THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

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Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section (precise instructions are below)—50 points in each section—for a total of 100 points. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are five (5) pages, including this one.

SECTION I

Please answer any **two** of the three questions from this section.

I.1 [25 pts] Short answers

(a) Suppose that there are two goods, $1, 2$. Suppose that a consumer has a utility function that is strictly quasiconcave, homogeneous of degree one, and strongly monotone.[1] Let $x(p, w) = (x_1(p, w), x_2(p, w))$ denote the demands for the two goods. Is it necessarily true that when $p_1 \geq p'_1$, then $x_1(p_1, 1, w) \leq x_1(p'_1, 1, w)$?

(b) Consider the following utility function over lotteries with two outcomes $\{a, b\}$:

$$U(p) = \min \{8p(a) + 4p(b), 4p(a) + 8p(b)\}.$$

Do these preferences satisfy the independence axiom?

I.2 **[25 pts]** There are two goods in the economy, (a)pples and (b)ananas. The government is contemplating a variety of tax policies on apples in the upcoming season and would like to estimate the tax revenues accruing from these policies. To do this, it relies on past choice data of this consumer:

price of a	price of b	wealth	consumption of a	consumption of b
6	1	40	6	4

Now suppose for the upcoming season, the price of apples fell to 1 and the price of bananas increased to 2. The wealth remained the same for the consumer. At the same time, the government is contemplating the introduction of a new tax on apples that would increase the price of apples from 1 to $1 + y$.

(a) Suppose that $y = 1$. Then what is the worst-case estimate of tax revenue from this particular consumer? In other words, if the consumer is a utility maximizer with a locally non-satiated, strictly quasiconcave utility function, then what is the lowest tax revenue that the government can expect to obtain from this consumer from the tax on apples?

(b) Suppose now that $y = 5$. Then what is the the worst-case amount of tax revenue?

(c) If the government cared about the worst-case tax revenue as in parts a and b, what is the optimal y? What is the worst-case amount of tax revenue under the optimal y?

[1] Recall that a utility function is homogeneous of degree one if for all $\alpha > 0$ and any x, $u(\alpha x) = \alpha u(x)$.

I.3 **[25 pts]** Consider the following two person economy with uncertainty. There are two equally likely states of nature, $\theta \in \{g, b\}$. There is only one physical good, denoted by x. Suppose that the agents are both expected utility maximizers with the following utility functions:

$$U_1(x_g, x_b) = \frac{3}{4}\log(x_g) + \frac{1}{4}\log(x_b),$$
$$U_2(x_g, x_b) = \frac{1}{2}\log(x_g) + \frac{1}{2}\log(x_b).$$

The respective endowments are: $(\omega_g^1, \omega_b^1) = (15, 15), (\omega_g^2, \omega_b^2) = (15, 15)$.

(a) Solve for the Arrow Debreu equilibrium of this economy. Do the consumers fully insure? Explain.

(b) Consider the Radner economy with the following two available assets:

$$r_1 = (1, 0), r_2 = (0, 1).$$

$r_i = (x, y)$ is an asset that pays x units of the commodity in state g and y units of the commodity in state b. Prove that there is a Radner equilibrium of this economy that implements the same allocation from part a. What are the prices of the assets and the portfolios purchased by the consumers in equilibrium?

(c) Now suppose instead that the two available assets are

$$r_1 = (1, 1), r_2 = (0, 1).$$

Prove again that there is a Radner equilibrium of this economy that implements the same allocation from part a. What are the prices of the assets and the portfolios purchased by the consumers in equilibrium?

SECTION II

Please answer any **two** of the three questions from this section.

II.1 **[25 pts]** An indivisible object is to be sold. The private value of buyer i for the object, X_i, is drawn from the *uniform* distribution on $[0, a_i]$ where $a_i > 0$.

(a) Derive the corresponding virtual value function ψ_i.

(b) Suppose there is a single buyer ($i = 1$), and the object is allocated via the optimal mechanism. How does a_1 affect the probability that the buyer gets the object?

(c) Next, suppose there are two buyers ($i = 1, 2$) such that $a_1 > a_2$. The object is allocated via the (Myerson) optimal mechanism.

  i. Which buyer has a higher probability of getting the good? (*Hint*: Draw a picture.)

  ii. Does the probability that 2 gets the object vanish as $a_2 \to 0$?

(d) Suppose the good is sold via a second-price auction with a single optimally chosen reserve price r, rather than the optimal mechanism. Does the probability that buyer 2 gets the object vanish in the limit as $a_2 \to 0$?

II.2 **[25 pts]** There are three states of nature, labelled α, β and γ. The prior probabilities of these states are $\frac{9}{13}, \frac{3}{13}$ and $\frac{1}{13}$, respectively. Two players are playing a game whose payoffs depend on the underlying state as follows:

	L	R
L	2, 2	0, 0
R	3, 0	1, 1

α

	L	R
L	2, 2	0, 0
R	0, 0	1, 1

β

	L	R
L	2, 2	0, 0
R	0, 0	1, 1

γ

Note that player 2's payoffs are the *same* in all three states while player 1's payoffs are the same in states β and γ. The players do not know whether the true state is α, β or γ but receive private informative signals $s_i \in \{0, 1\}$ about these as follows. Player 1 receives a signal $s_1 = 1$ if the state is α, and receives $s_1 = 0$ otherwise. Player 2 receives a signal $s_2 = 1$ if the state is α or β and receives $s_2 = 0$ otherwise. All this is commonly known. Only signals are private.

(a) Show that the incomplete information game above has a *unique* Nash equilibrium in which both player play R regardless of their private signals.

(b) Are there any rationalizable strategies other than the ones in part (a)?

II.3 **[25 pts]** Consider a matching problem between three (3) firms, each with one job opening, and three (3) workers. Let $F = \{f_1, f_2, f_3\}$ denote the set of firms and $W = \{w_1, w_2, w_3\}$ denote the set of workers. Each firm i has a (possibly weak) ranking R_i over the set $W \cup \{\emptyset\}$ where $\emptyset$ denotes that the firm keeps its position unfilled. Similarly, each worker j has a (possibly weak) ranking R_j over the set $F \cup \{\emptyset\}$ where $\emptyset$ denotes that the worker is unemployed. Suppose the preferences are:

Firm 1	Firm 2	Firm 3	Worker 1	Worker 2	Worker 3
w_2, w_3	w_2	w_3	f_1	f_1	f_1
w_1	w_1	w_1	f_2	f_2	f_3
$\emptyset$	$\emptyset$	$\emptyset$	f_3	$\emptyset$	$\emptyset$
	w_3	w_2	0	f_3	f_2

As an example, firm 1 ranks either w_2 or w_3 as best suited for the job, w_1 as second-best and having the vacancy unfilled as worst. Firm 1 is thus indifferent between w_2 and w_3. For firm 2, w_2 is best, w_1 is second-best but keeping the vacancy unfilled is better than hiring w_3. Similarly, worker 2 considers f_1 to be the best, f_2 the second-best but if neither is possible, he would rather be unemployed than work for firm f_3.

(a) Find two stable matchings of firms to workers for the preferences given above.

(b) Does there exist a stable matching that *all* firms weakly prefer to every other stable matching? Does there exist a stable matching that *all* workers weakly prefer to every other stable matching.