Macro Qualifier Exam

August, 2025

- You have 3.5 hours. There are five (multipart) questions corresponding to different parts of the first-year macro sequence: the first two for the first semester, the third one for the first half of the second semester, and the last two for the second half of the second semester. There are a total 180 points. If you progress at a rate of one point per minute, you will be able to complete the exam with some time to spare.
- Neither books nor notes are permitted.
- If you make any assumptions beyond what is in the text of the question, please state those assumptions clearly.
- If you need more space, request additional sheets of paper. Number the pages of your answer sheets, write your identification number instead of your name, and clearly label the question you are answering.
- Please write clearly. Show intermediate work for partial credit. Illegible work will not receive credit.

Good luck!

Question 1. Money in an OLG model with two types of agents. (45 points)

Consider an overlapping generations model with fiat money as the only store of value. A generation-t agent works when young at t and consumes when old at $t+1$, $t \geq 1$. All agents are of two types: $i \in \{H, L\}$. A generation-t type-i agent has productivity A^i when young: he produces $A^i n$ units of date-t goods if he chooses to work n units of time when young. At $t \geq 1$, a measure-1 of generation-t agents is born, of which a fraction λ agents have productivity A^H and the remainder have productivity A^L, $A^H > A^L > 0$, $\lambda \in (0, 1)$. There is no population growth.

A generation-t type-i agent's preference is given by

$$u(c_{t+1}^i) - g(n_t^i)$$

if he chooses to work n_t^i units of time at t in youth and to consume c_{t+1}^i units of date-$(t+1)$ goods when old, where $u(\cdot)$ is strictly increasing and strictly concave and $g(\cdot)$ is strictly increasing and convex.

At $t = 1$, there is a measure-1 initial old agent, each holds $m_0 > 0$ units of fiat money that are perfectly durable and have no consumption value regardless of his type. They are alive only at $t = 1$. A type-i initial old agent enjoys utility $u(c_1^i)$ if he consumes c_1^i units of date-1 goods.

There is a government that prints fiat money at a rate $\sigma \geq 0$ each period, $m_t = (1 + \sigma)m_{t-1}$, for $t \geq 1$. At date t, the newly printed money σm_{t-1} is transferred lump sum to low productivity generation-$(t-1)$ old agents, $t \geq 1$. That is, every generation of type-L old agents (including the initial old) receives a money transfer while the type-H old agents do not.

1. (12 Points) Formulate the optimization problem of a generation-t type-i agent and write down the corresponding first-order conditions with respect to n_t^i, $i = H, L$, $t \geq 1$.
2. (12 points) Define a monetary competitive equilibrium. Explicitly specify the set of equations that a competitive equilibrium must satisfy.
3. (21 Points) Now assume that $u(c) = \ln(c)$ and $g(n) = n$.
   (a) For a *stationary* monetary competitive equilibrium to exist, what conditions on the model parameters are needed?
   (b) Suppose that these conditions are satisfied, solve the stationary monetary equilibrium explicitly.
   (c) How does the steady-state n^i for a type-i agent depend on σ, $i = H, L$?

Question 2. Borrowing and lending in a model with preference shocks. (45 points)

There is a continuum of infinitely-lived agents with measure 1. At any date $t \geq 0$, each agent is endowed with $y > 0$ units of non-storable good. Each agent maximizes expected discounted utility with discount factor $\beta \in (0, 1)$ and date-t period utility $z_t u(c_t)$. Here, $u(\cdot)$ is bounded, strictly increasing, strictly concave and satisfies $u'(0) = \infty$, while $z_t \in \{z^l, z^h\}$, $0 < z^l < z^h$, and $z_t = z^i$ with probability $\pi^i \in (0, 1)$, $i \in \{l, h\}$, and $\pi^l + \pi^h = 1$. Realizations of z_t are i.i.d. over time for an agent and i.i.d. across agents. (There is no aggregate risk.)

At date $t \geq 0$, after seeing the realization of his z_t, an agent chooses (a_{t+1}, c_t) subject to

$$c_t + a_{t+1} q_t \leq a_t + y \quad \text{and} \quad a_{t+1} \geq \underline{a}$$

where $\underline{a} < 0$ is an exogenous lower bound on the agent's asset holdings, and q_t is the date-t price of an asset at t that will yield 1 unit of goods at $t+1$. The asset markets are perfectly competitive: all agents take q_t as given for all $t \geq 0$.

1. Suppose that $q_t = q$ for all $q_t = q \geq 0$, where $q > 0$ and satisfies $\underline{a} + y - q\underline{a} > 0$. Consider an agent's choice problem in this stationary environment.

   (a) (5 points) Formulate the agent's recursive problem.

   (b) (12 points) Show that there is a unique value function in the class of continuous and bounded functions that is the solution to the recursive problem in (a).

   (c) (10 points) Given z^i, if $a_2 > a_1 > \underline{a}$, compare the optimal consumption $c(a_1, z^i)$ and $c(a_2, z^i)$ for $i = l, h$. Does the ranking change for different preference shock z^i?

2. (18 points) Define a recursive competitive equilibrium.

Question 3. One-sided lack-of-commitment problem when $\beta R < 1$. (45 points)

The economy consists of a consumer and a money lender. The money lender is risk neutral and discounts the future at a rate R. The consumer has a utility function given by $u(c) = \log(c)$ and lacks commitment. Every period, the consumer receives an endowment (or income) which is stochastic, *iid*, with two possible realizations (states): $y_1 = 1$ with probability $\pi_1 = 1/2$, and $y_2 = \theta > 1$ with probability $\pi_2 = 1/2$. The consumer discounts the future at a rate β. The consumer lacks commitment in the sense that he can deviate at any point in time from the contract with the money lender, and if he deviates, he will remain in autarky forever. **Assume $\beta R < 1$.**

Let $P(v)$ be the value to the money lender in an optimal contract that promises the consumer a lifetime utility equal to v. Assume that $P(v)$ is strictly decreasing, concave, and continuously differentiable.

1. (3 points) Compute the autarky value of the consumer.
2. (5 points) Write down the Bellman equation that describes the recursive problem of the money lender, $P(v)$, that offers a contract to the consumer, v. Assume that, for every period, the money lender offers the contract **before the income is realized** (ex-ante). Thus, each period the money lender offers to the consumer a consumption for each state, c_1 and c_2, and the promised value for tomorrow for each state today, v_1 and v_2, given the promised value offered last period, v.
3. (3 points) Describe the three constraints included in the contracting problem above and why they are needed.
4. (5 points) Compute the first-order conditions and state the envelope condition.
5. (8 points) Show that consumption for each state, c_1 and c_2, and the promised value for each state, v_1 and v_2, are weakly increasing in v.
6. (5 points) Show that in the optimal solution to the lack-of-commitment problem, if the participation constraint at state 1 ($y_1 = 1$) **does not bind**, then $v_1(v) < v$. How does this compare to the case where $\beta R = 1$?
7. (3 points) Argue that if no participation constraint binds, then $c_1(v) = c_2(v)$ and $v_1(v) = v_2(v)$. How does this compare to the case where $\beta R = 1$?
8. (3 points) Show how to solve for c_2 and v_2 if the participation constraint for state 2 ($y_2 = \theta$) binds.
9. (5 points) Show that if the participation constraint for state 1 binds ($y_1 = 1$), then the participation constraint for state 2 ($y_2 = \theta$) binds as well.
10. (5 points) Show that if both participation constraints bind, then $v_1 < v_2$.

Question 4. RBC when labor is a risky investment. (30 points)

Consider an RBC model without physical capital. The output Y_t is produced by the *lagged* labor input L_{t-1} with linear technology:

$$Y_t = Z_t \, L_{t-1}.$$

That is, the fruits of labor effort made at $t-1$ become available only at t. Productivity Z_t follows a Markov process. In each period t, the social planner chooses labor input L_t and consumption C_t to maximize the expected conditional lifetime utility, $\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}, L_{t+s})\right]$, after the realization Z_t. The flow utility is given by:

$$U(C, L) = \frac{1}{1-\gamma} C^{1-\gamma} - \beta L, \quad \gamma > 1.$$

Note that the marginal disutility of labor is set equal to the time discount factor $\beta \in (0, 1)$ to simplify algebra.

1. (5 points) Set up the Bellman equation of the planner's problem.

2. (10 points) Use the first-order condition and the envelope condition to derive the Euler equation that characterizes the planner's optimal decision rule for labor.

3. (10 points) Now assume that the random process of Z_t takes the form:

   $$\ln Z_{t+1} = \rho \ln Z_t + \sigma \, \varepsilon_{t+1}, \quad \rho \in (0, 1), \ \sigma > 0$$

   where $\varepsilon_t \sim N(0, 1)$ is an iid shock, and σ captures the degree of uncertainty. Solve the planner's optimal decision rule. Specifically, write $\ln(L_t)$ as a function of $\ln(Z_t)$, γ, and σ.

   Hint: Suppose x is log-normal: $\ln x \sim N(\mu_x, \sigma_x)$, then $\mathbb{E}(x) = \exp(\mu_x + \sigma_x/2)$.

4. **(5 points)** Describe the effect of uncertainty σ on the planner's optimal labor choice. How does the effect depend on γ? What is the economics behind your reasoning?

Question 5: Long-run consumption under quadratic utility. (15 points)

A household maximizes the conditional expectation of its lifetime utility:

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right], \quad \text{where} \quad u(C_t) = \left(C_t - \frac{a}{2} {C_t}^2 \right), \quad 0 < \beta < 1, \quad a > 0,$$

by choosing consumption plans sequentially. Specifically, in each period $t \geq 0$, the household chooses consumption C_t and a non-contingent asset A_{t+1} after realizing a stochastic income Y_t subject to the sequential budget constraint,

$$C_t + A_{t+1} = (1 + r)A_t + Y_t,$$

and the no-Ponzi scheme condition. The variable $r > 0$ denotes the interest rate. The initial asset A_0 is given. When answering the questions below, assume that C_t is in the range where $u'(C_t)$ is positive, and A_{t+1} is strictly above the natural borrowing limit so that the first-order condition holds with equality.

1. (5 points) Derive the Euler equation for the household's problem.
2. (10 points) Suppose $\beta(1 + r) > 1$, what is the consumption path expected to converge to in long run? That is, solve $\lim_{s\to\infty} \mathbb{E}_t \left(C_{t+s}\right)$ and show your derivation.