquad\qquad - \int_t^T Z_\tau^X(x) \cdot dB_\tau . \end{cases}$$

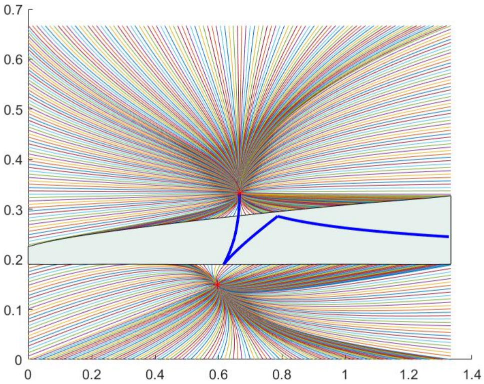
where $\gamma_0 = \exp\mu_0$.

Due to Ito's lemma and MRT which gives Z (think: $\dot{V}$).

The theory of FBSDEs gives conditions for unique existence of a solution (cf. Delarue (2002)) — No uniqueness with $T = \infty$ or $\sigma = 0$

Intuition: Equilibrium Uniqueness

Multiplicity arises when agents can coordinate on different long-run steady state



P-PAM

The domain of P-PAM

The domain of P-PAM are match probabilities $m_t(x, y)$

Remark

Data on unmatched populations $\mu_t^X(x), \mu_t^Y(y)$ and search frequencies $q_t(x, y)$ point-identify $m_t(x, y)$ up to a constant:

$$m_t(x, y) \propto \frac{q_t(x, y)}{\mu_t^X(x)\mu_t^Y(y)}$$

The domain of P-PAM has empirical content

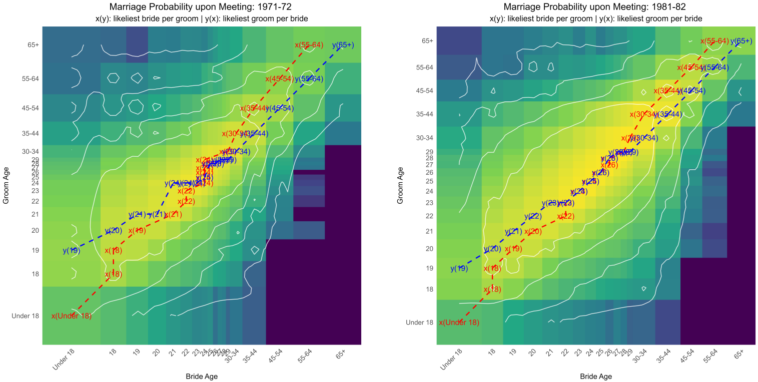


Figure: Heatmaps of $\propto m_t(x, y)$ replicating Choo-Siow's data on US marriages by age.

P-PAM = lattice + convex contour sets

To define P-PAM, begin with upper contour sets

$$U_t(p) = \{(x, y) : m_t(x, y) \geq p\}$$

Our definition of P-PAM demands the lattice property: given p-acceptable NAM matches, PAM matches must also be p-acceptable

Definition

$U_t(p)$ is a lattice if for all types $x_1 < x_2$ and $y_1 < y_2$

$$\begin{aligned} &(x_1, y_2) \in U_t(p) \text{ and } (x_2, y_1) \in U_t(p) \\ &\Rightarrow \quad (x_1, y_1) \in U_t(p) \text{ and } (x_2, y_2) \in U_t(p). \end{aligned}$$

Not original: Shimer & Smith (2000) say that matching is assortative if $U_t(1)$ is a lattice. P-PAM merely requires this for all $p \in [0, 1]$.

P-PAM = lattice + convex contour sets

Further define individual upper contour sets

$$U_t(x;p) = \{y : m_t(x,y) \geq p\}$$

Our definition of P-PAM demands that these be convex

Definition (PAM)

There is probabilistic PAM (P-PAM) if, for all $p \in [0,1]$,

(i) the upper contour matching set $U_t(p)$ is a lattice,

(ii) individual upper contour matching sets $U_t(x;p)$ are convex for all agent types.

To visualize P-PAM, draw lower and upper level lines

$$l_t(x;p) = \min\{y : m_t(x,y) \geq p\}$$
$$u_t(x;p) = \max\{y : m_t(x,y) \geq p\}$$

Proposition

Suppose that $U_t(x;p)$ is convex for all agent types. Then $U_t(p)$ is a lattice if and only if $x \mapsto l_t(x;p)$ and $x \mapsto u_t(x;p)$ are non-decreasing.

Axiomatic Characterization: SC + SP

Three axioms characterize P-PAM

First, consider single-crossing. P-PAM implies this:

$$m_t(x_1, y_2) > m_t(x_1, y_1) \quad \Rightarrow \quad m_t(x_2, y_2) \geq m_t(x_2, y_1)$$

for all $x_2 > x_1$, $y_2 > y_1$

Lemma

$m_t(x, y)$ satisfies single-crossing if upper contour sets $U_t(p)$ are a lattice for all p.

Single-peakedness also follows (from convexity):

Define $\triangle_y h(x, y) \equiv h(x, y_+) - h(x, y)$ the finite differences operator with y_+ the smallest type greater than y

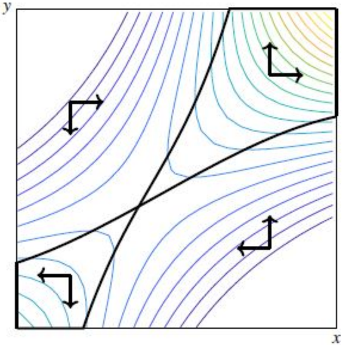
Definition (single-peakedness)

$y \mapsto m_t(x, y)$ is single-peaked if there exists $y_t(x)$ such that $\triangle_y m_t(x, y) \geq 0$ for $y < y_t(x)$ and $\leq$ for $y > y_t(x)$.

Axiomatic Characterization: SC + SP

Single-crossing + single-peaked matching patterns:

highest match probabilities center around *two* likeliest partner type correspondences



Axiomatic Characterization: Reciprocity

Single-peakedness and single-crossing alone do not imply P-PAM

Why? Compare matching patterns *within* a single population

The lattice property, by contrast, relates *across* populations

The missing condition: reciprocity

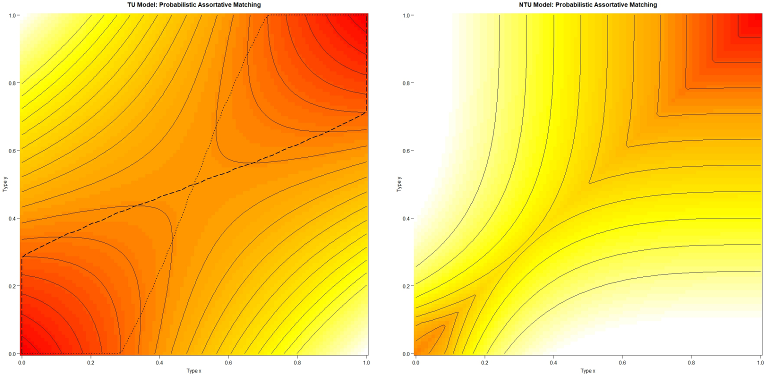
Definition (reciprocity)

$m_t(x, y)$ satisfies reciprocity if

$$y < \underline{y}_t(x) \Rightarrow \underline{x}_t(y) \leq x \quad \text{and} \quad y > \overline{y}_t(x) \Rightarrow \overline{x}_t(y) \geq x,$$

and vice versa where $\underline{y}_t(x) = \min y_t(x)$ and $\overline{y}_t(x) = \max y_t(x)$; likewise for $\underline{x}_t(y)$ and $\overline{x}_t(y)$.

Which of these satisfy P-PAM?



TU

Staring Down Value Functions

Mimicking Argument

Lemma

For all types $x', x'' \in X$ and time t the following holds in equilibrium:

$$V_t^X(x'') - V_t^X(x') \geq \sum_{y \in Y} \left[f(x'', y) - f(x', y) \right] Q_t^X(y|x').$$

$Q_t^X(y|x')$ is a non-negative measure over Y, adapted to $\mathcal{F}_t$, whose sum is strictly less than one.

Proof Sketch. Suppose that x'' matches when type x''s matches

Then x'''s expected match payoff conditional on matching with y is

$$\Pi_t^X(y|x'' \sim x') = \int_{-S_t(x',y)}^{\infty} \left[\pi_t^X(y|x'') + \alpha^X \xi \right] \frac{\Xi_t(d\xi)}{1 - \Xi_t(-S_t(x', y))}.$$

The difference in conditional match payoffs of types x'' and x':

$$\Pi_t^X(y|x'' \sim x') - \Pi_t^X(y|x') = \alpha^X \left[f(x'', y) - f(x', y) \right] + (1 - \alpha^X) \left[V_t^X(x'') - V_t^X(x') \right]$$

Matching is intratemporally efficient, so mimicking is undesirable:

$$
\begin{aligned}
&V_t^X(x'') - V_t^X(x') \\
&\geq \mathbb{E}\left[\int_t^T e^{-\rho(\tau-t)} \sum_{y\in Y} \left(\Pi_\tau^X(y|x'' \sim x') - \Pi_\tau^X(y|x') \right) \underbrace{P_t^X(d\tau)(y|x')}_{x'\text{'s matching probability}} \middle| \mathcal{F}_t \right] \\
&= \sum_{y\in Y} \left[f(x'', y) - f(x', y) \right] M_t^X(y|x') \\
&\quad + \mathbb{E}\left[\int_t^T \left(V_\tau^X(x'') - V_\tau^X(x') \right) e^{-\rho(\tau-t)} (1-\alpha^X) \bar{P}_t(d\tau)(x') \middle| \mathcal{F}_t \right]
\end{aligned}
$$

where

$$
M_t^X(y|x') = \alpha^X \mathbb{E}\left[\int_t^T e^{-\rho(\tau-t)} P_t^X(d\tau)(y|x') \middle| \mathcal{F}_t \right]
$$

$$
\bar{P}_t(\tau)(x') = \sum_y P_t(\tau)(y|x').
$$

Then iterate

Failure of Reciprocity

Search frictions prevent P-PAM. Highly-ranked types are likeliest to match with the highest —not complementary type

Proposition

Under supermodular output $f(x, y)$, the likeliest partners of highly ranked types x and y are $y_{\max}$ and $x_{\max}$ respectively.

Proof Sketch. By contradiction. If type x's modal partner were some $y < y_{\max}$, $y_{\max}$'s search value advantage over y would exceed the output advantage: $f(x, y_{\max}) - f(x, y) < V_t^Y(y_{\max}) - V_t^Y(y)$. And by the preceding lemma

$$f(x, y_{\max}) - f(x, y) < V_t^Y(y_{\max}) - V_t^Y(y) \leq \sum_{x' \in X} [f(x', y_{\max}) - f(x', y)] Q^Y(x' | y_{\max})$$

This inequality cannot hold for $x_{\max}$ since supermodularity makes output advantages largest at the top. With search frictions—smaller $Q^Y(x' | y_{\max})$—it also fails for large x close to $x_{\max}$

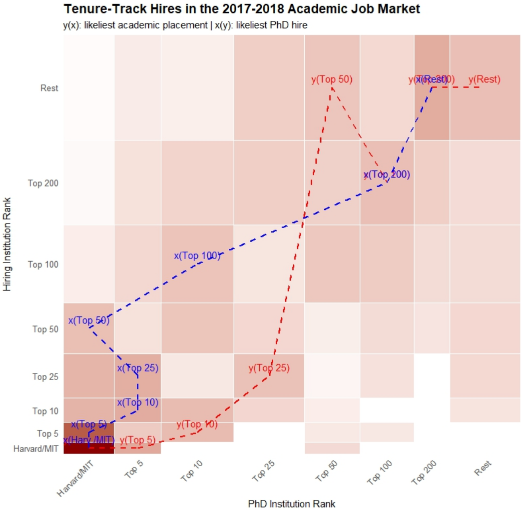


Figure: Reciprocity fails for junior economists (2017-2018 wiki data)

The modal TT hire in a top 5, 10, 25 or 50 department is a graduate from a top 5 department (MIT, Harvard, Stanford, Chicago, Princeton). And the modal academic placement of a PhD student from a top 10 department is a top 5 job

Single-Crossing

Proposition

If $f(x, y)$ is supermodular, then $m_t(x, y)$ satisfies single-crossing.

Proof.

$$
\begin{aligned}
& m_t(x_1, y_2) \geq m_t(x_1, y_1) \\
\Leftrightarrow & P[S_t(x_1, y_2) + \xi \geq 0] \geq P[S_t(x_1, y_1) + \xi \geq 0] \\
\Leftrightarrow & f(x_1, y_2) - V_t(x_1) - V_t(y_2) \geq f(x_1, y_1) - V_t(x_1) - V_t(y_1) \\
\Rightarrow & f(x_2, y_2) - V_t(x_1) - V_t(y_2) \geq f(x_2, y_1) - V_t(x_1) - V_t(y_1) \\
\Leftrightarrow & P[S_t(x_2, y_2) + \xi \geq 0] \geq P[S_t(x_2, y_1) + \xi \geq 0] \\
\Leftrightarrow & m_t(x_2, y_2) \geq m_t(x_2, y_1).
\end{aligned}
$$

Then single-crossing holds. □

Single-peakedness — Preliminaries

Definition (Log supermodularity)

A function $h(x, y)$ satisfies log supermodularity (LS) if for all $y_1 < y_2$ and $x_1 < x_2$

$$\frac{h(x_2, y_2)}{h(x_2, y_1)} \geq \frac{h(x_1, y_2)}{h(x_1, y_1)}.$$

Definition (Log supermodularity in differences)

A function $h(x, y)$ is log supermodular in differences (LSD) if $\triangle_y h(x, y)$ and $\triangle_y h(x, y)$ are log supermodular.

Single-peakedness — ansatz

Theorem

If $f(x, y_2) - f(x, y_1)$ is increasing in x (supermodular output) and satisfies LS + LSD, then $y \mapsto m_t(x, y)$ is single-peaked.

If not $\exists\ x_1$ and (adjacent) types $y_3 > y_2 > y_1$ such that

$$S_t(x_1, y_3) > S_t(x_1, y_2) \quad \text{and} \quad S_t(x_1, y_1) > S_t(x_1, y_2).$$

which is equivalent to

$$\begin{aligned} & V_t(y_3) - V_t(y_2) < f(x_1, y_3) - f(x_1, y_2) \\ \text{and} \ \ & V_t(y_2) - V_t(y_1) > f(x_1, y_2) - f(x_1, y_1). \end{aligned}$$

TU mimicking: there exist discounted match probabilities $Q_t(x|y_2)$ s.t.

$$\sum_{x' \in X} \left(f(x', y_3) - f(x', y_2) \right) Q_t(x'|y_2) \leq V_t(y_3) - V_t(y_2)$$

$$\sum_{x' \in X} \left(f(x', y_2) - f(x', y_1) \right) Q_t(x'|y_2) \geq V_t(y_2) - V_t(y_1).$$

NTU

NTU P-PAM $\Leftrightarrow$ Choosiness

Define the probability that x accepts a match with y

$$\beta_t^X(y|x) = Prob^{\xi}\left[\xi : \pi^X(y|x) + \alpha^X \xi \geq V_t^X(x)\right]$$

Proposition

Suppose that $x \mapsto \beta_t^X(y|x)$ and $y \mapsto \beta_t^Y(x|y)$ are non-increasing for all partner types. Then there is P-PAM.

Why must choosiness rise for P-PAM?

If higher-ranked types y_2 and x_2 accept lower shocks to match with x_1 and y_1, then match (x_1, y_1) was less likely than matches (x_1, y_2) and (x_2, y_1). Then the resulting match probability level curves would be decreasing in type

NTU Sorting Result

Definition (Log supermodularity)

X's payoffs satisfy log supermodularity (LS) if for all $y_1 < y_2$ and $x_1 < x_2$

$$\frac{\pi^X(y_2|x_2)}{\pi^X(y_1|x_2)} \geq \frac{\pi^X(y_2|x_1)}{\pi^X(y_1|x_1)}.$$

Captures higher types' greater patience.

Definition (Log supermodularity in differences)

X's payoffs satisfy log supermodularity in differences (LSD) if for all $y_1 < y_2 < y_3$ and $x_1 < x_2$

$$\frac{\pi^X(y_3|x_2) - \pi^X(y_2|x_2)}{\pi^X(y_2|x_2) - \pi^X(y_1|x_2)} \geq \frac{\pi^X(y_3|x_1) - \pi^X(y_2|x_1)}{\pi^X(y_2|x_1) - \pi^X(y_1|x_1)}.$$

Captures higher types' greater willingness to bear risk (Pratt).

Equivalently: $d_y\pi(y|x)$ is log supermodular.

No Complementarity $\Rightarrow$ P-PAM

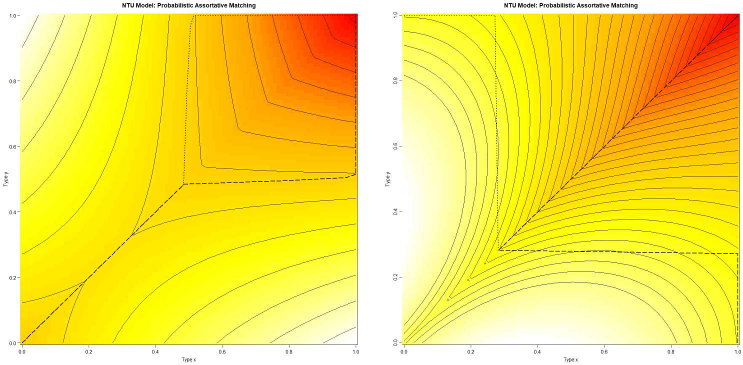
Definition

Population X's payoffs exhibit no complementarities if, for all $y_2 > y_1$ and $x_2 > x_1$,

$$\pi^X(y_2|x_2) - \pi^X(y_1|x_2) = \pi^X(y_2|x_1) - \pi^X(y_1|x_1).$$

Theorem

Suppose that payoffs satisfy log supermodularity and log supermodularity in differences and there is no complementarity. Then $y \mapsto \beta_t^X(y|x)$ and $x \mapsto \beta_t^Y(x|y)$ are non-increasing, hence there is P-PAM.



(left) supermodular payoffs $\pi^X(y|x) = xy$; (right) submodular payoffs $\pi^X(y|x) = \frac{y^2}{2} + y - xy$

Proof

Suppose by contradiction $\exists\ x_1 < x_2$, ξ and $\underline{y}$ such that

$$\pi^X(\underline{y}|x_1) + \alpha^X \xi < V_t^X(x_1) \quad \text{and} \quad \pi^X(\underline{y}|x_2) + \alpha^X \xi \geq V_t^X(x_2)$$

Mimicking entails that

$$V_t^X(x_1) = \int_0^1 \pi^X(y|x_1) Q_t^X(dy|x_1) + I_t^X(x_1)$$

$$V_t^X(x_2) \geq \int_0^1 \pi^X(y|x_2) Q_t^X(dy|x_1) + I_t^X(x_1)$$

where $I_t^X(x)$ is the discounted value of the expected match shock.

Then pick $\overline{y}$ so that $\pi^X(\overline{y}|x_1) = \pi^X(\underline{y}|x_1) + \alpha_t^X \xi - I_t^X(x_1)$. To proceed, we require that $\pi^X(\overline{y}|x_2) \geq \pi^X(\underline{y}|x_2) + \alpha_t^X \xi - I_t^X(x_1)$.

Two cases must be considered.

If $\alpha_t^X \xi \geq I_t^X(x_1)$ (plausible for low x_1), then $\overline{y} \geq \underline{y}$, and the inequality holds if π^X is supermodular.

If $\alpha_t^X \xi \leq I_t^X(x_1)$ (plausible for high x_1), then $\overline{y} \leq \underline{y}$, and the inequality holds if π^X is submodular.

Assuming no complementarities encompasses both cases. In effect, it holds that

$$\int_0^1 \pi^X(y|x_1) Q_t^X(dy|x_1) > \pi_t^X(\overline{y}|x_1)$$

$$\int_0^1 \pi^X(y|x_2) Q_t^X(dy|x_1) \leq \pi_t^X(\overline{y}|x_2).$$

The remainder of the proof verbatim repeats the proof of Proposition 3 (Equation (9) ff.) in Bonneton and Sandmann (2025) — greater high-type patience (LS) + less risk-aversion (LSD) suffice

Example

Consider marriage where types x and y correspond to female and male wages. Once matched, partners x and y choose hours worked a, b as to maximize household utility

$$f(x, y) = \max_{a,b \geq 0} ax + by - \frac{a^2}{2} - \frac{b^2}{2} - \gamma ab.$$

Here $\gamma \in (0, 1)$ represents coordination costs when both spouses work. The presence of coordination costs implies that household utility exhibits negative complementarity in wages.

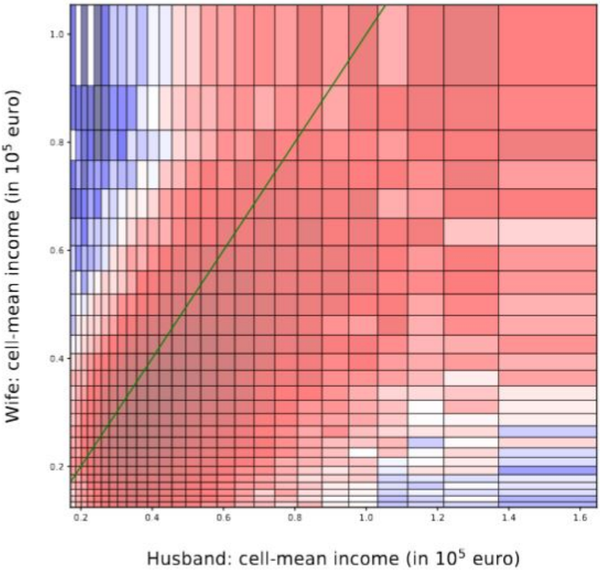


Figure: Chiappori, Fiorio, Galichon, Verzillo (WP): PAM on (pre-marriage) income — level lines flatten for high-low matches

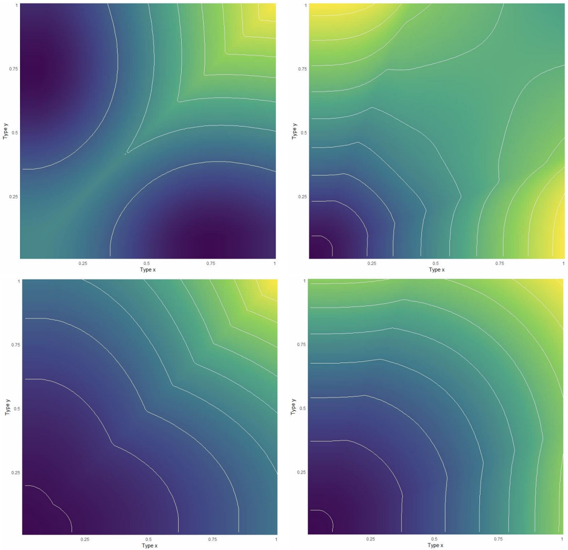


Figure: The marriage model. Heatmaps of $\propto m_t(x, y)$. (Left) NTU (Right) TU. NTU: negative complementarities give rise to patterns as in [?].

No complementarities Relative to Shocks

Remark

Consider the alternative model where x's payoff when matching with y at idiosyncratic shock ξ is given by $\pi^X(y+\xi|x)$. And posit that match payoffs $\pi^X(y+\xi|x)$ and $\pi^Y(y+\xi|y)$ are increasing in "partner" types $y+\xi$ and $x+\xi$, log supermodular, log supermodular in differences. Then there is probabilistic assortative matching.

Conclusion

- **Bias away from PAM:** In both paradigms and with complementarities, search frictions give rise to a tendency to match up rather than with complementary types — bargaining aggravates this

- **Noisy PAM:** Without complementarities, absent bargaining P-PAM occurs
  TU: P-PAM always fails: fundamental tension between matching with complementary partner and matching with who is cheap.
- **Noisy PAM = P-PAM:** Axioms give a new way to look at matching data

- **Comparative statics under dynamics are tractable:** two powerful mimicking arguments.